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Lecture 4, Exercise C

1a.

xyx^{-1}	1	r	r^2	r^3	s	sr	sr^2	sr^3
1	1	r	r^2	r^3	s	sr	sr^2	sr^3
r	1	r	r^2	r^3	sr^2	sr^3	s	sr
r^2	1	r	r^2	r^3	s	sr	sr^2	sr^3
r^3	1	r	r^2	r^3	sr^2	sr^3	s	sr
s	1	r^3	r^2	r	s	sr^3	sr^2	sr
sr	1	r^3	r^2	r	sr^2	sr	s	sr^3
sr^2	1	r^3	r^2	r	s	sr^3	sr^2	sr
sr^3	1	r^3	r^2	r	sr^2	sr	s	sr^3

x

b. i) $Z(D_8) = \{1, r^2\}$

ii) $C_{D_8}(r) = \{1, r, r^2, r^3\}$

iii) $C_{D_8}(\{r, s\}) = \{1, r^2\}$

iv) $C_{D_8}(\{r, r^3\}) = \{1, r, r^2, r^3\}$

v) $N_{D_8}(r) = \{1, r, r^2, r^3\}$

vi. $N_{D_8}(\{r, s\}) = \{1, r^2\}$

vii. $N_{D_8}(\{r, r^3\}) = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$

$= D_8$

C. All answers satisfy these conditions

d. i) Let $H = \{1, r^2\}$

Then $C_{D_8}(H) = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$

$= D_8$

ii) Let $H = \{1, s\}$

Then $C_{D_8}(H) = \{1, r^2, s, sr^2\}$

$\neq D_8$

iii) Let $H = \{1, r, r^2, r^3\}$

Then $N_{D_8}(H) = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$

$= D_8$

iv) Let $H = \{1, s\}$

Then $N_{D_8}(H) = \{1, r^2, s, sr^2\}$

$\neq D_8$

2. Note that $Z(G)$ is the set of elements that commute with everything in G.

Hence $\forall z \in Z(G)$, $g \in G$, we have

$$zg = gz$$

Now note that $C_G(Z(G)) = \{ g \in G \mid gzg^{-1} = z \ \forall z \in Z \}$

Since z commutes we have
$gzg^{-1} = gg^{-1}z = z \quad \forall g, g^{-1} \in G$

Hence $G = C_G(Z(G))$

We already know that
$C_G(Z(G)) \leq N_G(Z(G)) \leq G$, so we have

$$N_G(Z(G)) = G$$

3. Suppose H is a subgroup of G.