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Lecture 20 Ex A

1. They do not because they are not the identities of M_1, M_2, M_3
   - We have not - we only checked that there was an isomorphism

2. .5 Conjugacy Classes
   - $z_1 + \cdots + z_5 = 1$

3a. $F_{ij}^2 = \delta_{ij} F_{ij} \Rightarrow F_{ij}$ idempotent iff $i=j$

b. $F_3 = F_{11} + F_{22} + F_{33}$

$F_{11}(F_{12} + F_{33}) = 0$

(a) $a(a+b) = a^2 + ab = a$

$(a+b)a = a^2 + ba = a$

$b(a+b) = ba + b^2 = b$

$(a+b) = ab + b^2 = b$

a $F_{11} = a$

$F_{11}\, a = a$

$b F_{11} = b$

$F_{11}\, b = b$

$\Rightarrow a_{ij} = 0 \quad \forall i \neq j, \quad a_{ij} = 1 \quad , i = j$

$b_{ij} = 0 \quad \forall i \neq j, \quad b_{ij} = 1 \quad , i = j$

$a_{11}^2 = a_{11} \quad b_{11}^2 = b_{11}$

$\Rightarrow a_{11}\ b_{11}$ idempotents in $\mathbb{C}$

$\Rightarrow a_{11}, b_{11} \in \{0, 1\}$

$a+b = E_{11} \Rightarrow a_{11} + b_{11} = 1$

$\Rightarrow a_{11} = 0, b_{11} = 1$ or $a_{11} = 1, b_{11} = 0$

So either $a = 0$ or $b = 0$

4a. $Z(\mathbb{C}) = \mathbb{C}$ so $Z(M_3(\mathbb{C})) = \mathbb{C} I_3$

$a, b \in Z(A) \Rightarrow a = \alpha I_3, \; b = \beta I_3$

$a^2 = a, \; b^2 = b \Rightarrow \alpha, \beta \in \{0, 1\}$

$a + b = I_3 \Rightarrow \alpha + \beta = 1$

$\Rightarrow \alpha = 0, \; \beta = 1$

or $\alpha = 1, \beta = 0$

$\Rightarrow a = 0$ or $b = 0$

$\Rightarrow I_3$ prim. central

b: $\mathbb{C} I_3 \times \mathbb{C} I_3$