Statistical Theory II: Chapter 8 - Estimation

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Problem 8.12.

a) $E(\hat{\theta}) = \overline{Y} = \frac{\theta+(\theta+1)}{2} = \theta + \frac{1}{2}$
$B(\hat{\theta}) = E(\hat{\theta}) - \theta = \theta + \frac{1}{2} - \theta = \frac{1}{2}$

b) Let $E(\hat{\theta}) = \overline{Y} - \frac{1}{2} = \theta$

Then,
$B(\hat{\theta}) = E(\hat{\theta}) - \theta = \theta - \theta = 0$

c) $MSE(\overline{Y}) = V(\overline{Y}) + [B(\overline{Y})]^2 = \frac{1}{12n} + \frac{1}{2}$

Problem 8.22.

Let $b = 2\sigma_\mu = 2(\frac{\sigma}{\sqrt{n}}) \approx 2(\frac{\sigma_\mu}{\sqrt{2}}) = \frac{5.6}{\sqrt{200}} \approx 0.791960$.

Thus, the probability that $\epsilon \leq 0.791960$ is approximately 0.95. As a result, we expect the mean to fall in the range $[6.40804, 7.99196]$ with 95% certainty.

Problem 8.46.

a) $m_U(t) = E(e^{tU}) = E(e^{t\frac{2Y}{\theta}}) = m_Y(\frac{2t}{\theta})$

Since Y is distributed exponentially with mean θ, we know from Example 6.12 that $m_Y(t) = (1 - t\theta)^{-1}$

Thus,
$m_Y(\frac{2t}{\theta}) = (1 - \frac{2t}{\theta}\theta)^{-1} = (1 - 2t)^{-1}$

This is the moment generating function for a χ^2-distribution with two degrees of freedom. As a result, U has the same distribution. U is also a pivotal quantity because the distribution does not depend on θ.

b) From Appendix 3, Table 6 with two degrees of freedom:
$P(0.102587 \leq \frac{2Y}{\theta} \leq 5.99147) = 0.9$

This yields,
$(\frac{2Y}{5.99147}, \frac{2Y}{0.102587})$
as the 90% confidence interval for θ.

c) $\frac{2Y}{5.99147} \approx \frac{Y}{2.996}$ and $\frac{2Y}{0.102587} \approx \frac{Y}{0.051}$

Thus, the two confidence intervals are equivalent.

Problem 8.60.

a) From Example 8.6:

$\hat{\theta}_L = \hat{\theta} - z_{\alpha/2}\sigma_{\hat{\theta}}$ and $\hat{\theta}_U = \hat{\theta} + z_{\alpha/2}\sigma_{\hat{\theta}}$

Thus, with $\alpha = 0.01$,

$\hat{\theta}_L = 98.25 - 2.576(\frac{0.73}{\sqrt{130}}) \approx 98.0851$ and $\hat{\theta}_U = 98.25 + 2.576(\frac{0.73}{\sqrt{130}}) \approx 98.4149$

b) This confidence interval does not contain the value 98.6 degrees. Thus, we can say with 99% confidence that 98.6 degrees is not an accurate estimate for the average body temperature of a healthy human.

Problem 8.102.

$$\begin{aligned} s^2 &= [\frac{1}{n-1}]\sum_{i=1}^{n}(Y_i - \overline{Y})^2 \\ &= [\frac{1}{4}]\sum_{i=1}^{5}(Y_i - 57)^2 \\ &= \frac{289}{2} = 144.5 \end{aligned}$$

Thus, with $s^2 = 144.5$, $\frac{\alpha}{2} = 0.005$, and $df = 4$, Table 6, Appendix 3 gives $\chi^2_{0.995} = 0.206990$ and $\chi^2_{0.005} = 14.8602$. Hence, the 90% confidence interval for σ^2 is,

$(\frac{(4)(144.5)}{14.8602}, \frac{(4)(144.5)}{0.206990}) \approx (38.896, 2792.405)$

Since we are looking for the 90% confidence interval for $\sqrt{\sigma^2} = \sigma$, we take the square root of both endpoints of the above confidence interval, yielding:

$(\sqrt{38.896}, \sqrt{2792.405}) \approx (6.237, 52.843)$