# Dynamical Systems II: Homework 7

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# 1 Questions from Silva

## 1.1 Section 3.7

**Problem 2.**

Observe that the only $T$-invariant subset of $X$ is $X$ itself. Suppose we have a subset $A$ of $X$ such that $A \neq X$. Then there exists $a_j \in X$ such that $a_{(j+1) \bmod n} \notin A$ and hence, $T(a_j) \notin A$. Thus $A$ is not $T$-invariant. Hence, we must have $A = X$. But $A^C = X^C = \emptyset$. So $\mu(A^C) = 0$ whenever $A$ is $T$-invariant, and so $T$ is ergodic.

**Problem 3.**

Suppose $T$ is a totally ergodic measure-preserving transformation and suppose that $T$ is invertible. Since $T$ is ergodic, observe that $\mu(A) = 0$ or $\mu(A^C) = 0$ for any $T$-invariant set $A$. Now fix a $T$-invariant set $A$ and suppose $\mu(A) = 0$. Since $T$ is measure-preserving, we have that if $\mu(A) = 0$, then $\mu(T^{-1}(A)) = 0$.

For $T^{-2}(A)$, observe that we have

$$T^{-2}(A) = T^{-1}(T^{-1}(A)) = T^{-1}(B)$$

where $B = T^{-1}(A)$, a measure 0 set. Again, since $T$ is measure-preserving and $\mu(B) = 0$, we have that $\mu(T^{-1}(B)) = \mu(T^{-2}(A)) = 0$. Proceeding inductively, we get that if $\mu(A) = 0$, then $\mu(T^n(A)) = 0$ for all $n < 0$. Moreover, if $A$ is $T$-invariant, then $A$ is also $T^{-1}$-invariant, since $T$ is invertible. This again applies for $T^n$ with any $n < 0$, so we have that

**Problem 6.**

Recall the the two dimensional Baker's transformation is defined as

$$T(x, y) = \begin{cases} (2x, \frac{y}{2}), & \text{if } 0 \leq x < 1/2 \\ (2x - 1, \frac{y+1}{2}), & \text{if } 1/2 \leq x \leq 1 \end{cases}$$

For any subset $A$ of $[0, 1] \times [0, 1]$, we have that $\mu(A) =$

## 1.2 Section 3.10

**Problem 1.**

**Problem 3.**

**Problem 4.**

**Problem 6.**