Random Walks and Electric Networks: Exercise 1.3.7 Numerical Verification

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March 4, 2021

Define the transition probability matrix p as

```
##           a         b         c         d         e         f         g
## a 1.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## b 0.0000000 1.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## c 0.3333333 0.3333333 0.0000000 0.0000000 0.3333333 0.0000000 0.0000000
## d 0.3333333 0.3333333 0.0000000 0.0000000 0.0000000 0.3333333 0.0000000
## e 0.0000000 0.0000000 0.3333333 0.0000000 0.0000000 0.0000000 0.3333333
## f 0.0000000 0.0000000 0.0000000 0.3333333 0.0000000 0.0000000 0.3333333
## g 0.3333333 0.0000000 0.0000000 0.0000000 0.3333333 0.3333333 0.0000000
## h 0.0000000 0.3333333 0.0000000 0.0000000 0.3333333 0.3333333 0.0000000
##           h
## a 0.0000000
## b 0.0000000
## c 0.0000000
## d 0.0000000
## e 0.3333333
## f 0.3333333
## g 0.0000000
## h 0.0000000
```

Then the probabilities of reaching a or b first from each state are given by the following matrix,

```
##           a         b c d e f g h
## a 1.0000000 0.0000000 0 0 0 0 0 0
## b 0.0000000 1.0000000 0 0 0 0 0 0
## c 0.5000000 0.5000000 0 0 0 0 0 0
## d 0.5000000 0.5000000 0 0 0 0 0 0
## e 0.5000000 0.5000000 0 0 0 0 0 0
## f 0.5000000 0.5000000 0 0 0 0 0 0
## g 0.6666667 0.3333333 0 0 0 0 0 0
## h 0.3333333 0.6666667 0 0 0 0 0 0
```

where we have raised p to the power $10,000$. Since a has a $1/3$ chance of transitioning to c, d and g, we can write the probability that the chain will hit b before returning to a as $1/3 \cdot 0.5 + 1/3 \cdot 0.5 + 1/3 \cdot 0.33 \approx 0.44$