Chris Haydak 2/2/21

Lecture 1, Ex. C

If $\beta_b = 0$, then

$$\sum_{\substack{\gamma \in A' \\ \gamma \neq b}} \beta_\gamma \gamma = 0$$

Note that $A' - \{b\} = A - \{a\}$.

Since A is linearly independent, we have that $A - \{a\}$ (and therefore $A' - \{b\}$ is linearly independent as well.

Hence,

$$\sum_{\substack{\gamma \in A' \\ \gamma \neq b}} \beta_\gamma \gamma = 0 \implies \beta_\gamma = 0 \quad \forall \gamma \in V$$

Otherwise if $\beta_b \neq 0$,

$$0 = \beta_b + \sum_{\substack{\gamma \in A' \\ \gamma \neq b}} \beta_\gamma \gamma$$

$$\implies 0 = \sum_{x \in A} \alpha_x x + \sum_{\substack{\gamma \in A' \\ \gamma \neq b}} \beta_\gamma \gamma$$

$$= \alpha_a a + \sum_{\substack{\gamma \in A' \\ \gamma \neq b}} (\alpha_\gamma + \beta_\gamma) \gamma$$

$$\Rightarrow 0 = \sum_{\gamma \in A} (\alpha_\gamma + \beta_\gamma)\gamma \quad , \quad w/ \ \beta_a = 0$$

Since A is a basis, (thus linearly independent) the above implies

$$\alpha_a + \beta_a = \alpha_a = 0$$

But we chose a s.t. $\alpha_a \neq 0$, a contradiction.

Therefore A' is linearly independent and hence a basis.