

SOUTH CHINA UNIVERSITY OF TECHNOLOGY

SCUT_GUGUGU

TEMPLATE



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1 Graph Theory

1.1 Shortest Path

1.1.1 Dijkstra

```

1  typedef long long LL;
2  const int MAXN = ;
3  const int MAXM = ;
4  const LL DINF = ;
5  typedef pair<LL, int> P;
6  struct Edge {
7      int to, nxt;
8      LL w;
9  }e[MAXM];
10 int head[MAXN], ecnt;
11 LL d[MAXN];
12 priority_queue<P, vector<P>, greater<P> > q;
13 inline void addEdge(int x, int y, LL w) {
14     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
15 }
16 void dijkstra(int st, int n) {
17     for(int i = 0; i <= n; i++) d[i] = DINF;
18     d[st] = 0;
19     q.push(make_pair(0, st));
20     while(!q.empty()) {
21         P x = q.top(); q.pop();
22         int u = x.second;
23         if(d[u] != x.first) continue;
24         for(int i = head[u], v; i; i = e[i].nxt) {
25             v = e[i].to;
26             if(d[v] > d[u] + e[i].w) {
27                 d[v] = d[u] + e[i].w;
28                 q.push(make_pair(d[v], v));
29             }
30         }
31     }
32 }

```

1.1.2 SPFA

```

1  struct Edge {
2      int to, nxt;
3      LL w;
4  }e[MAXE];
5  int head[MAXN], ecnt;
6  LL d[MAXN];
7  bool exist[MAXN];
8  queue<int> q;
9  inline void addEdge(int x, int y, LL w) {
10     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
11 }
12 void SPFA(int st) {
13     memset(d, 0x3f, sizeof(d));
14     d[st] = 0;
15     q.push(st);
16     exist[st] = 1;
17     while(!q.empty()) {

```

```

18     int u = q.front(); q.pop();
19     exist[u] = 0;
20     for(int i = head[u], v; i; i = e[i].nxt) {
21         v = e[i].to;
22         if(d[v] > d[u] + e[i].w) {
23             d[v] = d[u] + e[i].w;
24             //pre[v] = u;
25             if(!exist[v]) {
26                 q.push(v);
27                 exist[v] = 1;
28             }
29         }
30     }
31 }
32 }

```

1.1.3 Johnson

```

1 void johnson() {
2     //全源带负权最短路,新建超级节点向全部点连权为0的边, 计算D, 利用Dij计算点对距离
3     for(int i = 1; i <= n; i++) addEdge(0, i, 0);
4     spfa(0);
5     for(int u = 1; u <= n; u++)
6         for(int i = head[u]; i; i = e[i].nxt)
7             e[i].w += D[e[i].from] - D[e[i].to];
8     dijkstra(s,n);
9     //ans = d[n] - D[s] + D[n];
10 }
11 }

```

1.1.4 K Shortest Path (A*)

```

1 //可重复走同一条边 利用了反向边表示, ecnt初始化为1
2 //调用 dijkstra(ed,n)跑反向图, 注意if(i < 1){松弛}
3 int shrt[MAXN];
4 LL A_star(int st, int ed, int k, int n) {
5     if(d[st] == d[0]) return -1;
6     if(st == ed) k++;
7     q.push(make_pair(d[st], st));
8     while(!q.empty()) {
9         P x = q.top(); q.pop();
10        int u = x.second;
11        LL xd = x.first;
12        ++shrt[u];
13        if(u == ed) {
14            if(shrt[ed] == k) return xd;
15        }
16        for(int i = head[u], v; i; i = e[i].nxt)
17            if(!(i & 1)) {
18                v = e[i].to;
19                if(shrt[v] < k) q.push(make_pair(xd-d[u]+e[i].w+d[v], v));
20            }
21    }
22    return -1;
23 }

```

1.1.5 K Shortest Path (Protractable Heap)

```

1  //可重复走同一条边
2  typedef double LD;
3  const int MAXN = ;
4  const int MAXM = ;
5  const int MAXLT = MAXM * 20;
6  const LD DINF = ;
7  const LD eps = ;
8
9  namespace LT{
10     int tcnt;
11     int ls[MAXLT], rs[MAXLT], dis[MAXLT];
12     int to[MAXLT];
13     LD val[MAXLT];
14
15     inline int newnode(LD w, int _to) {
16         ++tcnt;
17         ls[tcnt] = rs[tcnt] = 0; dis[tcnt] = 1;
18         val[tcnt] = w; to[tcnt] = _to;
19         return tcnt;
20     }
21     inline int copynode(int id) {
22         ++tcnt;
23         ls[tcnt] = ls[id]; rs[tcnt] = rs[id]; dis[tcnt] = dis[id];
24         val[tcnt] = val[id]; to[tcnt] = to[id];
25         return tcnt;
26     }
27     void push_up(int x) {
28         if(dis[ls[x]] < dis[rs[x]]) swap(ls[x], rs[x]);
29         dis[x] = dis[rs[x]] + 1;
30     }
31     int merge(int x, int y) {
32         if(!x || !y) return x^y;
33         if(val[x] - val[y] > eps) swap(x, y);
34         int p = copynode(x);
35         rs[p] = merge(rs[p], y);
36         push_up(p);
37         return p;
38     }
39 }
40 int rt[MAXN];
41
42 typedef pair<LD, int> P;
43 struct Edge {
44     int to, nxt;
45     LD w;
46 }e[MAXM];
47 int head[MAXN], ecnt;
48 int stan, sta[MAXN], fa[MAXN];
49 int vis[MAXN], cov[MAXM];
50 LD d[MAXN];
51 priority_queue<P, vector<P>, greater<P> > q;
52 inline void addEdge(int x, int y, LD w) {
53     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
54 }
55 void init(int n, int m) {
56     ecnt = 1; stan = 0;
57     for(int i = 1; i <= n; i++)
58         head[i] = cov[i] = fa[i] = 0;
59     for(int i = 1; i <= m; i++) {

```



```

60     int u, v; LD w;
61     scanf("%d%d%lf", &u,&v,&w);
62     addEdge(u, v, w);
63     addEdge(v, u, w);
64 }
65 }
66 void dijkstra(int st, int n) {
67     for(int i = 0; i <= n; i++) {d[i] = DINF; vis[i] = 0;}
68     while(!q.empty()) q.pop();
69     d[st] = 0;q.push(make_pair(0, st));
70     while(!q.empty()) {
71         P x = q.top(); q.pop();
72         int u = x.second;
73         if(vis[u]) continue;
74         vis[u] = 1;
75         for(int i = head[u], v; i; i = e[i].nxt)
76             if(i & 1) {
77                 v = e[i].to;
78                 if(d[v]- (d[u] + e[i].w) > eps) {
79                     d[v] = d[u] + e[i].w;
80                     q.push(make_pair(d[v], v));
81                 }
82             }
83     }
84 }
85 void buildT(int u) {
86     sta[++stan] = u;
87     vis[u] = 1;
88     for(int i = head[u], v; i; i = e[i].nxt)
89         if(i & 1) {
90             v = e[i].to;
91             if(fabs(d[v] - (d[u] + e[i].w)) < eps && !vis[v]) {
92                 fa[v] = u; cov[i^1] = 1; buildT(v);
93             }
94         }
95 }
96 void buildH(int st, int n) {
97     buildT(st);
98     for(int i = 2, u, v; i <= ecnt; i += 2) {
99         if(!cov[i]) {
100             u = e[i^1].to; v = e[i].to;
101             if(fabs(d[u] - d[0]) < eps || fabs(d[v] - d[0]) < eps) continue;
102             rt[u] = LT::merge(rt[u], LT::newnode(d[v]+e[i].w-d[u], v));
103         }
104     }
105     for(int i = 2, u; i <= n; i++)
106         if(fa[u = sta[i]]) rt[u] = LT::merge(rt[u], rt[fa[u]]);
107 }
108 //求前k短路,其和不超过W,问最大k int ans = 0;
109 //求第k短路,LD ans = 0;
110 void getKth(int st, int ed, LD W) {
111     while(!q.empty()) q.pop();
112     //最短路要记入答案,注意d[st]-W<eps与W-d[st]<eps
113     //!if(d[st] - W > eps) return;
114     //!else{W -= d[st]; ++ans;}
115     //注意st和ed相同
116     //if(st == ed) k++;
117     //if(d[st] == d[0]) {ans = -1; return;}
118     //if(--k == 0) {ans = d[st]; return;}
119
120     int u = rt[st], v;

```

```

121     if(u) q.push(make_pair(LT::val[u], u));
122     while (!q.empty()) {
123         u = q.top().second; LD cur = q.top().first;
124         q.pop();
125         //!if (cur + d[st] - W > eps) break;
126         //!else {W -= cur + d[st]; ++ans;}
127         //!#if (--k == 0) {ans = cur + d[st]; break;}
128         v = LT::ls[u];
129         if (v) q.push(make_pair(cur - LT::val[u] + LT::val[v], v));
130         v = LT::rs[u];
131         if (v) q.push(make_pair(cur - LT::val[u] + LT::val[v], v));
132         v = rt[LT::to[u]];
133         if (v) q.push(make_pair(cur + LT::val[v], v));
134     }
135 }
136 void sol() {
137     int n, m, st = , ed = ;
138     //LD W; int k;
139     init(n, m);
140     dijkstra(ed, n);
141     for(int i = 0; i <= n; i++) vis[i] = 0;
142     buildH(ed, n);
143     getKth(st, ed, W);
144     printf("%d\n", ans);
145 }

```

1.2 Network Flow

1.2.1 ISAP

```

1 namespace NWF {
2     struct Edge{
3         int to, nxt;LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], pre[MAXN], num[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(num, 0, (tot + 1) * sizeof(int));
11        memset(head, 0, (tot + 1) * sizeof(int));
12    }
13    inline void addEdge(int u, int v, LL f) {
14        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
15        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
16    }
17    void bfs() {
18        memset(dis, 0, (tot + 1) * sizeof(int));
19        q.push(T);
20        dis[T] = 1;
21        while(!q.empty()) {
22            int u = q.front(), v; q.pop();
23            num[dis[u]]++;
24            for(int i = cur[u] = head[u]; i; i = e[i].nxt) {
25                if(!dis[v = e[i].to]) {
26                    dis[v] = dis[u] + 1;
27                    q.push(v);
28                }
29            }

```

```

30     }
31 }
32 LL augment() {
33     LL flow = INF;
34     for(int i = S; i != T; i = e[cur[i]].to)
35         flow = min(flow, e[cur[i]].f);
36     for(int i = S; i != T; i = e[cur[i]].to) {
37         e[cur[i]].f -= flow;
38         e[cur[i] ^ 1].f += flow;
39     }
40     return flow;
41 }
42 LL isap() {
43     bfs();
44     int u = S, v;
45     LL flow = 0;
46     while(dis[S] <= tot) {
47         if(u == T) {
48             flow += augment();
49             u = S;
50         }
51         bool fg = 0;
52         for(int i = cur[u]; i; i = e[i].nxt) {
53             if(e[i].f && dis[u] > dis[v = e[i].to]) {
54                 pre[v] = u;
55                 cur[u] = i;
56                 u = v;
57                 fg = 1;
58                 break;
59             }
60         }
61         if(fg) continue;
62         if(!--num[dis[u]]) break;
63         int maxDis = tot;
64         for(int i = head[u]; i; i = e[i].nxt) {
65             if(e[i].f && maxDis > dis[v = e[i].to]) {
66                 maxDis = dis[v];
67                 cur[u] = i;
68             }
69         }
70         num[dis[u] = maxDis + 1]++;
71         if(u != S) u = pre[u];
72     }
73     return flow;
74 }
75 }

```

1.2.2 HLPP

```

1 namespace NWF{
2     struct Edge{
3         int to,nxt;LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], dis[MAXN], num[MAXN];
7     LL sumf[MAXN];
8     queue<int> q;
9     list<int> dep[MAXN];
10    void init(int _S,int _T,int _tot){
11        ecnt = 1;S = _S;T = _T;tot = _tot;

```

```

12     memset(num, 0, (tot + 1) * sizeof(int));
13     memset(head, 0, (tot + 1) * sizeof(int));
14     memset(sumf, 0, (tot + 1) * sizeof(LL));
15 }
16 void addEdge(int u,int v,LL f){
17     e[++ecnt] = (Edge) {v, head[u], f};head[u] = ecnt;
18     e[++ecnt] = (Edge) {u, head[v], 0};head[v] = ecnt;
19 }
20 void bfs(){
21     memset(dis, 0, (tot + 1) * sizeof(int));
22     q.push(T); dis[T] = 1;
23     while(!q.empty()){
24         int u=q.front(), v; q.pop();
25         for(int i = head[u]; i; i = e[i].nxt)
26             if(!dis[v = e[i].to]){
27                 dis[v] = dis[u] + 1;
28                 q.push(v);
29             }
30     }
31 }
32 LL hlpp(){
33     bfs();
34     dis[S] = tot + 1;
35     for(int i = 1;i <= tot; ++i)num[dis[i]]++;
36     for(int i = tot + 1; ~i; --i)dep[i].clear();
37     int maxd = dis[S];LL f;
38     dep[maxd].push_back(S);sumf[S] = INF;
39     for(;;){
40         while(maxd && dep[maxd].empty())maxd--;
41         if(!maxd)break;
42         int u = dep[maxd].back(), v;dep[maxd].pop_back();
43         int minDis = tot + 1;
44         for(int i = head[u]; i;i = e[i].nxt)
45             if(e[i].f){
46                 if(dis[u] > dis[v = e[i].to]){
47                     f = min(sumf[u], e[i].f);
48                     e[i].f -= f;e[i^1].f += f;
49                     if(sumf[u] != INF) sumf[u] -= f;
50                     if(sumf[v] != INF) sumf[v] += f;
51                     if(v!=S && v!=T && sumf[v] == f){
52                         maxd = max(maxd, dis[v]);
53                         dep[dis[v]].push_back(v);
54                     }
55                     if(!sumf[u])break;
56                 }else minDis=min(minDis, dis[v] + 1);
57             }
58         if(sumf[u]){
59             if(!--num[dis[u]]){
60                 for(int i = dis[u];i <= maxd;++i){
61                     while(!dep[i].empty()){
62                         --num[i];
63                         dis[dep[i].back()] = tot + 1;
64                         dep[i].pop_back();
65                     }
66                 }
67                 maxd = dis[u] - 1;dis[u] = tot + 1;
68             }else{
69                 dis[u] = minDis;
70                 if(minDis > tot)continue;
71                 num[minDis]++;
72                 maxd = max(maxd, minDis);

```

```

73         dep[minDis].push_back(u);
74     }
75 }
76 }
77 return sumf[T];
78 }
79 }

```

1.2.3 Dinic

注意当流为浮点数的时候，要判断 `eps` 以及不能使用 `sumf==tmpf`，否则 `1e18` 将不会发生改变

```

1 namespace NWF {
2     struct Edge {
3         int to, nxt; LL f;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(head, 0, (tot + 1) * sizeof(int));
11    }
12    void addEdge(int u, int v, LL f) {
13        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
14        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
15    }
16    bool bfs() {
17        memset(dis, 0, (tot + 1) * sizeof(int));
18        q.push(S); dis[S] = 1;
19        while (!q.empty()) {
20            int u = q.front(), v; q.pop();
21            for (int i = cur[u] = head[u]; i; i = e[i].nxt) {
22                if (e[i].f && !dis[v = e[i].to]) {
23                    q.push(v);
24                    dis[v] = dis[u] + 1;
25                }
26            }
27        }
28        return dis[T];
29    }
30    LL dfs(int u, LL maxf) {
31        if (u == T) return maxf;
32        LL sumf = maxf;
33        for (int &i = cur[u]; i; i = e[i].nxt) {
34            if (e[i].f && dis[e[i].to] > dis[u]) {
35                LL tmpf = dfs(e[i].to, min(sumf, e[i].f));
36                e[i].f -= tmpf; e[i ^ 1].f += tmpf;
37                sumf -= tmpf;
38                if (!sumf) return maxf;
39            }
40        }
41        return maxf - sumf;
42    }
43    LL dinic() {
44        LL ret = 0;
45        while (bfs()) ret += dfs(S, INF);
46        return ret;
47    }
48    void rebuild(){
49        //无向图采用  $e[i].f = e[i \wedge 1].f$  的方式建立图

```

```

50     for(int i = 2; i <= ecnt; i+=2) e[i].f = e[i^1].f = (e[i].f + e[i^1].f) >> 1;
51     //有向图
52     //for(int i = 2; i <= ecnt; i+=2) e[i].f += e[i^1].f, e[i^1].f = 0;
53 }
54 }

```

1.2.4 Bound Flow

```

1 namespace NWF{
2     //在 Edge 中添加下限, delta_flow[i]: 节点 i 的入流-出流, bound_flow 注意要处理除了 SS 和 TT 以
    外的点
3     int delta_flow[MAXN];
4     void addEdge(int u, int v, int mxf, int mnf) {
5         addEdge(u, v, mxf-mnf);
6         delta_flow[u] -= mnf; delta_flow[v] += mnf;
7     }
8     void delEdge(int u) {
9         for(int i = head[u]; i; i = e[i].nxt) e[i].f = e[i^1].f = 0;
10    }
11    int bound_flow() {
12        int SS = ++tot, TT = ++ tot, sum = 0;
13        head[SS] = head[TT] = 0;
14        for(int i = 1; i <= tot; ++i) {
15            if(delta_flow[i] < 0) addEdge(i, TT, -delta_flow[i]);
16            if(delta_flow[i] > 0){
17                sum += delta_flow[i];
18                addEdge(SS, i, delta_flow[i]);
19            }
20        }
21        addEdge(T, S, INF);
22        int tS = S, tT = T;
23        S = SS; T = TT;
24        if (dinic() == sum) {
25            delEdge(SS); delEdge(TT);
26            int flow = e[ecnt].f;
27            e[ecnt].f = e[ecnt^1].f = 0;
28            //S = tS; T = tT; // 有上下界有源汇最大流
29            //return flow + dinic();
30            S = tT; T = tS; // 有上下界有源汇最小流
31            return flow - dinic();
32        } else {
33            return -1;
34        }
35    }
36 }

```

1.2.5 Modeling Optimization

利用分治优化建模, 每个点 i 向 j 连边, 费用为 $|a_i - a_j|$

```

1 int pos[MAXN];
2 pair<int, int> tmp[MAXN];
3 void CDQ(int L, int R) {
4     if (L == R) return;
5     int mid = (L + R) >> 1;
6     CDQ(L, mid); CDQ(mid + 1, R);
7     inplace_merge(tmp + L, tmp + mid + 1, tmp + R + 1);
8     for (int i = L; i <= R; ++i) pos[tmp[i].second] = i;
9     for (int i = 2; i <= R - L + 1; ++i) {

```

```

10     addEdge(tot + i, tot + i - 1, INF, tmp[L + i - 1].first - tmp[L + i - 2].first);
11     addEdge(tot + i - 1, tot + i, INF, tmp[L + i - 1].first - tmp[L + i - 2].first);
12 }
13 for (int i = L; i <= R; ++i) {
14     if (i <= mid)
15         addEdge(i+i-1, tot + pos[i] - L + 1, 1, 0);
16     else
17         addEdge(tot + pos[i] - L + 1, i+i, 1, 0);
18 }
19 tot += R - L + 1;
20 }

```

1.2.6 Gomory-Hu Tree

两点间的割可以转为树上两点的距离

```

1 namespace NWF{
2     Edge Te[MAXN];
3     int Tcnt, Thead[MAXN];
4     void TaddEdge(int u, int v, LL f) {
5         Te[++Tcnt] = (Edge) {v, Thead[u], f}; Thead[u] = Tcnt;
6         Te[++Tcnt] = (Edge) {u, Thead[v], f}; Thead[v] = Tcnt;
7     }
8     int node[MAXN], tmp[MAXN];
9     void build(int l, int r) {
10         if (l == r) return;
11         S = node[l]; T = node[l+1];
12         rebuild();
13         LL cut = dinic();
14         TaddEdge(S, T, cut);
15         int tl = l, tr = r;
16         for(int i = l; i <= r; i++) {
17             if(dis[node[i]]) tmp[tl++] = node[i]; else tmp[tr--] = node[i];
18         }
19         for(int i=l; i<=r; i++) node[i] = tmp[i];
20         build(l,tl-1); build(tr+1,r);
21     }
22     int log2n;
23     int dep[MAXN], anc[MAXN][MAXS]; LL mnl[MAXN][MAXS]; //anc: 祖先; mnl: 最小边
24     void lca_dfs(int u, int _fa) {
25         for(int i=Thead[u], v; i; i=Te[i].nxt) {
26             if((v = Te[i].to) == _fa) continue;
27             dep[v] = dep[u] + 1;
28             anc[v][0] = u;
29             mnl[v][0] = Te[i].f;
30             lca_dfs(v, u);
31         }
32     }
33     void work() {
34         if(tot == 0) return;
35         log2n = log2(tot)+1; Tcnt = 1;
36         for(int i=1; i<=tot; i++) node[i]=i, Thead[i]=0;
37         build(1, tot);
38         dep[1] = 1; anc[1][0] = 0; mnl[1][0] = INF;
39         lca_dfs(1, -1);
40         for(int j = 1; j <= log2n; j++) {
41             for(int i = 1; i <= tot; i++) {
42                 anc[i][j] = anc[anc[i][j-1]][j-1];
43                 mnl[i][j] = min(mnl[i][j-1], mnl[anc[i][j-1]][j-1]);
44             }
45         }
46     }
47 }

```

```

46     }
47     LL get_cut(int u,int v) {
48         LL res=INF;
49         if(dep[u] < dep[v]) swap(u, v);
50         for(int i = log2n; i >= 0; i--) {
51             if(dep[anc[u][i]] >= dep[v]){
52                 res = min(res, mnl[u][i]);
53                 u = anc[u][i];
54             }
55         }
56         if(u == v) return res;
57         for(int i = log2n; i>=0 ; i--) {
58             if(anc[u][i] != anc[v][i]) {
59                 res = min(res, mnl[u][i]);
60                 res = min(res, mnl[v][i]);
61                 u = anc[u][i];
62                 v = anc[v][i];
63             }
64         }
65         res = min(res, mnl[u][0]);
66         res = min(res, mnl[v][0]);
67         return res;
68     }
69 }

```

1.2.7 MCMF

```

1 namespace NWF{
2     struct Edge {
3         int to, nxt;LL f, c;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN];LL dis[MAXN];
7     bool exist[MAXN];
8     queue<int> q;
9     void init(int _S, int _T, int _tot){
10         ecnt = 1; S = _S; T = _T; tot = _tot;
11         memset(head, 0, (tot + 1) * sizeof(int));
12     }
13     void addEdge(int u, int v, LL f, LL c) {
14         e[++ecnt] = (Edge) {v, head[u], f, c}; head[u] = ecnt;
15         e[++ecnt] = (Edge) {u, head[v], 0, -c}; head[v] = ecnt;
16     }
17     bool spfa() {
18         for(int i = 0; i <= tot; ++i){
19             dis[i] = INF; cur[i] = exist[i] = 0;
20         }
21         q.push(S);dis[S] = 0;exist[S] = 1;
22         while(!q.empty()) {
23             int u = q.front(), v; q.pop();exist[u] = 0;
24             for(int i = head[u]; i; i = e[i].nxt) {
25                 if(e[i].f && dis[v = e[i].to] > dis[u] + e[i].c) {
26                     dis[v] = dis[u] + e[i].c;
27                     cur[v] = i;
28                     if(!exist[v]) {
29                         q.push(v);
30                         exist[v] = 1;
31                     }
32                 }
33             }
34         }
35     }
36 }

```



```

34     }
35     return dis[T] != INF;
36 }
37 LL mcmf() {
38     LL cost = 0;
39     //while(spfa()) && dis[T] < 0) { //最小费用可行流
40     while(spfa()) {
41         LL flow = INF;
42         for(int i = T; i != S; i = e[cur[i] ^ 1].to)
43             flow = min(flow, e[cur[i]].f);
44         for(int i = T; i != S; i = e[cur[i] ^ 1].to) {
45             e[cur[i]].f -= flow;
46             e[cur[i] ^ 1].f += flow;
47         }
48         cost += flow * dis[T];
49     }
50     return cost;
51 }
52 }

```

1.3 Tree Related

1.3.1 Union Set

```

1 int fa[MAXN], rnk[MAXN];
2 int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
3 bool same(int x, int y){ return Find(x) == Find(y); }
4 void unite(int x, int y)
5 {
6     x = Find(x);
7     y = Find(y);
8     if(x == y) return;
9     if(rnk[x] < rnk[y]) {
10         fa[x] = y;
11     }
12     else {
13         fa[y] = x;
14         if(rnk[x] == rnk[y]) rnk[x]++;
15     }
16 }

```

1.3.2 Kruskal

```

1 namespace MST{
2     struct Edge{
3         int u,v; LL w;
4         bool operator < (const Edge& x) const { return w < x.w; }
5     }e[MAXM];
6     int ecnt, fa[MAXN];
7     void addEdge(int u, int v, LL w) {
8         e[++ecnt] = (Edge){v, u, w}; //headp[u] = ecnt;
9     }
10    int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
11    LL kruskal(int n) {
12        sort(e + 1, e + ecnt + 1);
13        for(int i = 1; i <= n; i++) fa[i] = i;
14        LL sum = 0;
15        for (int i = 1; i <= ecnt; i++){

```

```

16         int fu = Find(e[i].u), fv = Find(e[i].v);
17         if(fu != fv){
18             fa[fu] = fv;
19             sum += e[i].w;
20         }
21     }
22     return sum;
23 }
24 }

```

1.3.3 Prim

```

1 namespace MST {
2     struct Edge{
3         int to,nxt; LL w;
4     }e[MAXM];
5     int ecnt, head[MAXN], vis[MAXN]; // pre[MAXN];
6     LL dis[MAXN];
7     void addEdge(int u, int v, LL w){
8         e[++ecnt] = (Edge){v, head[u], w}; head[u] = ecnt;
9         e[++ecnt] = (Edge){u, head[v], w}; head[v] = ecnt;
10    }
11    LL Prim(int n){
12        for (int i = 1; i <= n; i++){
13            //pre[i] = 0;
14            vis[i] = 0;
15            dis[i] = INF;
16        }
17        vis[1] = 1;
18        LL sum = 0;
19        for (int i = head[1]; i; i = e[i].nxt)
20            dis[e[i].to] = min(dis[e[i].to],e[i].w);
21        for (int j = 1; j < n; j++){
22            int u; LL minDis = INF;
23            for (int i = 1; i <= n; ++i)
24                if (!vis[i] && dis[i] < minDis){
25                    minDis = dis[i];
26                    u = i;
27                }
28            if (minDis == INF) return -1;
29            vis[u] = 1;
30            sum += minDis;
31            for (int i = head[u], v; i; i = e[i].nxt)
32                if (!vis[v = e[i].to] && e[i].w < dis[v]){
33                    //pre[u] = v;
34                    dis[v] = e[i].w;
35                }
36        }
37        return sum;
38    }
39 }

```

1.3.4 Spanning Tree Calculation

关联矩阵 $B:n*m$ 的矩阵, 其中 $ek=(vi,vj)$, B_{ik} 和 B_{jk} 一个为 1 一个为 -1, 第 k 列其他元素为 0 度数矩阵 $D:n*n$ 的矩阵, 其中 $i!=j$ 时, $D[i][j]=0$; $i=j$ 时, $D[i][j]=vi$ 的度邻接矩阵 $A:n*n$ 的矩阵, vi,vj 有边相连, 为 1, 否则为 0 Kirchhoff 矩阵: $B^*B^T = D - A$ 即: 如果 $i=j$, 那么 a_{ij} 为点 $i(j)$ 的度数。如果 $i!=j$, 那么 A_{ij} 为 i 到 j 的边数的相反数。生成树个数: Kirchhoff 矩阵 $n-1$ 阶主子式的行列式值构造 Kirchhoff 矩阵, 调用 $\det(n)$

```

1 LL a[MAXN][MAXN];
2 void getSTC(int n, int m) {
3     for(int i = 1; i <= n; i++) {
4         for(int j = 1; j <= n; j++)
5             a[i][j] = a[j][i] = 0;
6     }
7     for(int i = 1, u, v; i <= m; i++) {
8         scanf("%d%d", &u, &v);
9         if(u == v) continue;
10        a[u][v] = --a[v][u];
11    }
12    for(int i = 1; i <= n; i++) {
13        int t = 0;
14        for(int j = 1; j <= n; j++)
15            t += a[i][j];
16        a[i][i] = - t;
17    }
18    LL ans = det(); //删掉一行一列以后求行列式的值
19 }

```

1.3.5 Minimum Spanning Tree Calculation

```

1 typedef long long LL;
2 const int MAXN = ;
3 const int MAXM = ;
4 int sum, ans1, ans2 = 1, Mod = ;
5 int fa1[MAXN], fa2[MAXN];
6 bool vis[MAXN];
7 struct Edge {int v, u, val;} e[MAXM];
8 bool cmp(Edge A, Edge B) {return A.val < B.val;}
9 int getfa1(int *fa, int x) {return fa[x] == x ? x : getfa1(fa, fa[x]);}
10 int getfa2(int *fa, int x) {return fa[x] == x ? x : getfa2(fa, fa[x]);}
11 void dfs(int tot, int l, int r) {
12     if(tot == 0) {++sum; return;}
13     for(int i = l, fx, fy; i <= r; ++i)
14         if(!vis[i]) {
15             vis[i] = true;
16             fx = getfa2(fa2, e[i].u); fy = getfa2(fa2, e[i].v);
17             if(fx != fy) {
18                 fa2[fx] = fy;
19                 dfs(tot - 1, i + 1, r);
20                 fa2[fx] = fx;
21             }
22             vis[i] = false;
23         }
24 }
25 void sol() {
26     int n, m;
27     scanf("%d %d", &n, &m);
28     for(int i = 1; i <= n; ++i) fa1[i] = fa2[i] = i;
29     for(int i = 1; i <= m; ++i)
30         scanf("%d %d %d", &e[i].v, &e[i].u, &e[i].val);
31     std::sort(e + 1, e + m + 1, cmp); e[m + 1].val = -1;
32     for(int i = 1, j = 1, fx, fy, tot; i <= m; ++i) {
33         for(; e[i].val == e[j].val; ++j);
34         tot = 0;
35         for(int k = i; k < j; ++k) {
36             fx = getfa1(fa1, e[k].u); fy = getfa1(fa1, e[k].v);
37             if(fx != fy) {++tot; ans1++; fa1[fx] = fy;}
38         }

```

```

39     if(!tot)continue;
40     sum=0;dfs(tot,i,j);
41     (ans2*=sum)%=Mod;
42     for(int k=i;k<j;++k){
43         fx=getfa1(fa2,e[k].u);fy=getfa1(fa2,e[k].v);
44         if(fx!=fy)fa2[fx]=fy;
45     }
46 }
47 if(ans1!=n-1)puts("0");else printf("%d",ans2);
48 }

```

1.3.6 Steiner Tree

```

1  const int MAXH = 128;
2  const int MAXW = 128;
3  const int MAXST = 1256;
4  namespace SteinerTree{
5      const int dx[4] = {0,0,1,-1};
6      const int dy[4] = {1,-1,0,0};
7      int n, m, k, stn;
8      bool vis[MAXH][MAXW];
9      LL G[MAXH][MAXW],f[MAXST][MAXH][MAXW];
10     queue<pair<int, int> > q;
11     struct PRE{int s, x, y;} pre[MAXST][MAXH][MAXW];
12
13     void spfa(int st) {
14         while(!q.empty()) {
15             int vx = q.front().first,vy = q.front().second;
16             q.pop();
17             vis[vx][vy] = 0;
18             for (int i = 0, ux,uy; i < 4; ++i) {
19                 ux = vx + dx[i];
20                 uy = vy + dy[i];
21                 if (ux==0||uy==0||ux==n+1||uy==m+1)continue;
22                 if (f[st][vx][vy]+G[ux][uy]<f[st][ux][uy]) {
23                     f[st][ux][uy] = f[st][vx][vy] + G[ux][uy];
24                     pre[st][ux][uy] = (PRE) {st, vx, vy};
25                     if (!vis[ux][uy]) {
26                         vis[ux][uy]=1;
27                         q.push(make_pair(ux,uy));
28                     }
29                 }
30             }
31         }
32     }
33     LL sum = 0;
34     void init() {
35         k = 0;
36         for(int i = 1; i <= n; ++i)
37             for(int j = 1; j <= m; ++j) {
38                 scanf("%d", &G[i][j]);
39                 if(G[i][j] == 0) k++;
40             }
41         stn = 1<<k;
42         for (int st = 0; st < stn; ++st)
43             for (int i = 1; i <= n; ++i)
44                 for (int j = 1; j <= m; ++j)
45                     f[st][i][j] = INF;
46         int tk = 0;
47         for(int i = 1; i <= n; ++i)

```

```

48         for(int j = 1; j <= m; ++j) {
49             vis[i][j] = 0;
50             if(G[i][j] == 0) {f[1<<tk][i][j] = 0; tk++;}
51         }
52     }
53     void dfs(int st, int x, int y) {
54         vis[x][y] = 1;
55         PRE tmp = pre[st][x][y];
56         if (tmp.x == 0 && tmp.y == 0) return ;
57         dfs(tmp.s, tmp.x, tmp.y);
58         if (tmp.x == x && tmp.y == y) dfs(st - tmp.s, tmp.x, tmp.y);
59     }
60     void sol(int _n, int _m) {
61         n = _n; m = _m;
62         init();
63         for (int st = 0; st < stn; ++st) {
64             for (int i = 1; i <= n; ++i)
65                 for (int j = 1; j <= m; ++j) {
66                     for (int s = st&(st-1); s; s = st&(s-1))
67                         if(f[st-s][i][j]+f[s][i][j]-G[i][j] < f[st][i][j]){
68                             f[st][i][j] = f[st-s][i][j]+f[s][i][j]-G[i][j];
69                             pre[st][i][j] = (PRE) {s, i, j};
70                         }
71                     if (f[st][i][j]!=INF) {
72                         q.push(make_pair(i,j));
73                         vis[i][j]=1;
74                     }
75                 }
76             spfa(st);
77         }
78         int ansx, ansy, fg = 0;
79         for(int i = 1; i <= n && !fg; ++i)
80             for(int j = 1; j <= m; ++j)
81                 if(!G[i][j]) {ansx = i; ansy = j; fg = 1; break;}
82         printf("%d\n", f[stn-1][ansx][ansy]);
83         memset(vis, 0, sizeof vis);
84         dfs(stn-1, ansx, ansy);
85         for(int i = 1; i <= n; i++, puts("")) {
86             for(int j = 1; j <= m; j++) {
87                 if(G[i][j] == 0) putchar('x');
88                 else if(vis[i][j]) putchar('o');
89                 else putchar('_');
90             }
91         }
92     }
93 }

```

1.3.7 Tree Divide and Conquer

```

1  struct Edge {
2      int to, nxt, w;
3  }e[MAXM];
4  int head[MAXN], ecnt;
5  int sz[MAXN];
6  int d[MAXN], t[5], ans;
7  bool vis[MAXN];
8  inline void add_edge(int u, int v, int w) {
9      e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
10     e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
11 }

```

```

12 int getsz(int x, int fa) {
13     sz[x] = 1;
14     for(int i = head[x]; i; i = e[i].nxt) {
15         int y = e[i].to;
16         if(vis[y] || y == fa) continue;
17         sz[x] += getsz(y, x);
18     }
19     return sz[x];
20 }
21 int getrt(int x) {
22     int tot = getsz(x, 0) >> 1;
23     while(1) {
24         int u = -1;
25         for(int i = head[x]; i; i = e[i].nxt) {
26             int y = e[i].to;
27             if(vis[y] || sz[y] > sz[x]) continue;
28             if(u == -1 || sz[y] > sz[u]) u = y;
29         }
30         if(~u && sz[u] > tot) x = u;
31         else break;
32     }
33     return x;
34 }
35 void getdep(int x, int fa) {
36     t[d[x]]++;
37     for(int i = head[x]; i; i = e[i].nxt) {
38         int y = e[i].to;
39         if(vis[y] || y == fa) continue;
40         d[y] = (d[x] + e[i].w) % 3;
41         getdep(y, x);
42     }
43 }
44 int cal(int x, int v) {
45     t[0] = t[1] = t[2] = 0;
46     d[x] = v % 3;
47     getdep(x, 0);
48     return t[0] * t[0] + t[1] * t[2] * 2;
49 }
50 void solve(int x) {
51     vis[x] = 1;
52     ans += cal(x, 0);
53     for(int i = head[x]; i; i = e[i].nxt) {
54         int y = e[i].to;
55         if(vis[y]) continue;
56         ans -= cal(y, e[i].w);
57         solve(getrt(y));
58     }
59 }
60 int main() {
61     solve(getrt(1));
62 }

```

1.3.8 Dominator Tree

```

1 #define LL long long
2 #define FILE "dagch"
3 using namespace std;
4
5 const int N = 200010;
6 struct Node{int to,next;}E[N<<1];

```

```

7  int n,m,q,head[N],tot,dfn[N],clo,rev[N],fa[N],semi[N],Ans[N];
8  vector<int>G[N];
9  struct Union_Merge_Set{
10     int fa[N],Mi[N];
11     inline void init(){
12         for(int i=0;i<=n;++i)
13             fa[i]=Mi[i]=semi[i]=i;
14     }
15     inline int find(int x){
16         if(x==fa[x])return x;
17         int fx=fa[x],y=find(fa[x]);
18         if(dfn[semi[Mi[fx]]]<dfn[semi[Mi[x]]])Mi[x]=Mi[fx];
19         return fa[x]=y;
20     }
21 }uset;
22
23 inline void tarjan(int u) {
24     rev[dfn[u] = ++tarjan_time] = u;
25     for(auto v : G[u])
26         if(!dfn[v]) {
27             fa[v] = u;
28             tarjan(v);
29         }
30 }
31 inline void get_semi() {
32     for(int i = tarjan_time; i >= 2; i--) {
33         int u = rev[i], tsemi = n;
34         for(auto v : rG[u]) {
35             if(!dfn[v]) continue;
36             if(dfn[v] < dfn[u]) tsemi = min(tsemi, dfn[v]);
37             else{
38                 uset.find(x);
39                 tsemi = min(tsemi, dfn[semi[uset.Mi[x]]]);
40             }
41         }
42         uset.fa[y] = fa[y];
43         semi[y] = rev[tsemi];
44         Ans[rev[tsemi]]++;
45     }
46 }
47
48 inline void solve() {
49     scanf("%d %d %d", &n, &m, &q);
50     fa[1]=1;
51     for(int i = 1, u, v; i <= m; ++i){
52         scanf("%d%d", &u,&v);
53         link(v,u);
54         G[u].push_back(v);
55     }
56     for(int i = 1; i <= n; i++)
57         if(G[i].size())
58             sort(G[i].begin(), G[i].end());
59
60     uset.init();
61
62     tarjan(1);
63     build();
64     for(int i=1;i<=q;++i)
65         printf("%d ",Ans[gi()]);
66     printf("\n");
67     for(int i=0;i<=n;++i){

```

```

68     G[i].clear();head[i]=0;
69     Ans[i]=semi[i]=fa[i]=0;
70 }
71 clo=tot=0;
72 }
73
74 int main() {
75     int T; scanf("%d", &T);
76     while(T--) solve();
77     return 0;
78 }

```

1.4 LCA

1.4.1 Tree Decomposition LCA

见树链剖分

1.4.2 Tarjan LCA

```

1  vector< pair<int,int> > G[MAXN],ask[MAXN];
2  int fa[MAXN], ans[MAXN], vis[MAXN],dis[MAXN];
3  int Find(int x){
4      return x == fa[x] ? x : fa[x] = Find(fa[x]);
5  }
6  void init(int n){
7      memset(ans, 0,sizeof ans);
8      memset(vis, 0,sizeof vis);
9      for(int i = 0; i <= n; i++){
10         G[i].clear();
11         ask[i].clear();
12     }
13 }
14 void LCA(int u){
15     int v;
16     fa[u] = u;
17     vis[u] = true;
18     for(auto it : ask[u])
19         if(vis[v = it.first])
20             ans[it.second] = dis[u] + dis[v] - 2 * dis[Find(it.first)];
21     for(auto it : G[u])
22         if(!vis[v = it.first]){
23             dis[v] = dis[u] + it.second;
24             LCA(v);
25             fa[v] = u;
26         }
27 }

```

1.5 Tarjan

1.5.1 SCC

```

1  namespace SCC{
2      vector<int> G[MAXN];
3      int dfs_clock, scc_cnt, dfn[MAXN], low[MAXN], sccno[MAXN];
4      stack<int> S;
5      void addEdge(int u, int v) {

```



```

6      G[u].push_back(v);
7  }
8  void tarjan(int u) {
9      dfn[u] = low[u] = ++dfs_clock;
10     S.push(u);
11     for(auto v : G[u]) {
12         if(!dfn[v]) {
13             tarjan(v);
14             low[u] = min(low[u], low[v]);
15         }else if(!sccno[v]) {
16             low[u] = min(low[u], dfn[v]);
17         }
18     }
19     if(dfn[u] == low[u]) {
20         scc_cnt++;
21         for(;;) {
22             int v = S.top(); S.pop();
23             sccno[v] = scc_cnt;
24             if(v == u) break;
25         }
26     }
27 }
28 void findSCC(int n) {
29     for(int i = 1; i <= n; i++)
30         if(!dfn[i]) tarjan(i);
31 }
32 void init(int n){
33     dfs_clock = scc_cnt = 0;
34     for(int i = 0; i <= n; ++i){
35         dfn[i] = low[i] = sccno[i] = 0;
36         G[i].clear();
37     }
38 }
39 }

```

1.5.2 BCC

```

1  namespace BCC{
2      struct Edge {
3          int to, nxt;
4      }e[MAXM << 1];
5      int ecnt, head[MAXN];
6      int dfs_clock, dfn[MAXN], low[MAXN];
7
8      int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
9      vector<int> vbcc[MAXN];
10     stack<int> vS;
11
12     int ebcc_cnt, ebccno[MAXN];
13     stack<int> eS;
14
15     inline void addEdge(int u, int v) {
16         e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
17         e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
18     }
19     inline void init(int n) {
20         ecnt = 1;
21         dfs_clock = 0;
22         vbcc_cnt = 0;
23         ebcc_cnt = 0;

```

```

24     for(int i = 1; i <= n; ++i){
25         head[i] = dfn[i] = low[i] = 0;
26         is_vertex[i] = 0;
27         vbccno[i] = 0;
28         ebccno[i] = 0;
29     }
30     while(!vS.empty()) vS.pop();
31 }
32 //root's edge = -1;
33 void tarjan(int u, int edge) {
34     dfn[u] = low[u] = ++dfs_clock;
35     int ch = 0;
36     vS.push(u);
37     eS.push(u);
38     for(int i = head[u], v; i; i = e[i].nxt) {
39         if(!dfn[v = e[i].to]) {
40             tarjan(v, i ^ 1);
41             low[u] = min(low[u], low[v]);
42             if(low[v] >= dfn[u]) {
43                 ++ch;
44                 if(edge > 0 || ch > 1) is_vertex[u] = 1;
45                 vbcc[++vbcc_cnt].clear();
46                 vbcc[vbcc_cnt].push_back(u);
47                 for(int x;;){
48                     x = vS.top();vS.pop();
49                     vbcc[vbcc_cnt].push_back(x);
50                     vbccno[x] = vbcc_cnt;
51                     if(x == v)break;
52                 }
53             }
54             if(low[v] > dfn[u]) {
55                 // i && i ^ 1 is bridge
56             }
57         }
58         else if(dfn[v] < dfn[u] && i != edge)
59             low[u] = min(low[u], dfn[v]);
60     }
61     if(dfn[u] == low[u]) {
62         ebcc_cnt++;
63         for(int v;;) {
64             v = eS.top(); eS.pop();
65             ebccno[v] = ebcc_cnt;
66             if(v == u) break;
67         }
68     }
69 }
70 void findBCC(int n){
71     for(int i = 1; i <= n; i++)
72         if(!dfn[i]) tarjan(i, -1);
73
74     //findBridge
75     for(int u = 1; u <= n; u++) {
76         for(int i = head[u], v; i; i = e[i].nxt)
77             if(ebccno[u] != ebccno[v = e[i].to]) {
78                 //is bridge
79             }
80     }
81 }
82 }

```

1.6 Cactus

1.6.1 Circle-Square Tree

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef pair<int, int> P;
4  const int MAXN = 2e4 + 5;
5  const int S = 15;
6  namespace Tree {
7      struct Edge {
8          int to, nxt, w;
9      }e[MAXN << 1];
10     int ecnt, head[MAXN];
11     int rt, isrt[MAXN], fa[MAXN][S + 3];
12     int sz[MAXN];
13     inline void addEdge(int u, int v, int w) {
14         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
15         fa[v][0] = u;
16     }
17 }
18 int n, m, Q;
19 namespace BCC {
20     struct Edge {
21         int to, nxt, w;
22     }e[MAXN << 1];
23     int ecnt, head[MAXN];
24     int dfs_clock, dfn[MAXN], low[MAXN];
25     int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
26     vector<P> vbcc[MAXN];
27     stack<P> vs;
28     int tag[MAXN];
29     inline void addEdge(int u, int v, int w) {
30         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
31         e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
32     }
33     inline void init(int n) {
34         ecnt = 1;
35         dfs_clock = 0;
36         vbcc_cnt = 0;
37         for(int i = 0; i <= 2 * n; i++){
38             head[i] = dfn[i] = low[i] = 0;
39             vbccno[i] = 0;
40             tag[i] = 0;
41         }
42         while(!vs.empty()) vs.pop();
43     }
44     //root's edge = -1;
45     void tarjan(int u, int edge) {
46         dfn[u] = low[u] = ++dfs_clock;
47         vs.push(P(u, e[edge ^ 1].w));
48         for(int i = head[u], v; i; i = e[i].nxt) {
49             if(!dfn[v = e[i].to]) {
50                 tarjan(v, i ^ 1);
51                 low[u] = min(low[u], low[v]);
52                 if(low[v] >= dfn[u]) {
53                     if(vs.top().first == v) {
54                         Tree::addEdge(u, v, vs.top().second);
55                         vs.pop();
56                         continue;

```

```

57         }
58         vbcc[++vbcc_cnt].clear();
59         vbcc[vbcc_cnt].push_back(P(u, 0));
60         Tree::isrt[u] = 1;
61         int &sz = Tree::sz[n + vbcc_cnt];
62         tag[vs.top().first] = n + vbcc_cnt;
63         //Tree::addEdge(u, rt, 0);
64         for(P x;;) {
65             x = vs.top(); vs.pop();
66             sz += x.second;
67             //Tree::addEdge(rt, x.first, sz);
68             vbcc[vbcc_cnt].push_back(x);
69             vbccno[x.first] = vbcc_cnt;
70             if(x.first == v) break;
71         }
72     }
73 }
74 else if(dfn[v] < dfn[u] && i != edge)
75     low[u] = min(low[u], dfn[v]);
76 }
77 for(int i = head[u], v; i; i = e[i].nxt) {
78     if(tag[v = e[i].to]) {
79         int r = tag[v]; Tree::sz[r] += e[i].w;
80         tag[v] = 0;
81     }
82 }
83 }
84 void findBCC(int n) {
85     for(int i = 1; i <= n; i++)
86         if(!dfn[i]) tarjan(i, -1);
87 }
88 }
89 namespace Tree {
90     int dis[MAXN], dep[MAXN], len[MAXN];
91     inline void init(int n) {
92         BCC::init(n);
93         rt = n;
94         ecnt = 1;
95         for(int i = 0; i <= 2 * n; i++) {
96             head[i] = 0;
97             fa[i][0] = isrt[i] = dis[i] = dep[i] = len[i] = 0;
98         }
99     }
100     void dfs(int x) {
101         for(int i = head[x], y; i; i = e[i].nxt) {
102             if(!dep[y = e[i].to]) {
103                 dep[y] = dep[x] + 1;
104                 dis[y] = dis[x] + e[i].w;
105                 dfs(y);
106             }
107         }
108     }
109     void pre() {
110         for(int k = 1; k <= BCC::vbcc_cnt; k++) {
111             rt++;
112             vector<P> &E = BCC::vbcc[k];
113             addEdge(E[0].first, rt, 0);
114             int cnt = 0;
115             for(int i = E.size() - 1; i >= 1; i--) {
116                 cnt += E[i].second;
117                 len[E[i].first] = cnt;

```

```

118         addEdge(rt, E[i].first, min(cnt, sz[rt] - cnt));
119     }
120 }
121 for(int k = 1; k <= S; k++) {
122     for(int i = 1; i <= rt; i++) {
123         fa[i][k] = fa[fa[i][k - 1]][k - 1];
124     }
125 }
126 dep[1] = 1;
127 dfs(1);
128 }
129 int up(int x, int d) {
130     for(int i = S; i >= 0; i--) {
131         if(dep[fa[x][i]] >= d) x = fa[x][i];
132     }
133     return x;
134 }
135 int lca(int u, int v) {
136     if(dep[u] > dep[v]) swap(u, v);
137     v = up(v, dep[u]);
138     if(u == v) return u;
139     for(int i = S; i >= 0; i--) {
140         if(fa[u][i] != fa[v][i]) {
141             u = fa[u][i], v = fa[v][i];
142         }
143     }
144     return fa[u][0];
145 }
146 int query(int u, int v) {
147     int l = lca(u, v);
148     if(l <= n) return dis[u] + dis[v] - 2 * dis[l];
149     int x = up(u, dep[l] + 1), y = up(v, dep[l] + 1);
150     int res = dis[u] - dis[x] + dis[v] - dis[y];
151     int tmp = abs(len[x] - len[y]);
152     return res + min(tmp, sz[l] - tmp);
153 }
154 }
155
156 int main() {
157     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
158     using namespace Tree;
159     cin >> n >> m >> Q;
160     init(n);
161     for(int i = 1, u, v, w; i <= m; i++) {
162         cin >> u >> v >> w;
163         BCC::addEdge(u, v, w);
164     }
165     BCC::findBCC(n);
166     pre();
167     int u, v;
168     while(Q--) {
169         cin >> u >> v;
170         cout << query(u, v) << endl;
171     }
172     return 0;
173 }

```

2 Data Structures

2.1 Basic Structures

2.1.1 RMQ

```

1 struct RMQ {
2     int d[MAXN][S + 2];
3     inline void init(int *a, int n) {
4         for(int i = 1; i <= n; i++) d[i][0] = a[i];
5         for(int k = 1; (1 << k) <= n; k++)
6             for(int i = 1; i + (1 << k) - 1 <= n; i++)
7                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
8     }
9     inline int query(int l, int r) {
10        if(l > r) swap(l, r);
11        int k = 0;
12        while((1 << (k + 1)) <= r - l + 1) k++;
13        return min(d[l][k], d[r - (1 << k) + 1][k]);
14    }
15 }rmq;
16 const int MAXM = 2e5 + 5, MAXN = 3e6 + 5, S = 22;
17 const LL INF = 1e18;
18 #define belong(x) (x / S + 1)
19 #define pos(x) (x % S + 1)
20 int Log[MAXN], sz;
21 struct RMQ {
22     LL a[MAXN];
23     LL d[MAXM][S + 2];
24     LL pre[MAXM][S + 2], aft[MAXM][S + 2];
25     inline void init(int n) {
26         sz = n / S + 1;
27         Log[0] = -1; for(int i = 1; i <= n; i++) Log[i] = Log[i / 2] + 1;
28         for(int i = 1; i <= sz; i++) {
29             pre[i][0] = aft[i][S + 1] = INF;
30         }
31         for(int i = 1; i <= n; i++) {
32             pre[belong(i)][pos(i)] = min(pre[belong(i)][pos(i) - 1], a[i]);
33         }
34         for(int i = n; i >= 1; i--) {
35             aft[belong(i)][pos(i)] = min(aft[belong(i)][pos(i) + 1], a[i]);
36         }
37         for(int i = 1; i <= sz; i++) {
38             d[i][0] = aft[i][1];
39         }
40         for(int k = 1; k <= S; k++)
41             for(int i = 1; i + (1 << k) <= sz; i++)
42                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
43     }
44     inline LL ask(int l, int r) {
45         assert(l <= r);
46         LL res = INF;
47         if(belong(l) == belong(r)) {
48             for(int i = l; i <= r; i++) res = min(res, a[i]);
49             return res;
50         }
51         res = min(aft[belong(l)][pos(l)], pre[belong(r)][pos(r)]);
52         int k = Log[belong(r) - belong(l) - 1];
53         if(~k) {

```

```

54         res = min(res, d[belong(l) + 1][k]);
55         res = min(res, d[belong(r) - (1 << k)][k]);
56     }
57     return res;
58 }
59 }rmq;

```

2.1.2 Divide Blocks

```

1  int belong[MAXN], l[MAXN], r[MAXN];
2  int sz, num;
3  void build(int n) {
4      sz = sqrt(n);
5      num = n / sz; if(n % sz) num++;
6      for(int i = 1; i <= num; i++) {
7          l[i] = (i - 1) * sz + 1;
8          r[i] = i * sz;
9      }
10     r[num] = n;
11     for(int i = 1; i <= n; i++) {
12         belong[i] = (i - 1) / sz + 1;
13     }
14 }

```

2.2 Heap Structures

2.2.1 Leftist Tree

```

1  const int MAXN = ;
2  namespace LeftistTree{
3      int ls[MAXN], rs[MAXN];
4      int dis[MAXN];
5      int fg[MAXN], sfa[MAXN], rt[MAXN]; //利用 rt 得到堆根节点
6      int val[MAXN];
7
8      void push_down(int x) {};
9      void push_up(int x) {
10         if(dis[ls[x]] < dis[rs[x]]) swap(ls[x], rs[x]);
11         dis[x] = dis[rs[x]] + 1;
12     }
13     int merge(int x, int y) {
14         if(!x || !y) return x^y;
15         if(val[x] > val[y] || (val[x] == val[y] && x > y)) swap(x, y);
16         push_down(x);
17         rs[x] = merge(rs[x], y);
18         push_up(x);
19         return x;
20     }
21     int getSfa(int x) {return sfa[x] == x ? x : sfa[x] = getSfa(sfa[x]);}
22     int uni(int x, int y) { //返回合并后的根
23         if(!x || !y) return x^y;
24         if(fg[x] || fg[y]) return;
25         x = getSfa(x); y = getSfa(y);
26         if(x == y) return;
27         int z = merge(x, y);
28         return sfa[x] = sfa[y] = sfa[z] = z;
29     }
30     void uni2(int a, int b) {

```

```

31 //val[a] >= 1;
32 int c = merge(ls[a], rs[a]);
33 ls[a] = rs[a] = dis[a] = 0;
34 int a1 = merge(c, a);
35 //val[b] >= 1;
36 c = merge(ls[b], rs[b]);
37 ls[b] = rs[b] = dis[b] = 0;
38 int b1 = merge(c, b);
39 c = merge(a1, b1);
40 sfa[a] = sfa[b] = sfa[c] = c;
41 printf("%d\n", val[c])
42 }
43 int pop(int x) { //返回堆顶值,也可以用于返回根
44     if(!x || fg[x]) return -1;
45     x = getSfa(x); fg[x] = 1;
46     push_down(x); //在删除堆顶时要下传标记
47     int y = merge(ls[x], rs[x]);
48     sfa[x] = sfa[y] = y;
49     return val[x];
50 }
51 void init(int n) {
52     for(int i = 1; i <= n; i++) {
53         sfa[i] = i;
54         ls[i] = rs[i] = dis[i] = fg[i] = 0;
55     }
56 }
57 }
58 //可持久化版本见k短路

```

2.3 Sequence Structures

2.3.1 Cartesian Tree

```

1 struct CartesianTree{
2     int rt, fa[MAXN], ls[MAXN], rs[MAXN];
3     int top, st[MAXN];
4     int cnt[MAXN];
5     void build(LL *a, int n) {
6         top = rt = 0;
7         for(int i = 1; i <= n; i++) {
8             ls[i] = rs[i] = fa[i] = 0;
9             while(top && a[st[top]] > a[i]) ls[i] = st[top--];
10            fa[i] = st[top];
11            if(ls[i]) fa[ls[i]] = i;
12            if(fa[i]) rs[fa[i]] = i; else rt = i;
13            st[++top] = i;
14        }
15    }
16    void dfs(int x) {
17        cnt[x] = 1;
18        if(ls[x]) {dfs(ls[x]); cnt[x] += cnt[ls[x]];}
19        if(rs[x]) {dfs(rs[x]); cnt[x] += cnt[rs[x]];}
20    }
21    LL getAns(LL *a, int n) {
22        //dfs(rt);
23        //_____
24        return res;
25    }
26 }T;

```


2.3.2 TreeArray

```

1 //树状数组上二分
2 int BS(int x) {
3     int res = 0;
4     for (int i = 1 << 18; i; i >>= 1)
5         if ((res | i) <= Tn && T[res | i] <= x)
6             x -= T[res | i];
7     return res;
8 }

```

2.3.3 Segment Tree

```

1 #define Ls(x) (x << 1)
2
3 #define Rs(x) (x << 1 | 1)
4 struct Tree {
5     int l, r, lazy;
6     LL sum, mx;
7 }tree[MAXN << 2];
8 int A[MAXN];
9 void push_up(int x) {
10     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
11     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
12 }
13 void push_down(int x) {
14     if(tree[x].lazy) {
15         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
16         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
17         tree[Ls(x)].mx += tree[x].lazy;
18         tree[Rs(x)].mx += tree[x].lazy;
19         tree[Ls(x)].lazy += tree[x].lazy;
20         tree[Rs(x)].lazy += tree[x].lazy;
21         tree[x].lazy = 0;
22     }
23 }
24 void build(int x, int L, int R) {
25     tree[x].lazy = 0;
26     tree[x].l = L; tree[x].r = R;
27     if(L == R) {
28         tree[x].sum = A[L];
29         tree[x].mx = A[L];
30
31         return;
32     }
33     int mid = (L + R) >> 1;
34     build(Ls(x), L, mid);
35     build(Rs(x), mid + 1, R);
36     push_up(x);
37 }
38 void update(int x, int L, int R, LL val) {
39     if(tree[x].l >= L && tree[x].r <= R) {
40         tree[x].lazy += val;
41         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
42         tree[x].mx += val;
43         return;
44     }
45     push_down(x);
46     int mid = (tree[x].l + tree[x].r) >> 1;

```

```

47     if(L <= mid) update(Ls(x), L, R, val);
48     if(R > mid) update(Rs(x), L, R, val);
49     push_up(x);
50 }
51 LL query(int x, int L, int R) {
52     if(tree[x].l >= L && tree[x].r <= R)
53         return tree[x].sum;
54     push_down(x);
55     int mid = (tree[x].l + tree[x].r) >> 1;
56     LL res = 0;
57     if(L <= mid) res += query(Ls(x), L, R);
58     if(R > mid) res += query(Rs(x), L, R);
59
60     return res;
61 }
62 LL query2(int x, int L, int R) {
63     if(tree[x].l >= L && tree[x].r <= R)
64         return tree[x].mx;
65     push_down(x);
66     int mid = (tree[x].l + tree[x].r) >> 1;
67     LL res = -INF;
68     if(L <= mid) res = max(res, query2(Ls(x), L, R));
69     if(R > mid) res = max(res, query2(Rs(x), L, R));
70     return res;
71 }

```

2.3.4 LiChao Tree

```

1  const double eps = 1e-12;
2  namespace LiT{
3      const int MLIMIT = 40000;
4      typedef double LD;
5      struct line{LD k,b;int l,r,id;} T[MAXN << 2];
6      //inline LD calc(line &a,int pos) {return a.k*vec[pos]+a.b;}
7      inline LD calc(line &a,int pos) {return a.k*pos+a.b;}
8      inline double cross(line &a,line &b) {
9          if(b.k == a.k) return -1e9;
10         return (double)(a.b-b.b)/(b.k-a.k);
11     }
12     void build(int v, int l, int r) {
13         T[v].k = 0;T[v].b = -1e18;
14         T[v].l = 0;T[v].r = MLIMIT;
15         T[v].id = 0;
16         if(l == r)return;
17         int mid = (l+r)>>1;
18         build(v<<1,l,mid);
19         build(v<<1|1,mid+1,r);
20     }
21     void ins(int v,int l,int r, line k) {
22         if(k.l <= l && r <= k.r) {
23             LD fl = calc(k, l), fr = calc(k, r);
24             LD gl = calc(T[v], l), gr = calc(T[v], r);
25             if(fl - gl > eps && fr - gr > eps) T[v] = k;
26             else if(fl - gl > eps || fr - gr > eps) {
27                 int mid = (l+r)>>1;
28                 if(calc(k, mid) - calc(T[v], mid) > eps) swap(k, T[v]);
29                 //if(vec[mid] - cross(k, T[v]) > eps)
30                 if(mid - cross(k, T[v]) > eps)
31                     ins(v<<1, l, mid, k);else ins(v<<1|1, mid+1, r, k);
32             }

```

```

33         return;
34     }
35     int mid=(l+r)>>1;
36     if(k.l <= mid) ins(v<<1, l, mid, k);
37     if(mid < k.r) ins(v<<1|1, mid+1, r, k);
38 }
39 LD ans;int ansid;
40 void que(int v, int l, int r, int x) {
41     LD tmp = calc(T[v], x);
42     if(tmp > ans || (tmp == ans && T[v].id < ansid)) {
43         ans = tmp;
44         ansid = T[v].id;
45     }
46     if(l == r) return;
47     int mid = (l+r)>>1;
48     if(x <= mid) que(v<<1,l,mid,x);else que(v<<1|1,mid+1,r,x);
49 }
50 };
51 //左闭右闭

```

2.3.5 Splay Tree

```

1 namespace splay{
2     int n, m, sz, rt;
3     int val[MAXN], id[MAXN];
4     int tr[MAXN][2], size[MAXN], fa[MAXN], rev[MAXN], s[MAXN], lazy[MAXN];
5     void push_up(int x) {
6         int l = tr[x][0], r = tr[x][1];
7         s[x] = max(val[x], max(s[l], s[r]));
8         size[x] = size[l] + size[r] + 1;
9     }
10    void push_down(int x) {
11        int l = tr[x][0], r = tr[x][1];
12        if(lazy[x]) {
13            if(l) {
14                lazy[l] += lazy[x];
15                s[l] += lazy[x];
16                val[l] += lazy[x];
17            }
18            if(r) {
19                lazy[r] += lazy[x];
20                s[r] += lazy[x];
21                val[r] += lazy[x];
22            }
23            lazy[x] = 0;
24        }
25        if(rev[x]) {
26            rev[x] = 0;
27            rev[l] ^= 1; rev[r] ^= 1;
28            swap(tr[x][0], tr[x][1]);
29        }
30    }
31    void rotate(int x, int &k) {
32        int y = fa[x];
33        int z = fa[y];
34        int l, r;
35        if(tr[y][0] == x) l = 0;
36        else l = 1;
37        r = l ^ 1;
38        if(y == k) k = x;

```

```

39     else {
40         if(tr[z][0] == y) tr[z][0] = x;
41         else tr[z][1] = x;
42     }
43     fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
44     tr[y][1] = tr[x][r]; tr[x][r] = y;
45     push_up(y); push_up(x);
46 }
47 void splay(int x, int &k) {
48     int y, z;
49     while(x != k) {
50         y = fa[x];
51         z = fa[y];
52         if(y != k) {
53             if((tr[y][0] == x) ^ (tr[z][0] == y)) rotate(x, k);
54
55             else rotate(y, k);
56         }
57         rotate(x, k);
58     }
59 }
60 int find(int x, int rank) {
61     push_down(x);
62
63     int l = tr[x][0], r = tr[x][1];
64     if(size[l] + 1 == rank) return x;
65     else if(size[l] >= rank) return find(l, rank);
66     else return find(r, rank - size[l] - 1);
67 }
68 void update(int l, int r, int v) {
69     int x = find(rt, l), y = find(rt, r + 2);
70     splay(x, rt); splay(y, tr[x][1]);
71     int z = tr[y][0];
72     lazy[z] += v;
73     val[z] += v;
74     s[z] += v;
75 }
76 void reverse(int l, int r) {
77     int x = find(rt, l), y = find(rt, r + 2);
78     splay(x, rt); splay(y, tr[x][1]);
79     int z = tr[y][0];
80     rev[z] ^= 1;
81 }
82 void query(int l, int r) {
83     int x = find(rt, l), y = find(rt, r + 2);
84     splay(x, rt); splay(y, tr[x][1]);
85     int z = tr[y][0];
86     printf("%d\n", s[z]);
87 }
88 void build(int l, int r, int f) {
89     if(l > r) return;
90     int now = id[l], last = id[f];
91     if(l == r) {
92         fa[now] = last; size[now] = 1;
93         if(l < f) tr[last][0] = now;
94         else tr[last][1] = now;
95         return;
96     }
97     int mid = (l + r) >> 1; now = id[mid];
98     build(l, mid - 1, mid); build(mid + 1, r, mid);
99     fa[now] = last;

```

```

100     push_up(now);
101     if(mid < f) tr[last][0] = now;
102     else tr[last][1] = now;
103 }
104 void init() {
105     s[0] = -INF;
106     scanf("%d%d", &n, &m);
107     for(int i = 1; i <= n + 2; i++) id[i] = ++sz;
108     build(1, n + 2, 0); rt = (n + 3) >> 1;
109 }
110 }
111 namespace splay{
112     //内存回收池见fhq_treap
113     int tcnt, root;
114     int sz[MAXN];
115     int tr[MAXN][2], fa[MAXN];
116     int val[MAXN];
117     int newnode(int w) {
118         ++tcnt;
119         sz[tcnt] = 1;
120         fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
121         //val[tcnt] = w;
122         return tcnt;
123     }
124     void push_up(int v) {
125         int l = tr[v][0], r = tr[v][1];
126         sz[v] = sz[l] + 1 + sz[r];
127     }
128     void push_down(int v) {
129         if(!v) return;
130     }
131     void init() {
132         tcnt = 2;
133         tr[root = fa[1] = 2][0] = 1;
134         sz[1] = 1; sz[2] = 2;
135         //val[1] = -INF; val[2] = INF; //权值平衡树
136         //val[1] = val[2] = 0; //位置平衡树
137         //1,2为哨兵节点,根据题意也可设置为n+1,n+2或1,n+1
138     }
139     void rotate(int x) {
140         int y = fa[x], z = fa[y];
141         push_down(y); push_down(x);
142         int lr = tr[y][1] == x;
143         if(z) tr[z][tr[z][1] == y] = x;
144         fa[x] = z;
145         fa[tr[y][lr]] = tr[x][lr^1] = y;
146         fa[tr[x][lr^1] = y] = x;
147         push_up(y); push_up(x);
148     }
149     void splay(int x, int k) {
150         for(int y, z; (y = fa[x]) != k; rotate(x)) {
151             if((z = fa[y]) != k) {
152                 if((tr[y][0] == x) ^ (tr[z][0] == y))
153                     rotate(x); else rotate(y);
154             }
155         }
156         if(!k) root = x;
157     }
158     int find(int x, int rank) {
159         push_down(x);
160         int l = tr[x][0], r = tr[x][1];

```

```

161     if(sz[l] + 1 == rank) return x;
162     if(sz[l] >= rank) return find(l, rank);
163     return find(r, rank - sz[l] - 1);
164 }
165 int build(int l, int r) {
166     if(l > r) return 0;
167     if(l == r) {
168         int num; scanf("%d", &num);
169         return newnode(num);
170     }
171     int mid = (l + r) >> 1;
172     int ls = build(l, mid-1);
173     int num; scanf("%d", &num);
174     int v = newnode(num);
175     int rs = build(mid+1, r);
176     if(ls) fa[ls] = v;
177     tr[v][0] = ls;
178     if(rs) fa[rs] = v;
179     tr[v][1] = rs;
180     push_up(v);
181     return v;
182 }
183 void insert(int pos, ...) {
184     int x = find(root, pos+1), y = find(root, pos+2);
185     splay(x, 0); splay(y, x);
186     //int z = newnode(w); //插入一个节点
187     //int z = build(1, n); //插入n个节点
188     fa[tr[y][0] = z] = y;
189     splay(z, 0);
190 }
191 void modifyOrQuery(int l, int r, int v) {
192     int x = find(root, l), y = find(root, r + 2);
193     splay(x, 0); splay(y, x);
194     int z = tr[y][0];
195     if(!z) return;
196     //标记对本身无效, 处理时将z点重新计算
197     splay(z, 0);
198 }
199 void display(int v) {
200     if(!v) return;
201     push_down(v);
202     display(tr[v][0]);
203     if(val[v]) printf("%d ", val[v]);
204     display(tr[v][1]);
205 }
206 /*int findValue(int v) {
207     int res = root;
208     for(int cur = root; cur; res = cur, cur = tr[cur][val[cur] <= v]);
209     return res;
210 }
211 void insert(int w) {
212     int y = findValue(w);
213     int z = newnode(w);
214     fa[tr[y][val[y] <= w] = z] = y;
215     splay(z, 0);
216 }*/
217 /*void split(int v) { //splay维护区间[l, r], 区间分裂为[l, k-1], [k, k], [k+1, r];
218     //ump查看标号是否出现, mp维护子区间左端点
219     if(ump.find(v) == ump.end()) {
220         auto it = mp.upper_bound(v); --it;
221         int z = it->second;

```

```

222     splay(z, 0);
223     int pos = sz[tr[z][0]];
224     int x = find(root, pos), y = find(root, pos+rc[z]-lc[z]+2);
225     splay(x, 0); splay(y, x);
226     z = tr[y][0];
227     if(lc[z] != v) {
228         tr[z][0] = newnode(lc[z], v-1);
229         fa[tr[z][0]] = z;
230         mp[lc[z]] = tr[z][0];
231     }
232     if(rc[z] != v) {
233         tr[z][1] = newnode(v+1, rc[z]);
234         fa[tr[z][1]] = z;
235         mp[v+1] = tr[z][1];
236     }
237     lc[z] = rc[z] = v;
238     splay(z, 0);
239     mp[v] = z;
240     ump[v] = z;
241 }
242 }*/
243
244 }

```

2.3.6 Scapegoat Tree

```

1 struct ScapegoatTree{
2     int Tsn; queue<int> q;
3     int val[MAXM], ext[MAXM];
4     int sz[MAXM], tsz[MAXM];
5     int fa[MAXM], tr[MAXM][2];
6     int root;
7     double alp;
8     void init() {
9         root = 0;
10        alp = 0.7;
11    }
12    int newnode(int x) {
13        if(q.empty()) q.push(++Tsn);
14        int tcnt = q.front(); q.pop();
15        val[tcnt] = x; ext[tcnt] = 1;
16        fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
17        sz[tcnt] = tsz[tcnt] = 0;
18        return tcnt;
19    }
20    void push_up(int v) {
21        sz[v] = ext[v]; tsz[v] = 1;
22        if(tr[v][0]) {
23            sz[v] += sz[tr[v][0]];
24            tsz[v] += tsz[tr[v][0]];
25        }
26        if(tr[v][1]) {
27            sz[v] += sz[tr[v][1]];
28            tsz[v] += tsz[tr[v][1]];
29        }
30    }
31    bool isBad(int v) {
32        return (double(tsz[ tr[v][0] ]) > double(tsz[v]) * alp) ||
33            (double(tsz[ tr[v][1] ]) > double(tsz[v]) * alp) ||
34            (sz[v] * 2 < tsz[v]);

```

```

35     }
36     vector<int> vec;
37     void rRecycle(int v) {
38         if(tr[v][0]) rRecycle(tr[v][0]);
39         if(ext[v]) vec.push_back(v); else q.push(v);
40         if(tr[v][1]) rRecycle(tr[v][1]);
41     }
42     int rBuild(int l, int r) {
43         int mid = (l + r) >> 1, v = vec[mid];
44         tr[v][0] = (l <= mid-1) ? rBuild(l, mid - 1) : 0;
45         if(tr[v][0]) fa[tr[v][0]] = v;
46         tr[v][1] = (mid+1 <= r) ? rBuild(mid + 1, r) : 0;
47         if(tr[v][1]) fa[tr[v][1]] = v;
48         push_up(v);
49         return v;
50     }
51     void rebuild(int x) {
52         int v = 0;
53         for(;x; x= fa[x]) {
54             push_up(x);
55             if(isBad(x)) v = x;
56         }
57         if(v && isBad(v)){
58             vec.clear();
59             int u = fa[v], lr = tr[u][1] == v;
60             rRecycle(v);
61             if(vec.size()) v = rBuild(0, vec.size() - 1); else v = 0;
62             if(u == 0) fa[root = v] = 0;
63             else{
64                 tr[u][lr] = v;
65                 if(v) fa[v] = u;
66             }
67         }
68     }
69     void ins(int x) {
70         int p = root, q = root;
71         for(;p && val[p] != x; q = p, p = tr[p][ val[p] < x]) ;
72         if(!q) {
73             p = root = newnode(x);
74         }else if(p) {
75             ext[p]++;
76         }else{
77             fa[p = tr[q][val[q] < x] = newnode(x) ] = q;
78         }
79         rebuild(p);
80     }
81     void del(int x) {
82         int p = root;
83         for(;p && val[p] != x; p = tr[p][ val[p] < x]);
84         if(p && ext[p]){
85             --ext[p];
86             rebuild(p);
87         }
88     }
89     int get_rank(int x) {
90         int ret = 0;
91         for(int p = root;p;) {
92             if(val[p] < x) {
93                 ret += sz[tr[p][0]] + ext[p];
94                 p = tr[p][1];
95             }else p = tr[p][0];

```



```

96     }
97     return ret + 1;
98 }
99 int get_Kth(int p, int k) {
100     if(sz[tr[p][0]] >= k) return get_Kth(tr[p][0], k);
101     k -= sz[tr[p][0]];
102     if(ext[p] >= k) return val[p];
103     k -= ext[p];
104     return get_Kth(tr[p][1], k);
105 }
106 int pre(int x) {
107     int id = get_rank(x);
108     return get_Kth(root, id - 1);
109 }
110 int nxt(int x) {
111     int id = get_rank(x + 1);
112     return get_Kth(root, id);
113 }
114 void display(int v) {
115     if(tr[v][0]) display(tr[v][0]);
116     cerr<<val[v]<<" ";
117     if(tr[v][1]) display(tr[v][1]);
118 }
119 }T;

```

2.3.7 FHQ Treap

```

1 namespace fhq_treap{
2     int Tsz; queue<int> q; //内存回收池
3     int tcnt, root;
4     //int rt[MAXN]; //可持久化时使用rt,维护版本号(int &root), 空间开大
5     int sz[MAXN], rnd[MAXN];
6     int tr[MAXN][2];
7     //int fa[MAXN]; //维护fa时除了在pushup更新v节点左右孩子父节点信息还要在split和merge结
8     //束时维护root的fa信息, fa[root]=0;
9     int val[MAXN], rev[MAXN];
10    void init() {
11        srand(time(0));
12        Tsz = tcnt = root = 0;
13    }
14    int newnode(int v) {
15        if(q.empty()) q.push(++Tsz);
16        tcnt = q.front(); q.pop();
17        sz[tcnt] = 1;
18        rnd[tcnt] = rand();
19        tr[tcnt][0] = tr[tcnt][1] = 0;
20        //val[tcnt] = v;
21        return tcnt;
22    }
23    /*int copynode(int id) {
24        //++tcnt; //获取一个新的节点编号
25        sz[tcnt] = sz[id];
26        rnd[tcnt] = rnd[id];
27        tr[tcnt][0] = tr[id][0];
28        tr[tcnt][1] = tr[id][1];
29        //val[tcnt] = val[id];
30        return tcnt;
31    }*/
32    void push_up(int v) {
33        int l = tr[v][0], r = tr[v][1];

```

```

33     sz[v] = sz[l] + 1 + sz[r];
34 }
35 void push_down(int v) {
36     if(!v) return;
37     int l = tr[v][0], r = tr[v][1];
38     //if(l) ;
39     //if(r) ;
40     //swap时候注意交换 tr[v][0]和tr[v][1],而不是l和r;
41 }
42 /* //可持久化在push_down是要新建节点,否者历史版本有可能被多次下传,以rev为例
43 void push_down(int v) {
44     if(!v || !rev[v]) return;
45     int &l = tr[v][0], &r = tr[v][1];
46     if(l) {
47         l = copynode(l);
48         rev[l]^=1;
49         swap(tr[l][0], tr[l][1]);
50     };
51     if(r) {
52         r = copynode(r);
53         rev[r]^=1;
54         swap(tr[r][0], tr[r][1]);
55     };
56     rev[v] = 0;
57 }*/
58 void split(int v,int k,int &x,int &y) {
59     if(!v) {x=y=0;return;}
60     push_down(v);
61     //v = copynode(v); //可持久化时复制节点
62     /*if(k > sz[tr[v][0]]) {
63         x = v;
64         split(tr[v][1], k-sz[tr[v][0]]-1, tr[v][1], y);
65     }else{
66         y = v;
67         split(tr[v][0], k, x, tr[v][0]);
68     }*/
69     if(val[v] <= k) {
70         x = v;
71         split(tr[v][1], k, tr[v][1], y);
72     }else{
73         y = v;
74         split(tr[v][0], k, x, tr[v][0]);
75     }
76     push_up(v);
77 }
78 int merge(int x,int y) { //x堆所有值均小于y堆
79     if(!x || !y) return x|y;
80     push_down(x); push_down(y);
81     if(rnd[x]<rnd[y]){
82         //x = copynode(x); //可持久化时复制节点,可不写
83         tr[x][1] = merge(tr[x][1],y);
84         push_up(x);
85         return x;
86     }else{
87         //y = copynode(y); //可持久化时复制节点,可不写
88         tr[y][0] = merge(x,tr[y][0]);
89         push_up(y);
90         return y;
91     }
92 }
93 void insert(int k) {

```

```

94     int x,y;
95     split(root,k,x,y);
96     root = merge(merge(x,newnode(k)),y);
97 }
98 void recycle(int v) { //回收一颗 treap 上所有节点
99     if(!v) return;
100    q.push(v);
101    recycle(tr[v][0]); recycle(tr[v][1]);
102 }
103 void erase(int k) {
104     int x,y,z;
105     split(root,k,x,y);
106     split(x,k-1,x,z);
107     z = merge(tr[z][0],tr[z][1]);
108     root = merge(x,merge(z,y));
109 }
110 void krank(int k) {
111     int x,y;
112     split(root,k-1,x,y);
113     printf("%d\n",sz[x]+1);
114     root = merge(x,y);
115 }
116 int find(int v,int k) {
117     if(sz[tr[v][0]]==k-1) return val[v];
118     if(sz[tr[v][0]]>=k) return find(tr[v][0],k);
119     return find(tr[v][1],k-sz[tr[v][0]]-1);
120 }
121 void pre(int k) {
122     int x,y;
123     split(root,k-1,x,y);
124     printf("%d\n",find(x,sz[x]));
125     root=merge(x,y);
126 }
127 void nxt(int k){
128     int x,y;
129     split(root,k,x,y);
130     printf("%d\n",find(y,1));
131     root=merge(x,y);
132 }
133 void reverse(int l,int r){
134     int x,y,z;
135     split(root, r, x, y);
136     split(x, l-1, x, z);
137     //rev[z] ^= 1; 标记对本身无效,处理时将z点重新计算
138     root = merge(merge(x,z),y);
139 }
140 /*int getRank(int S) {
141     int res = sz[tr[S][0]]+1;
142     for (; fa[S]; S = fa[S])
143         if (tr[fa[S]][1] == S) res += sz[tr[fa[S]][0]] + 1;
144     return res;
145 }*/
146 void display(int v) {
147     if(!v) return;
148     push_down(v);
149     display(tr[v][0]);
150     printf("%d ",val[v]);
151     display(tr[v][1]);
152 }
153 }

```

154 //一种可持久化平衡树的替代(非强制在线),由历史版本向当前版本连边,在dfs遍历中利用权值树状数组,普通平衡树等获取答案

2.4 Persistent Data Structures

2.4.1 Chairman Tree

```

1 struct Node {
2     int l, r;
3     LL sum;
4 }t[MAXN * 40];
5 int cnt, n;
6 int rt[MAXN];
7 void update(int pre, int &x, int l, int r, int v) {
8     x = ++cnt; t[x] = t[pre]; t[x].sum++;
9     if(l == r) return;
10    int mid = (l + r) >> 1;
11    if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
12    else update(t[pre].r, t[x].r, mid + 1, r, v);
13 }
14 int query(int x, int y, int l, int r, int v) {
15     if(l == r) return l;
16     int mid = (l + r) >> 1;
17     int sum = t[t[y].l].sum - t[t[x].l].sum;
18     if(sum >= v) return query(t[x].l, t[y].l, l, mid, v);
19     else return query(t[x].r, t[y].r, mid + 1, r, v - sum);
20 }

```

2.4.2 Unite Chairman Tree

```

1 //Q x到y路径第k大
2 //L link(x, y)
3 #include <bits/stdc++.h>
4 using namespace std;
5 typedef int LL;
6 const int MAXN = 8e4 + 5;
7 const int S = 18;
8 struct Node {
9     int l, r;
10    LL sum;
11 }t[MAXN * 800]; //2 * log^2(n)
12 int n, m, Q;
13 int cnt;
14 int rt[MAXN], sz[MAXN];
15 void update(int pre, int &x, int l, int r, int v) {
16     x = ++cnt; t[x] = t[pre]; t[x].sum++;
17     if(l == r) return;
18     int mid = (l + r) >> 1;
19     if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
20     else update(t[pre].r, t[x].r, mid + 1, r, v);
21 }
22 int query(int x, int y, int z, int w, int l, int r, int v) {
23     if(l == r) return l;
24     int mid = (l + r) >> 1;
25     int sum = t[t[x].l].sum + t[t[y].l].sum - t[t[z].l].sum - t[t[w].l].sum;
26     if(sum >= v) return query(t[x].l, t[y].l, t[z].l, t[w].l, l, mid, v);
27     return query(t[x].r, t[y].r, t[z].r, t[w].r, mid + 1, r, v - sum);
28 }

```

```

29 int fa[MAXN][S + 3], dep[MAXN];
30 int val[MAXN];
31 vector<int> G[MAXN];
32 inline void addEdge(int x, int y) {
33     G[x].push_back(y);
34     G[y].push_back(x);
35 }
36 inline void upd(int x) {
37     update(rt[fa[x][0]], rt[x], 1, n, val[x]);
38     for(int i = 1; i <= S; i++) fa[x][i] = fa[fa[x][i - 1]][i - 1];
39 }
40 inline void Go(int &x, int step) {
41     for(int i = S; i >= 0; i--) if(step >> i & 1) x = fa[x][i];
42 }
43 int lca(int x, int y) {
44     if(dep[x] < dep[y]) swap(x, y);
45     Go(x, dep[x] - dep[y]);
46     if(x == y) return x;
47     for(int i = S; i >= 0; i--) if(fa[x][i] != fa[y][i]) {
48         x = fa[x][i], y = fa[y][i];
49     }
50     return fa[x][0];
51 }
52 int get_rt(int x) {
53     for(int i = S; i >= 0; i--) if(fa[x][i]) x = fa[x][i];
54     return x;
55 }
56 void dfs(int x, int f) {
57     fa[x][0] = f;
58     dep[x] = dep[f] + 1;
59     upd(x);
60     for(auto y : G[x]) {
61         if(y == f) continue;
62         dfs(y, x);
63     }
64 }
65 void unite(int x, int y) {
66     int rx = get_rt(x), ry = get_rt(y);
67     if(sz[rx] > sz[ry]) swap(x, y), swap(rx, ry);
68     addEdge(x, y);
69     dfs(x, y);
70     sz[ry] += cnt - rt[x] + 1;
71 }
72 void init() {
73     cnt = 0;
74     for(int i = 1; i <= n; i++) {
75         rt[i] = sz[i] = 0;
76         G[i].clear();
77         dep[i] = 0;
78     }
79 }
80 int main() {
81     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
82     int T; cin >> T;
83     while(T--) {
84         cin >> n >> m >> Q;
85         init();
86         vector<int> b;
87         map<int, int> mp;
88         for(int i = 1; i <= n; i++) {
89             cin >> val[i];

```

```

90         b.push_back(val[i]);
91     }
92     sort(b.begin(), b.end());
93     b.erase(unique(b.begin(), b.end()), b.end());
94     for(int i = 1, tmp; i <= n; i++) {
95         tmp = val[i];
96         val[i] = lower_bound(b.begin(), b.end(), val[i]) - b.begin() + 1;
97         mp[val[i]] = tmp;
98     }
99     for(int i = 1, u, v; i <= m; i++) {
100         cin >> u >> v;
101         addEdge(u, v);
102     }
103     for(int i = 1; i <= n; i++) if(!dep[i]) {
104         dep[i] = 1;
105         dfs(i, 0);
106         sz[i] = cnt - rt[i] + 1;
107     }
108     char s[3]; int x, y, z, k, ans = 0;
109     while(Q--) {
110         cin >> s >> x >> y;
111         x ^= ans; y ^= ans;
112         if(s[0] == 'Q') {
113             cin >> k; k ^= ans;
114             z = lca(x, y);
115             ans = query(rt[x], rt[y], rt[z], rt[fa[z][0]], 1, n, k);
116             ans = mp[ans];
117             cout << ans << endl;
118         }
119         else {
120             unite(x, y);
121         }
122     }
123 }
124 return 0;
125 }

```

2.4.3 Persistent Trie

```

1  //区间异或最值查询
2  const int N=5e4+10;
3  int t[N];
4  int ch[N*32][2],val[N*32];
5  int cnt;
6  void init(){
7      mem(ch,0);
8      mem(val,0);
9      cnt=1;
10 }
11 int add(int root,int x){
12     int newroot=cnt++,ret=newroot;
13     for(int i=30;i>=0;i--){
14         ch[newroot][0]=ch[root][0];
15         ch[newroot][1]=ch[root][1];
16         int now=(x>>i)&1;
17         root=ch[root][now];
18
19         ch[newroot][now]=cnt++;
20         newroot=ch[newroot][now];
21         val[newroot]=val[root]+1;

```

```

22     }
23
24     return ret;
25 }
26 int query(int lt,int rt,int x){
27     int ans=0;
28     for(int i=30;i>=0;i--){
29         int now=(x>>i)&1;
30         if(val[ch[rt][now^1]]-val[ch[lt][now^1]]){
31             ans|=(1<<i);
32             rt=ch[rt][now^1];
33             lt=ch[lt][now^1];
34         } else{
35             rt=ch[rt][now];
36             lt=ch[lt][now];
37         }
38     }
39     return ans;
40 }

```

2.4.4 SGT in BBST

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int MAXN = 1e5;
4  const int MAXM = 2e7;
5  const int LM = 0;
6  const int RM = 70005;
7  namespace T{
8      int Tsz; queue<int> q;
9      int ls[MAXM], rs[MAXM], val[MAXM];
10     int Tan, Ta[105], Tbn, Tb[105];
11     int newnode() {
12         if(q.empty()) q.push(++Tsz);
13         int x = q.front(); q.pop();
14         ls[x] = rs[x] = val[x] = 0;
15         return x;
16     }
17     void insert(int &x, int l, int r, int k, int f) {
18         if(!x) x = newnode();
19         val[x] += f;
20         if(l == r) return;
21         int mid = (l + r) >> 1;
22         if(k <= mid) {
23             insert(ls[x], l, mid, k, f);
24         } else{
25             insert(rs[x], mid+1, r, k, f);
26         }
27     }
28     int query(int l, int r, int y) {
29         if(l == r) return l;
30         int mid = (l + r) >> 1;
31         int sum = 0;
32         for(int i = 0; i <= Tan; i++) sum -= val[ls[Ta[i]]];
33         for(int i = 0; i <= Tbn; i++) sum += val[ls[Tb[i]]];
34
35         if(y <= sum) {
36             for(int i = 0; i <= Tan; i++) Ta[i] = ls[Ta[i]];
37             for(int i = 0; i <= Tbn; i++) Tb[i] = ls[Tb[i]];
38             return query(l, mid, y);

```

```

39     }
40     for(int i = 0; i <= Tan; i++) Ta[i] = rs[Ta[i]];
41     for(int i = 0; i <= Tbn; i++) Tb[i] = rs[Tb[i]];
42     return query(mid+1, r, y - sum);
43 }
44 void recycle(int v) {
45     if(!v) return;
46     q.push(v);
47     recycle(ls[v]);
48     recycle(rs[v]);
49 }
50 };
51 namespace TT{
52     int Tsn;
53     int val[MAXN], Trt[MAXN];
54     int sz[MAXN];
55     int fa[MAXN], tr[MAXN][2];
56
57     int root;
58     double alp;
59
60     int newnode(int x) {
61         int tcnt = ++Tsn;
62         val[tcnt] = x; Trt[tcnt] = 0;
63         fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
64         sz[tcnt] = 0;
65         return tcnt;
66     }
67     bool isBad(int v) {
68         return (double(sz[ tr[v][0] ]) > double(sz[v]) * alp) ||
69             (double(sz[ tr[v][1] ]) > double(sz[v]) * alp);
70     }
71     vector<int> vec;
72     void rRecycle(int v) {
73         if(tr[v][0]) rRecycle(tr[v][0]);
74         vec.push_back(v);
75         T::recycle(Trt[v]);
76         if(tr[v][1]) rRecycle(tr[v][1]);
77     }
78     int rBuild(int l, int r) {
79         int mid = (l + r) >> 1, v = vec[mid];
80         Trt[v] = 0; sz[v] = r - l + 1;
81         for(int i = l; i <= mid; i++) {
82             T::insert(Trt[v], LM, RM, val[vec[i]], 1);
83         }
84         tr[v][0] = (l <= mid-1) ? rBuild(l, mid - 1) : 0;
85         if(tr[v][0]) fa[tr[v][0]] = v;
86         tr[v][1] = (mid+1 <= r) ? rBuild(mid + 1, r) : 0;
87         if(tr[v][1]) fa[tr[v][1]] = v;
88         return v;
89     }
90     void rebuild(int v) {
91         if(isBad(v)) {
92             vec.clear();
93             int u = fa[v], lr = tr[u][1] == v;
94             rRecycle(v);
95             if(vec.size()) v = rBuild(0, vec.size() - 1); else v = 0;
96             if(u == 0) fa[root = v] = 0;
97             else{
98                 tr[u][lr] = v;
99                 if(v) fa[v] = u;

```



```

100     }
101     }
102 }
103 int find(int x, int k) {
104     int l = tr[x][0], r = tr[x][1];
105     if(sz[l] + 1 == k) return x;
106     if(sz[l] >= k) return find(l, k);
107     return find(r, k - sz[l] - 1);
108 }
109 void ins(int x, int y) {
110     int v = find(root, x);
111     int p = tr[v][1], q = v;
112     if(p) {
113         for(;p; q = p, p = tr[p][0]);
114         fa[p = tr[q][0] = newnode(y)] = q;
115     } else {
116         fa[p = tr[q][1] = newnode(y)] = q;
117     }
118     int fg = 0;
119     T::insert(Trt[p], LM, RM, y, 1);
120     sz[p] = 1;
121     for(;fa[p]; p = fa[p]) {
122         if(tr[fa[p]][0] == p)
123             T::insert(Trt[fa[p]], LM, RM, y, 1);
124         sz[fa[p]]++;
125         if(isBad(fa[p])) fg = fa[p];
126     }
127     rebuild(fg);
128 }
129 void upd(int x, int y) {
130     int p = find(root, x+1);
131     int ty = val[p]; val[p] = y;
132     T::insert(Trt[p], LM, RM, ty, -1);
133     T::insert(Trt[p], LM, RM, y, 1);
134     for(;fa[p]; p = fa[p]) {
135         if(tr[fa[p]][0] == p) {
136             T::insert(Trt[fa[p]], LM, RM, ty, -1);
137             T::insert(Trt[fa[p]], LM, RM, y, 1);
138         }
139     }
140 }
141 int que(int x, int y, int z) {
142     x = find(root, x);
143     T::Ta[T::Tan = 0] = Trt[x];
144     for(;fa[x]; x = fa[x])
145         if(tr[fa[x]][1] == x) {
146             T::Ta[++T::Tan] = Trt[fa[x]];
147         }
148     y = find(root, y+1);
149     T::Tb[T::Tbn = 0] = Trt[y];
150     for(;fa[y]; y = fa[y]) {
151         if(tr[fa[y]][1] == y) {
152             T::Tb[++T::Tbn] = Trt[fa[y]];
153         }
154     }
155     return T::query(LM, RM, z);
156 }
157 void init(int n) {
158     alp = 0.7;
159     vec.clear();
160     vec.push_back(newnode(RM));

```

```

161     for(int i = 1, a; i <= n; i++) {
162         scanf("%d", &a);
163         vec.push_back(newnode(a));
164     }
165     root = rBuild(0,vec.size() - 1);
166     fa[root] = 0;
167 }
168 void display(int v) {
169     if(tr[v][0]) display(tr[v][0]);
170     cerr<<val[v]<<" ";
171     if(tr[v][1]) display(tr[v][1]);
172 }
173 };

```

2.5 Tree Structures

2.5.1 dsu on tree

```

1  const int MAXN = 1e5 + 7;
2  vector<int> G[MAXN];
3  int bgison, dfs_clock, sz[MAXN], st[MAXN], bt[MAXN], et[MAXN];
4  int fg[MAXN], col[MAXN];
5  long long ans[MAXN];
6  void dfs1(int u, int fa) {
7      sz[u] = 1;
8      st[bt[u] = ++dfs_clock] = u;
9      for(auto v : G[u])
10         if(v != fa) {
11             dfs1(v, u);
12             sz[u] += sz[v];
13         }
14         et[u] = dfs_clock;
15 }
16 int maxx = 0;
17 void dfs2(int u, int fa, int keep) {
18     int mx = -1, bigson = -1;
19     for(auto &v : G[u])
20         if(v != fa) {
21             if(sz[v] > mx)
22                 mx = sz[v], bigson = v;
23         }
24     for(auto &v : G[u])
25         if(v != fa && v != bigson)
26             dfs2(v,u,0);
27     if(bigson != -1) {
28         dfs2(bigson, u, 1);
29         ans[u] = ans[bigson];
30         for(int &v : G[u])
31             if(v != fa && v != bigson)
32                 for(int i = bt[v]; i <= et[v]; i++) {
33                     ++fg[col[st[i]]];
34                     if(fg[ col[st[i]] ] > maxx) maxx=fg[col[st[i]]], ans[u] = 0;
35                     if(fg[ col[st[i]] ] == maxx) ans[u] += col[st[i]];
36                 }
37     }
38     ++fg[col[u]];
39     if(fg[col[u]] > maxx) maxx = fg[col[u]], ans[u] = 0;
40     if(fg[col[u]] == maxx) ans[u] += col[u];
41     if(keep == 0) {

```

```

42     maxx = 0;
43     for(int i = bt[u]; i <= et[u]; i++)
44         fg[col[st[i]]] = 0;
45     }
46 }

```

2.5.2 Vitural Tree

```

1  const int MAXN = ;
2  const int MAXM = ;
3  const LL INF = ;
4  const int S = 19;
5  int ecnt, head[MAXN];
6  struct Edge{int to, nxt; LL w;} e[MAXM], ve[MAXM];
7  inline void addEdgeT(int x, int y, LL w) {
8      e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
9  }
10 int dep[MAXN], dfn_time, dfn[MAXN], fa[MAXN][S+1];
11 LL dis[MAXN][S+1];
12 void dfs(int v, int _fa) {
13     dfn[v] = ++dfn_time;
14     dep[v] = dep[_fa] + 1;
15     fa[v][0] = _fa;
16     for(int i = 1; i <= S; i++) {
17         fa[v][i] = fa[fa[v][i-1]][i-1];
18         dis[v][i] = min(dis[v][i-1], dis[fa[v][i-1]][i-1]);
19     }
20     for(int i = head[v], u; i; i = e[i].nxt)
21         if((u = e[i].to) != _fa) {
22             dis[u][0] = e[i].w;
23             dfs(u, v);
24         }
25 }
26 int getLca(int u, int v) {
27     if(dep[u] < dep[v]) swap(u, v);
28     for(int i = S; i >= 0; i--)
29         if(dep[fa[u][i]] >= dep[v]) u = fa[u][i];
30     if(u == v) return u;
31     for(int i = S; i >= 0; i--)
32         if(fa[u][i] != fa[v][i])
33             u = fa[u][i], v = fa[v][i];
34     return fa[u][0];
35 }
36 LL getDis(int u, int v) {
37     if(dep[u] < dep[v]) swap(u, v);
38     LL res = INF;
39     for(int i = S; i >= 0; i--)
40         if(dep[fa[u][i]] >= dep[v]) {
41             res = min(res, dis[u][i]);
42             u = fa[u][i];
43         }
44     return res;
45 }
46 namespace VituralTree{
47     int hn, h[MAXN];
48     int vecnt, vhead[MAXN];
49     Edge ve[MAXM];
50     int top, st[MAXN], cln, cl[MAXN];
51     int fgn, fg[MAXN]; //利用 fg[i]==fgn 判断是否为当前有效点
52     void addEdgeVT(int x, int y, LL w) {

```

```

53     ve[++vecnt] = (Edge) {y, vhead[x], w}; vhead[x] = vecnt;
54 }
55 inline void link(int u, int v) {
56     LL w = getDis(u, v);
57     addEdgeVT(u, v, w);
58     addEdgeVT(v, u, w);
59 }
60 inline bool cmp(int a, int b) {return dfn[a] < dfn[b];}
61 void build() {
62     ++fgn;
63     for(int i = 1; i <= hn; i++) fg[h[i]] = fgn;
64     sort(h + 1, h + hn + 1, cmp);
65     cl[cln = 1] = st[top = 1] = 1;
66     for(int i = 1; i <= hn; i++) {
67         int rem = getlca(st[top], h[i]);
68         if(rem == st[top]) {
69             if(rem != st[top]) cl[++cln] = st[++top] = h[i];
70             continue;
71         }
72         while(top > 1 && dep[st[top - 1]] >= dep[rem]) {
73             link(st[top - 1], st[top]); top--;
74         }
75         if(dep[st[top]] > dep[rem]) {
76             link(rem, st[top]), top--;
77         }
78         if(rem != st[top]) cl[++cln] = st[++top] = rem;
79         if(h[i] != st[top]) cl[++cln] = st[++top] = h[i];
80     }
81     while(top > 1) {
82         link(st[top - 1], st[top]), top--;
83     }
84 }
85 void clear() {
86     vecnt = 0;
87     for(;cln; --cln) vhead[cl[cln]] = 0;
88 }
89 void sol() {
90     build();
91     //注意一号节点可能在虚树外面
92     clear();
93 }
94 }

```

2.5.3 Tree Decomposition

```

1  int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
2  struct Edge {
3      int to, nxt;
4  }e[MAXN << 1];
5  int head[MAXN], ecnt;
6  int n, m, Q;
7  #define Ls(x) (x << 1)
8  #define Rs(x) (x << 1 | 1)
9  struct Tree {
10     int l, r, lazy;
11     LL sum, mx;
12 }tree[MAXN << 2];
13 int A[MAXN], B[MAXN];
14 void push_up(int x) {
15     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;

```

```

16     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
17 }
18 void push_down(int x) {
19     if(tree[x].lazy) {
20         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
21         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
22         tree[Ls(x)].mx += tree[x].lazy;
23         tree[Rs(x)].mx += tree[x].lazy;
24         tree[Ls(x)].lazy += tree[x].lazy;
25         tree[Rs(x)].lazy += tree[x].lazy;
26         tree[x].lazy = 0;
27     }
28 }
29 void build(int x, int L, int R) {
30     tree[x].lazy = 0;
31     tree[x].l = L; tree[x].r = R;
32     if(L == R) {
33         tree[x].sum = B[L];
34         tree[x].mx = B[L];
35         return;
36     }
37     int mid = (L + R) >> 1;
38     build(Ls(x), L, mid);
39     build(Rs(x), mid + 1, R);
40     push_up(x);
41 }
42 void update(int x, int L, int R, LL val) {
43     if(tree[x].l >= L && tree[x].r <= R) {
44         tree[x].lazy += val;
45         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
46         tree[x].mx += val;
47         return;
48     }
49     push_down(x);
50     int mid = (tree[x].l + tree[x].r) >> 1;
51     if(L <= mid) update(Ls(x), L, R, val);
52     if(R > mid) update(Rs(x), L, R, val);
53     push_up(x);
54 }
55 LL query(int x, int L, int R) {
56     if(tree[x].l >= L && tree[x].r <= R)
57         return tree[x].sum;
58     push_down(x);
59     int mid = (tree[x].l + tree[x].r) >> 1;
60     LL res = 0;
61     if(L <= mid) res += query(Ls(x), L, R);
62     if(R > mid) res += query(Rs(x), L, R);
63     return res;
64 }
65 LL query2(int x, int L, int R) {
66     if(tree[x].l >= L && tree[x].r <= R)
67         return tree[x].mx;
68     push_down(x);
69     int mid = (tree[x].l + tree[x].r) >> 1;
70     LL res = -INF;
71     if(L <= mid) res = max(res, query2(Ls(x), L, R));
72     if(R > mid) res = max(res, query2(Rs(x), L, R));
73     return res;
74 }
75 inline void add_edge(int x, int y) {
76     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;

```

```

77 }
78 void dfs1(int x) {
79     sz[x] = 1; son[x] = 0;
80     for(int i = head[x]; i; i = e[i].nxt) {
81         int v = e[i].to;
82         if(v == fa[x]) continue;
83         fa[v] = x;
84         dep[v] = dep[x] + 1;
85         dfs1(v);
86         sz[x] += sz[v];
87         if(sz[v] > sz[son[x]]) son[x] = v;
88     }
89 }
90 void dfs2(int x) {
91     B[num[x]] = A[x];
92     if(son[x]) {
93         top[son[x]] = top[x];
94         num[son[x]] = ++totw;
95         dfs2(son[x]);
96     }
97     for(int i = head[x]; i; i = e[i].nxt) {
98         int v = e[i].to;
99         if(v == fa[x] || v == son[x]) continue;
100         top[v] = v;
101         num[v] = ++totw;
102         dfs2(v);
103     }
104 }
105 void up(int a, int b, int c) {
106     int f1 = top[a], f2 = top[b];
107     while(f1 != f2) {
108         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
109         update(1, num[f1], num[a], c);
110         a = fa[f1];
111         f1 = top[a];
112     }
113     if(dep[a] > dep[b]) swap(a, b);
114     update(1, num[a], num[b], c);
115 }
116 int qsum(int a, int b) {
117     if(a == b) return query(1, num[a], num[a]);
118     int f1 = top[a], f2 = top[b];
119     int res = 0;
120     while(f1 != f2) {
121         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
122         res += query(1, num[f1], num[a]);
123         a = fa[f1];
124         f1 = top[a];
125     }
126     if(dep[a] > dep[b]) swap(a, b);
127     res += query(1, num[a], num[b]);
128     return res;
129 }
130 int qmax(int a, int b) {
131     if(a == b) return query2(1, num[a], num[a]);
132     int f1 = top[a], f2 = top[b];
133     int res = -1000000000;
134     while(f1 != f2) {
135         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
136         res = max(res, query2(1, num[f1], num[a]));
137         a = fa[f1];

```

```

138     f1 = top[a];
139 }
140 if(dep[a] > dep[b]) swap(a, b);
141 res = max(res, query2(1, num[a], num[b]));
142 return res;
143 }
144 inline void init() {
145     memset(head, 0, sizeof(head)); ecnt = 0;
146     fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
147 }
148 inline void pre() {
149     dfs1(1); dfs2(1); build(1, 1, totw);
150 }
151 //-----
152 const int MAXN = ;
153 const int INF = ;
154 int A[MAXN], B[MAXN], C[MAXN];
155 struct SGT{
156     int sL, sR;
157     LL sW, mx[MAXN << 2];
158     void push_down(int v) {
159     }
160     void push_up(int v) {
161         mx[v] = max(mx[v << 1], mx[v<<1|1]);
162     }
163     void build(int v, int l, int r) {
164         if(l == r) {
165             mx[v] = B[l];
166             return;
167         }
168         int mid = (l + r) >> 1;
169         build(v << 1, l, mid);
170         build(v<<1|1, mid+1, r);
171         push_up(v);
172     }
173     void upd(int v, int l, int r) {
174         if(sL <= l && r <= sR) {
175             //sum[v] = sW *(r - l + 1);
176             mx[v] = sW;
177             return;
178         }
179         push_down(v);
180         int mid = (l + r) >> 1;
181         if(sL <= mid) upd(v << 1, l, mid);
182         if(mid < sR) upd(v<<1|1, mid+1, r);
183         push_up(v);
184     }
185     void qmax(int v, int l, int r) {
186         if(sL <= l && r <= sR) {
187             sW = max(sW, mx[v]);
188             return;
189         }
190         push_down(v);
191         int mid = (l + r) >> 1;
192         if(sL <= mid) qmax(v << 1, l, mid);
193         if(mid < sR) qmax(v<<1|1, mid+1, r);
194     }
195 }T;
196 namespace TD{
197     struct Edge {int to, nxt;}e[MAXN << 1];
198     int ecnt, head[MAXN];

```

```

199 inline void addEdge(int x, int y) {
200     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
201     e[++ecnt] = (Edge) {x, head[y]}; head[y] = ecnt;
202 }
203 int fa[MAXN], sz[MAXN], dep[MAXN], son[MAXN], top[MAXN];
204 int dfn_time, dfn[MAXN], rnk[MAXN];
205 void dfs1(int u) {
206     sz[u] = 1; son[u] = -1;
207     for(int i = head[u], v; i; i = e[i].nxt) {
208         v = e[i].to;
209         if(v == fa[u]) continue;
210         fa[v] = u;
211         dep[v] = dep[u] + 1;
212         dfs1(v);
213         sz[u] += sz[v];
214         if(son[u] == -1 || sz[v] > sz[son[u]]) son[u] = v;
215     }
216 }
217 void dfs2(int u) {
218     dfn[u] = ++dfn_time;
219     //rnk[dfn_time] = u;
220     B[dfn[u]] = A[u];
221     if(son[u] == -1) return;
222     top[son[u]] = top[u];
223     dfs2(son[u]);
224     for(int i = head[u], v; i; i = e[i].nxt) {
225         v = e[i].to;
226         if(v == fa[u] || v == son[u]) continue;
227         top[v] = v;
228         dfs2(v);
229     }
230 }
231 inline void init(int n) {
232     ecnt = 1;
233     for(int i = 0; i <= n; i++) head[i] = 0;
234 }
235 inline void pre(int rt) {
236     fa[rt] = -1; dep[rt] = 1;
237     top[rt] = rt; dfn_time = 0;
238     dfs1(rt); dfs2(rt);
239     // edge -> point
240     //for(int i = 2; i <= ecnt; i += 2) {
241     //    if(e[i].to == fa[e[i^1].to]) {
242     //        C[i / 2] = e[i^1].to;
243     //    } else {
244     //        C[i / 2] = e[i].to;
245     //    }
246     //    B[dfn[C[i / 2]]] = e[i].w;
247     //}
248     T.build(1, 1, dfn_time);
249 }
250 int lca(int u, int v) {
251     while(top[u] != top[v]) {
252         if(dep[top[u]] > dep[top[v]]) swap(u, v);
253         v = fa[top[v]];
254     }
255     if(dep[u] > dep[v]) swap(u, v);
256     return u;
257 }
258 void upd(int a, int b, int c) {
259     int ta = top[a], tb = top[b];

```



```

260     while(ta != tb) {
261         if(dep[ta] < dep[tb]) { swap(a, b); swap(ta, tb); }
262         T.sL = dfn[ta]; T.sR = dfn[a]; T.sW = c;
263         T.upd(1, 1, dfn_time);
264         a = fa[ta]; ta = top[a];
265     }
266     if(dep[a] > dep[b]) swap(a, b); // point
267     //if(a == b) return; // edge
268     //if(dep[a] > dep[b]) swap(a, b); // edge
269     //a = son[a]; // edge
270     T.sL = dfn[a]; T.sR = dfn[b]; T.sW = c;
271     T.upd(1, 1, dfn_time);
272 }
273 //更新子树, 由于dfs2中有if(son[u] == -1) return; 小心使用end[u];
274 void upd2(int a, int c) {
275     T.sL = dfn[a]; T.sR = dfn[a] + sz[a] - 1; T.sW = c;
276     T.upd(1, 1, dfn_time);
277 }
278 int qmax(int a, int b) {
279     int ta = top[a], tb = top[b]; int res = - INF;
280     while(ta != tb) {
281         if(dep[ta] < dep[tb]) { swap(a, b); swap(ta, tb); }
282         T.sL = dfn[ta]; T.sR = dfn[a]; T.sW = - INF;
283         T.qmax(1, 1, dfn_time); res = max(T.sW, res);
284         a = fa[ta]; ta = top[a];
285     }
286     if(dep[a] > dep[b]) swap(a, b); // point
287     //if(a == b) return res; // edge
288     //if(dep[a] > dep[b]) swap(a, b); // edge
289     //a = son[a]; // edge
290     T.sL = dfn[a]; T.sR = dfn[b]; T.sW = -INF;
291     T.qmax(1, 1, dfn_time); res = max(T.sW, res);
292     return res;
293 }
294 }

```

2.5.4 Link-Cut Tree

```

1 namespace LCT {
2     int tcnt; //动态开点
3     int fa[MAXN], tr[MAXN][2], rev[MAXN];
4     //int val[MAXN], sval[MAXN];
5     //void clear(int id) { //或者改为int newnode() {}
6     //     fa[id] = tr[id][0] = tr[id][1] = 0;
7     //     rev[id] = 0; //sz[id] = 1;
8     //}
9     void Rev(int x) {
10         rev[x] ^= 1; swap(tr[x][0], tr[x][1]);
11     }
12     void push_up(int x) {
13         //int l = tr[x][0], r = tr[x][1];
14         //sval[x] = sval[l] + val[x] + sval[r];
15     }
16     void push_down(int x) {
17         int l = tr[x][0], r = tr[x][1];
18         if(rev[x]) {
19             if(l) Rev(l);
20             if(r) Rev(r);
21             rev[x] = 0;
22         }

```

```

23     }
24     bool isroot(int x) {
25         return tr[fa[x]][0] != x && tr[fa[x]][1] != x;
26     }
27     void pre(int x) {
28         if(!isroot(x)) pre(fa[x]);
29         push_down(x);
30     }
31     void rotate(int x) {
32         int y = fa[x], z = fa[y], lr = tr[y][1] == x;
33         if(!isroot(y)) tr[z][tr[z][1] == y] = x;
34         fa[x] = z;
35         fa[tr[y][lr]] = tr[x][lr^1] = y;
36         fa[tr[x][lr^1] = y] = x;
37         push_up(y);
38     }
39     inline void splay(int x) {
40         pre(x);
41         for (int y, z; !isroot(x); rotate(x)) {
42             y = fa[x]; z = fa[y];
43             if (!isroot(y)) rotate((tr[z][0] == y) ^ (tr[y][0] == x) ? x : y);
44         }
45         push_up(x);
46     }
47     inline int access(int x) {
48         int y = 0;
49         for (; x; y = x, x = fa[x]) {
50             splay(x);
51             //sz2[x] += sz[tr[x][1]] - sz[y]; //subtree
52             tr[x][1] = y;
53             push_up(x);
54         }
55         return y; //不求LCA不必
56     }
57     inline void makeroot(int x) {
58         access(x); splay(x); Rev(x);
59     }
60     inline int findroot(int x) {
61         access(x); splay(x);
62         for(; tr[x][0]; x = tr[x][0]) push_down(x);
63         splay(x);
64         return x;
65     }
66     inline void lnk(int x, int y) {
67         makeroot(x);
68         if(findroot(y) != x) fa[x] = y;
69         //sz2[y] += sz[x]; //subtree
70     }
71     inline void cut(int x, int y) {
72         makeroot(x); //access(y); splay(x);
73         if(findroot(y) == x && fa[y] == x && !tr[y][0]){
74             fa[y] = tr[x][1] = 0;
75             push_up(x);
76         }
77     }
78     inline void cut(int y) { //有根树断开与父节点连边
79         access(y); splay(y);
80         fa[tr[y][0]] = 0;
81         tr[y][0] = 0;
82         push_up(y);
83     }

```

```

84     inline int lca(int u, int v) {
85         access(u);
86         return access(v);
87     }
88     void split(int x, int y) {
89         makeroot(x); access(y); splay(y);
90     }
91     //维护节点或者维护路径
92     //例如：染色（注意tag_rev）、tag_add的区间信息、splay维护连续端最远位置
93     void upd(int x, int y) {
94         makeroot(x); val[x] = y; push_up(x);
95     }
96     int que(int x, int y) {
97         split(x, y);
98         //return sval[y];
99     }
100    //维护边权(y -> eid -> x)，需要初始化 vcnt, ecnt, 可用map(注意双向维护)维护eid信息
101    //注意 if(x == y) continue;
102    struct LCTEdge{int u, v; int w;} e[MAXN];
103    void addEdge(int eid) { //e[eid = ++ecnt] = (Edge){x, y, w};
104        lnk(e[eid].u, vcnt + eid);
105        lnk(vcnt + eid, e[eid].v);
106    }
107    void delEdge(int eid) {
108        cut(e[eid].u, vcnt + eid);
109        cut(vcnt + eid, e[eid].v);
110    }
111    //维护边双连通分量
112    //并查集,所有的fa[x]改为Find(fa[x]),public调用函数使用前Find(x),需要保证只在shrink进行过合并
113    //维护边双时,节点自身信息在unite中维护,路径等信息在push_up和push_down中维护
114    int Rt[MAXN];
115    int findroot(int x) {return Rt[x] == x ? x : Rt[x] = findroot(Rt[x]);}
116    void unite_dfs(int x) {
117        push_down(x);
118        if (tr[x][0]) unite_dfs(tr[x][0]), unite(tr[x][0], x);
119        if (tr[x][1]) unite_dfs(tr[x][1]), unite(tr[x][1], x);
120    }
121    void shrink(int x, int y) {
122        split(x, y);
123        unite_dfs(y);
124        int z = Find(y);
125        fa[z] = fa[y]; tr[z][0] = tr[z][1] = 0;
126        push_up(z);
127    }
128    void addEdge(int x, int y) {
129        x = Find(x); y = Find(y);
130        if(findroot(x) != findroot(y)) {
131            lnk(x, y); Rt[findroot(x)] = Rt[findroot(y)];
132        }else shrink(x, y);
133    }
134 };
135 //未连成树、初始化节点时,需要调用LCT::push_up 维护节点其余信息
136 //维护的信息要有 可减性,如子树结点数,子树权值和,但不能直接维护子树最大最小值,因为在
    将一条虚边变成实边时要排除原先虚边的贡献。
137 //新建一个附加值存储虚子树的贡献,在统计时将其加入本结点答案,在改变边的虚实时及时维护。
138 //其余部分同普通 LCT,在统计子树信息时一定要将其作为根节点。
139 //如果维护的信息没有可减性,如维护区间最值,可以对每个结点开一个平衡树维护结点的虚子树中的
    最值。
140 if(x == y) continue;
141 if(Find(x) != Find(y)) {

```

```

142     unite(x, y);
143     addEdge(mp[{x, y}]);
144 }else{
145     int eid = que(x, y);
146     int id = mp[{x, y}];
147     if(val[eid] > e[id].w) {
148         delEdge(eid - vcnt);
149         addEdge(id);
150     }
151 }

```

2.5.5 Divide Combine Tree

1. $[i, i+1]$ 构造依赖, 利用线段树辅助建图, 利用 tarjan 求 scc, 利用 rmq 求最左最右边界 2. 定义 (i, j) 为一个好二元组, 当且仅当 $a[i]-a[j]=1$ 这样的两项的二元组在 $[l, r]$ 中恰好有 $r-l$ 个线段树维护 $val + 1 = r$, 其中 val 是区间 $[l, r]$ 中好二元组的个数离线 3. 析合树维护 $mx - mn = r - l \Leftrightarrow fx = (mx - mn) - (r - l)$

```

1 namespace DCT{
2     struct RMQ {
3         int lg[MAXN], mn[MAXN][S+1], mx[MAXN][S+1];
4         inline void init(int *a, int n) {
5             for (int i = 2; i <= n; i++) lg[i] = lg[i >> 1] + 1;
6             for (int i = 1; i <= n; i++) mn[i][0] = mx[i][0] = a[i];
7             for (int k = 1; (1 << k) <= n; k++)
8                 for (int i = 1; i + (1 << k) - 1 <= n; i++) {
9                     mn[i][k] = min(mn[i][k - 1], mn[i + (1 << (k - 1))][k - 1]);
10                    mx[i][k] = max(mx[i][k - 1], mx[i + (1 << (k - 1))][k - 1]);
11                }
12        }
13        inline int Min(int l, int r) {
14            int len = lg[r - l + 1];
15            return min(mn[l][len], mn[r - (1 << len) + 1][len]);
16        }
17        inline int Max(int l, int r) {
18            int len = lg[r - l + 1];
19            return max(mx[l][len], mx[r - (1 << len) + 1][len]);
20        }
21    } D;
22
23    struct SEG {
24        int setL, setR, setW;
25        int mn[MAXN << 2], tag[MAXN << 2];
26
27        inline void pushup(int x) {
28            mn[x] = min(mn[x << 1], mn[x << 1 | 1]);
29        }
30        inline void pushdown(int x) {
31            if(!tag[x]) return;
32            mn[x << 1] += tag[x]; mn[x << 1 | 1] += tag[x];
33            tag[x << 1] += tag[x]; tag[x << 1 | 1] += tag[x]; tag[x] = 0;
34        }
35        void init(int x, int l, int r) {
36            mn[x] = tag[x] = 0;
37            if (l == r) return;
38            int mid = (l + r) >> 1;
39            init(x << 1, l, mid);
40            init(x << 1 | 1, mid + 1, r);
41        }
42        void upt(int x, int l, int r) {
43            if (setL <= l && r <= setR) {

```

```

44         tag[x] += setW; mn[x] += setW;
45         return;
46     }
47     pushdown(x);
48     int mid = (l + r) >> 1;
49     if (setL <= mid) upt(x << 1, l, mid);
50     if (mid < setR) upt(x << 1 | 1, mid+1, r);
51     pushup(x);
52 }
53 int que(int x, int l, int r) {
54     if (l == r) return l;
55     pushdown(x);
56     int mid = (l+r)>>1;
57     if (!mn[x << 1]) return que(x << 1, l, mid);
58     return que(x << 1 | 1, mid+1, r);
59 }
60 } T;
61
62 int tpmn, stmn[MAXN], tpmx, stmx[MAXN], tpk, stk[MAXN];
63 int ncnt, type[MAXN<<1], L[MAXN<<1], R[MAXN<<1], M[MAXN<<1];
64 int dep[MAXN<<1], fa[MAXN<<1][S+1], C[MAXN<<1];
65 int id[MAXN << 1];
66 int newnode(int _type, int _L, int _R, int _M = 0) {
67     ++ncnt; type[ncnt] = _type;
68     L[ncnt] = _L; R[ncnt] = _R; M[ncnt] = _M;
69     C[ncnt] = 0;
70     return ncnt;
71 }
72
73 inline bool judge(int l, int r) {
74     return D.Max(l, r) - D.Min(l, r) == r - l;
75 }
76
77 int ecnt, head[MAXN << 1];
78 struct Edge{int to, nxt;} e[MAXN<<1];
79 inline void addEdge(int x, int y) {
80     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
81     fa[y][0] = x; C[x]++;
82 }
83 void dfs(int u) {
84     for(int j = 0; j < S; j++) fa[u][j+1] = fa[fa[u][j]][j];
85     for(int i = head[u]; i; i = e[i].nxt) {
86         dep[e[i].to] = dep[u] + 1;
87         dfs(e[i].to);
88     }
89 }
90
91 inline void init(int n) {
92     ecnt = 0;
93     for(int i = 0; i <= n; i++) head[i] = 0;
94 }
95 void buildT(int *a, int n) {
96     init(n);
97     D.init(a, n);
98     T.init(1, 1, n);
99     tpmn = tpmx = tpk = 0;
100    stmn[0] = stmx[0] = stk[0] = 0;
101    for (int i = 1; i <= n; i++) {
102        for (; tpmn && a[i] <= a[stmn[tpmn]]; --tpmn) {
103            T.setL = stmn[tpmn - 1] + 1; T.setR = stmn[tpmn]; T.setW = a[stmn[tpmn]
]]];

```

```

104         T.upt(1, 1, n);
105     }
106     T.setL = stmn[tpmn] + 1; T.setR = i; T.setW = -a[i];
107     T.upt(1, 1, n);
108     stmn[++tpmn] = i;
109
110     for (; tpmx && a[i] >= a[stmx[tpmx]]; --tpmx) {
111         T.setL = stmx[tpmx - 1] + 1; T.setR = stmx[tpmx]; T.setW = -a[stmx[tpmx]
112     ]];
113         T.upt(1, 1, n);
114     }
115     T.setL = stmx[tpmx] + 1; T.setR = i; T.setW = a[i];
116     T.upt(1, 1, n);
117     stmx[++tpmx] = i;
118
119     int Li = T.que(1, 1, n), np = id[i] = newnode(0, i, i), nq, nw;
120     while (tpk && L[nq = stk[tpk]] >= Li) {
121         if (type[nq] && judge(M[nq], i)) {
122             R[nq] = i;
123             addEdge(nq, np);
124             np = nq; tpk--;
125         } else if (judge(L[nq], i)) {
126             nw = newnode(1, L[nq], i, L[np]);
127             addEdge(nw, nq); addEdge(nw, np);
128             np = nw; tpk--;
129         } else {
130             nw = newnode(0, -1, i);
131             addEdge(nw, np);
132             do {
133                 addEdge(nw, nq);
134                 nq = stk[--tpk];
135             } while (tpk && !judge(L[nq], i));
136             addEdge(nw, nq);
137             L[nw] = L[nq]; R[nw] = i;
138             np = nw; --tpk;
139         }
140     }
141     stk[++tpk] = np;
142     T.setL = 1; T.setR = i; T.setW = -1;
143     T.upt(1, 1, n);
144 }
145 assert(tpk == 1);
146 dfs(stk[tpk]);
147
148 void lca(int u, int v, int &aL, int &bR) {
149     if(u == v) {
150         aL = L[u]; bR = R[v];
151         return;
152     }
153     if(dep[u] > dep[v]) swap(u, v);
154     for(int i = S; i >= 0; i--)
155         if(dep[fa[v][i]] >= dep[u]) v = fa[v][i];
156     assert(u != v);
157     for(int i = S; i >= 0; i--)
158         if(fa[u][i] != fa[v][i]) {
159             u = fa[u][i]; v = fa[v][i];
160         }
161     if(type[fa[u][0]]) {
162         aL = min(L[v], L[u]);
163         bR = max(R[v], R[u]);
164     } else {

```

```
164         aL = L[fa[u][0]];
165         bR = R[fa[u][0]];
166     }
167 }
168 };
```

3 String

3.1 Basics

3.1.1 Hash

```

1  const LL p1 = 201, p2 = 301, mod1 = 1200000319, mod2 = 2147483647;
2  struct Hash {
3      LL a, b;
4      void append(Hash pre, int v) {
5          a = (pre.a * p1 + v) % mod1;
6          b = (pre.b * p2 + v) % mod2;
7      }
8      void init(string S) {
9          a = b = 0;
10         for(int i = 0; i < S.size(); i++) append(*this, S[i]);
11     }
12     bool operator == (const Hash &x) const {
13         return a == x.a && b == x.b;
14     }
15     bool operator < (const Hash &x) const {
16         return a < x.a || (a == x.a && b < x.b);
17     }
18 };

```

3.1.2 Minimum String

```

1  namespace minstring{
2      int getmin(char *s, int sn) {
3          int i = 0, j = 1, k = 0, t;
4          while(i < sn && j < sn && k < sn) {
5              t = s[(i + k) % sn] - s[(j + k) % sn];
6              if(!t) k++;
7              else {
8                  if(t > 0) i += k + 1; else j += k + 1;
9                  if(i == j) j++;
10                 k = 0;
11             }
12         }
13         return i < j ? i : j;
14     }
15 }

```

3.2 String Matching

3.2.1 Bitset Match

```

1  namespace BitMatch{
2      const int S = 26;
3      bitset<MAXN> bs[S], ret;
4      char s[MAXN];
5      inline int idx(char c) { return c - 'a'; }
6      inline void init() {
7          for(int i = 0; i < 26; i++) bs[i].reset();
8          scanf("%s", s);
9          int sn = strlen(s);

```



```

10     for(int i = 0; i < sn; i++) bs[idx(s[i])].set(i);
11 }
12 void modify(int p, char ch) {
13     bs[idx(s[p])].reset(p);
14     s[p] = ch;
15     bs[idx(s[p])].set(p);
16 }
17 int match(char *t, int tn) { //返回 t 串在 s 串中出现的次数
18     ret = bs[idx(t[0])];
19     for(int i = 1; i < tn; i++) {
20         ret <<= 1;
21         ret &= bs[idx(t[i])];
22     }
23     return ret.count();
24 }
25 }

```

3.2.2 KMP && exKMP

```

1 判断循环子串的充要条件:  $i/(i-fa[i]) > 1$  &&  $i\%(i-fa[i])==0$  (i是长度)且去除下面fa的优化
2 namespace KMP {
3     int fa[MAXN];
4     void get_fail(char* t, int tn) {
5         fa[0] = -1;
6         int i = 0, j = -1;
7         while(i < tn) {
8             if (j == -1 || t[i] == t[j]) {
9                 ++i; ++j;
10                fa[i] = t[i] != t[j] ? j : fa[j];
11            }else{
12                j = fa[j];
13            }
14        }
15    }
16    void kmp(char* s, int sn, char* t, int tn) {
17        int i = 0, j = 0;
18        while(i < sn) {
19            if (j == -1 || s[i] == t[j]) {
20                i++;j++;
21                if(j == tn) {
22                }
23            }else j = fa[j];
24        }
25    }
26 }
27 nxt[i]是t和从i开始的t的后缀的最大公共前缀长度。
28 t,s的第一个字符下标为0,最后一个字符下标为n-1。
29 namespace exKMP {
30     int nxt[MAXN], ext[MAXN];
31     void get_nxt(char* t, int tn) {
32         int j = 0, mx = 0;
33         nxt[0] = tn;
34         for(int i = 1; i < tn; i++) {
35             if(i >= mx || i + nxt[i - j] >= mx) {
36                 if(i > mx) mx = i;
37                 while(mx < tn && t[mx] == t[mx - i]) mx++;
38                 nxt[i] = mx - i;
39                 j = i;
40             }else nxt[i] = nxt[i - j];
41         }

```

```

42     }
43     void exkmp(char *s, int sn, char *t, int tn) {
44         int j = 0, mx = 0;
45         for(int i = 0; i < sn; i++) {
46             if(i >= mx || i + nxt[i - j] >= mx) {
47                 if(i > mx) mx = i;
48                 while(mx < sn && mx - i < tn && s[mx] == t[mx - i]) mx++;
49                 ext[i] = mx - i;
50                 j = i;
51             }else ext[i] = nxt[i - j];
52         }
53     }
54 }

```

3.2.3 AC Automaton

```

1 namespace AC {
2     int ch[MAXN][sigma_size], last[MAXN];
3     int val[MAXN], f[MAXN], sz;
4     inline void init() { sz = 1; memset(ch[0], 0, sizeof(ch[0])); }
5     inline int idx(char c) { return c - 'a'; }
6     void insert(string s, int v) {
7         int u = 0;
8         for(int i = 0; i < s.size(); i++) {
9             int c = idx(s[i]);
10            if(!ch[u][c]) {
11                memset(ch[sz], 0, sizeof(ch[sz]));
12                val[sz] = 0;
13                ch[u][c] = sz++;
14            }
15            u = ch[u][c];
16        }
17        val[u] = v;
18    }
19    void get_fail() {
20        queue<int> q;
21        f[0] = 0;
22        for(int c = 0; c < sigma_size; c++) {
23            int u = ch[0][c];
24            if(u) { f[u] = 0; q.push(u); last[u] = 0; }
25        }
26        while(!q.empty()) {
27            int r = q.front(); q.pop();
28            for(int c = 0; c < sigma_size; c++) {
29                int u = ch[r][c];
30                if(!u) { ch[r][c] = ch[f[r]][c]; continue; }
31                q.push(u);
32                int v = f[r];
33                while(v && !ch[v][c]) v = f[v];
34                f[u] = ch[v][c];
35                last[u] = val[f[u]] ? f[u] : last[f[u]];
36            }
37        }
38    }
39    inline void solve(int j) {
40        if(j) {
41            ans += val[j];
42            solve(last[j]);
43        }
44    }

```

```

45     void find(string T) {
46         int j = 0;
47         for(int i = 0; i < T.size(); i++) {
48             int c = idx(T[i]);
49             j = ch[j][c];
50             if(val[j]) solve(j);
51             else if(last[j]) solve(last[j]);
52         }
53     }
54 }
55 namespace AC {
56     int root, tcnt;
57     int ch[MAXN][sigma_size], fa[MAXN];
58     inline int newnode() {
59         fa[++tcnt] = 0;
60         for(int i = 0; i < sigma_size; ++i) ch[tcnt][i] = 0;
61         return tcnt;
62     }
63     inline void init() {
64         tcnt = -1;
65         root = newnode();
66     }
67     inline int idx(char c) { return c - 'a'; }
68     void extend(char *s, int sn) {
69         int cur = root;
70         for(int i = 0, c; i < sn; i++) {
71             if(!ch[cur][c = idx(s[i])])
72                 ch[cur][c] = newnode();
73             cur = ch[cur][c];
74         }
75     }
76     int q[MAXN], qh, qt;
77     void get_fail() {
78         qh = 1; qt = 0;
79         fa[root] = 0;
80         for(int c = 0, now; c < sigma_size; c++)
81             if((now = ch[root][c]) != 0)
82                 q[++qt] = now;
83         while(qh <= qt) {
84             int cur = q[qh++];
85             for(int c = 0, now; c < sigma_size; c++)
86                 if((now = ch[cur][c]) != 0) {
87                     fa[now] = ch[fa[cur]][c];
88                     q[++qt] = now;
89                 }else
90                     ch[cur][c] = ch[fa[cur]][c];
91         }
92     }
93     //统计模板串出现次数，每个模板串只计算一次
94     //     int cur = root, ans = 0;
95     //     for(int i = 0; i < sn; ++i) {
96     //         cur = ch[cur][idx(s[i])];
97     //         for(int j = cur; j && cnt[j] != -1; j = fa[j]) {
98     //             ans += cnt[j];
99     //             cnt[j] = -1;
100        //     }
101        // }
102    }

```

3.3 Suffix Related

3.3.1 Suffix Array

```

1 namespace SA {
2     char s[MAXN];
3     int sa[MAXN], rank[MAXN], height[MAXN];
4     int t[MAXN], t2[MAXN], c[MAXN], n;
5     void clear() { n = 0; memset(sa, 0, sizeof(sa)); }
6     void build(int m) {
7         int *x = t, *y = t2;
8         for(int i = 0; i < m; i++) c[i] = 0;
9         for(int i = 0; i < n; i++) c[x[i] = s[i]]++;
10        for(int i = 1; i < m; i++) c[i] += c[i - 1];
11        for(int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
12        for(int k = 1; k <= n; k <= 1) {
13            int p = 0;
14            for(int i = n - k; i < n; i++) y[p++] = i;
15            for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
16            for(int i = 0; i < m; i++) c[i] = 0;
17            for(int i = 0; i < n; i++) c[x[y[i]]]++;
18            for(int i = 1; i < m; i++) c[i] += c[i - 1];
19            for(int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
20            swap(x, y);
21            p = 1; x[sa[0]] = 0;
22            for(int i = 1; i < n; i++)
23                x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k]
? p - 1 : p++;
24            if(p >= n) break;
25            m = p;
26        }
27    }
28    void buildHeight() {
29        int k = 0;
30        for(int i = 0; i < n; i++) rank[sa[i]] = i;
31        for(int i = 0; i < n; i++) {
32            if(k) k--;
33            int j = sa[rank[i] - 1];
34            while(s[i + k] == s[j + k]) k++;
35            height[rank[i]] = k;
36        }
37    }
38    void init() {
39        n = strlen(s) + 1;
40        build('z' + 1);
41        buildHeight();
42    }
43 }

```

3.3.2 Suffix Automaton

```

1 //root = 1
2 //len[i]: i表示的状态集中最长的字符串长度
3 namespace SAM{
4     int scnt, root, last;
5     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
6     int sc[MAXN<<1], tml[MAXN<<1], minl[MAXN<<1];
7
8     int newnode(int _len, int q = 0) {

```

```

9      fa[++scnt] = fa[q]; len[scnt] = _len;
10     sc[scnt] = 0; tmp[scnt] = 0; minl[scnt] = INF;
11     for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
12     return scnt;
13 }
14 void init() {
15     scnt = 0;
16     root = last = newnode(0);
17 }
18 void extend(int c) {
19     int p = last, np = newnode(len[p] + 1);
20     for(; p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
21     if(!p) fa[np] = root;
22     else{
23         int q = ch[p][c];
24         if(len[p] + 1 == len[q]) fa[np] = q;
25         else{
26             int nq = newnode(len[p] + 1, q);
27             fa[np] = fa[q] = nq;
28             for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
29         }
30     }
31     last = np;
32 }
33 int c[MAXN], rs[MAXN << 1];
34 void radix_sort(int n){
35     for(int i = 0; i <= n; i++) c[i] = 0;
36     for(int i = 1; i <= scnt; i++) c[len[i]]++;
37     for(int i = 1; i <= n; i++) c[i] += c[i-1];
38     for(int i = scnt; i >= 1; i--) rs[c[len[i]]--] = i;
39 }
40 void go(){
41     scanf("%s",s);
42     int n = strlen(s);
43     for(int i = 0; i < n; ++i)
44         extend(s[i] - 'a');
45     radix_sort(n);
46     //以下sc集合意义不同
47     //预处理从每个节点出发，还有多少本质不同的子串可以到达，注意sc 要开 long long
48     for(int i = scnt; i >= 1; i--) {
49         LL S = 0;
50         for(int j = 0; j < 26; j++)
51             S += sc[ ch[rs[i]][j] ];
52         sc[rs[i]] = S + 1;
53     }
54 }
55 { //right集合大小
56     int cur = root;
57     for(int i = 0; i < n; ++i) {
58         cur = ch[cur][s[i] - 'a'];
59         sc[cur]++;
60     }
61     for(int i = scnt; i >= 1; --i) {
62         sc[ fa[rs[i]] ] += sc[rs[i]];
63     }
64 }
65 //公共子串
66 //tmp, 当前字符串: 在状态cur, 与模板串的最长公共后缀
67 //minl, 多个字符串: 在状态cur, 与模板串的最长公共后缀
68 //注意: 在状态cur匹配成功时, cur的祖先状态与字符串的最长公共后缀
69 for(; ~scanf("%s",s);) {

```

```

70     int cur = root, Blen = 0;
71     for(int i = 0; i <= scnt; i++)
72         tmp1[i] = 0;
73     n = strlen(s);
74     for(int i = 0, x; i < n; i++) {
75         x = s[i] - 'a';
76         if(ch[cur][x]) {
77             ++Blen;
78             cur = ch[cur][x];
79         }else{
80             for(;cur && ch[cur][x] == 0; cur = fa[cur]);
81             if(cur) {
82                 Blen = len[cur] + 1;
83                 cur = ch[cur][x];
84             }else{
85                 cur = root; Blen = 0;
86             }
87         }
88         tmp1[cur] = max(tmp1[cur], Blen);
89     }
90     for(int i = scnt; i ; --i) {
91         if( tmp1[ fa[rs[i]] ] < tmp1[ rs[i] ])
92             tmp1[ fa[rs[i]] ] = len[ fa[rs[i]] ];
93         min1[ rs[i] ] = min(min1[ rs[i] ], tmp1[ rs[i] ]);
94     }
95 }
96
97 namespace exSAM{
98     int scnt, root;
99     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
100     int sc[MAXN<<1], tmp1[MAXN<<1], min1[MAXN<<1];
101
102     int newnode(int _len, int q = 0) {
103         fa[++scnt] = fa[q]; len[scnt] = _len;
104         sc[scnt] = 0; tmp1[scnt] = 0; min1[scnt] = INF;
105         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
106         return scnt;
107     }
108
109     void init() {
110         scnt = 0;
111         root = newnode(0);
112     }
113     int work(int p, int c){
114         int q = ch[p][c];
115         int nq = newnode(len[p] + 1, q);
116         fa[q] = nq;
117         for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
118         return nq;
119     }
120     int extend(int p, int c) {
121         if (ch[p][c]){
122             int q = ch[p][c];
123             if (len[p] + 1 == len[q]) return q;
124             return work(p, c);
125         }
126         int np = newnode(len[p] + 1);
127         for(;p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
128         if (!p) fa[np] = root;
129         else{
130             int q = ch[p][c];

```

```

131         if (len[p] + 1 == len[q]) fa[np] = q;
132         else fa[np] = work(p, c);
133     }
134     return np;
135 }
136 void solve() {
137     int n; scanf("%d",&n);
138     for(int i = 1; i <= n; i++) {
139         scanf("%s", s);
140         int sn = strlen(s);
141         int last = root;
142         for(int j = 0; j < sn; ++j)
143             last = extend(last, s[j] - 'a');
144     }
145 }
146 }

```

3.4 Palindrome Related

3.4.1 Manacher

```

1 namespace Manacher {
2     char S[MAXN << 1];
3     int scnt, ans;
4     int p[MAXN << 1]; //p[i] - 1
5     void init(char *s0, int sn0) {
6         S[0] = '$'; S[1] = '#';
7         for(int i = 0; i < sn0; i++) {
8             S[2 * i + 2] = s0[i];
9             S[2 * i + 3] = '#';
10        }
11        scnt = sn0 * 2 + 2;
12        S[scnt] = '&';
13    }
14    void manacher() {
15        int id = 0, mx = 0;
16        for(int i = 1; i < scnt; i++) {
17            p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
18            while(S[i + p[i]] == S[i - p[i]]) p[i]++;
19            if(i + p[i] > mx) {
20                mx = i + p[i];
21                id = i;
22            }
23        }
24    }
25 }

```

3.4.2 Palindromic Automaton

pcnt 本质不同的回文串的个数 len[u] 状态 u 代表的串的长度 fail[u] 状态 u 所代表的回文串的最长回文后缀 trans[u] 小于等于当前节点长度一半的最长回文后缀 cnt[u] 包含状态 u 表示的回文串的出现的个数 (调用 count()) num[u] 以状态 u 表示的回文串的右端点为回文串结尾的回文串个数 [...][...] diff[u] 表示节点 u 和 fail[u] 所代表的回文串的长度差, 即 len[u] - len[fail[u]] slink[u] 表示 u 一直沿着 fail 向上跳到第一个节点 v, 使得 diff[v] != diff[u], 也就是 u 所在等差数列中长度最小的那个节点

```

1 namespace PAM {
2     int scnt, S[MAXN];

```

```

3  int pcnt, last, len[MAXN], fail[MAXN], ch[MAXN][26];
4  int cnt[MAXN], num[MAXN], trans[MAXN], diff[MAXN], slink[MAXN];
5  int newnode(int _len) {
6      len[pcnt] = _len;
7      cnt[pcnt] = num[pcnt] = 0;
8      for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
9      return pcnt++;
10 }
11 inline void init() {
12     S[scnt = 0] = -1;
13     pcnt = 0; newnode(0); newnode(-1);
14     fail[0] = 1; last = 0;
15 }
16 int getfail(int x) {
17     while(S[scnt - len[x] - 1] != S[scnt]) x = fail[x];
18     return x;
19 }
20 void extend(int c) {
21     S[++scnt] = c;
22     int cur = getfail(last);
23     if(!ch[cur][c]) {
24         int now = newnode(len[cur] + 2);
25         fail[now] = ch[getfail(fail[cur])][c];
26         ch[cur][c] = now;
27         num[now] = num[fail[now]] + 1;
28         if (len[now] <= 2) trans[now] = fail[now];
29         else{
30             int x = trans[cur];
31             while(S[scnt - len[x] - 1] != S[scnt] || (len[x]+2) * 2 > len[now]) x =
fail[x];
32             trans[now] = ch[x][c];
33         }
34         diff[now] = len[now] - len[fail[now]];
35         slink[now] = (diff[now] == diff[fail[now]]) ? slink[fail[now]] : fail[now];
36     }
37     last = ch[cur][c];
38     cnt[last]++;
39 }
40 void count() {
41     for(int i = pcnt - 1; i >= 0; i--) cnt[fail[i]] += cnt[i];
42 }
43 };

```

支持前后插入不基于势能分析的构造法可以实现持久化，比如在 Trie 上实现

```

1  namespace PAM {
2      int sL, sR, S[MAXN<<1];
3      int pcnt, lastL, lastR;
4      int len[MAXN<<1], fa[MAXN<<1], quick[MAXN<<1][26], ch[MAXN<<1][26];
5      int cnt[MAXN<<1], num[MAXN<<1];
6      int newnode(int _len) {
7          len[pcnt] = _len;
8          cnt[pcnt] = num[pcnt] = 0;
9          for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
10         return pcnt++;
11     }
12     inline void init() {
13         pcnt = 0; newnode(0); newnode(-1);
14         fa[0] = 1;
15         for(int i = 0; i < 26; i++) quick[0][i] = quick[1][i] = 1;
16         lastL = lastR = 0;
17         sL = MAXN; sR = MAXN-1;

```



```

18     S[sL] = S[sR] = -1;
19 }
20 void push_front(int c) {
21     S[--sL] = c; S[sL-1]=-1;
22     int p = lastL;
23     if (S[sL+len[p]+1] ^ c) p = quick[p][c];
24     if (!ch[p][c]) {
25         int np = newnode(len[p]+2), q = fa[p];
26         if (S[sL+len[q]+1] ^ c) q = quick[q][c];
27         fa[np] = ch[q][c];
28         memcpy(quick[np], quick[fa[np]], sizeof(quick[np]));
29         quick[np][S[sL+len[fa[np]]]] = fa[np];
30         ch[p][c] = np;
31         num[np] = num[fa[np]] + 1;
32     }
33     lastL = ch[p][c];
34     cnt[lastL]++;
35     if (len[lastL] == sR-sL+1) lastR = lastL;
36 }
37 void push_back(int c) {
38     S[++sR] = c; S[sR+1]=-1;
39     int p = lastR;
40     if (S[sR-len[p]-1] ^ c) p = quick[p][c];
41     if (!ch[p][c]) {
42         int np = newnode(len[p]+2), q = fa[p];
43         if (S[sR-len[q]-1] ^ c) q = quick[q][c];
44         fa[np] = ch[q][c];
45         memcpy(quick[np], quick[fa[np]], sizeof(quick[np]));
46         quick[np][S[sR-len[fa[np]]]] = fa[np];
47         ch[p][c] = np;
48         num[np] = num[fa[np]] + 1;
49     }
50     lastR = ch[p][c];
51     cnt[lastR]++;
52     if (len[lastR] == sR-sL+1) lastL = lastR;
53 }
54 int c[MAXN<<1], rs[MAXN<<2];
55 void count() {
56     for (int i = 0; i < pcnt; i++) c[i] = 0;
57     for (int i = 2; i < pcnt; i++) c[len[i]]++;
58     for (int i = 1; i < pcnt; i++) c[i] += c[i-1];
59     for (int i = 2; i < pcnt; i++) rs[c[len[i]]--] = i;
60     for (int i = pcnt-1; i; i--) cnt[fa[rs[i]]]+=cnt[rs[i]];
61 }
62 };

```

3.5 Substring Automaton

```

1  for(int j = 0; j < 26; j++)
2      ch[n][j] = ch[n+1][j] = n + 1; //或者 -1
3  for(int i = n; i >= 1; i--) {
4      for(int j = 0; j < 26; j++)
5          ch[i-1][j] = ch[i][j];
6      ch[i-1][s[i]- 'a'] = i;
7  }

```

当字符集过大时使用主席树维护 ch

4 Math

4.1 Algebra

4.1.1 FFT

```

1  //不预处理精度
2  const double pi = acos(-1.0);
3  const int MAXN = 300003;
4  struct comp {
5      double x, y;
6      comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
7      comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
8      comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
          y * a.x}; }
9  };
10 int rev[MAXN], T;
11 comp tmp;
12 void fft(comp *a, int r) {
13     if(r == -1) for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
14     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
15     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
16         comp step = (comp) {cos(pi / mid), r * sin(pi / mid)};
17         for(int j = 0; j < T; j += i) {
18             comp cur = (comp) {1, 0};
19             for(int k = j; k < j + mid; k++, cur = cur * step) {
20                 tmp = a[k + mid] * cur;
21                 a[k + mid] = a[k] - tmp;
22                 a[k] = a[k] + tmp;
23             }
24         }
25     }
26     if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
27 }
28 comp A[MAXN];
29 void init(int n) {
30     //A[0] = (comp) {0, 0};
31     for(T = 1; T <= n; T <= 1);
32     for(int i = 1; i < T; i++) {
33         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
34         else rev[i] = rev[i >> 1] >> 1;
35         //A[i] = (comp) {0, 0};
36     }
37 }
38 //预处理精度
39 int rev[MAXN], T;
40 comp Sin[MAXN], tmp;
41 void fft(comp *a, int r) {
42     if(r == -1) {
43         for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
44         for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
45     }
46     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
47     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
48         for(int j = 0; j < T; j += i) {
49             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
50                 tmp = a[k + mid] * Sin[cur];
51                 a[k + mid] = a[k] - tmp;
52                 a[k] = a[k] + tmp;

```

```

53     }
54 }
55 }
56 if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
57 }
58 comp A[MAXN];
59 void init(int n) {
60     for(T = 1; T <= n; T <= 1);
61     for(int i = 0; i < T; i++) {
62         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
63         else rev[i] = rev[i >> 1] >> 1;
64         //A[i] = (comp) {0, 0};
65     }
66     for(int i = 0; i < (T >> 1); i++) {
67         Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
68     }
69 }
70 int main() {
71     scanf("%d%d", &n, &m);
72     init(n + m);
73     for(int i = 0; i <= n; i++) scanf("%lf", &A[i].x);
74     for(int i = 0; i <= m; i++) scanf("%lf", &A[i].y);
75     fft(A, 1);
76     fft(A, -1);
77     for(int i = 0; i <= n + m; i++) printf("%d%c", (int)(A[i].y), i == n + m ? '\n' : ' ');
78     return 0;
79 }

```

4.1.2 NTT

4.常用NTT模数:

以下模数的共同 $g = 3189$

$p = r \times 2^k + 1$	k	g
104857601	22	3
167772161	25	3
469762049	26	3
950009857	21	7
998244353	23	3
1004535809	21	3
2013265921	27	31
2281701377	27	3
3221225473	30	5

```

1  const int MAXN = 300005, G = 3, mod = 998244353; //or (479LL<<21) + 1
2  int rev[MAXN], T;
3  LL qpow(LL x, LL y) {
4      LL res = 1;
5      while(y) {
6          if(y & 1) res = res * x % mod;
7          x = x * x % mod;
8          y >>= 1;
9      }
10     return res;
11 }
12 LL A[MAXN], B[MAXN];

```

```

13 void ntt(LL *a, int r) {
14     if(r == -1) for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
15     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17         LL gn = qpow(G, (mod - 1) / i);
18         if(r == -1) gn = qpow(gn, mod - 2);
19         for(int j = 0; j < T; j += i) {
20             LL cur = 1, tmp;
21             for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
22                 tmp = a[k + mid] * cur % mod;
23                 a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
24                 a[k] = (a[k] + tmp) % mod;
25             }
26         }
27     }
28     if(r == -1) {
29         LL inv = qpow(T, mod - 2);
30         for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
31     }
32 }
33 void init(int n) {
34     for(T = 1; T <= n; T <= 1);
35     for(int i = 0; i < T; i++) {
36         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
37         else rev[i] = rev[i >> 1] >> 1;
38     }
39 }

```

4.1.3 MTT

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  const double pi = acos(-1.0);
5  const int MAXN = 300003;
6  struct comp {
7      double x, y;
8      comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
9      comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
10     comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
11         y * a.x}; }
12 };
13 #define conj(a) ((comp){a.x, -a.y})
14 int rev[MAXN], T;
15 comp Sin[MAXN], tmp;
16 void fft(comp *a, int r) {
17     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
18     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
19         for(int j = 0; j < T; j += i) {
20             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
21                 tmp = a[k + mid] * Sin[cur];
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27 }
28 void init(int n) {
29     for(T = 1; T <= n; T <= 1);
30     for(int i = 0; i < T; i++) {

```

```

30     if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
31     else rev[i] = rev[i >> 1] >> 1;
32 }
33 for(int i = 0; i < (T >> 1); i++) {
34     Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
35 }
36 }
37 int n, m, mod;
38 void mtt(int *x, int *y) {
39     for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
40     static comp a[MAXN], b[MAXN];
41     static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
42     for(int i = 0; i < T; i++) {
43         a[i] = {x[i] & 0x7fff, x[i] >> 15};
44         b[i] = {y[i] & 0x7fff, y[i] >> 15};
45     }
46     fft(a, 1); fft(b, 1);
47     for(int i = 0; i < T; i++) {
48         int j = (T - i) & (T - 1);
49         static comp da, db, dc, dd;
50         da = (a[i] + conj(a[j])) * (comp){0.5, 0};
51         db = (a[i] - conj(a[j])) * (comp){0, -0.5};
52         dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
53         dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
54         dfta[j] = da * dc;
55         dftb[j] = da * dd;
56         dftc[j] = db * dc;
57         dftd[j] = db * dd;
58     }
59     for(int i = 0; i < T; i++) {
60         a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
61         b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
62     }
63     //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
64     fft(a, -1); fft(b, -1);
65     for(int i = 0; i < T; i++) {
66         static int da, db, dc, dd;
67         da = (LL)(a[i].x / T + 0.5) % mod;
68         db = (LL)(a[i].y / T + 0.5) % mod;
69         dc = (LL)(b[i].x / T + 0.5) % mod;
70         dd = (LL)(b[i].y / T + 0.5) % mod;
71         x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
72     }
73 }
74 int main() {
75     static int a[MAXN], b[MAXN];
76     scanf("%d%d%d", &n, &m, &mod);
77     for(int i = 0; i <= n; i++) scanf("%d", a + i);
78     for(int i = 0; i <= m; i++) scanf("%d", b + i);
79     init(n + m);
80     mtt(a, b);
81     for(int i = 0; i <= n + m; i++) printf("%d%c", a[i], i == n + m ? '\n' : ' ');
82     return 0;
83 }

```

4.1.4 FWT

Hint: $n = 2^k$

```

1 void FWT(LL *a, int n) {
2     for(int i = 2; i <= n; i <= 1) {

```

```

3     for(int j = 0; j < n; j += i) {
4         for(int d = 0, w = i >> 1; d < w; d++){
5             LL u = a[j + d], v = a[j + d + w];
6             //xor: a[j + d] = u + v, a[j + d + w] = u - v;
7             //and: a[j + d] = u + v;
8             //or : a[j + d + w] = u + v;
9         }
10    }
11 }
12 }
13 void UFWT(LL *a, int n) {
14     for(int i = 2; i <= n; i <= 1) {
15         for(int j = 0; j < n; j += i) {
16             for(int d = 0, w = i >> 1; d < w; d++) {
17                 LL u = a[j + d], v = a[j + d + w];
18                 //xor: a[j + d] = (u + v) / 2, a[j + d + w] = (u - v) / 2;
19                 //and: a[j + d] = u + v;
20                 //or : a[j + d + w] = v - u;
21             }
22         }
23     }
24 }
25 void solve(int n) {
26     FWT(a, n); FWT(b, n);
27     for(int i = 0; i < n; i++) a[i] = a[i] * b[i];
28     UFWT(a, n);
29 }

```

4.1.5 FFT Divide and Conquer

$$f_i = \sum_{j=1}^{i-1} f_j \cdot g_{i-j}$$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  const int MAXN = 300005, G = 3, mod = 998244353;
5  namespace NTT {
6      LL A[MAXN], B[MAXN]
7      int rev[MAXN], T;
8      LL qpow(LL x, LL y) {
9          LL res = 1;
10         while(y) {
11             if(y & 1) res = res * x % mod;
12             x = x * x % mod;
13             y >>= 1;
14         }
15         return res;
16     }
17     void ntt(LL *a, int r) {
18         for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
19         for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
20             LL gn = qpow(G, (mod - 1) / i);
21             if(r == -1) gn = qpow(gn, mod - 2);
22             for(int j = 0; j < T; j += i) {
23                 LL cur = 1, tmp;
24                 for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
25                     tmp = a[k + mid] * cur % mod;

```

```

26         a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
27         a[k] = (a[k] + tmp) % mod;
28     }
29 }
30 }
31 if(r == -1) {
32     LL inv = qpow(T, mod - 2);
33     for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
34 }
35 }
36 void init(int n) {
37     for(T = 1; T <= n; T <= 1);
38     for(int i = 0; i < T; i++) {
39         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
40         else rev[i] = rev[i >> 1] >> 1;
41     }
42 }
43 }
44 LL f[MAXN], g[MAXN];
45 using namespace NTT;
46 void solve(int l, int r) {
47     if(l == r) return;
48     int mid = (l + r) >> 1;
49     solve(l, mid);
50     init(r - l);
51     for(int i = 0; i < T; i++) A[i] = B[i] = 0;
52     for(int i = 0; i <= mid - 1; i++) A[i] = f[i + 1];
53     for(int i = 0; i <= r - 1; i++) B[i] = g[i];
54     ntt(A, 1); ntt(B, 1);
55     for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
56     ntt(A, -1);
57     for(int i = mid + 1; i <= r; i++) f[i] = (f[i] + A[i - 1]) % mod;
58     solve(mid + 1, r);
59 }
60 int main() {
61     int n; scanf("%d", &n);
62     for(int i = 1; i < n; i++) scanf("%lld", g + i);
63     f[0] = 1;
64     solve(0, n - 1);
65     for(int i = 0; i < n; i++) printf("%lld%c", f[i], i == n - 1 ? '\n' : ' ');
66     return 0;
67 }

```

4.1.6 Linear Basis

```

1 //dynamic
2 const int D = 60;
3 struct Basis {
4     vector<int> ind;
5     vector<LL> base;
6     Basis() {
7         ind.resize(D, -1);
8         base.resize(D);
9     }
10    bool update(LL x, int id) {
11        for(int i = 0; i < D; i++) if(~ind[i] && x >> i & 1) {
12            x ^= base[i];
13        }
14        if(!x) return 1;
15        int pos = __builtin_ctzll(x);

```

```

16     ind[pos] = id;
17     base[pos] = x;
18     return 0;
19 }
20 };
21 //array
22 int Gauss(int n, int m) {
23     int num = 1;
24     for(int x = 1; x <= n && x <= m; x++) {
25         int t = 0;
26         for(int j = x; j <= m; j++) if(g[j][x]) { t = j; break; }
27         if(t) {
28             swap(g[x], g[t]);
29             for(int i = x + 1; i <= n; i++) {
30                 if(g[i][x]) {
31                     for(int k = 1; k <= m; k++) g[i][k] ^= g[x][k];
32                 }
33             }
34             num++;
35         }
36     }
37     return --num;
38 }
39 //long long
40 int Gauss() {
41     int num = 1;
42     for(int k = 61; k >= 0; k--) {
43         int t = 0;
44         for(int j = num; j <= cnt; j++) if((A[j] >> k) & 1) { t = j; break; }
45         if(t) {
46             swap(A[t], A[num]);
47             for(int j = num + 1; j <= cnt; j++) if((A[j] >> k) & 1) A[j] ^= A[num];
48             num++;
49         }
50     }
51     return --num;
52 }
53 //det
54 LL det(int n){
55     LL ret = 1;
56     for(int i = 1; i < n; i++){
57         for(int j = i + 1; j < n; j++){
58             while(a[j][i]){
59                 LL t = (LL)a[i][i] / a[j][i];
60                 for(int k = i; k < n; k++) {
61                     a[i][k] = (a[i][k]-a[j][k] * t);
62                     swap(a[i][k], a[j][k]);
63                 }
64                 ret = -ret;
65             }
66             if(a[i][i] == 0)return 0;
67             ret = ret * a[i][i];
68         }
69         if(ret<0)ret = -ret;
70         return ret;
71     }
72 //bitset求逆
73 #include <bits/stdc++.h>
74 using namespace std;
75 bitset<1005> A[505];
76 int Gauss(int n) {

```



```

77     int num = 0;
78     for(int k = 0; k < n; k++) {
79         int t = -1;
80         for(int j = num; j <= n; j++) if(A[j][k] == 1) { t = j; break; }
81         if(~t) {
82             swap(A[t], A[num]);
83             for(int j = 0; j <= n; j++) if(j != num && A[j][k] == 1) A[j] ^= A[num];
84             num++;
85         }
86     }
87     return num;
88 }
89 int main() {
90     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
91     int n;
92     cin >> n;
93     for(int i = 0, k; i < n; i++) {
94         for(int j = 0; j < n; j++) {
95             cin >> k;
96             A[i][j] = k;
97         }
98         A[i][i + n] = 1;
99     }
100     if(Gauss(n) < n) cout << "-1" << endl;
101     else {
102         for(int i = 0; i < n; i++) {
103             for(int j = 0; j < n; j++) {
104                 if(A[i][j + n]) cout << j + 1 << " ";
105             }
106             cout << endl;
107         }
108     }
109     return 0;
110 }

```

4.1.7 Polynomial

Inverse:

$$A_x * B'_x \equiv 1 \pmod{x^{\frac{n}{2}}}$$

$$A_x * B_x \equiv 1 \pmod{x^n}$$

$$B_x \equiv 2 \cdot B'_x - A_x \cdot B'^2_x \pmod{x^n}$$

Division:

$$A_r(x) = x^n A\left(\frac{1}{x}\right) \Rightarrow A_r(x)[i] = A(x)[n - i]$$

$$A(x) = B(x) * Q(x) + R(x)$$

$$Q_r(x) = A_r(x) * B_r^{-1}(x)$$

$$R(x) = A(x) - B(x) * Q(x)$$

```

1 //NTT模数
2 #include <bits/stdc++.h>
3 using namespace std;
4 typedef long long LL;

```

```

5  const int MAXN = 300005, G = 3, mod = 998244353;
6  namespace NTT {
7      int rev[MAXN], T;
8      LL qpow(LL x, LL y) {
9          LL res = 1;
10         while(y) {
11             if(y & 1) res = res * x % mod;
12             x = x * x % mod;
13             y >>= 1;
14         }
15         return res;
16     }
17     void ntt(LL *a, int r) {
18         for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
19         for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
20             LL gn = qpow(G, (mod - 1) / i);
21             if(r == -1) gn = qpow(gn, mod - 2);
22             for(int j = 0; j < T; j += i) {
23                 LL cur = 1, tmp;
24                 for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
25                     tmp = a[k + mid] * cur % mod;
26                     a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
27                     a[k] = (a[k] + tmp) % mod;
28                 }
29             }
30         }
31         if(r == -1) {
32             LL inv = qpow(T, mod - 2);
33             for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
34         }
35     }
36     void init(int n) {
37         for(T = 1; T <= n; T <= 1);
38         for(int i = 0; i < T; i++) {
39             if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
40             else rev[i] = rev[i >> 1] >> 1;
41         }
42     }
43 }
44 namespace poly {
45 using namespace NTT;
46 void mul(LL *a, LL *b, LL *c, int n, int m) {
47     init(n + m);
48     static LL x[MAXN], y[MAXN];
49     for(int i = 0; i < T; i++) {
50         x[i] = i <= n ? a[i] : 0;
51         y[i] = i <= m ? b[i] : 0;
52     }
53     ntt(x, 1); ntt(y, 1);
54     for(int i = 0; i < T; i++) c[i] = x[i] * y[i] % mod;
55     ntt(c, -1);
56 }
57 void poly_inv(LL *a, LL *b, int n) {
58     if(n == 1) {
59         b[0] = qpow(a[0], mod - 2);
60         return;
61     }
62     static LL c[MAXN], d[MAXN];
63     memset(c, 0, n * sizeof(LL));
64     poly_inv(a, c, n >> 1);
65     for(int i = 0; i < n; i++) {

```

```

66         d[i] = a[i];
67     }
68     init(n);
69     ntt(c, 1); ntt(d, 1);
70     for(int i = 0; i < T; i++) b[i] = c[i] * (2 + mod - d[i] * c[i] % mod) % mod;
71     ntt(b, -1);
72     for(int i = n; i < T; i++) b[i] = 0;
73 }
74 void inv(LL *a, LL *b, int n) {//A must be different from B
75     init(n);
76     poly_inv(a, b, T);
77 }
78 //A_x = B_x * Q_x + R_x;
79 void div(LL *A, LL *B, LL *Q, LL *R, int n, int m) {
80     static LL f[MAXN], g[MAXN], inv_g[MAXN];
81     for(int i = 0; i <= n; i++) f[n - i] = A[i];
82     for(int i = 0; i <= m; i++) g[m - i] = B[i];
83     //inv(G_r)
84     for(int i = n - m + 1; i <= m; i++) g[i] = 0;
85     inv(g, inv_g, n - m);
86     //Q
87     mul(f, inv_g, f, n, n - m);
88     for(int i = 0; i <= n - m; i++) Q[i] = f[n - m - i];
89     //R
90     mul(Q, B, f, n - m, m);
91     for(int i = 0; i < m; i++) R[i] = (A[i] + mod - f[i]) % mod;
92 }
93 }
94 LL A[MAXN], B[MAXN];
95 LL Q[MAXN], R[MAXN];
96 int main() {
97     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
98     int n, m; cin >> n >> m;
99     for(int i = 0; i <= n; i++) cin >> A[i];
100    for(int i = 0; i <= m; i++) cin >> B[i];
101    poly::div(A, B, Q, R, n, m);
102    for(int i = 0; i <= n - m; i++) cout << Q[i] << " "; cout << endl;
103    for(int i = 0; i < m; i++) cout << R[i] << " "; cout << endl;
104    return 0;
105 }
106 //非NTT模数求逆
107 #include <bits/stdc++.h>
108 using namespace std;
109 typedef long long LL;
110 const double pi = acos(-1.0);
111 const int MAXN = 300003;
112 struct comp {
113     double x, y;
114     comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
115     comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
116     comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
117         y * a.x}; }
118 };
119 #define conj(a) ((comp){a.x, -a.y})
120 int rev[MAXN], T;
121 comp Sin[MAXN], tmp;
122 void fft(comp *a, int r) {
123     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
124     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
125         for(int j = 0; j < T; j += i) {
126             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {

```

```

126         tmp = a[k + mid] * Sin[cur];
127         a[k + mid] = a[k] - tmp;
128         a[k] = a[k] + tmp;
129     }
130 }
131 }
132 }
133 void init(int n) {
134     //for(T = 1; T <= n; T <<= 1);
135     T = n << 1;
136     for(int i = 0; i < T; i++) {
137         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
138         else rev[i] = rev[i >> 1] >> 1;
139     }
140     for(int i = 0; i < (T >> 1); i++) {
141         Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
142     }
143 }
144 const int mod = 1e9 + 7;
145 void mtt(int *x, int *y) {
146     for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
147     static comp a[MAXN], b[MAXN];
148     static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
149     for(int i = 0; i < T; i++) {
150         a[i] = {x[i] & 0x7fff, x[i] >> 15};
151         b[i] = {y[i] & 0x7fff, y[i] >> 15};
152     }
153     fft(a, 1); fft(b, 1);
154     for(int i = 0; i < T; i++) {
155         int j = (T - i) & (T - 1);
156         static comp da, db, dc, dd;
157         da = (a[i] + conj(a[j])) * (comp){0.5, 0};
158         db = (a[i] - conj(a[j])) * (comp){0, -0.5};
159         dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
160         dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
161         dfta[j] = da * dc;
162         dftb[j] = da * dd;
163         dftc[j] = db * dc;
164         dftd[j] = db * dd;
165     }
166     for(int i = 0; i < T; i++) {
167         a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
168         b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
169     }
170     //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
171     fft(a, -1); fft(b, -1);
172     for(int i = 0; i < T; i++) {
173         static int da, db, dc, dd;
174         da = (LL)(a[i].x / T + 0.5) % mod;
175         db = (LL)(a[i].y / T + 0.5) % mod;
176         dc = (LL)(b[i].x / T + 0.5) % mod;
177         dd = (LL)(b[i].y / T + 0.5) % mod;
178         x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
179     }
180 }
181 LL qpow(LL x, LL y) {
182     LL res = 1;
183     while(y) {
184         if(y & 1) res = res * x % mod;
185         x = x * x % mod;
186         y >>= 1;

```

```

187     }
188     return res;
189 }
190 void poly_inv(int *a, int *b, int n) {
191     if(n == 1) {
192         b[0] = qpow(a[0], mod - 2);
193         return;
194     }
195     static int c[MAXN], cc[MAXN], d[MAXN];
196     memset(c, 0, n * sizeof(int)); memset(cc, 0, n * sizeof(int)); memset(d, 0, n *
sizeof(int));
197     poly_inv(a, c, n >> 1);
198     for(int i = 0; i < n; i++) cc[i] = c[i], d[i] = a[i];
199     init(n);
200     mtt(cc, c);
201     mtt(d, cc);
202     for(int i = 0; i < T; i++) b[i] = (2 * c[i] % mod + mod - d[i]) % mod;
203     for(int i = 0; i < n; i++) b[n + i] = 0;
204 }
205 int A[MAXN], B[MAXN];
206 int main() {
207     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
208     int n; cin >> n; n--;
209     for(T = 1; T <= n; T <= 1);
210     for(int i = 0; i <= n; i++) cin >> A[i];
211     poly_inv(A, B, T);
212     for(int i = 0; i <= n; i++) cout << B[i] << " ";
213     cout << endl;
214     return 0;
215 }
216
217 //LOJ挑战多项式(备用)
218 #include <bits/stdc++.h>
219 #define rep(i, a, b) for (int i = (a); i <= (b); i++)
220 #define per(i, a, b) for (int i = (a); i >= (b); i--)
221 #define REP(i, n) for (int i = (0); i < (n); i++)
222 #define fi first
223 #define se second
224 #define mp make_pair
225 #define pb push_back
226 using namespace std;
227 typedef unsigned long long ull;
228 typedef pair<int, int> pii;
229 typedef vector<int> poly;
230 typedef long long ll;
231 struct ano {
232     char a[1 << 25], *s;
233     char b[1 << 25], *t;
234     ano() : s(a), t(b) { a[fread(a, 1, sizeof a, stdin)] = 0; }
235     ~ano() { fwrite(b, 1, t - b, stdout); }
236     operator int() {
237         int x = 0;
238         while (*s < 48) ++s;
239         while (*s > 32) x = x * 10 + *s++ - 48;
240         return x;
241     }
242     ll in() {
243         ll x = 0;
244         while (*s < 48) ++s;
245         while (*s > 32) x = x * 10 + *s++ - 48;
246         return x;

```

```

247     }
248     void out(int x, char e = ' ') {
249         static char c[12];
250         char *i = c;
251         if (!x)
252             *t++ = 48;
253         else {
254             while (x) {
255                 int y = x / 10;
256                 *i++ = x - y * 10 + 48, x = y;
257             }
258             while (i != c) *t++ = *--i;
259         }
260         *t++ = e;
261     }
262 } buf;
263 const int mod = 998244353;
264 namespace Poly {
265     const int N = (1 << 20) + 5, g = 3;
266     inline int power(int x, int p) {
267         int res = 1;
268         for (; p; p >>= 1, x = (ll)x * x % mod)
269             if (p & 1)
270                 res = (ll)res * x % mod;
271         return res;
272     }
273     inline int fix(const int x) { return x >= mod ? x - mod : x; }
274     void dft(poly &A, int n) {
275         static ull W[N << 1], *H[30], *las = W, mx = 0;
276         for (; mx < n; mx++) {
277             H[mx] = las;
278             ull w = 1, wn = power(g, (mod - 1) >> (mx + 1));
279             REP(i, 1 << n) *las++ = w, w = w * wn % mod;
280         }
281         if (A.size() != (1 << n))
282             A.resize(1 << n);
283         static ull a[N];
284         for (int i = 0, j = 0; i < (1 << n); ++i) {
285             a[i] = A[j];
286             for (int k = 1 << (n - 1); (j ^ k) < k; k >>= 1)
287                 ;
288         }
289         for (int k = 0, d = 1; k < n; k++, d <= 1)
290             for (int i = 0; i < (1 << n); i += (d < 1)) {
291                 ull *l = a + i, *r = a + i + d, *w = H[k], t;
292                 for (int j = 0; j < d; j++, l++, r++) {
293                     t = (*r) * (*w++) % mod;
294                     *r = *l + mod - t, *l += t;
295                 }
296             }
297         REP(i, 1 << n) A[i] = a[i] % mod;
298     }
299     void idft(poly &a, int n) {
300         a.resize(1 << n, reverse(a.begin() + 1, a.end()));
301         dft(a, n);
302         int inv = power(1 << n, mod - 2);
303         REP(i, 1 << n) a[i] = (ll)a[i] * inv % mod;
304     }
305     poly FIX(poly a) {
306         while (!a.empty() && !a.back()) a.pop_back();
307         return a;

```

```

308 }
309 poly add(poly a, poly b, int op = 0) {
310     a.resize(max(a.size(), b.size()));
311     REP(i, b.size()) a[i] = fix(op ? a[i] + mod - b[i] : a[i] + b[i]);
312     return FIX(a);
313 }
314 poly mul(poly a, poly b, int t = 1) {
315     if (t == 1 && a.size() + b.size() <= 24) {
316         poly c(a.size() + b.size(), 0);
317         REP(i, a.size()) REP(j, b.size()) c[i + j] = (c[i + j] + (ll)a[i] * b[j]) % mod;
318         return FIX(c);
319     }
320     int n = 1, aim = a.size() * t + b.size();
321     while ((1 << n) <= aim) n++;
322     dft(a, n), dft(b, n);
323     if (t == 1)
324         REP(i, 1 << n) a[i] = (ll)a[i] * b[i] % mod;
325     else
326         REP(i, 1 << n) a[i] = (ll)a[i] * a[i] % mod * b[i] % mod;
327     idft(a, n), a.resize(aim);
328     return FIX(a);
329 }
330 poly mul(poly a, int b) {
331     REP(i, a.size()) a[i] = (ll)a[i] * b % mod;
332     return FIX(a);
333 }
334
335 poly inv(poly a, int n) { // a[0] != 0
336     a.resize(n);
337     poly b;
338     if (n == 1) {
339         b.pb(power(a[0], mod - 2));
340         return b;
341     }
342     b = inv(a, n + 1 >> 1);
343     b = add(mul(b, 2), mul(b, a, 2), 1);
344     return b.resize(n), b;
345 }
346
347 poly Der(poly a) {
348     REP(i, a.size() - 1) a[i] = (ll)(i + 1) * a[i + 1] % mod;
349     return a.pop_back(), a;
350 }
351 poly Int(poly a) {
352     static int inv[N];
353     inv[1] = 1;
354     a.pb(0);
355     rep(i, 2, a.size()) inv[i] = (ll)(mod - mod / i) * inv[mod % i] % mod;
356     per(i, a.size() - 1, 1) a[i] = (ll)a[i - 1] * inv[i] % mod;
357     return a[0] = 0, a;
358 }
359 poly Ln(poly a, int n) { // a[0] = 1
360     a = mul(Der(a), inv(a, n)), a.resize(n - 1);
361     return FIX(Int(a));
362 }
363 poly Exp(poly a, int n) { // a[0] = 0
364     a.resize(n);
365     poly b, one(1, 1);
366     if (n == 1)
367         return one;
368     b = Exp(a, n + 1 >> 1);

```

```

369     b = mul(b, add(add(a, Ln(b, n), 1), one));
370     return b.resize(n), b;
371 }
372
373 poly Div(poly a, poly b) {
374     poly c;
375     int n = a.size() - 1, m = b.size() - 1;
376     if (n < m)
377         return c;
378     reverse(a.begin(), a.end());
379     a.resize(n - m + 1);
380     reverse(b.begin(), b.end());
381     b.resize(n - m + 1);
382     c = mul(a, inv(b, n - m + 1));
383     c.resize(n - m + 1);
384     return reverse(c.begin(), c.end()), c;
385 }
386 poly Mod(poly a, poly b) { return FIX(add(a, mul(Div(a, b), b), 1)); }
387 inline int chk(int x) { return power(x, (mod - 1) / 2) == 1; }
388 inline int R() { return rand() % mod; }
389 inline pii mul(pii a, pii b, int w) {
390     return mp(((ll)a.fi * b.fi + (ll)a.se * b.se % mod * w) % mod, ((ll)a.fi * b.se + (
391         ll)a.se * b.fi) % mod);
392 }
393 inline int Sqrt(int x) {
394     if (!chk(x))
395         return -1;
396     int a = R();
397     while (chk(((ll)a * a - x + mod) % mod)) a = R();
398     int w = ((ll)a * a - x + mod) % mod, p = (mod + 1) / 2;
399     pii res = mp(1, 0), t = mp(a, 1);
400     for (; p >= 1, t = mul(t, t, w))
401         if (p & 1)
402             res = mul(res, t, w);
403     assert(!res.se);
404     return min(res.fi, mod - res.fi);
405 }
406 poly Sqrt(poly a, int n) {
407     if (n == 1) {
408         poly b(1, Sqrt(a[0]));
409         return b;
410     }
411     a.resize(n);
412     poly b = Sqrt(a, n + 1 >> 1);
413     b = mul(add(b, mul(a, inv(b, n))), (mod + 1) / 2);
414     return b.resize(n), b;
415 }
416 poly fastpow(poly a, ll k, int n) {
417     a.resize(n), a = FIX(a);
418     if (!a.size())
419         return a;
420     int st = 0, base = 0;
421     while (!a[st]) ++st;
422     if (st * k >= n)
423         return a.resize(0), a;
424     REP(i, a.size() - st) a[i] = a[i + st];
425     if (st)
426         a.resize(a.size() - st);
427     base = a[0];
428     ll inv = power(base, mod - 2);
429     REP(i, a.size()) a[i] = a[i] * inv % mod;

```



```

429     a = FIX(Exp(mul(Ln(a, n), k % mod), n));
430     ;
431     if (st) {
432         reverse(a.begin(), a.end());
433         a.resize(a.size() + st * k);
434         reverse(a.begin(), a.end());
435         a.resize(n), a = FIX(a);
436     }
437     base = power(base, k);
438     REP(i, a.size()) a[i] = (ll)a[i] * base % mod;
439     return FIX(a);
440 }
441 } // namespace Poly
442 using namespace Poly;
443 int main() {
444     int n = buf + 1, K = buf;
445     poly a(n, 0), b;
446     REP(i, n) a[i] = buf;
447     b = add(a, Exp(Int(inv(Sqrt(a, n), n)), n), 1);
448     b[0] = (b[0] + 2 - a[0] + mod) % mod;
449     a = Ln(b, n);
450     a[0]++;
451     b = Der(fastpow(a, K, n));
452     b.resize(n - 1);
453     REP(i, n - 1) buf.out(b[i]);
454     return 0;
455 }

```

4.1.8 Lagrange Polynomial

$$L(x) = \sum_{i=0}^n y_i \prod_{j=0, j \neq i}^n \frac{x - x_j}{x_i - x_j}$$

```

1 //O(n^2)
2 #include <bits/stdc++.h>
3 using namespace std;
4 typedef long long LL;
5 typedef pair<int, int> P;
6 const int MAXN = 3005, mod = 998244353;
7 int exgcd(int a, int b, int &x, int &y) {
8     int d = a;
9     if(b != 0) {
10         d = exgcd(b, a % b, y, x);
11         y -= (a / b) * x;
12     }
13     else {
14         x = 1; y = 0;
15     }
16     return d;
17 }
18 int inv(int a) {
19     int x, y;
20     exgcd(a, mod, x, y);
21     return (x % mod + mod) % mod;
22 }
23 struct Lagrange {
24     int n, a[MAXN][2];
25     void init() {

```

```

26     for(int i = 0; i <= n; i++) a[i][0] = a[i][1] = 0;
27     n = 0;
28     a[0][1] = 1;
29 }
30 int query(int x, int q = 0) {
31     int res = 0;
32     for(int i = n; i >= 0; i--) res = ((LL)res * x + a[i][q]) % mod;
33     return res;
34 }
35 void update(int x, int y) {
36     a[n][0] = 0;
37     int v = (LL)(y - query(x) + mod) % mod * inv(query(x, 1)) % mod;
38     for(int i = 0; i <= n; i++) a[i][0] = (a[i][0] + (LL)a[i][1] * v) % mod;
39     a[++n][1] = 0;
40     for(int i = n; i; i--) a[i][1] = (a[i - 1][1] + (LL)a[i][1] * (mod - x)) % mod;
41     a[0][1] = (LL)a[0][1] * (mod - x) % mod;
42 }
43 }p;
44 int main() {
45     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
46     int Q;
47     cin >> Q;
48     int op, x, y;
49     p.n = 0;
50     p.init();
51     while(Q--) {
52         cin >> op >> x;
53         if(op == 1) {
54             cin >> y;
55             p.update(x, y);
56         }
57         else cout << p.query(x) << endl;
58     }
59     return 0;
60 }
61 //f(x) = y, 求f(z_i)
62 #include <bits/stdc++.h>
63 using namespace std;
64 #define ll long long
65 int n, m;
66 vector<int> x, y, z;
67 namespace Poly{
68     const int P = 998244353;
69     vector<int> ans; // for Evaluate()
70     vector<vector<int>> p; // for Evaluate() & Interpolate()
71     inline int Pow(ll x, int y=P-2){ // x^y
72         int ans=1;
73         for(; y; y>>=1, x=x*x%P) if(y&1) ans=ans*x%P;
74         return ans;
75     }
76     inline int Ge(int x){ int n=1; while(n<=x) n<<=1; return n;}
77     inline int Mod(int x){ return x<P?x:x-P;}
78     inline void NTT(vector<int> &f, int g, int n){
79         f.resize(n);
80         for(int i=0, j=0; i<n; ++i){
81             if(i>j) swap(f[i], f[j]);
82             for(int k=n>>1; (j^=k)<k; k>>=1);
83         }
84         vector<int> w(n>>1);
85         for(int i=1; i<n; i<<=1){
86             for(int j=w[0]=1, w0=(g==1?Pow(3, (P-1)/i/2):Pow(Pow(3, (P-1)/i/2))); j<i;

```

```

++j) w[j]=(ll)w[j-1]*w0%P;
87     for(int j=0; j<n; j+=i<<1){
88         for(int k=j; k<j+i; ++k){
89             int t=(ll)f[k+i]*w[k-j]%P;
90             f[k+i]=Mod(f[k]-t+P);
91             f[k]=Mod(f[k]+t);
92         }
93     }
94 }
95 if(g== -1) for(int i=0, I=Pow(n); i<n; ++i) f[i]=(ll)f[i]*I%P;
96 }
97 inline vector<int> Add(const vector<int> &f, const vector<int> &g){
98     vector<int> ans=f;
99     for(unsigned i=0; i<f.size(); ++i) (ans[i]+=g[i])%P;
100    return ans;
101 }
102 inline vector<int> Mul(const vector<int> &f, const vector<int> &g){ // f*g
103     vector<int> F=f, G=g;
104     int p=Ge(f.size()+g.size()-2);
105     NTT(F, 1, p), NTT(G, 1, p);
106     for(int i=0; i<p; ++i) F[i]=(ll)F[i]*G[i]%P;
107     NTT(F, -1, p);
108     return F.resize(f.size()+g.size()-1), F;
109 }
110 inline vector<int> PolyInv(const vector<int> &f, int n=-1){ // 1/f
111     if(n== -1) n=f.size();
112     vector<int> ans;
113     if(n==1) return ans.push_back(Pow(f[0])), ans;
114     ans=PolyInv(f, (n+1)/2);
115     vector<int> tmp(&f[0], &f[0]+n);
116     int p=Ge(n*2-2);
117     NTT(tmp, 1, p), NTT(ans, 1, p);
118     for(int i=0; i<p; ++i) ans[i]=(2-(ll)ans[i]*tmp[i]%P+P)*ans[i]%P;
119     NTT(ans, -1, p);
120     return ans.resize(n), ans;
121 }
122 inline void PolyDiv(const vector<int> &a, const vector<int> &b, vector<int> &d,
vector<int> &r){ // a=d*b+r
123     if(b.size()>a.size()) return (void)(d.clear(), r=a);
124
125     vector<int> A=a, B=b, iB;
126     int n=a.size(), m=b.size();
127     reverse(A.begin(), A.end()), reverse(B.begin(), B.end());
128     B.resize(n-m+1), iB=PolyInv(B, n-m+1);
129     d=Mul(A, iB);
130     d.resize(n-m+1), reverse(d.begin(), d.end());
131
132     r=Mul(b, d);
133     for(int i=0; i<m-1; ++i) r[i]=(P+a[i]-r[i])%P;
134     r.resize(m-1);
135 }
136 inline vector<int> Derivative(const vector<int> &a){ // a'
137     vector<int> ans;
138     ans.resize(a.size()-1);
139     for(unsigned i=1; i<a.size(); ++i) ans[i-1]=(ll)a[i]*i%P;
140     return ans;
141 }
142 void Evaluate_Interpolate_Init(int l, int r, int t, const vector<int> &a){
143     if(l==r) return p[t].clear(), p[t].push_back(P-a[l]), p[t].push_back(1);
144     int mid=(l+r)/2, k=t<<1;
145     Evaluate_Interpolate_Init(l, mid, k, a), Evaluate_Interpolate_Init(mid+1, r, k

```

```

146 |1, a);
147     p[t]=Mul(p[k], p[k|1]);
148 }
149 inline void Evaluate(int l, int r, int t, const vector<int> &f, const vector<int> &a
150 ){
151     if(r-l+1<=512){
152         for(int i=1; i<=r; ++i){
153             int x=0, j=f.size(), a1=a[i], a2=(1l)a[i]*a[i]%P, a3=(1l)a[i]*a2%P, a4=(
154             1l)a[i]*a3%P, a5=(1l)a[i]*a4%P, a6=(1l)a[i]*a5%P, a7=(1l)a[i]*a6%P, a8=(1l)a[i]*a7%P
155             ;
156             while(j>=8)
157                 x=((1l)x*a8+(1l)f[j-1]*a7+(1l)f[j-2]*a6+(1l)f[j-3]*a5+(1l)f[j-4]*a4+(1l)
158                 f[j-5]*a3+(1l)f[j-6]*a2+(1l)f[j-7]*a1+f[j-8])%P, j-=8;
159             while(j--) x=((1l)x*a[i]+f[j])%P;
160             ans.push_back(x);
161         }
162         return;
163     }
164     vector<int> tmp;
165     PolyDiv(f, p[t], tmp, tmp);
166     Evaluate(1, (l+r)/2, t<<1, tmp, a), Evaluate((l+r)/2+1, r, t<<1|1, tmp, a);
167 }
168 inline vector<int> Evaluate(const vector<int> &f, const vector<int> &a, int flag=-1)
169 {
170     // f(a_i)
171     if(flag== -1) p.resize(a.size()<<2), Evaluate_Interpolate_Init(0, a.size()-1, 1,
172     a);
173     ans.clear(), Evaluate(0, a.size()-1, 1, f, a);
174     return ans;
175 }
176 vector<int> Interpolate(int l, int r, int t, const vector<int> &x, const vector<int>
177 &f){
178     if(l==r){
179         vector<int> ans;
180         return ans.push_back(f[l]), ans;
181     }
182     int mid=(l+r)/2, k=t<<1;
183     return Add(Mul(Interpolate(l, mid, k, x, f), p[k|1]), Mul(Interpolate(mid+1, r,
184     k|1, x, f), p[k]));
185 }
186 inline vector<int> Interpolate(const vector<int> &x, const vector<int> &y){
187     // (x_i,
188     y_i)
189     int n=x.size();
190     p.resize(n<<2), Evaluate_Interpolate_Init(0, n-1, 1, x);
191     vector<int> f=Evaluate(Derivative(p[1]), x, 0);
192     for(int i=0; i<n; ++i) f[i]=(1l)y[i]*Pow(f[i])%P;
193     return Interpolate(0, n-1, 1, x, f);
194 }
195 }
196 using namespace Poly;
197 int main() {
198     cin >> n; x.resize(n), y.resize(n);
199     for(int i=0; i<n; ++i) cin >> x[i], cin >> y[i];
200     cin >> m, z.resize(m);
201     for(int i=0; i<m; ++i) cin >> z[i];
202     x=Evaluate(Interpolate(x, y), z);
203     for(int i:x) cout << i << " ";
204     return 0;
205 }

```

4.1.9 BM Alogrithm

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  #define rep(i,a,n) for (int i=a;i<n;i++)
4  #define per(i,a,n) for (int i=n-1;i>=a;i--)
5  #define pb push_back
6  #define mp make_pair
7  #define all(x) (x).begin(),(x).end()
8  #define fi first
9  #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector<int> VI;
12 typedef long long ll;
13 typedef pair<int,int> PII;
14 const ll mod=1000000007;
15 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b>=>1){if(b&1)res=res*a%mod;
    a=a*a%mod;}return res;}
16 // head
17 namespace linear_seq {
18     const int N=10010;
19     ll res[N],base[N],_c[N],_md[N];
20
21     vector<int> Md;
22     void mul(ll *a,ll *b,int k) {
23         rep(i,0,k+k) _c[i]=0;
24         rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
25         for (int i=k+k-1;i>=k;i--) if (_c[i])
26             rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
27         rep(i,0,k) a[i]=_c[i];
28     }
29     int solve(ll n,VI a,VI b) { // a 系数 b 初值 b[n+1]=a[0]*b[n]+...
30         // printf("%d\n",SZ(b));
31         ll ans=0,pnt=0;
32         int k=SZ(a);
33         assert(SZ(a)==SZ(b));
34         rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
35         Md.clear();
36         rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
37         rep(i,0,k) res[i]=base[i]=0;
38         res[0]=1;
39         while ((1ll<pnt)<=n) pnt++;
40         for (int p=pnt;p>=0;p--) {
41             mul(res,res,k);
42             if ((n>p)&1) {
43                 for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
44                 rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
45             }
46         }
47         rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
48         if (ans<0) ans+=mod;
49         return ans;
50     }
51     VI BM(VI s) {
52         VI C(1,1),B(1,1);
53         int L=0,m=1,b=1;
54         rep(n,0,SZ(s)) {
55             ll d=0;
56             rep(i,0,L+1) d=(d+(ll)C[i]*s[n-i])%mod;
57             if (d==0) ++m;
58             else if (2*L<=n) {

```

```

59         VI T=C;
60         ll c=mod-d*powmod(b,mod-2)%mod;
61         while (SZ(C)<SZ(B)+m) C.pb(0);
62         rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
63         L=n+1-L; B=T; b=d; m=1;
64     } else {
65         ll c=mod-d*powmod(b,mod-2)%mod;
66         while (SZ(C)<SZ(B)+m) C.pb(0);
67         rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
68         ++m;
69     }
70 }
71 return C;
72 }
73 int gao(VI a,ll n) {
74     VI c=BM(a);
75     c.erase(c.begin());
76     rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
77     return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
78 }
79 };
80
81 int main() {
82     while (~scanf("%d",&n)) {
83         vector<int>v;
84         v.push_back(1);
85         v.push_back(2);
86         v.push_back(4);
87         v.push_back(7);
88         v.push_back(13);
89         v.push_back(24);
90         //VI{1,2,4,7,13,24}
91         printf("%d\n",linear_seq::gao(v,n-1));
92     }
93 }

```

4.2 Math Theory

4.2.1 Inverse

```

1  //O(logn)求n的逆元
2  const int mod = 1e6 + 3;
3  int exgcd(int a, int b, int &x, int &y) {
4      int d = a;
5      if(b != 0) {
6          d = exgcd(b, a % b, y, x);
7          y -= (a / b) * x;
8      }
9      else {
10         x = 1; y = 0;
11     }
12     return d;
13 }
14 int inverse(int a) {
15     int x, y;
16     exgcd(a, mod, x, y);
17     return (x % mod + mod) % mod;
18 }
19 int inverse(int a) { return qpow(a, mod - 2); }

```

```

20 //O(n)求1~n的逆元
21 int inv[MAXN];
22 void init() {
23     inv[0] = inv[1] = 1;
24     for(int i = 2; i < MAXN; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
        mod;
25 }

```

4.2.2 Lucas

```

1 //mod很小可以预处理逆元的情况
2 void init() {
3     fac[0] = 1;
4     for(int i = 1; i < mod; i++) fac[i] = (long long)fac[i - 1] * i % mod;
5     inv[0] = inv[1] = 1;
6     for(int i = 2; i < mod; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
        mod;
7     for(int i = 1; i < mod; i++) inv[i] = (long long)inv[i] * inv[i - 1] % mod;
8 }
9 int C(int a, int b) {
10     if(b > a) return 0;
11     if(a < mod) return (long long)fac[a] * inv[b] % mod * inv[a - b] % mod;
12     return (long long)C(a / mod, b / mod) * C(a % mod, b % mod) % mod;
13 }
14 //mod过大不能预处理逆元的情况
15 LL qpow(LL x, LL y) {
16     LL res = 1;
17     while(y) {
18         if(y & 1) res = res * x % mod;
19         x = x * x % mod;
20         y >>= 1;
21     }
22     return res;
23 }
24 LL C(LL a, LL b) {
25     if(b > a) return 0;
26     if(b > a - b) b = a - b;
27     LL s1 = 1, s2 = 1;
28     for(LL i = 0; i < b; i++) {
29         s1 = s1 * (a - i) % mod;
30         s2 = s2 * (i + 1) % mod;
31     }
32     return s1 * qpow(s2, mod - 2) % mod;
33 }
34 LL lucas(LL a, LL b) {
35     if(a < mod) return C(a, b);
36     return lucas(a / mod, b / mod) * C(a % mod, b % mod);
37 }

```

4.2.3 CRT && exCRT

$$x \equiv a_i \pmod{m_i}$$

```

1 namespace CRT {
2     LL m[MAXN], a[MAXN];
3     LL exgcd(LL _a, LL _b, LL &x, LL &y) {
4         if(!_b) {

```

```

5         x = 1; y = 0;
6         return _a;
7     }
8     LL d = exgcd(_b, _a % _b, y, x);
9     y -= (_a / _b) * x;
10    return d;
11 }
12 LL crt(int n) {
13     LL M = 1, tmp, res = 0, x, y;
14     for(int i = 1; i <= n; i++) M *= m[i];
15     for(int i = 1; i <= n; i++) {
16         tmp = M / m[i];
17         exgcd(tmp, m[i], x, y);
18         x = (x + m[i]) % m[i];
19         res = (a[i] * x % M * tmp % M + res) % M;
20     }
21     return res;
22 }
23 }
24 namespace EXCRT {
25     LL m[MAXN], a[MAXN];
26     LL exgcd(LL _a, LL _b, LL &x, LL &y) {
27         if(!_b) {
28             x = 1; y = 0;
29             return _a;
30         }
31         LL d = exgcd(_b, _a % _b, y, x);
32         y -= (_a / _b) * x;
33         return d;
34     }
35     LL excrt(int n) {
36         LL M = m[1], A = a[1], x, y, d, tmp;
37         for(int i = 2; i <= n; i++) {
38             d = exgcd(M, m[i], x, y);
39             if((A - a[i]) % d) return -1; //No solution
40             tmp = M / d; M *= m[i] / d;
41             y = (A - a[i]) / d % M * y % M;
42             y = (y + tmp) % tmp;
43             A = (m[i] % M * y % M + a[i]) % M;
44             A = (A + M) % M;
45         }
46         return A;
47     }
48     LL inv(LL _a, LL _b) {
49         LL x, y;
50         exgcd(_a, _b, x, y);
51         return (x % _b + _b) % _b;
52     }
53     LL excrt(int n) {
54         LL M = m[1], A = a[1], x, y, d, c, tmp;
55         for(int i = 2; i <= n; i++) {
56             d = exgcd(M, m[i], x, y);
57             c = a[i] - A;
58             if(c % d) return -1;
59             c = (c % m[i] + m[i]) % m[i];
60             M /= d; m[i] /= d;
61             c = c / d * inv(M % m[i], m[i]) % m[i];
62             tmp = M;
63             M *= m[i] * d;
64             A = (c * tmp % M * d % M + A) % M;
65         }

```



```

66     return A;
67 }
68 //当  $a[i]$  一开始就是负数时, 转成正数  $a[i] = (a[i] \% m[i] + m[i]) \% m[i]$ ;
69 }

```

4.2.4 BSGS

```

1  const int MOD = 76543;
2  int hs[MOD + 5], head[MOD + 5], nxt[MOD + 5], id[MOD + 5], ecnt;
3  void insert(int x, int y) {
4      int k = x % MOD;
5      hs[ecnt] = x, id[ecnt] = y, nxt[ecnt] = head[k], head[k] = ecnt++;
6  }
7  int find(int x) {
8      int k = x % MOD;
9      for(int i = head[k]; i; i = nxt[i])
10         if(hs[i] == x)
11             return id[i];
12     return -1;
13 }
14 int BSGS(int a, int b, int c){
15     memset(head, 0, sizeof head); ecnt = 1;
16     if(b == 1) return 0;
17     int m = sqrt(c * 1.0), j;
18     LL x = 1, p = 1;
19     for(int i = 0; i < m; i++, p = p * a % c)
20         insert(p * b % c, i);
21     for(LL i = m; i += m){
22         if((j = find(x = x * p % c)) != -1) return i - j;
23         if(i > c) break;
24     }
25     return -1;
26 }

```

4.2.5 Miller-Rabin && PollardRho

```

1  LL ksc(LL a, LL n, LL mod){
2      LL ret=0;
3      for(;n>>=1){
4          if(n&1){ret+=a;if(ret>=mod)ret-=mod;}
5          a<<=1;if(a>=mod)a-=mod;
6      }
7      return ret;
8  }
9  LL ksm(LL a, LL n, LL mod){
10     LL ret = 1;
11     for(;n>>=1){
12         if(n&1)ret=ksc(ret,a,mod);
13         a=ksc(a,a,mod);
14     }
15     return ret;
16 }
17 int millerRabin(LL n){
18     if(n<2 || (n!=2 && !(n&1)))return 0;
19     LL d=n-1;for(;!(d&1);d>>=1);
20     for(int i=0;i<20;++i){
21         LL a=rand()%(n-1)+1;
22         LL t=d,m=ksm(a,d,n);

```

```

23     for(;t!=n-1 && m!=1 && m!=n-1;m=ksc(m,m,n),t<=1);
24     if(m!=n-1 && !(t&1)) return 0;
25 }
26 return 1;
27 }
28 LL cnt,fact[100];
29 LL gcd(LL a,LL b){return !b?a:gcd(b,a%b);}
30 LL pollardRho(LL n, int a){
31     LL x=rand()%n,y=x,d=1,k=0,i=1;
32     while(d==1){
33         ++k;
34         x=ksc(x,x,n)+a;if(x>=n)x-=n;
35         d=gcd(x>y?x-y:y-x,n);
36         if(k==i){y=x;i<=1;}
37     }
38     if(d==n)return pollardRho(n,a+1);
39     return d;
40 }
41 void findfac(LL n){
42     if(millerRabin(n)){fact[++cnt]=n;return;}
43     LL p=pollardRho(n,rand()%(n-1)+1);
44     findfac(p);
45     findfac(n/p);
46 }

```

4.2.6 $\varphi(n)$

```

1 int phi(int x) {
2     int res = x;
3     for(int i = 2; i * i <= x; i++) {
4         if(x % i == 0) {
5             res = res / i * (i - 1);
6             while(x % i == 0) x /= i;
7         }
8     }
9     if(x > 1) res = res / x * (x - 1);
10    return res;
11 }

```

4.2.7 Euler Sieve

```

1 int prime[MAXN], cnt, phi[MAXN], mu[MAXN];
2 bool isp[MAXN];
3
4 int min_pow[MAXN]; //最小质因子最高次幂
5 int min_sum[MAXN]; //1+p+p^2+...+p^k
6 int div_sum[MAXN]; //约数和
7
8 int min_index[MAXN]; //最小质因子的指数
9 int div_num[MAXN]; //约数个数
10 void Euler(int n) {
11     mu[1] = phi[1] = div_num[1] = div_sum[1] = 1;
12     for(int i = 2; i <= n; i++) {
13         if(!isp[i]) {
14             prime[++cnt] = min_pow[i] = i;
15             phi[i] = i - 1;
16             mu[i] = -1;
17             min_index[i] = 1; div_num[i] = 2;

```

```

18     div_sum[i] = min_sum[i] = i + 1;
19 }
20 for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
21     isp[i * prime[j]] = 1;
22     if(i % prime[j] == 0) {
23         phi[i * prime[j]] = phi[i] * prime[j];
24         mu[i * prime[j]] = 0;
25
26         min_index[i * prime[j]] = min_index[i] + 1;
27         div_num[i * prime[j]] = div_num[i] / (min_index[i] + 1) * (min_index[i] *
prime[j]] + 1);
28
29         min_sum[i * prime[j]] = min_sum[i] + min_pow[i] * prime[j];
30         div_sum[i * prime[j]] = div_sum[i] / min_sum[i] * min_sum[i * prime[j]];
31         min_pow[i * prime[j]] = min_pow[i] * prime[j];
32         break;
33     }
34     phi[i * prime[j]] = phi[i] * (prime[j] - 1);
35     mu[i * prime[j]] = -mu[i];
36
37     div_num[i * prime[j]] = div_num[i] << 1;
38     min_index[i * prime[j]] = 1;
39
40     div_sum[i * prime[j]] = div_sum[i] * (prime[j] + 1);
41     min_pow[i * prime[j]] = prime[j];
42     min_sum[i * prime[j]] = prime[j] + 1;
43 }
44 }
45 }

```

4.2.8 DuJiao Sieve

$$\sum_{i=1}^n \varphi(i)$$

```

1 vector<int> prime;
2 int phi[MAXN], P[MAXN];
3 bool isp[MAXN];
4 unordered_map<LL, int> mp;
5 void Euler(int n) {
6     phi[1] = 1;
7     for(int i = 2; i <= n; i++) {
8         if(!isp[i]) {
9             prime.push_back(i);
10            phi[i] = i - 1;
11        }
12        for(auto x : prime) {
13            if(i * x > n) break;
14            isp[i * x] = 1;
15            if(i % x == 0) {
16                phi[i * x] = phi[i] * x;
17                break;
18            }
19            phi[i * x] = phi[i] * (x - 1);
20        }
21    }
22    for(int i = 1; i <= n; i++) P[i] = (P[i - 1] + phi[i]) % mod;
23 }

```

```

24 LL cal(LL n) {
25     if(n < MAXN) return P[n];
26     if(mp.count(n)) return mp[n];
27     LL res = 0;
28     for(LL i = 2, last; i <= n; i = last + 1) {
29         last = n / (n / i);
30         res += (last - i + 1) % mod * cal(n / i) % mod;
31         res %= mod;
32     }
33     mp[n] = ((__int128)n * (n + 1) / 2 % mod + mod - res) % mod;
34     return mp[n];
35 }

```

$$\sum_{i=1}^n \mu(i)$$

```

1 LL cal(LL n) {
2     if(n < MAXN) return M[n];
3     if(mp.count(n)) return mp[n];
4     LL res = 0;
5     for(LL i = 2, last; i <= n; i = last + 1) {
6         last = n / (n / i);
7         res += (last - i + 1) * cal(n / i);
8     }
9     mp[n] = 1 - res;
10    return 1 - res;
11 }

```

4.2.9 Min_25 Sieve

思路为把结果分为 i 为质数的和, i 为合数的和, $i=1$ 的和

$g(n, j)$ 表示从 1 累加到 n 的 $f(i)$, 其中的 i 满足要么 i 自己是质数, 要么 i 的最小质因子大于第 j 个质数

要求 $f(p)(p \text{ is prime})$ 可被多项式表示, $f(p^k)$ 可快速计算

分别计算多项式 x^0, x^1, x^2 的 $g(n, j)$ 和 $h(n, j)$

h 由欧拉筛计算, 递归起点 $g(n, 0)$

公式

$$\sum_i^n f(i)$$

$$g(n, j) = \begin{cases} g(n, j-1) & p_j^2 > n \\ g(n, j-1) - f(p_j)[g(\frac{n}{p_j}, j-1) - g(p_j-1, j-1)] & p_j^2 \leq n \end{cases}$$

其中

$$g(p_j-1, j-1) = \sum_i^{j-1} f(p_i)$$

可用 h 表示

$S(n, j)$ 表示从 1 累加到 n 的 $f(i)$, 同样的, i 满足要么 i 自己是质数, 要么 i 的最小质因子大于第 j 个质数
和 g 顺序相反, 最后答案为 $S(n, 1)$, 递推式为

$$S(n, j) = g(n, |P|) - \sum_{i=1}^j f(p_i) + \sum_{k \geq j} \sum_{p_k^{e+1} < n} (f(p_k^e) S(\frac{n}{p_k^e}, k+1) + f(p_k^{e+1}))$$

前一部分 $g(n, |P|) - \sum_{i=1}^j f(p_i)$ 是相应的质数部分和。

例题：欧拉函数前缀和

$$\sum_{i=1}^n \varphi(i)$$

$g_{k,n}$ and $h_{k,n}$ Count

$$\sum_{i=1}^n i^k$$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  const int MAXN = 1e6 + 5, mod = 1e9 + 7;
5  const int inv2 = (mod + 1) / 2, inv6 = (mod + 1) / 6;
6  int prime[MAXN], isp[MAXN], cnt;
7  LL g[3][MAXN << 1], h[3][MAXN << 1];
8  LL w[MAXN << 1];
9  int id1[MAXN], id2[MAXN];
10 inline int MOD(LL x) { return x >= mod ? x - mod : x; }
11 //inline int MOD(LL x) { return x % mod; }
12 inline int add(LL x, LL y) { return MOD(MOD(x) + MOD(y)); }
13 void Euler(int n) {
14     for(int i = 2; i <= n; i++) {
15         if(!isp[i]) {
16             prime[++cnt] = i;
17             h[0][cnt] = h[0][cnt - 1] + 1;
18             h[1][cnt] = add(h[1][cnt - 1], i);
19             h[2][cnt] = add(h[2][cnt - 1], (LL)i * i % mod);
20         }
21         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
22             isp[i * prime[j]] = 1;
23             if(i % prime[j] == 0) {
24                 break;
25             }
26         }
27     }
28 }
29 LL n;
30 int sz, m;
31 inline int id(LL x) {
32     return x <= sz ? id1[x] : id2[n / x];
33 }
34 //f(p ^ k)
35 inline int f(int p, LL pk) {
36     return pk / p * (p - 1) % mod;
37 }
38 LL S(LL x, int y) {
39     if(x <= 1 || prime[y] > x) return 0;
40     //G(x) - H(j - 1) (first part)
41     LL res = add(add(g[1][id(x)], mod - g[0][id(x)]), mod - add(h[1][y - 1], mod - h[0][y - 1]));
42     for(int j = y, k = 1; j <= cnt && (LL)prime[j] * prime[j] <= x; j++, k = 1) {
43         for(LL pk = prime[j]; pk * prime[j] <= x; pk *= prime[j], k++) {
44             res = add(res, S(x / pk, j + 1) * f(prime[j], pk) % mod + f(prime[j], pk * prime[j]));
45         }
46     }
47     return res;
48 }

```

```

49 int main() {
50     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
51     cin >> n;
52     sz = sqrt(n);
53     Euler(sz);
54     for(LL i = 1, last, t; i <= n; i = last + 1) {
55         last = n / (n / i);
56         w[++m] = n / i, t = n / i % mod;
57         w[m] <= sz ? id1[w[m]] = m : id2[last] = m;
58         g[0][m] = MOD(t + mod - 1);
59         g[1][m] = add(t * (t + 1) % mod * inv2 % mod, mod - 1);
60         g[2][m] = add((2 * t + 1) % mod * t * (t + 1) % mod * inv6 % mod, mod - 1);
61     }
62     for(int j = 1; j <= cnt; j++) {
63         for(int i = 1; i <= m && (LL)prime[j] * prime[j] <= w[i]; i++) {
64             g[0][i] = MOD(g[0][i] + mod - (g[0][id(w[i] / prime[j])] - h[0][j - 1]));
65             g[1][i] = MOD(g[1][i] + mod - ((LL)prime[j] * MOD(g[1][id(w[i] / prime[j])]
+ mod - h[1][j - 1]) % mod));
66             g[2][i] = MOD(g[2][i] + mod - ((LL)prime[j] * prime[j] % mod * MOD(g[2][id(w
[i] / prime[j])] + mod - h[2][j - 1]) % mod));
67         }
68     }
69     //S(n, 1) + F(1);
70     LL ans = MOD(S(n, 1) + 1);
71     cout << ans << endl;
72     return 0;
73 }

```

$$\sum_{i=1}^n \mu(i)$$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  const int MAXN = 1e6 + 5;
5  int prime[MAXN], isp[MAXN], cnt;
6  LL g[3][MAXN << 1], h[3][MAXN << 1];
7  LL w[MAXN << 1];
8  int id1[MAXN], id2[MAXN];
9  void Euler(int n) {
10     for(int i = 2; i <= n; i++) {
11         if(!isp[i]) {
12             prime[++cnt] = i;
13             h[0][cnt] = h[0][cnt - 1] + 1;
14         }
15         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
16             isp[i * prime[j]] = 1;
17             if(i % prime[j] == 0) {
18                 break;
19             }
20         }
21     }
22 }
23 LL a, b;
24 LL n;
25 int sz, m;
26 inline int id(LL x) {
27     return x <= sz ? id1[x] : id2[n / x];
28 }
29 //f(p ^ k)
30 inline int f(int p, int k) {

```

```

31     return k == 1 ? -1 : 0;
32 }
33 LL S(LL x, int y) {
34     if(x <= 1 || prime[y] > x) return 0;
35     //g(x) - h(j - 1)
36     LL res = - g[0][id(x)] + h[0][y - 1];
37     for(int j = y, k = 1; j <= cnt && (LL)prime[j] * prime[j] <= x; j++, k = 1) {
38         for(LL pk = prime[j]; pk * prime[j] <= x; pk *= prime[j], k++) {
39             res += S(x / pk, j + 1) * f(prime[j], k) + f(prime[j], k + 1);
40         }
41     }
42     return res;
43 }
44 LL cal(LL x) {
45     n = x;
46     m = 0;
47     sz = sqrt(n);
48     for(LL i = 1, last, t; i <= n; i = last + 1) {
49         last = n / (n / i);
50         w[++m] = n / i, t = n / i;
51         w[m] <= sz ? id1[w[m]] = m : id2[last] = m;
52         g[0][m] = t - 1;
53     }
54     for(int j = 1; j <= cnt; j++) {
55         for(int i = 1; i <= m && (LL)prime[j] * prime[j] <= w[i]; i++) {
56             g[0][i] = g[0][i] - (g[0][id(w[i] / prime[j])] - h[0][j - 1]);
57         }
58     }
59     return S(x, 1) + 1;
60 }
61 int main() {
62     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
63     cin >> a >> b;
64     Euler(sqrt(b));
65
66     //S(n, 1) + F(1);
67     cout << cal(b) - cal(a - 1) << endl;
68     return 0;
69 }

```

4.2.10 Möbius Inversion

$$\sum_i^n \sum_j^m lcm(i, j) \pmod{p}$$

```

1  int mu[MAXN], prime[MAXN], sum[MAXN], cnt;
2  bool isp[MAXN];
3  void getmu(int n) {
4      mu[1] = 1;
5      for(int i = 2; i <= n; i++) {
6          if(!isp[i]) {
7              mu[i] = -1;
8              prime[++cnt] = i;
9          }
10         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
11             isp[i * prime[j]] = 1;
12             if(i % prime[j] == 0) {
13                 mu[i * prime[j]] = 0;

```

```

14         break;
15     }
16     mu[i * prime[j]] = -mu[i];
17 }
18 }
19 }
20 ll n, m, ans;
21 ll query(ll x, ll y) { return (x * (x + 1) / 2 % mod) * (y * (y + 1) / 2 % mod) % mod; }
22 ll F(ll x, ll y) {
23     ll res = 0, last;
24     for(ll i = 1; i <= min(x, y); i = last + 1) {
25         last = min(x / (x / i), y / (y / i));
26         res = (res + (sum[last] - sum[i - 1]) * query(x / i, y / i) % mod) % mod;
27     }
28     return res;
29 }
30 int main() {
31     cin >> n >> m;
32     getmu(min(n, m));
33     for(ll i = 1; i <= min(n, m); i++) sum[i] = (sum[i - 1] + (i * i * mu[i]) % mod) % mod;
34     ll last;
35     for(ll d = 1; d <= min(n, m); d = last + 1) {
36         last = min(n / (n / d), m / (m / d));
37         ans = (ans + (last - d + 1) * (d + last) / 2 % mod * F(n / d, m / d) % mod) % mod;
38     }
39     ans = (ans + mod) % mod;
40     cout << ans << endl;
41     return 0;
42 }

```

4.2.11 Primitive Root

简易版如果模 m 有原根，则有 $\varphi(\varphi(m))$ 个原根当 d 遍历 $\varphi(m)$ 的简化剩余系时， g^d 遍历完 m 的全部原根

```

1  const int MAXN = ;
2  LL qpow(LL a, LL b, LL mod) {
3      LL res = 1;
4      for (; b >>= 1) {
5          if(b&1) res = res * a % mod;
6          a = a * a % mod;
7      }
8      return res;
9  }
10 LL phi(LL x) {
11     LL res = x;
12     for(int i = 2; i <= x / i; i++) {
13         if(x % i == 0) {
14             res = res / i * (i - 1);
15             while(x % i == 0) x /= i;
16         }
17     }
18     if(x > 1) res = res / x * (x - 1);
19     return res;
20 }
21 int has_primitive_root(int p) {
22     if (p == 4) return 1;
23     if (p % 2 == 0) p /= 2;
24     if (p % 2 == 0) return false;
25     for (int i = 2; i <= p/i; i++)

```



```

26     if (p % i == 0) {
27         while(p % i == 0) p /= i;
28         return p == 1 ? i : 0;
29     }
30     return p;
31 }
32 //int indg[MAXN];
33 int get_g_init(LL p) {
34     //p : 2 or 4 or (p**n) or (2 * p**n); else return -1;
35     if(p == 2) {puts("1");return 1;}
36     if(p == 4) {puts("3"); return 3;}
37     if(!has_primitive_root(p)) {puts("-1"); return -1;}
38     vector<int> p_fact;
39     LL p_phi = phi(p), tp = p_phi ;
40     for(int i = 2, in = sqrt(p) + 0.5; i <= in; i++)
41         if(tp % i == 0) {
42             p_fact.push_back(p_phi / i);
43             while(tp % i == 0) tp /= i;
44         }
45     if(tp != 1) p_fact.push_back(p_phi / tp);
46     int g = 1;
47     for (bool fg = 0; !fg ;) {
48         fg = 1; g++;
49         if(qpow(g, p_phi, p) != 1) {fg = 0; continue;}
50         for(auto it : p_fact)
51             if(qpow(g, it, p) == 1) {fg = 0; break;}
52     }
53     //for(int i = 0, tg = 1; i < p_phi; i++, tg = tg * g % p) indg[tg] = i;
54     /*vector<int> fac;
55     for(int d = 1, tg = g; d < p_phi; d++, tg = tg * g % p)
56         if(__gcd((LL)d, p_phi) == 1) fac.push_back(tg);
57     sort(fac.begin(), fac.end());
58     int ed = fac.back();
59     for(auto it : fac)
60         printf("%d%c", it, it != ed ? ' ' : '\n');*/
61     return g;
62 }

```

5 Geometry

5.1 Commonly Definition and Functions

5.1.1 Const and Functions

```

1 namespace CG{
2     #define Point Vector
3     const double pi=acos(-1.0);
4     const double inf=1e100;
5     const double eps=1e-9;
6     template <typename T> inline T Abs(T x){return x>0?x:-x;}
7     template <typename T> inline bool operator == (T x,T y){return Abs(x-y)<eps;}
8     int sgn(double x){
9         if (Abs(x)<eps) return 0;
10        if (x>0) return 1;
11        else return -1;
12    }
13 }

```

5.1.2 Point Definition

```

1 namespace CG{
2     struct Point{
3         double x,y;
4         Point(double x=0,double y=0):x(x),y(y){}
5     };
6     Vector operator + (const Vector a,const Vector b){return Vector(a.x+b.x,a.y+b.y);}
7     Vector operator - (const Vector a,const Vector b){return Vector(a.x-b.x,a.y-b.y);}
8     Vector operator * (const Vector a,const double k){return Vector(a.x*k,a.y*k);}
9     Vector operator / (const Vector a,const double k){return Vector(a.x/k,a.y/k);}
10    bool operator < (const Vector a,const Vector b) {return a.x==b.x?a.y<b.y:a.x<b.x;}
11    bool operator == (const Vector a,const Vector b) {return a.x==b.x && a.y==b.y;}
12    double Dot(const Vector a,const Vector b){return a.x*b.x+a.y*b.y;}
13    double Cross(const Vector a,const Vector b){return a.x*b.y-a.y*b.x;}
14    double mult_Cross(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.
y-c.y)-(b.x-c.x)*(a.y-c.y);}
15    double mult_Dot(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.x-
c.x)+(a.y-c.y)*(b.y-c.y);}
16    double Norm(const Vector a){return sqrt(Dot(a,a));}
17    double Angle(const Vector a,const Vector b){return acos(Dot(a,b)/Norm(a)/Norm(b));}
18    Vector Rotate(const Vector a,const double theta){return Vector(a.x*cos(theta)-a.y*
sin(theta),a.x*sin(theta)+a.y*cos(theta));}
19    boolToLeftTest(const Vector a,const Vector b){return Cross(a,b)<0;}
20    double DisPP(const Vector a,const Vector b){return sqrt((a.x-b.x)*(a.x-b.x)+(a.y-b.y
)*(a.y-b.y));}
21 }

```

5.1.3 Line Definition

```

1 namespace CG{
2     struct Line{
3         Point p0,v,p1;
4         double t,theta;
5         Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),v(_v),t(_t){p1=p0+v*t; theta=
atan2(v.y,v.x);}

```

```

6      // Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),p1(_v){v=(p1-p0)/t; theta=
      atan2(v.y,v.x);}
7  };
8  bool operator < (const Line n,const Line m) {return n.theta<m.theta;}
9  Point GetIntersection(const Line n,const Line m){return n.p0+n.v*Cross(m.v,(n.p0-m.
p0))/Cross(n.v,m.v);}
10 bool OnLine(const Vector a,const Line l){return Cross(l.p0-a,l.p1-a)==0;}
11 bool OnSegment(const Point a,const Line l){return sgn(Cross(l.p0-a,l.p1-a))==0 &&
sgn(Dot(l.p0-a,l.p1-a))<0;}
12 double DisPL(const Point a,const Line l){return Abs(Cross(l.p1-l.p0,a-l.p0)/Norm(l.
p1-l.p0));}
13 double DisPS(const Point a,const Line l){
14     if (l.p0==l.p1) return Norm(a-l.p0);
15     Vector v1=l.p1-l.p0,v2=a-l.p0,v3=a-l.p1;
16     if (sgn(Dot(v1,v2))<0) return Norm(v2);
17     if (sgn(Dot(v1,v3))>0) return Norm(v3);
18     return DisPL(a,l);
19 }
20 Point GetProjection(const Point a,const Line l){
21     Vector v=l.p1-l.p0;
22     return l.p0+v*(Dot(v,a-l.p0)/Dot(v,v));
23 }
24 bool SegmentIntersection(const Line n,const Line m,bool p){
25     double c1=Cross(n.p1-n.p0,m.p0-n.p0);
26     double c2=Cross(n.p1-n.p0,m.p1-n.p0);
27     double c3=Cross(m.p1-m.p0,n.p0-m.p0);
28     double c4=Cross(m.p1-m.p0,n.p1-m.p0);
29     if (p){
30         if (!sgn(c1) || !sgn(c2) || !sgn(c3) || !sgn(c4)){
31             return OnSegment(n.p0,m) || OnSegment(n.p1,m) || OnSegment(m.p0,n) ||
OnSegment(m.p1,n);
32         }
33     }
34     return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);
35 }
36 }
37 }

```

5.1.4 Get Area

```

1 namespace CG{
2     double GetArea(Point *p,int n){
3         double area=Cross(p[n],p[1]);
4         for (int i=2;i<=n;i++) area+=0.5*Cross(p[i-1],p[i]);
5         return Abs(area);
6     }
7 }

```

5.1.5 Get Circumference

```

1 namespace CG{
2     double GetCircumference(Point *p,int n){
3         double Circumference=DisPP(p[n],p[1]);
4         for (int i=2;i<=n;i++) Circumference+=DisPP(p[i-1],p[i]);
5         return Circumference;
6     }
7 }

```

5.1.6 Anticlockwise Sort

```

1 namespace CG{
2     \\p为一个凸包,只是不知其点集是否为逆时针
3     void clockwise_sort(Point *p,int n){
4         for(int i=0;i<n-2;i++){
5             double tmp = mult_Cross(p[i+1],p[i+2],p[i]);
6             if(tmp>0) return;
7             else if(tmp<0){
8                 reverse(p,p+n);
9                 return;
10            }
11        }
12    }
13 }

```

5.2 Convex Hull

5.2.1 Get Convex Hull

```

1 namespace CG{
2     Point p[MAXN],s[MAXN]; // both based from 0
3     int ConvexHull(Point *p,int n,Point *s){
4         sort(p,p+n,cmp); //x从小到大,y从小到大;
5         int m=0;
6         for (int i=0;i<n;i++){
7             for (;m>=2 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
8             s[m++]=p[i];
9         }
10        int k=m;
11        for (int i=n-2;i;i--){
12            for (;m>=k+1 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
13            s[m++]=p[i];
14        }
15        return m-1;
16    }
17 }

```

5.2.2 Point in Convex Hull

```

1 namespace CG{
2     bool PointInConvexHull(Point A){
3         int l=1,r=tot-2,mid;
4         while(l<=r){
5             mid=(l+r)>>1;
6             double a1=Cross(p[mid]-p[0],A-p[0]);
7             double a2=Cross(p[mid+1]-p[0],A-p[0]);
8             if(a1>=0 && a2<=0){
9                 if(Cross(p[mid+1]-p[mid],A-p[mid])>=0) return true;
10                return false;
11            }
12            else if(a1<0) r=mid-1;
13            else l=mid+1;
14        }
15        return false;
16    }
17 }

```

5.3 Minkowski Sum

```

1 namespace CG{
2     void Minkowski(Point *C1,int n,Point *C2,int m){
3         for(int i=1;i<=n;i++) s1[i]=C1[i]-C1[i-1];
4         for(int i=1;i<=m;i++) s2[i]=C2[i]-C2[i-1];
5         A[tot=1]=C1[1]+C2[1];
6         int p1=1,p2=1;
7         while (p1<=n && p2<=m) ++tot,A[tot]=A[tot-1]+(s1[p1]*s2[p2]>=0?s1[p1++]:s2[p2
++]);
8         while (p1<=n) ++tot,A[tot]=A[tot-1]+s1[p1++];
9         while (p2<=m) ++tot,A[tot]=A[tot-1]+s2[p2++];
10        tot=ConvexHull(A,tot);
11    }
12 }

```

5.4 Rotating Calipers

5.4.1 The Diameter of Convex Hull

```

1 namespace CG{
2     double RotatingCalipers(Point *p,int n){
3         double dis=0;
4         for(int i=0,j=2;i<n;++i){
5             while (abs(Cross(p[i+1]-p[i],p[j]-p[i]))<abs(Cross(p[i+1]-p[i],p[j+1]-p[i])))
6             ) j=(j+1)%n;
7             dis=max(dis,max(DisPP(p[j],p[i]),DisPP(p[j],p[i+1])));
8         }
9         return dis;
10    }

```

5.4.2 The Min Distance Between two Convex Hull

```

1 namespace CG{
2     ///点c到线段ab的最短距离
3     double GetDist(Point a,Point b,Point c){
4         if(dis(a,b)<esp) return dis(b,c); ///a,b是同一个点
5         if(mult_Dot(b,c,a)<-esp) return dis(a,c); ///投影
6         if(mult_Dot(a,c,b)<-esp) return dis(b,c);
7         return fabs(mult_Cross(b,c,a)/dis(a,b));
8     }
9
10    ///求一条线段ab的两端点到另外一条线段bc的距离，反过来一样，共4种情况
11    double MinDist(Point a,Point b,Point c,Point d){
12        return min(min(GetDist(a,b,c),GetDist(a,b,d)),min(GetDist(c,d,a),GetDist(c,d,b))
13    );
14    }
15    double RotatingCalipers(Point *p,int n,Point *q,int m){
16        int yminP = 0,ymaxQ=0;
17        for(int i=1;i<n;i++){ ///找到点集p组成的凸包的左下角
18            if(p[i].y<p[yminP].y||((p[i].y==p[yminP].y)&&(p[i].x<p[yminP].x)) yminP = i;
19        }
20        for(int i=1;i<m;i++){ ///找到点集q组成的凸包的右上角
21            if(q[i].y>q[ymaxQ].y||((q[i].y==q[ymaxQ].y)&&(q[i].x>q[ymaxQ].x)) ymaxQ = i;
22        }
23        double ans = DisPP(p[yminP],q[ymaxQ]); ///距离(yminP,ymaxQ)维护为当前最小值。

```

```

23     for(int i=0;i<n;i++){
24         double tmp;
25         while(tmp=(mult_Cross(q[ymaxQ+1],p[yminP],p[yminP+1])-mult_Cross(q[ymaxQ],p[
yminP],p[yminP+1]))>esp)
26             ymaxQ = (ymaxQ+1)%m;
27         if(tmp<-esp) ans = min(ans,GetDist(p[yminP],p[yminP+1],q[ymaxQ]));
28         else ans=min(ans,MinDist(p[yminP],p[yminP+1],q[ymaxQ],q[ymaxQ+1]));
29         yminP = (yminP+1)%n;
30     }
31     return ans;
32 }
33 }

```

5.5 Half Plane Intersection

```

1  namespace CG{
2      void HalfPlaneIntersection(Line l[],int n){
3          deque<Point> p;
4          sort(l+1,l+1+n);
5          deque<Line> q;
6          q.push_back(l[1]);
7          for (int i=2;i<=n;i++){
8              for (;!p.empty() && !ToLeftTest(p.back()-l[i].p0,l[i].v);q.pop_back(),p.
pop_back());
9              for (;!p.empty() && !ToLeftTest(p.front()-l[i].p0,l[i].v);q.pop_front(),p.
pop_front());
10             if (sgn(Cross(l[i].v,q.back().v))==0)
11                 if (ToLeftTest(l[i].p0-q.back().p0),q.back().v){
12                     q.pop_back();
13                     if (!p.empty()) p.pop_back();
14                 }
15             if (!q.empty()) p.push_back(GetIntersection(q.back(),l[i]));
16             q.push_back(l[i]);
17         }
18         for (;!p.empty() && !ToLeftTest(p.back()-q.front().p0,q.front().v);q.pop_back(),
p.pop_back());
19         p.push_back(GetIntersection(q.back(),q.front()));
20         if (p.size() < 3) printf("0\n");
21         else{
22             cerr << "!" << endl;
23             double area = 0.5 * Cross(p.back(), p.front());
24             Point last = p.front();
25             for (p.pop_front(); !p.empty(); last = p.front(), p.pop_front())
26                 area += 0.5 * Cross(last, p.front());
27             printf("%.8lf\n", fabs(area));
28         }
29     }
30 }

```

5.6 Min Circle Cover

```

1  namespace CG{
2      Point GetCircleCenter(const Point a,const Point b,const Point c){
3          Point p=(a+b)/2.0,q=(a+c)/2.0;
4          Vector v=Rotate(b-a,pi/2.0),w=Rotate(c-a,pi/2.0);
5          if (sgn(Norm(Cross(v,w)))==0){
6              if (sgn(Norm(a-b)+Norm(b-c)-Norm(a-c))==0) return (a+c)/2;
7              if (sgn(Norm(b-a)+Norm(a-c)-Norm(b-c))==0) return (b+c)/2;

```

```

8         if (sgn(Norm(a-c)+Norm(c-b)-Norm(a-b))==0) return (a+c)/2;
9     }
10    return GetIntersection(Line(p,v),Line(q,w));
11 }
12 void MinCircleCover(Point p[],int n){
13     random_shuffle(p+1,p+1+n);
14     Point c=p[1];
15     double r=0;
16     for (int i=2;i<=n;i++){
17         if (sgn(Norm(c-p[i])-r)>0){
18             c=p[i],r=0;
19             for (int j=1;j<i;j++){
20                 if (sgn(Norm(c-p[j])-r)>0){
21                     c=(p[i]+p[j])/2.0;
22                     r=Norm(c-p[i]);
23                     for (int k=1;k<j;k++){
24                         if (sgn(Norm(c-p[k])-r)>0){
25                             c=GetCircleCenter(p[i],p[j],p[k]);
26                             r=Norm(c-p[i]);
27                         }
28                     }
29                 }
30             }
31             printf("%.10f\n%.10f %.10f",r,c.x,c.y);
32 }

```

5.7 Circle Union Area

```

1 //k次覆盖
2 //圆并去重后s[0]
3 typedef pair<double, int> P;
4 const double pi = acos(-1.0);
5 const int MAXN = 10003;
6 P arc[MAXN << 1];
7 int acnt, cnt;
8 double s[1003];
9 bool del[1003];
10 void add(double st, double en) {
11     if(st < -pi) {
12         add(st + 2 * pi, pi);
13         add(-pi, en);
14         return;
15     }
16     if(en > pi) {
17         add(st, pi);
18         add(-pi, en - 2 * pi);
19         return;
20     }
21     arc[++acnt] = P(st, 1);
22     arc[++acnt] = P(en, -1);
23 }
24 double F(double x) {
25     return (x - sin(x)) / 2;
26 }
27 struct Node {
28     int x, y, r;
29     Node(int _x = 0, int _y = 0, int _r = 0):x(_x), y(_y), r(_r) {}
30     bool operator == (const Node& t) {
31         return x == t.x && y == t.y && r == t.r;

```

```

32     }
33     inline void read() {
34         scanf("%d%d%d", &x, &y, &r);
35     }
36 }a[1003];
37 int main() {
38     int n;
39     scanf("%d", &n);
40     for(int i = 1; i <= n; i++) a[i].read();
41     /*
42     //去重
43     int nn = 0;
44     for(int i = 1; i <= n; i++) {
45         bool same = 0;
46         for(int j = 1; j < i; j++) {
47             if(a[i] == a[j]) {
48                 same = 1; break;
49             }
50         }
51         if(!same) a[++nn] = a[i];
52     }
53     n = nn;
54     //去包含
55     for(int i = 1; i <= n; i++) {
56         for(int j = 1; j <= n; j++) if(i != j) {
57             if(hypot(a[i].x - a[j].x, a[i].y - a[j].y) < (double)(a[i].r - a[j].r)) del[
j] = 1;
58         }
59     }
60     nn = 0;
61     for(int i = 1; i <= n; i++) if(!del[i]) {
62         a[++nn] = a[i];
63     }
64     n = nn;
65     */
66     for(int i = 1; i <= n; i++) {
67         acnt = 0;
68         for(int j = 1; j <= n; j++) if(i != j) {
69             int dis = (a[i].x - a[j].x) * (a[i].x - a[j].x) + (a[i].y - a[j].y) * (a[i].
y - a[j].y);
70             if(a[j].r > a[i].r && dis <= (a[j].r - a[i].r) * (a[j].r - a[i].r)) add(-pi,
pi);
71             else if(dis > (a[i].r - a[j].r) * (a[i].r - a[j].r) && dis < (a[i].r + a[j].
r) * (a[i].r + a[j].r)){
72                 double c = sqrt(dis);
73                 double angle = acos((a[i].r * a[i].r + c * c - a[j].r * a[j].r) / (2 * a
[i].r * c));
74                 double k = atan2(a[j].y - a[i].y, a[j].x - a[i].x);
75                 add(k - angle, k + angle);
76             }
77         }
78         arc[++acnt] = P(pi, -1);
79         sort(arc + 1, arc + acnt + 1);
80         cnt = 0;
81         double last = -pi;
82         for(int j = 1; j <= acnt; j++) {
83             s[cnt] += F(arc[j].first - last) * a[i].r * a[i].r; //扇形 - 三角形
84             double xa = a[i].x + a[i].r * cos(last);
85             double ya = a[i].y + a[i].r * sin(last);
86             last = arc[j].first;
87             double xb = a[i].x + a[i].r * cos(last);

```



```

88         double yb = a[i].y + a[i].r * sin(last);
89         s[cnt] += (xa * yb - xb * ya) / 2; //到圆心的三角形面积
90         cnt += arc[j].second;
91     }
92 }
93 //printf("%.3f\n", s[0]);
94 for (int i = 0; i < n; i++) {
95     printf("[%d] = %.3f\n", i + 1, s[i] - s[i + 1]);
96 }
97 return 0;
98 }

```

5.8 Simpson Integrate

```

1 double Simpson(double l, double r){
2     return (r-l)*(F(l)+4*(F((l+r)/2))+F(r))/6;
3 }
4 double Integrate(double l, double r, double S){
5     double mid=(l+r)/2;
6     double A=Simpson(l, mid);
7     double B=Simpson(mid, r);
8     if(A+B-S<eps) return S;
9     return Integrate(l, mid, A)+Integrate(mid, r, B);
10 }

```

5.9 Closest Point

```

1 vector <Point> P;
2 DB CP(int l, int r) {
3     if (l == r) return DB_INF;
4     if (l + 1 == r) return DisPP(P[l], P[r]);
5     int mid = (l + r) >> 1;
6     DB d = min(CP(l, mid), CP(mid + 1, r));
7     vector <Point> tmp;
8     for (int i = l; i <= r; i++)
9         if (fabs(P[mid].x - P[i].x) < d) tmp.push_back(P[i]);
10    sort(tmp.begin(), tmp.end(), cmp);
11    for (int i = 0; i < tmp.size(); i++)
12        for (int j = i + 1; j < tmp.size() && tmp[j].y - tmp[i].y < d; j++)
13            d = min(d, DisPP(tmp[i], tmp[j]));
14    return d;
15 }

```

5.10 K-D Tree

```

1 #include <iostream>
2 #include <algorithm>
3 #include <stack>
4 #include <math.h>
5 using namespace std;
6 /*function of this program: build a 2d tree using the input trainingdata
7 the input is exm_set which contains a list of tuples (x,y)
8 the output is a 2d tree pointer*/
9
10
11 struct data

```

```

12 {
13     double x = 0;
14     double y = 0;
15 };
16
17 struct Tnode
18 {
19     struct data dom_elt;
20     int split;
21     struct Tnode * left;
22     struct Tnode * right;
23 };
24
25 bool cmp1(data a, data b){
26     return a.x < b.x;
27 }
28
29 bool cmp2(data a, data b){
30     return a.y < b.y;
31 }
32
33 bool equal(data a, data b){
34     if (a.x == b.x && a.y == b.y)
35     {
36         return true;
37     }
38     else{
39         return false;
40     }
41 }
42
43 void ChooseSplit(data exm_set[], int size, int &split, data &SplitChoice{
44     /*compute the variance on every dimension. Set split as the dimension that have the
45     variance. Then choose the instance which is the median on this split dimension.*/
46     /*compute variance on the x,y dimension.  $DX=EX^2-(EX)^2$ */
47     double tmp1,tmp2;
48     tmp1 = tmp2 = 0;
49     for (int i = 0; i < size; ++i)
50     {
51         tmp1 += 1.0 / (double)size * exm_set[i].x * exm_set[i].x;
52         tmp2 += 1.0 / (double)size * exm_set[i].x;
53     }
54     double v1 = tmp1 - tmp2 * tmp2; //compute variance on the xdimension
55
56     tmp1 = tmp2 = 0;
57     for (int i = 0; i < size; ++i)
58     {
59         tmp1 += 1.0 / (double)size * exm_set[i].y * exm_set[i].y;
60         tmp2 += 1.0 / (double)size * exm_set[i].y;
61     }
62     double v2 = tmp1 - tmp2 * tmp2; //compute variance on the ydimension
63
64     split = v1 > v2 ? 0:1; //set the split dimension
65
66     if (split == 0)
67     {
68         sort(exm_set,exm_set + size, cmp1);
69     }
70     else{
71         sort(exm_set,exm_set + size, cmp2);

```

```

72     }
73
74     //set the split point value
75     SplitChoice.x = exm_set[size / 2].x;
76     SplitChoice.y = exm_set[size / 2].y;
77
78 }
79
80 Tnode* build_kdtree(data exm_set[], int size, Tnode* T){
81     //call function ChooseSplit to choose the split dimension and splitpoint
82     if (size == 0){
83         return NULL;
84     }
85     else{
86         int split;
87         data dom_elt;
88         ChooseSplit(exm_set, size, split, dom_elt);
89         data exm_set_right [100];
90         data exm_set_left [100];
91         int sizeleft ,sizeright;
92         sizeleft = sizeright = 0;
93
94         if (split == 0)
95         {
96             for (int i = 0; i < size; ++i)
97             {
98
99                 if (!equal(exm_set[i],dom_elt) && exm_set[i].x <=dom_elt.x)
100                 {
101                     exm_set_left[sizeleft].x = exm_set[i].x;
102                     exm_set_left[sizeleft].y = exm_set[i].y;
103                     sizeleft++;
104                 }
105                 else if (!equal(exm_set[i],dom_elt) && exm_set[i].x >dom_elt.x)
106                 {
107                     exm_set_right[sizeright].x = exm_set[i].x;
108                     exm_set_right[sizeright].y = exm_set[i].y;
109                     sizeright++;
110                 }
111             }
112         }
113         else{
114             for (int i = 0; i < size; ++i)
115             {
116
117                 if (!equal(exm_set[i],dom_elt) && exm_set[i].y <=dom_elt.y)
118                 {
119                     exm_set_left[sizeleft].x = exm_set[i].x;
120                     exm_set_left[sizeleft].y = exm_set[i].y;
121                     sizeleft++;
122                 }
123                 else if (!equal(exm_set[i],dom_elt) && exm_set[i].y >dom_elt.y)
124                 {
125                     exm_set_right[sizeright].x = exm_set[i].x;
126                     exm_set_right[sizeright].y = exm_set[i].y;
127                     sizeright++;
128                 }
129             }
130         }
131         T = new Tnode;
132         T->dom_elt.x = dom_elt.x;

```

```

133     T->dom_elt.y = dom_elt.y;
134     T->split = split;
135     T->left = build_kdtree(exm_set_left, sizeleft, T->left);
136     T->right = build_kdtree(exm_set_right, sizeright, T->right);
137     return T;
138
139 }
140 }
141
142
143 double Distance(data a, data b){
144     double tmp = (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
145     return sqrt(tmp);
146 }
147
148
149 void searchNearest(Tnode * Kd, data target, data &nearestpoint, double &distance){
150
151     //1. 如果Kd是空的, 则设 dist为无穷大返回
152
153     //2. 向下搜索直到叶子结点
154
155     stack<Tnode*> search_path;
156     Tnode* pSearch = Kd;
157     data nearest;
158     double dist;
159
160     while(pSearch != NULL)
161     {
162         //pSearch加入到 search_path 中;
163         search_path.push(pSearch);
164
165         if (pSearch->split == 0)
166         {
167             if(target.x <= pSearch->dom_elt.x) /* 如果小于就进入左子树 */
168             {
169                 pSearch = pSearch->left;
170             }
171             else
172             {
173                 pSearch = pSearch->right;
174             }
175         }
176         else{
177             if(target.y <= pSearch->dom_elt.y) /* 如果小于就进入左子树 */
178             {
179                 pSearch = pSearch->left;
180             }
181             else
182             {
183                 pSearch = pSearch->right;
184             }
185         }
186     }
187     //取出 search_path 最后一个赋给 nearest
188     nearest.x = search_path.top()->dom_elt.x;
189     nearest.y = search_path.top()->dom_elt.y;
190     search_path.pop();
191
192
193     dist = Distance(nearest, target);

```

```

194 //3. 回溯搜索路径
195
196 Tnode* pBack;
197
198 while(search_path.size() != 0)
199 {
200     //取出 search_path 最后一个结点赋给 pBack
201     pBack = search_path.top();
202     search_path.pop();
203
204     if(pBack->left == NULL && pBack->right == NULL) /* 如果 pBack 为子结点 */
205     {
206
207         if( Distance(nearest, target) > Distance(pBack->dom_elt, target) )
208         {
209             nearest = pBack->dom_elt;
210             dist = Distance(pBack->dom_elt, target);
211         }
212     }
213
214     else
215     {
216
217         int s = pBack->split;
218         if (s == 0)
219         {
220             if( fabs(pBack->dom_elt.x - target.x) < dist) /* 如果 target 为中心的圆
221             (球或超球), 半径为 dist 的圆与分割超平面交, 那么就要跳到另一边的子空间去搜索 */
222             {
223                 if( Distance(nearest, target) > Distance(pBack->dom_elt, target) )
224                 {
225                     nearest = pBack->dom_elt;
226                     dist = Distance(pBack->dom_elt, target);
227                 }
228                 if(target.x <= pBack->dom_elt.x) /* 如果 target 位于 pBack 的左子空间, 那
229                 么就要跳到右子空间去搜索 */
230                     pSearch = pBack->right;
231                 else
232                     pSearch = pBack->left; /* 如果 target 位于 pBack 的子空间, 那么就要
233                 跳到左子空间去搜索 */
234                 if(pSearch != NULL)
235                     //pSearch 加入到 search_path 中
236                     search_path.push(pSearch);
237             }
238         }
239         else {
240             if( fabs(pBack->dom_elt.y - target.y) < dist) /* 如果 target 为中心的圆
241             (球或超球), 半径为 dist 的圆与分割超平面交, 那么就要跳到另一边的子空间去搜索 */
242             {
243                 if( Distance(nearest, target) > Distance(pBack->dom_elt, target) )
244                 {
245                     nearest = pBack->dom_elt;
246                     dist = Distance(pBack->dom_elt, target);
247                 }
248                 if(target.y <= pBack->dom_elt.y) /* 如果 target 位于 pBack 的左子空间, 那
249                 么就要跳到右子空间去搜索 */
250                     pSearch = pBack->right;
251                 else

```

```

250         pSearch = pBack->left; /* 如果target位于pBack的子空间, 那么就要
跳到左子空间去搜索 */
251         if(pSearch != NULL)
252             // pSearch加入到search_path中
253             search_path.push(pSearch);
254     }
255 }
256
257 }
258 }
259
260 nearestpoint.x = nearest.x;
261 nearestpoint.y = nearest.y;
262 distance = dist;
263
264 }
265
266 int main(){
267     data exm_set[100]; //assume the max training set size is 100
268     double x,y;
269     int id = 0;
270     cout<<"Please input the training data in the form x y. One instance per line. Enter
-1 -1 to stop."<<endl;
271     while (cin>>x>>y){
272         if (x == -1)
273         {
274             break;
275         }
276         else{
277             exm_set[id].x = x;
278             exm_set[id].y = y;
279             id++;
280         }
281     }
282     struct Tnode * root = NULL;
283     root = build_kdtree(exm_set, id, root);
284
285     data nearestpoint;
286     double distance;
287     data target;
288     cout <<"Enter search point"<<endl;
289     while (cin>>target.x>>target.y)
290     {
291         searchNearest(root, target, nearestpoint, distance);
292         cout<<"The nearest distance is "<<distance<<","and the nearestpoint is "<<
nearestpoint.x<<","<<nearestpoint.y<<endl;
293         cout <<"Enter search point"<<endl;
294     }
295 }
296 }

```

6 Others

6.1 Offline Algorithm

6.1.1 CDQ Divide and Conquer

```

1 struct Node {
2     int x, y, z, ans;
3     Node() {}
4     Node(int _x, int _y, int _z):x(_x), y(_y), z(_z) {}
5     bool operator < (const Node &b) const {
6         if(y == b.y) {
7             if(z == b.z) return x < b.x;
8             return z < b.z;
9         }
10        return y < b.y;
11    }
12 }A[MAXN], B[MAXN], C[MAXN];
13 int bit[MAXN];
14 void add(int k, int v) {
15     for(; k <= m; k += k & -k) bit[k] = max(bit[k], v);
16 }
17 void clear(int k) {
18     for(; k <= m; k += k & -k) bit[k] = 0;
19 }
20 int sum(int k) {
21     int res = 0;
22     for(; k; k -= k & -k) res = max(res, bit[k]);
23     return res;
24 }
25 void solve(int l, int r) {
26     if(l == r) {
27         B[l] = A[l];
28         return;
29     }
30     int mid = (l + r) >> 1;
31     solve(l, mid);
32     for(int i = mid + 1; i <= r; i++) B[i] = A[i];
33     //sort(B + l, B + mid + 1);
34     sort(B + mid + 1, B + r + 1);
35     int L = 1;
36     for(int R = mid + 1; R <= r; R++) {
37         while(L <= mid && B[L].y < B[R].y) add(B[L].z, B[L].ans), L++;
38         A[B[R].x].ans = max(A[B[R].x].ans, sum(B[R].z - 1) + 1);
39         B[R].ans = A[B[R].x].ans;
40     }
41     for(int i = 1; i <= L; i++) clear(B[i].z);
42     solve(mid + 1, r);
43     L = 1;
44     int p = 1, q = mid + 1;
45     while(p <= mid || q <= r) {
46         if(q > r || (p <= mid && B[p].y <= B[q].y)) C[L++] = B[p++];
47         else C[L++] = B[q++];
48     }
49     for(int i = 1; i <= r; i++) B[i] = C[i];
50 }

```

6.1.2 Mo' s Algorithm

```

1 struct Node{
2     int l, r, t, id;
3     bool operator < (const Node& a) const {
4         if(l / sz == a.l / sz) {
5             if(r == a.r) return t < a.t;
6             return r < a.r;
7         }
8         return l / sz < a.l / sz;
9     }
10 }q[MAXN];
11 void solve() {
12     while (t < q[i].t) addTime(t++, 1);
13     while (t > q[i].t) addTime(--t, -1);
14     while(L < q[i].l) add(L++, -1);
15     while(L > q[i].l) add(--L, 1);
16     while(R < q[i].r) add(++R, 1);
17     while(R > q[i].r) add(R--, -1);
18 }

```

6.1.3 Mo's Algorithm On Tree

```

1 struct Edge {
2     int to, nxt;
3 }e[MAXN << 1];
4 int head[MAXN], ecnt;
5 int stack[MAXN], top, belong[MAXN], cnt, sz;
6 struct Node {
7     int l, r, id, ti;
8     bool operator < (const Node &x) const {
9         return belong[l] < belong[x.l] || (belong[l] == belong[x.l] && belong[r] <
10         belong[x.r]) || (belong[l] == belong[x.l] && belong[r] == belong[x.r] && ti < x.ti);
11 }q[MAXN];
12 struct Node2 {
13     int l, r, ti;
14 }qq[MAXN];
15 int n, m, Q, Q0, Q1;
16 int V[MAXN], W[MAXN], C[MAXN];
17 int fa[MAXN][S + 3], dep[MAXN];
18 long long ans[MAXN], tans;
19 int vis[MAXN], cur[MAXN];
20 long long sum[MAXN];
21 int l, r, tm;
22 inline int read() {
23     int x = 0; char ch = getchar(); bool fg = 0;
24     while(ch < '0' || ch > '9') { if(ch == '-') fg = 1; ch = getchar(); }
25     while(ch >= '0' && ch <= '9') { x = x * 10 + ch - '0'; ch = getchar(); }
26     return fg ? -x : x;
27 }
28 inline void add_edge(int u, int v) {
29     e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
30     e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
31 }
32 void dfs(int u, int f) {
33     fa[u][0] = f;
34     dep[u] = dep[f] + 1;
35     int bot = top;

```



```

36     for(int i = head[u]; i; i = e[i].nxt) {
37         int v = e[i].to;
38         if(v == f) continue;
39         dfs(v, u);
40         if(top - bot >= sz) {
41             cnt++;
42             while(top != bot) belong[stack[top--]] = cnt;
43         }
44     }
45     stack[++top] = u;
46 }
47 void G(int &u, int step) {
48     for(int i = 0; i < S; i++) if((1 << i) & step) u = fa[u][i];
49 }
50 int lca(int u, int v) {
51     if(dep[u] > dep[v]) swap(u, v);
52     G(v, dep[v] - dep[u]);
53     if(u == v) return u;
54     for(int i = S; i >= 0; i--) if(fa[u][i] != fa[v][i]) {
55         u = fa[u][i]; v = fa[v][i];
56     }
57     return fa[u][0];
58 }
59 inline void modify(int u) {
60     tans -= V[C[u]] * sum[cur[C[u]]];
61     cur[C[u]] += vis[u];
62     vis[u] = -vis[u];
63     tans += V[C[u]] * sum[cur[C[u]]];
64 }
65 inline void update(int u, int v) {
66     if(u == v) return;
67     if(dep[u] > dep[v]) swap(u, v);
68     while(dep[v] > dep[u]) {
69         modify(v);
70         v = fa[v][0];
71     }
72     while(u != v) {
73         modify(u); modify(v);
74         u = fa[u][0]; v = fa[v][0];
75     }
76 }
77 inline void upd(int t) {
78     if(vis[q[t].l] == -1) {
79         modify(q[t].l);
80         swap(C[q[t].l], q[t].r);
81         modify(q[t].l);
82     }
83     else swap(C[q[t].l], q[t].r);
84 }
85 inline void moveto(int u, int v) {
86     update(l, u); update(r, v);
87     l = u; r = v;
88 }
89 int main() {
90     n = read(); m = read(); Q = read();
91     sz = (int)pow(n, 2.0 / 3.0);
92     for(int i = 1; i <= m; i++) V[i] = read();
93     for(int i = 1; i <= n; i++) W[i] = read();
94     for(int i = 1, u, v; i < n; i++) {
95         u = read(); v = read();
96         add_edge(u, v);

```

```

97     }
98     for(int i = 1; i <= n; i++) {
99         C[i] = read();
100        vis[i] = 1;
101        sum[i] = sum[i - 1] + W[i];
102    }
103    for(int i = 1, tp; i <= Q; i++) {
104        tp = read();
105        if(tp) {
106            ++Q1;
107            q[Q1].l = read(); q[Q1].r = read();
108            q[Q1].id = Q1;
109            q[Q1].ti = i;
110        }
111        else {
112            ++Q0;
113            qq[Q0].l = read(); qq[Q0].r = read();
114            qq[Q0].ti = i;
115        }
116    }
117    dfs(1, 0);
118    while(top) belong[stack[top--]] = cnt;
119    sort(q + 1, q + Q1 + 1);
120    for(int k = 1; k <= S; k++) {
121        for(int i = 1; i <= n; i++) {
122            fa[i][k] = fa[fa[i][k - 1]][k - 1];
123        }
124    }
125    for(int i = 1; i <= Q1; i++) {
126        if(belong[q[i].l] > belong[q[i].r]) swap(q[i].l, q[i].r);
127        moveto(q[i].l, q[i].r);
128        int lc = lca(l, r);
129        modify(lc);
130        while(qq[tm + 1].ti < q[i].ti && tm < Q0) upd(++tm);
131        while(qq[tm].ti > q[i].ti) upd(tm--);
132        ans[q[i].id] = tans;
133        modify(lc);
134    }
135    for(int i = 1; i <= Q1; i++) printf("%lld\n", ans[i]);
136    return 0;
137 }

```

6.2 Randomized Algorithm

6.2.1 Simulated Annealing

```

1 void solve() {
2     while(T > eps) {
3         double alpha = ((rand() % 30001) / 15000.0) * pi;
4         double theta = ((rand() % 10001) / 10000.0) * pi;
5         tmp.x = cur.x + T * sin(theta) * cos(alpha);
6         tmp.y = cur.y + T * sin(theta) * sin(alpha);
7         tmp.z = cur.z + T * cos(theta);
8         tmp.dis = cal(tmp);
9         if(tmp.dis < cur.dis || (tmp.dis * 0.999 < cur.dis && (rand() & 7) == 7)) cur =
tmp;
10        //if(exp((cur.d - tmp.d) / T) > ((double)rand() / RAND_MAX)) cur = tmp;
11
12        T *= 0.999;

```

```

13     }
14 }

```

6.3 Other Method

6.3.1 Enumerate Subset

```

1  for(int i = 0; i < (1 << k); i++) {
2      for(int j = i; ; --j &= i) {
3          // work();
4          if(j == 0) break;
5      }
6  }

```

6.3.2 Enumerate $\lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$

```

1  int cal(int n, int m) {
2      if(n > m) swap(n, m);
3      int res = 0, last;
4      for(int i = 1; i <= n; i = last + 1) {
5          last = min(n / (n / i), m / (m / i));
6          res += (n / i) * (m / i) * (sum(last) - sum(i - 1));
7      }
8      return res;
9  }

```

6.3.3 Find Primitive Root Modulo N

```

1  for i in range(1,mod):
2      if 3 ** i % mod == 1:
3          if i == mod - 1:
4              print("yes")
5              break
6      print("no")

```

7 Samples

7.1 vimrc

```

1 set cindent
2 set number
3 set mouse=a
4 set tabstop=4
5 set shiftwidth=4
6 syntax on
7 inoremap { {}<left>
8 map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>
9 "ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;

1 set nocompatible
2 source $VIMRUNTIME/vimrc_example.vim
3 source $VIMRUNTIME/mswin.vim
4 se cin nu mouse=a ts=4 sw=4 ww=b,s,<,>[,]
5 syntax on
6 inoremap { {}<left>
7 map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>
8 func! AddTitle()
9     call append(0,"// Cease to struggle and you cease to live")
10    call append(1,"// Created by hjj")
11    call append(2,"#include <bits/stdc++.h>")
12    call append(3,"using namespace std;")
13    call append(4,"")
14    call append(5,"int main() {")
15    call append(6,"")
16    call append(7,"    return 0;")
17    call append(8,"}")
18 endfunc
19 map <F8> :call AddTitle()<CR>

```

7.2 Check

Linux

```

1 while true; do
2     ./data > in
3     ./tmp < in > out
4     ./std < in > ans
5     diff out ans
6     if [ $? -ne 0 ] ; then exit; fi
7     echo Passed
8 done

```

windows

```

1 @echo off
2 :loop
3     rand.exe > data.in
4     std.exe < data.in > std.out
5     my.exe < data.in > my.out
6     fc my.out std.out
7 if not errorlevel 1 goto loop
8 pause
9 goto loop

```

7.3 Random

```

1 mt19937_64 mt(chrono::steady_clock::now().time_since_epoch().count());
2 shuffle(per.begin(), per.end(), mt);
3 //random_shuffle(per.begin(), per.end());

```

7.4 FastIO

```

1 //普通情况
2 namespace IO {
3     const int MB = 1048576;
4     const int RMAX = 16 * MB;
5     const int WMAX = 16 * MB;
6     #define getchar() *(rp++)
7     #define putchar(x) (*(wp++) = (x))
8     char rb[RMAX], *rp = rb, wb[WMAX], *wp = wb;
9     inline void init() {
10         fread(rb, sizeof(char), RMAX, stdin);
11     }
12     template <class _T> inline void read(_T &a) {
13         _a = 0; bool _f = 0; int _c = getchar();
14         while (_c < '0' || _c > '9') _f |= _c == '-', _c = getchar();
15         while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getchar();
16         _a = _f ? -_a : _a;
17     }
18     template <class _T> inline void write(_T _a) {
19         static char buf[20], *top = buf;
20         if (_a) {
21             while (_a) {
22                 _T tm = _a / 10;
23                 *(++top) = char(_a - tm * 10) | '0';
24                 _a = tm;
25             }
26             while (top != buf) putchar(*(top--));
27         }
28         else putchar('0');
29     }
30     void output() {
31         fwrite(wb, sizeof(char), wp - wb, stdout);
32     }
33 }
34 //EOF结尾+分块读入
35 #define likely(x) __builtin_expect(!!(x), 1)
36 #define unlikely(x) __builtin_expect(!!(x), 0)
37 namespace IO {
38     const int MB = 1048576;
39     const int RMAX = 4 * MB;
40     const int WMAX = 4 * MB;
41     unsigned long long filesize;
42     #define putchar(x) (*(wp++) = (x))
43     char rb[RMAX], wb[WMAX], *wp = wb;
44     int rp = 0;
45     inline void init() {
46         filesize = fread(rb, sizeof(char), RMAX, stdin);
47         rp = 0;
48         wp = wb;
49     }
50     void output() {
51         fwrite(wb, sizeof(char), wp - wb, stdout);

```

```

52     }
53     inline char getCHAR(){
54         if(unlikely(rp == filesize)){
55             fwrite(wb, sizeof(char), wp - wb, stdout);
56             init();
57             if(unlikely(filesize == 0)) {
58                 //cerr << 1.0 * (clock() - st) / CLOCKS_PER_SEC << endl;
59                 exit(0);
60             }
61         }
62         return rb[rp++];
63     }
64     template <class _T> inline void read(_T &a) {
65         _a = 0; static bool _f = 0; static int _c;
66         _f = 0; _c = getCHAR();
67         while (_c < '0' || _c > '9') _f |= _c == '-', _c = getCHAR();
68         while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getCHAR();
69         _a = _f ? -_a : _a;
70     }
71     template <class _T> inline void write(_T _a) {
72         static char buf[20], *top = buf;
73         if (_a) {
74             while (_a) {
75                 _T tm = _a / 10;
76                 *(&top) = char(_a - tm * 10) | '0';
77                 _a = tm;
78             }
79             while (top != buf) putchar(*(top--));
80         }
81         else putchar('0');
82         putchar('\n');
83     }
84 }

```

7.5 Java BigNum

```

1  import java.math.*;
2  import java.util.*;
3  import java.lang.*;
4
5  public class Main{
6      public static void main(String []args){}
7  }
8  //IO
9  Scanner in = new Scanner(System.in);
10 while(in.hasNext()){ //EOF
11     //fast-IO
12     public static void main(String argv[]) throws IOException{
13         StreamTokenizer cin = new StreamTokenizer(new BufferedReader(new InputStreamReader(
14             System.in)));
15         PrintWriter cout = new PrintWriter(new OutputStreamWriter(System.out));
16         while(cin.nextToken() != StreamTokenizer.TT_EOF) ;//EOF
17         cin.nextToken();int n = (int)cin.nval;String s = cin.sval;
18         cout.println( Type );cout.flush();
19         cin.ordinaryChar( '/' );
20         BufferedReader br = new BufferedReader(new InputStreamReader(System.in));
21         br.ready();//EOF
22         while ((valueString=br.readLine())!=null);

```

```

23 br.close();
24 //true fast-IO
25 static class InputReader {
26     public BufferedReader reader;
27     public StringTokenizer tokenizer;
28
29     public InputReader(InputStream stream) {
30         reader = new BufferedReader(new InputStreamReader(stream), 32768);
31         tokenizer = null;
32     }
33
34     public String next() {
35         while (tokenizer == null || !tokenizer.hasMoreTokens()) {
36             try {
37                 tokenizer = new StringTokenizer(reader.readLine());
38             } catch (IOException e) {
39                 throw new RuntimeException(e);
40             }
41         }
42         return tokenizer.nextToken();
43     }
44
45     public int nextInt() {
46         return Integer.parseInt(next());
47     }
48 }
49
50 //类 Number
51 //doubleValue()
52 //intValue()
53 //longValue()
54 //shortValue()
55 //类 BigDecimal
56 //ROUND_CEILING 接近正无穷大的舍入模式。
57 //ROUND_FLOOR 接近负无穷大的舍入模式。
58 //ROUND_DOWN 接近零的舍入模式
59 //ROUND_HALF_UP 四舍五入 >=0.5向上舍入
60 //ROUND_HALF_DOWN 四舍五入 >0.5向上舍入
61 //BigDecimal(BigInteger val)
62 //BigDecimal(BigInteger unscaledVal, int scale)
63 //BigDecimal(char[] in, int offset, int len, MathContext mc)
64 //BigDecimal(double val, MathContext mc)不建议
65 //BigDecimal(int val, MathContext mc)
66 //BigDecimal(long val, MathContext mc)
67 //BigDecimal(String val, MathContext mc)
68 //abs()
69 //add(BigDecimal augend, MathContext mc)
70 //compareTo(BigDecimal val)
71 //divide(BigDecimal divisor, MathContext mc)
72 //divideToIntegralValue(BigDecimal divisor, MathContext mc)
73 //max(BigDecimal val)
74 //min(BigDecimal val)
75 //multiply(BigDecimal multiplicand, MathContext mc)
76 //negate() 其值为 (-this), 其标度为 this.scale()
77 //pow(int n)
78 //remainder(BigDecimal divisor) 返回其值为 (this % divisor) 的 BigDecimal
79 //round(MathContext mc) 返回根据 MathContext 设置进行舍入后的 BigDecimal。
80 //scaleByPowerOfTen(int n) 返回其数值等于 (this * 10^n) 的 BigDecimal。
81 //subtract(BigDecimal subtrahend, MathContext mc)
82 //setScale(int newScale, RoundingMode roundingMode)
83 //toString()

```

```

84 //ulp()返回此 BigDecimal 的 ulp (最后一位的单位) 的大小
85 //String s = b.stripTrailingZeros().toPlainString();让 bigdecimal不用科学计数法显示
86 //类 BigInteger
87 //parseInt
88 //BigInteger zero = BigInteger.valueOf(0);
89 //BigInteger a = in.nextBigInteger();
90 //abs()
91 //and(BigInteger val) 返回其值为 (this & val)
92 //or(BigInteger val) 返回其值为 (this | val)
93 //andNot(BigInteger val) 返回其值为 (this & ~val)
94 //compareTo(BigInteger val)
95 //add(BigInteger val)
96 //divide(BigInteger val)
97 //BigInteger[] divideAndRemainder(BigInteger val) 返回包含 (this / val) 后跟 (this %
    val) 的两个 BigInteger 的数组。
98 //equals(Object x)
99 //gcd(BigInteger val)
100 //isProbablePrime(int certainty) e.g.: a.isProbablePrime(4)
101 //max(BigInteger val) min(BigInteger val)
102 //mod(BigInteger m)
103 //modInverse(BigInteger m) 返回其值为 (this-1 mod m)
104 //modPow(BigInteger exponent, BigInteger m) 返回其值为 (thisexponent mod m)
105 //multiply(BigInteger val)
106 //not() 返回其值为 (~this)
107 //shiftLeft(int n) 返回其值为 (this << n)
108 //shiftRight(int n) 返回其值为 (this >> n)
109 //toString()
110 //valueOf(long val)
111 //xor(BigInteger val) 返回其值为 (this ^ val)
112 //other
113 //Arrays.sort(array);

```

7.6 pb_ds

```

1 //P.S.:无脑正确使用pb_ds代替std::set/map/priority_queue不会变慢
2 //可持久化平衡树,不过时间和空间都不太行
3 #include <ext/rope>
4 using namespace __gnu_cxx;
5 int a[1000];
6 rope<int> x;
7 rope<int> x(a,a + n);
8 rope<int> a(x);
9 x->at(10);x[10];
10 x->push_back(x) // 在末尾添加x
11 x->insert(pos,x) // 在pos插入x
12 x->erase(pos,x) // 从pos开始删除x个
13 x->replace(pos,x) // 从pos开始换成x
14 x->substr(pos,x) // 提取pos开始x个
15
16 //树
17 //不支持低级操作 (如交换左右子树)
18 #include <ext/pb_ds/assoc_container.hpp>
19 #include <ext/pb_ds/tree_policy.hpp>
20 using namespace __gnu_pbds;
21 定义一颗红黑树
22 tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update>t, other;
23 int 关键字类型
24 null_type
25 无映射(低版本g++为null_mapped_type)(无映射为类似set,有映射类似map)

```



```

26 less<int> 从小到大排序
27 rb_tree_tag 红黑树 (splay_tree_tag)
28 tree_order_statistics_node_update 结点更新(统计子树size, 可自写), 不写不支持order_of_key
    以及find_by_order
29 插入:t.insert();
30 删除:t.erase();
31 比x小的个数:t.order_of_key(x);
32 第x+1值:t.find_by_order(x);
33 前驱:t.lower_bound();
34 后继:t.upper_bound();
35 合并:t.join(other); (other和*this值域不能相交)
36 分裂:t.split(x, other); (清空other, 将t中比x小的元素移至other)
37
38 //自定义节点更新
39 template <class Node_CIttr , class Node_Itr , class Cmp_Fn , class _Alloc >
40 struct my_node_update {
41     virtual Node_CIttr node_begin () const = 0;
42     virtual Node_CIttr node_end() const = 0;
43     typedef char metadata_type; //节点上记录的额外信息的类型
44     //以上为固定格式
45
46     //operator()的功能是将节点it的信息更新为其左右孩子的信息之和, 传入的end_it表示空节点
47     //对Node_Itr可以做的事情有: 用get_l_child, get_r_child获取左右孩子, 用两个星号获取节
    点信息, 用get_metadata获取节点额外信息
48     inline void operator()(Node_Itr it, Node_CIttr end_it) {
49         Node_Itr l = it.get_l_child(), r = it.get_r_child();
50         int left = 0, right = 0;
51         if(l != end_it) left = l.get_metadata();
52         if(r != end_it) right = r.get_metadata();
53         const_cast<metadata_type &>(it.get_metadata()) = left + right + (*it)->second;
54         //it是node_Itr, 取*后变为iterator, 再取->second变成mapped_value
55     }
56     inline int prefix_sum(int x) {
57         int ans = 0;
58         Node_CIttr it = node_begin ();
59         while(it != node_end()) {
60             Node_CIttr l = it.get_l_child(), r = it.get_r_child();
61             if(Cmp_Fn()(x, (*it)->first)) it = l;
62             else {
63                 ans += (*it)->second;
64                 if(l != node_end()) ans += l.get_metadata();
65                 it = r;
66             }
67         }
68         return ans;
69     }
70     inline int interval_sum(int l, int r) {
71         return prefix_sum(r) - prefix_sum(l - 1);
72     }
73 };
74
75 tree<int, char, less<int>, rb_tree_tag, my_node_update> T; //map
76 int main() {
77     T[2] = 'a'; T[3] = 'b'; T[4] = 1;
78     cout << (char)T.interval_sum(3, 4) << endl; //c
79     return 0;
80 }
81 //堆
82 #include <ext/pb_ds/priority_queue.hpp>
83 using namespace __gnu_pbds;
84 __gnu_pbds::priority_queue<int, std::less<int>, __gnu_pbds::pairing_heap_tag> q;

```

```

85
86 template <typename Value_Type ,
87 typename Cmp_Fn = std::less<Value_Type>,
88 typename Tag = pairing_heap_tag ,
89 typename Allocator = std::allocator<char> >
90 class priority_queue
91 Tag可以是binary_heap_tag (二叉堆) binomial_heap_tag (二项堆) rc_binomial_heap_tag
    pairing_heap_tag (配对堆) thin_heap_tag
92 用begin()和end()获取迭代器从而遍历
93 删除单个元素 void erase(point_iterator)
94 更改一个元素的值 void modify(point_iterator, const_reference)
95 合并 void join(priority_queue &other), 把other合并到*this, 并把other清空
96 push()会返回迭代器
97 五种操作: push、pop、modify、erase、join
98 • pairing_heap_tag: push和join为O(1), 其余均摊O(logn)
99 • binary_heap_tag: 只支持push和pop, 均为均摊O(logn)
100 • binomial_heap_tag: push为均摊O(1), 其余为O(logn)
101 • rc_binomial_heap_tag: push为O(1), 其余为O(logn)
102 • thin_heap_tag: push为O(1), 不支持join, 其余为O(logn); 但是如果只有increase_key, modify
    均摊O(1)
103 • 不支持不是不能用, 而是用起来很慢
104 经过实践检测得到的结论:
105 • Dijkstra算法中应用pairing_heap_tag, 速度与手写数据结构相当。
106 • binary_heap_tag在绝大多数情况下优于std::priority_queue
107 • pairing_heap_tag在绝大多数情况优于binomial_heap_tag和rc_binomial_heap_tag
108 • 只有push, pop和join操作时, binary_heap_tag速度较快
109 • 有modify操作时, 可以考虑thin_heap_tag或者pairing_heap_tag, 或手写数据结构。
110
111 //hash_table
112 #include <ext/pb_ds/assoc_container.hpp>
113 #include <ext/pb_ds/hash_policy.hpp>
114 using namespace __gnu_pbds;
115 __gnu_pbds::cc_hash_table <Key, Mapped> mp; //使用链地址法解决哈希冲突
116 __gnu_pbds::gp_hash_table <Key, Mapped> mp; //使用探测法解决哈希冲突
117 //用法和map一样

```