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Lec 15, Ex. A

1a. $U = \{(v,v) \mid v \in V\}$

$V \neq \emptyset$ so $U \neq \emptyset$

$x, y \in U$, $a \in A$

ie. $x = (v_1, v_1)$ $y = (v_2, v_2)$

$x + ay = (v_1, v_1) + a(v_2, v_2)$

$= (v_1, v_1) + (av_2, av_2)$

$= (v_1 + av_2, v_1 + av_2)$

$v_1 + av_2 \in V$, so $x + ay \in U$

$\Rightarrow$ U submodule of M

Define $\varphi: U \to V$

$\varphi: (v,v) \mapsto v$

Then $\varphi(x+y) = \varphi((v_1+v_2, v_1+v_2))$

$= v_1 + v_2$

$= \varphi(x) + \varphi(y)$ ✓

$\varphi(ax) = \varphi((av_1, av_1)) = av_1 = a\varphi(x)$ ✓

So $U \cong V$

b. $(0,0) = (y \cdot 0, z \cdot 0) \in U \Rightarrow U \neq \emptyset$

$x_1 \in U$, $x_2 \in U$, $a \in A$

$x_1 = (yu_1, zu_1)$ $x_2 = (yu_2, zu_2)$

$$x_1 + ax_2 = (yu_1, zu_1) + (ayu_2, azu_2)$$
$$= (yu_1 + ayu_2, zu_1 + azu_2)$$
$$= (y(u_1 + au_2), z(u_1 + au_2))$$

$au_2 \in V$ so,

$x_1 + ax_2 \in U$

$\Rightarrow U$ submodule of M

Define $\phi : U \to V$, $\phi : (yv, zv) \mapsto v$

20. $T_1 = \{(\alpha, 0, 0) \mid \alpha \in \mathbb{C}\}$

$T_2 = \{(0, \alpha, 0) \mid \alpha \in \mathbb{C}\}$

$S_1 = \{(0, 0, \alpha) \mid \alpha \in \mathbb{C}\}$