3/23

Lec 14 – Ex. B

1. $\exists$ ses of the form:

$$0 \to Q \xrightarrow{f} Y \xrightarrow{g} Z \to 0$$

Z is projective. Hence Q injective.

$\exists$ SES of form:

$$0 \to X \xrightarrow{f} Y \xrightarrow{g} P \to 0$$

so X injective. Thus P projective.

2. $\alpha F = F \quad \forall \alpha \in F - \{0\} \Rightarrow F$ divisible

$\Rightarrow F$ injective F-module

3a. $$a(x)\,\mathbb{Q}(x) = a(x)\left(\frac{1}{a(x)}\,\mathbb{Q}(x)\right)$$

$$= (a(x)/a(x))\,\mathbb{Q}(x)$$

$$= \mathbb{Q}(x)$$

$\Rightarrow M = \mathbb{Q}(x)$ is injective.

b. $x \in A$ acts on $m \in \mathbb{Q}$ by

$$x \cdot m = x \frac{1}{x} = 0_m$$
$$= 0_m$$
$$= 0$$

So $x \cdot \mathbb{Q} = 0 \neq \mathbb{Q}$

4a.

b i) $\bar{0} = f(2) = \Phi(i(2))$
$$= \Phi(2) = 2\Phi(1)$$

Similarly,

$$\overline{12} = f(x) = \Phi(i(x))$$
$$= \Phi(x)$$
$$= x\,\Phi(1)$$

ii) $\Phi(1) = \alpha + \mathbb{Z}[x]$ for $\alpha \in \mathbb{Q}(x)$. Then

$$2\,\Phi(1) = \overline{0}$$

means $2\alpha \in \mathbb{Z}[x]$

$\Rightarrow$ α polynomial

$\Rightarrow$ $x\,\Phi(1) = \overline{1/2}$

$\Rightarrow$ $x\alpha - \frac{1}{2} \in \mathbb{Z}[x]$

$\Rightarrow$ $x\alpha$ doesn't have constant term

Thus $x\alpha - \frac{1}{2}$ polynomial w/ constant term $-\frac{1}{2}$ $\Rightarrow$ $x\alpha - \frac{1}{2} \notin \mathbb{Z}[x]$

$\Rightarrow$ no such $\alpha \in \mathbb{Q}(x)$