Chris Haydук

11/17

Lecture 20, Ex. A

1. Let $x \in \mathbb{Z}$ and let $a = 2$.

Suppose x is even. Then

$$x = 2q_1 + 0$$

for some q_1.

Suppose x odd. Then

$$x = 2q_2 + 1$$

for some q_2.

Hence, $r = 0$ or a unit as required.

2a. $\omega = \frac{1}{2}(1 + \sqrt{-19})$

b. $a + b\omega = (a + b/2) + b/2\sqrt{-19}$

$$N((a+b/2) + b/2\sqrt{-19})$$

$=$

i. $b=0, \; a=\pm 1$

ii. $B=5$

iii. $N(0)=0$

$N(\pm 1)=1$

$N(\pm 2)=4$

c i. $x=2$

ii. $qQ=2$

$\Rightarrow 4=N(2)=N(q)N(Q)$

Since Q not unit.

$\Rightarrow Q=\pm 2$

$qa=3$

$\Rightarrow 9=N(3)=N(q)N(Q)$

$qa=1$

$\Rightarrow 1=N(1)=N(q)N(\alpha)$

3a. $\frac{\alpha}{\beta} \in \mathbb{Q}(\sqrt{-19})$

b. $N(S\alpha + T\beta) < N(\beta)$

$\Rightarrow \frac{N(S\alpha + T\beta)}{N(\beta)} < 1$

$\Rightarrow N(\frac{\alpha}{\beta} S + T) < 1$

c. (a, b, c)

$1 \in (a, b, c)$

So $\exists x, y, z$ s.t.

$ax + by + cz = 1$

d. $r \le |B|/2 \Rightarrow r' = r$

Else:

$r' = r - 1$

ei. $\frac{\alpha}{\beta} S + T = \frac{1}{c}(r + 1\sqrt{-19})$

ii $N(\frac{\alpha}{\beta}S + T) = N(\frac{1}{c}(r + \sqrt{-19}))$

$$= \frac{r^2 + 19}{c^2}$$

iii $r \leq c/2$

$$\Rightarrow \frac{r^2+19}{c^2} \leq \frac{(c/2)^2 + 19}{c^2}$$

$$= \frac{1}{4} + \frac{19}{c^2}$$

f i $S\frac{\alpha}{\beta} + T = \frac{1}{2}$

ii a, b multiples of 3

iii