Chris Haydluk 10/6

Lecture 10, Exercise A

1a. $1N = \{1, y^3, y^6, y^9\} = N$

$yN = \{y, y^4, y^7, y^{10}\}$

$y^2N = \{y^2, y^5, y^8, y^{11}\}$

$y^3N = \{y^3, y^6, y^9, 1\} = 1N$

$y^4N = y(y^3N) = yN$

$y^5N = y^2(y^3N) = y^2N$

$y^6N = y^3N = 1N$

$y^7N = yN$

$y^8N = y^2N$

$y^9N = y^3N = 1N$

$y^{10}N = yN$

$y^{11}N = y^2N$

b. $\pi: G \to G/N \qquad g \mapsto gN$

$\pi(1) = N$

$\pi(y) = yN$

$\pi(y^2) = y^2N$

$\pi(y^3) = N$

$\pi(y^4) = yN$

$\pi(y^5) = y^2N$

$\pi(y^6) = N$

$\pi(y^7) = yN$

$\pi(y^8) = y^2N$

$\pi(y^9) = N$

$\pi(y^{10}) = yN$

$\pi(y^{11}) = y^2N$

c. $\ker(\pi) = \{1, r^3, r^6, r^9\}$

2 $G = S_3 = \{1, (123), (132), (23), (12), (31)\}$

$N = \langle (123) \rangle = \{1, (123), (132)\}$

$N \neq \emptyset$, $N \subseteq G$, and

$(123)(132)^{-1}$

$= (123)(231)$

$= (213) = (132)$

$\in G$

$(132)(123)^{-1}$

$= (132)(321)$

$= (312) = (123) \in G$

So N is a subgroup of G

We check $gN = Ng \quad \forall g \in G$

$$1N = N1$$

$$(123)N = \{(123), (132), 1\}$$
$$= N(123)$$

$$(132)N = \{(132), 1, (123)\}$$
$$= N(132)$$

$$(23)N = \{(23), (13), (12)\}$$
$$= N(23)$$

$$(12)N = N(12)$$

$$\vdots$$

So $N \trianglelefteq G$

$\text{Ker}(\pi) = \{1, (123), (132)\} = N$

3. Fiber over $gN = gN$

4a. Suppose $\phi: G \to H$ injective homomorphism.

Then $\phi(x) = \phi(y)$

$\Rightarrow x = y$

And we have

$G/\text{Ker}(\phi) \cong \phi(G)$

Since ϕ injective, then

$|G| = |\phi(G)|$

So $|G/\ker(\phi)| = |G|$

Hence, the # of fibers of G = # of elements of G.

By Prop 1.1: $\phi(1_G) = 1_H$

Hence, $X_{1_H} = \{1_G\}$

6. We have $G/\ker(\phi) \cong \phi(G)$

Thus, $\exists$ a bijection between $G/\ker(\phi)$ and $\phi(G)$ and we have

$$|G/\ker(\phi)| = \phi(G)$$