Chris Haydak 2/8

Lecture 3, Ex. C

1. $A, B \in M_n(F)$, $A = (\alpha_{ij})$, $B = (\beta_{ij})$

$$tr(A+B) = \sum_{i=1}^{n} (\alpha_{ii} + \beta_{ii})$$
$$= \sum_{i=1}^{n} \alpha_{ii} + \sum_{i=1}^{n} \beta_{ii}$$
$$= tr(A) + tr(B) \quad \checkmark$$

$\alpha \in F$,
$$tr(\alpha A) = \sum_{i=1}^{n} (\alpha\, \alpha_{ii})$$
$$= \alpha \sum_{i=1}^{n} \alpha_{ii}$$
$$= \alpha\, tr(A) \quad \checkmark$$

2a. tr encoded as matrix in $M_{1,n^2}(F)$ because

$$\dim(M_n(F)) = n^2$$
$$\dim(F) = 1$$

b. $tr(v_1) = 1$, $tr(v_2) = 0$, $tr(v_3) = 0$, $tr(v_4) = 1$

$\Rightarrow M_{\mathcal{E}}^{F}(tr) = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}$

c. $\begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{pmatrix} = \alpha_{1,1} E_{1,1} + \alpha_{1,2} E_{1,2} + \alpha_{2,1} E_{2,1} + \alpha_{2,2} E_{2,2}$

$$tr(A) = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{1,1} \\ \alpha_{1,2} \\ \alpha_{2,1} \\ \alpha_{2,2} \end{pmatrix}$$

$$= \alpha_{1,1} + \alpha_{2,2}$$

3. $tr(M_B^B(\varphi)) = tr\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} = 2$

$tr(M_{B'}^{B'}(\varphi)) = tr\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = 2$

$\Rightarrow 2 = tr(\varphi)$

$tr(M_{B'}^{B}(\varphi)) = tr\begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix} = 0 \neq 2$