Dynamical Systems II: Homework 2

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1 Questions from Silva

1.1 Section 2.3

Problem 1.

Suppose A is measurable. Then for any $\epsilon >)$, there is an open set $G = G_\epsilon$ such that $A \subset G$ and $\lambda^*(G \setminus A) < \epsilon$.

Problem 2.

Suppose A is a null set. Then for any $\epsilon > 0$, there exists a sequence of intervals I_j such that $A \subset \bigcup_{j=1}^{\infty} I_j$ and $\sum_{j=1}^{\infty} |I_j| < \epsilon$. Now consider $A^2 = \{a^2 : \; a \in A\}$. Note that if $a \in I_k = (x, y)$ for some k, then

$$x < a < y$$

Problem 3.

Suppose A is any set, B is measurable, and $\lambda^*(A \triangle B) = 0$. Note that $A \triangle B = (A \setminus B) \cup (B \setminus A)$. Also note that $(A \setminus B) \cup (B \setminus A) = \emptyset$. So we have,

$$\begin{aligned} \lambda^*(A \triangle B) &= \lambda * (() \cup (B \setminus A)) \\ &= \lambda^*(A \setminus B) + \lambda^*(B \setminus A) \\ &= 0 \end{aligned}$$

Thus, we have that $\lambda^*(A \setminus B) = -\lambda^*(B \setminus A)$. Since outer measure is non-negative, we must have that,

$$\lambda^*(A \setminus B) = 0 = \lambda^*(B \setminus A)$$

So we have that $A \setminus B$ is a null set and is thus measurable. Now note that $A = (A \cap B) \cup (A \setminus B)$ and that $(A \cap B) \cap (A \setminus B) = \emptyset$. Since B is measurable, we have that there exists an open set G such that $B \subset G$ and $\lambda^*(G \setminus B) < \epsilon$

Problem 6.

Problem 9.

1.2 Section 2.4

Problem 2.

Problem 4.

1.3 Section 2.5

Problem 1.

Problem 6.

Problem 7.

Problem 10.

Problem 13.