Chris Haydyk 10/8

Lecture 11, Exercise A

1a. Since $A \subseteq B$, ~~ta~~ we have

$$aN = bN$$

for $a = b \in B$

Hence,

$$\{aN \mid a \in A\} \subseteq \{bN \mid b \in B\}$$

b. $A/N \subseteq B/N \Rightarrow \forall\, aN \in A/N$, exists $b \in B$ w/

$$aN = bN$$

Also $aN \in A/N \Rightarrow a \in A$

$aN = bN \Rightarrow b^{-1}a \in N$

$\Rightarrow b^{-1}a = n$ for some $n \in N$

So $a = bn \in B$ since $n \in N \subseteq B$

2a. $\bar{b}\bar{A} = bN\{aN \mid a \in A\}$

$= \{bNaN \mid a \in A\}$

$= \{baN \mid a \in A\}$

because $N \trianglelefteq B$.

b. $\bar{x}\bar{A} = \bar{y}\bar{A}$

$\iff \overline{y^{-1}x} = \bar{y}^{-1}\bar{x} \in \bar{A}$

$\iff y^{-1}xN = aN$ for some $a \in A$

$\iff a^{-1}y^{-1}x \in N$

$\iff a^{-1}y^{-1}x = n$ for some $n \in N$

$\iff y^{-1}x = an$

$\iff y^{-1}x \in A$

c. $y^{-1}xN = aN$ for some $a \in N$

$\Rightarrow a^{-1}(y^{-1}x)N = N$

$\Rightarrow a^{-1}(y$

2. $bA \mapsto \bar{b}\bar{A}$

3. $$\overline{\langle A, B\rangle} = \pi\Big(\bigcap_{\substack{H \le G \\ A, B \subseteq H}} H\Big)$$

$$= \bigcap_{\substack{H \le G \\ A, B \subseteq H}} \pi(H)$$

$$= \bigcap_{\substack{H \le G \\ A, B \subseteq H}} \bar{H}$$

and $$\langle \bar{A}, \bar{B}\rangle = \bigcap_{\substack{\bar{H} \le G \\ \bar{A}, \bar{B} \subseteq \bar{H}}} \bar{H}$$

We have $A, B \subseteq H \Rightarrow \bar{A}, \bar{B} \le \bar{H}$

So

$$\bigcap_{\substack{H \le G \\ A, B \subseteq H}} \bar{H} = \bigcap_{\substack{\bar{H} \le G \\ \bar{A}, \bar{B} \subseteq \bar{H}}} \bar{H}$$

So $\overline{\langle A, B\rangle} = \langle \bar{A}, \bar{B}\rangle$