1 Bicompletion

The bicompletion of a small category $\mathcal{C}$ is a category $\Lambda\mathcal{C}$ with a functor $j : \mathcal{C} \to \Lambda\mathcal{C}$ containing all small colimits and limits, and with the property that for any bicomplete category $\mathcal{D}$, with a functor $F : \mathcal{C} \to \mathcal{D}$, there is a unique bicontinuous functor $F' : \mathcal{C} \to \mathcal{D}$ such that $j \circ F' \cong F$.

The functor j is fully faithful. Everything in this document will be proven from the fully faithfulness of j and the universal property of $\Lambda\mathcal{C}$.

There is a map from $[\mathcal{C}^{op}, Set] \to \Lambda\mathcal{C}$, and from $[\mathcal{C}, Set]^{op} \to \Lambda\mathcal{C}$, definable using the universal property of the presheaf category as the (co)completion of a small category.

The bicompletion of the opposite category is the opposite of the bicompletion. i.e. $\Lambda\mathcal{C}^{op} \cong (\Lambda\mathcal{C})^{op}$.

1.1 Adjunctions

Let F be a functor $\mathcal{C} \to \mathcal{D}$, where $\mathcal{D}$ and $\mathcal{D}$ are small categories. Then $F^* : \Lambda\mathcal{C} \to \Lambda\mathcal{D}$ Then F^* has a left adjoint. Proof. We can define a functor $h : \mathcal{D} \to [C^{op}, Set]$ by setting $h(d, c) := \hom_{\mathcal{C}}(F(c), d)$.