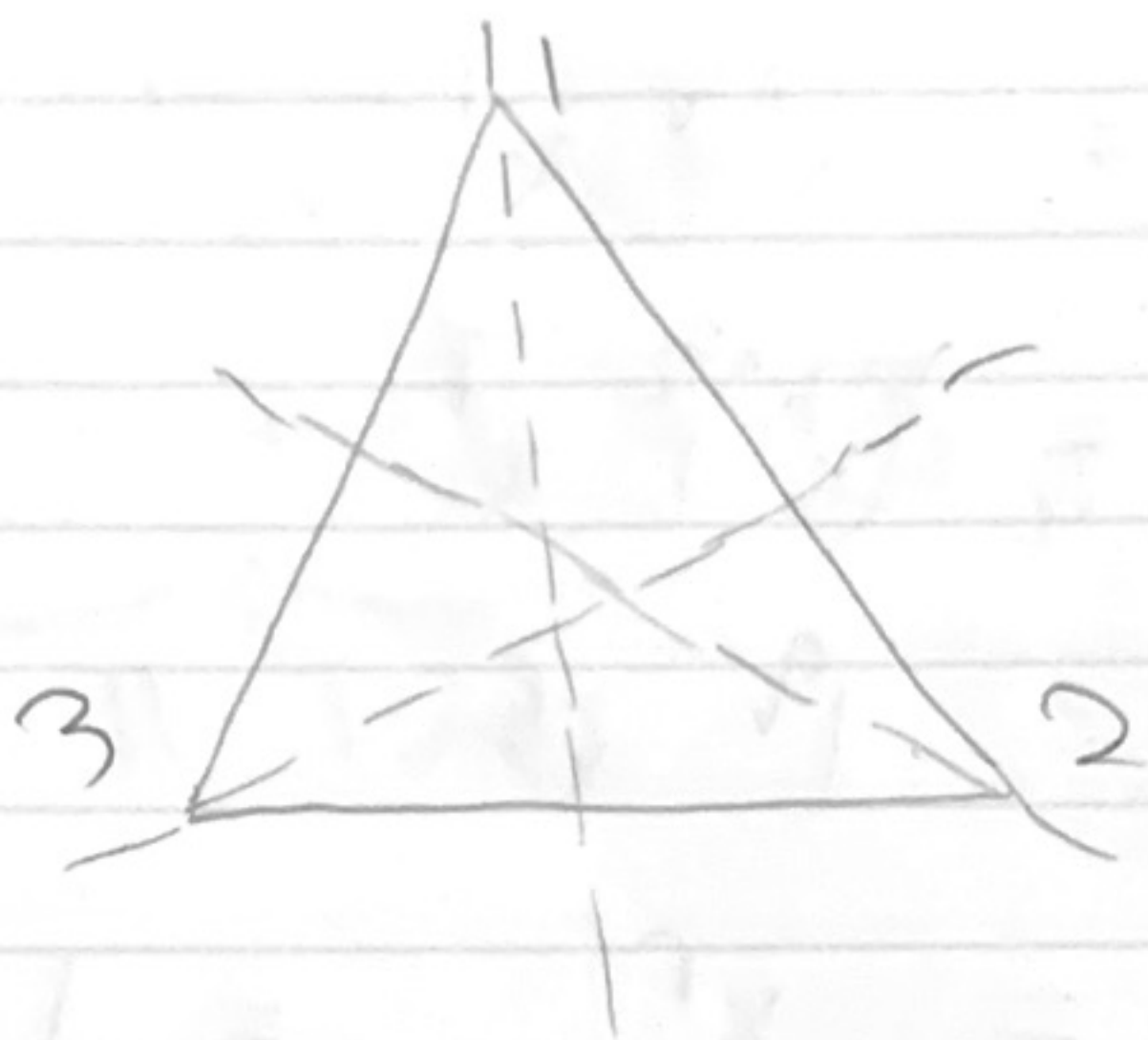


Chris Hayduk

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Lecture 2, Exercise B

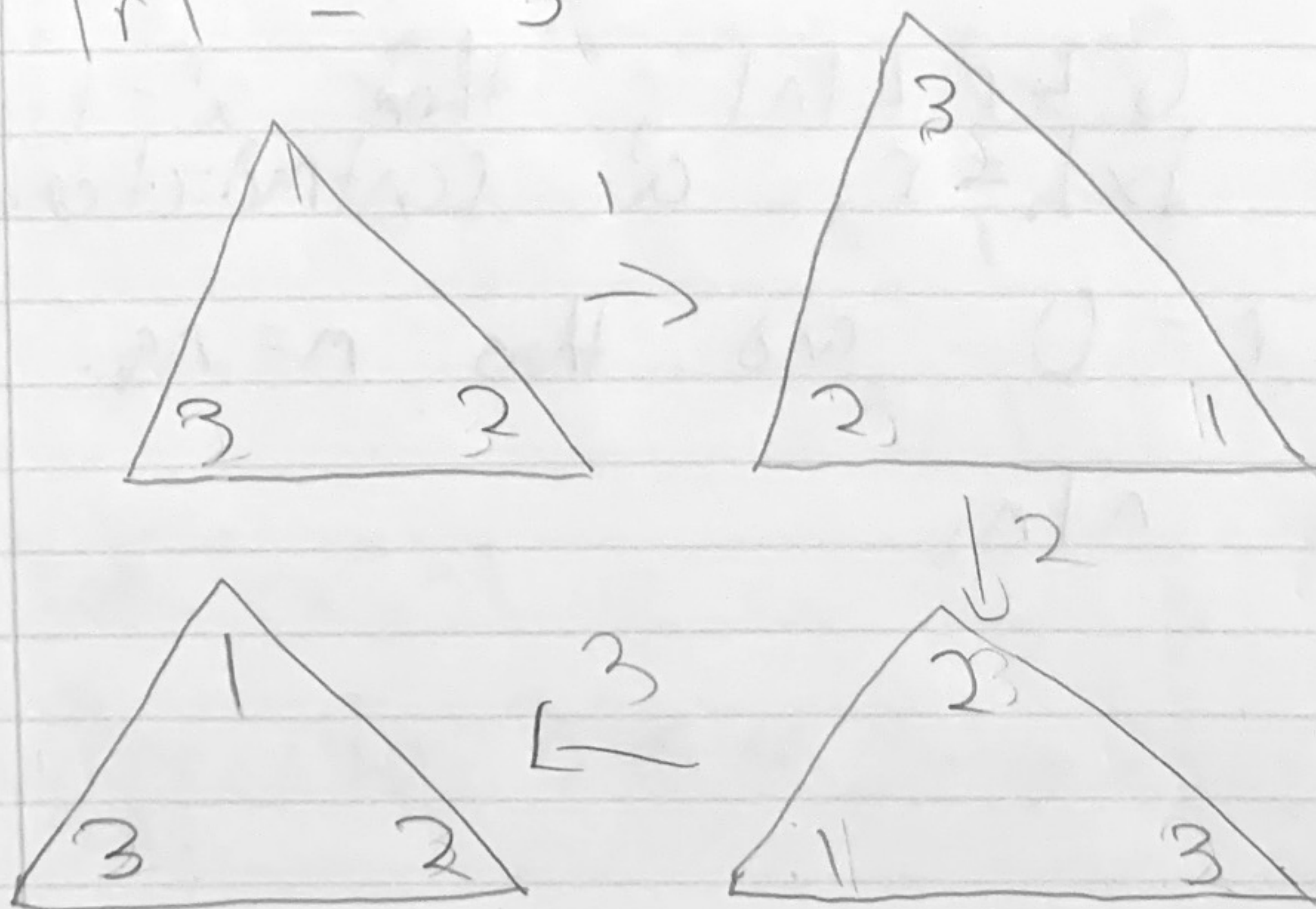
1a.



There are 6 symmetries:

3 rotations and 3 reflections

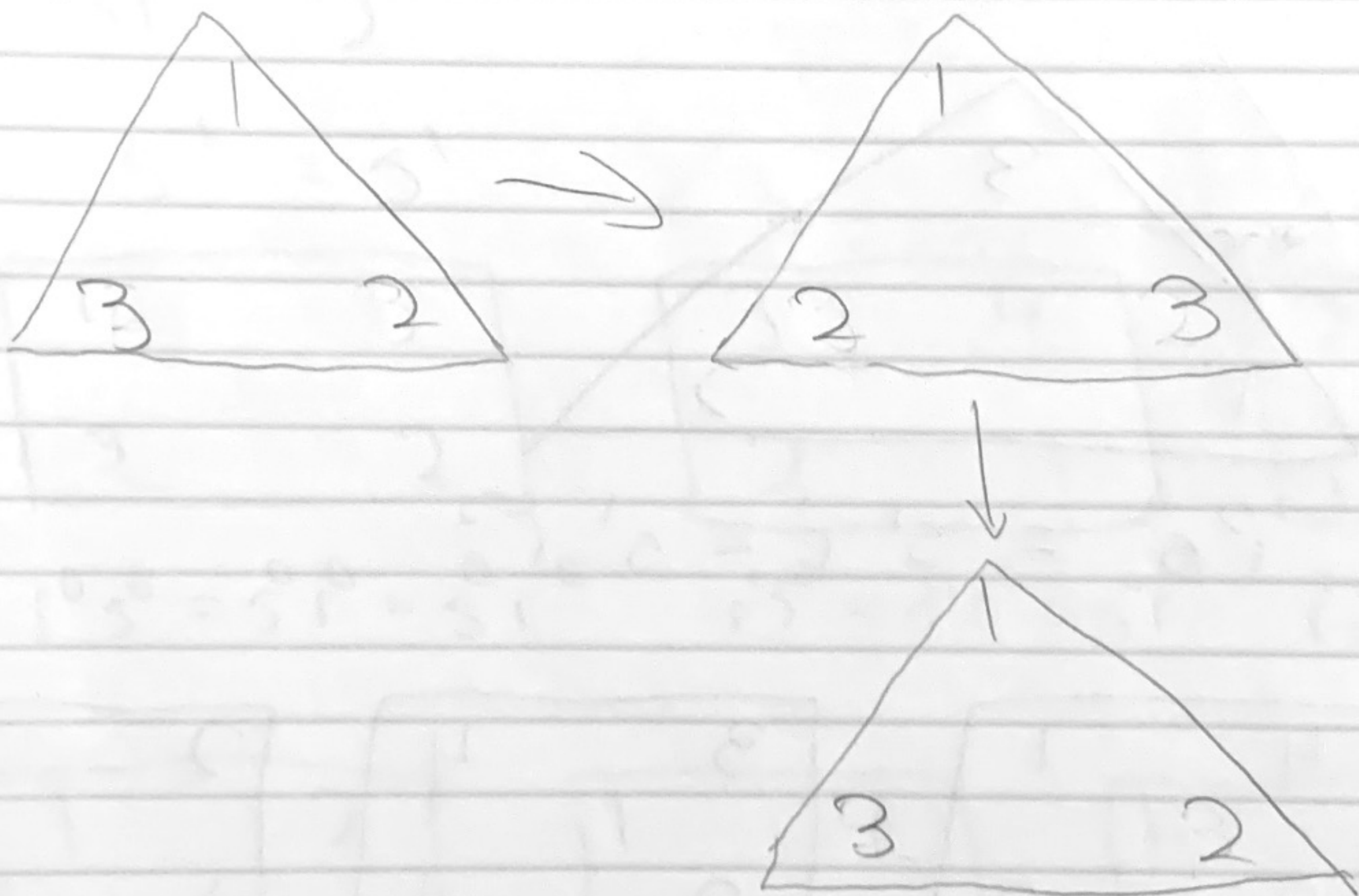
b. $|r| = 3$



$$r^{-1} = r^2$$

That is, 2 clockwise rotations are equivalent to 1 counter clockwise rotation

c. $|s| = 2$



$$s^{-1} = s'$$

d.

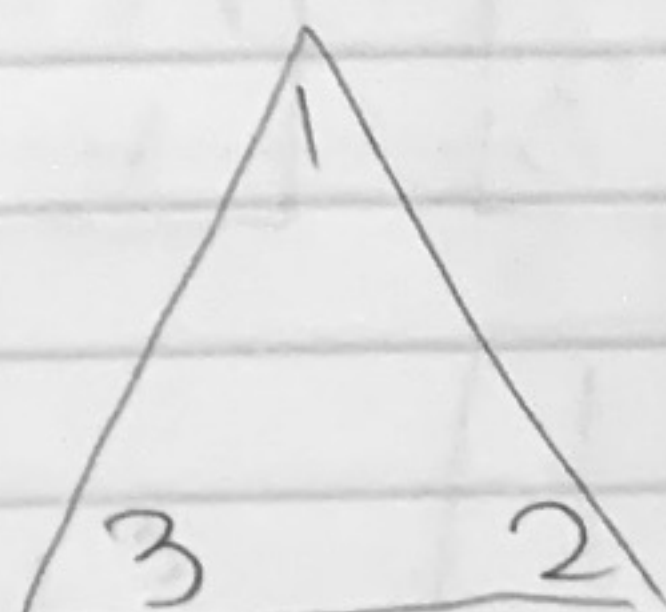


Diagram of a triangle with vertices 1, 2, and 3. Below it, the following equations are written:

$$r^0 s^0 = s^0 r^0$$

$$= s^0 r^0$$

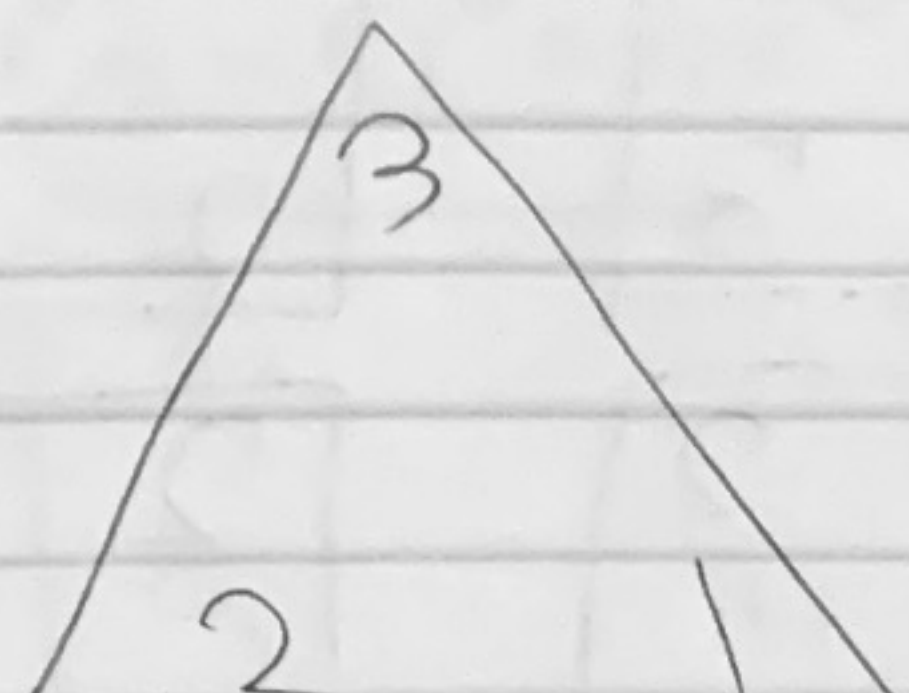


Diagram of a triangle with vertices 3, 2, and 1. Below it, the following equations are written:

$$r^1 s^0 = s^0 r^1$$

$$= s^1 r^3$$

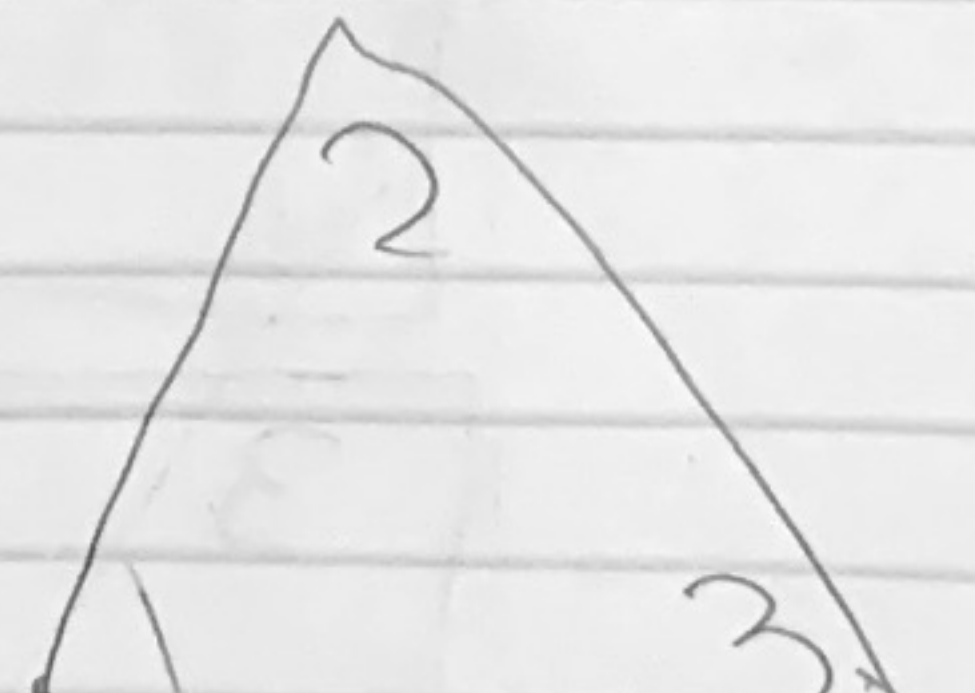


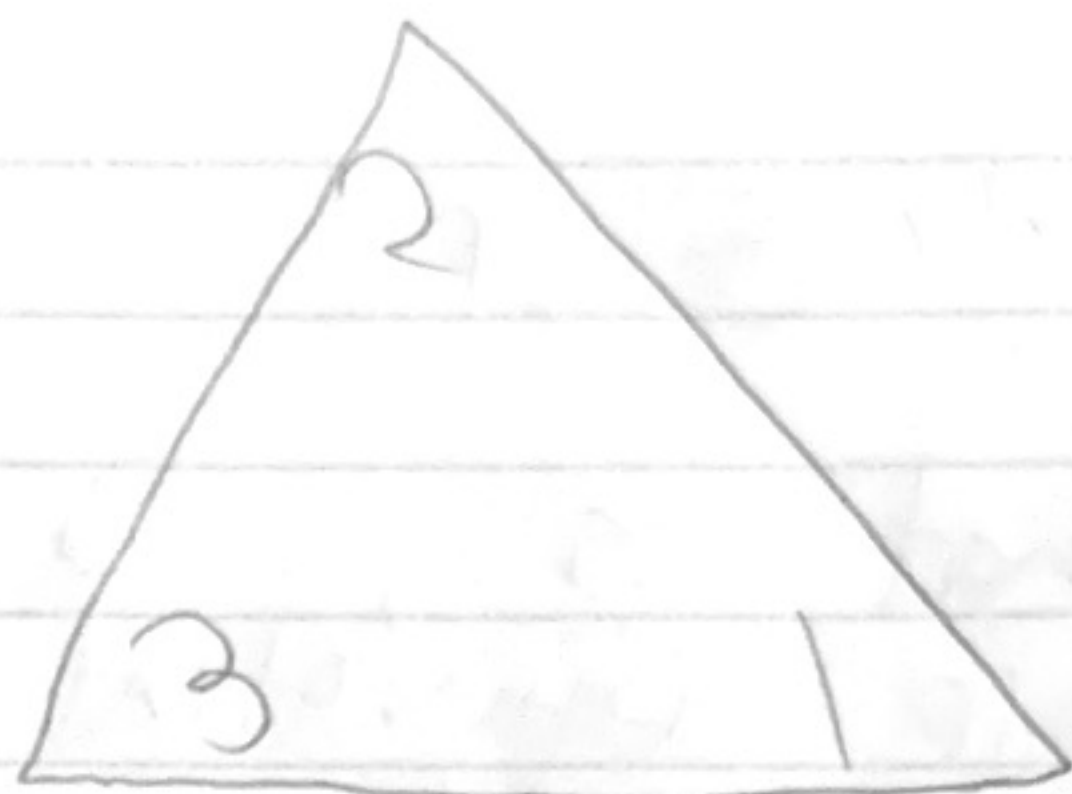
Diagram of a triangle with vertices 2, 1, and 3. Below it, the following equations are written:

$$r^2 s^0 = s^0 r^2$$

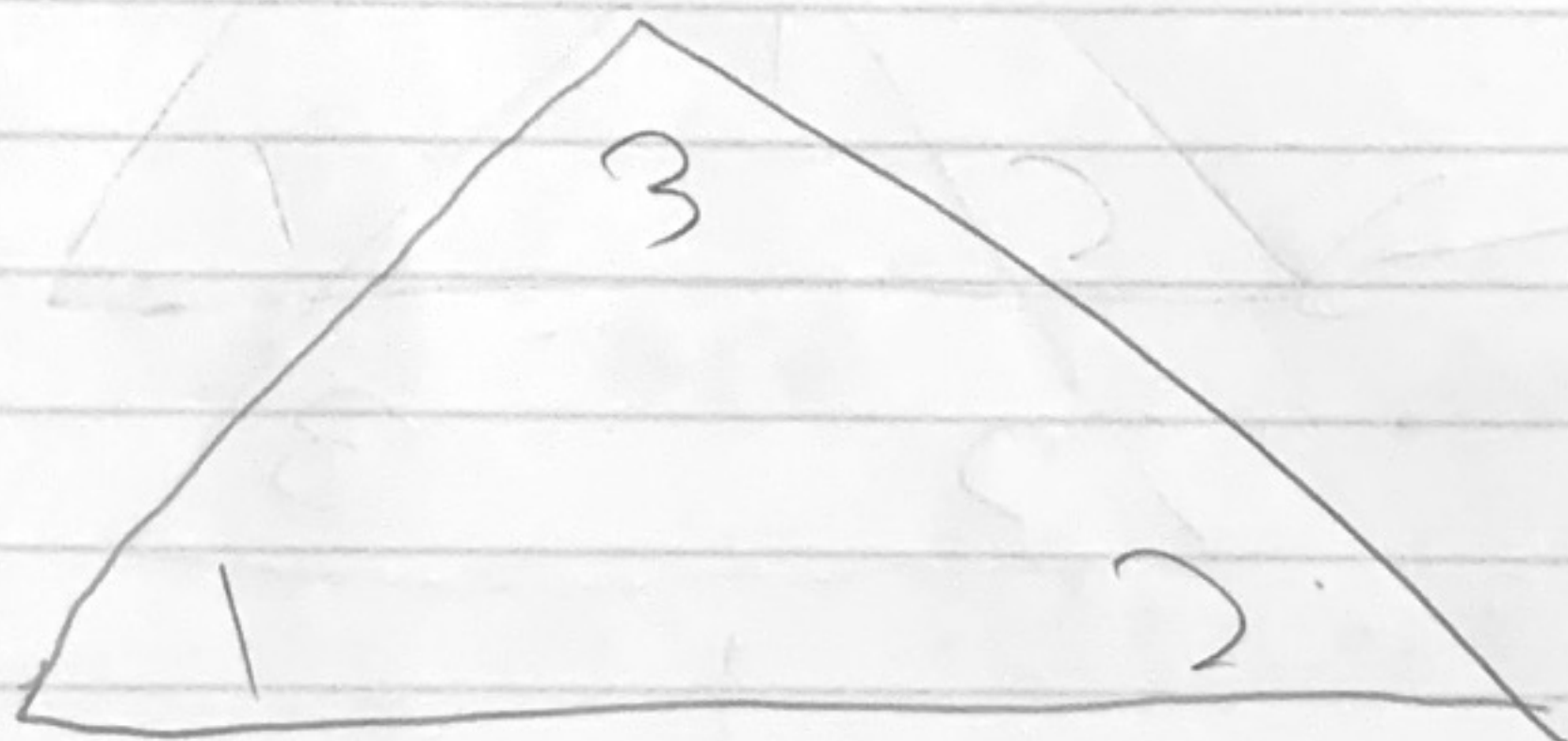
$$= s^0 r^{-1}$$



$$r^0 s = s' r^0 = s' r^{-0}$$

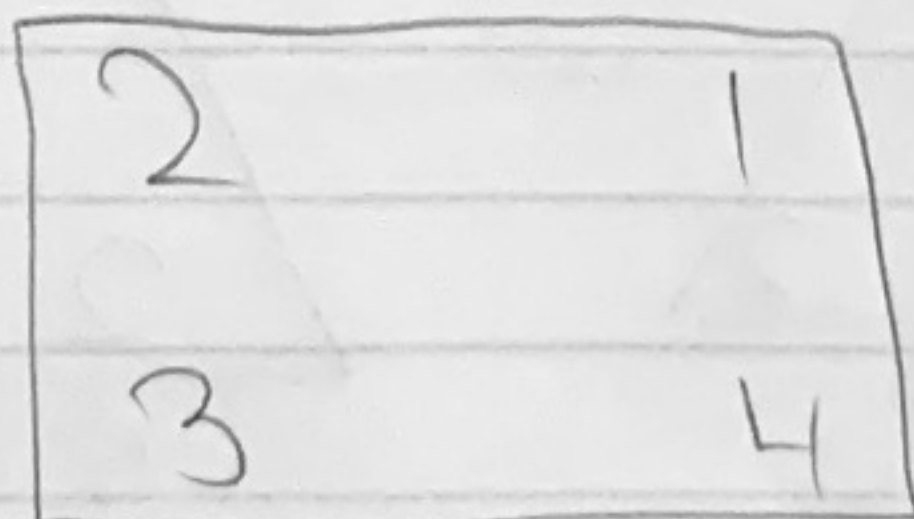
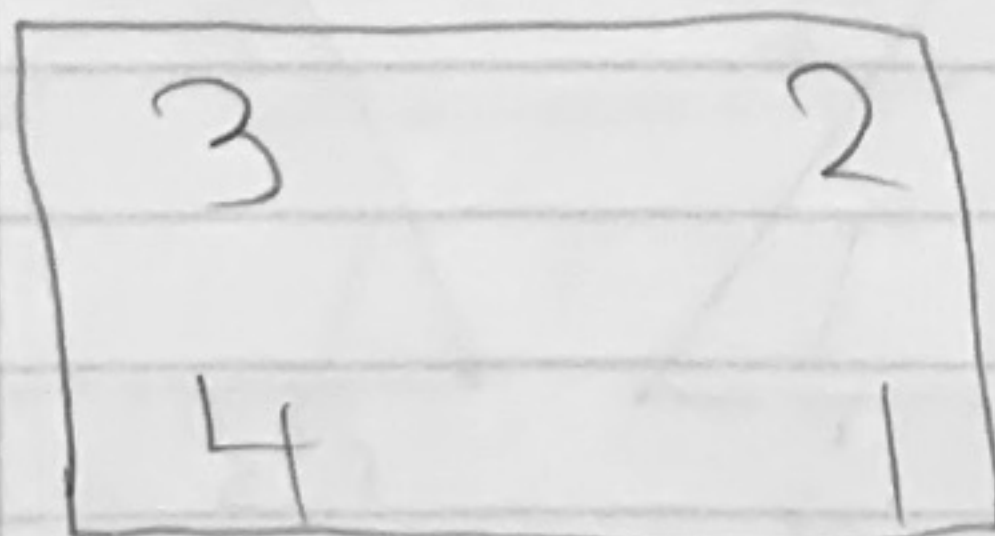
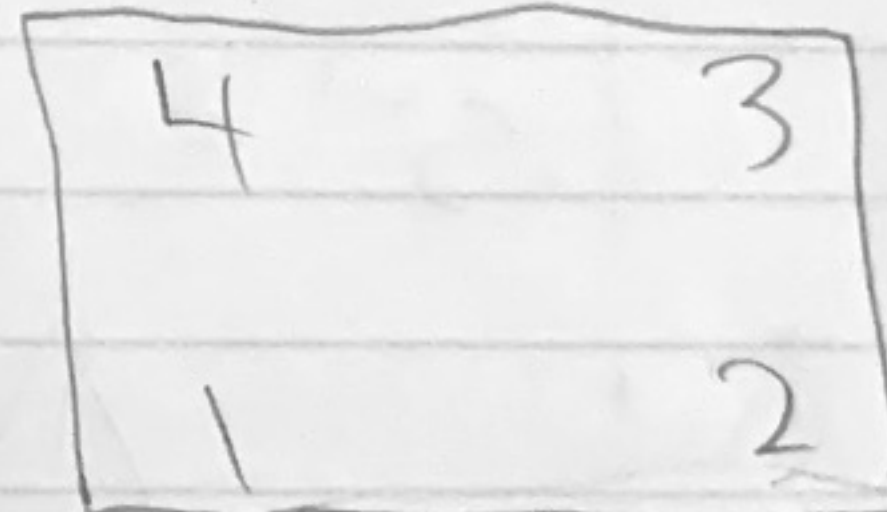
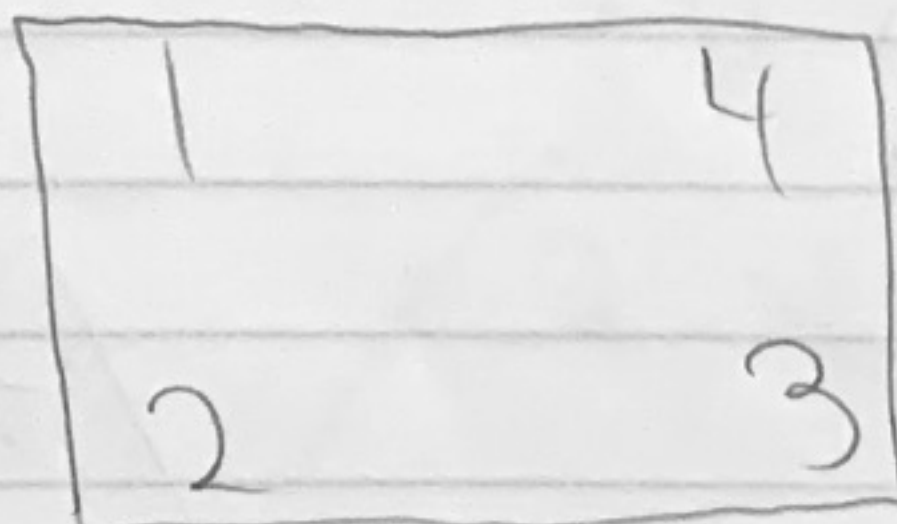
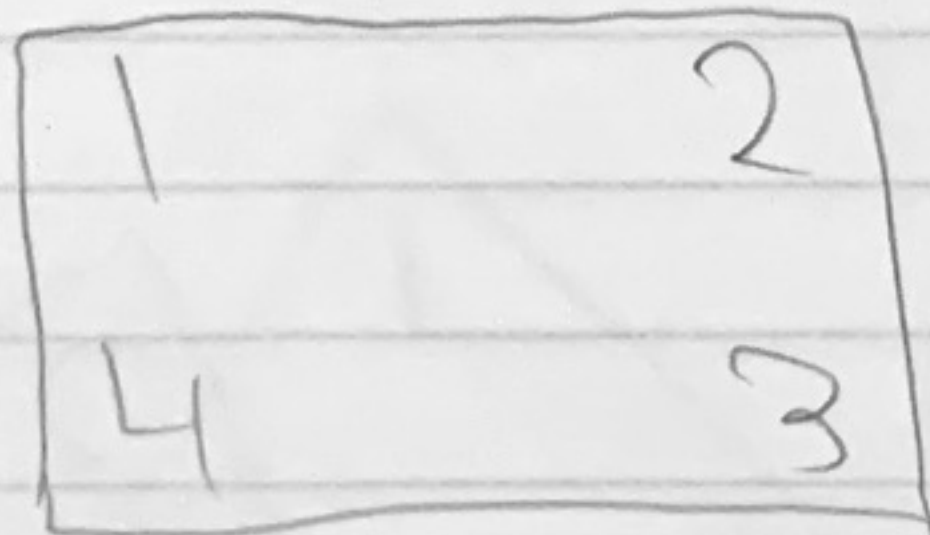
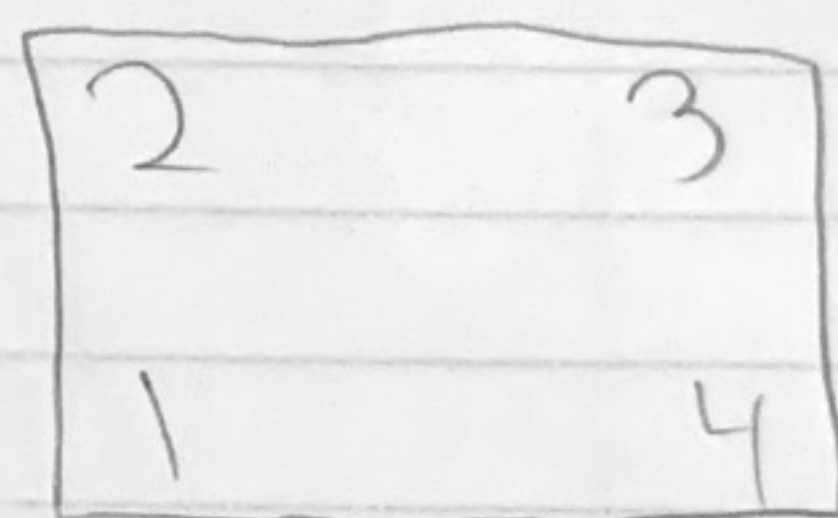
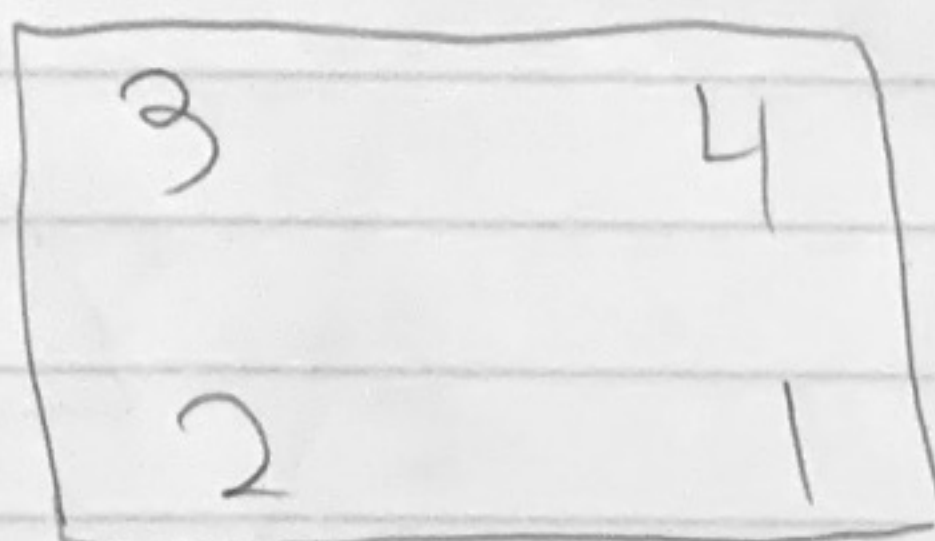
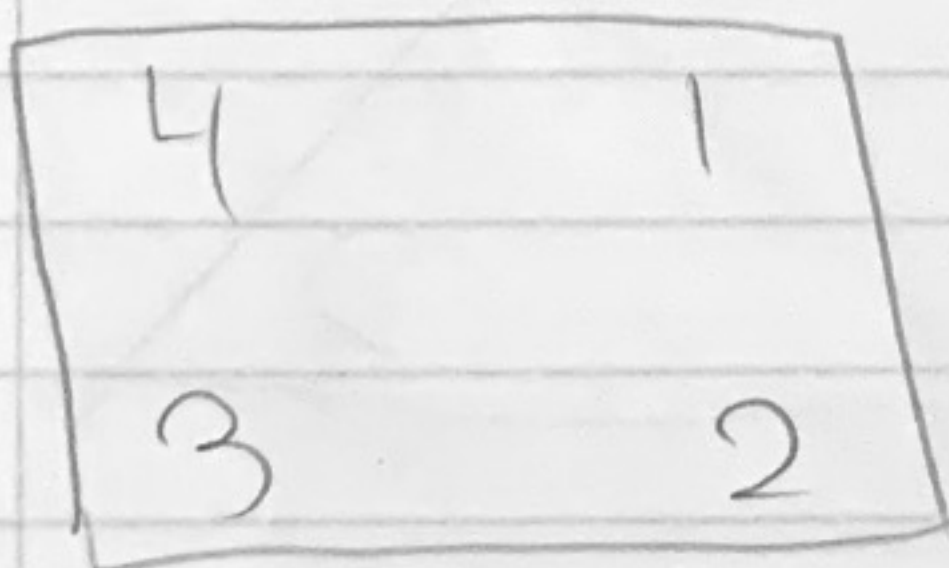


$$r^1 s = s' r^1 = s' r^{-1}$$



$$r^2 s = s' r^2 = s' r^{-2}$$

2a.



There are 8 symmetries of the square

b. $|r| = 4$

$$r^{-1} = r^3$$

c. $|s| = 2$

$$s^{-1} = s^1$$

d.

4	1
3	2

$$r^0 s^0 = s^0 r^0 = s^0 r^{-0}$$

3	4
2	1

$$r^1 s^0 = s^0 r^1 = s^0 r^{-3}$$

2	3
1	4

$$r^2 s^0 = s^0 r^2 = s^0 r^{-2}$$

1	2
4	3

$$r^3 s^0 = s^0 r^3 = s^0 r^{-1}$$

1	4
2	3

$$r^0 s^1 = s^1 r^0 = s^1 r^{-0}$$

4	3
1	2

$$r^1 s^1 = s^1 r^1 = s^1 r^{-3}$$

3	2
4	1

$$r^2 s^1 = s^1 r^2 = s^1 r^{-2}$$

2	1
3	4

$$r^3 s^1 = s^1 r^3 = s^1 r^{-1}$$

3a. $s \neq r^i \nmid i$ (from 1d and 2d)

b. $sr^i \neq sr^j \nmid 0 \leq i, j \leq n-1$
with $i \neq j$ (from 1d and 2d)

So the number of symmetries of a regular n -gon is $2n$ and that set can be written as

$$D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, \dots, sr^{n-1}\}$$