Chris Haydutk 2/27

Lec 8, Ex A

1. $s_1 = e_1 - e_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$s_2 = e_2 - e_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

1 element matrix

$\mathbb{1} = (1)$

$\rho(s_1) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $\rho(s_2) = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$

$\mathbb{1} \oplus \rho(s_1) = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \rho(s_1) \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\mathbb{1} \oplus \rho(s_2) = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \rho(s_2) \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -1/2 \end{pmatrix}$

$$\varphi(\mathbb{1} \oplus \rho)(s_1)\varphi^{-1}$$

$$= \begin{pmatrix} 1 & \sqrt{3} & 1 \\ 1 & -\sqrt{3} & 1 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2\sqrt{3} & -1/2\sqrt{3} & 0 \\ 1/6 & 1/6 & -1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\varphi(\mathbb{1} \oplus \rho)(s_2)\varphi^{-1}$$

$$= \begin{pmatrix} 1 & \sqrt{3} & 1 \\ 1 & -\sqrt{3} & 1 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -1/2 \end{pmatrix} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2\sqrt{3} & -1/2\sqrt{3} & 0 \\ 1/6 & 1/6 & -1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Note $\pi(s_1) = \pi(e_1) - \pi(e_2)$

$=$

2. Consider $\rho(s_1)$

weight vectors:

$$\alpha e_1 , \alpha e_2$$

Neither work for s_2

3a. $|\mathbb{F}|^3 = 3^3 = 27 = |V|$

b. $\forall \sigma \in S_3, \sigma v = v$

$\Rightarrow \mathbb{F}_3 v$ submodule

(a)

(b) $v_1 - 1w_2 = (1,-1,0) - (0,1,-1)$

$= (1,-2,1)$

$\equiv_3 (1,1,1) = v$

(c)

4a. Let ϕ be the identity map

b. If $\rho_1 \cong \rho_2$, $\exists$ isomorphism $\phi : V \to V$
s.t.

$$\rho_2(a) = \phi \rho_1(a) \phi^{-1} \quad \forall a$$

$$\Rightarrow \phi^{-1} \rho_2(a) \phi = \rho_1(a) \quad \forall a$$

ϕ^{-1}, ϕ are both isomorphisms, so

$$\rho_2 \cong \rho_1$$

c. Let $\rho_1 \cong \rho_2$ and $\rho_2 \cong \rho_3$

Then $\exists$ isomorphisms $\varphi, \psi : V \to V$
s.t. $\forall a$,

$$\varphi \rho_1(a) \varphi^{-1} = \rho_2(a)$$

$$\psi \rho_2(a) \psi^{-1} = \rho_3(a)$$

$$\Rightarrow \quad \psi(\varphi \rho_1(a) \varphi^{-1}) \psi^{-1} = \rho_3(a)$$

$$\Rightarrow \quad (\psi \varphi) \rho_1(a) (\varphi^{-1} \psi^{-1}) = \rho_3(a)$$

$$\Rightarrow \quad (\psi \varphi) \rho_1(a) (\psi \varphi)^{-1} = \rho_3(a)$$