Chris Haydut 9/21

Lecture 8, Exercise C

1. We know,

$$\Phi(\sigma\tau) = X_{\sigma\tau} = X_\sigma \cdot X_\tau$$
$$= \Phi(\sigma) \cdot \Phi(\tau)$$

So Φ is homomorphism

Φ is also a bijection, so Φ is isomorphism

2a. $g \in X_h \Rightarrow \phi(g) = h$

$k \in K \Rightarrow \phi(k) = 1$

Since ϕ is a homomorphism, $\forall k \in K$, we have

$$\phi(kg) = \phi(k)\,\phi(g)$$
$$= 1 \cdot h = h$$

and

$$\phi(gk) = \phi(g)\,\phi(k)$$
$$= h \cdot 1 = h$$

So $Kg_1, gK \in X_2$ and $Kg, gK \subset X_2$

b) We have

$xy = x$

$y^2 = 1$

$\Rightarrow \phi(xy) = \phi(x)$

$\phi(yx) = \phi(x)$

$\Rightarrow \phi(x)\phi(y) = \phi(x)$

$\phi(x)\phi(y) = \phi(x)$

$\Rightarrow \phi(x)1 = \phi(x)$

$\phi(x)1 = \phi(x)$

$\Rightarrow 1 = \phi(x)\phi(x) = \phi(x)\phi(x)$

3) $D_8 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$

$\mathbb{Z}_2 = \{1, x\}$

$\phi(\{1, r^2, s, sr^2\}) = 1$

$\phi(\{r, r^3, sr, sr^3\}) = x$

ϕ hence well defined and surjective

We also have:

$\phi(r)^4 = x^4 = 1 = \phi(1)$,

$\phi(s)^2 = 1^2 = 1$

(a)

$\phi(r)\phi(s) = x \cdot 1 = 1 \cdot x = \phi(s)\phi(r)$

so ϕ a homomorphism as well

The fibers are as given above

$X_1 = \ker(\phi) = \{1, r^2, s, sr^2\}$

$X_x = \{r, r^3, sr, sr^3\}$

We have $X_1 = K_{e_1}(\varphi) = K$. So,

$$X_1 = 1K = K \cdot 1$$

and

$$X_x = rK = Kr$$