Chris Haydnk 10/6

Lecture 10, Part B

1a. $A \cap B \subseteq A, B, G$

and $A \cap B \leq G$

So $|A \cap B|$ divides $|A|$, $|B|$, $|G|$

Common factors of 12, 20:

2, 4

So $|A \cap B| = 2$ or $|A \cap B| = 4$

b. $\text{lcm}(12, 20) = 60$

$|G| \geq 60$

2a. Yes, orders are

$1 \to 4 \to 8 \to 16$

b. Only prime subgroup is $\{1\}$

Cyclic bc $|1| = 1$

c No;

$\langle r^4 \rangle = \{1, r^4, r^8, r^{12}\}$

and $|\langle r^4 \rangle| = 4 = 2.2$

3a. $|\langle (12)(34), (13)(24) \rangle|$ divides 12 and is divided by 2

So $|\langle (12)(34), (13)(24) \rangle| = 4$ or $= 6$

b. $|G| = 12$ and $6 \mid 12$, but there is no subgroup of order 6

4a $A = \{(12), 1\}$

$B = \{(23), 1\}$

i) $AB = \{1, (12), (23), (23)(12)\}$

$BA = \{1, (12), (23), (132)\}$

ii) $AB \neq BA$

iii) $(12)(23)^{-1} = (12)(32) = (312) \notin AB$ so $AB \not\leq G$

$(12)(23)^{-1} \notin BA$, so $BA \not\leq G$

iv) $|AB| = 4$

$|A| = 2$

$|B| = 2$

$|A \cap B| = 1$

So

$4 = 2 \cdot 2 / 1$

$= 4$ ✓

5. Let $\phi(xH) = x^{-1}(xH)y$