Real Analysis I: Proof from Notes

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Problem 1. Prove the number of equivalence classes for the relation $x \sim y$ if $x - y \in \mathbb{Q}$.

Suppose $x, y \in \mathbb{Q} \cap [0, 1)$. Then $\exists a, b, m, n \in \mathbb{Z}$ such that,

$$x = \frac{a}{b}$$
$$y = \frac{m}{n}$$

Thus, we have,

$$
\begin{aligned}
x - y &= \frac{a}{b} - \frac{m}{n} \\
&= \frac{an}{bn} - \frac{mb}{bn} \\
&= \frac{an - mb}{bn}
\end{aligned}
$$

We know that the integers are closed under multiplication and addition, so $(an - mb), bn \in \mathbb{Z}$. Thus, $x - y \in \mathbb{Q}$.

Now suppose $x \in \mathbb{Q} \cap [0, 1)$ and $y \in \mathbb{I} \cap [0, 1)$.