Statistical Theory II: Chapter 9 - Properties of Point Estimators and Methods of Estimation

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Problem 9.20.

We know that $\frac{Y}{n}$ is an unbiased estimator of p with $V(\frac{Y}{n}) = \frac{pq}{n}$ from Ch. 8.

Since pq is a constant,

$$\lim_{n\to\infty} \frac{pq}{n} = pq \lim_{n\to\infty} \frac{1}{n} = pq(0) = 0$$

By Theorem 9.1, $\frac{Y}{n}$ is thus a consistent estimator of p.

Problem 9.46.

We know that an exponential distribution has the following pdf:

$$f(x;\beta) = \begin{cases} \frac{1}{\beta}e^{-\frac{x}{\beta}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Thus,

$$\begin{aligned} L(y_1, y_2, ..., y_n|\beta) &= f(y_1, y_2, ..., y_n|\beta) \\ &= f(y_1|\beta) * f(y_2|\beta) * ... * f(y_n|\beta) \\ &= \frac{1}{\beta}e^{-\frac{y_1}{\beta}} * ... * \frac{1}{\beta}e^{-\frac{y_n}{\beta}} \\ &= \left(\frac{1}{\beta}\right)^n e^{-\Sigma y_i/\beta} \\ &= \left(\frac{1}{\beta}\right)^n e^{-n\overline{y}/\beta} \end{aligned}$$

If we let $g(\overline{y}, \beta) = \left(\frac{1}{\beta}\right)^n e^{-n\overline{y}/\beta}$ nd $h(y_1, y_2, ..., y_n) = 1$, we can see that,

$$L(y_1, ..., y_n|\beta) = g(\overline{y}, \beta) * h(y_1, ..., y_n)$$

This satisfies Theorem 9.4, and thus $\overline{Y}$ is a sufficient statistic for the parameter β.

Problem 9.56.

We know from Exercise 9.38(b) that $\Sigma(Y_i - \mu)^2$ is sufficient for σ^2.

We also know that $\hat{\sigma}^2 = \frac{1}{n-1}\Sigma(y_i - \mu)^2$ is an unbiased estimator for σ^2 from Ch. 8.4. Since $\hat{\sigma}^2$ is unbiased and is a function of the sufficient statistic, it is an MVUE for σ^2.

Problem 9.70.

We need to estimate one parameter, λ, so we only need to find $\mu_1' = m_1'$.

The value of μ_1' for a Poisson random variable is,

$$\mu_1' = \mu = \lambda$$

.

The first sample moment is,

$$m_1' = \frac{1}{n}\Sigma Y_i = \overline{Y}$$

.

Thus,

$$\mu_1' = \lambda = \overline{Y}$$

.

Problem 9.82.

a) By Theorem 9.4, a sufficient statistic for θ is ΣY_i^r.

b) $ln(L(\theta)) = -nln(\theta) + nln(r) + (r-1)ln(\Pi y_i) - \Sigma y_i^r/\theta$
After taking the derivative with regards to θ and setting the equation equal to 0, we get:

$$\hat{\theta} = \frac{1}{n}\Sigma Y_i^r$$

.

c) We know that $\hat{\theta}$ is a function of the sufficient statistic.
$E(Y^r) = \theta$, so $\hat{\theta}$ is unbiased and, by Theorem 9.5, $\hat{\theta}$ is the MVUE for θ.