Chris Haydук 10/29

Lecture 15, Exercise A

1. $Z(G_1 \times \cdots \times G_n) = \{(g_1, \ldots, g_n) \in G_1 \times \cdots \times G_n \mid (g_1, \ldots, g_n) \cdot (g_1', \ldots, g_n') = (g_1', \ldots, g_n') \cdot (g_1, \ldots, g_n) \; \forall (g_1', \ldots, g_n') \in G_1 \times \cdots \times G_n\}$

Note then that

$$g_1 g_1' = g_1' g_1$$
$$\vdots$$
$$g_n g_n' = g_n' g_n$$

Since g_i' ranges over all elements in G_i we have that g_i commutes with every element in G_i.

2. $D_8 \cong G_1 \times \cdots \times G_n$ non-trivial

We can have

$n=1$

$n=2$

$n=3$

But every group order 2, 4 is Abelian, can only have $n=1$

3. $\hat{N}_i \cong N_i \times 1$

$N_i = G_1 \times \cdots \times G_i$

4. We know $S_3 \times 1$ and $1 \times S_3$ from class

5. $\mathbb{Z}_4 = \langle x \rangle$

$\langle i, x \rangle = \{1, (i, x), (-1, x^2), (-i, x^3)\}$

6. $h, h' \in H, \quad k, k' \in K$

$hk = h'k'$

$\Rightarrow h'^{-1} h = k (k')^{-1}$
$\quad \in H \qquad \in K$

$\Rightarrow h'^{-1}h, \; k(k')^{-1} \in H \cap K = 1$

$\Rightarrow h'^{-1}h = 1, \; k(k')^{-1} = 1$

$\Rightarrow h = h', \; k = k'$