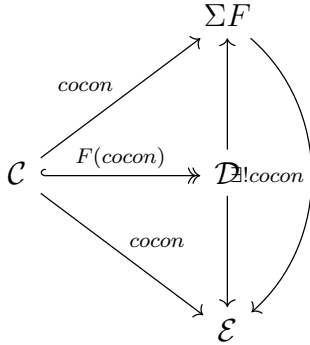


1 Cocompletion of a Category

If \mathcal{C} is a small category then $\mathcal{C}^{op} \rightarrow \text{Type}$ is its cocompletion. Intuitively this is because every element of $\mathcal{C}^{op} \rightarrow \text{Type}$ is a small colimit of a diagram in \mathcal{C} . If \mathcal{C} is a large category, then the cocompletion is the subcategory of $\mathcal{C}^{op} \rightarrow \text{Type}$ consisting of elements which are a small colimit of objects in \mathcal{C} . (Check this works, does the map from this subcategory preserve colimits)? We use the notation $\Sigma\mathcal{C}$ for the cocompletion of \mathcal{C} and $\Pi\mathcal{C}$ for the completion.

2 Partial Cocompletion of a Category

We have large categories \mathcal{C} and \mathcal{D} and a fully faithful cocontinuous functor $F : \mathcal{C} \rightarrow \mathcal{D}$, such that everything in \mathcal{D} is a limit of objects in \mathcal{C} . Then ΣF is a cocomplete category such that there is a cocontinuous map $i_{\mathcal{C}} : \mathcal{C} \rightarrow \Sigma F$, and a functor $i_{\mathcal{D}} : \mathcal{D} \rightarrow \Sigma F$ such that $i_{\mathcal{D}} \circ F = i_{\mathcal{C}}$, and for any category \mathcal{E} with a cocontinuous map $j_{\mathcal{C}} : \mathcal{C} \rightarrow \mathcal{E}$ and a map $j_{\mathcal{D}} : \mathcal{D} \rightarrow \mathcal{E}$ such that $j_{\mathcal{D}} \circ F = j_{\mathcal{C}}$, then there is a unique cocontinuous map $\Sigma F \rightarrow \mathcal{E}$ making everything commute.



When we construct ΣF it should be obvious that $i_{\mathcal{D}}$ is fully faithful and continuous. It will be constructed as a subcategory of $PSh(\mathcal{D})$.

2.1 Constructing the Partial Cocompletion

We define an adjunction between $\Sigma\mathcal{D}$ and $\Pi\mathcal{D}$. It is inspired by Isbell conjugation, if F is the identity functor, then it is Isbell conjugation.