Chris Haydak 2/16

Lecture 5, Ex. A

1. $B^{(i)} = \{v_1^{(i)}, \dots, v_{d_i}^{(i)}\}$ of V_i, $i = 1, \dots, n$

$B = \{v_1^{(1)}, \dots, v_{d_1}^{(1)}, \dots, v_1^{(n)}, \dots, v_{d_n}^{(n)}\}$

$A = \sum_{i=1}^{n} \sum_{j=1}^{d_i} a_j^{(i)} v_j^{(i)} \mapsto (a_1^{(1)}, \dots, a_{d_1}^{(1)}, a_1^{(2)}, \dots, a_{d_2}^{(2)}, \dots, a_1^{(n)}, \dots, a_{d_n}^{(n)})$

2. $b_2 = (e_2^{(1)}, 0)$ $\mapsto 2b_1 - 3b_2$

$$\begin{pmatrix} 1 & 2 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = ((2, 3), (0, 0, 0))$$

$b_3 = (0, e_1^{(2)})$

$$\begin{pmatrix} 0 & -7 & 5 \\ 1 & 0 & 4 \\ 0 & 2 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \\ 2 \end{pmatrix}$$

$((0, 0), (-7, 0, 2)) = -7b_3 + 2b_5$

$b_5 = (0, e_3^{(3)})$

$$\begin{pmatrix} 0 & -7 & 5 \\ 1 & 0 & 4 \\ 0 & 2 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix}$$

$((0,0), (5, 4, 2)) = 5b_3 + 4b_4 + 2b_5$

3a $(\varphi \oplus \psi)((u,v)) = (\varphi(u), \psi(v)) \in W \oplus X$

b. $(\varphi \oplus \psi)(\alpha(u,v)) = (\varphi \oplus \psi)((\alpha u, \alpha v))$

$= (\varphi(\alpha u), \psi(\alpha v))$

$= (\alpha\varphi(u), \alpha\psi(v))$

$= \alpha(\varphi(u), \psi(v))$

$= \alpha(\varphi \oplus \psi)((u,v))$

c. $(\phi \oplus \psi)((u,v) + (u',v'))$

$$= (\phi \oplus \psi)((u+u', v+v'))$$

$$= \phi(u+u') + \psi(v+v')$$

$$= \phi(u) + \phi(u') + \psi(v) + \psi(v')$$

$$= \phi(u) + \psi(v) + \phi(u') + \psi(v')$$

$$= (\phi \oplus \psi)(u,v) + (\phi \oplus \psi)(u',v')$$

4a. matches prob 2

b.i) $(\phi \oplus \psi)(e_i^{(1)}) = (\phi(a_i), 0)$

$$\in F\{f_i^{(1)} \mid i = 1, \ldots, m\}$$

ii. $(\phi \oplus \psi)(e_i^{(1)}) = (\phi(a_i), 0)$

$$= \left(\sum_{j=1}^{m} \alpha_{j,i} c_j, 0\right)$$

$$= \sum_{j=1}^{m} \alpha_{j,i} (c_j, 0)$$

$$= \sum_{j=1}^{n} \alpha_{j,i} f_j^{(1)}$$

iii) $\phi(b_i) = \sum_{j=1}^{n} \beta_{j,i} d_j$

$$(\phi \oplus \psi)(e_i^{(2)}) = (0, \psi(b_i))$$
$$= (0, \sum_{j=1}^{n} \beta_{j,i} d_j)$$
$$= \sum_{j=1}^{n} \beta_{j,i} f_j^{(2)}$$