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Lecture 15, Exercise B

$$\begin{aligned} 1a. (1,1)(a,b) &= (1(1 \cdot a), 1b) \\ &= (1a, b) \\ &= (a, b) \end{aligned}$$

$$\begin{aligned} (a,b)(1,1) &= (a(b \cdot 1), b1) \\ &= (a1, b) \\ &= (a, b) \end{aligned}$$

$$b. (b^{-1} \cdot a^{-1})(b^{-1} \cdot 1) = b^{-1} \cdot (a a^{-1}) = b^{-1} \cdot 1$$

$\approx 1$

$$\Rightarrow (b^{-1} \cdot a^{-1}, b^{-1})(a, b) = \left( \begin{matrix} b^{-1} \cdot a^{-1} \\ b^{-1} \cdot b \end{matrix} \right) (b^{-1} \cdot 1),$$
$$\approx (1, 1)$$



$$1. (a, 1)^{-1} = (1^{-1} \cdot a^{-1}, 1^{-1})$$

$$= (a^{-1}, 1)$$

$$(1, b)^{-1} = (b^{-1} \cdot 1, b^{-1})$$

$$= (1, b^{-1})$$

Now plug in

$$2. 1 = x^3 \rightarrow (123)^3 = 1$$

$$1 = y^2 \rightarrow (12)^2 = 1$$

and

$$yx = (1, y)(x, 1) = (1(y \cdot x), y \cdot 1)$$

$$= (x^{-1}, y)$$

$$= x^{-1}y$$



$$3. |A \times B| = |A| |B|$$

$$\Rightarrow |A| = 2 \text{ or}$$

$$|B| = 4$$

$$4. a b a^{-1} = (a, 1)(1, b)(a^{-1}, 1)$$

$$= (a, b)(a^{-1}, 1)$$

$$= (a(b \cdot a^{-1}), b)$$