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Lecture 1, Exercise B

1a. Note that:

$$a-b = a+(-b)$$

and that $-b \in \mathbb{Z}$ since $b \in \mathbb{Z}$

We know $+ : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is associative. Therefore $- : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is as well

b. $(a*b)*c = (a+b+ab)*c$

$= a+b+ab + c + ac + bc + abc$

$a*(b*c) = a*(b+c+bc)$

$= a+b+c+bc+ab+ac+abc$

$= a+b+ab+c+ac+bc+abc$

$= (a*b)*c$

So it is associative.

2. $\mathbb{Z}/n\mathbb{Z}$ is not a group under multiplication because not every element has an inverse.

For example, in $\mathbb{Z}/4\mathbb{Z}$, $\bar{1}$ is the identity element, but multiplying $\bar{0}$ by any other element of $\mathbb{Z}/n\mathbb{Z}$ does not yield 1

3. $\mathbb{Z}/n\mathbb{Z} - \{\bar{0}\}$ is a group under multiplication when n is odd.

Take $(\mathbb{Z}/4\mathbb{Z} - \{\bar{0}\}, \times)$.

$\times$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{2}$	$\bar{2}$	$\bar{0}$	$\bar{2}$
$\bar{3}$	$\bar{3}$	$\bar{2}$	$\bar{1}$

There is no way to multiply $\bar{2}$ by an element that yields
$\bar{1}$

Now take $(\mathbb{Z}/3\mathbb{Z} - \{\bar{0}\}, \times)$:

$\times$	$\bar{1}$	$\bar{2}$
$\bar{1}$	$\bar{1}$	$\bar{2}$
$\bar{2}$	$\bar{2}$	$\bar{1}$

4. The group identity is c

$c^{-1} = c$

$b^{-1} = a$

$a^{-1} = b$

5. First table:

$(b * c) * c = c * c = a$

$b * (c * c) = b * a = b$

Not associative

Second table:

No identity element