Chris Hoyduk 2/23

Lec 7, Ex. A

1. Vec spaces are additive

$\rho: A \to End(V) \Rightarrow a \cdot v = \rho(a) v \in V$

$a \cdot (b \cdot v) = \rho(a)(\rho(b) v)$

$= (\rho(a)\rho(b)) v$

$= \rho(ab) v$

$= (ab) \cdot v$ ✓

$(a+b) \cdot v = \rho(a+b) \cdot v$

$= (\rho(a) + \rho(b)) \cdot v$

$= \rho(a) v + \rho(b) v$

$= a \cdot v + b \cdot v$ ✓

$1 \cdot v = \rho(1) v = id\ v = v$ ✓

$a \cdot (u+v) = \rho(a)(u+v)$

$= \rho(a) u + \rho(a) v$

$= a \cdot u + a \cdot v$ ✓

2a. L subgroup of M and closed under group action

b. ... φ preserves additive structure and group action

c. ... additive group of cosets

... $r \cdot (m+L) = (r \cdot m) + L$

... L subgroup of L and $RL \subset L$

d. ... contains no non-trivial proper submodules

3. $r = -1 \Rightarrow x - y \in N \quad \forall x, y \in N$

$x = 0 \Rightarrow ry \in N \quad \forall r \in R, y \in N$

4a.