Statistical Rethinking: Chapter 4 - Linear Models

Chris Hayduk

January 2, 2019

1 Easy

Problem 4E1.
In the model definition below, which line is the likelihood?

1. $\mathrm{y_i} \sim \mathrm{Normal}(\mu,\, \sigma)$
2. $\mu \sim \mathrm{Normal}(0,\, 10)$
3. $\sigma \sim \mathrm{Uniform}(0,\, 10)$

Line 1 represents the likelihood.

Problem 4E2.
In the model definition just above, how many parameters are in the posterior distribution?

There are two parameters: μ and σ.

Problem 4E3.
Using the model definition above, write down the appropriate form of Bayes' theorem that includes the proper likelihood and priors.

$$\mathrm{Pr}(\mu,\, \sigma|\mathrm{y}) = \frac{\Pi_\mathrm{i}\mathrm{Normal}(\mathrm{y_i}|\mu,\, \sigma)\ \mathrm{Normal}(\mu|0,\, 10)\ \mathrm{Uniform}(\sigma|0,\, 10)}{\int\int\Pi_\mathrm{i}\mathrm{Normal}(\mathrm{y_i}|\mu,\, \sigma)\ \mathrm{Normal}(\mu|0,\, 10)\ \mathrm{Uniform}(\sigma|0,\, 10)\ d\mu d\sigma}$$

Problem 4E4.
In the model definition below, which line is the linear model?

1. $\mathrm{y_i} \sim \mathrm{Normal}(\mu,\, \sigma)$
2. $\mu_\mathrm{i} = \alpha + \beta \mathrm{x_i}$
3. $\alpha \sim \mathrm{Normal}(0,\, 10)$
4. $\beta \sim \mathrm{Normal}(0,\, 10)$
5. $\sigma \sim \mathrm{Uniform}(0,\, 10)$

Line 2 represents the linear model.

Problem 4E5.
In the model definition just above, how many parameters are in the posterior distribution?

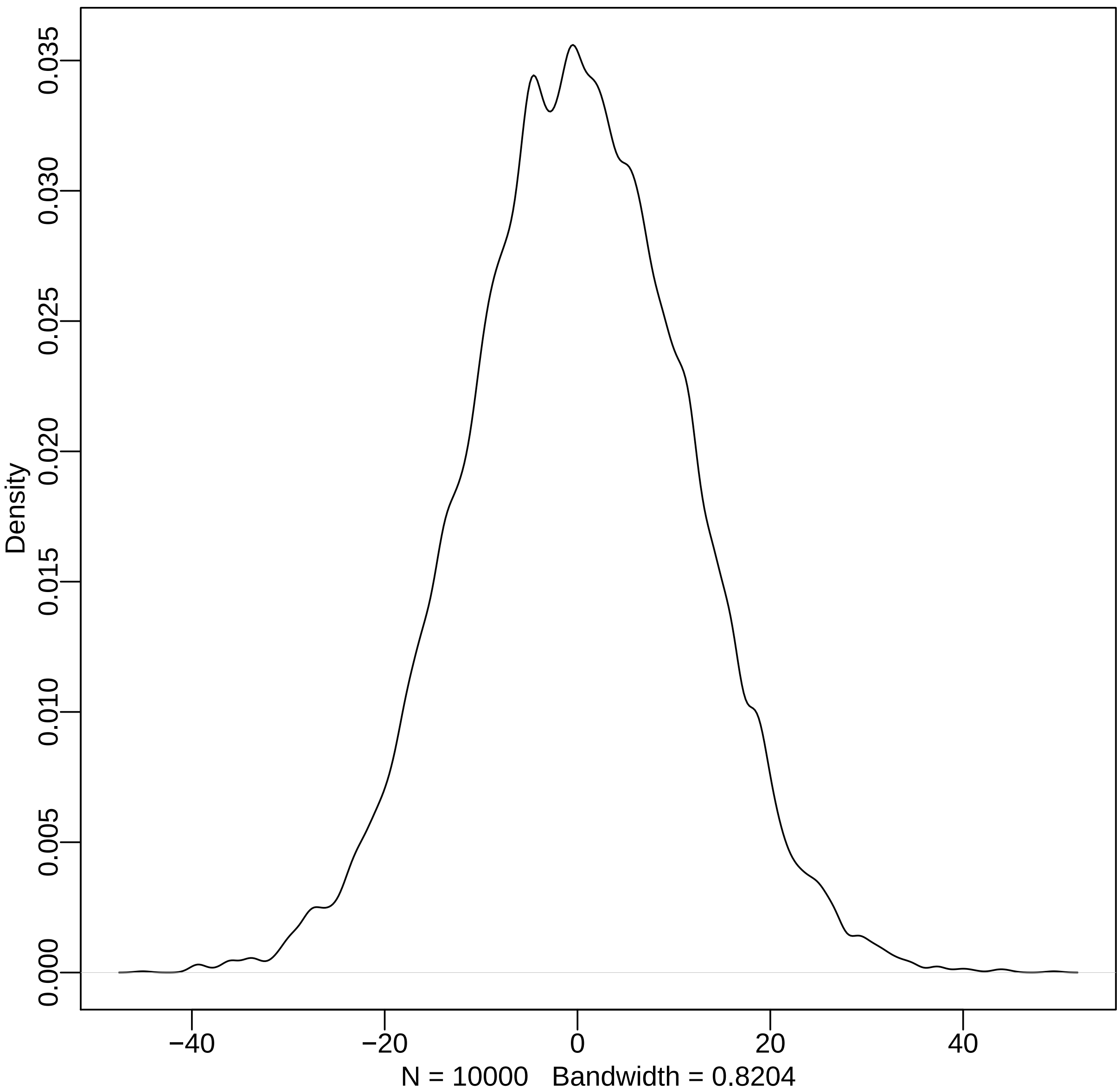
There are three parameters: α, β, and σ.

2 Medium

Problem 4M1.
For the model definition below, simulate the observed heights from the prior (not the posterior).

$$
\begin{aligned}
\mathrm{y_i} &\sim \mathrm{Normal}(\mu, \sigma) \\
\mu &\sim \mathrm{Normal}(0, 10) \\
\sigma &\sim \mathrm{Uniform}(0, 10)
\end{aligned}
$$

```
sample_mu <- rnorm(1e4, 0, 10)
sample_sigma <- runif(1e4, 0, 10)
prior_h <- rnorm(1e4, sample_mu, sample_sigma)
dens(prior_h)
```



Problem 4M2.
Translate the model just above into a map formula.

```
flist <- alist(
  y ~ dnorm(mu, sigma),
  mu ~ dnorm(0, 10),
  sigma ~ dunif(0, 10)
)
```

Problem 4M2.
Translate the map model formula below into a mathematical model definition.

```r
flist <- alist(
  y ~ dnorm(mu, sigma),
  mu <- a +b*x,
  a ~ dnorm(0, 50),
  b ~ dunif(0, 10),
  sigma ~ dunif(0, 50)
)
```

Model:

$$
\begin{gathered}
\mathrm{y_i} \sim \text{Normal}(\mu, \sigma) \\
\mu = \alpha + \beta^{*}\mathrm{x_i} \\
\alpha \sim \text{Normal}(0, 50) \\
\beta \sim \text{Uniform}(0, 10) \\
\sigma \sim \text{Uniform}(0, 50)
\end{gathered}
$$