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Lecture 2, Exercise C

1a. Note that $a = a^{-1}$

We have $a = s$ and $b = rs$

$$b = rs$$

$$\Rightarrow b = ra^{-1}$$

$$\Rightarrow ba = ra^{-1}a$$

$$\Rightarrow ba = r$$

$$b^3 = (ba)^3$$

$$= baba$$

$$= b(aba)ba$$

$$= bbabba$$

$$= b^2 a b^2 a$$

$$= aa = a^2 = 1 \quad \checkmark$$

$$s^2 = (a)^2$$

$$= a^2 = 1$$

$$rs = baq$$

$$r^{-1} = ab$$

$$\text{because } rr^{-1} = (ba)ab$$

$$= ba^2b$$

$$= bb = b^2 = 1$$

$$\text{So } rsr^{-1} = aqab$$

$$= a^2b$$

$$= b$$

$$= ba^2 \quad (\text{because } a^2 = 1)$$

$$= baq$$

$$= rs$$

$$2. \quad rs = baq$$

$$rs^2 = baqbaq$$

$$r = baqaba$$

$$= baqbaq = baq = baq = 1$$

$$|rs| = 2$$

$$r^2 = baba = abaa$$

$$= ab$$

$$(r^2)^2 = abab$$

$$= aaba$$

$$= ba$$

$$(r^2)^3 = (r^2)^2 r^2$$

$$= baab$$

$$= bb = 1$$

$$\text{So } |r^2| = 3$$

$$sr^2 = ababa$$

$$= aba$$

$$(sr^2)^2 = abaaba$$

$$= abba = aa = 1$$

$$|sr^2| = 2$$

3. $+i,$

$$r^2 r^i = r^{2+i} = r^{i+2} = r^i r^2$$