Chris Haydukk 9/10

Lecture 5, Exercise B

1a. Let $n \in \mathbb{N}$ and $x \in G$.

For $n > 1$, suppose $\varphi(x^{n-1}) = \varphi(x)^{n-1}$

We have:

$$\varphi(x^n) = \varphi(x^{n-1}x)$$

$$= \varphi(x^{n-1})\varphi(x)$$

Since we assumed the property holds for $\varphi(x^{n-1})$, we get

$$= \varphi(x)^{n-1}\varphi(x)$$

$$= \varphi(x)^n$$

6. $$\varphi(x^{-n}) = \varphi(x^{-n+1}x^{-1})$$
$$= \varphi(x^{-n+1})\varphi(x^{-1})$$
$$= \varphi(x^{-n+1})\varphi(x)^{-1}$$
$$= \varphi(x^{-n+2})\varphi(x)^{-1}\varphi(x)^{-1}$$
$$= \varphi(x^{-n+2})\varphi(x)^{-2}$$
$$\vdots$$
$$= \varphi(x)^{-n}$$

2. $\varphi(1_G) = 1_H \Rightarrow 1_G \in \ker(\varphi)$,
so

$$\ker(\varphi) \neq \emptyset$$

Now suppose $g_1, g_2 \in \ker(\varphi)$.
Then we have:
$$\varphi(g_1 g_2^{-1}) = \varphi(g_1)\varphi(g_2)^{-1}$$
$$= 1_H \cdot 1_H = 1_H$$

So $g_1 g_2^{-1} \in \ker(\varphi)$

So $\ker(\varphi)$ satisfies the subgroup criterion and is hence a subgroup of G.