

Expository Paper Outline: Galois Theory

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Abstract

Abstract goes here

1 Preliminaries

1.1 Polynomials

Introduce solving polynomial equations:

1. [4, Sec. 1.3]
2. [1, Ch. 1]

Solution by radicals:

1. [4, Sec. 1.4]
2. [1, Ch. 1, 2]

Example of solutions to cubic and quartic polynomials: [2, Ch. 1]

Problem of polynomials with degree ≥ 5 :

1. [1, Ch. 1]
2. [5]

The Fundamental Theorem of Algebra:

1. [4, Section 2.2]
2. [1, Ch. 4]

1.2 Field Theory

Define field: [3, Sec. 7.1]

Define characteristic of a field: [3, Sec. 13.1]

Define prime subfield: [3, Sec. 13.1]

Define field extension: [3, Sec. 13.1]

Discuss field extension properties:

1. [3, Sec. 13.1]
2. [4, Ch. 4]

Discuss simple extensions: [4, Ch. 5]

Discuss the degree of extensions [4, Ch. 6]

2 Galois Theory

2.1 Basics

Overview of Galois theory: [5]

Define automorphism: [3, Sec. 14.1]

Define Galois extension: [3, Sec. 14.1]

Define Galois group: [3, 14. 1]

2.2 The Fundamental Theorem of Galois Theory

Summarize the Fundamental Theorem of Galois Theory: [6]

Define character of a group: [3, Sec. 14.2]

Define linearly independent characters: [3, Sec. 14.2]

State and prove linear independence of characters theorem: [3, Sec. 14.2]

State and prove Fundamental Theorem of Galois Theory:

1. [3, Sec. 14.2]
2. [2, Ch. 9]
3. [4, Ch. 12]

Compute some examples using Galois Extensions and the Fundamental Theorem of Galois Theory:

1. [3, Sec. 14.2]
2. [2, Ch. 9]
3. [4, Ch. 13]

2.3 Soluble Groups

Define soluble group: [4, Sec. 14.1]

State and prove theorem about solubility of subgroups: [4, Sec. 14.1]

2.4 The General Polynomial Equation

Define symmetric polynomials: [4, Sec. 18.2]

State elementary symmetric polynomial theorem: [4, 18.2]

State and prove theorem that a polynomial is soluble by radicals if and only if it has a soluble Galois group: [4, 18.4]

2.5 Finite Fields

Summarize properties of finite fields:

1. [3, Sec. 14.3]
2. [4, Ch. 19]

2.6 Galois Groups of Polynomials

Give examples of Galois groups of polynomials of degree 2, 3, 4: [3, Sec. 14.6]

3 Conclusion

Summarize key ideas here.

References

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