Dynamical Systems II: Homework 11

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May 4, 2021

1 Questions from Silva

1.1 Section 4.5

Problem 2.

Problem 3.

1.2 Section 5.1

Problem 2.

Let $T : X \to X$ be a measure-preserving transformation with $\mu(X) = 1$ and suppose that for every measurable set A the limit

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{I}_A(T^i(x))$$

exists and equals $\mu(A)$ a.e. We want to show that T is ergodic. Let us fix two measurable sets A, B. Other than on two sets of measure 0 (whose union is measure 0), we have,

$$\begin{aligned}
\mu(A)\mu(B) &= \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{I}_A(T^i(x)) \cdot \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{I}_B(T^i(x)) \\
&= \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \mathbb{I}_A(T^i(x))\mathbb{I}_B(T^j(x)) \\
&= \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{I}_{A\cap B}(T^i(x)) \\
&=
\end{aligned}$$

Problem 3.

1.3 Section 5.2

Problem 2.

Problem 4.

Problem 5.