South China University of Technology

SCUT_gugugu

TEMPLATE

Last build at October 28, 2020

Contents

1 Graph Theory

1.1 Shortest Path

1.1.1 Dijkstra

```
typedef long long LL;
const int MAXN = ;
const int MAXM = ;
const LL DINF = ;
typedef pair<LL, int> P;
struct Edge {
    int to, nxt;
    LL w;
}e[MAXM];
int head[MAXN], ecnt;
LL d[MAXN];
priority_queue<P, vector<P>, greater<P> > q;
inline void addEdge(int x, int y, LL w) {
    e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
}
void dijkstra(int st, int n) {
    for(int i = 0; i <= n; i++) d[i] = DINF;
    d[st] = 0;
    q.push(make_pair(0, st));
    while(!q.empty()) {
        P x = q.top(); q.pop();
        int u = x.second;
        if(d[u] != x.first) continue;
        for(int i = head[u], v; i; i = e[i].nxt) {
            v = e[i].to;
            if(d[v] > d[u] + e[i].w) {
                d[v] = d[u] + e[i].w;
                q.push(make_pair(d[v], v));
            }
        }
    }
}
```

1.1.2 SPFA

```
struct Edge {
    int to, nxt;
    LL w;
}e[MAXE];
int head[MAXN], ecnt;
LL d[MAXN];
bool exist[MAXN];
queue<int> q;
inline void addEdge(int x, int y, LL w) {
   e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
}
void SPFA(int st) {
    memset(d,0x3f,sizeof(d));
    d[st] = 0;
    q.push(st);
    exist[st] = 1;
    while(!q.empty()) {
```

```
        int u = q.front(); q.pop();
        exist[u] = 0;
        for(int i = head[u], v; i; i = e[i].nxt) {
            v = e[i].to;
            if(d[v] > d[u] + e[i].w) {
                d[v] = d[u] + e[i].w;
                //pre[v] = u;
                if(!exist[v]) {
                    q.push(v);
                    exist[v] = 1;
                }
            }
        }
    }
}
```

1.1.3 Johnson

```
void johnson() {
    //全源带负权最短路,新建超级节点向全部点连权为0的边, 计算D, 利用Dij计算点对距离
    for(int i = 1; i <= n; i++) addEdge(0, i, 0);
    spfa(0);
    for(int u = 1; u <= n; u++)
        for(int i = head[u]; i; i = e[i].nxt)
            e[i].w += D[e[i].from] - D[e[i].to];
    dijkstra(s,n);
    //ans  = d[n] - D[s] + D[n];

}
```

1.1.4 K Shortest Path (A*)

```
//可重复走同一条边 利用了反向边表示,ecnt初始化为1
//调用dijkstra(ed,n)跑反向图,注意if(i&1){松弛}
int shrt[MAXN];
LL A_star(int st, int ed, int k, int n) {
    if(d[st] == d[0]) return -1;
    if(st == ed) k++;
    q.push(make_pair(d[st], st));
    while(!q.empty()) {
        P x = q.top(); q.pop();
        int u = x.second;
        LL xd = x.first;
        ++shrt[u];
        if(u == ed) {
            if(shrt[ed] == k) return xd;
        }
        for(int i = head[u], v; i; i = e[i].nxt)
        if(!(i & 1)) {
            v = e[i].to;
            if(shrt[v] < k)q.push(make_pair(xd-d[u]+e[i].w+d[v], v));
        }
    }
    return -1;
}
```

1.1.5 K Shortest Path (Protractable Heap)

```
//可重复走同一条边
typedef double LD;
const int MAXN = ;
const int MAXM = ;
const int MAXLT = MAXM * 20;
const LD DINF = ;
const LD eps = ;

namespace LT{
    int tcnt;
    int ls[MAXLT], rs[MAXLT], dis[MAXLT];
    int to[MAXLT];
    LD val[MAXLT];

    inline int newnode(LD w, int _to) {
        ++tcnt;
        ls[tcnt] = rs[tcnt] = 0; dis[tcnt] = 1;
        val[tcnt] = w; to[tcnt] = _to;
        return tcnt;
    }
    inline int copynode(int id) {
        ++tcnt;
        ls[tcnt] = ls[id]; rs[tcnt] = rs[id]; dis[tcnt] = dis[id];
        val[tcnt] = val[id]; to[tcnt] = to[id];
        return tcnt;
    }
    void push_up(int x) {
        if(dis[ls[x]] < dis[rs[x]]) swap(ls[x], rs[x]);
        dis[x] = dis[rs[x]] + 1;
    }
    int merge(int x, int y) {
        if(!x || !y) return x^y;
        if(val[x] - val[y] > eps) swap(x, y);
        int p = copynode(x);
        rs[p] = merge(rs[p],y);
        push_up(p);
        return p;
    }
}
int rt[MAXN];

typedef pair<LD, int> P;
struct Edge {
    int to, nxt;
    LD w;
}e[MAXM];
int head[MAXN], ecnt;
int stan, sta[MAXN], fa[MAXN];
int vis[MAXN], cov[MAXM];
LD d[MAXN];
priority_queue<P, vector<P>, greater<P> > q;
inline void addEdge(int x, int y, LD w) {
    e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
}
void init(int n, int m) {
    ecnt = 1; stan = 0;
    for(int i = 1; i <= n; i++)
        head[i] = cov[i] = fa[i] = 0;
    for(int i = 1; i <= m; i++) {
```

```
        int u, v; LD w;
        scanf("%d%d%lf", &u,&v,&w);
        addEdge(u, v, w);
        addEdge(v, u, w);
    }
}
void dijkstra(int st, int n) {
    for(int i = 0; i <= n; i++) {d[i] = DINF; vis[i] = 0;}
    while(!q.empty()) q.pop();
    d[st] = 0;q.push(make_pair(0, st));
    while(!q.empty()) {
        P x = q.top(); q.pop();
        int u = x.second;
        if(vis[u]) continue;
        vis[u] = 1;
        for(int i = head[u], v; i; i = e[i].nxt)
            if(i & 1) {
                v = e[i].to;
                if(d[v]- (d[u] + e[i].w) > eps) {
                    d[v] = d[u] + e[i].w;
                    q.push(make_pair(d[v], v));
                }
            }
    }
}
void buildT(int u) {
    sta[++stan] = u;
    vis[u] = 1;
    for(int i = head[u], v; i; i = e[i].nxt)
    if(i & 1) {
        v = e[i].to;
        if(fabs(d[v] - (d[u] + e[i].w)) < eps && !vis[v]) {
            fa[v] = u; cov[i^1] = 1; buildT(v);
        }
    }
}
void buildH(int st, int n) {
    buildT(st);
    for(int i = 2, u, v; i <= ecnt; i += 2) {
        if(!cov[i]) {
            u = e[i^1].to; v = e[i].to;
            if(fabs(d[u] - d[0]) < eps || fabs(d[v] - d[0]) < eps) continue;
            rt[u] = LT::merge(rt[u], LT::newnode(d[v]+e[i].w-d[u], v));
        }
    }
    for(int i = 2, u; i <= n; i++)
        if(fa[u = sta[i]]) rt[u] = LT::merge(rt[u], rt[fa[u]]);
}
//求前k短路径,其和不超过W,问最大k int ans = 0;
//求第k短路径,LD ans = 0;
void getKth(int st, int ed, LD W) {
    while(!q.empty()) q.pop();
    //最短路要记入答案,注意d[st]-W<eps与W-d[st]<eps
    //!if(d[st] - W > eps) return;
    //!else{W -= d[st]; ++ans;}
    //注意st和ed相同
    //#if(st == ed) k++;
    //#if(d[st] == d[0]) {ans = -1; return;}
    //#if(--k == 0) {ans = d[st]; return;}

    int u = rt[st], v;
```

```
    if(u) q.push(make_pair(LT::val[u], u));
    while (!q.empty()) {
        u = q.top().second; LD cur = q.top().first;
        q.pop();
        //!if (cur + d[st] - W > eps) break;
        //!else {W-= cur + d[st]; ++ans;}
        //#if (--k == 0) {ans = cur + d[st]; break;}
        v = LT::ls[u];
        if (v) q.push(make_pair(cur - LT::val[u] + LT::val[v], v));
        v = LT::rs[u];
        if (v) q.push(make_pair(cur - LT::val[u] + LT::val[v], v));
        v = rt[LT::to[u]];
        if (v) q.push(make_pair(cur + LT::val[v], v));
    }
}
void sol() {
    int n, m, st = , ed = ;
    //LD W; int k;
    init(n, m);
    dijkstra(ed, n);
    for(int i = 0; i <= n; i++) vis[i] = 0;
    buildH(ed, n);
    getKth(st, ed, W);
    printf("%d\n", ans);
}
```

1.2 Network Flow

1.2.1 ISAP

```
namespace NWF {
    struct Edge{
        int to, nxt;LL f;
    }e[MAXM << 1];
    int S, T, tot;
    int ecnt, head[MAXN], cur[MAXN], pre[MAXN], num[MAXN], dis[MAXN];
    queue<int> q;
    void init(int _S, int _T, int _tot){
        ecnt = 1; S = _S; T = _T; tot = _tot;
        memset(num,  0, (tot + 1) * sizeof(int));
        memset(head, 0, (tot + 1) * sizeof(int));
    }
    inline void addEdge(int u, int v, LL f) {
        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
    }
    void bfs() {
        memset(dis, 0, (tot + 1) * sizeof(int));
        q.push(T);
        dis[T] = 1;
        while(!q.empty()) {
            int u = q.front(), v; q.pop();
            num[dis[u]]++;
            for(int i = cur[u] = head[u]; i; i = e[i].nxt) {
                if(!dis[v = e[i].to]) {
                    dis[v] = dis[u] + 1;
                    q.push(v);
                }
            }
```

```
        }
    }
    LL augment() {
        LL flow = INF;
        for(int i = S; i != T; i = e[cur[i]].to)
            flow = min(flow, e[cur[i]].f);
        for(int i = S; i != T; i = e[cur[i]].to) {
            e[cur[i]].f -= flow;
            e[cur[i] ^ 1].f += flow;
        }
        return flow;
    }
    LL isap() {
        bfs();
        int u = S, v;
        LL flow = 0;
        while(dis[S] <= tot) {
            if(u == T) {
                flow += augment();
                u = S;
            }
            bool fg = 0;
            for(int i = cur[u]; i; i = e[i].nxt) {
                if(e[i].f && dis[u] > dis[v = e[i].to]) {
                    pre[v] = u;
                    cur[u] = i;
                    u = v;
                    fg = 1;
                    break;
                }
            }
            if(fg) continue;
            if(!--num[dis[u]]) break;
            int maxDis = tot;
            for(int i = head[u]; i; i = e[i].nxt) {
                if(e[i].f && maxDis > dis[v = e[i].to]) {
                    maxDis = dis[v];
                    cur[u] = i;
                }
            }
            num[dis[u] = maxDis + 1]++;
            if(u != S) u = pre[u];
        }
        return flow;
    }
}
```

1.2.2 HLPP

```
namespace NWF{
    struct Edge{
        int to,nxt;LL f;
    }e[MAXM << 1];
    int S, T, tot;
    int ecnt, head[MAXN], dis[MAXN], num[MAXN];
    LL sumf[MAXN];
    queue<int> q;
    list<int> dep[MAXN];
    void init(int _S,int _T,int _tot){
        ecnt = 1;S = _S;T = _T;tot = _tot;
```

```
        memset(num,  0, (tot + 1) * sizeof(int));
        memset(head, 0, (tot + 1) * sizeof(int));
        memset(sumf, 0, (tot + 1) * sizeof(LL));
    }
    void addEdge(int u,int v,LL f){
        e[++ecnt] = (Edge) {v, head[u], f};head[u] = ecnt;
        e[++ecnt] = (Edge) {u, head[v], 0};head[v] = ecnt;
    }
    void bfs(){
        memset(dis, 0, (tot + 1) * sizeof(int));
        q.push(T); dis[T] = 1;
        while(!q.empty()){
            int u=q.front(), v; q.pop();
            for(int i = head[u]; i; i = e[i].nxt)
            if(!dis[v = e[i].to]){
                dis[v] = dis[u] + 1;
                q.push(v);
            }
        }
    }
    LL hlpp(){
        bfs();
        dis[S] = tot + 1;
        for(int i = 1;i <= tot; ++i)num[dis[i]]++;
        for(int i = tot + 1; ~i; --i)dep[i].clear();
        int maxd = dis[S];LL f;
        dep[maxd].push_back(S);sumf[S] = INF;
        for(;;){
            while(maxd && dep[maxd].empty())maxd--;
            if(!maxd)break;
            int u = dep[maxd].back(), v;dep[maxd].pop_back();
            int minDis = tot + 1;
            for(int i = head[u]; i;i = e[i].nxt)
            if(e[i].f){
                if(dis[u] > dis[v = e[i].to]){
                    f = min(sumf[u], e[i].f);
                    e[i].f -= f;e[i^1].f += f;
                    if(sumf[u] != INF) sumf[u] -= f;
                    if(sumf[v] != INF) sumf[v] += f;
                    if(v!=S && v!=T && sumf[v] == f){
                        maxd = max(maxd, dis[v]);
                        dep[dis[v]].push_back(v);
                    }
                    if(!sumf[u])break;
                }else minDis=min(minDis, dis[v] + 1);
            }
            if(sumf[u]){
                if(!--num[dis[u]]){
                    for(int i = dis[u];i <= maxd;++i){
                        while(!dep[i].empty()){
                            --num[i];
                            dis[dep[i].back()] = tot + 1;
                            dep[i].pop_back();
                        }
                    }
                    maxd = dis[u] - 1;dis[u] = tot + 1;
                }else{
                    dis[u] = minDis;
                    if(minDis > tot)continue;
                    num[minDis]++;
                    maxd = max(maxd, minDis);
```

```
                    dep[minDis].push_back(u);
                }
            }
        }
        return sumf[T];
    }
}
```

1.2.3 Dinic

注意当流为浮点数的时候，要判断 eps 以及不能使用 sumf-=tmpf，否者 1e18 将不会发生改变

```
namespace NWF {
    struct Edge {
        int to, nxt;LL f;
    } e[MAXM << 1];
    int S, T, tot;
    int ecnt, head[MAXN], cur[MAXN], dis[MAXN];
    queue<int> q;
    void init(int _S, int _T, int _tot){
        ecnt = 1; S = _S; T = _T; tot = _tot;
        memset(head, 0, (tot + 1) * sizeof(int));
    }
    void addEdge(int u, int v, LL f) {
        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
    }
    bool bfs() {
        memset(dis, 0, (tot + 1) * sizeof(int));
        q.push(S); dis[S] = 1;
        while (!q.empty()) {
            int u = q.front(), v; q.pop();
            for (int i = cur[u] = head[u]; i ; i = e[i].nxt) {
                if (e[i].f && !dis[v = e[i].to]) {
                    q.push(v);
                    dis[v] = dis[u] + 1;
                }
            }
        }
        return dis[T];
    }
    LL dfs(int u, LL maxf) {
        if (u == T) return maxf;
        LL sumf = maxf;
        for (int &i = cur[u]; i; i = e[i].nxt) {
            if (e[i].f && dis[e[i].to] > dis[u]) {
                LL tmpf = dfs(e[i].to, min(sumf, e[i].f));
                e[i].f -= tmpf; e[i ^ 1].f += tmpf;
                sumf -= tmpf;
                if (!sumf) return maxf;
            }
        }
        return maxf - sumf;
    }
    LL dinic() {
        LL ret = 0;
        while (bfs()) ret += dfs(S, INF);
        return ret;
    }
    void rebuild(){
        //无向图采用e[i].f = e[i^1].f的方式建立图
```

```
        for(int i = 2; i <= ecnt; i+=2) e[i].f = e[i^1].f = (e[i].f + e[i^1].f) >> 1;
        //有向图
        //for(int i = 2; i <= ecnt; i+=2) e[i].f += e[i^1].f, e[i^1].f = 0;
    }
}
```

1.2.4 Bound Flow

```
namespace NWF{
    //在Edge中添加下限，delta_flow[i]:节点i的入流−出流，bound_flow注意要处理除了SS和TT以外的点
    int delta_flow[MAXN];
    void addEdge(int u, int v, int mxf, int mnf) {
        addEdge(u, v, mxf-mnf);
        delta_flow[u] -= mnf; delta_flow[v] += mnf;
    }
    void delEdge(int u) {
        for(int i = head[u]; i; i = e[i].nxt) e[i].f = e[i^1].f = 0;
    }
    int bound_flow() {
        int SS = ++tot, TT = ++ tot, sum = 0;
        head[SS] = head[TT] = 0;
        for(int i =1; i <= tot;++i) {
            if(delta_flow[i]<0) addEdge(i, TT, -delta_flow[i]);
            if(delta_flow[i]>0){
                sum+=delta_flow[i];
                addEdge(SS, i, delta_flow[i]);
            }
        }
        addEdge(T,S, INF);
        int tS = S, tT = T;
        S = SS; T = TT;
        if (dinic()==sum) {
            delEdge(SS); delEdge(TT);
            int flow = e[ecnt].f;
            e[ecnt].f = e[ecnt^1].f = 0;
            //S = tS; T = tT;// 有上下界有源汇最大流
            //return flow + dinic();
            S = tT; T = tS;// 有上下界有源汇最小流
            return flow - dinic();
        }else{
            return -1;
        }
    }
}
```

1.2.5 Modeling Optimization

利用分治优化建模，每个点 i 向 j 连边，费用为 |ai-aj|

```
int pos[MAXN];
pair<int, int> tmp[MAXN];
void CDQ(int L, int R) {
    if (L == R) return;
    int mid = (L + R) >> 1;
    CDQ(L, mid); CDQ(mid + 1, R);
    inplace_merge(tmp + L, tmp + mid + 1, tmp + R + 1);
    for (int i = L; i <= R; ++i) pos[tmp[i].second] = i;
    for (int i = 2; i <= R - L + 1; ++i) {
```

```
        addEdge(tot + i, tot + i - 1, INF, tmp[L + i - 1].first - tmp[L + i - 2].first);
        addEdge(tot + i - 1, tot + i, INF, tmp[L + i - 1].first - tmp[L + i - 2].first);
    }
    for (int i = L; i <= R; ++i) {
        if (i <= mid)
            addEdge(i+i-1, tot + pos[i] - L + 1, 1, 0);
        else
            addEdge(tot + pos[i] - L + 1, i+i, 1, 0);
    }
    tot += R - L + 1;
}
```

1.2.6 Gomory-Hu Tree

两点间的割可以转为树上两点的距离

```
namespace NWF{
    Edge Te[MAXN];
    int Tcnt, Thead[MAXN];
    void TaddEdge(int u, int v, LL f) {
        Te[++Tcnt] = (Edge) {v, Thead[u], f}; Thead[u] = Tcnt;
        Te[++Tcnt] = (Edge) {u, Thead[v], f}; Thead[v] = Tcnt;
    }
    int node[MAXN], tmp[MAXN];
    void build(int l, int r) {
        if (l == r) return;
        S = node[l]; T = node[l+1];
        rebuild();
        LL cut = dinic();
        TaddEdge(S, T, cut);
        int tl = l, tr = r;
        for(int i = l; i <= r; i++) {
            if(dis[node[i]]) tmp[tl++] = node[i]; else tmp[tr--] = node[i];
        }
        for(int i=l; i<=r; i++) node[i] = tmp[i];
        build(l,tl-1); build(tr+1,r);
    }
    int log2n;
    int dep[MAXN], anc[MAXN][MAXS];LL mnl[MAXN][MAXS];//anc: 祖先; mnl: 最小边
    void lca_dfs(int u, int _fa) {
        for(int i=Thead[u], v; i; i=Te[i].nxt) {
            if((v = Te[i].to) == _fa) continue;
            dep[v] = dep[u] + 1;
            anc[v][0] = u;
            mnl[v][0] = Te[i].f;
            lca_dfs(v, u);
        }
    }
    void work() {
        if(tot == 0) return;
        log2n = log2(tot)+1; Tcnt = 1;
        for(int i=1; i<=tot; i++) node[i]=i, Thead[i]=0;
        build(1, tot);
        dep[1] = 1; anc[1][0] = 0; mnl[1][0] = INF;
        lca_dfs(1, -1);
        for(int j = 1; j <= log2n; j++) {
            for(int i = 1; i <= tot; i++) {
                anc[i][j] = anc[anc[i][j-1]][j-1];
                mnl[i][j] = min(mnl[i][j-1], mnl[anc[i][j-1]][j-1]);
            }
        }
```

```
    }
    LL get_cut(int u,int v) {
        LL res=INF;
        if(dep[u] < dep[v]) swap(u, v);
        for(int i = log2n; i >= 0; i--) {
            if(dep[anc[u][i]] >= dep[v]){
                res = min(res, mnl[u][i]);
                u = anc[u][i];
            }
        }
        if(u == v) return res;
        for(int i = log2n; i>=0 ; i--) {
            if(anc[u][i] != anc[v][i]) {
                res = min(res, mnl[u][i]);
                res = min(res, mnl[v][i]);
                u = anc[u][i];
                v = anc[v][i];
            }
        }
        res = min(res, mnl[u][0]);
        res = min(res, mnl[v][0]);
        return res;
    }
}
```

1.2.7 MCMF

```
namespace NWF{
    struct Edge {
        int to, nxt;LL f, c;
    } e[MAXM << 1];
    int S, T, tot;
    int ecnt, head[MAXN], cur[MAXN];LL dis[MAXN];
    bool exist[MAXN];
    queue<int> q;
    void init(int _S, int _T, int _tot){
        ecnt = 1; S = _S; T = _T; tot = _tot;
        memset(head, 0, (tot + 1) * sizeof(int));
    }
    void addEdge(int u, int v, LL f, LL c) {
        e[++ecnt] = (Edge) {v, head[u], f, c}; head[u] = ecnt;
        e[++ecnt] = (Edge) {u, head[v], 0,-c}; head[v] = ecnt;
    }
    bool spfa() {
        for(int i = 0;i <= tot; ++i){
            dis[i] = INF; cur[i] = exist[i] = 0;
        }
        q.push(S);dis[S] = 0;exist[S] = 1;
        while(!q.empty()) {
            int u = q.front(), v; q.pop();exist[u] = 0;
            for(int i = head[u]; i; i = e[i].nxt) {
                if(e[i].f && dis[v = e[i].to] > dis[u] + e[i].c) {
                    dis[v] = dis[u] + e[i].c;
                    cur[v] = i;
                    if(!exist[v]) {
                        q.push(v);
                        exist[v] = 1;
                    }
                }
            }
```

```
        }
        return dis[T] != INF;
    }
    LL mcmf() {
        LL cost = 0;
        //while(spfa() && dis[T] < 0) {//最小费用可行流
        while(spfa()) {
            LL flow = INF;
            for(int i = T; i != S; i = e[cur[i] ^ 1].to)
                flow = min(flow, e[cur[i]].f);
            for(int i = T; i != S; i = e[cur[i] ^ 1].to) {
                e[cur[i]].f -= flow;
                e[cur[i] ^ 1].f += flow;
            }
            cost += flow * dis[T];
        }
        return cost;
    }
}
```

1.3 Tree Related

1.3.1 Union Set

```
int fa[MAXN], rnk[MAXN];
int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
bool same(int x, int y){ return Find(x) == Find(y); }
void unite(int x, int y)
{
    x = Find(x);
    y = Find(y);
    if(x == y) return;
    if(rnk[x] < rnk[y]) {
        fa[x] = y;
    }
    else {
        fa[y] = x;
        if(rnk[x] == rnk[y]) rnk[x]++;
    }
}
```

1.3.2 Kruskal

```
namespace MST{
    struct Edge{
        int u,v; LL w;
        bool operator < (const Edge& x) const { return w < x.w; }
    }e[MAXM];
    int ecnt, fa[MAXN];
    void addEdge(int u, int v, LL w) {
        e[++ecnt] = (Edge){v, u, w}; //headp[u] = ecnt;
    }
    int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
    LL kruskal(int n) {
        sort(e + 1, e + ecnt + 1);
        for(int i = 1; i <= n; i++) fa[i] = i;
        LL sum = 0;
        for (int i = 1; i <= ecnt; i++){
```

```
            int fu = Find(e[i].u), fv = Find(e[i].v);
            if(fu != fv){
                fa[fu] = fv;
                sum += e[i].w;
            }
        }
        return sum;
    }
}
```

1.3.3 Prim

```
namespace MST {
    struct Edge{
        int to,nxt; LL w;
    }e[MAXM];
    int ecnt, head[MAXN], vis[MAXN]; // pre[MAXN];
    LL dis[MAXN];
    void addEdge(int u, int v, LL w){
        e[++ecnt] = (Edge){v, head[u], w}; head[u] = ecnt;
        e[++ecnt] = (Edge){u, head[v], w}; head[v] = ecnt;
    }
    LL Prim(int n){
        for (int i = 1; i <= n; i++){
            //pre[i] = 0;
            vis[i] = 0;
            dis[i] = INF;
        }
        vis[1] = 1;
        LL sum = 0;
        for (int i = head[1]; i; i = e[i].nxt)
            dis[e[i].to] = min(dis[e[i].to],e[i].w);
        for (int j = 1; j < n; j++){
            int u; LL minDis = INF;
            for (int i = 1; i <= n; ++i)
                if (!vis[i] && dis[i] < minDis){
                    minDis = dis[i];
                    u = i;
                }
            if (minDis == INF) return -1;
            vis[u] = 1;
            sum += minDis;
            for (int i = head[u], v; i; i = e[i].nxt)
            if (!vis[v = e[i].to] && e[i].w < dis[v]){
                //pre[u] = v;
                dis[v] = e[i].w;
            }
        }
        return sum;
    }
}
```

1.3.4 Spanning Tree Calculation

关联矩阵 B:n*m 的矩阵, 其中 ek=(vi,vj),Bik 和 Bjk 一个为 1 一个为-1, 第 k 列其他元素为 0 度数矩阵 D:n*n 的矩阵, 其中 i!=j 时,D[i][j]=0;i==j 时,D[i][j]=vi 的度邻接矩阵 A:n*n 的矩阵,vi,vj 有边相连, 为 1, 否则为 0 Kirchhoff 矩阵: B*BT = D - A 即：如果 i=j，那么 aij 为点 i(j) 的度数。如果 i!=j，那么 Aij 为 i 到 j 的边数的相反数。生成树个数:Kirchhoff 矩阵 n-1 阶主子式的行列式值构造 Kirchhoff 矩阵, 调用 det(n)

```
LL a[MAXN][MAXN];
void getSTC(int n, int m) {
    for(int i = 1; i <= n; i++) {
        for(int j = 1; j <= n; j++)
            a[i][j] = a[j][i] = 0;
    }
    for(int i = 1,u, v; i <= m; i++) {
        scanf("%d%d", &u,&v);
        if(u == v) continue;
        a[u][v] = --a[v][u];
    }
    for(int i = 1; i <= n; i++) {
        int t = 0;
        for(int j = 1; j <= n; j++)
            t += a[i][j];
        a[i][i] = - t;
    }
    LL ans = det();//删掉一行一列以后求行列式的值
}
```

1.3.5 Minimum Spanning Tree Calculation

```
typedef long long LL;
const int MAXN = ;
const int MAXM = ;
int sum,ans1,ans2=1, Mod=;
int fa1[MAXN],fa2[MAXN];
bool vis[MAXN];
struct Edge{int v,u,val;}e[MAXM];
bool cmp(Edge A,Edge B){return A.val<B.val;}
int getfa1(int *fa,int x){return fa[x]=fa[x]==x?x:getfa1(fa,fa[x]);}
int getfa2(int *fa,int x){return fa[x]==x?x:getfa2(fa,fa[x]);}
void dfs(int tot,int l,int r){
    if(tot==0){++sum;return;}
    for(int i=l,fx,fy;i<r;++i)
        if(!vis[i]){
            vis[i]=true;
            fx=getfa2(fa2,e[i].u);fy=getfa2(fa2,e[i].v);
            if(fx!=fy){
                fa2[fx]=fy;
                dfs(tot-1,i+1,r);
                fa2[fx]=fx;
            }
            vis[i]=false;
        }
}
void sol(){
    int n,m;
    scanf("%d %d",&n,&m);
    for(int i=1;i<=n;++i)fa1[i]=fa2[i]=i;
    for(int i=1;i<=m;++i)
        scanf("%d %d %d",&e[i].v,&e[i].u,&e[i].val);
    std::sort(e+1,e+m+1,cmp);e[m+1].val=-1;
    for(int i=1,j=1,fx,fy,tot;i<=m;++i){
        for(;e[i].val==e[j].val;++j);
        tot=0;
        for(int k=i;k<j;++k){
            fx=getfa1(fa1,e[k].u);fy=getfa1(fa1,e[k].v);
            if(fx!=fy){++tot;ans1++;fa1[fx]=fy;}
        }
```

```
        if(!tot)continue;
        sum=0;dfs(tot,i,j);
        (ans2*=sum)%=Mod;
        for(int k=i;k<j;++k){
            fx=getfa1(fa2,e[k].u);fy=getfa1(fa2,e[k].v);
            if(fx!=fy)fa2[fx]=fy;
        }
    }
    if(ans1!=n-1)puts("0");else printf("%d",ans2);
}
```

1.3.6 Steiner Tree

```
const int MAXH = 128;
const int MAXW = 128;
const int MAXST = 1256;
namespace SteinerTree{
    const int dx[4] = {0,0,1,-1};
    const int dy[4] = {1,-1,0,0};
    int n, m, k, stn;
    bool vis[MAXH][MAXW];
    LL G[MAXH][MAXW],f[MAXST][MAXH][MAXW];
    queue<pair<int, int> > q;
    struct PRE{int s, x, y;} pre[MAXST][MAXH][MAXW];

    void spfa(int st) {
        while(!q.empty()) {
            int vx = q.front().first,vy = q.front().second;
            q.pop();
            vis[vx][vy] = 0;
            for (int i = 0, ux,uy; i < 4; ++i) {
                ux = vx + dx[i];
                uy = vy + dy[i];
                if (ux==0||uy==0||ux==n+1||uy==m+1)continue;
                if (f[st][vx][vy]+G[ux][uy]<f[st][ux][uy]) {
                    f[st][ux][uy] = f[st][vx][vy] + G[ux][uy];
                    pre[st][ux][uy] = (PRE) {st, vx, vy};
                    if (!vis[ux][uy]) {
                        vis[ux][uy]=1;
                        q.push(make_pair(ux,uy));
                    }
                }
            }
        }
    }
    LL sum = 0;
    void init() {
        k = 0;
        for(int i = 1; i <= n; ++i)
            for(int j = 1; j <= m; ++j) {
                scanf("%d", &G[i][j]);
                if(G[i][j] == 0)  k++;
            }
        stn = 1<<k;
        for (int st = 0; st < stn; ++st)
            for (int i = 1; i <= n; ++i)
                for (int j = 1; j <= m; ++j)
                    f[st][i][j] = INF;
        int tk = 0;
        for(int i = 1; i <= n; ++i)
```

```
            for(int j = 1; j <= m; ++j) {
                vis[i][j] = 0;
                if(G[i][j] == 0) {f[1<<tk][i][j] = 0; tk++;}
            }
    }
    void dfs(int st, int x, int y) {
        vis[x][y] = 1;
        PRE tmp = pre[st][x][y];
        if (tmp.x == 0 && tmp.y == 0) return ;
        dfs(tmp.s, tmp.x, tmp.y);
        if (tmp.x == x && tmp.y == y) dfs(st - tmp.s, tmp.x, tmp.y);
    }
    void sol(int _n, int _m) {
        n = _n; m = _m;
        init();
        for (int st = 0; st < stn; ++st) {
            for (int i = 1; i <= n; ++i)
                for (int j = 1; j <= m; ++j) {
                    for (int s = st&(st-1); s; s = st&(s-1))
                    if(f[st-s][i][j]+f[s][i][j]-G[i][j] < f[st][i][j]){
                        f[st][i][j] = f[st-s][i][j]+f[s][i][j]-G[i][j];
                        pre[st][i][j] = (PRE) {s, i, j};
                    }
                    if (f[st][i][j]!=INF) {
                        q.push(make_pair(i,j));
                        vis[i][j]=1;
                    }
                }
            spfa(st);
        }
        int ansx, ansy, fg = 0;
        for(int i = 1; i <= n && !fg; ++i)
           for(int j = 1; j <= m; ++j)
           if(!G[i][j]) {ansx = i; ansy = j; fg = 1; break;}
        printf("%d\n", f[stn-1][ansx][ansy]);
        memset(vis, 0, sizeof vis);
        dfs(stn-1, ansx, ansy);
        for(int i = 1; i <= n; i++, puts("")) {
            for(int j = 1; j <= m; j++) {
                if(G[i][j] == 0) putchar('x');
                else if(vis[i][j]) putchar('o');
                else putchar('_');
            }
        }
    }
}
```

1.3.7 Tree Divide and Conquer

```
struct Edge {
    int to, nxt, w;
}e[MAXM];
int head[MAXN], ecnt;
int sz[MAXN];
int d[MAXN], t[5], ans;
bool vis[MAXN];
inline void add_edge(int u, int v, int w) {
    e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
    e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
}
```

```
int getsz(int x, int fa) {
    sz[x] = 1;
    for(int i = head[x]; i; i = e[i].nxt) {
        int y = e[i].to;
        if(vis[y] || y == fa) continue;
        sz[x] += getsz(y, x);
    }
    return sz[x];
}
int getrt(int x) {
    int tot = getsz(x, 0) >> 1;
    while(1) {
        int u = -1;
        for(int i = head[x]; i; i = e[i].nxt) {
            int y = e[i].to;
            if(vis[y] || sz[y] > sz[x]) continue;
            if(u == -1 || sz[y] > sz[u]) u = y;
        }
        if(~u && sz[u] > tot) x = u;
        else break;
    }
    return x;
}
void getdep(int x, int fa) {
    t[d[x]]++;
    for(int i = head[x]; i; i = e[i].nxt) {
        int y = e[i].to;
        if(vis[y] || y == fa) continue;
        d[y] = (d[x] + e[i].w) % 3;
        getdep(y, x);
    }
}
int cal(int x, int v) {
    t[0] = t[1] = t[2] = 0;
    d[x] = v % 3;
    getdep(x, 0);
    return t[0] * t[0] + t[1] * t[2] * 2;
}
void solve(int x) {
    vis[x] = 1;
    ans += cal(x, 0);
    for(int i = head[x]; i; i = e[i].nxt) {
        int y = e[i].to;
        if(vis[y]) continue;
        ans -= cal(y, e[i].w);
        solve(getrt(y));
    }
}
int main() {
    solve(getrt(1));
}
```

1.3.8 Dominator Tree

```
#define LL long long
#define FILE "dagch"
using namespace std;

const int N = 200010;
struct Node{int to,next;}E[N<<1];
```

```
int n,m,q,head[N],tot,dfn[N],clo,rev[N],fa[N],semi[N],Ans[N];
vector<int>G[N];
struct Union_Merge_Set{
    int fa[N],Mi[N];
    inline void init(){
        for(int i=0;i<=n;++i)
            fa[i]=Mi[i]=semi[i]=i;
    }
    inline int find(int x){
        if(x==fa[x])return x;
        int fx=fa[x],y=find(fa[x]);
        if(dfn[semi[Mi[fx]]]<dfn[semi[Mi[x]]])Mi[x]=Mi[fx];
        return fa[x]=y;
    }
}uset;

inline void tarjan(int u) {
    rev[dfn[u] = ++tarjan_time] = u;
    for(auto v : G[u])
    if(!dfn[v]) {
        fa[v] = u;
        tarjan(v);
    }
}
inline void get_semi() {
    for(int i = tarjan_time; i >= 2; i--) {
        int u = rev[i], tsemi = n;
        for(auto v : rG[u]) {
            if(!dfn[v]) continue;
            if(dfn[v] < dfn[u]) tsemi = min(tsemi, dfn[x]);
            else{
                uset.find(x);
                tsemi = min(tsemi, dfn[semi[uset.Mi[x]]]);
            }
        }
        uset.fa[y] = fa[y];
        semi[y] = rev[tsemi];
        Ans[rev[tsemi]]++;
    }
}

inline void solve() {
    scanf("%d %d %d", &n, &m, &q);
    fa[1]=1;
    for(int i = 1, u, v; i <= m; ++i){
        scanf("%d%d", &u,&v);
        link(v,u);
        G[u].push_back(v);
    }
    for(int i = 1; i <= n; i++)
        if(G[i].size())
            sort(G[i].begin(), G[i].end());

    uset.init();

    tarjan(1);
    build();
    for(int i=1;i<=q;++i)
        printf("%d ",Ans[gi()]);
    printf("\n");
    for(int i=0;i<=n;++i){
```

```
        G[i].clear();head[i]=0;
        Ans[i]=semi[i]=fa[i]=0;
    }
    clo=tot=0;
}

int main() {
    int T; scanf("%d", &T);
    while(T--) solve();
    return 0;
}
```

1.4 LCA

1.4.1 Tree Decomposition LCA

见树链剖分

1.4.2 Tarjan LCA

```
vector< pair<int,int> > G[MAXN],ask[MAXN];
int fa[MAXN], ans[MAXN], vis[MAXN] ,dis[MAXN];
int Find(int x){
    return x == fa[x] ? x : fa[x] = Find(fa[x]);
}
void init(int n){
    memset(ans, 0,sizeof ans);
    memset(vis, 0,sizeof vis);
    for(int i = 0; i <= n; i++){
        G[i].clear();
        ask[i].clear();
    }
}
void LCA(int u){
    int v;
    fa[u] = u;
    vis[u] = true;
    for(auto it : ask[u])
        if(vis[v = it.first])
            ans[it.second] = dis[u] + dis[v] - 2 * dis[Find(it.first)];
    for(auto it : G[u])
    if(!vis[v = it.first]){
        dis[v] = dis[u] + it.second;
        LCA(v);
        fa[v] = u;
    }
}
```

1.5 Tarjan

1.5.1 SCC

```
namespace SCC{
    vector<int> G[MAXN];
    int dfs_clock, scc_cnt, dfn[MAXN], low[MAXN], sccno[MAXN];
    stack<int> S;
    void addEdge(int u, int v) {
```

```
        G[u].push_back(v);
    }
    void tarjan(int u) {
        dfn[u] = low[u] = ++dfs_clock;
        S.push(u);
        for(auto v : G[u]) {
            if(!dfn[v]) {
                tarjan(v);
                low[u] = min(low[u], low[v]);
            }else if(!sccno[v]) {
                low[u] = min(low[u], dfn[v]);
            }
        }
        if(dfn[u] == low[u]) {
            scc_cnt++;
            for(;;) {
                int v = S.top(); S.pop();
                sccno[v] = scc_cnt;
                if(v == u) break;
            }
        }
    }
    void findSCC(int n) {
        for(int i = 1; i <= n; i++)
            if(!dfn[i]) tarjan(i);
    }
    void init(int n){
        dfs_clock = scc_cnt = 0;
        for(int i = 0;i <= n;++i){
            dfn[i] = low[i] = sccno[i] = 0;
            G[i].clear();
        }
    }
}
```

1.5.2 BCC

```
namespace BCC{
    struct Edge {
        int to, nxt;
    }e[MAXM << 1];
    int ecnt, head[MAXN];
    int dfs_clock, dfn[MAXN], low[MAXN];

    int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
    vector<int> vbcc[MAXN];
    stack<int> vS;

    int ebcc_cnt, ebccno[MAXN];
    stack<int> eS;

    inline void addEdge(int u, int v) {
        e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
        e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
    }
    inline void init(int n) {
        ecnt = 1;
        dfs_clock = 0;
        vbcc_cnt = 0;
        ebcc_cnt = 0;
```

```
        for(int i = 1; i <= n; ++i){
            head[i] = dfn[i] = low[i] = 0;
            is_vertex[i] = 0;
            vbccno[i] = 0;
            ebccno[i] = 0;
        }
        while(!vS.empty()) vS.pop();
    }
    //root's edge = -1;
    void tarjan(int u, int edge) {
        dfn[u] = low[u] = ++dfs_clock;
        int ch = 0;
        vS.push(u);
        eS.push(u);
        for(int i = head[u], v; i; i = e[i].nxt) {
            if(!dfn[v = e[i].to]) {
                tarjan(v, i ^ 1);
                low[u] = min(low[u], low[v]);
                if(low[v] >= dfn[u]) {
                    ++ch;
                    if(edge > 0 || ch > 1) is_vertex[u] = 1;
                    vbcc[++vbcc_cnt].clear();
                    vbcc[vbcc_cnt].push_back(u);
                    for(int x;;){
                        x = vS.top();vS.pop();
                        vbcc[vbcc_cnt].push_back(x);
                        vbccno[x] = vbcc_cnt;
                        if(x == v)break;
                    }
                }
                if(low[v] > dfn[u]) {
                // i && i ^ 1 is bridge
                }
            }
            else if(dfn[v] < dfn[u] && i != edge)
                low[u] = min(low[u], dfn[v]);
        }
        if(dfn[u] == low[u]) {
            ebcc_cnt++;
            for(int v;;) {
                v = eS.top(); eS.pop();
                ebccno[v] = ebcc_cnt;
                if(v == u) break;
            }
        }
    }
    void findBCC(int n){
        for(int i = 1; i <= n; i++)
            if(!dfn[i]) tarjan(i, -1);

        //findBridge
        for(int u = 1; u <= n; u++) {
            for(int i = head[u], v; i; i = e[i].nxt)
            if(ebccno[u] != ebccno[v = e[i].to]) {
                //is bridge
            }
        }
    }
}
```

1.6 Cactus

1.6.1 Circle-Square Tree

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> P;
const int MAXN = 2e4 + 5;
const int S = 15;
namespace Tree {
    struct Edge {
        int to, nxt, w;
    }e[MAXN << 1];
    int ecnt, head[MAXN];
    int rt, isrt[MAXN], fa[MAXN][S + 3];
    int sz[MAXN];
    inline void addEdge(int u, int v, int w) {
        e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
        fa[v][0] = u;
    }
}
int n, m, Q;
namespace BCC {
    struct Edge {
        int to, nxt, w;
    }e[MAXN << 1];
    int ecnt, head[MAXN];
    int dfs_clock, dfn[MAXN], low[MAXN];
    int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
    vector<P> vbcc[MAXN];
    stack<P> vs;
    int tag[MAXN];
    inline void addEdge(int u, int v, int w) {
        e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
        e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
    }
    inline void init(int n) {
        ecnt = 1;
        dfs_clock = 0;
        vbcc_cnt = 0;
        for(int i = 0; i <= 2 * n; i++){
            head[i] = dfn[i] = low[i] = 0;
            vbccno[i] = 0;
            tag[i] = 0;
        }
        while(!vs.empty()) vs.pop();
    }
    //root's edge = -1;
    void tarjan(int u, int edge) {
        dfn[u] = low[u] = ++dfs_clock;
        vs.push(P(u, e[edge ^ 1].w));
        for(int i = head[u], v; i; i = e[i].nxt) {
            if(!dfn[v = e[i].to]) {
                tarjan(v, i ^ 1);
                low[u] = min(low[u], low[v]);
                if(low[v] >= dfn[u]) {
                    if(vs.top().first == v) {
                        Tree::addEdge(u, v, vs.top().second);
                        vs.pop();
                        continue;
```

```
                    }
                    vbcc[++vbcc_cnt].clear();
                    vbcc[vbcc_cnt].push_back(P(u, 0));
                    Tree::isrt[u] = 1;
                    int &sz = Tree::sz[n + vbcc_cnt];
                    tag[vs.top().first] = n + vbcc_cnt;
                    //Tree::addEdge(u, rt, 0);
                    for(P x;;) {
                        x = vs.top(); vs.pop();
                        sz += x.second;
                        //Tree::addEdge(rt, x.first, sz);
                        vbcc[vbcc_cnt].push_back(x);
                        vbccno[x.first] = vbcc_cnt;
                        if(x.first == v) break;
                    }
                }
            }
            else if(dfn[v] < dfn[u] && i != edge)
                low[u] = min(low[u], dfn[v]);
        }
        for(int i = head[u], v; i; i = e[i].nxt) {
            if(tag[v = e[i].to]) {
                int r = tag[v]; Tree::sz[r] += e[i].w;
                tag[v] = 0;
            }
        }
    }
    void findBCC(int n) {
        for(int i = 1; i <= n; i++)
            if(!dfn[i]) tarjan(i, -1);
    }
}
namespace Tree {
    int dis[MAXN], dep[MAXN], len[MAXN];
    inline void init(int n) {
        BCC::init(n);
        rt = n;
        ecnt = 1;
        for(int i = 0; i <= 2 * n; i++) {
            head[i] = 0;
            fa[i][0] = isrt[i] = dis[i] = dep[i]  = len[i] = 0;
        }
    }
    void dfs(int x) {
        for(int i = head[x], y; i; i = e[i].nxt) {
            if(!dep[y = e[i].to]) {
                dep[y] = dep[x] + 1;
                dis[y] = dis[x] + e[i].w;
                dfs(y);
            }
        }
    }
    void pre() {
        for(int k = 1; k <= BCC::vbcc_cnt; k++) {
            rt++;
            vector<P> &E = BCC::vbcc[k];
            addEdge(E[0].first, rt, 0);
            int cnt = 0;
            for(int i = E.size() - 1; i >= 1; i--) {
                cnt += E[i].second;
                len[E[i].first] = cnt;
```

```
                addEdge(rt, E[i].first, min(cnt, sz[rt] - cnt));
            }
        }
        for(int k = 1; k <= S; k++) {
            for(int i = 1; i <= rt; i++) {
                fa[i][k] = fa[fa[i][k - 1]][k - 1];
            }
        }
        dep[1] = 1;
        dfs(1);
    }
    int up(int x, int d) {
        for(int i = S; i >= 0; i--) {
            if(dep[fa[x][i]] >= d) x = fa[x][i];
        }
        return x;
    }
    int lca(int u, int v) {
        if(dep[u] > dep[v]) swap(u, v);
        v = up(v, dep[u]);
        if(u == v) return u;
        for(int i = S; i >= 0; i--) {
            if(fa[u][i] != fa[v][i]) {
                u = fa[u][i], v = fa[v][i];
            }
        }
        return fa[u][0];
    }
    int query(int u, int v) {
        int l = lca(u, v);
        if(l <= n) return dis[u] + dis[v] - 2 * dis[l];
        int x = up(u, dep[l] + 1), y = up(v, dep[l] + 1);
        int res = dis[u] - dis[x] + dis[v] - dis[y];
        int tmp = abs(len[x] - len[y]);
        return res + min(tmp, sz[l] - tmp);
    }
}

int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    using namespace Tree;
    cin >> n >> m >> Q;
    init(n);
    for(int i = 1, u, v, w; i <= m; i++) {
        cin >> u >> v >> w;
        BCC::addEdge(u, v, w);
    }
    BCC::findBCC(n);
    pre();
    int u, v;
    while(Q--) {
        cin >> u >> v;
        cout << query(u, v) << endl;
    }
    return 0;
}
```

2 Data Structures

2.1 Basic Structures

2.1.1 RMQ

```
struct RMQ {
    int d[MAXN][S + 2];
    inline void init(int *a, int n) {
        for(int i = 1; i <= n; i++) d[i][0] = a[i];
        for(int k = 1; (1 << k) <= n; k++)
            for(int i = 1; i + (1 << k) - 1 <= n; i++)
                d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
    }
    inline int query(int l, int r) {
        if(l > r) swap(l, r);
        int k = 0;
        while((1 << (k + 1)) <= r - l + 1) k++;
        return min(d[l][k], d[r - (1 << k) + 1][k]);
    }
}rmq;
const int MAXM = 2e5 + 5, MAXN = 3e6 + 5, S = 22;
const LL INF = 1e18;
#define belong(x) (x / S + 1)
#define pos(x) (x % S + 1)
int Log[MAXN], sz;
struct RMQ {
    LL a[MAXN];
    LL d[MAXM][S + 2];
    LL pre[MAXM][S + 2], aft[MAXM][S + 2];
    inline void init(int n) {
        sz = n / S + 1;
        Log[0] = -1; for(int i = 1; i <= n; i++) Log[i] = Log[i / 2] + 1;
        for(int i = 1; i <= sz; i++) {
            pre[i][0] = aft[i][S + 1] = INF;
        }
        for(int i = 1; i <= n; i++) {
            pre[belong(i)][pos(i)] = min(pre[belong(i)][pos(i) - 1], a[i]);
        }
        for(int i = n; i >= 1; i--) {
            aft[belong(i)][pos(i)] = min(aft[belong(i)][pos(i) + 1], a[i]);
        }
        for(int i = 1; i <= sz; i++) {
            d[i][0] = aft[i][1];
        }
        for(int k = 1; k <= S; k++)
            for(int i = 1; i + (1 << k) <= sz; i++)
                d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
    }
    inline LL ask(int l, int r) {
        assert(l <= r);
        LL res = INF;
        if(belong(l) == belong(r)) {
            for(int i = l; i <= r; i++) res = min(res, a[i]);
            return res;
        }
        res = min(aft[belong(l)][pos(l)], pre[belong(r)][pos(r)]);
        int k = Log[belong(r) - belong(l) - 1];
        if(~k) {
```

```
            res = min(res, d[belong(l) + 1][k]);
            res = min(res, d[belong(r) - (1 << k)][k]);
        }
        return res;
    }
}rmq;
```

2.1.2 Divide Blocks

```
int belong[MAXN], l[MAXN], r[MAXN];
int sz, num;
void build(int n) {
    sz = sqrt(n);
    num = n / sz; if(n % sz) num++;
    for(int i = 1; i <= num; i++) {
        l[i] = (i - 1) * sz + 1;
        r[i] = i * sz;
    }
    r[num] = n;
    for(int i = 1; i <= n; i++) {
        belong[i] = (i - 1) / sz + 1;
    }
}
```

2.2 Heap Structures

2.2.1 Leftist Tree

```
const int MAXN = ;
namespace LeftistTree{
    int ls[MAXN], rs[MAXN];
    int dis[MAXN];
    int fg[MAXN], sfa[MAXN], rt[MAXN];//利用rt得到堆根节点
    int val[MAXN];

    void push_down(int x) {};
    void push_up(int x) {
        if(dis[ls[x]] < dis[rs[x]]) swap(ls[x], rs[x]);
        dis[x] = dis[rs[x]] + 1;
    }
    int merge(int x, int y) {
        if(!x || !y) return x^y;
        if(val[x] > val[y] || (val[x] == val[y] && x > y)) swap(x, y);
        push_down(x);
        rs[x]  = merge(rs[x], y);
        push_up(x);
        return x;
    }
    int getSfa(int x) {return sfa[x] == x ? x : sfa[x] = getSfa(sfa[x]);}
    int uni(int x, int y) {//返回合并后的根
        if(!x || !y) return x^y;
        if(fg[x] || fg[y]) return;
        x = getSfa(x); y = getSfa(y);
        if(x == y) return;
        int z = merge(x, y);
        return sfa[x] = sfa[y] = sfa[z] = z;
    }
    void uni2(int a, int b) {
```

```
        //val[a] >>= 1;
        int c = merge(ls[a], rs[a]);
        ls[a] = rs[a] = dis[a] = 0;
        int a1 = merge(c, a);
        //val[b] >>= 1;
        c = merge(ls[b], rs[b]);
        ls[b] = rs[b] = dis[b] = 0;
        int b1 = merge(c, b);
        c = merge(a1, b1);
        sfa[a] = sfa[b] = sfa[c] = c;
        printf("%d\n", val[c])
    }
    int pop(int x) {//返回堆顶值,也可以用于返回根
        if(!x || fg[x]) return -1;
        x = getSfa(x); fg[x] = 1;
        push_down(x);//在删除堆顶时要下传标记
        int y = merge(ls[x], rs[x]);
        sfa[x] = sfa[y] = y;
        return val[x];
    }
    void init(int n) {
        for(int i = 1; i <= n; i++) {
            sfa[i] = i;
            ls[i] = rs[i] = dis[i] = fg[i] = 0;
        }
    }
}
//可持久化版本见k短路
```

2.3 Sequence Structures

2.3.1 Cartesian Tree

```
struct CartesianTree{
    int rt, fa[MAXN], ls[MAXN], rs[MAXN];
    int top, st[MAXN];
    int cnt[MAXN];
    void build(LL *a,int n) {
        top = rt = 0;
        for(int i = 1; i <= n; i++) {
            ls[i] = rs[i] = fa[i] = 0;
            while(top && a[st[top]] > a[i]) ls[i] = st[top--];
            fa[i] = st[top];
            if(ls[i]) fa[ls[i]] = i;
            if(fa[i]) rs[fa[i]] = i; else rt = i;
            st[++top] = i;
        }
    }
    void dfs(int x) {
        cnt[x] = 1;
        if(ls[x]) {dfs(ls[x]); cnt[x] += cnt[ls[x]];}
        if(rs[x]) {dfs(rs[x]); cnt[x] += cnt[rs[x]];}
    }
    LL getAns(LL *a, int n) {
        //dfs(rt);
        //————————
        return res;
    }
}T;
```

2.3.2 TreeArray

```
//树状数组上二分
int BS(int x) {
    int res = 0;
    for (int i = 1 << 18; i; i >>= 1)
        if ((res | i) <= Tn && T[res | i] <= x)
            x -= T[res |= i];
    return res;
}
```

2.3.3 Segment Tree

```
#define Ls(x) (x << 1)

#define Rs(x) (x << 1 | 1)
struct Tree {
    int l, r, lazy;
    LL sum, mx;
}tree[MAXN << 2];
int A[MAXN];
void push_up(int x) {
    tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
    tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
}
void push_down(int x) {
    if(tree[x].lazy) {
        tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
        tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
        tree[Ls(x)].mx += tree[x].lazy;
        tree[Rs(x)].mx += tree[x].lazy;
        tree[Ls(x)].lazy += tree[x].lazy;
        tree[Rs(x)].lazy += tree[x].lazy;
        tree[x].lazy = 0;
    }
}
void build(int x, int L, int R) {
    tree[x].lazy = 0;
    tree[x].l = L; tree[x].r = R;
    if(L == R) {
        tree[x].sum = A[L];
        tree[x].mx = A[L];

        return;
    }
    int mid = (L + R) >> 1;
    build(Ls(x), L, mid);
    build(Rs(x), mid + 1, R);
    push_up(x);
}
void update(int x, int L, int R, LL val) {
    if(tree[x].l >= L && tree[x].r <= R) {
        tree[x].lazy += val;
        tree[x].sum += val * (tree[x].r - tree[x].l + 1);
        tree[x].mx += val;
        return;
    }
    push_down(x);
    int mid = (tree[x].l + tree[x].r) >> 1;
```

```
    if(L <= mid) update(Ls(x), L, R, val);
    if(R > mid) update(Rs(x), L, R, val);
    push_up(x);
}
LL query(int x, int L, int R) {
    if(tree[x].l >= L && tree[x].r <= R)
        return tree[x].sum;
    push_down(x);
    int mid = (tree[x].l + tree[x].r) >> 1;
    LL res = 0;
    if(L <= mid) res += query(Ls(x), L, R);
    if(R > mid) res += query(Rs(x), L, R);

    return res;
}
LL query2(int x, int L, int R) {
    if(tree[x].l >= L && tree[x].r <= R)
        return tree[x].mx;
    push_down(x);
    int mid = (tree[x].l + tree[x].r) >> 1;
    LL res = -INF;
    if(L <= mid) res = max(res, query2(Ls(x), L, R));
    if(R > mid) res = max(res, query2(Rs(x), L, R));
    return res;
}
```

2.3.4 LiChao Tree

```
const double eps = 1e-12;
namespace LiT{
    const int MLIMIT = 40000;
    typedef double LD;
    struct line{LD k,b;int l,r,id;} T[MAXN << 2];
    //inline LD calc(line &a,int pos) {return a.k*vec[pos]+a.b;}
    inline LD calc(line &a,int pos) {return a.k*pos+a.b;}
    inline double cross(line &a,line &b) {
        if(b.k == a.k) return -1e9;
        return (double)(a.b-b.b)/(b.k-a.k);
    }
    void build(int v, int l, int r) {
        T[v].k = 0;T[v].b = -1e18;
        T[v].l = 0;T[v].r = MLIMIT;
        T[v].id = 0;
        if(l == r)return;
        int mid = (l+r)>>1;
        build(v<<1,l,mid);
        build(v<<1|1,mid+1,r);
    }
    void ins(int v,int l,int r, line k) {
        if(k.l <= l && r <= k.r) {
            LD fl = calc(k, l), fr = calc(k, r);
            LD gl = calc(T[v], l), gr = calc(T[v], r);
            if(fl - gl > eps && fr - gr > eps) T[v] = k;
            else if(fl - gl > eps || fr - gr > eps) {
                int mid = (l+r)>>1;
                if(calc(k, mid) - calc(T[v], mid) > eps) swap(k, T[v]);
                //if(vec[mid] - cross(k, T[v]) > eps)
                if(mid - cross(k, T[v]) > eps)
                    ins(v<<1, l, mid, k);else ins(v<<1|1, mid+1, r, k);
            }
```

```
            return;
        }
        int mid=(l+r)>>1;
        if(k.l <= mid) ins(v<<1, l, mid, k);
        if(mid < k.r)  ins(v<<1|1, mid+1, r, k);
    }
    LD ans;int ansid;
    void que(int v, int l, int r, int x) {
        LD tmp = calc(T[v], x);
        if(tmp > ans || (tmp == ans && T[v].id < ansid)) {
            ans = tmp;
            ansid = T[v].id;
        }
        if(l == r) return;
        int mid = (l+r)>>1;
        if(x <= mid) que(v<<1,l,mid,x);else que(v<<1|1,mid+1,r,x);
    }
};
//左闭右闭
```

2.3.5 Splay Tree

```
namespace splay{
    int n, m, sz, rt;
    int val[MAXN], id[MAXN];
    int tr[MAXN][2], size[MAXN], fa[MAXN], rev[MAXN], s[MAXN], lazy[MAXN];
    void push_up(int x) {
        int l = tr[x][0], r = tr[x][1];
        s[x] = max(val[x], max(s[l], s[r]));
        size[x] = size[l] + size[r] + 1;
    }
    void push_down(int x) {
        int l = tr[x][0], r = tr[x][1];
        if(lazy[x]) {
            if(l) {
                lazy[l] += lazy[x];
                s[l] += lazy[x];
                val[l] += lazy[x];
            }
            if(r) {
                lazy[r] += lazy[x];
                s[r] += lazy[x];
                val[r] += lazy[x];
            }
            lazy[x] = 0;
        }
        if(rev[x]) {
            rev[x] = 0;
            rev[l] ^= 1; rev[r] ^= 1;
            swap(tr[x][0], tr[x][1]);
        }
    }
    void rotate(int x, int &k) {
        int y = fa[x];
        int z = fa[y];
        int l, r;
        if(tr[y][0] == x) l = 0;
        else l = 1;
        r = l ^ 1;
        if(y == k) k = x;
```

```
        else {
            if(tr[z][0] == y) tr[z][0] = x;
            else tr[z][1] = x;
        }
        fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
        tr[y][l] = tr[x][r]; tr[x][r] = y;
        push_up(y); push_up(x);
    }
    void splay(int x, int &k) {
        int y, z;
        while(x != k) {
            y = fa[x];
            z = fa[y];
            if(y != k) {
                if((tr[y][0] == x) ^ (tr[z][0] == y)) rotate(x, k);

                else rotate(y, k);
            }
            rotate(x, k);
        }
    }
    int find(int x, int rank) {
        push_down(x);

        int l = tr[x][0], r = tr[x][1];
        if(size[l] + 1 == rank) return x;
        else if(size[l] >= rank) return find(l, rank);
        else return find(r, rank - size[l] - 1);
    }
    void update(int l, int r, int v) {
        int x = find(rt, l), y = find(rt, r + 2);
        splay(x, rt); splay(y, tr[x][1]);
        int z = tr[y][0];
        lazy[z] += v;
        val[z] += v;
        s[z] += v;
    }
    void reverse(int l, int r) {
        int x = find(rt, l), y = find(rt, r + 2);
        splay(x, rt); splay(y, tr[x][1]);
        int z = tr[y][0];
        rev[z] ^= 1;
    }
    void query(int l, int r) {
        int x = find(rt, l), y = find(rt, r + 2);
        splay(x, rt); splay(y, tr[x][1]);
        int z = tr[y][0];
        printf("%d\n", s[z]);
    }
    void build(int l, int r, int f) {
        if(l > r) return;
        int now = id[l], last = id[f];
        if(l == r) {
            fa[now] = last; size[now] = 1;
            if(l < f) tr[last][0] = now;
            else tr[last][1] = now;
            return;
        }
        int mid = (l + r) >> 1; now = id[mid];
        build(l, mid - 1, mid); build(mid + 1, r, mid);
        fa[now] = last;
```

```
100         push_up(now);
101         if(mid < f) tr[last][0] = now;
102         else tr[last][1] = now;
103     }
104     void init() {
105         s[0] = -INF;
106         scanf("%d%d", &n, &m);
107         for(int i = 1; i <= n + 2; i++) id[i] = ++sz;
108         build(1, n + 2, 0); rt = (n + 3) >> 1;
109     }
110 }
111 namespace splay{
112     //内存回收池见fhq_treap
113     int tcnt, root;
114     int sz[MAXN];
115     int tr[MAXN][2], fa[MAXN];
116     int val[MAXN];
117     int newnode(int w) {
118         ++tcnt;
119         sz[tcnt] = 1;
120         fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
121         //val[tcnt] = w;
122         return tcnt;
123     }
124     void push_up(int v) {
125         int l = tr[v][0], r = tr[v][1];
126         sz[v] = sz[l] + 1 + sz[r];
127     }
128     void push_down(int v) {
129         if(!v) return;
130     }
131     void init() {
132         tcnt = 2;
133         tr[root = fa[1] = 2][0] = 1;
134         sz[1] = 1; sz[2] = 2;
135         //val[1] = -INF; val[2] = INF;//权值平衡树
136         //val[1] = val[2] = 0;//位置平衡树
137         //1,2为哨兵节点,根据题意也可设置为n+1,n+2或1,n+1
138     }
139     void rotate(int x) {
140         int y = fa[x], z = fa[y];
141         push_down(y);push_down(x);
142         int lr = tr[y][1] == x;
143         if(z) tr[z][tr[z][1]==y] = x;
144         fa[x] = z;
145         fa[tr[y][lr] = tr[x][lr^1]] = y;
146         fa[tr[x][lr^1] = y] = x;
147         push_up(y); push_up(x);
148     }
149     void splay(int x, int k) {
150         for(int y, z; (y = fa[x]) != k; rotate(x)) {
151             if((z = fa[y]) != k) {
152                 if((tr[y][0] == x) ^ (tr[z][0] == y))
153                     rotate(x); else rotate(y);
154             }
155         }
156         if(!k) root = x;
157     }
158     int find(int x, int rank) {
159         push_down(x);
160         int l = tr[x][0], r = tr[x][1];
```

```
        if(sz[l] + 1 == rank) return x;
        if(sz[l] >= rank) return find(l, rank);
        return find(r, rank - sz[l] - 1);
    }
    int build(int l, int r) {
        if(l > r) return 0;
        if(l == r) {
            int num;scanf("%d", &num);
            return newnode(num);
        }
        int mid = (l + r) >> 1;
        int ls = build(l, mid-1);
        int num; scanf("%d", &num);
        int v = newnode(num);
        int rs = build(mid+1, r);
        if(ls) fa[ls] = v;
        tr[v][0] = ls;
        if(rs) fa[rs] = v;
        tr[v][1] = rs;
        push_up(v);
        return v;
    }
    void insert(int pos, ...) {
        int x = find(root, pos+1), y = find(root, pos+2);
        splay(x, 0); splay(y, x);
        //int z = newnode(w);//插入一个节点
        //int z = build(1, n);//插入n个节点
        fa[tr[y][0] = z] = y;
        splay(z,0);
    }
    void modifyOrQuery(int l, int r, int v) {
        int x = find(root, l), y = find(root, r + 2);
        splay(x, 0); splay(y, x);
        int z = tr[y][0];
        if(!z) return;
        //标记对本身无效,处理时将z点重新计算
        splay(z,0);
    }
    void display(int v) {
        if(!v) return;
        push_down(v);
        display(tr[v][0]);
        if(val[v]) printf("%d ", val[v]);
        display(tr[v][1]);
    }
    /*int findValue(int v) {
        int res = root;
        for(int cur = root;cur; res = cur, cur = tr[cur][val[cur] <= v]);
        return res;
    }
    void insert(int w) {
        int y = findValue(w);
        int z = newnode(w);
        fa[tr[y][val[y] <= w] = z] = y;
        splay(z,0);
    }*/
    /*void split(int v) {//splay维护区间[l,r],区间分裂为[l,k-1],[k,k],[k+1,r];
        //ump查看标号是否出现,mp维护子区间左端点
        if(ump.find(v) == ump.end()) {
            auto it = mp.upper_bound(v); --it;
            int z = it->second;
```

```
            splay(z, 0);
            int pos = sz[tr[z][0]];
            int x = find(root,pos), y = find(root,pos+rc[z]-lc[z]+2);
            splay(x, 0); splay(y, x);
            z = tr[y][0];
            if(lc[z] != v) {
                tr[z][0] = newnode(lc[z],v-1);
                fa[tr[z][0]] = z;
                mp[lc[z]] = tr[z][0];
            }
            if(rc[z] != v) {
                tr[z][1] = newnode(v+1,rc[z]);
                fa[tr[z][1]] = z;
                mp[v+1] = tr[z][1];
            }
            lc[z] = rc[z] = v;
            splay(z,0);
            mp[v] = z;
            ump[v] = z;
        }
    }*/

}
```

2.3.6 Scapegoat Tree

```
struct ScapegoatTree{
    int Tsn; queue<int> q;
    int val[MAXM], ext[MAXM];
    int sz[MAXM], tsz[MAXM];
    int fa[MAXM], tr[MAXM][2];
    int root;
    double alp;
    void init() {
        root = 0;
        alp = 0.7;
    }
    int newnode(int x) {
        if(q.empty()) q.push(++Tsn);
        int tcnt = q.front(); q.pop();
        val[tcnt] = x; ext[tcnt] = 1;
        fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
        sz[tcnt] = tsz[tcnt] = 0;
        return tcnt;
    }
    void push_up(int v) {
        sz[v] = ext[v];tsz[v] = 1;
        if(tr[v][0]) {
            sz[v] += sz[tr[v][0]];
            tsz[v] += tsz[tr[v][0]];
        }
        if(tr[v][1]) {
            sz[v] += sz[tr[v][1]];
            tsz[v] += tsz[tr[v][1]];
        }
    }
    bool isBad(int v) {
        return (double(tsz[ tr[v][0] ]) > double(tsz[v]) * alp) ||
            (double(tsz[ tr[v][1] ]) > double(tsz[v]) * alp) ||
            (sz[v] * 2 < tsz[v]);
```

```
    }
    vector<int> vec;
    void rRecycle(int v) {
        if(tr[v][0]) rRecycle(tr[v][0]);
        if(ext[v]) vec.push_back(v); else q.push(v);
        if(tr[v][1]) rRecycle(tr[v][1]);
    }
    int rBuild(int l, int r) {
        int mid = (l + r) >> 1, v = vec[mid];
        tr[v][0] = (l <= mid-1) ? rBuild(l, mid - 1) : 0;
        if(tr[v][0]) fa[tr[v][0]] = v;
        tr[v][1] = (mid+1 <= r) ? rBuild(mid + 1, r) : 0;
        if(tr[v][1]) fa[tr[v][1]] = v;
        push_up(v);
        return v;
    }
    void rebuild(int x) {
        int v = 0;
        for(;x; x= fa[x]) {
            push_up(x);
            if(isBad(x)) v = x;
        }
        if(v && isBad(v)){
            vec.clear();
            int u = fa[v], lr = tr[u][1] == v;
            rRecycle(v);
            if(vec.size()) v = rBuild(0, vec.size() - 1); else v = 0;
            if(u == 0) fa[root = v] = 0;
            else{
                tr[u][lr] = v;
                if(v) fa[v] = u;
            }
        }
    }
    void ins(int x) {
        int p = root, q = root;
        for(;p && val[p] != x; q = p, p = tr[p][ val[p] < x]) ;
        if(!q) {
            p = root = newnode(x);
        }else if(p) {
            ext[p]++;
        }else{
            fa[p = tr[q][val[q] < x] = newnode(x) ] = q;
        }
        rebuild(p);
    }
    void del(int x) {
        int p = root;
        for(;p && val[p] != x; p = tr[p][ val[p] < x]);
        if(p && ext[p]){
            --ext[p];
            rebuild(p);
        }
    }
    int get_rank(int x) {
        int ret = 0;
        for(int p = root;p;) {
            if(val[p] < x) {
                ret += sz[tr[p][0]] + ext[p];
                p = tr[p][1];
            }else p = tr[p][0];
```

```
        }
        return ret + 1;
    }
    int get_Kth(int p, int k) {
        if(sz[tr[p][0]] >= k) return get_Kth(tr[p][0] ,k);
        k -= sz[tr[p][0]];
        if(ext[p] >= k) return val[p];
        k -= ext[p];
        return get_Kth(tr[p][1], k);
    }
    int pre(int x) {
        int id = get_rank(x);
        return get_Kth(root, id - 1);
    }
    int nxt(int x) {
        int id = get_rank(x + 1);
        return get_Kth(root ,id);
    }
    void display(int v) {
        if(tr[v][0]) display(tr[v][0]);
        cerr<<val[v]<<" ";
        if(tr[v][1]) display(tr[v][1]);
    }
}T;
```

2.3.7 FHQ Treap

```
namespace fhq_treap{
    int Tsz; queue<int> q; //内存回收池
    int tcnt, root;
    //int rt[MAXN]; //可持久化时使用rt,维护版本号(int &root), 空间开大
    int sz[MAXN], rnd[MAXN];
    int tr[MAXN][2];
    //int fa[MAXN];//维护fa时除了在pushup更新v节点左右孩子父节点信息还要在split和merge结束时维护root的fa信息,fa[root]=0;
    int val[MAXN], rev[MAXN];
    void init() {
        srand(time(0));
        Tsz = tcnt = root = 0;
    }
    int newnode(int v) {
        if(q.empty()) q.push(++Tsz);
        tcnt = q.front(); q.pop();
        sz[tcnt] = 1;
        rnd[tcnt] = rand();
        tr[tcnt][0] = tr[tcnt][1] = 0;
        //val[tcnt] = v;
        return tcnt;
    }
    /*int copynode(int id) {
        //++tcnt;//获取一个新的节点编号
        sz[tcnt] = sz[id];
        rnd[tcnt] = rnd[id];
        tr[tcnt][0] = tr[id][0];
        tr[tcnt][1] = tr[id][1];
        //val[tcnt] = val[id];
        return tcnt;
    }*/
    void push_up(int v) {
        int l = tr[v][0], r = tr[v][1];
```

```
33          sz[v] = sz[l] + 1 + sz[r];
34      }
35      void push_down(int v) {
36          if(!v) return;
37          int l = tr[v][0], r = tr[v][1];
38          //if(l) ;
39          //if(r) ;
40          //swap时候注意交换tr[v][0]和tr[v][1],而不是l和r;
41      }
42      /* //可持久化在push_down是要新建节点,否者历史版本有可能被多次下传,以rev为例
43      void push_down(int v) {
44          if(!v || !rev[v]) return;
45          int &l = tr[v][0], &r = tr[v][1];
46          if(l) {
47              l = copynode(l);
48              rev[l]^=1;
49              swap(tr[l][0],tr[l][1]);
50          };
51          if(r) {
52              r = copynode(r);
53              rev[r]^=1;
54              swap(tr[r][0],tr[r][1]);
55          };
56          rev[v] = 0;
57      }*/
58      void split(int v,int k,int &x,int &y) {
59          if(!v) {x=y=0;return;}
60          push_down(v);
61          //v = copynode(v);//可持久化时复制节点
62          /*if(k > sz[tr[v][0]]) {
63              x = v;
64              split(tr[v][1], k-sz[tr[v][0]]-1, tr[v][1], y);
65          }else{
66              y = v;
67              split(tr[v][0], k , x ,tr[v][0]);
68          }*/
69          if(val[v] <= k) {
70              x = v;
71              split(tr[v][1], k, tr[v][1], y);
72          }else{
73              y = v;
74              split(tr[v][0], k, x, tr[v][0]);
75          }
76          push_up(v);
77      }
78      int merge(int x,int y) {//x堆所有值均小于y堆
79          if(!x || !y) return x|y;
80          push_down(x); push_down(y);
81          if(rnd[x]<rnd[y]){
82              //x = copynode(x);//可持久化时复制节点,可不写
83              tr[x][1] = merge(tr[x][1],y);
84              push_up(x);
85              return x;
86          }else{
87              //y = copynode(y);//可持久化时复制节点,可不写
88              tr[y][0] = merge(x,tr[y][0]);
89              push_up(y);
90              return y;
91          }
92      }
93      void insert(int k) {
```

```
94          int x,y;
95          split(root,k,x,y);
96          root = merge(merge(x,newnode(k)),y);
97      }
98      void recycle(int v) {//回收一颗treap上所有节点
99          if(!v) return;
100         q.push(v);
101         recycle(tr[v][0]); recycle(tr[v][1]);
102     }
103     void erase(int k) {
104         int x,y,z;
105         split(root,k,x,y);
106         split(x,k-1,x,z);
107         z = merge(tr[z][0],tr[z][1]);
108         root = merge(x,merge(z,y));
109     }
110     void krank(int k) {
111         int x,y;
112         split(root,k-1,x,y);
113         printf("%d\n",sz[x]+1);
114         root = merge(x,y);
115     }
116     int find(int v,int k) {
117         if(sz[tr[v][0]]==k-1) return val[v];
118         if(sz[tr[v][0]]>=k) return find(tr[v][0],k);
119         return find(tr[v][1],k-sz[tr[v][0]]-1);
120     }
121     void pre(int k) {
122         int x,y;
123         split(root,k-1,x,y);
124         printf("%d\n",find(x,sz[x]));
125         root=merge(x,y);
126     }
127     void nxt(int k){
128         int x,y;
129         split(root,k,x,y);
130         printf("%d\n",find(y,1));
131         root=merge(x,y);
132     }
133     void reverse(int l,int r){
134         int x,y,z;
135         split(root, r, x, y);
136         split(x, l-1, x, z);
137         //rev[z] ^= 1;标记对本身无效,处理时将z点重新计算
138         root = merge(merge(x,z),y);
139     }
140     /*int getRank(int S) {
141         int res = sz[tr[S][0]]+1;
142         for(;fa[S];S = fa[S])
143             if(tr[fa[S]][1] == S) res += sz[tr[fa[S]][0]] + 1;
144        return res;
145     }*/
146     void display(int v) {
147         if(!v) return;
148         push_down(v);
149         display(tr[v][0]);
150         printf("%d ",val[v]);
151         display(tr[v][1]);
152     }
153 }
```

```
154 //一种可持久化平衡树的替代(非强制在线),由历史版本向当前版本连边,在dfs遍历中利用权值树状数组,普通平衡树等获取答案
```

2.4 Persistent Data Structures

2.4.1 Chairman Tree

```
1  struct Node {
2      int l, r;
3      LL sum;
4  }t[MAXN * 40];
5  int cnt, n;
6  int rt[MAXN];
7  void update(int pre, int &x, int l, int r, int v) {
8      x = ++cnt; t[x] = t[pre]; t[x].sum++;
9      if(l == r) return;
10     int mid = (l + r) >> 1;
11     if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
12     else update(t[pre].r, t[x].r, mid + 1, r, v);
13 }
14 int query(int x, int y, int l, int r, int v) {
15     if(l == r) return l;
16     int mid = (l + r) >> 1;
17     int sum = t[t[y].l].sum - t[t[x].l].sum;
18     if(sum >= v) return query(t[x].l, t[y].l, l, mid, v);
19     else return query(t[x].r, t[y].r, mid + 1, r, v - sum);
20 }
```

2.4.2 Unite Chairman Tree

```
1  //Q x到y路径第k大
2  //L link(x, y)
3  #include <bits/stdc++.h>
4  using namespace std;
5  typedef int LL;
6  const int MAXN = 8e4 + 5;
7  const int S = 18;
8  struct Node {
9      int l, r;
10     LL sum;
11 }t[MAXN * 800]; //2 * log^2(n)
12 int n, m, Q;
13 int cnt;
14 int rt[MAXN], sz[MAXN];
15 void update(int pre, int &x, int l, int r, int v) {
16     x = ++cnt; t[x] = t[pre]; t[x].sum++;
17     if(l == r) return;
18     int mid = (l + r) >> 1;
19     if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
20     else update(t[pre].r, t[x].r, mid + 1, r, v);
21 }
22 int query(int x, int y, int z, int w, int l, int r, int v) {
23     if(l == r) return l;
24     int mid = (l + r) >> 1;
25     int sum = t[t[x].l].sum + t[t[y].l].sum - t[t[z].l].sum - t[t[w].l].sum;
26     if(sum >= v) return query(t[x].l, t[y].l, t[z].l, t[w].l, l, mid, v);
27     return query(t[x].r, t[y].r, t[z].r, t[w].r, mid + 1, r, v - sum);
28 }
```

```cpp
int fa[MAXN][S + 3], dep[MAXN];
int val[MAXN];
vector<int> G[MAXN];
inline void addEdge(int x, int y) {
    G[x].push_back(y);
    G[y].push_back(x);
}
inline void upd(int x) {
    update(rt[fa[x][0]], rt[x], 1, n, val[x]);
    for(int i = 1; i <= S; i++) fa[x][i] = fa[fa[x][i - 1]][i - 1];
}
inline void Go(int &x, int step) {
    for(int i = S; i >= 0; i--) if(step >> i & 1) x = fa[x][i];
}
int lca(int x, int y) {
    if(dep[x] < dep[y]) swap(x, y);
    Go(x, dep[x] - dep[y]);
    if(x == y) return x;
    for(int i = S; i >= 0; i--) if(fa[x][i] != fa[y][i]) {
        x = fa[x][i], y = fa[y][i];
    }
    return fa[x][0];
}
int get_rt(int x) {
    for(int i = S; i >= 0; i--) if(fa[x][i]) x = fa[x][i];
    return x;
}
void dfs(int x, int f) {
    fa[x][0] = f;
    dep[x] = dep[f] + 1;
    upd(x);
    for(auto y : G[x]) {
        if(y == f) continue;
        dfs(y, x);
    }
}
void unite(int x, int y) {
    int rx = get_rt(x), ry = get_rt(y);
    if(sz[rx] > sz[ry]) swap(x, y), swap(rx, ry);
    addEdge(x, y);
    dfs(x, y);
    sz[ry] += cnt - rt[x] + 1;
}
void init() {
    cnt = 0;
    for(int i = 1; i <= n; i++) {
        rt[i] = sz[i] = 0;
        G[i].clear();
        dep[i] = 0;
    }
}
int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    int T; cin >> T;
    while(T--) {
        cin >> n >> m >> Q;
        init();
        vector<int> b;
        map<int, int> mp;
        for(int i = 1; i <= n; i++) {
            cin >> val[i];
```

```
            b.push_back(val[i]);
        }
        sort(b.begin(), b.end());
        b.erase(unique(b.begin(), b.end()), b.end());
        for(int i = 1, tmp; i <= n; i++) {
            tmp = val[i];
            val[i] = lower_bound(b.begin(), b.end(), val[i]) - b.begin() + 1;
            mp[val[i]] = tmp;
        }
        for(int i = 1, u, v; i <= m; i++) {
            cin >> u >> v;
            addEdge(u, v);
        }
        for(int i = 1; i <= n; i++) if(!dep[i]) {
            dep[i] = 1;
            dfs(i, 0);
            sz[i] = cnt - rt[i] + 1;
        }
        char s[3]; int x, y, z, k, ans = 0;
        while(Q--) {
            cin >> s >> x >> y;
            x ^= ans; y ^= ans;
            if(s[0] == 'Q') {
                cin >> k; k ^= ans;
                z = lca(x, y);
                ans = query(rt[x], rt[y], rt[z], rt[fa[z][0]], 1, n, k);
                ans = mp[ans];
                cout << ans << endl;
            }
            else {
                unite(x, y);
            }
        }
    }
    return 0;
}
```

2.4.3 Persistent Trie

```
//区间异或最值查询
const int N=5e4+10;
int t[N];
int ch[N*32][2],val[N*32];
int cnt;
void init(){
    mem(ch,0);
    mem(val,0);
    cnt=1;
}
int add(int root,int x){
    int newroot=cnt++,ret=newroot;
    for(int i=30;i>=0;i--){
        ch[newroot][0]=ch[root][0];
        ch[newroot][1]=ch[root][1];
        int now=(x>>i)&1;
        root=ch[root][now];

        ch[newroot][now]=cnt++;
        newroot=ch[newroot][now];
        val[newroot]=val[root]+1;
```

```
    }

    return ret;
}
int query(int lt,int rt,int x){
    int ans=0;
    for(int i=30;i>=0;i--){
        int now=(x>>i)&1;
        if(val[ch[rt][now^1]]-val[ch[lt][now^1]]){
            ans|=(1<<i);
            rt=ch[rt][now^1];
            lt=ch[lt][now^1];
            } else{
            rt=ch[rt][now];
            lt=ch[lt][now];
        }
    }
    return ans;
}
```

2.4.4 SGT in BBST

```
#include <bits/stdc++.h>
using namespace std;
const int MAXN = 1e5;
const int MAXM = 2e7;
const int LM = 0;
const int RM = 70005;
namespace T{
    int Tsz; queue<int> q;
    int ls[MAXM], rs[MAXM], val[MAXM];
    int Tan, Ta[105], Tbn, Tb[105];
    int newnode() {
        if(q.empty()) q.push(++Tsz);
        int x = q.front(); q.pop();
        ls[x] = rs[x] = val[x] = 0;
        return x;
    }
    void insert(int &x, int l, int r, int k, int f) {
        if(!x) x = newnode();
        val[x] += f;
        if(l == r) return;
        int mid = (l + r) >> 1;
        if(k <= mid) {
            insert(ls[x], l, mid, k,f);
        }else{
            insert(rs[x], mid+1, r, k,f);
        }
    }
    int query(int l, int r, int y) {
        if(l == r) return l;
        int mid = (l + r) >> 1;
        int sum = 0;
        for(int i = 0; i <= Tan; i++) sum -= val[ls[Ta[i]]];
        for(int i = 0; i <= Tbn; i++) sum += val[ls[Tb[i]]];

        if(y <= sum) {
            for(int i = 0; i <= Tan; i++) Ta[i] = ls[Ta[i]];
            for(int i = 0; i <= Tbn; i++) Tb[i] = ls[Tb[i]];
            return query(l, mid, y);
```

```
        }
        for(int i = 0; i <= Tan; i++) Ta[i] = rs[Ta[i]];
        for(int i = 0; i <= Tbn; i++) Tb[i] = rs[Tb[i]];
        return query(mid+1, r, y - sum);
    }
    void recycle(int v) {
        if(!v) return;
        q.push(v);
        recycle(ls[v]);
        recycle(rs[v]);
    }
};
namespace TT{
    int Tsn;
    int val[MAXN], Trt[MAXN];
    int sz[MAXN];
    int fa[MAXN], tr[MAXN][2];

    int root;
    double alp;

    int newnode(int x) {
        int tcnt = ++Tsn;
        val[tcnt] = x; Trt[tcnt] = 0;
        fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
        sz[tcnt] = 0;
        return tcnt;
    }
    bool isBad(int v) {
        return (double(sz[ tr[v][0] ]) > double(sz[v]) * alp) ||
            (double(sz[ tr[v][1] ]) > double(sz[v]) * alp);
    }
    vector<int> vec;
    void rRecycle(int v) {
        if(tr[v][0]) rRecycle(tr[v][0]);
        vec.push_back(v);
        T::recycle(Trt[v]);
        if(tr[v][1]) rRecycle(tr[v][1]);
    }
    int rBuild(int l, int r) {
        int mid = (l + r) >> 1, v = vec[mid];
        Trt[v] = 0; sz[v] = r - l + 1;
        for(int i = l; i <= mid; i++) {
            T::insert(Trt[v], LM, RM, val[vec[i]], 1);
        }
        tr[v][0] = (l <= mid-1) ? rBuild(l, mid - 1) : 0;
        if(tr[v][0]) fa[tr[v][0]] = v;
        tr[v][1] = (mid+1 <= r) ? rBuild(mid + 1, r) : 0;
        if(tr[v][1]) fa[tr[v][1]] = v;
        return v;
    }
    void rebuild(int v) {
        if(isBad(v)) {
            vec.clear();
            int u = fa[v], lr = tr[u][1] == v;
            rRecycle(v);
            if(vec.size()) v = rBuild(0, vec.size() - 1); else v = 0;
            if(u == 0) fa[root = v] = 0;
            else{
                tr[u][lr] = v;
                if(v) fa[v] = u;
```

```
            }
        }
    }
    int find(int x, int k) {
        int l = tr[x][0], r = tr[x][1];
        if(sz[l] + 1 == k) return x;
        if(sz[l] >= k) return find(l, k);
        return find(r, k - sz[l] - 1);
    }
    void ins(int x, int y) {
        int v = find(root, x);
        int p = tr[v][1], q = v;
        if(p) {
            for(;p; q = p, p = tr[p][0]);
            fa[p = tr[q][0] = newnode(y)] = q;
        }else {
            fa[p = tr[q][1] = newnode(y)] = q;
        }
        int fg = 0;
        T::insert(Trt[p], LM, RM, y, 1);
        sz[p] = 1;
        for(;fa[p]; p = fa[p]) {
            if(tr[fa[p]][0] == p)
                T::insert(Trt[fa[p]], LM, RM, y, 1);
            sz[fa[p]]++;
            if(isBad(fa[p])) fg = fa[p];
        }
        rebuild(fg);
    }
    void upd(int x, int y) {
        int p = find(root, x+1);
        int ty = val[p]; val[p] = y;
        T::insert(Trt[p], LM, RM, ty, -1);
        T::insert(Trt[p], LM, RM, y, 1);
        for(;fa[p]; p = fa[p]) {
            if(tr[fa[p]][0] == p) {
                T::insert(Trt[fa[p]], LM, RM, ty, -1);
                T::insert(Trt[fa[p]], LM, RM, y, 1);
            }
        }
    }
    int que(int x, int y,int z) {
        x = find(root, x);
        T::Ta[T::Tan = 0] = Trt[x];
        for(;fa[x]; x = fa[x])
            if(tr[fa[x]][1] == x) {
                T::Ta[++T::Tan] = Trt[fa[x]];
            }
        y = find(root, y+1);
        T::Tb[T::Tbn = 0] = Trt[y];
        for(;fa[y]; y = fa[y]) {
            if(tr[fa[y]][1] == y) {
                T::Tb[++T::Tbn] = Trt[fa[y]];
            }
        }
        return T::query(LM, RM, z);
    }
    void init(int n) {
        alp = 0.7;
        vec.clear();
        vec.push_back(newnode(RM));
```

```
        for(int i = 1, a; i <= n; i++) {
            scanf("%d", &a);
            vec.push_back(newnode(a));
        }
        root = rBuild(0,vec.size() - 1);
        fa[root] = 0;
    }
    void display(int v) {
        if(tr[v][0]) display(tr[v][0]);
        cerr<<val[v]<<" ";
        if(tr[v][1]) display(tr[v][1]);
    }
};
```

2.5 Tree Structures

2.5.1 dsu on tree

```
const int MAXN = 1e5 + 7;
vector<int> G[MAXN];
int bgison, dfs_clock, sz[MAXN], st[MAXN], bt[MAXN], et[MAXN];
int fg[MAXN], col[MAXN];
long long ans[MAXN];
void dfs1(int u, int fa) {
    sz[u] = 1;
    st[bt[u] = ++dfs_clock] = u;
    for(auto v : G[u])
    if(v != fa) {
        dfs1(v, u);
        sz[u] += sz[v];
    }
    et[u] = dfs_clock;
}
int maxx = 0;
void dfs2(int u, int fa, int keep) {
    int mx = -1, bigson = -1;
    for(auto &v : G[u])
    if(v != fa) {
        if(sz[v] > mx)
            mx = sz[v], bigson = v;
    }
    for(auto &v : G[u])
    if(v != fa && v != bigson)
        dfs2(v,u,0);
    if(bigson != -1) {
        dfs2(bigson, u, 1);
        ans[u] = ans[bigson];
        for(int &v : G[u])
            if(v != fa && v != bigson)
            for(int i = bt[v]; i <= et[v]; i++) {
                ++fg[col[st[i]]];
                if(fg[ col[st[i]] ] > maxx) maxx=fg[col[st[i]]], ans[u] = 0;
                if(fg[ col[st[i]] ] == maxx) ans[u] += col[st[i]];
            }
    }
    ++fg[col[u]];
    if(fg[col[u]] > maxx) maxx = fg[col[u]], ans[u] = 0;
    if(fg[col[u]] == maxx) ans[u] += col[u];
    if(keep == 0) {
```

```
        maxx = 0;
        for(int i = bt[u]; i <= et[u]; i++)
            fg[col[st[i]]] = 0;
    }
}
```

2.5.2 Vitural Tree

```
const int MAXN = ;
const int MAXM = ;
const LL INF = ;
const int S = 19;
int ecnt, head[MAXN];
struct Edge{int to, nxt; LL w;} e[MAXM], ve[MAXM];
inline void addEdgeT(int x, int y, LL w) {
    e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
}
int dep[MAXN], dfn_time, dfn[MAXN], fa[MAXN][S+1];
LL dis[MAXN][S+1];
void dfs(int v,int _fa) {
    dfn[v] = ++dfn_time;
    dep[v] = dep[_fa] + 1;
    fa[v][0] = _fa;
    for(int i = 1; i <= S; i++) {
        fa[v][i] = fa[fa[v][i-1]][i-1];
        dis[v][i] = min(dis[v][i-1], dis[fa[v][i-1]][i-1]);
    }
    for(int i = head[v], u; i; i = e[i].nxt)
    if((u = e[i].to) != _fa) {
        dis[u][0] = e[i].w;
        dfs(u, v);
    }
}
int getLca(int u, int v) {
    if(dep[u] < dep[v]) swap(u, v);
    for(int i = S; i >= 0; i--)
        if(dep[fa[u][i]] >= dep[v]) u = fa[u][i];
    if(u == v) return u;
    for(int i = S; i >= 0; i--)
        if(fa[u][i] != fa[v][i])
            u = fa[u][i], v = fa[v][i];
    return fa[u][0];
}
LL getDis(int u, int v) {
    if(dep[u] < dep[v]) swap(u, v);
    LL res = INF;
    for(int i = S; i >= 0; i--)
        if(dep[fa[u][i]] >= dep[v]) {
            res = min(res, dis[u][i]);
            u = fa[u][i];
        }
    return res;
}
namespace VituralTree{
    int hn, h[MAXN];
    int vecnt, vhead[MAXN];
    Edge ve[MAXM];
    int top, st[MAXN], cln, cl[MAXN];
    int fgn, fg[MAXN];//利用fg[i]==fgn判断是否为当前有效点
    void addEdgeVT(int x, int y, LL w) {
```

```
        ve[++vecnt] = (Edge) {y, vhead[x], w}; vhead[x] = vecnt;
    }
    inline void link(int u, int v) {
        LL w = getDis(u, v);
        addEdgeVT(u, v, w);
        addEdgeVT(v, u, w);
    }
    inline bool cmp(int a, int b) {return dfn[a] < dfn[b];}
    void build() {
        ++fgn;
        for(int i = 1; i <= hn; i++) fg[h[i]] = fgn;
        sort(h + 1, h + hn + 1, cmp);
        cl[cln = 1] = st[top = 1] = 1;
        for(int i = 1; i <= hn; i++) {
            int rem = getLca(st[top], h[i]);
            if(rem == st[top]) {
                if(rem != st[top]) cl[++cln] = st[++top] = h[i];
                continue;
            }
            while(top > 1 && dep[st[top - 1]] >= dep[rem]) {
                link(st[top - 1], st[top]); top--;
            }
            if(dep[st[top]] > dep[rem]) {
                link(rem, st[top]), top --;
            }
            if(rem != st[top]) cl[++cln] = st[++top] = rem;
            if(h[i] != st[top]) cl[++cln] = st[++ top] = h[i];
        }
        while(top > 1) {
            link(st[top - 1], st[top]), top --;
        }
    }
    void clear() {
        vecnt = 0;
        for(;cln; --cln) vhead[cl[cln]] = 0;
    }
    void sol() {
        build();
        //注意一号节点可能在虚树外面
        clear();
    }
}
```

2.5.3 Tree Decomposition

```
int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
struct Edge {
    int to, nxt;
}e[MAXN << 1];
int head[MAXN], ecnt;
int n, m, Q;
#define Ls(x) (x << 1)
#define Rs(x) (x << 1 | 1)
struct Tree {
    int l, r, lazy;
    LL sum, mx;
}tree[MAXN << 2];
int A[MAXN], B[MAXN];
void push_up(int x) {
    tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
```

```
    tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
}
void push_down(int x) {
    if(tree[x].lazy) {
        tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
        tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
        tree[Ls(x)].mx += tree[x].lazy;
        tree[Rs(x)].mx += tree[x].lazy;
        tree[Ls(x)].lazy += tree[x].lazy;
        tree[Rs(x)].lazy += tree[x].lazy;
        tree[x].lazy = 0;
    }
}
void build(int x, int L, int R) {
    tree[x].lazy = 0;
    tree[x].l = L; tree[x].r = R;
    if(L == R) {
        tree[x].sum = B[L];
        tree[x].mx = B[L];
        return;
    }
    int mid = (L + R) >> 1;
    build(Ls(x), L, mid);
    build(Rs(x), mid + 1, R);
    push_up(x);
}
void update(int x, int L, int R, LL val) {
    if(tree[x].l >= L && tree[x].r <= R) {
        tree[x].lazy += val;
        tree[x].sum += val * (tree[x].r - tree[x].l + 1);
        tree[x].mx += val;
        return;
    }
    push_down(x);
    int mid = (tree[x].l + tree[x].r) >> 1;
    if(L <= mid) update(Ls(x), L, R, val);
    if(R > mid) update(Rs(x), L, R, val);
    push_up(x);
}
LL query(int x, int L, int R) {
    if(tree[x].l >= L && tree[x].r <= R)
        return tree[x].sum;
    push_down(x);
    int mid = (tree[x].l + tree[x].r) >> 1;
    LL res = 0;
    if(L <= mid) res += query(Ls(x), L, R);
    if(R > mid) res += query(Rs(x), L, R);
    return res;
}
LL query2(int x, int L, int R) {
    if(tree[x].l >= L && tree[x].r <= R)
        return tree[x].mx;
    push_down(x);
    int mid = (tree[x].l + tree[x].r) >> 1;
    LL res = -INF;
    if(L <= mid) res = max(res, query2(Ls(x), L, R));
    if(R > mid) res = max(res, query2(Rs(x), L, R));
    return res;
}
inline void add_edge(int x, int y) {
    e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
```

```
}
void dfs1(int x) {
    sz[x] = 1; son[x] = 0;
    for(int i = head[x]; i; i = e[i].nxt) {
        int v = e[i].to;
        if(v == fa[x]) continue;
        fa[v] = x;
        dep[v] = dep[x] + 1;
        dfs1(v);
        sz[x] += sz[v];
        if(sz[v] > sz[son[x]]) son[x] = v;
    }
}
void dfs2(int x) {
    B[num[x]] = A[x];
    if(son[x]) {
        top[son[x]] = top[x];
        num[son[x]] = ++totw;
        dfs2(son[x]);
    }
    for(int i = head[x]; i; i = e[i].nxt) {
        int v = e[i].to;
        if(v == fa[x] || v == son[x]) continue;
        top[v] = v;
        num[v] = ++totw;
        dfs2(v);
    }
}
void up(int a, int b, int c) {
    int f1 = top[a], f2 = top[b];
    while(f1 != f2) {
        if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
        update(1, num[f1], num[a], c);
        a = fa[f1];
        f1 = top[a];
    }
    if(dep[a] > dep[b]) swap(a, b);
    update(1, num[a], num[b], c);
}
int qsum(int a, int b) {
    if(a == b) return query(1, num[a], num[a]);
    int f1 = top[a], f2 = top[b];
    int res = 0;
    while(f1 != f2) {
        if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
        res += query(1, num[f1], num[a]);
        a = fa[f1];
        f1 = top[a];
    }
    if(dep[a] > dep[b]) swap(a, b);
    res += query(1, num[a], num[b]);
    return res;
}
int qmax(int a, int b) {
    if(a == b) return query2(1, num[a], num[a]);
    int f1 = top[a], f2 = top[b];
    int res = -1000000000;
    while(f1 != f2) {
        if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
        res = max(res, query2(1, num[f1], num[a]));
        a = fa[f1];
```

```
        f1 = top[a];
    }
    if(dep[a] > dep[b]) swap(a, b);
    res = max(res, query2(1, num[a], num[b]));
    return res;
}
inline void init() {
    memset(head, 0, sizeof(head)); ecnt = 0;
    fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
}
inline void pre() {
    dfs1(1); dfs2(1); build(1, 1, totw);
}
//-------------------------------------------------------------------------------
const int MAXN = ;
const int INF = ;
int A[MAXN], B[MAXN], C[MAXN];
struct SGT{
    int sL, sR;
    LL sW, mx[MAXN << 2];
    void push_down(int v) {
    }
    void push_up(int  v) {
        mx[v] = max(mx[v << 1], mx[v<<1|1]);
    }
    void build(int v, int l, int r) {
        if(l == r) {
            mx[v] = B[l];
            return;
        }
        int mid = (l + r) >> 1;
        build(v << 1, l, mid);
        build(v<<1|1, mid+1,r);
        push_up(v);
    }
    void upd(int v, int l, int r) {
        if(sL <= l && r <= sR) {
            //sum[v] = sW *(r - l + 1);
            mx[v] = sW;
            return;
        }
        push_down(v);
        int mid = (l + r) >> 1;
        if(sL <= mid) upd(v << 1, l, mid);
        if(mid < sR)  upd(v<<1|1, mid+1,r);
        push_up(v);
    }
    void qmax(int v, int l, int r) {
        if(sL <= l && r <= sR) {
            sW = max(sW, mx[v]);
            return;
        }
        push_down(v);
        int mid = (l + r) >> 1;
        if(sL <= mid) qmax(v << 1, l, mid);
        if(mid < sR)  qmax(v<<1|1, mid+1, r);
    }
}T;
namespace TD{
    struct Edge {int to, nxt;}e[MAXN << 1];
    int ecnt, head[MAXN];
```

```
199     inline void addEdge(int x, int y) {
200         e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
201         e[++ecnt] = (Edge) {x, head[y]}; head[y] = ecnt;
202     }
203     int fa[MAXN], sz[MAXN], dep[MAXN], son[MAXN], top[MAXN];
204     int dfn_time, dfn[MAXN], rnk[MAXN];
205     void dfs1(int u) {
206         sz[u] = 1; son[u] = -1;
207         for(int i = head[u], v; i; i = e[i].nxt) {
208             v = e[i].to;
209             if(v == fa[u]) continue;
210             fa[v] = u;
211             dep[v] = dep[u] + 1;
212             dfs1(v);
213             sz[u] += sz[v];
214             if(son[u] == -1 || sz[v] > sz[son[u]]) son[u] = v;
215         }
216     }
217     void dfs2(int u) {
218         dfn[u] = ++dfn_time;
219         //rnk[dfn_time] = u;
220         B[dfn[u]] = A[u];
221         if(son[u] == -1) return;
222         top[son[u]] = top[u];
223         dfs2(son[u]);
224         for(int i = head[u], v; i; i = e[i].nxt) {
225             v = e[i].to;
226             if(v == fa[u] || v == son[u]) continue;
227             top[v] = v;
228             dfs2(v);
229         }
230     }
231     inline void init(int n) {
232         ecnt = 1;
233         for(int i = 0; i <= n; i++) head[i] = 0;
234     }
235     inline void pre(int rt) {
236         fa[rt] = -1; dep[rt] = 1;
237         top[rt] = rt; dfn_time = 0;
238         dfs1(rt); dfs2(rt);
239         // edge -> point
240         //for(int i = 2; i <= ecnt; i += 2)  {
241         //    if(e[i].to == fa[e[i^1].to]) {
242         //        C[i / 2] = e[i^1].to;
243         //    }else{
244         //        C[i / 2] = e[i].to;
245         //    }
246         //    B[dfn[C[i / 2]]] = e[i].w;
247         //}
248         T.build(1, 1, dfn_time);
249     }
250     int lca(int u, int v) {
251         while(top[u] != top[v]) {
252             if(dep[top[u]] > dep[top[v]]) swap(u, v);
253             v = fa[top[v]];
254         }
255         if(dep[u] > dep[v]) swap(u, v);
256         return u;
257     }
258     void upd(int a, int b, int c) {
259         int ta = top[a], tb = top[b];
```

```
        while(ta != tb) {
            if(dep[ta] < dep[tb]) { swap(a, b); swap(ta, tb); }
            T.sL = dfn[ta]; T.sR = dfn[a]; T.sW = c;
            T.upd(1, 1, dfn_time);
            a = fa[ta]; ta = top[a];
        }
        if(dep[a] > dep[b]) swap(a, b);                 // point
        //if(a == b) return;                            // edge
        //if(dep[a] > dep[b]) swap(a, b);               // edge
        //a = son[a];                                   // edge
        T.sL = dfn[a]; T.sR = dfn[b]; T.sW = c;
        T.upd(1, 1, dfn_time);
    }
    //更新子树, 由于dfs2中有if(son[u] == -1) return; 小心使用end[u];
    void upd2(int a, int c) {
        T.sL = dfn[a]; T.sR = dfn[a] + sz[a] - 1; T.sW = c;
        T.upd(1, 1, dfn_time);
    }
    int qmax(int a, int b) {
        int ta = top[a], tb = top[b]; int res = - INF;
        while(ta != tb) {
            if(dep[ta] < dep[tb]) { swap(a, b); swap(ta, tb); }
            T.sL = dfn[ta]; T.sR = dfn[a]; T.sW = - INF;
            T.qmax(1, 1, dfn_time); res = max(T.sW, res);
            a = fa[ta];ta = top[a];
        }
        if(dep[a] > dep[b]) swap(a, b);                 // point
        //if(a == b) return res;                        // edge
        //if(dep[a] > dep[b]) swap(a, b);               // edge
        //a = son[a];                                   // edge
        T.sL = dfn[a]; T.sR = dfn[b]; T.sW = -INF;
        T.qmax(1, 1, dfn_time); res = max(T.sW, res);
        return res;
    }
}
```

2.5.4 Link-Cut Tree

```
namespace LCT {
    int tcnt; //动态开点
    int fa[MAXN], tr[MAXN][2], rev[MAXN];
    //int val[MAXN], sval[MAXN];
    //void clear(int id) { //或者改为int newnode() {}
    //    fa[id] = tr[id][0] = tr[id][1] = 0;
    //    rev[id] = 0; //sz[id] = 1;
    //}
    void Rev(int x) {
        rev[x] ^= 1; swap(tr[x][0], tr[x][1]);
    }
    void push_up(int x) {
        //int l = tr[x][0], r = tr[x][1];
        //sval[x] = sval[l] + val[x] + sval[r];
    }
    void push_down(int x) {
        int l = tr[x][0], r = tr[x][1];
        if(rev[x]) {
            if(l) Rev(l);
            if(r) Rev(r);
            rev[x] = 0;
        }
```

```
    }
    bool isroot(int x) {
        return tr[fa[x]][0] != x && tr[fa[x]][1] != x;
    }
    void pre(int x) {
        if(!isroot(x)) pre(fa[x]);
        push_down(x);
    }
    void rotate(int x) {
        int y = fa[x], z = fa[y], lr = tr[y][1] == x;
        if(!isroot(y)) tr[z][tr[z][1] == y] = x;
        fa[x] = z;
        fa[tr[y][lr] = tr[x][lr^1]] = y;
        fa[tr[x][lr^1] = y] = x;
        push_up(y);
    }
    inline void splay(int x) {
        pre(x);
        for (int y, z; !isroot(x); rotate(x)) {
            y = fa[x]; z = fa[y];
            if (!isroot(y)) rotate((tr[z][0] == y) ^ (tr[y][0] == x) ? x : y);
        }
        push_up(x);
    }
    inline int access(int x) {
        int y = 0;
        for (; x; y = x, x = fa[x]) {
            splay(x);
            //sz2[x] += sz[tr[x][1]] - sz[y]; //subtree
            tr[x][1] = y;
            push_up(x);
        }
        return y;//不求LCA不必
    }
    inline void makeroot(int x) {
        access(x); splay(x); Rev(x);
    }
    inline int findroot(int x) {
        access(x); splay(x);
        for(;tr[x][0]; x = tr[x][0]) push_down(x);
        splay(x);
        return x;
    }
    inline void lnk(int x, int y) {
        makeroot(x);
        if(findroot(y) != x) fa[x] = y;
        //sz2[y] += sz[x]; //subtree
    }
    inline void cut(int x,int y) {
        makeroot(x); //access(y); splay(x);
        if(findroot(y) == x&&fa[y] == x&&!tr[y][0]){
            fa[y] = tr[x][1] = 0;
            push_up(x);
        }
    }
    inline void cut(int y) { //有根树断开与父节点连边
        access(y); splay(y);
        fa[tr[y][0]] = 0;
        tr[y][0] = 0;
        push_up(y);
    }
```

```
84      inline int lca(int u, int v) {
85          access(u);
86          return access(v);
87      }
88      void split(int x, int y) {
89          makeroot(x); access(y); splay(y);
90      }
91      //维护节点或者维护路径
92      //例如: 染色（注意tag_rev）、tag_add的区间信息、splay维护连续端最远位置
93      void upd(int x, int y) {
94          makeroot(x); val[x] = y; push_up(x);
95      }
96      int que(int x, int y) {
97          split(x, y);
98          //return sval[y];
99      }
100     //维护边权(y -> eid -> x) ， 需要初始化 vcnt, ecnt, 可用map(注意双向维护)维护eid信息
101     //注意 if(x == y) continue;
102     struct LCTEdge{int u, v; int w;} e[MAXN];
103     void addEdge(int eid) { //e[eid = ++ecnt] = (Edge){x, y, w};
104         lnk(e[eid].u, vcnt + eid);
105         lnk(vcnt + eid, e[eid].v);
106     }
107     void delEdge(int eid) {
108         cut(e[eid].u, vcnt + eid);
109         cut(vcnt + eid, e[eid].v);
110     }
111     //维护边双连通分量
112     //并查集,所有的fa[x]改为Find(fa[x]),public调用函数使用前Find(x), 需要保证只在shink进
    行过合并
113     //维护边双时, 节点自身信息在unite中维护, 路径等信息在push_up和push_down中维护
114     int Rt[MAXN];
115     int findroot(int x) {return Rt[x] == x ? x : Rt[x] = findroot(Rt[x]);}
116     void unite_dfs(int x) {
117         push_down(x);
118         if (tr[x][0]) unite_dfs(tr[x][0]), unite(tr[x][0], x);
119         if (tr[x][1]) unite_dfs(tr[x][1]), unite(tr[x][1], x);
120     }
121     void shrink(int x, int y) {
122         split(x, y);
123         unite_dfs(y);
124         int z = Find(y);
125         fa[z] = fa[y]; tr[z][0] = tr[z][1] = 0;
126         push_up(z);
127     }
128     void addEdge(int x, int y) {
129         x = Find(x);y = Find(y);
130         if(findroot(x) != findroot(y)) {
131             lnk(x, y);  Rt[findroot(x)] = Rt[findroot(y)];
132         }else shrink(x, y);
133     }
134 };
135 //未连成树、初始化节点时, 需要调用LCT::push_up 维护节点其余信息
136 //维护的信息要有 可减性 ，如子树结点数, 子树权值和, 但不能直接维护子树最大最小值, 因为在
    将一条虚边变成实边时要排除原先虚边的贡献。
137 //新建一个附加值存储虚子树的贡献, 在统计时将其加入本结点答案, 在改变边的虚实时及时维护。
138 //其余部分同普通 LCT, 在统计子树信息时一定将其作为根节点。
139 //如果维护的信息没有可减性, 如维护区间最值, 可以对每个结点开一个平衡树维护结点的虚子树中
    的最值。
140 if(x == y) continue;
141 if(Find(x) != Find(y)) {
```

```
    unite(x, y);
    addEdge(mp[{x, y}]);
}else{
    int eid = que(x, y);
    int id = mp[{x, y}];
    if(val[eid] > e[id].w) {
        delEdge(eid - vcnt);
        addEdge(id);
    }
}
```

2.5.5 Divide Combine Tree

1.[i,i+1] 构造依赖, 利用线段树辅助建图，利用 tarjan 求 scc，利用 rmq 求最左最右边界 2. 定义 (i,j) 为一个好二元组，当且仅当 a[i]-a[j]=1 这样的两项的二元组在 [l,r] 中恰好有 r-l 个线段树维护 val + l = r，其中 val 是区间 [l,r] 中好二元组的个数离线 3. 析合树维护 mx - mn = r - l <=> fx = (mx - mn) - (r - l)

```
namespace DCT{
    struct RMQ {
        int lg[MAXN], mn[MAXN][S+1], mx[MAXN][S+1];
        inline void init(int *a, int n) {
            for (int i = 2; i <= n; i++) lg[i] = lg[i >> 1] + 1;
            for (int i = 1; i <= n; i++) mn[i][0] = mx[i][0] = a[i];
            for (int k = 1; (1 << k) <= n; k++)
                for (int i = 1; i + (1 << k) - 1 <= n; i++) {
                    mn[i][k] = min(mn[i][k - 1], mn[i + (1 << (k - 1))][k - 1]);
                    mx[i][k] = max(mx[i][k - 1], mx[i + (1 << (k - 1))][k - 1]);
                }
        }
        inline int Min(int l, int r) {
            int len = lg[r - l + 1];
            return min(mn[l][len], mn[r - (1 << len) + 1][len]);
        }
        inline int Max(int l, int r) {
            int len = lg[r - l + 1];
            return max(mx[l][len], mx[r - (1 << len) + 1][len]);
        }
    } D;

    struct SEG {
        int setL, setR, setW;
        int mn[MAXN << 2], tag[MAXN << 2];

        inline void pushup(int x) {
            mn[x] = min(mn[x << 1], mn[x << 1 | 1]);
        }
        inline void pushdown(int x) {
            if(!tag[x]) return;
            mn[x << 1] += tag[x]; mn[x << 1 | 1] += tag[x];
            tag[x << 1] += tag[x]; tag[x << 1 | 1] += tag[x]; tag[x] = 0;
        }
        void init(int x, int l, int r) {
            mn[x] = tag[x] = 0;
            if (l == r) return;
            int mid = (l + r) >> 1;
            init(x << 1, l, mid);
            init(x << 1 | 1, mid + 1, r);
        }
        void upt(int x, int l, int r) {
            if (setL <= l && r <= setR) {
```

```
                tag[x] += setW; mn[x] += setW;
                return;
            }
            pushdown(x);
            int mid = (l + r) >> 1;
            if (setL <= mid) upt(x << 1, l, mid);
            if (mid < setR ) upt(x << 1 | 1, mid+1, r);
            pushup(x);
        }
        int que(int x, int l, int r) {
            if (l == r) return l;
            pushdown(x);
            int mid = (l+r)>>1;
            if (!mn[x << 1]) return que(x << 1, l, mid);
            return que(x << 1 | 1, mid+1, r);
        }
    } T;

    int tpmn, stmn[MAXN], tpmx, stmx[MAXN], tpk, stk[MAXN];
    int ncnt, type[MAXN<<1], L[MAXN<<1], R[MAXN<<1], M[MAXN<<1];
    int dep[MAXN<<1], fa[MAXN<<1][S+1], C[MAXN<<1];
    int id[MAXN << 1];
    int newnode(int _type, int _L, int _R, int _M = 0) {
        ++ncnt; type[ncnt] = _type;
        L[ncnt] = _L; R[ncnt] = _R; M[ncnt] = _M;
        C[ncnt] = 0;
        return ncnt;
    }

    inline bool judge(int l, int r) {
        return D.Max(l, r) - D.Min(l, r) == r - l;
    }

    int ecnt, head[MAXN << 1];
    struct Edge{int to, nxt;} e[MAXN<<1];
    inline void addEdge(int x, int y) {
        e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
        fa[y][0] = x; C[x]++;
    }
    void dfs(int u) {
        for(int j = 0; j < S; j++) fa[u][j+1] = fa[fa[u][j]][j];
        for(int i = head[u]; i; i = e[i].nxt) {
            dep[e[i].to] = dep[u] + 1;
            dfs(e[i].to);
        }
    }

    inline void init(int n) {
        ecnt = 0;
        for(int i = 0; i <= n; i++) head[i] = 0;
    }
    void buildT(int *a, int n) {
        init(n);
        D.init(a, n);
        T.init(1, 1, n);
        tpmn = tpmx = tpk = 0;
        stmn[0] = stmx[0] = stk[0] = 0;
        for (int i = 1; i <= n; i++) {
            for (;tpmn && a[i] <= a[stmn[tpmn]]; --tpmn) {
                T.setL = stmn[tpmn - 1] + 1; T.setR = stmn[tpmn]; T.setW = a[stmn[tpmn
]];
```

```
104                 T.upt(1, 1, n);
105             }
106             T.setL = stmn[tpmn] + 1; T.setR = i; T.setW = -a[i];
107             T.upt(1, 1, n);
108             stmn[++tpmn] = i;
109 
110             for (;tpmx && a[i] >= a[stmx[tpmx]]; --tpmx) {
111                 T.setL = stmx[tpmx - 1] + 1; T.setR = stmx[tpmx]; T.setW = -a[stmx[tpmx
    ]];
112                 T.upt(1, 1, n);
113             }
114             T.setL = stmx[tpmx] + 1; T.setR = i; T.setW = a[i];
115             T.upt(1, 1, n);
116             stmx[++tpmx] = i;
117 
118             int Li = T.que(1, 1, n), np = id[i] = newnode(0, i, i), nq, nw;
119             while (tpk && L[nq = stk[tpk]] >= Li) {
120                 if (type[nq] && judge(M[nq], i)) {
121                     R[nq] = i;
122                     addEdge(nq, np);
123                     np = nq; tpk--;
124                 } else if (judge(L[nq], i)) {
125                     nw = newnode(1, L[nq], i, L[np]);
126                     addEdge(nw, nq); addEdge(nw, np);
127                     np = nw; tpk--;
128                 } else {
129                     nw = newnode(0, -1, i);
130                     addEdge(nw, np);
131                     do {
132                         addEdge(nw, nq);
133                         nq = stk[--tpk];
134                     } while (tpk && !judge(L[nq], i));
135                     addEdge(nw, nq);
136                     L[nw] = L[nq]; R[nw] = i;
137                     np = nw; --tpk;
138                 }
139             }
140             stk[++tpk] = np;
141             T.setL = 1; T.setR = i; T.setW = -1;
142             T.upt(1, 1, n);
143         }
144         assert(tpk == 1);
145         dfs(stk[tpk]);
146     }
147     void lca(int u, int v, int &aL, int &bR) {
148         if(u == v) {
149             aL = L[u]; bR = R[v];
150             return;
151         }
152         if(dep[u] > dep[v]) swap(u, v);
153         for(int i = S; i >= 0; i--)
154             if(dep[fa[v][i]] >= dep[u]) v = fa[v][i];
155         assert(u != v);
156         for(int i = S; i >= 0; i--)
157             if(fa[u][i] != fa[v][i]) {
158                 u = fa[u][i]; v = fa[v][i];
159             }
160         if(type[fa[u][0]]) {
161             aL = min(L[v], L[u]);
162             bR = max(R[v], R[u]);
163         }else{
```

```
164             aL = L[fa[u][0]];
165             bR = R[fa[u][0]];
166         }
167     }
168 };
```

3 String

3.1 Basics

3.1.1 Hash

```
const LL p1 = 201, p2 = 301, mod1 = 1200000319, mod2 = 2147483647;
struct Hash {
    LL a, b;
    void append(Hash pre, int v) {
        a = (pre.a * p1 + v) % mod1;
        b = (pre.b * p2 + v) % mod2;
    }
    void init(string S) {
        a = b = 0;
        for(int i = 0; i < S.size(); i++) append(*this, S[i]);
    }
    bool operator == (const Hash &x) const {
        return a == x.a && b == x.b;
    }
    bool operator < (const Hash &x) const {
        return a < x.a || (a == x.a && b < x.b);
    }
};
```

3.1.2 Minimum String

```
namespace minstring{
    int getmin(char *s, int sn) {
        int i = 0, j = 1, k = 0, t;
        while(i < sn && j < sn && k < sn) {
            t = s[(i + k) % sn] - s[(j + k) % sn];
            if(!t) k++;
            else {
                if(t > 0) i += k + 1; else j += k + 1;
                if(i == j) j++;
                k = 0;
            }
        }
        return i < j ? i : j;
    }
}
```

3.2 String Matching

3.2.1 Bitset Match

```
namespace BitMatch{
    const int S = 26;
    bitset<MAXN> bs[S], ret;
    char s[MAXN];
    inline int idx(char c) { return c - 'a'; }
    inline void init() {
        for(int i = 0; i < 26; i++) bs[i].reset();
        scanf("%s", s);
        int sn = strlen(s);
```

```
        for(int i = 0; i < sn; i++) bs[idx(s[i])].set(i);
    }
    void modify(int p, char ch) {
        bs[idx(s[p])].reset(p);
        s[p] = ch;
        bs[idx(s[p])].set(p);
    }
    int match(char *t, int tn) {//返回t串在s串中出现的次数
        ret = bs[idx(t[0])];
        for(int i = 1; i < tn; i++) {
            ret <<= 1;
            ret &= bs[idx(t[i])];
        }
        return ret.count();
    }
}
```

3.2.2 KMP && exKMP

```
判断循环子串的充要条件: i/(i-fa[i]) > 1 && i%(i-fa[i])==0 (i是长度)且去除下面fa的优化
namespace KMP {
    int fa[MAXN];
    void get_fail(char* t, int tn) {
        fa[0] = -1;
        int i = 0, j = -1;
        while(i < tn) {
            if (j == -1 || t[i] == t[j]) {
                ++i; ++j;
                fa[i] = t[i] != t[j] ? j : fa[j];
            }else{
                j = fa[j];
            }
        }
    }
    void kmp(char* s, int sn, char* t, int tn) {
        int i = 0, j = 0;
        while(i < sn) {
            if (j == -1 || s[i] == t[j]) {
                i++;j++;
                if(j == tn) {
                }
            }else j = fa[j];
        }
    }
}
namespace exKMP {
    int nxt[MAXN], ext[MAXN];
    void get_nxt(char* t, int tn) {
        int j = 0, mx = 0;
        nxt[0] = tn;
        for(int i = 1; i < tn; i++) {
            if(i >= mx || i + nxt[i - j] >= mx) {
                if(i > mx) mx = i;
                while(mx < tn && t[mx] == t[mx - i]) mx++;
                nxt[i] = mx - i;
                j = i;
            }else nxt[i] = nxt[i - j];
        }
    }
    void exkmp(char *s, int sn, char *t, int tn) {
```

```
        int j = 0, mx = 0;
        for(int i = 0; i < sn; i++) {
            if(i >= mx || i + nxt[i - j] >= mx) {
                if(i > mx) mx = i;
                while(mx < sn && mx - i < tn && s[mx] == t[mx - i]) mx++;
                ext[i] = mx - i;
                j = i;
            }else ext[i] = nxt[i - j];
        }
    }
}
```

3.2.3 AC Automaton

```
namespace AC {
    int ch[MAXN][sigma_size], last[MAXN];
    int val[MAXN], f[MAXN], sz;
    inline void init() { sz = 1; memset(ch[0], 0, sizeof(ch[0])); }
    inline int idx(char c) { return c - 'a'; }
    void insert(string s, int v) {
        int u = 0;
        for(int i = 0; i < s.size(); i++) {
            int c = idx(s[i]);
            if(!ch[u][c]) {
                memset(ch[sz], 0, sizeof(ch[sz]));
                val[sz] = 0;
                ch[u][c] = sz++;
            }
            u = ch[u][c];
        }
        val[u] = v;
    }
    void get_fail() {
        queue<int> q;
        f[0] = 0;
        for(int c = 0; c < sigma_size; c++) {
            int u = ch[0][c];
            if(u) { f[u] = 0; q.push(u); last[u] = 0; }
        }
        while(!q.empty()) {
            int r = q.front(); q.pop();
            for(int c = 0; c < sigma_size; c++) {
                int u = ch[r][c];
                if(!u) { ch[r][c] = ch[f[r]][c]; continue; }
                q.push(u);
                int v = f[r];
                while(v && !ch[v][c]) v = f[v];
                f[u] = ch[v][c];
                last[u] = val[f[u]] ? f[u] : last[f[u]];
            }
        }
    }
    inline void solve(int j) {
        if(j) {
            ans += val[j];
            solve(last[j]);
        }
    }
    void find(string T) {
        int j = 0;
```

```
47          for(int i = 0; i < T.size(); i++) {
48              int c = idx(T[i]);
49              j = ch[j][c];
50              if(val[j]) solve(j);
51              else if(last[j]) solve(last[j]);
52          }
53      }
54  }
55  namespace AC {
56      int root, tcnt;
57      int ch[MAXN][sigma_size], fa[MAXN];
58      inline int newnode() {
59          fa[++tcnt] = 0;
60          for(int i = 0; i < sigma_size; ++i) ch[tcnt][i] = 0;
61          return tcnt;
62      }
63      inline void init() {
64          tcnt = -1;
65          root = newnode();
66      }
67      inline int idx(char c) { return c - 'a'; }
68      void extend(char *s, int sn) {
69          int cur = root;
70          for(int i = 0, c; i < sn; i++) {
71              if(!ch[cur][c = idx(s[i])])
72                  ch[cur][c] = newnode();
73              cur = ch[cur][c];
74          }
75      }
76      int q[MAXN], qh, qt;
77      void get_fail() {
78          qh = 1; qt = 0;
79          fa[root] = 0;
80          for(int c = 0, now; c < sigma_size; c++)
81              if((now = ch[root][c]) != 0)
82                  q[++qt] = now;
83          while(qh <= qt) {
84              int cur = q[qh++];
85              for(int c = 0, now; c < sigma_size; c++)
86                  if((now = ch[cur][c]) != 0) {
87                      fa[now] = ch[fa[cur]][c];
88                      q[++qt] = now;
89                  }else
90                      ch[cur][c] = ch[fa[cur]][c];
91          }
92      }
93  //统计模板串出现次数, 每个模板串只计算一次
94  //      int cur = root, ans = 0;
95  //      for(int i = 0; i < sn; ++i) {
96  //          cur = ch[cur][idx(s[i])];
97  //          for(int j = cur; j && cnt[j] != -1; j = fa[j]) {
98  //              ans += cnt[j];
99  //              cnt[j] = -1;
100 //          }
101 //      }
102
103 }
```

3.3 Suffix Related

3.3.1 Suffix Array

```
namespace SA {
    char s[MAXN];
    int sa[MAXN], rank[MAXN], height[MAXN];
    int t[MAXN], t2[MAXN], c[MAXN], n;
    void clear() { n = 0; memset(sa, 0, sizeof(sa)); }
    void build(int m) {
        int *x = t, *y = t2;
        for(int i = 0; i < m; i++) c[i] = 0;
        for(int i = 0; i < n; i++) c[x[i] = s[i]]++;
        for(int i = 1; i < m; i++) c[i] += c[i - 1];
        for(int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
        for(int k = 1; k <= n; k <<= 1) {
            int p = 0;
            for(int i = n - k; i < n; i++) y[p++] = i;
            for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
            for(int i = 0; i < m; i++) c[i] = 0;
            for(int i = 0; i < n; i++) c[x[y[i]]]++;
            for(int i = 1; i < m; i++) c[i] += c[i - 1];
            for(int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
            swap(x, y);
            p = 1; x[sa[0]] = 0;
            for(int i = 1; i < n; i++)
                x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k]
    ? p - 1 : p++;
            if(p >= n) break;
            m = p;
        }
    }
    void buildHeight() {
        int k = 0;
        for(int i = 0; i < n; i++) rank[sa[i]] = i;
        for(int i = 0; i < n; i++) {
            if(k) k--;
            int j = sa[rank[i] - 1];
            while(s[i + k] == s[j + k]) k++;
            height[rank[i]] = k;
        }
    }
    void init() {
        n = strlen(s) + 1;
        build('z' + 1);
        buildHeight();
    }
}
```

3.3.2 Suffix Automaton

```
namespace SAM{
    int scnt, root, last;
    int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
    int sc[MAXN<<1], tmpl[MAXN<<1], minl[MAXN<<1];

    int newnode(int _len, int q = 0) {
        fa[++scnt] = fa[q]; len[scnt] = _len;
        sc[scnt] = 0;tmpl[scnt] = 0; minl[scnt] = INF;
```

```
        for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
        return scnt;
    }
    void init() {
        scnt = 0;
        root = last = newnode(0);
    }
    void extend(int c) {
        int p = last, np = newnode(len[p] + 1);
        for(;p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
        if(!p) fa[np] = root;
        else{
            int q = ch[p][c];
            if(len[p] + 1 == len[q]) fa[np] = q;
            else{
                int nq = newnode(len[p] + 1, q);
                fa[np] = fa[q] = nq;
                for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
            }
        }
        last = np;
    }
    int c[MAXN], rs[MAXN << 1];
    void radix_sort(int n){
        for(int i = 0; i <= n; i++) c[i] = 0;
        for(int i = 1; i <= scnt; i++) c[len[i]]++;
        for(int i = 1; i <= n; i++) c[i] += c[i-1];
        for(int i = scnt; i >= 1; i--) rs[c[len[i]]--] = i;
    }
    void go(){
        scanf("%s",s);
        int n = strlen(s);
        for(int i = 0; i < n; ++i)
            extend(s[i] - 'a');
        radix_sort(n);
        //以下sc集合意义不同
        {//每个节点对应的位置之后有多少个不同子串
            for(int i = scnt; i >= 1; i--) {
                int S = 0;
                for(int j = 0; j < 26; j++)
                    S += sc[ ch[rs[i]][j] ];
                sc[rs[i]] = S + 1;
            }
        }
        {//right集合大小
            int cur = root;
            for(int i = 0; i < n; ++i) {
                cur = ch[cur][s[i]-'a'];
                sc[cur]++;
            }
            for(int i = scnt; i >= 1; --i) {
                sc[ fa[rs[i]] ] += sc[rs[i]];
            }
        }
        //公共子串
        //tmpl,当前字符串:在状态cur, 与模板串的最长公共后缀
        //minl, 多个字符串:在状态cur,与模板串的最长公共后缀
        //注意:在状态cur匹配成功时, cur的祖先状态与字符串的最长公共后缀
        for(; ~scanf("%s",s);) {
            int cur = root, Blen = 0;
            for(int i = 0; i <= scnt; i++)
```

```
                tmpl[i] = 0;
            n = strlen(s);
            for(int i = 0, x; i < n; i++) {
                x = s[i] - 'a';
                if(ch[cur][x]) {
                    ++Blen;
                    cur = ch[cur][x];
                }else{
                    for(;cur && ch[cur][x] == 0; cur = fa[cur]);
                    if(cur) {
                        Blen = len[cur] + 1;
                        cur = ch[cur][x];
                    }else{
                        cur = root; Blen = 0;
                    }
                }
                tmpl[cur] = max(tmpl[cur], Blen);
            }
            for(int i = scnt; i ; --i) {
                if( tmpl[ fa[rs[i]] ] < tmpl[ rs[i] ])
                    tmpl[ fa[rs[i]] ] = len[ fa[rs[i]] ];
                minl[ rs[i] ] = min(minl[ rs[i] ], tmpl[ rs[i] ]);
            }
        }
    }
}
namespace exSAM{
    int scnt, root;
    int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
    int sc[MAXN<<1], tmpl[MAXN<<1], minl[MAXN<<1];

    int newnode(int _len, int q = 0) {
        fa[++scnt] = fa[q]; len[scnt] = _len;
        sc[scnt] = 0;tmpl[scnt] = 0; minl[scnt] = INF;
        for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
        return scnt;
    }
    void init() {
        scnt = 0;
        root = newnode(0);
    }
    int work(int p,int c){
        int q = ch[p][c];
        int nq = newnode(len[p] + 1, q);
        fa[q] = nq;
        for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
        return nq;
    }
    int extend(int p, int c) {
        if (ch[p][c]){
            int q = ch[p][c];
            if (len[p] + 1 == len[q]) return q;
            return work(p, c);
        }
        int np = newnode(len[p] + 1);
        for(;p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
        if (!p) fa[np] = root;
        else{
            int q = ch[p][c];
            if (len[p] + 1 == len[q]) fa[np] = q;
            else fa[np] = work(p, c);
```

```
        }
        return np;
    }
    void solve() {
        int n; scanf("%d",&n);
        for(int i = 1; i <= n; i++) {
            scanf("%s", s);
            int sn = strlen(s);
            int last = root;
            for(int j = 0; j < sn; ++j)
                last = extend(last, s[j] - 'a');
        }
    }
}
```

3.4 Palindrome Related

3.4.1 Manacher

```
namespace Manachar {
    char S[MAXN << 1];
    int scnt, ans;
    int p[MAXN << 1]; //p[i] - 1
    void init(char *s0, int sn0) {
        S[0] = '$'; S[1] = '#';
        for(int i = 0; i < sn0; i++) {
            S[2 * i + 2] = s0[i];
            S[2 * i + 3] = '#';
        }
        scnt = sn0 * 2 + 2;
        S[scnt] = '&';
    }
    void manachar() {
        int id = 0, mx = 0;
        for(int i = 1; i < scnt; i++) {
            p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
            while(S[i + p[i]] == S[i - p[i]]) p[i]++;
            if(i + p[i] > mx) {
                mx = i + p[i];
                id = i;
            }
        }
    }
}
```

3.4.2 Palindromic Automaton

pcnt 本质不同的回文串的个数 len[u] 状态 u 代表的串的长度 fail[u] 状态 u 所代表的回文串的最长回文后缀 trans[u] 小于等于当前节点长度一半的最长回文后缀 cnt[u] 包含状态 u 表示的回文串的出现的个数 (调用 count()) num[u] 以状态 u 表示的回文串的右端点为回文串结尾的回文串个数 [..[....[] diff[u] 表示节点 u 和 fail[u] 所代表的回文串的长度差，即 len[u] - len[fail[u] slink[u] 表示 u 一直沿着 fail 向上跳到第一个节点 v, 使得 diff[v]!=diff[u]，也就是 u 所在等差数列中长度最小的那个节点

```
namespace PAM {
    int scnt, S[MAXN];
    int pcnt, last, len[MAXN], fail[MAXN], ch[MAXN][26];
    int cnt[MAXN], num[MAXN], trans[MAXN], diff[MAXN], slink[MAXN];
```

```
    int newnode(int _len) {
        len[pcnt] = _len;
        cnt[pcnt] = num[pcnt] = 0;
        for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
        return pcnt++;
    }
    inline void init() {
        S[scnt = 0] = -1;
        pcnt = 0;newnode(0);newnode(-1);
        fail[0] = 1;last = 0;
    }
    int getfail(int x) {
        while(S[scnt - len[x] - 1] != S[scnt]) x = fail[x];
        return x;
    }
    void extend(int c) {
        S[++scnt] = c;
        int cur = getfail(last);
        if(!ch[cur][c]) {
            int now = newnode(len[cur] + 2);
            fail[now] = ch[getfail(fail[cur])][c];
            ch[cur][c] = now;
            num[now] = num[fail[now]] + 1;
            if (len[now] <= 2) trans[now] = fail[now];
            else{
                int x = trans[cur];
                while(S[scnt - len[x] - 1] != S[scnt] || (len[x]+2) * 2 > len[now]) x = fail[x];
                trans[now] = ch[x][c];
            }
            diff[now] = len[now] - len[fail[now]];
            slink[now] = (diff[now] == diff[fail[now]]) ? slink[fail[now]] : fail[now];
        }
        last = ch[cur][c];
        cnt[last]++;
    }
    void count() {
        for(int i = pcnt - 1; i >= 0; i--) cnt[fail[i]] += cnt[i];
    }
};
```

支持前后插入不基于势能分析的构造法可以实现持久化，比如在 Trie 上实现

```
namespace PAM {
    int sL, sR, S[MAXN<<1];
    int pcnt, lastL, lastR;
    int len[MAXN<<1], fa[MAXN<<1], quick[MAXN<<1][26], ch[MAXN<<1][26];
    int cnt[MAXN<<1], num[MAXN<<1];
    int newnode(int _len) {
        len[pcnt] = _len;
        cnt[pcnt] = num[pcnt] = 0;
        for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
        return pcnt++;
    }
    inline void init() {
        pcnt = 0;newnode(0);newnode(-1);
        fa[0] = 1;
        for(int i = 0; i < 26; i++) quick[0][i] = quick[1][i] = 1;
        lastL = lastR = 0;
        sL = MAXN; sR = MAXN-1;
        S[sL] = S[sR] = -1;
    }
```

```
    void push_front(int c) {
        S[--sL] = c; S[sL-1]=-1;
        int p = lastL;
        if (S[sL+len[p]+1] ^ c) p = quick[p][c];
        if (!ch[p][c]) {
            int np = newnode(len[p]+2), q = fa[p];
            if (S[sL+len[q]+1] ^ c) q = quick[q][c];
            fa[np] = ch[q][c];
            memcpy(quick[np], quick[fa[np]], sizeof(quick[np]));
            quick[np][S[sL+len[fa[np]]]] = fa[np];
            ch[p][c] = np;
            num[np] = num[fa[np]] + 1;
        }
        lastL = ch[p][c];
        cnt[lastL]++;
        if (len[lastL] == sR-sL+1) lastR = lastL;
    }
    void push_back(int c) {
        S[++sR] = c; S[sR+1]=-1;
        int p = lastR;
        if (S[sR-len[p]-1] ^ c) p = quick[p][c];
        if (!ch[p][c]) {
            int np = newnode(len[p]+2), q = fa[p];
            if (S[sR-len[q]-1] ^ c) q = quick[q][c];
            fa[np] = ch[q][c];
            memcpy(quick[np],quick[fa[np]], sizeof(quick[np]));
            quick[np][S[sR-len[fa[np]]]] = fa[np];
            ch[p][c] = np;
            num[np] = num[fa[np]] + 1;
        }
        lastR = ch[p][c];
        cnt[lastR]++;
        if (len[lastR] == sR-sL+1) lastL = lastR;
    }
    int c[MAXN<<1], rs[MAXN<<2];
    void count() {
        for (int i = 0; i < pcnt; i++) c[i] = 0;
        for (int i = 2; i < pcnt; i++) c[len[i]]++;
        for (int i = 1; i < pcnt; i++) c[i] += c[i-1];
        for (int i = 2; i < pcnt; i++) rs[c[len[i]]--] = i;
        for (int i = pcnt-1; i; i--)  cnt[fa[rs[i]]]+=cnt[rs[i]];
    }
};
```

3.5 Substring Automaton

```
for(int j = 0; j < 26; j++)
    ch[n][j] = ch[n+1][j] = n + 1; //或者 -1
for(int i = n; i >= 1; i--) {
    for(int j = 0; j < 26; j++)
        ch[i-1][j] = ch[i][j];
    ch[i-1][s[i]-'a'] = i;
}
```

当字符集过大时使用主席树维护 ch

4 Math

4.1 Algebra

4.1.1 FFT

```
//不预处理精度
const double pi = acos(-1.0);
const int MAXN = 300003;
struct comp {
    double x, y;
    comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
    comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
    comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y + y * a.x}; }
};
int rev[MAXN], T;
comp tmp;
void fft(comp *a, int r) {
    if(r == -1) for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
    for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
    for(int i = 2, mid = 1; i <= T; mid = i, i <<= 1) {
        comp step = (comp) {cos(pi / mid), r * sin(pi / mid)};
        for(int j = 0; j < T; j += i) {
            comp cur = (comp) {1, 0};
            for(int k = j; k < j + mid; k++, cur = cur * step) {
                tmp = a[k + mid] * cur;
                a[k + mid] = a[k] - tmp;
                a[k] = a[k] + tmp;
            }
        }
    }
    if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
}
comp A[MAXN];
void init(int n) {
    //A[0] = (comp) {0, 0};
    for(T = 1; T <= n; T <<= 1);
    for(int i = 1; i < T; i++) {
        if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
        else rev[i] = rev[i >> 1] >> 1;
        //A[i] = (comp) {0, 0};
    }
}
//预处理精度
int rev[MAXN], T;
comp Sin[MAXN], tmp;
void fft(comp *a, int r) {
    if(r == -1) {
        for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
        for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
    }
    for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
    for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <<= 1, s >>= 1) {
        for(int j = 0; j < T; j += i) {
            for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
                tmp = a[k + mid] * Sin[cur];
                a[k + mid] = a[k] - tmp;
                a[k] = a[k] + tmp;
```

```
            }
        }
    }
    if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
}
comp A[MAXN];
void init(int n) {
    for(T = 1; T <= n; T <<= 1);
    for(int i = 0; i < T; i++) {
        if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
        else rev[i] = rev[i >> 1] >> 1;
        //A[i] = (comp) {0, 0};
    }
    for(int i = 0; i < (T >> 1); i++) {
        Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
    }
}
int main() {
    scanf("%d%d", &n, &m);
    init(n + m);
    for(int i = 0; i <= n; i++) scanf("%lf", &A[i].x);
    for(int i = 0; i <= m; i++) scanf("%lf", &A[i].y);
    fft(A, 1);
    fft(A, -1);
    for(int i = 0; i <= n + m; i++) printf("%d%c", (int)(A[i].y), i == n + m ? '\n' : '
    ');
    return 0;
}
```

4.1.2 NTT

4.常用NTT模数：

以下模数的共同$g = 3189$

$p = r \times 2^k + 1$	k	g
104857601	22	3
167772161	25	3
469762049	26	3
950009857	21	7
998244353	23	3
1004535809	21	3
2013265921	27	31
2281701377	27	3
3221225473	30	5

```
const int MAXN = 300005, G = 3, mod = 998244353; //or (479LL<<21) + 1
int rev[MAXN], T;
LL qpow(LL x, LL y) {
    LL res = 1;
    while(y) {
        if(y & 1) res = res * x % mod;
        x = x * x % mod;
        y >>= 1;
    }
    return res;
}
LL A[MAXN], B[MAXN];
```

```
void ntt(LL *a, int r) {
    if(r == -1) for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
    for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
    for(int i = 2, mid = 1; i <= T; mid = i, i <<= 1) {
        LL gn = qpow(G, (mod - 1) / i);
        if(r == -1) gn = qpow(gn, mod - 2);
        for(int j = 0; j < T; j += i) {
            LL cur = 1, tmp;
            for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
                tmp = a[k + mid] * cur % mod;
                a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
                a[k] = (a[k] + tmp) % mod;
            }
        }
    }
    if(r == -1) {
        LL inv = qpow(T, mod - 2);
        for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
    }
}
void init(int n) {
    for(T = 1; T <= n; T <<= 1);
    for(int i = 0; i < T; i++) {
        if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
        else rev[i] = rev[i >> 1] >> 1;
    }
}
```

4.1.3 MTT

```
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
const double pi = acos(-1.0);
const int MAXN = 300003;
struct comp {
    double x, y;
    comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
    comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
    comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y + y * a.x}; }
};
#define conj(a) ((comp){a.x, -a.y})
int rev[MAXN], T;
comp Sin[MAXN], tmp;
void fft(comp *a, int r) {
    for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
    for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <<= 1, s >>= 1) {
        for(int j = 0; j < T; j += i) {
            for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
                tmp = a[k + mid] * Sin[cur];
                a[k + mid] = a[k] - tmp;
                a[k] = a[k] + tmp;
            }
        }
    }
}
void init(int n) {
    for(T = 1; T <= n; T <<= 1);
    for(int i = 0; i < T; i++) {
```

```
        if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
        else rev[i] = rev[i >> 1] >> 1;
    }
    for(int i = 0; i < (T >> 1); i++) {
        Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
    }
}
int n, m, mod;
void mtt(int *x, int *y) {
    for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
    static comp a[MAXN], b[MAXN];
    static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
    for(int i = 0; i < T; i++) {
        a[i] = {x[i] & 0x7fff, x[i] >> 15};
        b[i] = {y[i] & 0x7fff, y[i] >> 15};
    }
    fft(a, 1); fft(b, 1);
    for(int i = 0; i < T; i++) {
        int j = (T - i) & (T - 1);
        static comp da, db, dc, dd;
        da = (a[i] + conj(a[j])) * (comp){0.5, 0};
        db = (a[i] - conj(a[j])) * (comp){0, -0.5};
        dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
        dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
        dfta[j] = da * dc;
        dftb[j] = da * dd;
        dftc[j] = db * dc;
        dftd[j] = db * dd;
    }
    for(int i = 0; i < T; i++) {
        a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
        b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
    }
    //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
    fft(a, -1); fft(b, -1);
    for(int i = 0; i < T; i++) {
        static int da, db, dc, dd;
        da = (LL)(a[i].x / T + 0.5) % mod;
        db = (LL)(a[i].y / T + 0.5) % mod;
        dc = (LL)(b[i].x / T + 0.5) % mod;
        dd = (LL)(b[i].y / T + 0.5) % mod;
        x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
    }
}
int main() {
    static int a[MAXN], b[MAXN];
    scanf("%d%d%d", &n, &m, &mod);
    for(int i = 0; i <= n; i++) scanf("%d", a + i);
    for(int i = 0; i <= m; i++) scanf("%d", b + i);
    init(n + m);
    mtt(a, b);
    for(int i = 0; i <= n + m; i++) printf("%d%c", a[i], i == n + m ? '\n' : ' ');
    return 0;
}
```

4.1.4 FWT

```
void FWT(LL *a,int n) {
    for(int i = 2;i <= n; i <<= 1) {
        for(int j = 0; j < n; j += i) {
```

```
            for(int d = 0, w = i >> 1; d < w; d++){
                LL u = a[j + d], v = a[j + d + w];
                //xor: a[j + d] = u + v, a[j + d + w] = u - v;
                //and: a[j + d] = u + v;
                //or : a[j + d + w] = u + v;
            }
        }
    }
}
void UFWT(LL *a, int n) {
    for(int i = 2; i <= n; i <<= 1) {
        for(int j = 0; j < n; j += i) {
            for(int d = 0, w = i >> 1; d < w; d++) {
                LL u = a[j + d], v = a[j + d + w];
                //xor: a[j + d] = (u + v) / 2, a[j + d + w] = (u - v) / 2;
                //and: a[j + d] = u - v;
                //or : a[j + d + w] = v - u;
            }
        }
    }
}
void solve(int n) {
    FWT(a, n); FWT(b, n);
    for(int i = 0; i < n; i++) a[i] = a[i] * b[i];
    UFWT(a, n);
}
```

4.1.5 FFT Divide and Conquer

$$f_i = \sum_{j=1}^{i-1} f_j \cdot g_{i-j}$$

```
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
const int MAXN = 300005, G = 3, mod = 998244353;
namespace NTT {
    LL A[MAXN], B[MAXN]
    int rev[MAXN], T;
    LL qpow(LL x, LL y) {
        LL res = 1;
        while(y) {
            if(y & 1) res = res * x % mod;
            x = x * x % mod;
            y >>= 1;
        }
        return res;
    }
    void ntt(LL *a, int r) {
        for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
        for(int i = 2, mid = 1; i <= T; mid = i, i <<= 1) {
            LL gn = qpow(G, (mod - 1) / i);
            if(r == -1) gn = qpow(gn, mod - 2);
            for(int j = 0; j < T; j += i) {
                LL cur = 1, tmp;
                for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
                    tmp = a[k + mid] * cur % mod;
                    a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
```

```
                    a[k] = (a[k] + tmp) % mod;
                }
            }
        }
        if(r == -1) {
            LL inv = qpow(T, mod - 2);
            for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
        }
    }
    void init(int n) {
        for(T = 1; T <= n; T <<= 1);
        for(int i = 0; i < T; i++) {
            if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
            else rev[i] = rev[i >> 1] >> 1;
        }
    }
}
LL f[MAXN], g[MAXN];
using namespace NTT;
void solve(int l, int r) {
    if(l == r) return;
    int mid = (l + r) >> 1;
    solve(l, mid);
    init(r - l);
    for(int i = 0; i < T; i++) A[i] = B[i] = 0;
    for(int i = 0; i <= mid - l; i++) A[i] = f[i + l];
    for(int i = 0; i <= r - l; i++) B[i] = g[i];
    ntt(A, 1); ntt(B, 1);
    for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
    ntt(A, -1);
    for(int i = mid + 1; i <= r; i++) f[i] =(f[i] + A[i - l]) % mod;
    solve(mid + 1, r);
}
int main() {
    int n; scanf("%d", &n);
    for(int i = 1; i < n; i++) scanf("%lld", g + i);
    f[0] = 1;
    solve(0, n - 1);
    for(int i = 0; i < n; i++) printf("%lld%c", f[i], i == n - 1 ? '\n' : ' ');
    return 0;
}
```

4.1.6 Linear Basis

```
//dynamic
const int D = 60;
struct Basis {
    vector<int> ind;
    vector<LL> base;
    Basis() {
        ind.resize(D, -1);
        base.resize(D);
    }
    bool update(LL x, int id) {
        for(int i = 0; i < D; i++) if(~ind[i] && x >> i & 1) {
            x ^= base[i];
        }
        if(!x) return 1;
        int pos = __builtin_ctzll(x);
        ind[pos] = id;
```

```
        base[pos] = x;
        return 0;
    }
};
//array
int Gauss(int n, int m) {
    int num = 1;
    for(int x = 1; x <= n && x <= m; x++) {
        int t = 0;
        for(int j = x; j <= m; j++) if(g[j][x]) { t = j; break; }
        if(t) {
            swap(g[x], g[t]);
            for(int i = x + 1; i <= n; i++) {
                if(g[i][x]) {
                    for(int k = 1; k <= m; k++) g[i][k] ^= g[x][k];
                }
            }
            num++;
        }
    }
    return --num;
}
//long long
int Gauss() {
    int num = 1;
    for(int k = 61; k >= 0; k--) {
        int t = 0;
        for(int j = num; j <= cnt; j++) if((A[j] >> k) & 1) { t = j; break; }
        if(t) {
            swap(A[t], A[num]);
            for(int j = num + 1; j <= cnt; j++) if((A[j] >> k) & 1) A[j] ^= A[num];
            num++;
        }
    }
    return --num;
}
//det
LL det(int n){
    LL ret = 1;
    for(int i = 1; i < n; i++){
        for(int j = i + 1; j < n; j++)
            while(a[j][i]){
                LL t = (LL)a[i][i] / a[j][i];
                for(int k = i; k < n; k++) {
                    a[i][k] = (a[i][k]-a[j][k] * t);
                    swap(a[i][k], a[j][k]);
                }
                ret = -ret;
            }
        if(a[i][i] == 0)return 0;
        ret = ret * a[i][i];
    }
    if(ret<0)ret = -ret;
    return ret;
}
```

4.1.7 Polynomial

Inverse:

$$\begin{aligned} A_x * B'_x &\equiv 1 && (\bmod\ x^{\frac{n}{2}}) \\ A_x * B_x &\equiv 1 && (\bmod\ x^n) \\ B_x &\equiv 2 \cdot B'_x - A_x \cdot {B'}_x^2 && (\bmod\ x^n) \end{aligned}$$

Division:

$$A_r(x) = x^n A(\frac{1}{x}) \Rightarrow A_r(x)[i] = A(x)[n-i]$$

$$\begin{aligned} A(x) &= B(x) * Q(x) + R(x) \\ Q_r(x) &= A_r(x) * B_r^{-1}(x) \\ R(x) &= A(x) - B(x) * Q(x) \end{aligned}$$

```cpp
//NTT模数
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
const int MAXN = 300005, G = 3, mod = 998244353;
namespace NTT {
    int rev[MAXN], T;
    LL qpow(LL x, LL y) {
        LL res = 1;
        while(y) {
            if(y & 1) res = res * x % mod;
            x = x * x % mod;
            y >>= 1;
        }
        return res;
    }
    void ntt(LL *a, int r) {
        for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
        for(int i = 2, mid = 1; i <= T; mid = i, i <<= 1) {
            LL gn = qpow(G, (mod - 1) / i);
            if(r == -1) gn = qpow(gn, mod - 2);
            for(int j = 0; j < T; j += i) {
                LL cur = 1, tmp;
                for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
                    tmp = a[k + mid] * cur % mod;
                    a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
                    a[k] = (a[k] + tmp) % mod;
                }
            }
        }
        if(r == -1) {
            LL inv = qpow(T, mod - 2);
            for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
        }
    }
    void init(int n) {
        for(T = 1; T <= n; T <<= 1);
        for(int i = 0; i < T; i++) {
            if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
            else rev[i] = rev[i >> 1] >> 1;
```

```
        }
    }
}
namespace poly {
using namespace NTT;
    void mul(LL *a, LL *b, LL *c, int n, int m) {
        init(n + m);
        static LL x[MAXN], y[MAXN];
        for(int i = 0; i < T; i++) {
            x[i] = i <= n ? a[i] : 0;
            y[i] = i <= m ? b[i] : 0;
        }
        ntt(x, 1); ntt(y, 1);
        for(int i = 0; i < T; i++) c[i] = x[i] * y[i] % mod;
        ntt(c, -1);
    }
    void poly_inv(LL *a, LL *b, int n) {
        if(n == 1) {
            b[0] = qpow(a[0], mod - 2);
            return;
        }
        static LL c[MAXN], d[MAXN];
        memset(c, 0, n * sizeof(LL));
        poly_inv(a, c, n >> 1);
        for(int i = 0; i < n; i++) {
            d[i] = a[i];
        }
        init(n);
        ntt(c, 1); ntt(d, 1);
        for(int i = 0; i < T; i++) b[i] = c[i] * (2 + mod - d[i] * c[i] % mod) % mod;
        ntt(b, -1);
        for(int i = n; i < T; i++) b[i] = 0;
    }
    void inv(LL *a, LL *b, int n) {//A must be different from B
        init(n);
        poly_inv(a, b, T);
    }
    //A_x = B_x * Q_x + R_x;
    void div(LL *A, LL *B, LL *Q, LL *R, int n, int m) {
        static LL f[MAXN], g[MAXN], inv_g[MAXN];
        for(int i = 0; i <= n; i++) f[n - i] = A[i];
        for(int i = 0; i <= m; i++) g[m - i] = B[i];
        //inv(G_r)
        for(int i = n - m + 1; i <= m; i++) g[i] = 0;
        inv(g, inv_g, n - m);
        //Q
        mul(f, inv_g, f, n, n - m);
        for(int i = 0; i <= n - m; i++) Q[i] = f[n - m - i];
        //R
        mul(Q, B, f, n - m, m);
        for(int i = 0; i < m; i++) R[i] = (A[i] + mod - f[i]) % mod;
    }
}
LL A[MAXN], B[MAXN];
LL Q[MAXN], R[MAXN];
int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    int n, m; cin >> n >> m;
    for(int i = 0; i <= n; i++) cin >> A[i];
    for(int i = 0; i <= m; i++) cin >> B[i];
    poly::div(A, B, Q, R, n, m);
```

```
    for(int i = 0; i <= n - m; i++) cout << Q[i] << " "; cout << endl;
    for(int i = 0; i < m; i++) cout << R[i] << " "; cout << endl;
    return 0;
}
//非NTT模数求逆
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
const double pi = acos(-1.0);
const int MAXN = 300003;
struct comp {
    double x, y;
    comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
    comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
    comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y + y * a.x}; }
};
#define conj(a) ((comp){a.x, -a.y})
int rev[MAXN], T;
comp Sin[MAXN], tmp;
void fft(comp *a, int r) {
    for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
    for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <<= 1, s >>= 1) {
        for(int j = 0; j < T; j += i) {
            for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
                tmp = a[k + mid] * Sin[cur];
                a[k + mid] = a[k] - tmp;
                a[k] = a[k] + tmp;
            }
        }
    }
}
void init(int n) {
    //for(T = 1; T <= n; T <<= 1);
    T = n << 1;
    for(int i = 0; i < T; i++) {
        if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
        else rev[i] = rev[i >> 1] >> 1;
    }
    for(int i = 0; i < (T >> 1); i++) {
        Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
    }
}
const int mod = 1e9 + 7;
void mtt(int *x, int *y) {
    for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
    static comp a[MAXN], b[MAXN];
    static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
    for(int i = 0; i < T; i++) {
        a[i] = {x[i] & 0x7fff, x[i] >> 15};
        b[i] = {y[i] & 0x7fff, y[i] >> 15};
    }
    fft(a, 1); fft(b, 1);
    for(int i = 0; i < T; i++) {
        int j = (T - i) & (T - 1);
        static comp da, db, dc, dd;
        da = (a[i] + conj(a[j])) * (comp){0.5, 0};
        db = (a[i] - conj(a[j])) * (comp){0, -0.5};
        dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
        dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
        dfta[j] = da * dc;
```

```
        dftb[j] = da * dd;
        dftc[j] = db * dc;
        dftd[j] = db * dd;
    }
    for(int i = 0; i < T; i++) {
        a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
        b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
    }
    //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
    fft(a, -1); fft(b, -1);
    for(int i = 0; i < T; i++) {
        static int da, db, dc, dd;
        da = (LL)(a[i].x / T + 0.5) % mod;
        db = (LL)(a[i].y / T + 0.5) % mod;
        dc = (LL)(b[i].x / T + 0.5) % mod;
        dd = (LL)(b[i].y / T + 0.5) % mod;
        x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
    }
}
LL qpow(LL x, LL y) {
    LL res = 1;
    while(y) {
        if(y & 1) res = res * x % mod;
        x = x * x % mod;
        y >>= 1;
    }
    return res;
}
void poly_inv(int *a, int *b, int n) {
    if(n == 1) {
        b[0] = qpow(a[0], mod - 2);
        return;
    }
    static int c[MAXN], cc[MAXN], d[MAXN];
    memset(c, 0, n * sizeof(int)); memset(cc, 0, n * sizeof(int)); memset(d, 0, n *
    sizeof(int));
    poly_inv(a, c, n >> 1);
    for(int i = 0; i < n; i++) cc[i] = c[i], d[i] = a[i];
    init(n);
    mtt(cc, c);
    mtt(d, cc);
    for(int i = 0; i < T; i++) b[i] = (2 * c[i] % mod + mod - d[i]) % mod;
    for(int i = 0; i < n; i++) b[n + i] = 0;
}
int A[MAXN], B[MAXN];
int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    int n; cin >> n; n--;
    for(T = 1; T <= n; T <<= 1);
    for(int i = 0; i <= n; i++) cin >> A[i];
    poly_inv(A, B, T);
    for(int i = 0; i <= n; i++) cout << B[i] << " ";
    cout << endl;
    return 0;
}

//LOJ挑战多项式(备用)
#include <bits/stdc++.h>
#define rep(i, a, b) for (int i = (a); i <= (b); i++)
#define per(i, a, b) for (int i = (a); i >= (b); i--)
#define REP(i, n) for (int i = (0); i < (n); i++)
```

```
#define fi first
#define se second
#define mp make_pair
#define pb push_back
using namespace std;
typedef unsigned long long ull;
typedef pair<int, int> pii;
typedef vector<int> poly;
typedef long long ll;
struct ano {
    char a[1 << 25], *s;
    char b[1 << 25], *t;
    ano() : s(a), t(b) { a[fread(a, 1, sizeof a, stdin)] = 0; }
    ~ano() { fwrite(b, 1, t - b, stdout); }
    operator int() {
        int x = 0;
        while (*s < 48) ++s;
        while (*s > 32) x = x * 10 + *s++ - 48;
        return x;
    }
    ll in() {
        ll x = 0;
        while (*s < 48) ++s;
        while (*s > 32) x = x * 10 + *s++ - 48;
        return x;
    }
    void out(int x, char e = ' ') {
        static char c[12];
        char *i = c;
        if (!x)
            *t++ = 48;
        else {
            while (x) {
                int y = x / 10;
                *i++ = x - y * 10 + 48, x = y;
            }
            while (i != c) *t++ = *--i;
        }
        *t++ = e;
    }
} buf;
const int mod = 998244353;
namespace Poly {
const int N = (1 << 20) + 5, g = 3;
inline int power(int x, int p) {
    int res = 1;
    for (; p; p >>= 1, x = (ll)x * x % mod)
        if (p & 1)
            res = (ll)res * x % mod;
    return res;
}
inline int fix(const int x) { return x >= mod ? x - mod : x; }
void dft(poly &A, int n) {
    static ull W[N << 1], *H[30], *las = W, mx = 0;
    for (; mx < n; mx++) {
        H[mx] = las;
        ull w = 1, wn = power(g, (mod - 1) >> (mx + 1));
        REP(i, 1 << n) *las++ = w, w = w * wn % mod;
    }
    if (A.size() != (1 << n))
        A.resize(1 << n);
```

```
    static ull a[N];
    for (int i = 0, j = 0; i < (1 << n); ++i) {
        a[i] = A[j];
        for (int k = 1 << (n - 1); (j ^= k) < k; k >>= 1)
            ;
    }
    for (int k = 0, d = 1; k < n; k++, d <<= 1)
        for (int i = 0; i < (1 << n); i += (d << 1)) {
            ull *l = a + i, *r = a + i + d, *w = H[k], t;
            for (int j = 0; j < d; j++, l++, r++) {
                t = (*r) * (*w++) % mod;
                *r = *l + mod - t, *l += t;
            }
        }
    REP(i, 1 << n) A[i] = a[i] % mod;
}
void idft(poly &a, int n) {
    a.resize(1 << n), reverse(a.begin() + 1, a.end());
    dft(a, n);
    int inv = power(1 << n, mod - 2);
    REP(i, 1 << n) a[i] = (ll)a[i] * inv % mod;
}
poly FIX(poly a) {
    while (!a.empty() && !a.back()) a.pop_back();
    return a;
}
poly add(poly a, poly b, int op = 0) {
    a.resize(max(a.size(), b.size()));
    REP(i, b.size()) a[i] = fix(op ? a[i] + mod - b[i] : a[i] + b[i]);
    return FIX(a);
}
poly mul(poly a, poly b, int t = 1) {
    if (t == 1 && a.size() + b.size() <= 24) {
        poly c(a.size() + b.size(), 0);
        REP(i, a.size()) REP(j, b.size()) c[i + j] = (c[i + j] + (ll)a[i] * b[j]) % mod;
        return FIX(c);
    }
    int n = 1, aim = a.size() * t + b.size();
    while ((1 << n) <= aim) n++;
    dft(a, n), dft(b, n);
    if (t == 1)
        REP(i, 1 << n) a[i] = (ll)a[i] * b[i] % mod;
    else
        REP(i, 1 << n) a[i] = (ll)a[i] * a[i] % mod * b[i] % mod;
    idft(a, n), a.resize(aim);
    return FIX(a);
}
poly mul(poly a, int b) {
    REP(i, a.size()) a[i] = (ll)a[i] * b % mod;
    return FIX(a);
}

poly inv(poly a, int n) {  // a[0] != 0
    a.resize(n);
    poly b;
    if (n == 1) {
        b.pb(power(a[0], mod - 2));
        return b;
    }
    b = inv(a, n + 1 >> 1);
    b = add(mul(b, 2), mul(b, a, 2), 1);
```

```
344     return b.resize(n), b;
345 }
346
347 poly Der(poly a) {
348     REP(i, a.size() - 1) a[i] = (ll)(i + 1) * a[i + 1] % mod;
349     return a.pop_back(), a;
350 }
351 poly Int(poly a) {
352     static int inv[N];
353     inv[1] = 1;
354     a.pb(0);
355     rep(i, 2, a.size()) inv[i] = (ll)(mod - mod / i) * inv[mod % i] % mod;
356     per(i, a.size() - 1, 1) a[i] = (ll)a[i - 1] * inv[i] % mod;
357     return a[0] = 0, a;
358 }
359 poly Ln(poly a, int n) {  // a[0] = 1
360     a = mul(Der(a), inv(a, n)), a.resize(n - 1);
361     return FIX(Int(a));
362 }
363 poly Exp(poly a, int n) {  // a[0] = 0
364     a.resize(n);
365     poly b, one(1, 1);
366     if (n == 1)
367         return one;
368     b = Exp(a, n + 1 >> 1);
369     b = mul(b, add(add(a, Ln(b, n), 1), one));
370     return b.resize(n), b;
371 }
372
373 poly Div(poly a, poly b) {
374     poly c;
375     int n = a.size() - 1, m = b.size() - 1;
376     if (n < m)
377         return c;
378     reverse(a.begin(), a.end());
379     a.resize(n - m + 1);
380     reverse(b.begin(), b.end());
381     b.resize(n - m + 1);
382     c = mul(a, inv(b, n - m + 1));
383     c.resize(n - m + 1);
384     return reverse(c.begin(), c.end()), c;
385 }
386 poly Mod(poly a, poly b) { return FIX(add(a, mul(Div(a, b), b), 1)); }
387 inline int chk(int x) { return power(x, (mod - 1) / 2) == 1; }
388 inline int R() { return rand() % mod; }
389 inline pii mul(pii a, pii b, int w) {
390     return mp(((ll)a.fi * b.fi + (ll)a.se * b.se % mod * w) % mod, ((ll)a.fi * b.se + (
    ll)a.se * b.fi) % mod);
391 }
392 inline int Sqrt(int x) {
393     if (!chk(x))
394         return -1;
395     int a = R();
396     while (chk(((ll)a * a - x + mod) % mod)) a = R();
397     int w = ((ll)a * a - x + mod) % mod, p = (mod + 1) / 2;
398     pii res = mp(1, 0), t = mp(a, 1);
399     for (; p; p >>= 1, t = mul(t, t, w))
400         if (p & 1)
401             res = mul(res, t, w);
402     assert(!res.se);
403     return min(res.fi, mod - res.fi);
```

```
}
poly Sqrt(poly a, int n) {
    if (n == 1) {
        poly b(1, Sqrt(a[0]));
        return b;
    }
    a.resize(n);
    poly b = Sqrt(a, n + 1 >> 1);
    b = mul(add(b, mul(a, inv(b, n))), (mod + 1) / 2);
    return b.resize(n), b;
}
poly fastpow(poly a, ll k, int n) {
    a.resize(n), a = FIX(a);
    if (!a.size())
        return a;
    int st = 0, base = 0;
    while (!a[st]) ++st;
    if (st * k >= n)
        return a.resize(0), a;
    REP(i, a.size() - st) a[i] = a[i + st];
    if (st)
        a.resize(a.size() - st);
    base = a[0];
    ll inv = power(base, mod - 2);
    REP(i, a.size()) a[i] = a[i] * inv % mod;
    a = FIX(Exp(mul(Ln(a, n), k % mod), n));
    ;
    if (st) {
        reverse(a.begin(), a.end());
        a.resize(a.size() + st * k);
        reverse(a.begin(), a.end());
        a.resize(n), a = FIX(a);
    }
    base = power(base, k);
    REP(i, a.size()) a[i] = (ll)a[i] * base % mod;
    return FIX(a);
}
}  // namespace Poly
using namespace Poly;
int main() {
    int n = buf + 1, K = buf;
    poly a(n, 0), b;
    REP(i, n) a[i] = buf;
    b = add(a, Exp(Int(inv(Sqrt(a, n), n)), n), 1);
    b[0] = (b[0] + 2 - a[0] + mod) % mod;
    a = Ln(b, n);
    a[0]++;
    b = Der(fastpow(a, K, n));
    b.resize(n - 1);
    REP(i, n - 1) buf.out(b[i]);
    return 0;
}
```

4.1.8 Lagrange Polynomial

$$L(x) = \sum_{i=0}^{n} y_i \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

```cpp
//O(n^2)
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
typedef pair<int, int> P;
const int MAXN = 3005, mod = 998244353;
int exgcd(int a, int b, int &x, int &y) {
    int d = a;
    if(b != 0) {
        d = exgcd(b, a % b, y, x);
        y -= (a / b) * x;
    }
    else {
        x = 1; y = 0;
    }
    return d;
}
int inv(int a) {
    int x, y;
    exgcd(a, mod, x, y);
    return (x % mod + mod) % mod;
}
struct Lagrange {
    int n, a[MAXN][2];
    void init() {
        for(int i = 0; i <= n; i++) a[i][0] = a[i][1] = 0;
        n = 0;
        a[0][1] = 1;
    }
    int query(int x, int q = 0) {
        int res = 0;
        for(int i = n; i >= 0; i--) res = ((LL)res * x + a[i][q]) % mod;
        return res;
    }
    void update(int x, int y) {
        a[n][0] = 0;
        int v = (LL)(y - query(x) + mod) % mod * inv(query(x, 1)) % mod;
        for(int i = 0; i <= n; i++) a[i][0] = (a[i][0] + (LL)a[i][1] * v) % mod;
        a[++n][1] = 0;
        for(int i = n; i; i--) a[i][1] = (a[i - 1][1] + (LL)a[i][1] * (mod - x)) % mod;
        a[0][1] = (LL)a[0][1] * (mod - x) % mod;
    }
}p;
int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    int Q;
    cin >> Q;
    int op, x, y;
    p.n = 0;
    p.init();
    while(Q--) {
        cin >> op >> x;
        if(op == 1) {
            cin >> y;
            p.update(x, y);
        }
        else cout << p.query(x) << endl;
    }
    return 0;
}
//f(x) = y, 求f(z_i)
```

```cpp
#include <bits/stdc++.h>
using namespace std;
#define ll long long
int n, m;
vector<int> x, y, z;
namespace Poly{
    const int P = 998244353;
    vector<int> ans;// for Evaluate()
    vector<vector<int>> p;// for Evaluate() & Interpolate()
    inline int Pow(ll x, int y=P-2){ // x^y
        int ans=1;
        for(; y; y>>=1, x=x*x%P) if(y&1) ans=ans*x%P;
        return ans;
    }
    inline int Ge(int x){ int n=1; while(n<=x) n<<=1; return n;}
    inline int Mod(int x){ return x<P?x:x-P;}
    inline void NTT(vector<int> &f, int g, int n){
        f.resize(n);
        for(int i=0, j=0; i<n; ++i){
            if(i>j) swap(f[i], f[j]);
            for(int k=n>>1; (j^=k)<k; k>>=1);
        }
        vector<int> w(n>>1);
        for(int i=1; i<n; i<<=1){
            for(int j=w[0]=1, w0=(g==1?Pow(3, (P-1)/i/2):Pow(Pow(3, (P-1)/i/2))); j<i; ++j) w[j]=(ll)w[j-1]*w0%P;
            for(int j=0; j<n; j+=i<<1){
                for(int k=j; k<j+i; ++k){
                    int t=(ll)f[k+i]*w[k-j]%P;
                    f[k+i]=Mod(f[k]-t+P);
                    f[k]=Mod(f[k]+t);
                }
            }
        }
        if(g==-1) for(int i=0, I=Pow(n); i<n; ++i) f[i]=(ll)f[i]*I%P;
    }
    inline vector<int> Add(const vector<int> &f, const vector<int> &g){
        vector<int> ans=f;
        for(unsigned i=0; i<f.size(); ++i) (ans[i]+=g[i])%=P;
        return ans;
    }
    inline vector<int> Mul(const vector<int> &f, const vector<int> &g){// f*g
        vector<int> F=f, G=g;
        int p=Ge(f.size()+g.size()-2);
        NTT(F, 1, p), NTT(G, 1, p);
        for(int i=0; i<p; ++i) F[i]=(ll)F[i]*G[i]%P;
        NTT(F, -1, p);
        return F.resize(f.size()+g.size()-1), F;
    }
    inline vector<int> PolyInv(const vector<int> &f, int n=-1){// 1/f
        if(n==-1) n=f.size();
        vector<int> ans;
        if(n==1) return ans.push_back(Pow(f[0])), ans;
        ans=PolyInv(f, (n+1)/2);
        vector<int> tmp(&f[0], &f[0]+n);
        int p=Ge(n*2-2);
        NTT(tmp, 1, p), NTT(ans, 1, p);
        for(int i=0; i<p; ++i) ans[i]=(2-(ll)ans[i]*tmp[i]%P+P)*ans[i]%P;
        NTT(ans, -1, p);
        return ans.resize(n), ans;
    }
```

```
122     inline void PolyDiv(const vector<int> &a, const vector<int> &b, vector<int> &d,
        vector<int> &r){//a=d*b+r
123         if(b.size()>a.size()) return (void)(d.clear(), r=a);
124 
125         vector<int> A=a, B=b, iB;
126         int n=a.size(), m=b.size();
127         reverse(A.begin(), A.end()), reverse(B.begin(), B.end());
128         B.resize(n-m+1), iB=PolyInv(B, n-m+1);
129         d=Mul(A, iB);
130         d.resize(n-m+1), reverse(d.begin(), d.end());
131 
132         r=Mul(b, d);
133         for(int i=0; i<m-1; ++i) r[i]=(P+a[i]-r[i])%P;
134         r.resize(m-1);
135     }
136     inline vector<int> Derivative(const vector<int> &a){ // a'
137         vector<int> ans;
138         ans.resize(a.size()-1);
139         for(unsigned i=1; i<a.size(); ++i) ans[i-1]=(ll)a[i]*i%P;
140         return ans;
141     }
142     void Evaluate_Interpolate_Init(int l, int r, int t, const vector<int> &a){
143         if(l==r) return p[t].clear(), p[t].push_back(P-a[l]), p[t].push_back(1);
144         int mid=(l+r)/2, k=t<<1;
145         Evaluate_Interpolate_Init(l, mid, k, a), Evaluate_Interpolate_Init(mid+1, r, k
        |1, a);
146         p[t]=Mul(p[k], p[k|1]);
147     }
148     inline void Evaluate(int l, int r, int t, const vector<int> &f, const vector<int> &a
        ){
149         if(r-l+1<=512){
150             for(int i=l; i<=r; ++i){
151                 int x=0, j=f.size(), a1=a[i], a2=(ll)a[i]*a[i]%P, a3=(ll)a[i]*a2%P, a4=(
            ll)a[i]*a3%P, a5=(ll)a[i]*a4%P, a6=(ll)a[i]*a5%P, a7=(ll)a[i]*a6%P, a8=(ll)a[i]*a7%P
            ;
152                 while(j>=8)
153                 x=((ll)x*a8+(ll)f[j-1]*a7+(ll)f[j-2]*a6+(ll)f[j-3]*a5+(ll)f[j-4]*a4+(ll)
            f[j-5]*a3+(ll)f[j-6]*a2+(ll)f[j-7]*a1+f[j-8])%P, j-=8;
154                 while(j--) x=((ll)x*a[i]+f[j])%P;
155                 ans.push_back(x);
156             }
157             return;
158         }
159         vector<int> tmp;
160         PolyDiv(f, p[t], tmp, tmp);
161         Evaluate(l, (l+r)/2, t<<1, tmp, a), Evaluate((l+r)/2+1, r, t<<1|1, tmp, a);
162     }
163     inline vector<int> Evaluate(const vector<int> &f, const vector<int> &a, int flag=-1)
        {// f(a_i)
164         if(flag==-1) p.resize(a.size()<<2), Evaluate_Interpolate_Init(0, a.size()-1, 1,
        a);
165         ans.clear(), Evaluate(0, a.size()-1, 1, f, a);
166         return ans;
167     }
168     vector<int> Interpolate(int l, int r, int t, const vector<int> &x, const vector<int>
         &f){
169         if(l==r){
170             vector<int> ans;
171             return ans.push_back(f[l]), ans;
172         }
173         int mid=(l+r)/2, k=t<<1;
```

```
174         return Add(Mul(Interpolate(l, mid, k, x, f), p[k|1]), Mul(Interpolate(mid+1, r,
        k|1, x, f), p[k]));
175     }
176     inline vector<int> Interpolate(const vector<int> &x, const vector<int> &y){// (x_i,
        y_i)
177         int n=x.size();
178         p.resize(n<<2), Evaluate_Interpolate_Init(0, n-1, 1, x);
179         vector<int> f=Evaluate(Derivative(p[1]), x, 0);
180         for(int i=0; i<n; ++i) f[i]=(ll)y[i]*Pow(f[i])%P;
181         return Interpolate(0, n-1, 1, x, f);
182     }
183 }
184 using namespace Poly;
185 int main() {
186     cin >> n; x.resize(n), y.resize(n);
187     for(int i=0; i<n; ++i) cin >> x[i], cin >> y[i];
188     cin >> m, z.resize(m);
189     for(int i=0; i<m; ++i) cin >> z[i];
190     x=Evaluate(Interpolate(x, y), z);
191     for(int i:x) cout << i << " ";
192     return 0;
193 }
```

4.1.9 BM Alogrithm

```
1  #include<bits/stdc++.h>
2  using namespace std;
3  #define rep(i,a,n) for (int i=a;i<n;i++)
4  #define per(i,a,n) for (int i=n-1;i>=a;i--)
5  #define pb push_back
6  #define mp make_pair
7  #define all(x) (x).begin(),(x).end()
8  #define fi first
9  #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector<int> VI;
12 typedef long long ll;
13 typedef pair<int,int> PII;
14 const ll mod=1000000007;
15 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b;b>>=1){if(b&1)res=res*a%mod;
       a=a*a%mod;}return res;}
16 // head
17 namespace linear_seq {
18     const int N=10010;
19     ll res[N],base[N],_c[N],_md[N];
20
21     vector<int> Md;
22     void mul(ll *a,ll *b,int k) {
23         rep(i,0,k+k) _c[i]=0;
24         rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
25         for (int i=k+k-1;i>=k;i--) if (_c[i])
26             rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
27         rep(i,0,k) a[i]=_c[i];
28     }
29     int solve(ll n,VI a,VI b) { // a 系数 b 初值 b[n+1]=a[0]*b[n]+...
30 //        printf("%d\n",SZ(b));
31         ll ans=0,pnt=0;
32         int k=SZ(a);
33         assert(SZ(a)==SZ(b));
34         rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
```

```
        Md.clear();
        rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
        rep(i,0,k) res[i]=base[i]=0;
        res[0]=1;
        while ((1ll<<pnt)<=n) pnt++;
        for (int p=pnt;p>=0;p--) {
            mul(res,res,k);
            if ((n>>p)&1) {
                for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
                rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
            }
        }
        rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
        if (ans<0) ans+=mod;
        return ans;
    }
    VI BM(VI s) {
        VI C(1,1),B(1,1);
        int L=0,m=1,b=1;
        rep(n,0,SZ(s)) {
            ll d=0;
            rep(i,0,L+1) d=(d+(ll)C[i]*s[n-i])%mod;
            if (d==0) ++m;
            else if (2*L<=n) {
                VI T=C;
                ll c=mod-d*powmod(b,mod-2)%mod;
                while (SZ(C)<SZ(B)+m) C.pb(0);
                rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
                L=n+1-L; B=T; b=d; m=1;
            } else {
                ll c=mod-d*powmod(b,mod-2)%mod;
                while (SZ(C)<SZ(B)+m) C.pb(0);
                rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
                ++m;
            }
        }
        return C;
    }
    int gao(VI a,ll n) {
        VI c=BM(a);
        c.erase(c.begin());
        rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
        return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
    }
};

int main() {
    while (~scanf("%d",&n)) {
        vector<int>v;
        v.push_back(1);
        v.push_back(2);
        v.push_back(4);
        v.push_back(7);
        v.push_back(13);
        v.push_back(24);
        //VI{1,2,4,7,13,24}
        printf("%d\n",linear_seq::gao(v,n-1));
    }
}
```

4.2 Math Theory

4.2.1 Inverse

```
//O(logn)求n的逆元
const int mod = 1e6 + 3;
int exgcd(int a, int b, int &x, int &y) {
    int d = a;
    if(b != 0) {
        d = exgcd(b, a % b, y, x);
        y -= (a / b) * x;
    }
    else {
        x = 1; y = 0;
    }
    return d;
}
int inverse(int a) {
    int x, y;
    exgcd(a, mod, x, y);
    return (x % mod + mod) % mod;
}
int inverse(int a) { return qpow(a, mod - 2); }
//O(n)求1~n的逆元
int inv[MAXN];
void init() {
    inv[0] = inv[1] = 1;
    for(int i = 2; i < MAXN; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
    mod;
}
```

4.2.2 Lucas

```
//mod很小可以预处理逆元的情况
void init() {
    fac[0] = 1;
    for(int i = 1; i < mod; i++) fac[i] = (long long)fac[i - 1] * i % mod;
    inv[0] = inv[1] = 1;
    for(int i = 2; i < mod; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
    mod;
    for(int i = 1; i < mod; i++) inv[i] = (long long)inv[i] * inv[i - 1] % mod;
}
int C(int a, int b) {
    if(b > a) return 0;
    if(a < mod) return (long long)fac[a] * inv[b] % mod * inv[a - b] % mod;
    return (long long)C(a / mod, b / mod) * C(a % mod, b % mod) % mod;
}
//mod过大不能预处理逆元的情况
LL qpow(LL x, LL y) {
    LL res = 1;
    while(y) {
        if(y & 1) res = res * x % mod;
        x = x * x % mod;
        y >>= 1;
    }
    return res;
}
LL C(LL a, LL b) {
    if(b > a) return 0;
```

```
    if(b > a - b) b = a - b;
    LL s1 = 1, s2 = 1;
    for(LL i = 0; i < b; i++) {
        s1 = s1 * (a - i) % mod;
        s2 = s2 * (i + 1) % mod;
    }
    return s1 * qpow(s2, mod - 2) % mod;
}
LL lucas(LL a, LL b) {
    if(a < mod) return C(a, b);
    return lucas(a / mod, b / mod) * C(a % mod, b % mod);
}
```

4.2.3 CRT && exCRT

$$x \equiv a_i \pmod{m_i}$$

```
namespace CRT {
    LL m[MAXN], a[MAXN];
    LL exgcd(LL _a, LL _b, LL &x, LL &y) {
        if(!_b) {
            x = 1; y = 0;
            return _a;
        }
        LL d = exgcd(_b, _a % _b, y, x);
        y -= (_a / _b) * x;
        return d;
    }
    LL crt(int n) {
        LL M = 1, tmp, res = 0, x, y;
        for(int i = 1; i <= n; i++) M *= m[i];
        for(int i = 1; i <= n; i++) {
            tmp = M / m[i];
            exgcd(tmp, m[i], x, y);
            x = (x + m[i]) % m[i];
            res = (a[i] * x % M * tmp % M + res) % M;
        }
        return res;
    }
}
namespace EXCRT {
    LL m[MAXN], a[MAXN];
    LL exgcd(LL _a, LL _b, LL &x, LL &y) {
        if(!_b) {
            x = 1; y = 0;
            return _a;
        }
        LL d = exgcd(_b, _a % _b, y, x);
        y -= (_a / _b) * x;
        return d;
    }
    LL excrt(int n) {
        LL M = m[1], A = a[1], x, y, d, tmp;
        for(int i = 2; i <= n; i++) {
            d = exgcd(M, m[i], x, y);
            if((A - a[i]) % d) return -1; //No solution
            tmp = M / d; M *= m[i] / d;
            y = (A - a[i]) / d % M * y % M;
```

```
            y = (y + tmp) % tmp;
            A = (m[i] % M * y % M + a[i]) % M;
            A = (A + M) % M;
        }
        return A;
    }
    LL inv(LL _a, LL _b) {
        LL x, y;
        exgcd(_a, _b, x, y);
        return (x % _b + _b) % _b;
    }
    LL excrt(int n) {
        LL M = m[1], A = a[1], x, y, d, c, tmp;
        for(int i = 2; i <= n; i++) {
            d = exgcd(M, m[i], x, y);
            c = a[i] - A;
            if(c % d) return -1;
            c = (c % m[i] + m[i]) % m[i];
            M /= d; m[i] /= d;
            c = c / d * inv(M % m[i], m[i]) % m[i];
            tmp = M;
            M *= m[i] * d;
            A = (c * tmp % M * d % M + A) % M;
        }
        return A;
    }
    //当a[i] 一开始就是负数时，转成正数a[i]=(a[i]%m[i]+m[i])%m[i];
}
```

4.2.4 BSGS

```
const int MOD = 76543;
int hs[MOD + 5], head[MOD + 5], nxt[MOD + 5], id[MOD + 5], ecnt;
void insert(int x, int y) {
    int k = x % MOD;
    hs[ecnt] = x, id[ecnt] = y, nxt[ecnt] = head[k], head[k] = ecnt++;
}
int find(int x) {
    int k = x % MOD;
    for(int i = head[k]; i; i = nxt[i])
        if(hs[i] == x)
            return id[i];
    return -1;
}
int BSGS(int a, int b, int c){
    memset(head, 0, sizeof head); ecnt = 1;
    if(b == 1) return 0;
    int m = sqrt(c * 1.0), j;
    LL x = 1, p = 1;
    for(int i = 0; i < m; i++, p = p * a % c)
        insert(p * b % c, i);
    for(LL i = m; ;i += m){
        if((j = find(x = x * p % c)) != -1) return i - j;
        if(i > c) break;
    }
    return -1;
}
```

4.2.5 Miller-Rabin && PollardRho

```
LL ksc(LL a,LL n,LL mod){
    LL ret=0;
    for(;n;n>>=1){
        if(n&1){ret+=a;if(ret>=mod)ret-=mod;}
        a<<=1;if(a>=mod)a-=mod;
    }
    return ret;
}
LL ksm(LL a,LL n,LL mod){
    LL ret = 1;
    for(;n;n>>=1){
        if(n&1)ret=ksc(ret,a,mod);
        a=ksc(a,a,mod);
    }
    return ret;
}
int millerRabin(LL n){
    if(n<2 || (n!=2 && !(n&1)))return 0;
    LL d=n-1;for(;!(d&1);d>>=1);
    for(int i=0;i<20;++i){
        LL a=rand()%(n-1)+1;
        LL t=d,m=ksm(a,d,n);
        for(;t!=n-1 && m!=1 && m!=n-1;m=ksc(m,m,n),t<<=1);
        if(m!=n-1 && !(t&1)) return 0;
    }
    return 1;
}
LL cnt,fact[100];
LL gcd(LL a,LL b){return !b?a:gcd(b,a%b);}
LL pollardRho(LL n, int a){
    LL x=rand()%n,y=x,d=1,k=0,i=1;
    while(d==1){
        ++k;
        x=ksc(x,x,n)+a;if(x>=n)x-=n;
        d=gcd(x>y?x-y:y-x,n);
        if(k==i){y=x;i<<=1;}
    }
    if(d==n)return pollardRho(n,a+1);
    return d;
}
void findfac(LL n){
    if(millerRabin(n)){fact[++cnt]=n;return;}
    LL p=pollardRho(n,rand()%(n-1)+1);
    findfac(p);
    findfac(n/p);
}
```

4.2.6 $\varphi(n)$

```
int phi(int x) {
    int res = x;
    for(int i = 2; i * i <= x; i++) {
        if(x % i == 0) {
            res = res / i * (i - 1);
            while(x % i == 0) x /= i;
        }
    }
```

```
    if(x > 1) res = res / x * (x - 1);
    return res;
}
```

4.2.7 Euler Sieve

```
int prime[MAXN], cnt, phi[MAXN], mu[MAXN];
bool isp[MAXN];

int min_pow[MAXN];   //最小质因子最高次幂
int min_sum[MAXN];   //1+p+p^2+...+p^k
int div_sum[MAXN];   //约数和

int min_index[MAXN]; //最小质因子的指数
int div_num[MAXN];   //约数个数
void Euler(int n) {
    mu[1] = phi[1] = div_num[1] = div_sum[1] = 1;
    for(int i = 2; i <= n; i++) {
        if(!isp[i]) {
            prime[++cnt] = min_pow[i] = i;
            phi[i] = i - 1;
            mu[i] = -1;
            min_index[i] = 1; div_num[i] = 2;
            div_sum[i] = min_sum[i] = i + 1;
        }
        for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
            isp[i * prime[j]] = 1;
            if(i % prime[j] == 0) {
                phi[i * prime[j]] = phi[i] * prime[j];
                mu[i * prime[j]] = 0;

                min_index[i * prime[j]] = min_index[i] + 1;
                div_num[i * prime[j]] = div_num[i] / (min_index[i] + 1) * (min_index[i *
    prime[j]] + 1);

                min_sum[i * prime[j]] = min_sum[i] + min_pow[i] * prime[j];
                div_sum[i * prime[j]] = div_sum[i] / min_sum[i] * min_sum[i * prime[j]];
                min_pow[i * prime[j]] = min_pow[i] * prime[j];
                break;
            }
            phi[i * prime[j]] = phi[i] * (prime[j] - 1);
            mu[i * prime[j]] = -mu[i];

            div_num[i * prime[j]] = div_num[i] << 1;
            min_index[i * prime[j]] = 1;

            div_sum[i * prime[j]] = div_sum[i] * (prime[j] + 1);
            min_pow[i * prime[j]] = prime[j];
            min_sum[i * prime[j]] = prime[j] + 1;
        }
    }
}
```

4.2.8 DuJiao Sieve

$$\sum_{i=1}^{n} \varphi(i)$$

```cpp
vector<int> prime;
int phi[MAXN], P[MAXN];
bool isp[MAXN];
unordered_map<LL, int> mp;
void Euler(int n) {
    phi[1] = 1;
    for(int i = 2; i <= n; i++) {
        if(!isp[i]) {
            prime.push_back(i);
            phi[i] = i - 1;
        }
        for(auto x : prime) {
            if(i * x > n) break;
            isp[i * x] = 1;
            if(i % x == 0) {
                phi[i * x] = phi[i] * x;
                break;
            }
            phi[i * x] = phi[i] * (x - 1);
        }
    }
    for(int i = 1; i <= n; i++) P[i] = (P[i - 1] + phi[i]) % mod;
}
LL cal(LL n) {
    if(n < MAXN) return P[n];
    if(mp.count(n)) return mp[n];
    LL res = 0;
    for(LL i = 2, last; i <= n; i = last + 1) {
        last = n / (n / i);
        res += (last - i + 1) % mod * cal(n / i) % mod;
        res %= mod;
    }
    mp[n] = ((__int128)n * (n + 1) / 2 % mod + mod - res) % mod;
    return mp[n];
}
```

$$\sum_{i=1}^{n} \mu(i)$$

```cpp
LL cal(LL n) {
    if(n < MAXN) return M[n];
    if(mp.count(n)) return mp[n];
    LL res = 0;
    for(LL i = 2, last; i <= n; i = last + 1) {
        last = n / (n / i);
        res += (last - i + 1) * cal(n / i);
    }
    mp[n] = 1 - res;
    return 1 - res;
}
```

4.2.9 Min_25 Sieve

思路为把结果分为 i 为质数的和，i 为合数的和，i=1 的和
g(n, j) 表示从 1 累加到 n 的 f(i)，其中的 i 满足要么 i 自己是质数，要么 i 的最小质因子大于第 j 个质数
要求 $f(p)(p\ is\ prime)$ 可被多项式表示,$f(p^k)$ 可快速计算
分别计算多项式 x^0, x^1, x^2 的 g(n,j) 和 h(n,j)
h 由欧拉筛计算，递归起点 g(n, 0)

公式

$$\sum_{i}^{n} f(i)$$

$$g(n,j) = \begin{cases} g(n,j-1) & p_j^2 > n \\ g(n,j-1) - f(p_j)[g(\frac{n}{p_j}, j-1) - g(p_j - 1, j-1)] & p_j^2 \le n \end{cases}$$

其中

$$g(p_j - 1, j-1) = \sum_{i}^{j-1} f(p_i)$$

可用 h 表示

S(n,j) 表示从 1 累加到 n 的 f(i)，同样的，i 满足要么 i 自己是质数，要么 i 的最小质因子大于第 j 个质数和 g 顺序相反，最后答案为 S(n, 1)，递推式为

$$S(n,j) = g(n,|P|) - \sum_{i=1}^{j} f(p_i) + \sum_{k \ge j} \sum_{p_k^{e+1} < n} (f(p_k^e) S(\frac{n}{p_k^e}, k+1) + f(p_k^{e+1}))$$

前一部分 $g(n,|P|) - \sum_{i=1}^{j} f(p_i)$ 是相应的质数部分和。

例题：欧拉函数前缀和

$$\sum_{i=1}^{n} \varphi(i)$$

$$g_{k,n} \text{ and } h_{k,n} \text{ Count}$$

$$\sum_{i=1}^{n} i^k$$

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
const int MAXN = 1e6 + 5, mod = 1e9 + 7;
const int inv2 = (mod + 1) / 2, inv6 = (mod + 1) / 6;
int prime[MAXN], isp[MAXN], cnt;
LL g[3][MAXN << 1], h[3][MAXN << 1];
LL w[MAXN << 1];
int id1[MAXN], id2[MAXN];
inline int MOD(LL x) { return x >= mod ? x - mod : x; }
//inline int MOD(LL x) { return x % mod; }
inline int add(LL x, LL y) { return MOD(MOD(x) + MOD(y)); }
void Euler(int n) {
    for(int i = 2; i <= n; i++) {
        if(!isp[i]) {
            prime[++cnt] = i;
            h[0][cnt] = h[0][cnt - 1] + 1;
            h[1][cnt] = add(h[1][cnt - 1], i);
            h[2][cnt] = add(h[2][cnt - 1], (LL)i * i % mod);
        }
        for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
            isp[i * prime[j]] = 1;
            if(i % prime[j] == 0) {
                break;
            }
        }
```

```
    }
}
LL n;
int sz, m;
inline int id(LL x) {
    return x <= sz ? id1[x] : id2[n / x];
}
//f(p ^ k)
inline int f(int p, LL pk) {
    return pk / p * (p - 1) % mod;
}
LL S(LL x, int y) {
    if(x <= 1 || prime[y] > x) return 0;
    //G(x) - H(j - 1) (first part)
    LL res = add(add(g[1][id(x)], mod - g[0][id(x)]), mod - add(h[1][y - 1], mod - h[0][y - 1]));
    for(int j = y, k = 1; j <= cnt && (LL)prime[j] * prime[j] <= x; j++, k = 1) {
        for(LL pk = prime[j]; pk * prime[j] <= x; pk *= prime[j], k++) {
            res = add(res, S(x / pk, j + 1) * f(prime[j], pk) % mod + f(prime[j], pk * prime[j]));
        }
    }
    return res;
}
int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    cin >> n;
    sz = sqrt(n);
    Euler(sz);
    for(LL i = 1, last, t; i <= n; i = last + 1) {
        last = n / (n / i);
        w[++m] = n / i, t = n / i % mod;
        w[m] <= sz ? id1[w[m]] = m : id2[last] = m;
        g[0][m] = MOD(t + mod - 1);
        g[1][m] = add(t * (t + 1) % mod * inv2 % mod, mod - 1);
        g[2][m] = add((2 * t + 1) % mod * t * (t + 1) % mod * inv6 % mod, mod - 1);
    }
    for(int j = 1; j <= cnt; j++) {
        for(int i = 1; i <= m && (LL)prime[j] * prime[j] <= w[i]; i++) {
            g[0][i] = MOD(g[0][i] + mod - (g[0][id(w[i] / prime[j])] - h[0][j - 1]));
            g[1][i] = MOD(g[1][i] + mod - ((LL)prime[j] * MOD(g[1][id(w[i] / prime[j])] + mod - h[1][j - 1]) % mod));
            g[2][i] = MOD(g[2][i] + mod - ((LL)prime[j] * prime[j] % mod * MOD(g[2][id(w[i] / prime[j])] + mod - h[2][j - 1]) % mod));
        }
    }
    //S(n, 1) + F(1);
    LL ans = MOD(S(n, 1) + 1);
    cout << ans << endl;
    return 0;
}
```

$$\sum_{i=1}^{n} \mu(i)$$

```
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
const int MAXN = 1e6 + 5;
int prime[MAXN], isp[MAXN], cnt;
LL g[3][MAXN << 1], h[3][MAXN << 1];
```

```
LL w[MAXN << 1];
int id1[MAXN], id2[MAXN];
void Euler(int n) {
    for(int i = 2; i <= n; i++) {
        if(!isp[i]) {
            prime[++cnt] = i;
            h[0][cnt] = h[0][cnt - 1] + 1;
        }
        for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
            isp[i * prime[j]] = 1;
            if(i % prime[j] == 0) {
                break;
            }
        }
    }
}
LL a, b;
LL n;
int sz, m;
inline int id(LL x) {
    return x <= sz ? id1[x] : id2[n / x];
}
//f(p ^ k)
inline int f(int p, int k) {
    return k == 1 ? -1 : 0;
}
LL S(LL x, int y) {
    if(x <= 1 || prime[y] > x) return 0;
    //g(x) - h(j - 1)
    LL res = - g[0][id(x)] + h[0][y - 1];
    for(int j = y, k = 1; j <= cnt && (LL)prime[j] * prime[j] <= x; j++, k = 1) {
        for(LL pk = prime[j]; pk * prime[j] <= x; pk *= prime[j], k++) {
            res += S(x / pk, j + 1) * f(prime[j], k) + f(prime[j], k + 1);
        }
    }
    return res;
}
LL cal(LL x) {
    n = x;
    m = 0;
    sz = sqrt(n);
    for(LL i = 1, last, t; i <= n; i = last + 1) {
        last = n / (n / i);
        w[++m] = n / i, t = n / i;
        w[m] <= sz ? id1[w[m]] = m : id2[last] = m;
        g[0][m] = t - 1;
    }
    for(int j = 1; j <= cnt; j++) {
        for(int i = 1; i <= m && (LL)prime[j] * prime[j] <= w[i]; i++) {
            g[0][i] = g[0][i] - (g[0][id(w[i] / prime[j])] - h[0][j - 1]);
        }
    }
    return S(x, 1) + 1;
}
int main() {
    ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
    cin >> a >> b;
    Euler(sqrt(b));

    //S(n, 1) + F(1);
    cout << cal(b) - cal(a - 1) << endl;
```

```
    return 0;
}
```

4.2.10 Möbius Inversion

$$\sum_{i}^{n}\sum_{j}^{m} lcm(i,j) \pmod p$$

```
int mu[MAXN], prime[MAXN], sum[MAXN], cnt;
bool isp[MAXN];
void getmu(int n) {
    mu[1] = 1;
    for(int i = 2; i <= n; i++) {
        if(!isp[i]) {
            mu[i] = -1;
            prime[++cnt] = i;
        }
        for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
            isp[i * prime[j]] = 1;
            if(i % prime[j] == 0) {
                mu[i * prime[j]] = 0;
                break;
            }
            mu[i * prime[j]] = -mu[i];
        }
    }
}
ll n, m, ans;
ll query(ll x, ll y) { return (x * (x + 1) / 2 % mod) * (y * (y + 1) / 2 % mod) % mod; }
ll F(ll x, ll y) {
    ll res = 0, last;
    for(ll i = 1; i <= min(x, y); i = last + 1) {
        last = min(x / (x / i), y / (y / i));
        res = (res + (sum[last] - sum[i - 1]) * query(x / i, y / i) % mod) % mod;
    }
    return res;
}
int main() {
    cin>>n>>m;
    getmu(min(n, m));
    for(ll i = 1; i <= min(n, m); i++) sum[i] = (sum[i - 1] + (i * i * mu[i]) % mod) %
    mod;
    ll last;
    for(ll d = 1; d <= min(n, m); d = last + 1) {
        last = min(n / (n / d), m / (m / d));
        ans = (ans + (last - d + 1) * (d + last) / 2 % mod * F(n / d, m / d) % mod) %
    mod;
    }
    ans = (ans + mod) % mod;
    cout<<ans<<endl;
    return 0;
}
```

4.2.11 Primitive Root

简易版如果模 m 有原根，则有 $\varphi(\varphi(m))$ 个原根当 d 遍历 $\varphi(m)$ 的简化剩余系时，g^d 遍历完 m 的全部原根

```cpp
const int MAXN = ;
LL qpow(LL a, LL b, LL mod) {
    LL res = 1;
    for (;b; b >>= 1) {
        if(b&1) res = res * a % mod;
        a = a * a % mod;
    }
    return res;
}
LL phi(LL x) {
    LL res = x;
    for(int i = 2; i <= x / i; i++) {
        if(x % i == 0) {
            res = res / i * (i - 1);
            while(x % i == 0) x /= i;
        }
    }
    if(x > 1) res = res / x * (x - 1);
    return res;
}
int has_primitive_root(int p) {
    if (p == 4) return 1;
    if (p % 2 == 0) p /= 2;
    if (p % 2 == 0) return false;
    for (int i = 2; i <= p/i; i++)
    if (p % i == 0) {
        while(p % i == 0) p /= i;
        return p == 1 ? i : 0;
    }
    return p;
}
//int indg[MAXN];
int get_g_init(LL p) {
    //p : 2 or 4 or (p**n) or (2 * p**n); else return -1;
    if(p == 2) {puts("1");return 1;}
    if(p == 4) {puts("3"); return 3;}
    if(!has_primitive_root(p)) {puts("-1"); return -1;}
    vector<int> p_fact;
    LL p_phi = phi(p), tp = p_phi ;
    for(int i = 2, in = sqrt(p) + 0.5; i <= in; i++)
    if(tp % i == 0) {
        p_fact.push_back(p_phi / i);
        while(tp % i == 0) tp /= i;
    }
    if(tp != 1) p_fact.push_back(p_phi / tp);
    int g = 1;
    for (bool fg = 0; !fg ;) {
        fg = 1; g++;
        if(qpow(g, p_phi, p) != 1) {fg = 0; continue;}
        for(auto it : p_fact)
            if(qpow(g, it, p) == 1) {fg = 0; break;}
    }
    //for(int i = 0, tg = 1; i < p_phi; i++, tg = tg * g % p) indg[tg] = i;
    /*vector<int> fac;
    for(int d = 1, tg = g; d < p_phi; d++, tg = tg * g % p)
        if(__gcd((LL)d, p_phi) == 1) fac.push_back(tg);
    sort(fac.begin(), fac.end());
    int ed = fac.back();
    for(auto it : fac)
        printf("%d%c", it, it != ed ? ' ':'\n');*/
    return g;
```

```
62 }
```

5 Geometry

5.1 Commonly Definition and Functions

5.1.1 Const and Functions

```
namespace CG{
    #define Point Vector
    const double pi=acos(-1.0);
    const double inf=1e100;
    const double eps=1e-9;
    template <typename T> inline T Abs(T x){return x>0?x:-x;}
    template <typename T> inline bool operator == (T x,T y){return Abs(x-y)<eps;}
    int sgn(double x){
        if (Abs(x)<eps) return 0;
        if (x>0) return 1;
        else return -1;
    }
}
```

5.1.2 Point Definition

```
namespace CG{
    struct Point{
        double x,y;
        Point(double x=0,double y=0):x(x),y(y){}
    };
    Vector operator + (const Vector a,const Vector b){return Vector(a.x+b.x,a.y+b.y);}
    Vector operator - (const Vector a,const Vector b){return Vector(a.x-b.x,a.y-b.y);}
    Vector operator * (const Vector a,const double k){return Vector(a.x*k,a.y*k);}
    Vector operator / (const Vector a,const double k){return Vector(a.x/k,a.y/k);}
    bool operator < (const Vector a,const Vector b) {return a.x==b.x?a.y<b.y:a.x<b.x;}
    bool operator == (const Vector a,const Vector b) {return a.x==b.x && a.y==b.y;}
    double Dot(const Vector a,const Vector b){return a.x*b.x+a.y*b.y;}
    double Cross(const Vector a,const Vector b){return a.x*b.y-a.y*b.x;}
    double mult_Cross(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.
    y-c.y)-(b.x-c.x)*(a.y-c.y);}
    double mult_Dot(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.x-
    c.x)+(a.y-c.y)*(b.y-c.y);}
    double Norm(const Vector a){return sqrt(Dot(a,a));}
    double Angle(const Vector a,const Vector b){return acos(Dot(a,b)/Norm(a)/Norm(b));}
    Vector Rotate(const Vector a,const double theta){return Vector(a.x*cos(theta)-a.y*
    sin(theta),a.x*sin(theta)+a.y*cos(theta));}
    bool ToLeftTest(const Vector a,const Vector b){return Cross(a,b)<0;}
    double DisPP(const Vector a,const Vector b){return sqrt((a.x-b.x)*(a.x-b.x)+(a.y-b.y
    )*(a.y-b.y));}
}
```

5.1.3 Line Definition

```
namespace CG{
    struct Line{
        Point p0,v,p1;
        double t,theta;
        Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),v(_v),t(_t){p1=p0+v*t; theta=
    atan2(v.y,v.x);}
```

```
        // Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),p1(_v){v=(p1-p0)/t; theta=atan2(v.y,v.x);}
    };
    bool operator < (const Line n,const Line m) {return n.theta<m.theta;}
    Point GetIntersection(const Line n,const Line m){return n.p0+n.v*Cross(m.v,(n.p0-m.p0))/Cross(n.v,m.v);}
    bool OnLine(const Vector a,const Line l){return Cross(l.p0-a,l.p1-a)==0;}
    bool OnSegment(const Point a,const Line l){return sgn(Cross(l.p0-a,l.p1-a))==0 && sgn(Dot(l.p0-a,l.p1-a))<0;}
    double DisPL(const Point a,const Line l){return Abs(Cross(l.p1-l.p0,a-l.p0)/Norm(l.p1-l.p0));}
    double DisPS(const Point a,const Line l){
        if (l.p0==l.p1) return Norm(a-l.p0);
        Vector v1=l.p1-l.p0,v2=a-l.p0,v3=a-l.p1;
        if (sgn(Dot(v1,v2))<0) return Norm(v2);
        if (sgn(Dot(v1,v3))>0) return Norm(v3);
        return DisPL(a,l);
    }
    Point GetProjection(const Point a,const Line l){
        Vector v=l.p1-l.p0;
        return l.p0+v*(Dot(v,a-l.p0)/Dot(v,v));
    }
    bool SegmentIntersection(const Line n,const Line m,bool p){
        double c1=Cross(n.p1-n.p0,m.p0-n.p0);
        double c2=Cross(n.p1-n.p0,m.p1-n.p0);
        double c3=Cross(m.p1-m.p0,n.p0-m.p0);
        double c4=Cross(m.p1-m.p0,n.p1-m.p0);
        if (p){
            if (!sgn(c1) || !sgn(c2) || !sgn(c3) || !sgn(c4)){
                return OnSegment(n.p0,m) || OnSegment(n.p1,m) || OnSegment(m.p0,n) || OnSegment(m.p0,m);

            }
        }
        return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);
    }
}
```

5.1.4 Get Area

```
namespace CG{
    double GetArea(Point *p,int n){
        double area=Cross(p[n],p[1]);
        for (int i=2;i<=n;i++) area+=0.5*Cross(p[i-1],p[i]);
        return Abs(area);
    }
}
```

5.1.5 Get Circumference

```
namespace CG{
    double GetCircumference(Point *p,int n){
        double Circumference=DisPP(p[n],p[1]);
        for (int i=2;i<=n;i++) Circumference+=DisPP(p[i-1],p[i]);
        return Circumference;
    }
}
```

5.1.6 Anticlockwise Sort

```
namespace CG{
    \\p为一个凸包,只是不知其点集是否为逆时针
    void clockwise_sort(Point *p,int n){
        for(int i=0;i<n-2;i++){
            double tmp = mult_Cross(p[i+1],p[i+2],p[i]);
            if(tmp>0) return;
            else if(tmp<0){
                reverse(p,p+n);
                return;
            }
        }
    }
}
```

5.2 Convex Hull

5.2.1 Get Convex Hull

```
namespace CG{
    Point p[MAXN],s[MAXN]; // both based from 0
    int ConvexHull(Point *p,int n,Point *s){
        sort(p,p+n,cmp); //x从小到大,y从小到大;
        int m=0;
        for (int i=0;i<n;i++){
            for (;m>=2 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
            s[m++]=p[i];
        }
        int k=m;
        for (int i=n-2;i;i--){
            for (;m>=k+1 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
            s[m++]=p[i];
        }
        return m-1;
    }
}
```

5.2.2 Point in Convex Hull

```
namespace CG{
    bool PointInConvexHull(Point A){
        int l=1,r=tot-2,mid;
        while(l<=r){
            mid=(l+r)>>1;
            double a1=Cross(p[mid]-p[0],A-p[0]);
            double a2=Cross(p[mid+1]-p[0],A-p[0]);
            if(a1>=0 && a2<=0){
                if(Cross(p[mid+1]-p[mid],A-p[mid])>=0) return true;
                return false;
            }
            else if(a1<0) r=mid-1;
            else l=mid+1;
        }
        return false;
    }
}
```

5.3 Minkowski Sum

```
namespace CG{
    void Minkowski(Point *C1,int n,Point *C2,int m){
        for(int i=1;i<=n;i++) s1[i]=C1[i]-C1[i-1];
        for(int i=1;i<=m;i++) s2[i]=C2[i]-C2[i-1];
        A[tot=1]=C1[1]+C2[1];
        int p1=1,p2=1;
        while (p1<=n && p2<=m) ++tot,A[tot]=A[tot-1]+(s1[p1]*s2[p2]>=0?s1[p1++]:s2[p2
    ++]);
        while (p1<=n) ++tot,A[tot]=A[tot-1]+s1[p1++];
        while (p2<=m) ++tot,A[tot]=A[tot-1]+s2[p2++];
        tot=ConvexHull(A,tot);
    }
}
```

5.4 Rotating Calipers

5.4.1 The Diameter of Convex Hull

```
namespace CG{
    double RotatingCalipers(Point *p,int n){
        double dis=0;
        for(int i=0,j=2;i<n;++i){
            while (abs(Cross(p[i+1]-p[i],p[j]-p[i]))<abs(Cross(p[i+1]-p[i],p[j+1]-p[i]))
    ) j=(j+1)%n;
            dis=max(dis,max(DisPP(p[j],p[i]),DisPP(p[j],p[i+1])));
        }
        return dis;
    }
}
```

5.4.2 The Min Distance Bewteen two Convex Hull

```
namespace CG{
    ///点c到线段ab的最短距离
    double GetDist(Point a,Point b,Point c){
        if(dis(a,b)<esp) return dis(b,c); ///a,b是同一个点
        if(mult_Dot(b,c,a)<-esp) return dis(a,c); ///投影
        if(mult_Dot(a,c,b)<-esp) return dis(b,c);
        return fabs(mult_Cross(b,c,a)/dis(a,b));

    }
    ///求一条线段ab的两端点到另外一条线段bc的距离，反过来一样，共4种情况
    double MinDist(Point a,Point b,Point c,Point d){
        return min(min(GetDist(a,b,c),GetDist(a,b,d)),min(GetDist(c,d,a),GetDist(c,d,b))
    );
    }
    double RotatingCalipers(Point *p,int n,Point *q,int m){
        int yminP = 0,ymaxQ=0;
        for(int i=1;i<n;i++){  ///找到点集p组成的凸包的左下角
            if(p[i].y<p[yminP].y||(p[i].y==p[yminP].y)&&(p[i].x<p[yminP].x)) yminP = i;
        }
        for(int i=1;i<m;i++){  ///找到点集q组成的凸包的右上角
            if(q[i].y>q[ymaxQ].y||(q[i].y==q[ymaxQ].y)&&(q[i].x>q[ymaxQ].x)) ymaxQ = i;
        }
        double ans = DisPP(p[yminP],q[ymaxQ]); ///距离(yminP,ymaxQ)维护为当前最小值。
```

```
        for(int i=0;i<n;i++){
            double tmp;
            while(tmp=(mult_Cross(q[ymaxQ+1],p[yminP],p[yminP+1])-mult_Cross(q[ymaxQ],p[yminP],p[yminP+1]))>esp)
                ymaxQ = (ymaxQ+1)%m;
            if(tmp<-esp) ans = min(ans,GetDist(p[yminP],p[yminP+1],q[ymaxQ]));
            else ans=min(ans,MinDist(p[yminP],p[yminP+1],q[ymaxQ],q[ymaxQ+1]));
            yminP = (yminP+1)%n;
        }
        return ans;
    }
}
```

5.5 Half Plane Intersection

```
namespace CG{
    void HalfPlaneIntersection(Line l[],int n){
        deque <Point> p;
        sort(l+1,l+1+n);
        deque <Line> q;
        q.push_back(l[1]);
        for (int i=2;i<=n;i++){
            for (;!p.empty() && !ToLeftTest(p.back()-l[i].p0,l[i].v);q.pop_back(),p.pop_back());
            for (;!p.empty() && !ToLeftTest(p.front()-l[i].p0,l[i].v);q.pop_front(),p.pop_front());
            if (sgn(Cross(l[i].v,q.back().v))==0)
                if (ToLeftTest(l[i].p0-q.back().p0),q.back().v){
                    q.pop_back();
                    if (!p.empty()) p.pop_back();
                }
            if (!q.empty()) p.push_back(GetIntersection(q.back(),l[i]));
            q.push_back(l[i]);
        }
        for (;!p.empty() && !ToLeftTest(p.back()-q.front().p0,q.front().v);q.pop_back(),p.pop_back());
        p.push_back(GetIntersection(q.back(),q.front()));
        if (p.size() < 3) printf("0\n");
        else{
            cerr << "!" << endl;
            double area = 0.5 * Cross(p.back(), p.front());
            Point last = p.front();
            for (p.pop_front(); !p.empty(); last = p.front(), p.pop_front())
                area += 0.5 * Cross(last, p.front());
            printf("%.8lf\n", fabs(area));
        }
    }
}
```

5.6 Min Circle Cover

```
namespace CG{
    Point GetCircleCenter(const Point a,const Point b,const Point c){
        Point p=(a+b)/2.0,q=(a+c)/2.0;
        Vector v=Rotate(b-a,pi/2.0),w=Rotate(c-a,pi/2.0);
        if (sgn(Norm(Cross(v,w)))==0){
            if (sgn(Norm(a-b)+Norm(b-c)-Norm(a-c))==0) return (a+c)/2;
            if (sgn(Norm(b-a)+Norm(a-c)-Norm(b-c))==0) return (b+c)/2;
```

```
            if (sgn(Norm(a-c)+Norm(c-b)-Norm(a-b))==0) return (a+c)/2;
        }
        return GetIntersection(Line(p,v),Line(q,w));
    }
    void MinCircleCover(Point p[],int n){
        random_shuffle(p+1,p+1+n);
        Point c=p[1];
        double r=0;
        for (int i=2;i<=n;i++)
            if (sgn(Norm(c-p[i])-r)>0){
                c=p[i],r=0;
                for (int j=1;j<i;j++)
                    if (sgn(Norm(c-p[j])-r)>0){
                        c=(p[i]+p[j])/2.0;
                        r=Norm(c-p[i]);
                        for (int k=1;k<j;k++)
                            if (sgn(Norm(c-p[k])-r)>0){
                                c=GetCircleCenter(p[i],p[j],p[k]);
                                r=Norm(c-p[i]);
                            }
                    }
            }
        printf("%.10f\n%.10f %.10f",r,c.x,c.y);
    }
}
```

5.7 Circle Union Area

```
//k次覆盖
//圆并去重后s[0]
typedef pair<double, int> P;
const double pi = acos(-1.0);
const int MAXN = 10003;
P arc[MAXN << 1];
int acnt, cnt;
double s[1003];
bool del[1003];
void add(double st, double en) {
    if(st < -pi) {
        add(st + 2 * pi, pi);
        add(-pi, en);
        return;
    }
    if(en > pi) {
        add(st, pi);
        add(-pi, en - 2 * pi);
        return;
    }
    arc[++acnt] = P(st, 1);
    arc[++acnt] = P(en, -1);
}
double F(double x) {
    return (x - sin(x)) / 2;
}
struct Node {
    int x, y, r;
    Node(int _x = 0, int _y = 0, int _r = 0):x(_x), y(_y), r(_r) {}
    bool operator == (const Node& t) {
        return x == t.x && y == t.y && r == t.r;
```

```
    }
    inline void read() {
        scanf("%d%d%d", &x, &y, &r);
    }
}a[1003];
int main() {
    int n;
    scanf("%d", &n);
    for(int i = 1; i <= n; i++) a[i].read();
    /*
    //去重
    int nn = 0;
    for(int i = 1; i <= n; i++) {
        bool same = 0;
        for(int j = 1; j < i; j++) {
            if(a[i] == a[j]) {
                same = 1; break;
            }
        }
        if(!same) a[++nn] = a[i];
    }
    n = nn;
    //去包含
    for(int i = 1; i <= n; i++) {
        for(int j = 1; j <= n; j++) if(i != j) {
            if(hypot(a[i].x - a[j].x, a[i].y - a[j].y) < (double)(a[i].r - a[j].r)) del[
    j] = 1;
        }
    }
    nn = 0;
    for(int i = 1; i <= n; i++) if(!del[i]) {
        a[++nn] = a[i];
    }
    n = nn;
    */
    for(int i = 1; i <= n; i++) {
        acnt = 0;
        for(int j = 1; j <= n; j++) if(i != j) {
            int dis = (a[i].x - a[j].x) * (a[i].x - a[j].x) + (a[i].y - a[j].y) * (a[i].
    y - a[j].y);
            if(a[j].r > a[i].r && dis <= (a[j].r - a[i].r) * (a[j].r - a[i].r)) add(-pi,
     pi);
            else if(dis > (a[i].r - a[j].r) * (a[i].r - a[j].r) && dis < (a[i].r + a[j].
    r) * (a[i].r + a[j].r)){
                double c = sqrt(dis);
                double angle = acos((a[i].r * a[i].r + c * c - a[j].r * a[j].r) / (2 * a
    [i].r * c));
                double k = atan2(a[j].y - a[i].y, a[j].x - a[i].x);
                add(k - angle, k + angle);
            }
        }
        arc[++acnt] = P(pi, -1);
        sort(arc + 1, arc + acnt + 1);
        cnt = 0;
        double last = -pi;
        for(int j = 1; j <= acnt; j++) {
            s[cnt] += F(arc[j].first - last) * a[i].r * a[i].r; //扇形 - 三角形
            double xa = a[i].x + a[i].r * cos(last);
            double ya = a[i].y + a[i].r * sin(last);
            last = arc[j].first;
            double xb = a[i].x + a[i].r * cos(last);
```

```
            double yb = a[i].y + a[i].r * sin(last);
            s[cnt] += (xa * yb - xb * ya) / 2; //到圆心的三角形面积
            cnt += arc[j].second;
        }
    }
    //printf("%.3f\n", s[0]);
    for (int i = 0; i < n; i++) {
        printf("[%d] = %.3f\n", i + 1, s[i] - s[i + 1]);
    }
    return 0;
}
```

5.8 Simpson Integrate

```
double Simpson(double l,double r){
    return (r-l)*(F(l)+4*F((l+r)/2)+F(r))/6;
}
double Integrate(double l,double r,double S){
    double mid=(l+r)/2;
    double A=Simpson(l,mid);
    double B=Simpson(mid,r);
    if(A+B-S<eps)return S;
    return Integrate(l,mid,A)+Integrate(mid,r,B);
}
```

5.9 Closest Point

```
vector <Point> P;
DB CP(int l, int r) {
    if (l == r) return DB_INF;
    if (l + 1 == r) return DisPP(P[l], P[r]);
    int mid = (l + r) >> 1;
    DB d = min(CP(l, mid), CP(mid + 1, r));
    vector <Point> tmp;
    for (int i = l; i <= r; i++)
        if (fabs(P[mid].x - P[i].x) < d) tmp.push_back(P[i]);
    sort(tmp.begin(), tmp.end(), cmpy);
    for (int i = 0; i < tmp.size(); i++)
        for (int j = i + 1; j < tmp.size() && tmp[j].y - tmp[i].y < d; j++)
            d = min(d, DisPP(tmp[i], tmp[j]));
    return d;
}
```

5.10 K-D Tree

```
#include <iostream>
#include <algorithm>
#include <stack>
#include <math.h>
using namespace std;
/*function of this program: build a 2d tree using the input trainingdata
 the input is exm_set which contains a list of tuples (x,y)
 the output is a 2d tree pointer*/


struct data
```

```
12 {
13     double x = 0;
14     double y = 0;
15 };
16
17 struct Tnode
18 {
19     struct data dom_elt;
20     int split;
21     struct Tnode * left;
22     struct Tnode * right;
23 };
24
25 bool cmp1(data a, data b){
26     return a.x < b.x;
27 }
28
29 bool cmp2(data a, data b){
30     return a.y < b.y;
31 }
32
33 bool equal(data a, data b){
34     if (a.x == b.x && a.y == b.y)
35     {
36         return true;
37     }
38     else{
39         return false;
40     }
41 }
42
43 void ChooseSplit(data exm_set[], int size, int &split, data &SplitChoice{
44     /*compute the variance on every dimension. Set split as thedismension that have the
       biggest
45      variance. Then choose the instance which is the median on thissplit dimension.*/
46     /*compute variance on the x,y dimension. DX=EX^2-(EX)^2*/
47     double tmp1,tmp2;
48     tmp1 = tmp2 = 0;
49     for (int i = 0; i < size; ++i)
50     {
51         tmp1 += 1.0 / (double)size * exm_set[i].x * exm_set[i].x;
52         tmp2 += 1.0 / (double)size * exm_set[i].x;
53     }
54     double v1 = tmp1 - tmp2 * tmp2;  //compute variance on the xdimension
55
56     tmp1 = tmp2 = 0;
57     for (int i = 0; i < size; ++i)
58     {
59         tmp1 += 1.0 / (double)size * exm_set[i].y * exm_set[i].y;
60         tmp2 += 1.0 / (double)size * exm_set[i].y;
61     }
62     double v2 = tmp1 - tmp2 * tmp2;  //compute variance on the ydimension
63
64     split = v1 > v2 ? 0:1; //set the split dimension
65
66     if (split == 0)
67     {
68         sort(exm_set,exm_set + size, cmp1);
69     }
70     else{
71         sort(exm_set,exm_set + size, cmp2);
```

```
72      }
73
74      //set the split point value
75      SplitChoice.x = exm_set[size / 2].x;
76      SplitChoice.y = exm_set[size / 2].y;
77
78  }
79
80  Tnode* build_kdtree(data exm_set[], int size, Tnode* T){
81      //call function ChooseSplit to choose the split dimension and splitpoint
82      if (size == 0){
83          return NULL;
84      }
85      else{
86          int split;
87          data dom_elt;
88          ChooseSplit(exm_set, size, split, dom_elt);
89          data exm_set_right [100];
90          data exm_set_left [100];
91          int sizeleft ,sizeright;
92          sizeleft = sizeright = 0;
93
94          if (split == 0)
95          {
96              for (int i = 0; i < size; ++i)
97              {
98
99                  if (!equal(exm_set[i],dom_elt) && exm_set[i].x <=dom_elt.x)
100                 {
101                     exm_set_left[sizeleft].x = exm_set[i].x;
102                     exm_set_left[sizeleft].y = exm_set[i].y;
103                     sizeleft++;
104                 }
105                 else if (!equal(exm_set[i],dom_elt) && exm_set[i].x >dom_elt.x)
106                 {
107                     exm_set_right[sizeright].x = exm_set[i].x;
108                     exm_set_right[sizeright].y = exm_set[i].y;
109                     sizeright++;
110                 }
111             }
112         }
113         else{
114             for (int i = 0; i < size; ++i)
115             {
116
117                 if (!equal(exm_set[i],dom_elt) && exm_set[i].y <=dom_elt.y)
118                 {
119                     exm_set_left[sizeleft].x = exm_set[i].x;
120                     exm_set_left[sizeleft].y = exm_set[i].y;
121                     sizeleft++;
122                 }
123                 else if (!equal(exm_set[i],dom_elt) && exm_set[i].y >dom_elt.y)
124                 {
125                     exm_set_right[sizeright].x = exm_set[i].x;
126                     exm_set_right[sizeright].y = exm_set[i].y;
127                     sizeright++;
128                 }
129             }
130         }
131         T = new Tnode;
132         T->dom_elt.x = dom_elt.x;
```

```
        T->dom_elt.y = dom_elt.y;
        T->split = split;
        T->left = build_kdtree(exm_set_left, sizeleft, T->left);
        T->right = build_kdtree(exm_set_right, sizeright, T->right);
        return T;

    }
}


double Distance(data a, data b){
    double tmp = (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
    return sqrt(tmp);
}


void searchNearest(Tnode * Kd, data target, data &nearestpoint, double &distance){

    //1. 如果Kd是空的, 则设dist为无穷大返回

    //2. 向下搜索直到叶子结点

    stack<Tnode*> search_path;
    Tnode* pSearch = Kd;
    data nearest;
    double dist;

    while(pSearch != NULL)
    {
        //pSearch加入到search_path中;
        search_path.push(pSearch);

        if (pSearch->split == 0)
        {
            if(target.x <= pSearch->dom_elt.x) /* 如果小于就进入左子树 */
            {
                pSearch = pSearch->left;
            }
            else
            {
                pSearch = pSearch->right;
            }
        }
        else{
            if(target.y <= pSearch->dom_elt.y) /* 如果小于就进入左子树 */
            {
                pSearch = pSearch->left;
            }
            else
            {
                pSearch = pSearch->right;
            }
        }
    }
    //取出search_path最后一个赋给nearest
    nearest.x = search_path.top()->dom_elt.x;
    nearest.y = search_path.top()->dom_elt.y;
    search_path.pop();


    dist = Distance(nearest, target);
```

```
194     //3. 回溯搜索路径
195
196     Tnode* pBack;
197
198     while(search_path.size() != 0)
199     {
200         //取出search_path最后一个结点赋给pBack
201         pBack = search_path.top();
202         search_path.pop();
203
204         if(pBack->left == NULL && pBack->right == NULL) /* 如果pBack为子结点 */
205
206         {
207
208             if( Distance(nearest, target) > Distance(pBack->dom_elt,target) )
209             {
210                 nearest = pBack->dom_elt;
211                 dist = Distance(pBack->dom_elt, target);
212             }
213
214         }
215
216         else
217
218         {
219
220             int s = pBack->split;
221             if (s == 0)
222             {
223                 if( fabs(pBack->dom_elt.x - target.x) < dist) /* 如果target为中心的圆
    （球或超球），半径为dist的圆与分割超平面交， 那么就要跳到另一边的子空间去搜索 */
224                 {
225                     if( Distance(nearest, target) > Distanc(pBack->dom_elt, target) )
226                     {
227                         nearest = pBack->dom_elt;
228                         dist = Distance(pBack->dom_elt, target);
229                     }
230                     if(target.x <= pBack->dom_elt.x) /* 如果target位pBack的左子空间，那
    么就要跳到右子空间去搜索 */
231                         pSearch = pBack->right;
232                     else
233                         pSearch = pBack->left; /* 如果target位于pBack的子空间，那么就要
    跳到左子空间去搜索 */
234                     if(pSearch != NULL)
235                         //pSearch加入到search_path中
236                         search_path.push(pSearch);
237                 }
238             }
239             else {
240                 if( fabs(pBack->dom_elt.y - target.y) < dist) /* 如果target为中心的圆
    （球或超球），半径为dist的圆与分割超平面交， 那么就要跳到另一边的子空间去搜索 */
241                 {
242                     if( Distance(nearest, target) > Distanc(pBack->dom_elt, target) )
243                     {
244                         nearest = pBack->dom_elt;
245                         dist = Distance(pBack->dom_elt, target);
246                     }
247                     if(target.y <= pBack->dom_elt.y) /* 如果target位pBack的左子空间，那
    么就要跳到右子空间去搜索 */
248                         pSearch = pBack->right;
249                     else
```

```
250                         pSearch = pBack->left; /* 如果target位于pBack的子空间，那么就要
    跳到左子空间去搜索 */
251                     if(pSearch != NULL)
252                         // pSearch加入到search_path中
253                         search_path.push(pSearch);
254                 }
255             }
256 
257         }
258     }
259 
260     nearestpoint.x = nearest.x;
261     nearestpoint.y = nearest.y;
262     distance = dist;
263 
264 }
265 
266 int main(){
267     data exm_set[100]; //assume the max training set size is 100
268     double x,y;
269     int id = 0;
270     cout<<"Please input the training data in the form x y. One instanceper line. Enter
    -1 -1 to stop."<<endl;
271     while (cin>>x>>y){
272         if (x == -1)
273         {
274             break;
275         }
276         else{
277             exm_set[id].x = x;
278             exm_set[id].y = y;
279             id++;
280         }
281     }
282     struct Tnode * root = NULL;
283     root = build_kdtree(exm_set, id, root);
284 
285     data nearestpoint;
286     double distance;
287     data target;
288     cout <<"Enter search point"<<endl;
289     while (cin>>target.x>>target.y)
290     {
291         searchNearest(root, target, nearestpoint, distance);
292         cout<<"The nearest distance is "<<distance<<",and the nearestpoint is "<<
    nearestpoint.x<<","<<nearestpoint.y<<endl;
293         cout <<"Enter search point"<<endl;
294 
295     }
296 }
```

6 Conclusion

6.1 Math

6.1.1 Euler's Theorem

$$a^b \equiv \begin{cases} a^{b\%\varphi(p)} & \gcd(a,p)=1 \\ a^b & \gcd(a,p)\neq 1, b<\varphi(p) \\ a^{b\%\varphi(p)+\varphi(p)} & \gcd(a,p)\neq 1, b\geq\varphi(p) \end{cases} (mod\ p)$$

6.1.2 Möbius Inversion

Dirichlet Convolution is $(f\times g)(N)=\sum_{d|N} f(d)*g(\frac{N}{d})$
Theorem:

$$\begin{cases} f=g\times 1 \\ g=f\times\mu \end{cases}$$

$$\begin{cases} id(n)=\sum\limits_{d|n}\varphi(d) \\ e(n)=\sum\limits_{d|n}\mu(d) \end{cases} \tag{1}$$

$$\begin{cases} \sum\limits_{i}^{n}\sum\limits_{j}^{m} gcd(i,j)=\sum\limits_{d}^{\max(n,m)}\varphi(d)*\lfloor\frac{n}{d}\rfloor\lfloor\frac{m}{d}\rfloor \\ \sum\limits_{i}^{n}\sum\limits_{j}^{m} e(gcd(i,j))=\sum\limits_{d}^{\min(n,m)}\mu(d)*\lfloor\frac{n}{d}\rfloor\lfloor\frac{m}{d}\rfloor \\ \sum\limits_{i=1}^{n}|\mu(i)|=\sum\limits_{i=1}^{\lfloor\sqrt{n}\rfloor}\mu(i)*\lfloor\frac{n}{i*i}\rfloor \end{cases} \tag{2}$$

$$\begin{cases} sum(x,y)=\sum\limits_{i}^{x}\sum\limits_{j}^{y} i*j=\frac{x*(x+1)}{2}*\frac{y*(y+1)}{2} \\ F(x,y)=\sum\limits_{i=1}^{\min(x,y)} i^2*\mu(i)*sum(\lfloor\frac{x}{i}\rfloor,\lfloor\frac{y}{i}\rfloor) \\ \sum\limits_{i}^{n}\sum\limits_{j}^{m} lcm(i,j)=\sum\limits_{i=1}^{\min(n,m)} d*F(\lfloor\frac{n}{i}\rfloor,\lfloor\frac{y}{i}\rfloor) \end{cases} \tag{3}$$

6.1.3 Sieve Tips

$$\varphi(nm)=\varphi(n)\cdot\varphi(m)\cdot\frac{gcd(n,m)}{\varphi(gcd(n,m))} \tag{4}$$

$$\varphi(n)=\sum_{i=1}^{n}[(n,i)=1]\cdot i=\frac{n*\varphi(n)+[n=1]}{2} \tag{5}$$

$$\begin{cases} id = \varphi \times 1 \\ \dfrac{n\cdot(n+1)}{2} = \displaystyle\sum_{i=1}^{n} i = \sum_{i=1}^{n}\sum_{d|i}\varphi(d) = \sum_{\frac{i}{d}=1}^{n}\sum_{d=1}^{\lfloor\frac{n}{\frac{i}{d}}\rfloor}\varphi(d) = \sum_{i=1}^{n}\phi(\lfloor\frac{n}{i}\rfloor) \end{cases} \tag{6}$$

$$\begin{cases} e = \mu \times 1 \\ 1 = \displaystyle\sum_{i=1}^{n}[i=1] = \sum_{i=1}^{n}\sum_{d|i}\mu(d) = \sum_{i=1}^{n}\sum_{d=1}^{\lfloor\frac{n}{i}\rfloor}\mu(d) = \sum_{i=1}^{n} M(\lfloor\frac{n}{i}\rfloor) \end{cases} \tag{7}$$

$$\begin{cases} id^2 = (id\cdot\varphi)\times id \\ \phi'(n) = \displaystyle\sum_{i=1}^{n} i\cdot\varphi(i) \\ \dfrac{n\cdot(n+1)\cdot(2n+1)}{6} = \displaystyle\sum_{i=1}^{n} i^2 = \sum_{i=1}^{n}\sum_{d|i} d\cdot\varphi(d)\cdot\frac{i}{d} = \sum_{\frac{i}{d}=1}^{n}\frac{i}{d}\sum_{d=1}^{\lfloor\frac{n}{\frac{i}{d}}\rfloor} d\cdot\varphi(d) = \sum_{i=1}^{n} i\cdot\phi'(\lfloor\frac{n}{i}\rfloor) \end{cases} \tag{8}$$

6.1.4 Newton's method

Iff

$$f^{'}(x) \neq 0 \;\&\&\; f^{''}(x) continuous$$

Then

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

6.1.5 Cantor Expansion

康托展开是一个全排列到一个自然数的双射，常用于构建哈希表时的空间压缩。康托展开的实质是计算当前排列在所有由小到大全排列中的顺序，因此是可逆的。以下称第 x 个全排列是都是指由小到大的顺序。

$$X = a_n(n-1)! + a_{n-1}(n-2)! + \cdots + a_1\cdot 0!$$

其中 a_i 为排列的第 i 个元素值
例如，3 5 7 4 1 2 9 6 8 展开为 98884。因为 X=2*8!+3*7!+4*6!+2*5!+0*4!+0*3!+2*2!+0*1!+0*0!=98884.
解释：
排列的第一位是 3，比 3 小的数有两个，以这样的数开始的排列有 8! 个，因此第一项为 2*8!
排列的第二位是 5，比 5 小的数有 1、2、3、4，由于 3 已经出现，因此共有 3 个比 5 小的数，这样的排列有 7! 个，因此第二项为 3*7!
以此类推，直至 0*0!
用途：
显然，n 位（0 n-1）全排列后，其康托展开唯一且最大约为 n!，因此可以由更小的空间来储存这些排列。由公式可将 X 逆推出唯一的一个排列。
逆运算：
如 n=5,x=96 时：
首先用 96-1 得到 95，说明 x 之前有 95 个排列.(将此数本身减去 1)
用 95 去除 4! 得到 3 余 23，说明有 3 个数比第 1 位小，所以第一位是 4.
用 23 去除 3! 得到 3 余 5，说明有 3 个数比第 2 位小，所以是 4，但是 4 已出现过，因此是 5.

用 5 去除 2! 得到 2 余 1，类似地，这一位是 3.
用 1 去除 1! 得到 1 余 0，这一位是 2.
最后一位只能是 1.
所以这个数是 45321.
按以上方法可以得出通用的算法。

6.1.6 $\sum_{i=1}^{n} i^k$

$$\sum_{i}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{i}^{n} i^3 = [\frac{n(n+1)}{2}]^2$$

差分和组合数:$O(k^2)$
考虑数列 $\{1_k, 2_k, \cdots, i_k\}$
相邻两项做差之后，得到的数列的每项应该是一个 k - 1 次关于 i 的多项式。
再次相邻两项做差之后，得到的数列的每项应该是一个 k - 2 次关于 i 的多项式。
如此进行 k 次，得到的数列的每项应该是一个 0 次关于 i 的多项式，即常数数列。
再次相邻两项做差之后，一定会得到一个全是 0 的数列。
假设经过 i 次差分数列之后的数列第一项为 r_i 那么答案就是

$$\sum_{i=0}^{k} r_i \binom{n}{i+1}$$

举例来说，考虑数列
1,8,27,64,125,216,...
求差分可得
7,19,37,61,91,...
12,18,24,30,...
6,6,6,...
0,0,...
每个数列的第一项为 1, 7, 12, 6。所以最终的答案即为

$$\binom{n}{1} + 7\binom{n}{2} + 12\binom{n}{3} + 6\binom{n}{4} = \frac{n^2(n+1)^2}{4}$$

拉格朗日插值:$O(k)$

6.1.7 Generating Function

生成函数

普通型

$$A(x) = \sum_{i=0}^{\infty} a_i x^i$$

已知 $\{a_i\}$, $\{b_i\}$ 的生成函数分别是 A(x), B(x)。
那么 $\{a_i \pm b_i\}$ 的生成函数是 A(x) ± B(x)。
值得注意的是数列 $\{a_i\}$, $\{b_i\}$ 的卷积的生成函数，恰好是 C(x) = A(x)B(x)。

$$c_i = \sum_j a_j b_{i-j}$$

关键公式

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_i x^i$$
$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + \cdots = \sum_i x^{2i}$$

推广的二项式定理

$$(1+x)^n = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \cdots = \sum_i \binom{n}{i}x^i$$

其中有 β 函数和 Γ 函数

$$\Gamma(s) = \int_0^{+\infty} x^{s-1}e^{-x}dx$$
$$B(p,q) = \binom{p+q}{p} = \int_0^1 x^{p-1}(1-x)^{q-1}dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

分别有递推公式

$$\begin{aligned}\Gamma(s+1) &= s\Gamma(s)\\ B(p,q) &= B(q,p)\\ &= \frac{q-1}{p+q-1}B(p,q-1)\\ &= \frac{p-1}{p+q-1}B(p-1,q)\\ &= \frac{(p-1)(q-1)}{(p+q-1)(p+q-2)}B(p-1,q-1)\end{aligned}$$

例子

Fibonacci 生成函数

构造一个与 f_n 有关得幂级数

$$F(x) = f_0 + f_1 x + f_2 x^2 + \cdots + f_n x^n + \cdots$$

亦即

$$F(x) = xF(x) + x^2F(x) + f_0 + (f_1 - f_0)x$$

带入 $f_0 = 0, f_1 = 1$

$$F(x) = \frac{x}{1-x-x^2}$$

如果想继续计算通项，我们需要解方程得到

$$\frac{x}{1-x-x^2} = \frac{a}{1-\alpha x} + \frac{b}{1-\beta x}$$

解出

$$\begin{cases} \alpha = \dfrac{1+\sqrt{5}}{2} \\ \beta = \dfrac{1-\sqrt{5}}{2} \end{cases}$$

带入方程有

$$a = \frac{1}{\sqrt{5}}, b = -\frac{1}{\sqrt{5}}$$

则

$$\begin{aligned} F(x) &= \frac{a}{1-\alpha x} + \frac{b}{1-\beta x} \\ &= a(1+\alpha x+\alpha^2 x^2+\cdots) + b(1+\beta x+\beta^2 x^2+\cdots) \end{aligned}$$

蕴含通项为

$$f_n = a\alpha^n + b\beta^n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$$

Catalan 数

$$c_n = \sum_{i=1}^{n-1} c_i c_{n-i}$$

生成函数

$$C(x) = c_1 x + c_2 x^2 + \cdots$$

亦即

$$C(x) = xC(x)^2 + 1$$

解得

$$C(x) = \frac{1-\sqrt{1-4x}}{2x}$$

利用推广的二项式定理展开 $\sqrt{1-4x}$ 可得通项

$$c_n = \frac{1}{n}\binom{2n-2}{n-1}$$

指数型生成函数

$$A(x) = \sum_{i=0}^{\infty} \frac{a_i x^i}{i!}$$

已知 $\{a_i\}$, $\{b_i\}$ 的生成函数分别是 A(x), B(x)。
那么 $\{a_i \pm b_i\}$ 的生成函数是 A(x) ± B(x)。
值得注意的是数列 $\{a_i\}$, $\{b_i\}$ 的卷积的生成函数，恰好是 C(x) = A(x)B(x)。

$$c_i = \sum_j \binom{i}{j} a_j b_{i-j}$$

关键公式

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots = \sum_i \frac{x^i}{i!}$$

$$e^{-x} = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots = \sum_i \frac{(-x)^i}{i!}$$

$$\frac{e^x + x^{-x}}{2} = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots = \sum_i \frac{x^{2i}}{(2i)!}$$

6.1.8 Polya

设 G 是 p 个对象的一个置换群，用 k 种颜色去染这 p 个对象，若一种染色方案在群 G 的作用下变为另一种方案，则这两个方案当作是同一种方案，这样的不同染色方案数为:

$$\frac{1}{|G|} \times \sum (k^{C(f)}), f \in G$$

C(f) 为循环节，|G| 表示群的置换方法数
对于有 n 个位置的手镯，有 n 种旋转置换和 n 种翻转置换
对于旋转置换:

$$C(f_i) = gcd(n, i)$$

i 表示一次转过 i 颗宝石，i = 0 时 c = n;
对于翻转置换:
如果 n 为偶数: 则有 $\frac{n}{2}$ 个置换 $C(f) = \frac{n}{2}$，有 $\frac{n}{2}$ 个置换 $C(f) = \frac{n}{2} + 1$
如果 n 为奇数: $C(f) = \frac{n}{2} + 1$

6.1.9 FWT

$$\begin{cases} C_k = \sum\limits_{i \oplus j = k} A_i * B_j \\ DWT(A)_i = \sum\limits_{j}^{n} A_j * f_{i,j} \\ DWT(C)_i = DWT(A)_i * DWT(B)_i \\ f_{i,j} \cdot f_{i,k} = f_{i,j \oplus k} \\ f_{i,j} = [i \ and \ j == i] \qquad (and) \\ f_{i,j} = [i \ and \ j == j] \qquad (or) \\ f_{i,j} = (-1)^{|i \ and \ j|} \qquad (xor) \end{cases}$$

6.2 Geometry

6.2.1 The Number of Ingeter Point on a Circle

Set $r = const$ is the radius of the circle.

$$r^2 = {p_1}^{a_1} + {p_2}^{a_2} + \cdots + {p_m}^{a_m} = \sum_{i=1}^{m} {p_i}^{a_i}$$

Define

$$\chi(n) = \begin{cases} 1 & n\%4 = 1 \\ -1 & n\%4 = 3 \\ 0 & n\%2 = 0 \end{cases}$$

By the way, $\chi(n)$ is a multiplicative function.

Define

$$\Gamma(p_i, a_i) = \sum_{j=0}^{a_i} \chi(p_i^j) = \begin{cases} 1 & p_i = 2 \quad || \quad (p_i\%4 = 3 \quad \&\& \quad a_i\%2 = 0) \\ 0 & p_i\%4 = 3 \quad \&\& \quad a_i\%2 = 1 \\ a_i + 1 & p_i\%4 = 1 \end{cases}$$

Define cnt is the number of integer point on circle

$$cnt(r) = 4 \prod_{i=1}^{m} \sum_{j=0}^{a_i} \chi(p_i^j) = 4 \prod_{i=1}^{m} \Gamma(p_i, a_i) = 4 \sum_{k|r^2} \chi(k)$$

Define CNT is the number of integer point in circle

$$CNT(r) = 1 + \sum_{i=1}^{r^2} cnt(i) = 1 + \sum_{i=1}^{r^2} \lfloor \frac{r^2}{i} \rfloor \chi(i)$$

6.3 Josephus

6.3.1 $J(n, m)$: The Last Surviving Person

based-0, $m - 1$ **was the first be killed.**

$$J_0(n, m) = \begin{cases} 0 & n = 1; \\ (J_0(n-1, m) + m)\%n & 1 < n < m; \\ \lfloor \frac{k((f(n', k) - n mod k) mod n')}{k-1} \rfloor \ where\ n' = n - \lfloor \frac{n}{k} \rfloor & m <= n \end{cases}$$

$$J_k(n, m) = (J_0(n, m) + k)\%n$$

```
int J0(int n, int m) {
    if (m == 1) return n - 1;
    int ans = 0;
    for (int i = 2; i <= n; ) {
        if (ans + m >= i) {
            ans = (ans + m) % i;
            i++;
            continue;
```

```
        }
        int step = (i - 1 - ans - 1) / (m - 1);
        if (i + step > n) {
            ans += (n - (i - 1)) * m;
            break;
        }
        i += step; ans += step * m;
    }
    return ans;
}
```

6.3.2 $aJ(n, 1, K)$: Survival Time of K-th Person

based-1, 1 was the first be killed.

```
int aJ1(int size, int be, int goal){
    if (be > size) be = (be - 1) % size + 1;
    if (goal % m == be % m) return (goal - be) / m + 1;
    return (size - be) / m + 1 + get(size - (size - be) / m - 1, (be + ((size - be) / m
    + 1) * m) - size, goal > be ? goal - (goal - be) / m - 1 : goal);
}
\\ size : the size of current group;
\\ be : the start person of current step;
\\ goal : index of the asked person.
```

7 Others

7.1 Offline Algorithm

7.1.1 CDQ Divide and Conquer

```
struct Node {
    int x, y, z, ans;
    Node() {}
    Node(int _x, int _y, int _z):x(_x), y(_y), z(_z) {}
    bool operator < (const Node &b) const {
        if(y == b.y) {
            if(z == b.z) return x < b.x;
            return z < b.z;
        }
        return y < b.y;
    }
}A[MAXN], B[MAXN], C[MAXN];
int bit[MAXN];
void add(int k, int v) {
    for(; k <= m; k += k & -k) bit[k] = max(bit[k], v);
}
void clear(int k) {
    for(; k <= m; k += k & -k) bit[k] = 0;
}
int sum(int k) {
    int res = 0;
    for(; k; k -= k & -k) res = max(res, bit[k]);
    return res;
}
void solve(int l, int r) {
    if(l == r) {
        B[l] = A[l];
        return;
    }
    int mid = (l + r) >> 1;
    solve(l, mid);
    for(int i = mid + 1; i <= r; i++) B[i] = A[i];
    //sort(B + l, B + mid + 1);
    sort(B + mid + 1, B + r + 1);
    int L = l;
    for(int R = mid + 1; R <= r; R++) {
        while(L <= mid && B[L].y < B[R].y) add(B[L].z, B[L].ans), L++;
        A[B[R].x].ans = max(A[B[R].x].ans, sum(B[R].z - 1) + 1);
        B[R].ans = A[B[R].x].ans;
    }
    for(int i = l; i <= L; i++) clear(B[i].z);
    solve(mid + 1, r);
    L = l;
    int p = l, q = mid + 1;
    while(p <= mid || q <= r) {
        if(q > r || (p <= mid && B[p].y <= B[q].y)) C[L++] = B[p++];
        else C[L++] = B[q++];
    }
    for(int i = l; i <= r; i++) B[i] = C[i];
}
```

7.1.2 Mo' s Algorithm

```
struct Node{
    int l, r, t, id;
    bool operator < (const Node& a) const {
        if(l /sz == a.l / sz) {
            if(r == a.r) return t < a.t;
            return r < a.r;
        }
        return l / sz < a.l / sz;
    }
}q[MAXN];
void solve() {
    while (t < q[i].t) addTime(t++, 1);
    while (t > q[i].t) addTime(--t, -1);
    while(L < q[i].l) add(L++, -1);
    while(L > q[i].l) add(--L, 1);
    while(R < q[i].r) add(++R, 1);
    while(R > q[i].r) add(R--, -1);
}
```

7.1.3 Mo's Algorithm On Tree

```
struct Edge {
    int to, nxt;
}e[MAXN << 1];
int head[MAXN], ecnt;
int stack[MAXN], top, belong[MAXN], cnt, sz;
struct Node {
    int l, r, id, ti;
    bool operator < (const Node &x) const {
        return belong[l] < belong[x.l] || (belong[l] == belong[x.l] && belong[r] <
    belong[x.r]) || (belong[l] == belong[x.l] && belong[r] == belong[x.r] && ti < x.ti);
    }
}q[MAXN];
struct Node2 {
    int l, r, ti;
}qq[MAXN];
int n, m, Q, Q0, Q1;
int V[MAXN], W[MAXN], C[MAXN];
int fa[MAXN][S + 3], dep[MAXN];
long long ans[MAXN], tans;
int vis[MAXN], cur[MAXN];
long long sum[MAXN];
int l, r, tm;
inline int read() {
    int x = 0; char ch = getchar(); bool fg = 0;
    while(ch < '0' || ch > '9') { if(ch == '-') fg = 1; ch = getchar(); }
    while(ch >= '0' && ch <= '9') { x = x * 10 + ch - '0'; ch = getchar(); }
    return fg ? -x : x;
}
inline void add_edge(int u, int v) {
    e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
    e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
}
void dfs(int u, int f) {
    fa[u][0] = f;
    dep[u] = dep[f] + 1;
    int bot = top;
```

```
36      for(int i = head[u]; i; i = e[i].nxt) {
37          int v = e[i].to;
38          if(v == f) continue;
39          dfs(v, u);
40          if(top - bot >= sz) {
41              cnt++;
42              while(top != bot) belong[stack[top--]] = cnt;
43          }
44      }
45      stack[++top] = u;
46  }
47  void G(int &u, int step) {
48      for(int i = 0; i < S; i++) if((1 << i) & step) u = fa[u][i];
49  }
50  int lca(int u, int v) {
51      if(dep[u] > dep[v]) swap(u, v);
52      G(v, dep[v] - dep[u]);
53      if(u == v) return u;
54      for(int i = S; i >= 0; i--) if(fa[u][i] != fa[v][i]) {
55          u = fa[u][i]; v = fa[v][i];
56      }
57      return fa[u][0];
58  }
59  inline void modify(int u) {
60      tans -= V[C[u]] * sum[cur[C[u]]];
61      cur[C[u]] += vis[u];
62      vis[u] = -vis[u];
63      tans += V[C[u]] * sum[cur[C[u]]];
64  }
65  inline void update(int u, int v) {
66      if(u == v) return;
67      if(dep[u] > dep[v]) swap(u, v);
68      while(dep[v] > dep[u]) {
69          modify(v);
70          v = fa[v][0];
71      }
72      while(u != v) {
73          modify(u); modify(v);
74          u = fa[u][0]; v = fa[v][0];
75      }
76  }
77  inline void upd(int t) {
78      if(vis[qq[t].l] == -1) {
79          modify(qq[t].l);
80          swap(C[qq[t].l], qq[t].r);
81          modify(qq[t].l);
82      }
83      else swap(C[qq[t].l], qq[t].r);
84  }
85  inline void moveto(int u, int v) {
86      update(l, u); update(r, v);
87      l = u; r = v;
88  }
89  int main() {
90      n = read(); m = read(); Q = read();
91      sz = (int)pow(n, 2.0 / 3.0);
92      for(int i = 1; i <= m; i++) V[i] = read();
93      for(int i = 1; i <= n; i++) W[i] = read();
94      for(int i = 1, u, v; i < n; i++) {
95          u = read(); v = read();
96          add_edge(u, v);
```

```
    }
    for(int i = 1; i <= n; i++) {
        C[i] = read();
        vis[i] = 1;
        sum[i] = sum[i - 1] + W[i];
    }
    for(int i = 1, tp; i <= Q; i++) {
        tp = read();
        if(tp) {
            ++Q1;
            q[Q1].l = read(); q[Q1].r = read();
            q[Q1].id = Q1;
            q[Q1].ti = i;
        }
        else {
            ++Q0;
            qq[Q0].l = read(); qq[Q0].r = read();
            qq[Q0].ti = i;
        }
    }
    dfs(1, 0);
    while(top) belong[stack[top--]] = cnt;
    sort(q + 1, q + Q1 + 1);
    for(int k = 1; k <= S; k++) {
        for(int i = 1; i <= n; i++) {
            fa[i][k] = fa[fa[i][k - 1]][k - 1];
        }
    }
    for(int i = 1; i <= Q1; i++) {
        if(belong[q[i].l] > belong[q[i].r]) swap(q[i].l, q[i].r);
        moveto(q[i].l, q[i].r);
        int lc = lca(l, r);
        modify(lc);
        while(qq[tm + 1].ti < q[i].ti && tm < Q0) upd(++tm);
        while(qq[tm].ti > q[i].ti) upd(tm--);
        ans[q[i].id] = tans;
        modify(lc);
    }
    for(int i = 1; i <= Q1; i++) printf("%lld\n", ans[i]);
    return 0;
}
```

7.2 Randomized Algorithm

7.2.1 Simulated Annealing

```
void solve() {
    while(T > eps) {
        double alpha = ((rand() % 30001) / 15000.0) * pi;
        double theta = ((rand() % 10001) / 10000.0) * pi;
        tmp.x = cur.x + T * sin(theta) * cos(alpha);
        tmp.y = cur.y + T * sin(theta) * sin(alpha);
        tmp.z = cur.z + T * cos(theta);
        tmp.dis = cal(tmp);
        if(tmp.dis < cur.dis || (tmp.dis * 0.999 < cur.dis && (rand() & 7) == 7)) cur =
    tmp;
        //if(exp((cur.d - tmp.d)  / T) > ((double)rand() / RAND_MAX)) cur = tmp;

        T *= 0.999;
```

```
    }
}
```

7.3 Other Method

7.3.1 Enumerate Subset

```
for(int i = 0; i < (1 << k); i++) {
    for(int j = i; ; --j &= i) {
        // work();
        if(j == 0) break;
    }
}
```

7.3.2 Enumerate $\lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$

```
int cal(int n, int m) {
    if(n > m) swap(n, m);
    int res = 0, last;
    for(int i = 1; i <= n; i = last + 1) {
        last = min(n / (n / i), m / (m / i));
        res += (n / i) * (m / i) * (sum(last) - sum(i - 1));
    }
    return res;
}
```

7.3.3 Find Primitive Root Modulo N

```
for i in range(1,mod):
    if 3 ** i % mod == 1:
        if i == mod - 1:
            print("yes")
            break
        print("no")
```

8 Samples

8.1 vimrc

```
set cindent
set number
set mouse=a
set tabstop=4
set shiftwidth=4
syntax on
inoremap { {}<left>
map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>
"ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
```

```
set nocompatible
source $VIMRUNTIME/vimrc_example.vim
source $VIMRUNTIME/mswin.vim
se cin nu mouse=a ts=4 sw=4 ww=b,s,<,>,[,]
syntax on
inoremap { {}<left>
map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>
func! AddTitle()
    call append(0,"// Cease to struggle and you cease to live")
    call append(1,"// Created by hjj")
    call append(2,"#include <bits/stdc++.h>")
    call append(3,"using namespace std;")
    call append(4,"")
    call append(5,"int main() {")
    call append(6,"")
    call append(7,"    return 0;")
    call append(8,"}")
endfunc
map <F8> :call AddTitle()<CR>
```

8.2 Check

Linux

```
while true; do
    ./data > in
    ./tmp < in > out
    ./std < in > ans
    diff out ans
    if [ $? -ne 0 ] ; then exit; fi
    echo Passed
done
```

windows

```
@echo off
:loop
    rand.exe > data.in
    std.exe < data.in > std.out
    my.exe < data.in > my.out
    fc my.out std.out
if not errorlevel 1 goto loop
pause
goto loop
```

8.3 Random

```
mt19937_64 mt(chrono::steady_clock::now().time_since_epoch().count());
shuffle(per.begin(), per.end(), mt);
//random_shuffle(per.begin(),per.end());
```

8.4 FastIO

```
//普通情况
namespace IO {
    const int MB = 1048576;
    const int RMAX = 16 * MB;
    const int WMAX = 16 * MB;
    #define getchar() *(rp++)
    #define putchar(x) (*(wp++) = (x))
    char rb[RMAX], *rp = rb,  wb[WMAX], *wp = wb;
    inline void init() {
        fread(rb, sizeof(char), RMAX, stdin);
    }
    template <class _T> inline void read(_T &_a) {
        _a = 0; bool _f = 0; int _c = getchar();
        while (_c < '0' || _c > '9') _f |= _c == '-', _c = getchar();
        while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getchar();
        _a = _f ? -_a : _a;
    }
    template <class _T> inline void write(_T _a) {
        static char buf[20], *top = buf;
        if (_a) {
            while (_a) {
                _T tm = _a / 10;
                *(++top) = char(_a - tm * 10) | '0';
                _a = tm;
            }
            while (top != buf) putchar(*(top--));
        }
        else putchar('0');
    }
    void output() {
        fwrite(wb, sizeof(char), wp - wb, stdout);
    }
}
//EOF结尾+分块读入
#define likely(x) __builtin_expect(!!(x), 1)
#define unlikely(x) __builtin_expect(!!(x), 0)
namespace IO {
    const int MB = 1048576;
    const int RMAX = 4 * MB;
    const int WMAX = 4 * MB;
    unsigned long long filesize;
    #define putchar(x) (*(wp++) = (x))
    char rb[RMAX], wb[WMAX], *wp = wb;
    int rp = 0;
    inline void init() {
        filesize = fread(rb, sizeof(char), RMAX, stdin);
        rp = 0;
        wp = wb;
    }
    void output() {
        fwrite(wb, sizeof(char), wp - wb, stdout);
```

```
    }
    inline char getCHAR(){
        if(unlikely(rp == filesize)){
            fwrite(wb, sizeof(char), wp - wb, stdout);
            init();
            if(unlikely(filesize == 0)) {
                //cerr << 1.0 * (clock() - st) / CLOCKS_PER_SEC << endl;
                exit(0);
            }
        }
        return rb[rp++];
    }
    template <class _T> inline void read(_T &_a) {
        _a = 0; static bool _f = 0; static int _c;
        _f = 0; _c = getCHAR();
        while (_c < '0' || _c > '9') _f |= _c == '-', _c = getCHAR();
        while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getCHAR();
        _a = _f ? -_a : _a;
    }
    template <class _T> inline void write(_T _a) {
        static char buf[20], *top = buf;
        if (_a) {
            while (_a) {
                _T tm = _a / 10;
                *(++top) = char(_a - tm * 10) | '0';
                _a = tm;
            }
            while (top != buf) putchar(*(top--));
        }
        else putchar('0');
        putchar('\n');
    }
}
```

8.5 Java BigNum

```
import java.math.*;
import java.util.*;
import java.lang.*;

public class Main{
    public static void main(String []args){}
}
//IO
Scanner in = new Scanner(System.in);
while(in.hasNext()){} //EOF
//fast-IO
public static void main(String argv[]) throws IOException{}
StreamTokenizer cin = new StreamTokenizer(new BufferedReader(new InputStreamReader(
    System.in)));
PrintWriter cout = new PrintWriter(new OutputStreamWriter(System.out));
while(cin.nextToken() != StreamTokenizer.TT_EOF) ;//EOF
cin.nextToken();int n = (int)cin.nval;String s = cin.sval;
cout.println( Type );cout.flush();
cin.ordinaryChar('/');

BufferedReader br = new BufferedReader(new InputStreamReader(System.in));
br.ready()//EOF
while ((valueString=bf.readLine())!=null);
```

```
23 br.close();
24 //true fast-IO
25 static class InputReader {
26     public BufferedReader reader;
27     public StringTokenizer tokenizer;
28 
29     public InputReader(InputStream stream) {
30         reader = new BufferedReader(new InputStreamReader(stream), 32768);
31         tokenizer = null;
32     }
33 
34     public String next() {
35         while (tokenizer == null || !tokenizer.hasMoreTokens()) {
36             try {
37                 tokenizer = new StringTokenizer(reader.readLine());
38             } catch (IOException e) {
39                 throw new RuntimeException(e);
40             }
41         }
42         return tokenizer.nextToken();
43     }
44 
45     public int nextInt() {
46         return Integer.parseInt(next());
47     }
48 
49 }
50 //类 Number
51 //doubleValue()
52 //intValue()
53 //longValue()
54 //shortValue()
55 //类 BigDecimal
56 //ROUND_CEILING 接近正无穷大的舍入模式。
57 //ROUND_FLOOR 接近负无穷大的舍入模式。
58 //ROUND_DOWN 接近零的舍入模式
59 //ROUND_HALF_UP 四舍五入 >=0.5向上舍入
60 //ROUND_HALF_DOWN 四舍五入 >0.5向上舍入
61 //BigDecimal(BigInteger val)
62 //BigDecimal(BigInteger unscaledVal, int scale)
63 //BigDecimal(char[] in, int offset, int len, MathContext mc)
64 //BigDecimal(double val, MathContext mc)不建议
65 //BigDecimal(int val, MathContext mc)
66 //BigDecimal(long val, MathContext mc)
67 //BigDecimal(String val, MathContext mc)
68 //abs()
69 //add(BigDecimal augend, MathContext mc)
70 //compareTo(BigDecimal val)
71 //divide(BigDecimal divisor,MathContext mc)
72 //divideToIntegralValue(BigDecimal divisor, MathContext mc)
73 //max(BigDecimal val)
74 //min(BigDecimal val)
75 //multiply(BigDecimal multiplicand, MathContext mc)
76 //negate() 其值为 (-this), 其标度为 this.scale()
77 //pow(int n)
78 //remainder(BigDecimal divisor) 返回其值为 (this % divisor) 的 BigDecimal
79 //round(MathContext mc) 返回根据 MathContext 设置进行舍入后的 BigDecimal。
80 //caleByPowerOfTen(int n) 返回其数值等于 (this * 10^n) 的 BigDecimal。
81 //subtract(BigDecimal subtrahend, MathContext mc)
82 //setScale(int newScale,RoundingMode roundingMode)
83 //toString()
```

```
//ulp()返回此 BigDecimal 的 ulp（最后一位的单位）的大小
//String s = b.stripTrailingZeros().toPlainString();让bigdecimal不用科学计数法显示
//类 BigInteger
//parseInt
//BigInteger zero = BigInteger.valueOf(0);
//BigInteger a = in.nextBigInteger();
//abs()
//and(BigInteger val) 返回其值为 (this & val)
//or(BigInteger val) 返回其值为 (this | val)
//andNot(BigInteger val) 返回其值为 (this & ~val)
//compareTo(BigInteger val)
//add(BigInteger val)
//divide(BigInteger val)
//BigInteger[] divideAndRemainder(BigInteger val) 返回包含 (this / val) 后跟 (this % val) 的两个 BigInteger 的数组。
//equals(Object x)
//gcd(BigInteger val)
//isProbablePrime(int certainty) e.g: a.isProbablePrime(4)
//max(BigInteger val) min(BigInteger val)
//mod(BigInteger m)
//modInverse(BigInteger m) 返回其值为 (this^-1 mod m)
//modPow(BigInteger exponent, BigInteger m) 返回其值为 (thisexponent mod m)
//multiply(BigInteger val)
//not() 返回其值为 (~this)
//shiftLeft(int n) 返回其值为 (this << n)
//shiftRight(int n) 返回其值为 (this >> n)
//toString()
//valueOf(long val)
//xor(BigInteger val) 返回其值为 (this ^ val)
//other
//Arrays.sort(array);
```

8.6 pb_ds

```
//P.S.:无脑正确使用pb_ds代替std::set/map/priority_queue不会变慢
//可持久化平衡树,不过时间和空间都不太行
#include <ext/rope>
using namespace __gnu_cxx;
int a[1000];
rope<int> x;
rope<int> x(a,a + n);
rope<int> a(x);
x->at(10);x[10];
x->push_back(x)      // 在末尾添加x
x->insert(pos,x)     // 在pos插入x
x->erase(pos,x)      // 从pos开始删除x个
x->replace(pos,x)    // 从pos开始换成x
x->substr(pos,x)     // 提取pos开始x个

//树
//不支持低级操作（如交换左右子树）
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
定义一颗红黑树
tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update>t, other;
int 关键字类型
null_type
无映射(低版本g++为null_mapped_type) (无映射为类似set, 有映射类似map)
```

```
less<int>从小到大排序
rb_tree_tag 红黑树（splay_tree_tag）
tree_order_statistics_node_update结点更新(统计子树size，可自写)，不写不支持order_of_key 以及find_by_order
插入:t.insert();
删除:t.erase();
比x小的个数:t.order_of_key(x);
第x+1值:t.find_by_order(x);
前驱:t.lower_bound();
后继:t.upper_bound();
合并:t.join(other); (other和*this值域不能相交)
分裂:t.split(x, other); (清空other，将t中比x小的元素移至other)

//自定义节点更新
template <class Node_CItr , class Node_Itr , class Cmp_Fn , class _Alloc >
struct my_node_update {
    virtual Node_CItr node_begin () const = 0;
    virtual Node_CItr node_end() const = 0;
    typedef char metadata_type;//节点上记录的额外信息的类型
    //以上为固定格式

    //operator()的功能是将节点it的信息更新为其左右孩子的信息之和，传入的end_it表示空节点
    //对Node_Itr可以做的事情有：用get_l_child,get_r_child获取左右孩子，用两个星号获取节点信息，用get_metadata获取节点额外信息
    inline void operator()(Node_Itr it, Node_CItr end_it) {
        Node_Itr l = it.get_l_child(), r = it.get_r_child();
        int left = 0, right = 0;
        if(l != end_it) left = l.get_metadata();
        if(r != end_it) right = r.get_metadata();
        const_cast<metadata_type &>(it.get_metadata()) = left + right + (*it)->second;
        //it是node_Itr，取*后变为iterator，再取->second变成mapped_value
    }
    inline int prefix_sum(int x) {
        int ans = 0;
        Node_CItr it = node_begin ();
        while(it != node_end()) {
            Node_CItr l = it.get_l_child(), r = it.get_r_child();
            if(Cmp_Fn()(x, (*it)->first)) it = l;
            else {
                ans += (*it)->second;
                if(l != node_end()) ans += l.get_metadata();
                it = r;
            }
        }
        return ans;
    }
    inline int interval_sum(int l, int r) {
        return prefix_sum(r) - prefix_sum(l - 1);
    }
};

tree<int, char, less<int>, rb_tree_tag, my_node_update> T;//map
int main() {
    T[2] = 'a'; T[3] = 'b'; T[4] = 1;
    cout << (char)T.interval_sum(3, 4) << endl;//c
    return 0;
}
//堆
#include <ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;
__gnu_pbds::priority_queue<int, std::less<int>, __gnu_pbds::pairing_heap_tag> q;
```

```

template <typename Value_Type ,
typename Cmp_Fn = std::less<Value_Type>,
typename Tag = pairing_heap_tag ,
typename Allocator = std::allocator<char> >
class priority_queue
Tag可以是binary_heap_tag（二叉堆）binomial_heap_tag（二项堆）rc_binomial_heap_tag
    pairing_heap_tag（配对堆）thin_heap_tag
用begin()和end()获取迭代器从而遍历
删除单个元素 void erase(point_iterator)
更改一个元素的值 void modify(point_iterator, const_reference)
合并 void join(priority_queue &other)，把other合并到*this，并把other清空
push()会返回迭代器
五种操作：push、pop、modify、erase、join
• pairing_heap_tag：push和joinO(1)，其余均摊O(logn)
• binary_heap_tag：只支持push和pop，均为均摊O(logn)
• binomial_heap_tag：push为均摊O(1)，其余为O(logn)
• rc_binomial_heap_tag：push为O(1)，其余为O(logn)
• thin_heap_tag：push为O(1)，不支持join，其余为O(logn)；但是如果只有increase_key，modify
    均摊O(1)
• 不支持不是不能用，而是用起来很慢
经过实践检测得到的结论：
• Dijkstra算法中应用pairing_heap_tag，速度与手写数据结构相当。
• binary_heap_tag在绝大多数情况下优于std::priority_queue
• pairing_heap_tag在绝大多数情况优于binomial_heap_tag和rc_binomial_heap_tag
• 只有push，pop和join操作时，binary_heap_tag速度较快
• 有modify操作时，可以考虑thin_heap_tag或者pairing_heap_tag，或手写数据结构。

//hash_table
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/hash_policy.hpp>
using namespace __gnu_pbds;
__gnu_pbds::cc_hash_table <Key, Mapped> mp; //使用链地址法解决哈希冲突
__gnu_pbds::gp_hash_table <Key, Mapped> mp; //使用探测法解决哈希冲突
//用法和map一样
```