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Lecture 7, Exercise A

1a. In D_8, we have

$$r^4 = 1$$

So

$$\langle r \rangle = \{1, r, r^2, r^3\}$$

$|\langle r \rangle| = 4$ from above

We also know $|r| = 4$ in D_8

So

$$|\langle r \rangle| = |r|$$

b. $\langle r^2 \rangle = \{1, r^2\}$

$|\langle r^2 \rangle| = 2$

We have $(r^2) r^2 = r^4 = 1$ in D_8, so

$$|\langle r^2 \rangle| = 2 = |r^2|$$

c. We have,

$$r^3$$

$$(r^3)^2 = r^6 = r^2 r^4 = r^2$$

$$(r^3)^3 = r r^4 r^4 = r$$

$$(r^3)^4 = r^{12} = r^4 r^4 r^4 = 1$$

$$(r^3)^5 = (r^3)^4 r^3 = r^3$$

So,

$$\langle r^3 \rangle = \{1, r, r^2, r^3\}$$

$$= \langle r \rangle$$

d. $\langle s \rangle = \{1, s\}$

$$\langle sr \rangle = \{1, sr\}$$

$$(sr)^2 = sr(sr) = ss(r^{-1}r) = 1$$

$$\langle sr^2 \rangle = \{1, sr^2\}$$

$$(sr^2)^2 = sr^2(sr^2) = ss(r^2 r^2) = 1$$

$$\langle sr^3 \rangle = \{1, sr^3\}$$

2.

cycle type	generic element	cyclic groups generated
1, 1, 1, 1	1	1
2, 1, 1	$(i\ j)$	$\{1, (i, j)\}$
2, 2	$(i\ j)(k\ l)$	$\{1, (ij)(kl)\}$
3, 1	$(i\ j\ k)$	$\{1, (i\ j\ k), (k\ j\ i)\}$
4	$(i\ j\ k\ l)$	$\{1, (l\ i\ j\ k), (l\ k\ j\ i), (l\ j)(i\ k)\}$

3. Let $Z_2 = \langle x \rangle = \{1, x\}$ and

$$Z_4 = \langle y \rangle = \{1, y, y^2, y^3\}$$

So $Z_2 \times Z_4 = \{(1,1), (1,y), (1,y^2), (1,y^3), (x,1), (x,y), (x,y^2), (x,y^3)\}$

The cyclic subgroups are:

$\langle (1,1) \rangle$ $\quad$ $\langle (x,1) \rangle$

$\langle (1,y) \rangle$ $\quad$ $\langle (x,y) \rangle$

$\langle (1,y^2) \rangle$ $\quad$ $\langle (x,y^2) \rangle$