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Lecture 8, Exercise A

1a. Cyclic subgroups of D_{12}:

$\{1, s\}$, $\{1, rs\}$, $\{1, r^2s\}$, $\{1, r^3s\}$, $\{1, r^4s\}$, $\{1, r^5s\}$, $\{1\}$, $\{1, r^3\}$, $\{1, r^2, r^4\}$, $\{1, r, r^2, r^3, r^4, r^5\}$

b. Two generators:

$\{1, r^2, r^4, s, sr^2, sr^4\}$, $\{1, r^3, s, sr^3\}$, $\{1, r^2, r^4, sr, sr^3, sr^5\}$, $\{1, r^3, sr, sr^4\}$, $\{1, r^3, sr^2, sr^5\}$, $\{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$

c. No new subgroups

d. Lattice would go here

2 $\langle r^3, s\rangle = \{1, r^3, s, sr^3\}$

$= \overline{\{r, s\}} \cap \overline{\{s, r^3\}}$

3 True

4 We have $\langle A\rangle = \bigcap_{\substack{A \subseteq H \\ H \leq G}} H$

a. Assume $H \leq G$ and $H \subseteq H$.

Hence $H \in \{K \mid K \subseteq H, H \leq G\}$

So $\bigcap_{K \in K} \{K \mid K \subseteq H, H \leq G\} \subseteq H$

$\Rightarrow \langle H \rangle \subseteq H$

In addition $H \subseteq \langle H \rangle$ by definition. Hence, we have $\langle H \rangle = H$

b. Let $A, B \subseteq G$ and $A \subseteq B$

$$\langle A \rangle = \bigcap_{\substack{A \subseteq H \\ H \leq G}} H$$

$$\langle B \rangle = \bigcap_{\substack{B \subseteq H \\ H \leq G}} H$$

Observe that since $A \subseteq B$ and $B \subseteq H$, we have $A \subseteq H$

Hence $\langle A \rangle = \langle B \rangle \cap \bigcap_{\substack{A \subseteq H \subseteq B \\ H \leq G}} H$

Thus, $\langle A \rangle \subseteq \langle B \rangle$ as required