Chris Hayduk

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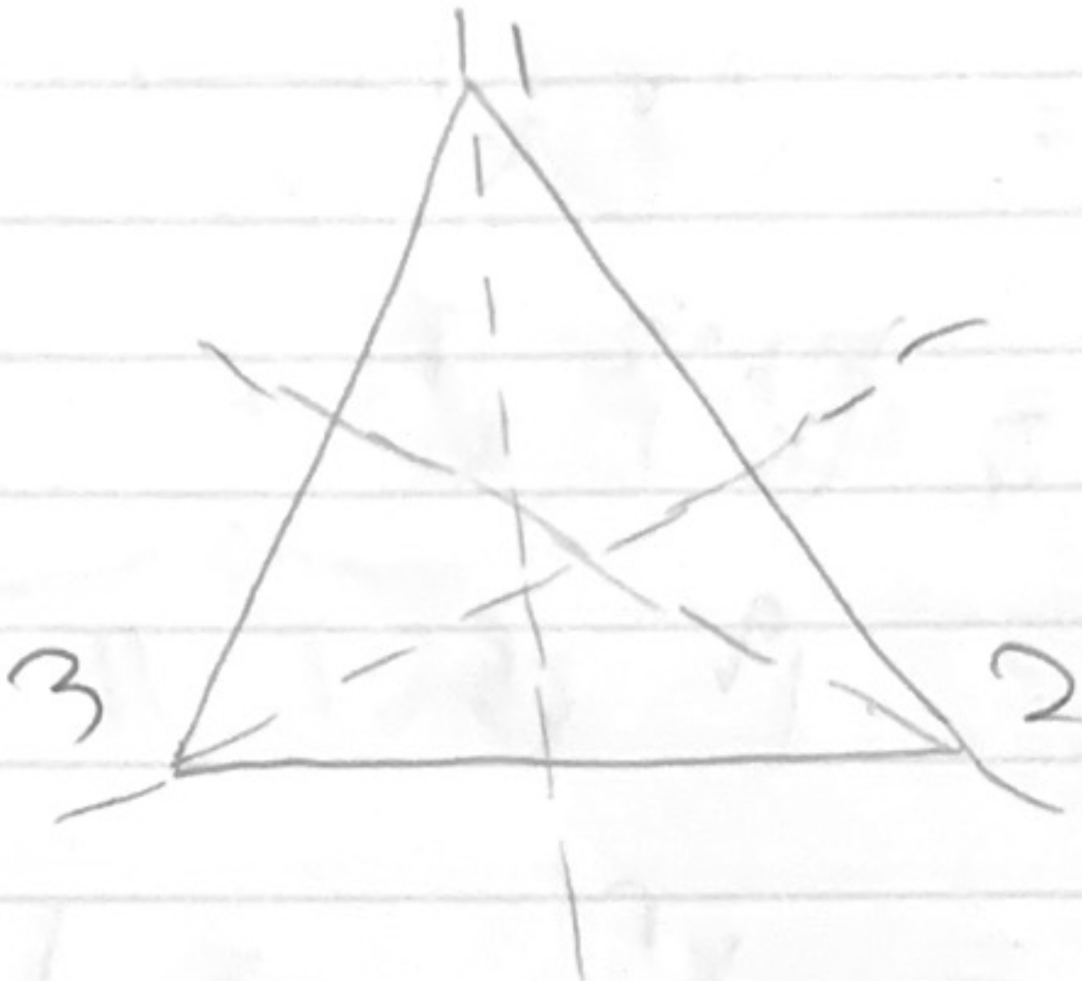
Lecture 2, Exercise B

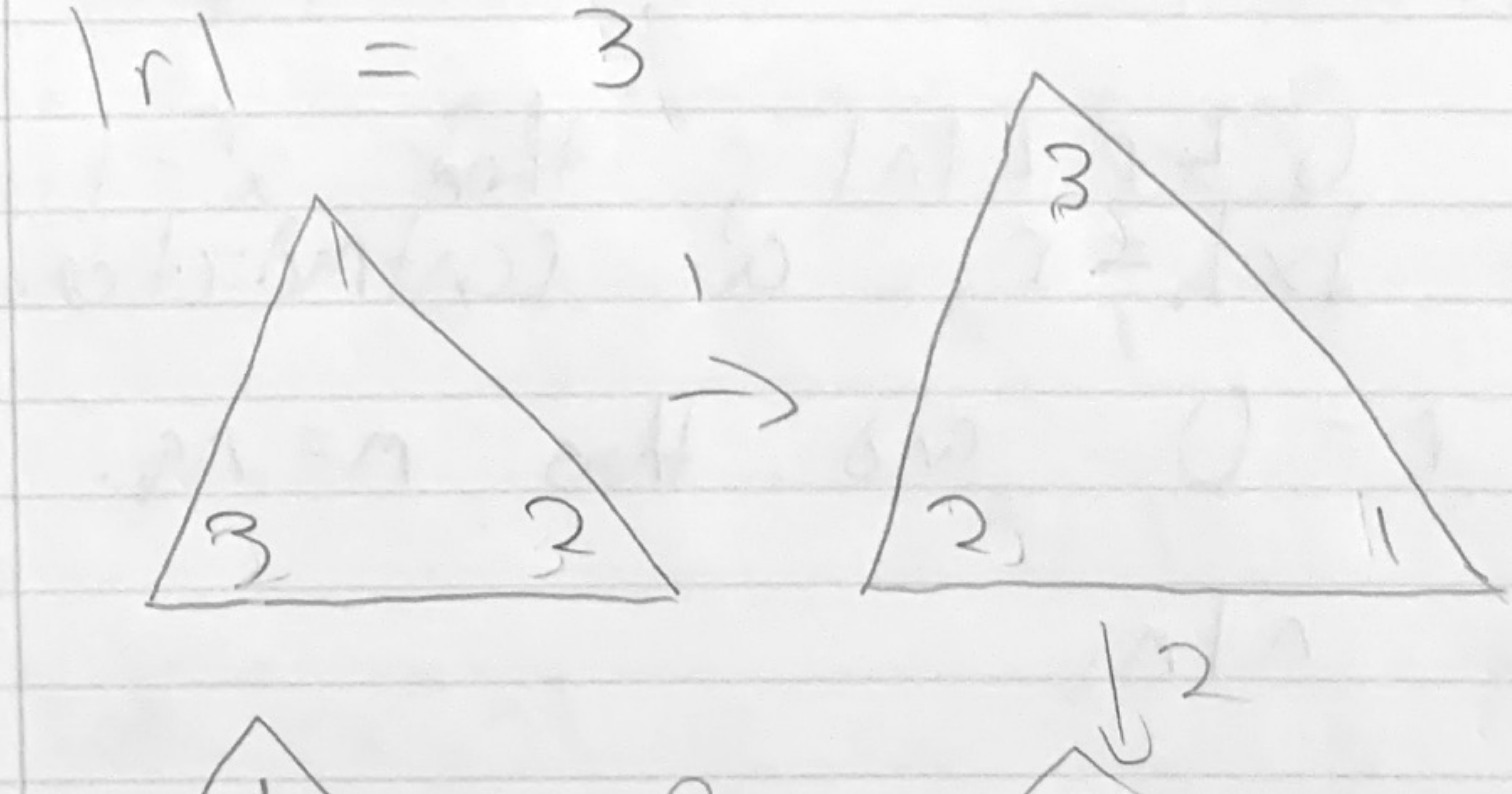
1a.



There are 6 symmetries:

3 rotations and 3 reflections

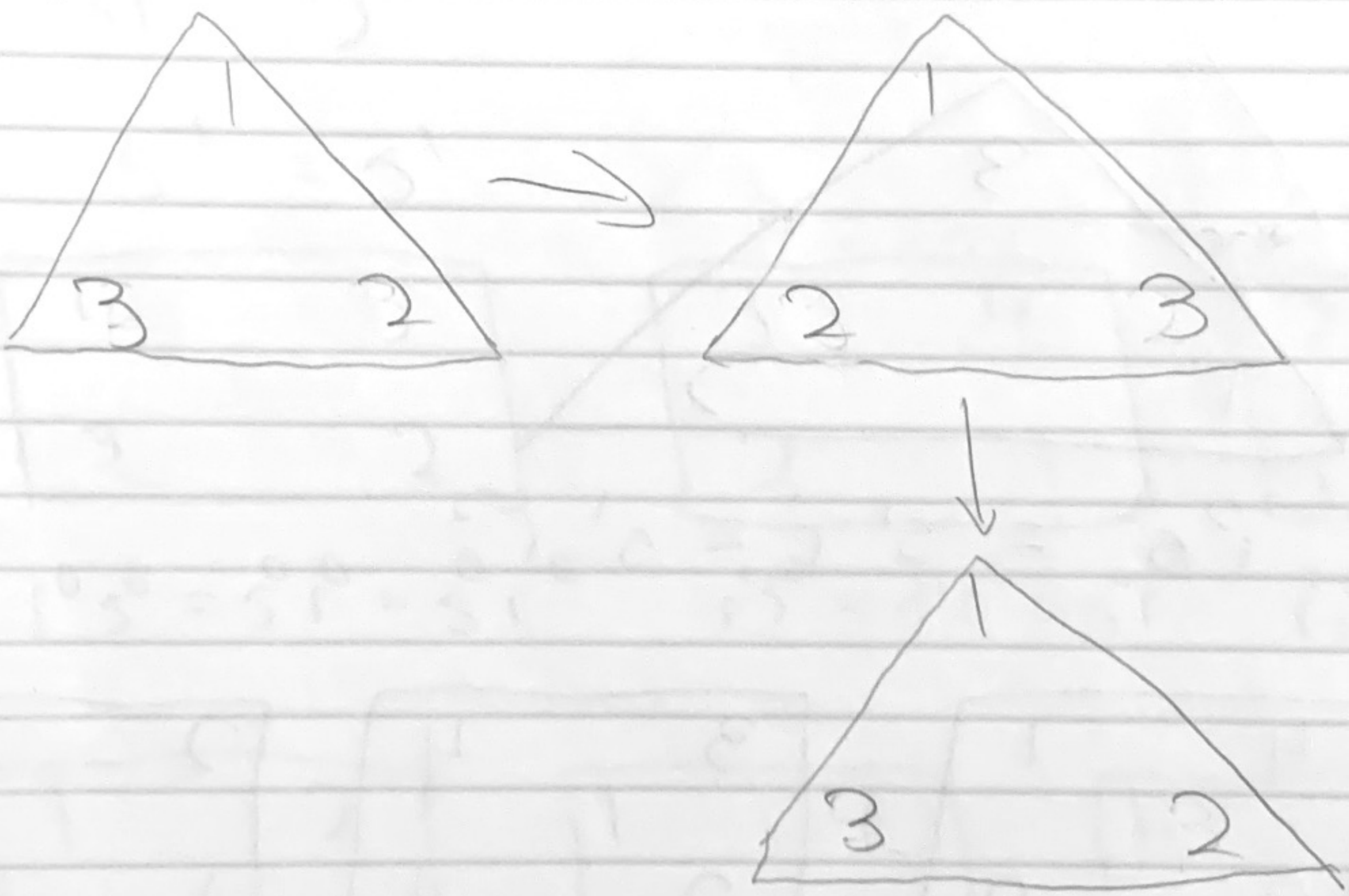
b. $|r| = 3$



$r^{-1} = r^2$

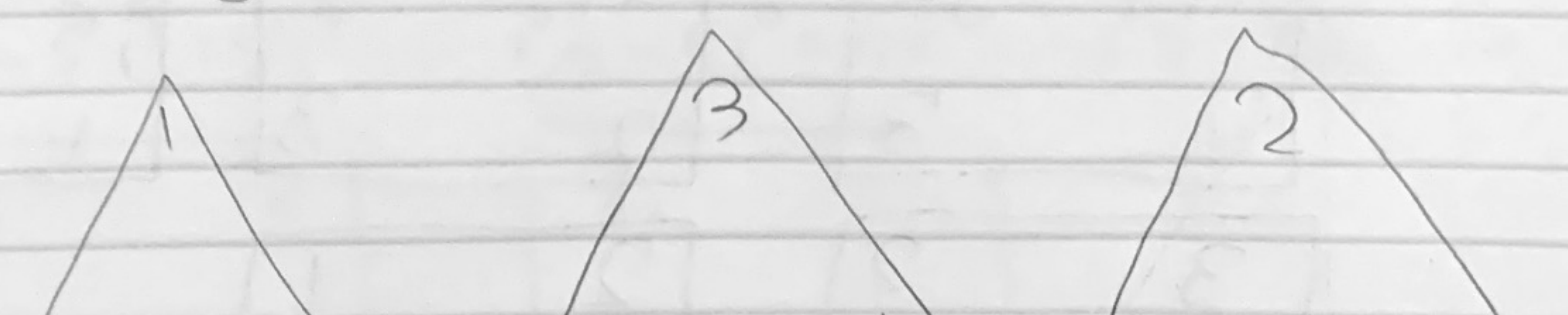
That is, 2 clockwise rotations are equivalent to 1 counter clockwise rotation

c. $|s| = 2$



$s^{-1} = s^1$

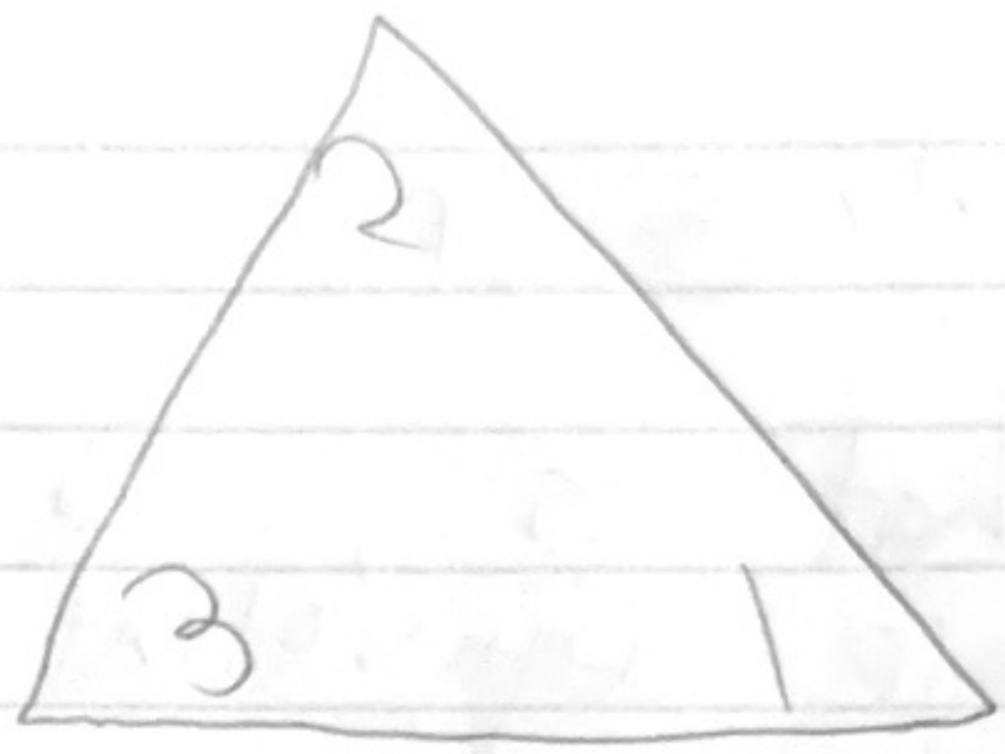
d.



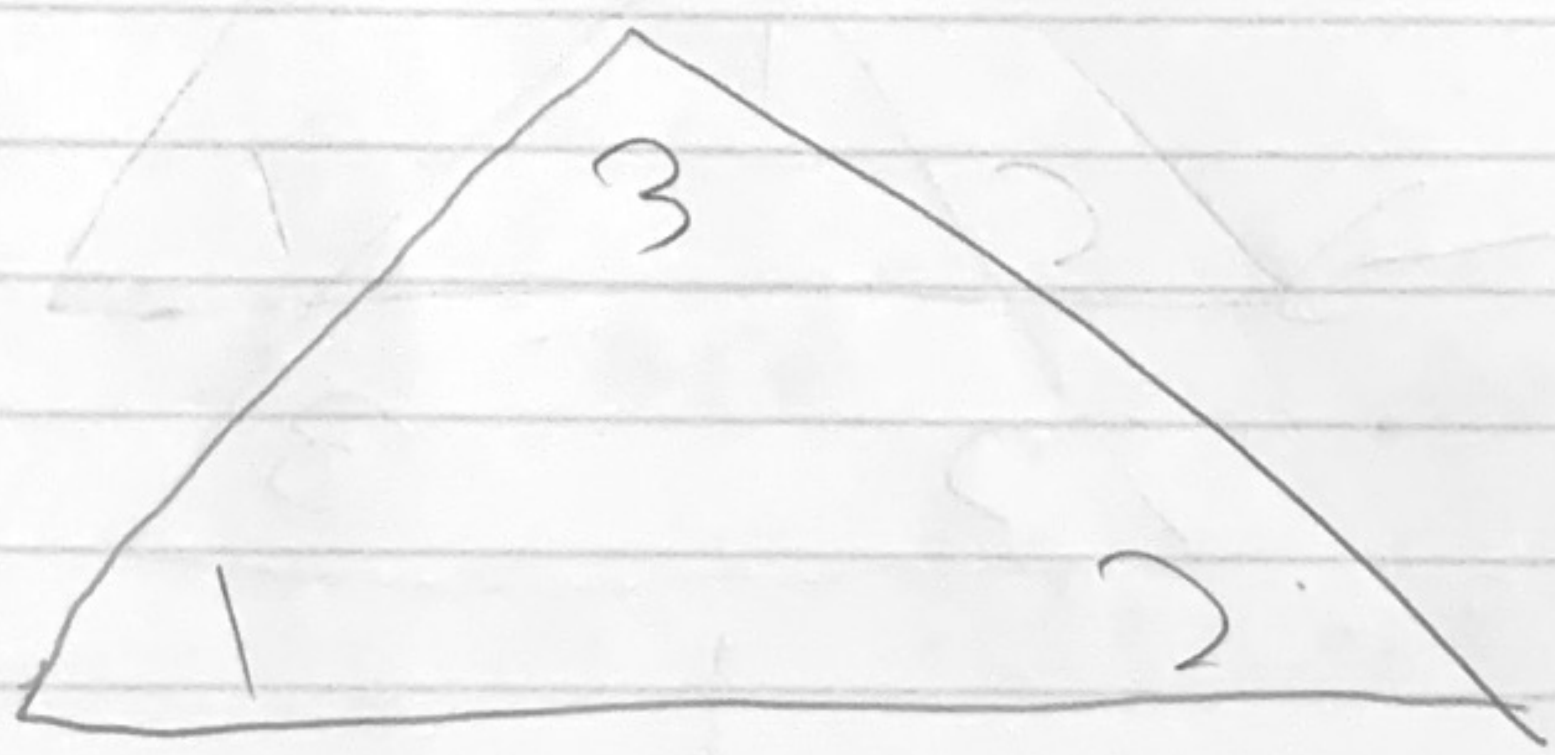
$r^0 s^0 = s^0 r^0$
$= s^0 r^{-0}$

$r^1 s^0 = s^0 r^1$
$= s^1 r^{-3}$

$r^2 s^0 = s^0 r^2$
$= s^0 r^{-1}$

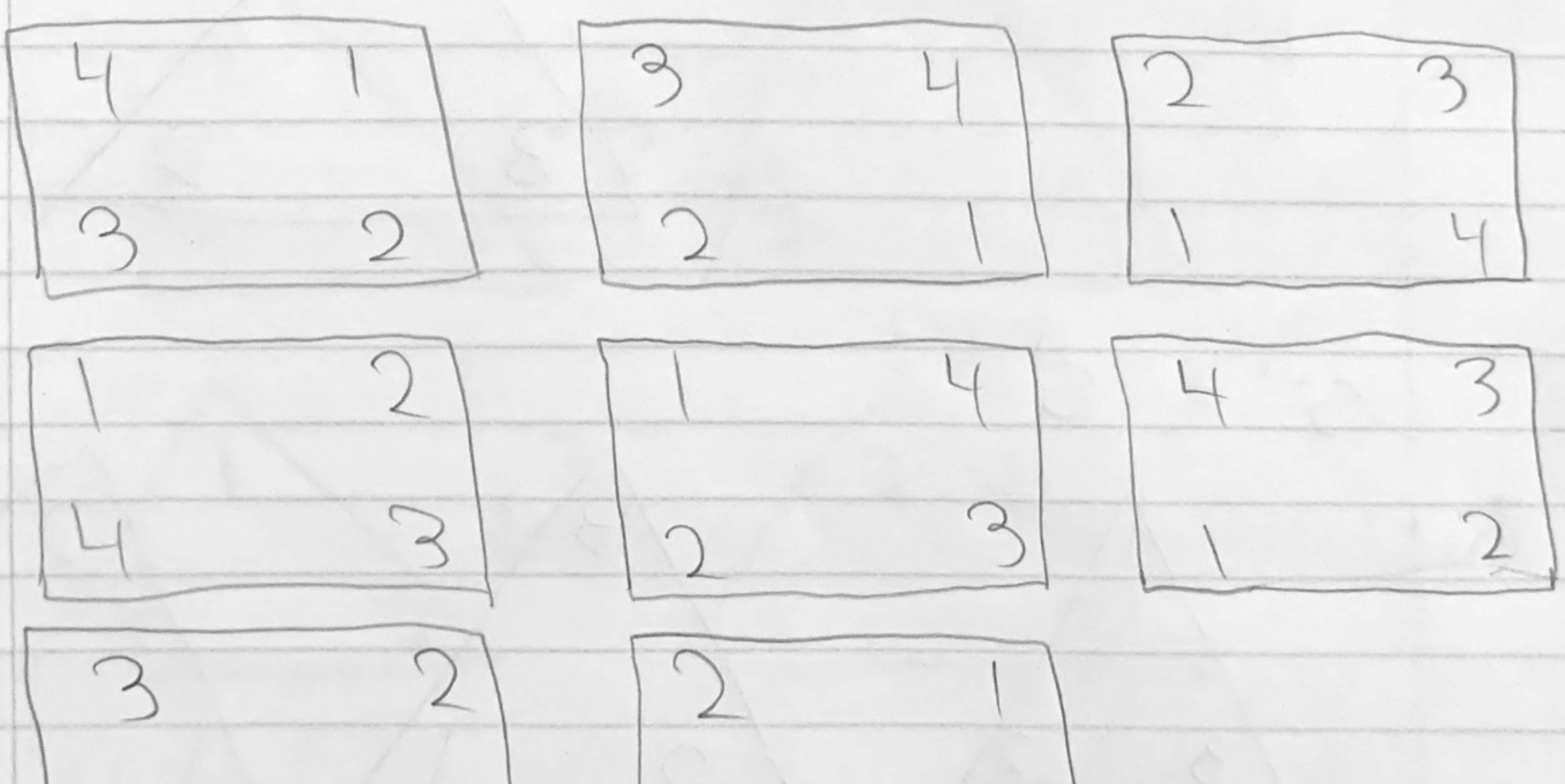


$r^0 s^1 = s^1 r^0 = s^1 r^{-0}$ $\quad$ $r^1 s^1 = s^1 r^1 = s^1 r^{-1}$



$r^2 s^1 = s^1 r^1 = s^1 r^{-2}$

2a.



There are 8 symetries of the square

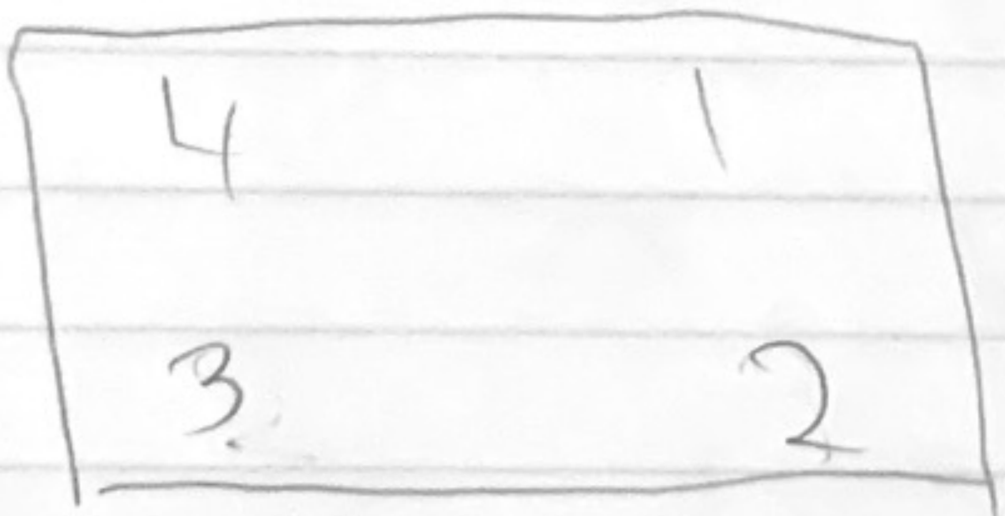
b. $|r| = 4$

$r^{-1} = r^3$

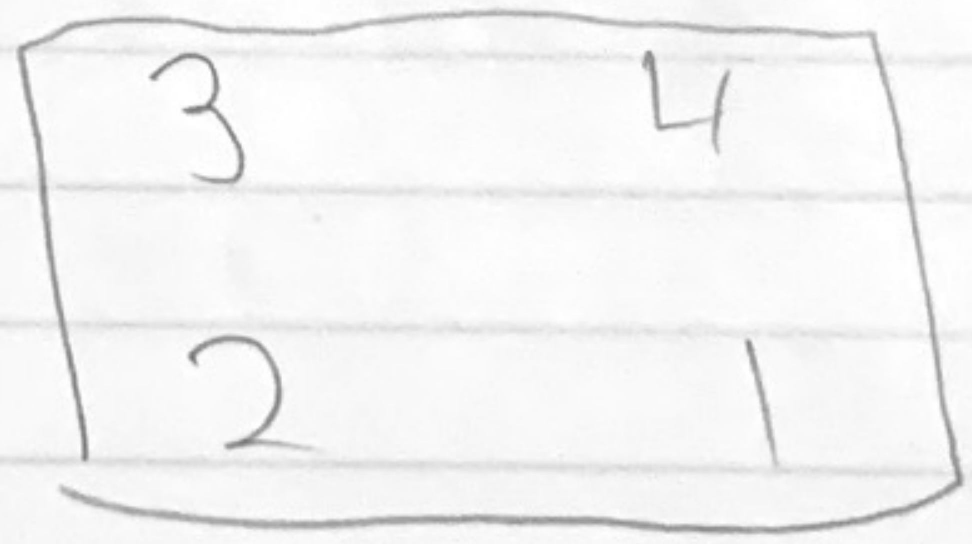
c. $|s| = 2$

$s^{-1} = s^1$

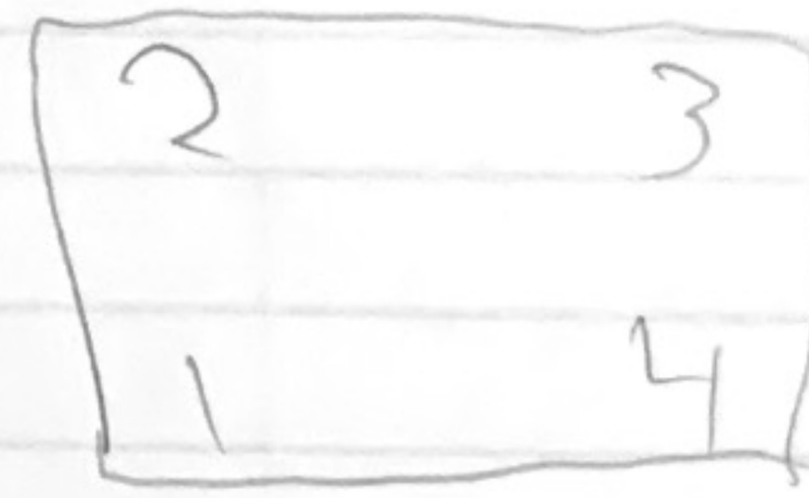
d.



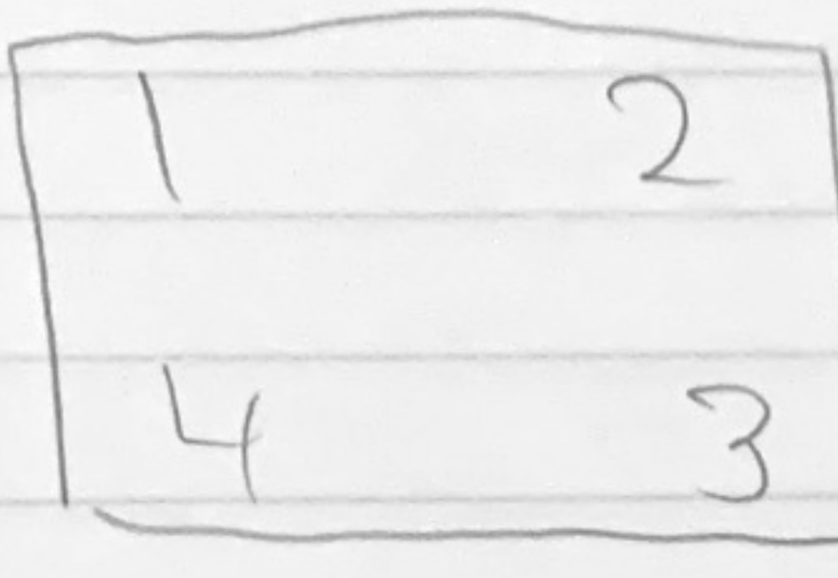
$r^0 s^0 = s^0 r^0 = s^0 r^{-0}$



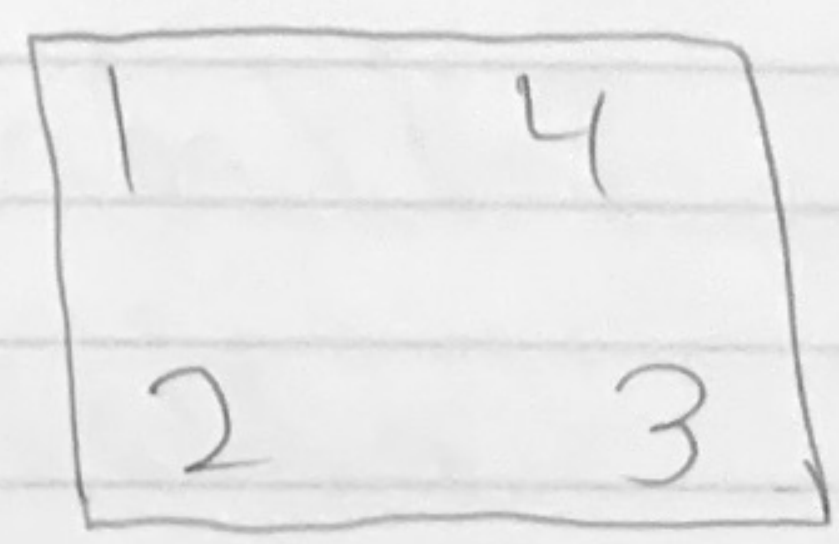
$r^1 s^0 = s^0 r^1 = s^0 r^{-3}$



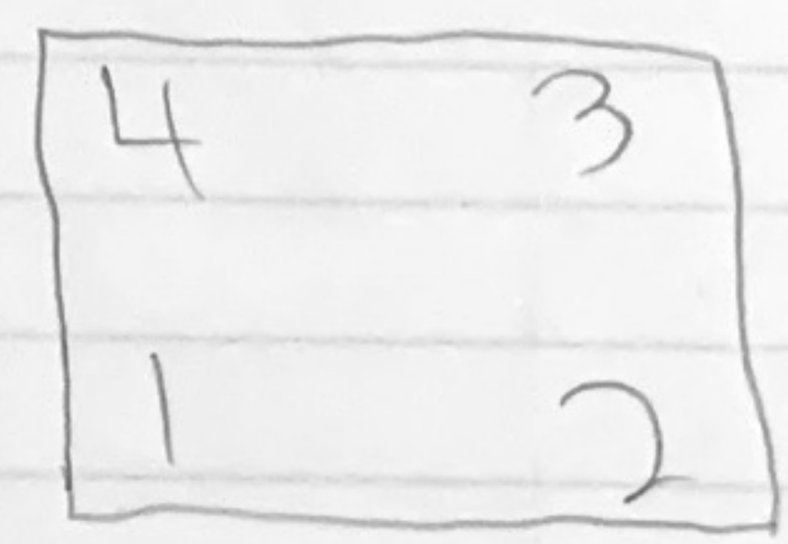
$r^2 s^0 = s^0 r^2$
$= s^0 r^{-2}$



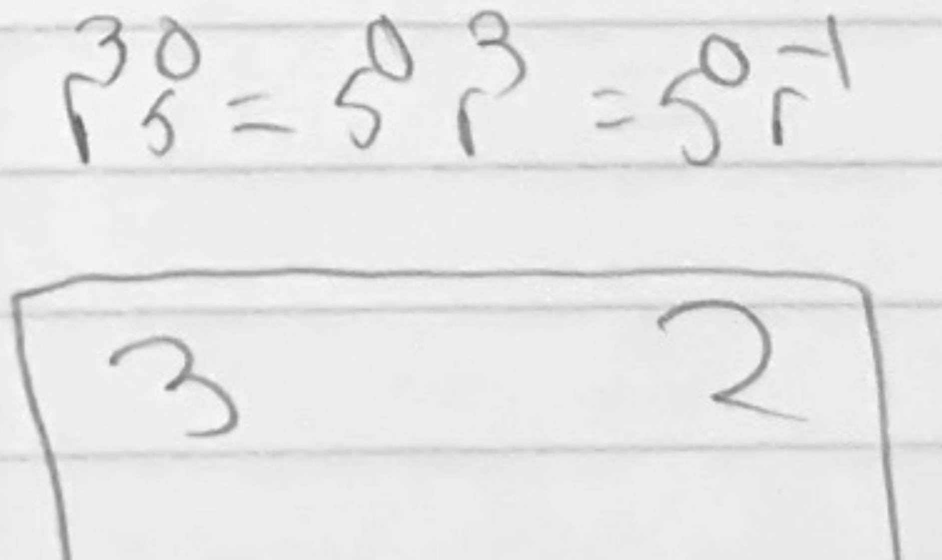
$r^3 s^0 = s^0 r^3 = s^0 r^{-1}$



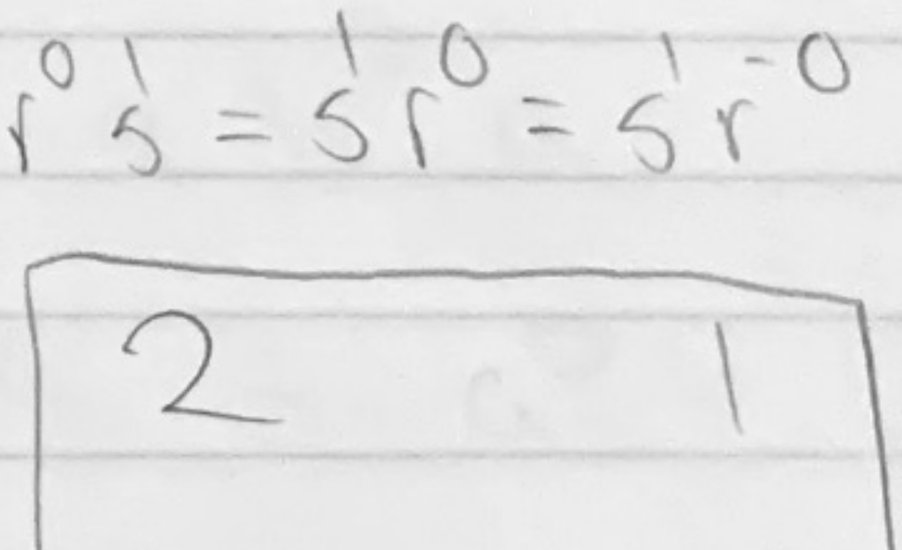
$r^0 s^1 = s^1 r^0 = s^1 r^{-0}$



$r^1 s^1 = s^1 r^1 = s^1 r^{-3}$



$r^2 s^1 = s^1 r^2$
$= s^1 r^{-2}$



$r^3 s^1 = s^1 r^3$
$= s^1 r^{-1}$

3a. $s \neq r^i \ \forall i$ (from 1d and 2d)

b. $sr^i \neq sr^j \ \forall \ 0 \leq i, j \leq n-1$ with $i \neq j$ (from 1d and 2d)

So the number of symmetries of a regular n-gon is $2n$ and that set can be written as

$$D_{2n} = \{1, r, r^2, \ldots, r^{n-1}, s, sr, \ldots, sr^{n-1}\}$$