Chris Hayduk 10/6

Lecture 10, Exercise C

1. $N_G(A) = \{g \in G : gAg^{-1} = A\}$

and

$N_B(A) = \{b \in B : bAb^{-1} = A\}$

If $N_G(A) = G$, then $\forall g \in G$, gAg^{-1}.

Since $B \leq G$, every $b \in B$ is also in G. So

$$bAb^{-1} = B \qquad \forall b \in B$$

Hence

$$N_B(A) = B$$

Thus $A \trianglelefteq B$.

2a. $\overline{B} \subseteq \overline{G}$ because B/A has fibers defined in $B \leq G$.

$\overline{B} \neq \emptyset$ because $p(1_B) = 1_A$, so $1_B \in \overline{B}$

Let $\overline{x}, \overline{y} \in \overline{B}$. Then they are distinct fibers and

$$p(x) \neq p(y)$$

If $\overline{y}^{-1} = \overline{x}$, then

$$\vdots$$

$\overline{B} \leq \overline{G}$

b. $\overline{g}, \overline{b}$ are fibers above some elements in A. $g \in G$, $b \in B$

c. $$\overline{g}\,\overline{b}\,\overline{g}^{-1} = (gA)(bA)(gA)^{-1}$$
$$= (gbg^{-1})A$$
$$= \overline{gbg^{-1}}$$

d. $gbg^{-1} \in B$

f. $gbg^{-1}A \in B/A$, so

$\bar{g}\bar{b}\bar{g}^{-1} \in B$

3a. $p: G/A \to G/B$

$gA \mapsto gB$