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Loc. 12, Ex. A

$$1a. (12) \cdot \bar{v}_2 = (12) \cdot (v_2 + T) = v_1 + T \\ = \bar{v}_1$$

$$(23) \cdot \bar{v}_1 = (23) \cdot (v_1 + T) = v_1 + T \\ = \bar{v}_1$$

$$b. (12) \mapsto \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(23) \mapsto \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$$

$$c. p((123)) = p((12)) p((23)) \\ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \\ = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

$$(123) \cdot (\alpha \bar{v}_1 + \beta \bar{v}_2) = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= -\beta \bar{v}_1 + (\alpha - \beta) \bar{v}_2$$

$$\varphi(3 - (12) + 5(123)) = 3\varphi(1) - \varphi((12)) + 5\varphi((123))$$

$$= 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + 5 \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -6 \\ 4 & -2 \end{pmatrix}$$

$$(3 - (12) + 5(123)) \cdot (\alpha \bar{v}_1 + \beta \bar{v}_2)$$

$$= \begin{pmatrix} 3 & -6 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= (3\alpha - 6\beta) \bar{v}_1 + (4\alpha - 2\beta) \bar{v}_2$$

$$d. \quad S_1 = P(12) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad S_2 = P(123) = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$$

$$2a. \quad \det(P(S_1 - \lambda)) = \det \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} \\ = \lambda^2 - 1$$

$$\Rightarrow \lambda_1 = 1, \lambda_2 = -1$$

$$V_1 = \ker \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} = \mathbb{F}_3(\bar{v}_1 + \bar{v}_2)$$

$$V_{-1} = \ker \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \mathbb{F}_3(\bar{v}_1 - \bar{v}_2)$$

b.

From (a),

$$\begin{aligned}\phi(s_2) u_1 &= \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ -1 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\phi(s_2) u_2 &= \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ -1 \end{pmatrix}\end{aligned}$$

c.

$$3. \quad s_1 \cdot u = s_2 \cdot u = -u \Rightarrow S \text{ is sign module}$$

$$4. \quad \bar{P}/S = \{u + S \mid u \in \bar{P}\}$$

5.