Real Analysis I: Assignment 1

Chris Hayduk

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Problem 1.

Suppose $f : X \to Y$ is onto.

Let $B \subset Y$ be nonempty. Thus, $\exists b \in B$.

Suppose $f^{-1}(B) = \emptyset$. This implies that there is no $x \in X$ such that $f(x) = b$.

However, by the definition of onto, $\forall y \in Y$, $\exists x \in X$ such that $f(x) = y$. Since $b \in B \subset Y$ and f is assumed to be onto, this definition applies.

Thus, we have a contradiction and $f^{-1}(B) \neq \emptyset$.

Problem 2.

Let $\mathscr{A}$ be a collection of sets and assume properties (ii) and (iii) from the question.

Let $A, B \in \mathscr{A}$. By property (ii), $\tilde{A}, \tilde{B} \in \mathscr{A}$ as well.

Thus, by property (iii),

$$\tilde{A} \cap \tilde{B} \in \mathscr{A}$$

Then, by (ii) and DeMorgan's Laws, we have,

$$\begin{aligned} &\widetilde{\tilde{A} \cap \tilde{B}} \in \mathscr{A} \\ \implies &\tilde{\tilde{A}} \cup \tilde{\tilde{B}} \in \mathscr{A} \\ \implies &A \cup B \in \mathscr{A} \end{aligned}$$

Thus, by properties (ii) and (iii), whenever $A, B \in \mathscr{A}$, $A \cup B \in \mathscr{A}$ as well.

Problem 3.