Chris Haydock 2/7/21

Lecture 3, Ex. A

1. $p(x) = \left(\sum_{i=1}^{n} a_i v_i^*\right)\left(\sum_{j=1}^{n} \beta_j v_j\right)$

$= \sum_{i=1}^{n} a_i \left(v_i^*\left(\sum_{j=1}^{n} \beta_j v_j\right)\right)$

$= \sum_{i=1}^{n} a_i \left(\sum_{j=1}^{n} \beta_j v_i^*(v_j)\right)$

$= \sum_{i=1}^{n} a_i \left(\sum_{j=1}^{n} \beta_j \delta_{i,j}\right)$ $\quad$ $=1$ when $j=i$, $=0$ otherwise

$= \sum_{i=1}^{n} a_i \beta_i$

2. $(a, b, c) \in V$, $\quad x, y, z \in Q$

$(a,b,c) = x v_1 + y v_2 + z v_3$

$= (x, 0, 0) + (y, -y, 0) + (2z, 0, z)$

$= (x+y+2z, -y, z)$

So,

$$\left.\begin{aligned} a &= x+y+2z \\ b &= -y \\ c &= z \end{aligned}\right\} \quad \begin{aligned} x &= a+b-2c \\ y &= -b \\ z &= c \end{aligned}$$

So $(a,b,c) = (a+2b-2c)v_1 - b v_2 + c v_3$

$v_1^*: \mathbb{Q}^3 \to \mathbb{Q}$

$(a,b,c) \mapsto a + 2b - 2c$

$v_2^*: \mathbb{Q}^3 \to \mathbb{Q}$

$(a,b,c) \mapsto -b$

$v_3^*: \mathbb{Q}^3 \to \mathbb{Q}$

$(a,b,c) \mapsto c$

3. Let $v_i \mapsto 1$ $\forall i$

Need to map infinitely many v_i to something non-zero.