Lecture 3, Exercise B

1a. $\sigma\tau = (1\,4\,2\,3\,5)$

$\tau\sigma = (1\,4\,5\,3\,2)$

b. $\sigma = (1\,4\,2)(3\,5)$

$\sigma^2 = (1\,4\,2)^2 (3\,5)^2$

$= (1\,2\,4)(3)(5)$

$= (1\,2\,4)$

$\sigma^3 = (1\,4\,2)^3 (3\,5)^3$

$= (3\,5)$

$\sigma^4 = (1\,4\,2)^4 (3\,5)^4$

$= (1\,4\,2)$

$\sigma^5 = (1\,4\,2)^5 (3\,5)^5$

$= (1\,2\,4)(3\,5)$

$\sigma^6 = (1\,4\,2)^6 (3\,5)^6$

$= 1$

So $|\sigma| = 6$

$\tau = (25)$

$\tau^2 = 1$

$|\tau| = 2$

$\sigma\tau = (14235)$

$(\sigma\tau)^2 = (12543)$

$(\sigma\tau)^3 = (13452)$

$(\sigma\tau)^4 = (15324)$

$(\sigma\tau)^5 = 1$

$|\sigma\tau| = 5$

$\tau\sigma = (14532)$

As above, since $\tau\sigma$ is
a single cycle the order
equals the length of the
cycle.

$$|\tau\sigma| = 5$$

2.. $\sigma \sigma^{-1} = (a_1 a_2 \cdots a_\ell)(a_\ell \cdots a_2 a_1)$

i.e. $\sigma \sigma^{-1} = 1$

$\sigma^{-1} \sigma = (a_\ell \cdots a_2 a_1)(a_1 a_2 \cdots a_\ell)$

$= 1$

3a. $xyx^{-1} = (abc) \, a \, (abc)^{-1}$

$= (abc) \, a \, (cba)$

$= ab \, (ca) \, cba$

$= abac \, cba$

$= ab \, aba$

$= a \, (bab) \, a$

$= a \, ab \, aa = b$

$$x^2 y x^2 = (abc)^2 \, a \, (abc)^2$$
$$= abc\,abc\,a\,cba\,cba$$
$$= ab\,ac\,b\,ac\,cb\,caba$$
$$= aba\,c\,bab\,c\,aba$$
$$= aba\,c\,aba\,c\,aba$$
$$= aba\,ac\,ba\,ac\,ba$$
$$= abc\,b\,cba$$
$$= abb\,c\,bba$$
$$= a\,c\,a$$
$$= c\,a\,a$$
$$= c$$

b. $a^2 = y^2 = 1$

$b^2 = (xyx^{-1})^2 = 1$

$c^2 = (x^2 y x^{-2})^2 = 1$

$ac = ca \Rightarrow yx^2 y x^2 = x^2 y x^2 y$

$$aba = bab \implies yxyx^{-1}y = xyx^{-1}yxyx^{-1}$$

$$xyx^{-1} x^2 y x^{-2} xyx^{-1} = x^2 y x^{-2} xyx^{-1} x^2 y x^{-2}$$

$\Updownarrow$ (from $bcb = cbc$)

c. $\quad y^2 = 1 \implies (xyx^{-1})^2 = 1$

$\quad y^2 = 1 \implies (x^2yx^{-2})^2 = 1$