Expository Paper Outline: Lie Algebra - Representations of $sl(2;\mathbb{C})$

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Abstract

Abstract goes here

1 Lie Groups

1.1 Introduction

General description of Lie Groups:

1. [4]
2. [5, Section 1.1]
3. [5, Section 1.6]

Basic definitions of Lie Groups:

1. [4]
2. [2, Section 1.1]
3. [2, Section 1.5]

1.2 Matrix Lie Groups

Matrix lie group examples: [2, Section 1.2]

Exponential of a matrix:

1. [2, Section 2.1]
2. [2, Section 2.2] - Example 2.5

2 Lie Algebras

2.1 Introduction

Overview of Lie Algebras: [3]

Motivation for Lie Algebras: [1, Section 8.1]

2.2 Basic Definitions

Definitions and examples:

1. [1, Section 8.1]
2. [2, Section 3.1]

2.3 Lie Algebra of a Matrix Lie Group

Definitions and theorem: [2, Section 3.3]

Examples: [2, Section 3.4]

2.4 The Exponential Map

Define exponential map: [2, Section 3.7]

State theorems regarding exponential map: [2, Section 3.7]

Compute the exponential map onto SO(2): [5, Section 4.1]

2.5 Classification of Lie Algebras

[1, Chapter 9]

3 Representations of $sl(2;\mathbb{C})$

Discuss complexification of $su(2)$ and that $su(2)$ is isomorphic to $so(3)$: [2, Section 4.6]

Discuss physical significance of the representations:

1. [2, Section 4.6]
2. [4, Section 5]
3. [3, Section 4.4]

Basis of $sl(2;\mathbb{C})$: [2, Section 4.6]

Characterize irreducible representations: [1, Section 11.1]

4 Conclusion

Summarize key ideas here.

References

[1] Fulton, William, and Joe Harris. *Representation Theory: A First Course*. Springer, 2004.

[2] Hall, Brian C. *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*. Springer, 2016.

[3] "Lie Algebra." *Wikipedia*, Wikimedia Foundation, 16 Apr. 2021, en.wikipedia.org/wiki/Lie_algebra.

[4] "Lie Group." *Wikipedia*, Wikimedia Foundation, 21 Apr. 2021, en.wikipedia.org/wiki/Lie_algebra.

[5] Stillwell, John. *Naive Lie Theory*. Springer, 2012.