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Lec 15 - Ex B

1. $1_X: X \rightarrow X$ $f_X: X \rightarrow X$

2. Directed graphs

- Objects: vertices of graph

- Morphisms: paths in graph

Rings

- Objects: rings

- Morphisms: ring homomorphisms

- Top

- Objects: topological spaces

- Morphisms: continuous functions

3a. Group mult associative \Rightarrow (i)

1 object \Rightarrow (iii)

$1_X = 1_G$, group identity

b. Transitivity $\Rightarrow x \leq y$ and $y \leq z \Rightarrow x \leq z$

Map well-defined

$$\begin{aligned} (w \rightarrow x) \circ (x \rightarrow y \circ y \rightarrow z) &= (w \rightarrow x) \circ (y \rightarrow z) \\ &= (w \rightarrow z) \end{aligned}$$

and

$$\begin{aligned} (w \rightarrow x \circ x \rightarrow y) \circ y \rightarrow z &= (w \rightarrow y) \circ y \rightarrow z \\ &= (w \rightarrow z) \end{aligned}$$

so assoc. holds

4. One object in \mathcal{G}

By axiom of group actions, identity morphism $1_x = 1_G$ mapped to identity map which is identity morphism $1_{f(x)}$ in Set .

Again by axioms of group actions,

$$\begin{aligned} f(gh)(x) &= (gh) \cdot x = g \cdot (h \cdot x) \\ &= (f(g) \circ f(h))(x) \end{aligned}$$