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Lecture 12 - Exercise B

1. Suppose $gGa = hGa$. Then $k^{-1}g \in Ga$.

So

$$a = (k^{-1}g) \cdot a$$

$$\Rightarrow h \cdot a = h \cdot (k^{-1}g) \cdot a$$

$$\Rightarrow h \cdot a = (h \cdot k^{-1} \cdot g) \cdot a$$

$$\Rightarrow h \cdot a = g \cdot a$$

2. We have from Ex A that

$$L(1,1) = \{(1,1), (2,2), (3,3)\}$$

Now we also have $G_{(1,1)} = \{1, (23)\}$.
We need to compute $gG_{(1,1)}$ for $g \in G$.

$$1G_{(1,1)} = \{1, (23)\}$$

$$(23)G_{(1,1)} = \{1, (23)\}$$

$$(12)G_{(1,1)} = \{(12), (123)\}$$

$$(123)G_{(1,1)} = \{(12), (123)\}$$

$$(13)G_{(1,1)} = \{(13), (132)\}$$

$$(132) G_{(1,1)} = \{(13), (132)\}$$

$$\text{So, } |[I(1,1)]| = 3 = |G : G_{(1,1)}|$$

6. We have

$$[I(1,2)] = \{(1,2), (2,1), (1,3), (3,2)\}$$

$$G_{(1,2)} = \{1\}$$

So,

$$|G_{(1,2)}| = 1$$

$$(12) G_{(1,2)} = \{(12)\}$$

$$(23) G_{(1,2)} = \{(23)\}$$

$$(13) G_{(1,2)} = \{(13)\}$$

$$(123) G_{(1,2)} = \{(123)\}$$

$$(132) G_{(1,2)} = \{(132)\}$$

$$3. \quad [E1, 23] = \{E1, 23, E2, 33, E3, 43, E1, 43\}$$

$$[E1, 33] = \{E1, 33, E2, 43\}$$

$$x \in [E1, 23],$$

$$|G_x| = 4 = 8/|x|$$

$$x \in [E1, 33],$$

$$|G_x| = 2 = 8/|x|$$

$$\begin{aligned} 4a. \quad \tau \cdot (\sigma \cdot \rho) &= \tau \cdot (12x_2^5 x_3^7 x_1 - 18x_3^6 x_4 \\ &\quad + 11x_2 x_3 x_4 x_1) \\ &= 12x_3^5 x_1^7 x_2 - 18x_1^6 x_4 \\ &\quad + 11x_3 x_1 x_4 x_2 \end{aligned}$$

$$\tau\sigma = (1342)$$

$$\begin{aligned} (\tau\sigma) \cdot \rho &= 12x_3^5 x_1^7 x_2 - 18x_1^6 x_4 \\ &\quad + 11x_3 x_1 x_4 x_2 \end{aligned}$$

$$= \tau \cdot (\sigma \cdot \rho)$$

b. We have,

$$1 \cdot p(x_1, x_2, \dots, x_n) \\ = p(x_1, x_2, \dots, x_n)$$

and,

$$g \cdot (h \cdot p(x_1, x_2, \dots, x_n)) \\ = g \cdot p(x_{h \cdot 1}, x_{h \cdot 2}, \dots, x_{h \cdot n}) \\ = p(x_{g \cdot h \cdot 1}, x_{g \cdot h \cdot 2}, \dots, x_{g \cdot h \cdot n}) \\ = (g \cdot h) \cdot p(x_1, x_2, \dots, x_n)$$

c. Fix $n=4$

i)