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Lecture 1, Ex. A

1. $\alpha = (12) - 2(23) + 5(13)$

$\beta = (12) + (23) - (13)$

$2\beta = 2(12) + 2(23) - 2(13)$

$\alpha + 2\beta = 3(12) + 3(13)$

$\alpha\beta = \big((12) - 2(23) + 5(13)\big) \cdot \big((12) + (23) - (13)\big)$

$= (12)^2 + (12)(23) - (12)(13) - 2(23)(12) - 2(23)^2 - 2(23)(13) + 5(13)(12) + 5(13)(23) - 5(13)^2$

$= \mathbb{1} + (132) - (123) - 2(231) - 2\mathbb{1} - 2(213) + 5(132) + 5(123) - 5\mathbb{1}$

$= 6(132) + 4(123) - 2(231) - 2(213) - 6\mathbb{1}$

2a. $r \in R^\times$, $g \in G$ and let $r' \in R$, $g' \in G$

$(rg)(r'g') = (rr')(gg')$

$= r(gg')$

6. $a = 1+x$, $b = -1+x-x^2$

$ab = (1+x)(-1+x-x^2)$

$= -1+x-x^2-x+x^2-1$

$= -2$

$= 1$

3. $a = 1-g$

$a \cdot (1+g+\dots+g^{n-1})$

$= (1+g+\dots+g^{n-1}) + (-g-g^2-\dots-g^n)$

$= 1-g^n = 1-1 = 0$

So G finite $\Rightarrow$ RG has at least one zero divisor

$\Rightarrow$ RG integral domain

4. $\left(\frac{1}{2}(1+t)\right)^2 = \left(\frac{1}{2}+\frac{1}{2}t\right)\cdot\left(\frac{1}{2}+\frac{1}{2}t\right)$

$= \frac{1}{4} + \frac{1}{4}t + \frac{1}{4}t + \frac{1}{4}t^2$

$= \frac{1}{4} + \frac{2}{4}t + \frac{1}{4}$

$= \frac{1}{2} + \frac{1}{2}t$

$= \frac{1}{2}(1+t) = e_+$

$$\left(\frac{1}{2}(1-t)\right)^2 = \left(\frac{1}{2} - \frac{1}{2}t\right) \cdot \left(\frac{1}{2} - \frac{1}{2}t\right)$$

$$= \frac{1}{4} - \frac{1}{4}t - \frac{1}{4}t + \frac{1}{4}t^2$$

$$= \frac{1}{2} - \frac{1}{2}t$$

$$= \frac{1}{2}(1-t) = e-$$