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Lec 14 - Ex A

1 __Free module__ - module which admits a basis, or the 0 module

- Same cardinality of basis $\Rightarrow$ isomorphic
- $M/aM \cong \bigoplus_{i \in I} Ax_i / ax_i$

__Short exact sequence__ - exact sequence of the form:

$$0 \to X \xrightarrow{\varphi} Y \xrightarrow{\psi} Z \to 0$$

- Sequence $X' \xrightarrow{\lambda} X \to X'' \to 0$ is exact iff

$$\mathrm{Hom}_A(X', Y) \leftarrow \mathrm{Hom}_A(X, Y) \leftarrow \mathrm{Hom}_A(X'', Y) \leftarrow 0$$

exact $\forall Y$.

- Sequence $0 \to Y' \to Y \to Y''$ exact iff

$$0 \to \mathrm{Hom}_A(X, Y') \to \mathrm{Hom}_A(X, Y) \to \mathrm{Hom}_A(X, Y'')$$

is exact $\forall X$

Split short exact sequence - A ring, X, Y A-modules,

$$0 \to X \xrightarrow{x \mapsto (x,0)} X \oplus Y \xrightarrow{(x,y) \mapsto y} Y \xrightarrow{0} 0$$

2a. Module is direct summand of itself

b. Left-reg module of $A \Rightarrow$ free module

3.

4a. T - trivial module

Projective but not free

b P_1 and P_2 are direct summands of left regular module $\Rightarrow$ free

But both not free

5a. Free $\mathbb{Z}$-module $\Rightarrow Fr(B) \cong \bigoplus_{b \in B} \mathbb{Z}$

$\Rightarrow$ no non-zero elements of finite order

G has at least 1 such element

$\Rightarrow$ G not direct summand

b $Hom_{\mathbb{Z}}(\mathbb{Z}/n\mathbb{Z})$ not exact b/c induced π not surjective