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Lecture 9, Exercise A 9/23

1. Suppose $xH = Hx$

Then

$$\{xh \mid h \in H\} = \{hx \mid h \in H\}$$

So fix $y \in xH, Hx$.

Can write

$$y = xh_1 = h_2 x \quad \text{(for some } h_1, h_2\text{)}$$

So

$$x h_1 x^{-1} = h_2$$

But $h_2 \in H$ so $xh_1x^{-1} \in H$.

So $xHx^{-1} = \{xhx^{-1} \mid h \in H\} \subset H$

Now let $h \in H$ and fix $x \in G$.

We know

$$xh = hx$$

$$\Rightarrow xhx^{-1} = h$$

So $\quad H \supseteq xHx^{-1}$ and

$$H \subset xHx^{-1}$$

and hence

$$xHx^{-1} = H$$

Now suppose $xHx^{-1} = H$

We have

$$(xHx^{-1})x = (H)x$$

$$\Rightarrow xH(x^{-1}x) = Hx$$

$$\Rightarrow xH = Hx$$

2. $\langle 1 \rangle$, $\langle r^2 \rangle$, $\langle s, r^2 \rangle$, $\langle r \rangle$, $\langle rs, r^2 \rangle$, D_8

3a. $\langle s \rangle = \{1, s\} = H$

$rH = \{r, rs\}$

$sr^3 H = \{sr^3, sr^3 s\}$

$= \{rs, (rs)s\}$

$= \{rs, r\}$

$= rH$

b. $(r \cdot r)H = r^2 H$

$= \{r^2, r^2 s\} \neq H$

$(sr^3 \cdot sr^3)H = (r^{-3} s s r^3)H$

$= (r^{-3} r^3)H$

$= 1H = H$

So $(r \cdot r)H \neq (sr^3 \cdot sr^3)H$

$= H$

4. $\langle 1 \rangle = \{1\}$. Let $G = D_8$ and

$$\varphi : D_8 \to G$$

$$\varphi(x) = x$$

Then $\ker(\varphi) = \{1\} = H$

$\langle r^2 \rangle = \{1, r^2\} = H$

Let $G = \langle r \rangle$ and

$$\varphi : D_8 \to G$$

$$\varphi =$$