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Lecture 2, Ex. B

1. $\phi(v_1) = (1, 1, 2)$

$= w_1 + w_2 + 2w_3$

$\phi(v_2) = 3w_2$

$$M_\beta^c(\phi) = \begin{bmatrix} 1 & 0 \\ 1 & 3 \\ 2 & 0 \end{bmatrix}$$

2. $\phi(v_1) = (1, 1, 2)$

$= v_1 + w_2 + 2w_3$

$\phi(v_2) = (0, 3, 0)$

$= 3w_1$

$$M_\beta^c(\phi) = \begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$$

3. Note that I_n has 1s in $I_n(i,i)$ for $1 \le i \le n$ and 0 everywhere else.

So column corresponding to b will have 1 in row with b and 0s elsewhere

4a. $v_1 = \frac{1}{2} v_1' + \frac{1}{2} v_2'$, $v_2 = -\frac{1}{2} v_1' + \frac{1}{2} v_2'$

$$P = M_B^{B'}(1) = \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$v_1' = v_1 - v_2$, $v_2' = v_1 + v_2$

$$Q = M_{B'}^{B}(1) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

b. $$PQ = \begin{pmatrix} 1/2 + 1/2 & 1/2 - 1/2 \\ 1/2 - 1/2 & 1/2 + 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

$$QP = \begin{pmatrix} 1/2 + 1/2 & -1/2 + 1/2 \\ -1/2 + 1/2 & 1/2 + 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

c. $A = M_B^B(\varphi) = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$

$B = M_{B'}^{B'}(\varphi) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$QBP = \begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}$

$= \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} = A$

$PAQ = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = B$