1 Questions

1.1 How to put category structure on each type?

1.2 What is the correct notion of parametric category?

There is a notion given by the

One answer to this is to stipulate that the relations on Types are all of the form $f(x) = g(y)$ for some type Z functions $f : X \to Z$ and $g : Y \to Z$. This is equivalent to saying that the relation satisfies $\forall x_1 x_2 y_1 y_2, R(x_1, y_1) \to R(x_1, y_2) \to R(x_2, y_1) \to R(x_2, y_2)$. I think it is true that for any $F : \mathit{Type} \to \mathit{Type}$, if $R : X \to Y \to \mathit{Prop}$ satisfies the above condition then so does the corresponding relation on $F(X)$ and $F(Y)$.

Then a “parametric category” could be defined to be a category with binary pullback, such that the ”edges” between objects were the pullbacks. This is better than defining an edge to be a subobject of the product both because it is simpler in a pure categorical language and because not all categories generated by parametricity have products but I cannot think of any that do not have binary pullbacks.

e.g. The category $\Sigma X : \mathit{Type}, X \times (X \to \mathit{bool})$ has pullbacks but not products.

1.3 Find the correct notion of morphism of a parametric category weaker than functor, but still having some structure on the map of relations