Chris Hayduk 4/22

Lecture 8, Exercise B

1. $\mathbb{Z}_2 = \{\bar{0}, \bar{1}\}$

$\mathbb{Z}_3 = \{\bar{0}, \bar{1}, \bar{2}\}$

$\mathbb{Z}_2 \times \mathbb{Z}_3 = \{ (\bar{0},\bar{0}), (\bar{0},\bar{1}), (\bar{0},\bar{2}), (\bar{1},\bar{0}), (\bar{1},\bar{1}), (\bar{1},\bar{2})\}$

$|(\bar{0},\bar{0})| = 1$

$|(\bar{0},\bar{1})| = 3$

$|(\bar{0},\bar{2})| = 3$

$|(\bar{1},\bar{0})| = 2$

$|(\bar{1},\bar{1})| = 6$

$|(\bar{1},\bar{2})| = 6$

$\mathbb{Z}_2 \times \mathbb{Z}_3 = \langle (\bar{1},\bar{1}) \rangle$

$\mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6$

2 $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{(\bar{0},\bar{0}), (\bar{0},\bar{1}), (\bar{1},\bar{0}), (\bar{1},\bar{1})\}$

$|(\bar{0},\bar{0})| = 1$

$|(\bar{0},\bar{1})| = 2$

$|(\bar{1},\bar{0})| = 2$

$|(\bar{1},\bar{1})| = 2$

$\langle(\bar{0},\bar{0})\rangle = \{(\bar{0},\bar{0})\}$

$\langle(\bar{0},\bar{1})\rangle = \{(\bar{0},\bar{0}), (\bar{0},\bar{1})\}$

$\langle(\bar{1},\bar{0})\rangle = \{(\bar{1},\bar{0}), (\bar{0},\bar{0})\}$

$\langle(\bar{1},\bar{1})\rangle = \{(\bar{0},\bar{0}), (\bar{1},\bar{1})\}$

So $\mathbb{Z}_2 \times \mathbb{Z}_2$ not cyclic.

3 Largest order for an element in $\mathbb{Z}_8 \times \mathbb{Z}_{12}$ is

$$\frac{8 \cdot 12}{\gcd(8,12)} = \frac{8 \cdot 12}{4} = 24$$

$|\langle(1,1)\rangle| = 24$