Binary Search

Binary Search

- In binary search, the idea of the algorithm is to divide and conquer, reducing the search area by half each time, trying to find a target number.
    - In order to leverage this power however, our array must first be sorted, else we cannot make assumptions about the array's contents.

Binary Search

<u>In pseudocode:</u>

- Repeat until the (sub)array is of size 0:
    - Calculate the middle point of the current (sub)array.
    - If the target is at the middle, stop.
    - Otherwise, if the target is less than what’s at the middle, repeat, changing the end point to be just to the left of the middle.
    - Otherwise, if the target is greater than what’s at the middle, repeat, changing the start point to be just to the right of the middle.

Binary Search

Target	Start	End	Middle

11	23	8	14	30	9	6	17	22	28	25	15	7	10	19

In pseudocode:

```
Repeat until the (sub)array is of size 0:
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```

Binary Search

Target	Start	End	Middle

11	23	8	14	30	9	6	17	22	28	25	15	7	10	19
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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    point to be just to the right of the middle.
```

Binary Search

Target	Start	End	Middle

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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```

Binary Search

Target	Start	End	Middle
19	0	14	

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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    point to be just to the right of the middle.
```

Binary Search

Target	Start	End	Middle
19	0	14	7

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

Target	Start	End	Middle
19	0	14	7

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

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19	8	14	7

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

Target	Start	End	Middle
19	8	14	11

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

Target	Start	End	Middle
19	8	14	11

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

Target	Start	End	Middle
19	8	10	11

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Target	Start	End	Middle
19	8	10	9

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

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16	0	14	

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

Target	Start	End	Middle
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6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Target	Start	End	Middle
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6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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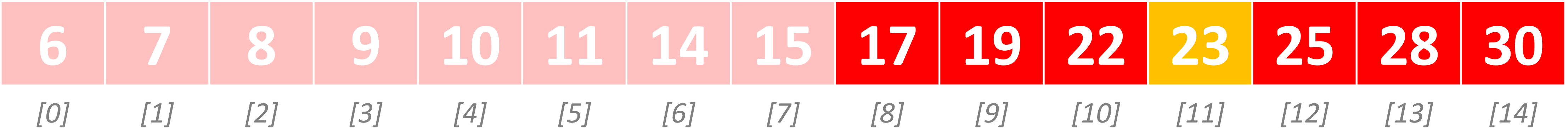
Target	Start	End	Middle
16	8	14	11



6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]

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16	8	10	11

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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16	8	10	9

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]

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16	8	8	9

6	7	8	9	10	11	14	15	17	19	22	23	25	28	30
[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]

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[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]

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[0]	*[1]*	*[2]*	*[3]*	*[4]*	*[5]*	*[6]*	*[7]*	*[8]*	*[9]*	*[10]*	*[11]*	*[12]*	*[13]*	*[14]*

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Binary Search

- **Worst-case scenario**: We have to divide a list of n elements in half repeatedly to find the target element, either because the target element will be found at the end of the last division or doesn’t exist in the array at all.

- **Best-case scenario**: The target element is at the midpoint of the full array, and so we can stop looking immediately after we start.

Binary Search

$$O(\mathit{log}\ n)$$

$$\Omega(1)$$