

SOUTH CHINA UNIVERSITY OF TECHNOLOGY

SCUT_GUGUGU

TEMPLATE



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1 Graph Theory

1.1 Shortest Path

1.1.1 Dijkstra

```

1  typedef long long LL;
2  const int MAXN = ;
3  const int MAXM = ;
4  const LL DINF = ;
5  typedef pair<LL, int> P;
6  struct Edge {
7      int to, nxt;
8      LL w;
9  }e[MAXM];
10 int head[MAXN], ecnt;
11 LL d[MAXN];
12 priority_queue<P, vector<P>, greater<P> > q;
13 inline void addEdge(int x, int y, LL w) {
14     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
15 }
16 void dijkstra(int st, int n) {
17     for(int i = 0; i <= n; i++) d[i] = DINF;
18     d[st] = 0;
19     q.push(make_pair(0, st));
20     while(!q.empty()) {
21         P x = q.top(); q.pop();
22         int u = x.second;
23         if(d[u] != x.first) continue;
24         for(int i = head[u], v; i; i = e[i].nxt) {
25             v = e[i].to;
26             if(d[v] > d[u] + e[i].w) {
27                 d[v] = d[u] + e[i].w;
28                 q.push(make_pair(d[v], v));
29             }
30         }
31     }
32 }

```

1.1.2 SPFA

```

1  struct Edge {
2      int to, nxt;
3      LL w;
4  }e[MAXE];
5  int head[MAXN], ecnt;
6  LL d[MAXN];
7  bool exist[MAXN];
8  queue<int> q;
9  inline void addEdge(int x, int y, LL w) {
10     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
11 }
12 void SPFA(int st) {
13     memset(d, 0x3f, sizeof(d));
14     d[st] = 0;
15     q.push(st);
16     exist[st] = 1;
17     while(!q.empty()) {

```

```

18     int u = q.front(); q.pop();
19     exist[u] = 0;
20     for(int i = head[u], v; i; i = e[i].nxt) {
21         v = e[i].to;
22         if(d[v] > d[u] + e[i].w) {
23             d[v] = d[u] + e[i].w;
24             //pre[v] = u;
25             if(!exist[v]) {
26                 q.push(v);
27                 exist[v] = 1;
28             }
29         }
30     }
31 }
32 }

```

1.2 Johnson

```

1 void johnson() {
2     //建图中, Edge需要from, w1, w2, 去掉w;
3     spfa(1);
4     for(int u = 1; u <= n; u++)
5         for(int i = head[u]; i; i = e[i].nxt)
6             e[i].w2 = e[i].w1 + d[e[i].from] - d[e[i].to];
7     dijkstra(s, n);
8 }

```

1.3 Network Flow

1.3.1 ISAP

```

1 namespace NWF {
2     struct Edge{
3         int to, nxt; LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], pre[MAXN], num[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(num, 0, (tot + 1) * sizeof(int));
11        memset(head, 0, (tot + 1) * sizeof(int));
12    }
13    inline void addEdge(int u, int v, LL f) {
14        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
15        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
16    }
17    void bfs() {
18        memset(dis, 0, (tot + 1) * sizeof(int));
19        q.push(T);
20        dis[T] = 1;
21        while(!q.empty()) {
22            int u = q.front(), v; q.pop();
23            num[dis[u]]++;
24            for(int i = cur[u] = head[u]; i; i = e[i].nxt) {
25                if(!dis[v = e[i].to]) {
26                    dis[v] = dis[u] + 1;

```

```

27         q.push(v);
28     }
29 }
30 }
31 }
32 LL augment() {
33     LL flow = INF;
34     for(int i = S; i != T; i = e[cur[i]].to)
35         flow = min(flow, e[cur[i]].f);
36     for(int i = S; i != T; i = e[cur[i]].to) {
37         e[cur[i]].f -= flow;
38         e[cur[i] ^ 1].f += flow;
39     }
40     return flow;
41 }
42 LL isap() {
43     bfs();
44     int u = S, v;
45     LL flow = 0;
46     while(dis[S] <= tot) {
47         if(u == T) {
48             flow += augment();
49             u = S;
50         }
51         bool fg = 0;
52         for(int i = cur[u]; i; i = e[i].nxt) {
53             if(e[i].f && dis[u] > dis[v = e[i].to]) {
54                 pre[v] = u;
55                 cur[u] = i;
56                 u = v;
57                 fg = 1;
58                 break;
59             }
60         }
61         if(fg) continue;
62         if(!--num[dis[u]]) break;
63         int maxDis = tot;
64         for(int i = head[u]; i; i = e[i].nxt) {
65             if(e[i].f && maxDis > dis[v = e[i].to]) {
66                 maxDis = dis[v];
67                 cur[u] = i;
68             }
69         }
70         num[dis[u] = maxDis + 1]++;
71         if(u != S) u = pre[u];
72     }
73     return flow;
74 }
75 }

```

1.3.2 HLPP

```

1 namespace NWF{
2     struct Edge{
3         int to,nxt;LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], dis[MAXN], num[MAXN];
7     LL sumf[MAXN];
8     queue<int> q;

```

```

9      list<int> dep[MAXN];
10     void init(int _S,int _T,int _tot){
11         ecnt = 1;S = _S;T = _T;tot = _tot;
12         memset(num, 0, (tot + 1) * sizeof(int));
13         memset(head, 0, (tot + 1) * sizeof(int));
14         memset(sumf, 0, (tot + 1) * sizeof(LL));
15     }
16     void addEdge(int u,int v,LL f){
17         e[++ecnt] = (Edge) {v, head[u], f};head[u] = ecnt;
18         e[++ecnt] = (Edge) {u, head[v], 0};head[v] = ecnt;
19     }
20     void bfs(){
21         memset(dis, 0, (tot + 1) * sizeof(int));
22         q.push(T); dis[T] = 1;
23         while(!q.empty()){
24             int u=q.front(), v; q.pop();
25             for(int i = head[u]; i; i = e[i].nxt)
26                 if(!dis[v = e[i].to]){
27                     dis[v] = dis[u] + 1;
28                     q.push(v);
29                 }
30         }
31     }
32     LL hlpp(){
33         bfs();
34         dis[S] = tot + 1;
35         for(int i = 1;i <= tot; ++i)num[dis[i]]++;
36         for(int i = tot + 1; ~i; --i)dep[i].clear();
37         int maxd = dis[S];LL f;
38         dep[maxd].push_back(S);sumf[S] = INF;
39         for(;;){
40             while(maxd && dep[maxd].empty())maxd--;
41             if(!maxd)break;
42             int u = dep[maxd].back(), v;dep[maxd].pop_back();
43             int minDis = tot + 1;
44             for(int i = head[u]; i; i = e[i].nxt)
45                 if(e[i].f){
46                     if(dis[u] > dis[v = e[i].to]){
47                         f = min(sumf[u], e[i].f);
48                         e[i].f -= f;e[i^1].f += f;
49                         if(sumf[u] != INF) sumf[u] -= f;
50                         if(sumf[v] != INF) sumf[v] += f;
51                         if(v!=S && v!=T && sumf[v] == f){
52                             maxd = max(maxd, dis[v]);
53                             dep[dis[v]].push_back(v);
54                         }
55                     }
56                     if(!sumf[u])break;
57                 }else minDis=min(minDis, dis[v] + 1);
58             }
59             if(sumf[u]){
60                 if(!--num[dis[u]]){
61                     for(int i = dis[u];i <= maxd;++i){
62                         while(!dep[i].empty()){
63                             --num[i];
64                             dis[dep[i].back()] = tot + 1;
65                             dep[i].pop_back();
66                         }
67                     }
68                     maxd = dis[u] - 1;dis[u] = tot + 1;
69                 }else{
68                     dis[u] = minDis;

```



```

70         if(minDis > tot)continue;
71         num[minDis]++;
72         maxd = max(maxd, minDis);
73         dep[minDis].push_back(u);
74     }
75 }
76 }
77 return sumf[T];
78 }
79 }

```

1.3.3 Dinic

```

1 namespace NWF {
2     struct Edge {
3         int to, nxt;LL f;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(head, 0, (tot + 1) * sizeof(int));
11    }
12    void addEdge(int u, int v, LL f) {
13        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
14        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
15    }
16    bool bfs() {
17        memset(dis, 0, (tot + 1) * sizeof(int));
18        q.push(S); dis[S] = 1;
19        while (!q.empty()) {
20            int u = q.front(), v; q.pop();
21            for (int i = cur[u] = head[u]; i ; i = e[i].nxt) {
22                if (e[i].f && !dis[v = e[i].to]) {
23                    q.push(v);
24                    dis[v] = dis[u] + 1;
25                }
26            }
27        }
28        return dis[T];
29    }
30    LL dfs(int u, LL maxf) {
31        if (u == T) return maxf;
32        LL sumf = maxf;
33        for (int &i = cur[u]; i; i = e[i].nxt) {
34            if (e[i].f && dis[e[i].to] > dis[u]) {
35                LL tmpf = dfs(e[i].to, min(sumf, e[i].f));
36                e[i].f -= tmpf; e[i ^ 1].f += tmpf;
37                sumf -= tmpf;
38                if (!sumf) return maxf;
39            }
40        }
41        return maxf - sumf;
42    }
43    LL dinic() {
44        LL ret = 0;
45        while (bfs()) ret += dfs(S, INF);
46        return ret;
47    }

```

48 }

1.3.4 MCMF

```

1 namespace NWF{
2     struct Edge {
3         int to, nxt; LL f, c;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN]; LL dis[MAXN];
7     bool exist[MAXN];
8     queue<int> q;
9     void init(int _S, int _T, int _tot){
10         ecnt = 1; S = _S; T = _T; tot = _tot;
11         memset(head, 0, (tot + 1) * sizeof(int));
12     }
13     void addEdge(int u, int v, LL f, LL c) {
14         e[++ecnt] = (Edge) {v, head[u], f, c}; head[u] = ecnt;
15         e[++ecnt] = (Edge) {u, head[v], 0, -c}; head[v] = ecnt;
16     }
17     bool spfa() {
18         for(int i = 0; i <= tot; ++i){
19             dis[i] = INF; exist[i] = cur[i] = 0;
20         }
21         q.push(S); dis[S] = 0; exist[S] = 1;
22         while(!q.empty()) {
23             int u = q.front(), v; q.pop(); exist[u] = 0;
24             for(int i = head[u]; i; i = e[i].nxt) {
25                 if(e[i].f && dis[v = e[i].to] > dis[u] + e[i].c) {
26                     dis[v] = dis[u] + e[i].c;
27                     cur[v] = i;
28                     if(!exist[v]) {
29                         q.push(v);
30                         exist[v] = 1;
31                     }
32                 }
33             }
34         }
35         return dis[T] != INF;
36     }
37     LL mcmf() {
38         LL cost = 0;
39         while(spfa()) {
40             LL flow = INF;
41             for(int i = T; i != S; i = e[cur[i] ^ 1].to)
42                 flow = min(flow, e[cur[i]].f);
43             for(int i = T; i != S; i = e[cur[i] ^ 1].to) {
44                 e[cur[i]].f -= flow;
45                 e[cur[i] ^ 1].f += flow;
46             }
47             cost += flow * dis[T];
48         }
49         return cost;
50     }
51 }

```

1.4 Tree Related

1.4.1 Union Set

```

1 int fa[MAXN], rnk[MAXN];
2 int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
3 bool same(int x, int y){ return Find(x) == Find(y); }
4 void unite(int x, int y)
5 {
6     x = Find(x);
7     y = Find(y);
8     if(x == y) return;
9     if(rnk[x] < rnk[y]) {
10         fa[x] = y;
11     }
12     else {
13         fa[y] = x;
14         if(rnk[x] == rnk[y]) rnk[x]++;
15     }
16 }

```

1.4.2 Kruskal

```

1 namespace MST{
2     struct Edge{
3         int u,v; LL w;
4         bool operator < (const Edge& x) const { return w < x.w; }
5     }e[MAXM];
6     int ecnt, fa[MAXN];
7     void addEdge(int u, int v, LL w) {
8         e[++ecnt] = (Edge){v, u, w}; headp[u] = ecnt;
9     }
10    int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
11    LL kruskal(int n) {
12        sort(e + 1, e + ecnt + 1);
13        for(int i = 1; i <= n; i++) fa[i] = i;
14        LL sum = 0;
15        for (int i = 1; i <= ecnt; i++){
16            int fu = Find(e[i].u), fv = Find(e[i].v);
17            if(fu != fv){
18                fa[fu] = fv;
19                sum += e[i].w;
20            }
21        }
22        return sum;
23    }
24 }

```

1.4.3 Prim

```

1 namespace MST {
2     struct Edge{
3         int to,nxt; LL w;
4     }e[MAXM];
5     int ecnt, head[MAXN], vis[MAXN]; // pre[MAXN];
6     LL dis[MAXN];
7     void addEdge(int u, int v, LL w){

```

```

8      e[++ecnt] = (Edge){v, head[u], w}; head[u] = ecnt;
9      e[++ecnt] = (Edge){u, head[v], w}; head[v] = ecnt;
10     }
11     LL Prim(int n){
12         for (int i = 1; i <= n; i++){
13             //pre[i] = 0;
14             vis[i] = 0;
15             dis[i] = INF;
16         }
17         vis[1] = 1;
18         LL sum = 0;
19         for (int i = head[1]; i; i = e[i].nxt)
20             dis[e[i].to] = min(dis[e[i].to], e[i].w);
21         for (int j = 1; j < n; j++){
22             int u; LL minDis = INF;
23             for (int i = 1; i <= n; ++i)
24                 if (!vis[i] && dis[i] < minDis){
25                     minDis = dis[i];
26                     u = i;
27                 }
28             if (minDis == INF) return -1;
29             vis[u] = 1;
30             sum += minDis;
31             for (int i = head[u]; i; i = e[i].nxt)
32                 if (!vis[v = e[i].to] && e[i].w < dis[v]){
33                     //pre[u] = v;
34                     dis[v] = e[i].w;
35                 }
36         }
37         return sum;
38     }
39 }

```

1.4.4 Tree Divide and Conquer

```

1 struct Edge {
2     int to, nxt, w;
3 }e[MAXM];
4 int head[MAXN], ecnt;
5 int sz[MAXN];
6 int d[MAXN], t[5], ans;
7 bool vis[MAXN];
8 inline void add_edge(int u, int v, int w) {
9     e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
10    e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
11 }
12 int getsz(int x, int fa) {
13     sz[x] = 1;
14     for(int i = head[x]; i; i = e[i].nxt) {
15         int y = e[i].to;
16         if(vis[y] || y == fa) continue;
17         sz[x] += getsz(y, x);
18     }
19     return sz[x];
20 }
21 int getrt(int x) {
22     int tot = getsz(x, 0) >> 1;
23     while(1) {
24         int u = -1;
25         for(int i = head[x]; i; i = e[i].nxt) {

```

```

26         int y = e[i].to;
27         if(vis[y] || sz[y] > sz[x]) continue;
28         if(u == -1 || sz[y] > sz[u]) u = y;
29     }
30     if(~u && sz[u] > tot) x = u;
31     else break;
32 }
33 return x;
34 }
35 void getdep(int x, int fa) {
36     t[d[x]]++;
37     for(int i = head[x]; i; i = e[i].nxt) {
38         int y = e[i].to;
39         if(vis[y] || y == fa) continue;
40         d[y] = (d[x] + e[i].w) % 3;
41         getdep(y, x);
42     }
43 }
44 int cal(int x, int v) {
45     t[0] = t[1] = t[2] = 0;
46     d[x] = v % 3;
47     getdep(x, 0);
48     return t[0] * t[0] + t[1] * t[2] * 2;
49 }
50 void solve(int x) {
51     vis[x] = 1;
52     ans += cal(x, 0);
53     for(int i = head[x]; i; i = e[i].nxt) {
54         int y = e[i].to;
55         if(vis[y]) continue;
56         ans -= cal(y, e[i].w);
57         solve(getrt(y));
58     }
59 }
60 int main() {
61     solve(getrt(1));
62 }

```

1.5 LCA

1.5.1 Tree Decomposition LCA

```

1  int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
2  struct Edge {
3      int to, nxt;
4  }e[MAXN << 1];
5  int head[MAXN], ecnt;
6  inline void add_edge(int x, int y) {
7      e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
8  }
9  void dfs1(int x) {
10     sz[x] = 1; son[x] = 0;
11     for(int i = head[x]; i; i = e[i].nxt) {
12         int v = e[i].to;
13         if(v == fa[x]) continue;
14         fa[v] = x;
15         dep[v] = dep[x] + 1;
16         dfs1(v);
17         sz[x] += sz[v];

```

```

18     if(sz[v] > sz[son[x]]) son[x] = v;
19 }
20 }
21 void dfs2(int x) {
22     B[num[x]] = A[x];
23     if(son[x]) {
24         top[son[x]] = top[x];
25         num[son[x]] = ++totw;
26         dfs2(son[x]);
27     }
28     for(int i = head[x]; i; i = e[i].nxt) {
29         int v = e[i].to;
30         if(v == fa[x] || v == son[x]) continue;
31         top[v] = v;
32         num[v] = ++totw;
33         dfs2(v);
34     }
35 }
36 int lca(int u, int v) {
37     if(u == v) return u;
38     while(top[u] != top[v]) {
39         if(dep[top[u]] > dep[top[v]]) swap(u, v);
40         v = fa[top[v]];
41     }
42     if(dep[u] > dep[v]) swap(u, v);
43     return u;
44 }
45 inline void init() {
46     memset(head, 0, sizeof(head)); ecnt = 0;
47     fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
48 }
49 inline void pre() {
50     dfs1(1); dfs2(1);
51 }

```

1.5.2 Tarjan LCA

```

1  vector< pair<int,int> > G[MAXN],ask[MAXN];
2  int fa[MAXN], ans[MAXN], vis[MAXN],dis[MAXN];
3  int Find(int x){
4      return x == fa[x] ? x : fa[x] = Find(fa[x]);
5  }
6  void init(int n){
7      memset(ans, 0, sizeof ans);
8      memset(vis, 0, sizeof vis);
9      for(int i = 0; i <= n; i++){
10         G[i].clear();
11         ask[i].clear();
12     }
13 }
14 void LCA(int u){
15     int v;
16     fa[u] = u;
17     vis[u] = true;
18     for(auto it : ask[u])
19         if(vis[v = it.first])
20             ans[it.second] = dis[u] + dis[v] - 2 * dis[Find(it.first)];
21     for(auto it : G[u])
22         if(!vis[v = it.first]){
23             dis[v] = dis[u] + it.second;

```

```

24     LCA(v);
25     fa[v] = u;
26 }
27 }

```

1.6 Tarjan

1.6.1 SCC

```

1 namespace SCC{
2     vector<int> G[MAXN];
3     int dfs_clock, scc_cn, dfn[MAXN], low[MAXN], sccno[MAXN];
4     stack<int> S;
5     void addEdge(int u, int v) {
6         G[u].push_back(v);
7     }
8     void tarjan(int u) {
9         dfn[u] = low[u] = ++dfs_clock;
10        S.push(u);
11        for(auto v : G[u]) {
12            if(!dfn[v]) {
13                tarjan(v);
14                low[u] = min(low[u], low[v]);
15            }else if(!sccno[v]) {
16                low[u] = min(low[u], dfn[v]);
17            }
18        }
19        if(dfn[u] == low[u]) {
20            scc_cnt++;
21            for(;;) {
22                int v = S.top(); S.pop();
23                sccno[v] = scc_cnt;
24                if(v == u) break;
25            }
26        }
27    }
28    void findSCC(int n) {
29        for(int i = 1; i <= n; i++)
30            if(!dfn[i]) tarjan(i);
31    }
32    void init(int n){
33        dfs_clock = scc_cnt = 0;
34        for(int i = 0; i <= n; ++i){
35            dfn[i] = low[i] = sccno[i] = 0;
36            G[i].clear();
37        }
38    }
39 }

```

1.6.2 BCC

```

1 namespace BCC{
2     struct Edge {
3         int to, nxt;
4     }e[MAXM << 1];
5     int ecnt, head[MAXN];
6     int dfs_clock, dfn[MAXN], low[MAXN];
7

```

```

8   int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
9   vector<int> vbcc[MAXN];
10  stack<int> vS;
11
12  int ebcc_cnt, ebccno[MAXN];
13  stack<int> eS;
14
15  inline void addEdge(int u, int v) {
16      e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
17      e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
18  }
19  inline void init(int n) {
20      ecnt = 1;
21      dfs_clock = 0;
22      vbcc_cnt = 0;
23      ebcc_cnt = 0;
24      for(int i = 1; i <= n; ++i){
25          head[i] = dfn[i] = low[i] = 0;
26          is_vertex[i] = 0;
27          vbccno[i] = 0;
28          ebccno[i] = 0;
29      }
30      while(!vS.empty()) vS.pop();
31  }
32  //root's edge = -1;
33  void tarjan(int u, int edge) {
34      dfn[u] = low[u] = ++dfs_clock;
35      int ch = 0;
36      vS.push(u);
37      eS.push(u);
38      for(int i = head[u], v; i; i = e[i].nxt) {
39          if(!dfn[v = e[i].to]) {
40              tarjan(v, i ^ 1);
41              low[u] = min(low[u], low[v]);
42              if(low[v] >= dfn[u]) {
43                  ++ch;
44                  if(edge > 0 || ch > 1) is_vertex[u] = 1;
45                  vbcc[++vbcc_cnt].clear();
46                  vbcc[vbcc_cnt].push_back(u);
47                  for(int x;;){
48                      x = vS.top();vS.pop();
49                      vbcc[vbcc_cnt].push_back(x);
50                      vbccno[x] = vbcc_cnt;
51                      if(x == v)break;
52                  }
53              }
54              if(low[v] > dfn[u]) {
55                  // i && i ^ 1 is bridge
56              }
57          }
58          else if(dfn[v] < dfn[u] && i != edge)
59              low[u] = min(low[u], dfn[v]);
60      }
61      if(dfn[u] == low[u]) {
62          ebcc_cnt++;
63          for(int v;;) {
64              v = eS.top(); eS.pop();
65              ebccno[v] = ebcc_cnt;
66              if(v == u) break;
67          }
68      }

```



```

69     }
70     void findBCC(int n){
71         for(int i = 1; i <= n; i++){
72             if(!dfn[i]) tarjan(i, -1);
73
74             //findBridge
75             for(int u = 1; u <= n; u++) {
76                 for(int i = head[u], v; i; i = e[i].nxt)
77                     if(ebccno[u] != ebccno[v = e[i].to]) {
78                         //is bridge
79                     }
80             }
81         }
82     }

```

1.7 Cactus

1.7.1 Circle-Square Tree

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef pair<int, int> P;
4  const int MAXN = 2e4 + 5;
5  const int S = 15;
6  namespace Tree {
7      struct Edge {
8          int to, nxt, w;
9      }e[MAXN << 1];
10     int ecnt, head[MAXN];
11     int rt, isrt[MAXN], fa[MAXN][S + 3];
12     int sz[MAXN];
13     inline void addEdge(int u, int v, int w) {
14         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
15         fa[v][0] = u;
16     }
17 }
18 int n, m, Q;
19 namespace BCC {
20     struct Edge {
21         int to, nxt, w;
22     }e[MAXN << 1];
23     int ecnt, head[MAXN];
24     int dfs_clock, dfn[MAXN], low[MAXN];
25     int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
26     vector<P> vbcc[MAXN];
27     stack<P> vs;
28     int tag[MAXN];
29     inline void addEdge(int u, int v, int w) {
30         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
31         e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
32     }
33     inline void init(int n) {
34         ecnt = 1;
35         dfs_clock = 0;
36         vbcc_cnt = 0;
37         for(int i = 0; i <= 2 * n; i++){
38             head[i] = dfn[i] = low[i] = 0;
39             vbccno[i] = 0;
40             tag[i] = 0;

```

```

41     }
42     while(!vs.empty()) vs.pop();
43 }
44 //root's edge = -1;
45 void tarjan(int u, int edge) {
46     dfn[u] = low[u] = ++dfs_clock;
47     vs.push(P(u, e[edge ^ 1].w));
48     for(int i = head[u], v; i; i = e[i].nxt) {
49         if(!dfn[v = e[i].to]) {
50             tarjan(v, i ^ 1);
51             low[u] = min(low[u], low[v]);
52             if(low[v] >= dfn[u]) {
53                 if(vs.top().first == v) {
54                     Tree::addEdge(u, v, vs.top().second);
55                     vs.pop();
56                     continue;
57                 }
58                 vbcc[++vbcc_cnt].clear();
59                 vbcc[vbcc_cnt].push_back(P(u, 0));
60                 Tree::isrt[u] = 1;
61                 int &sz = Tree::sz[n + vbcc_cnt];
62                 tag[vs.top().first] = n + vbcc_cnt;
63                 //Tree::addEdge(u, rt, 0);
64                 for(P x;;) {
65                     x = vs.top(); vs.pop();
66                     sz += x.second;
67                     //Tree::addEdge(rt, x.first, sz);
68                     vbcc[vbcc_cnt].push_back(x);
69                     vbccno[x.first] = vbcc_cnt;
70                     if(x.first == v) break;
71                 }
72             }
73         }
74         else if(dfn[v] < dfn[u] && i != edge)
75             low[u] = min(low[u], dfn[v]);
76     }
77     for(int i = head[u], v; i; i = e[i].nxt) {
78         if(tag[v = e[i].to]) {
79             int r = tag[v]; Tree::sz[r] += e[i].w;
80             tag[v] = 0;
81         }
82     }
83 }
84 void findBCC(int n) {
85     for(int i = 1; i <= n; i++)
86         if(!dfn[i]) tarjan(i, -1);
87 }
88 }
89 namespace Tree {
90     int dis[MAXN], dep[MAXN], len[MAXN];
91     inline void init(int n) {
92         BCC::init(n);
93         rt = n;
94         ecnt = 1;
95         for(int i = 0; i <= 2 * n; i++) {
96             head[i] = 0;
97             fa[i][0] = isrt[i] = dis[i] = dep[i] = len[i] = 0;
98         }
99     }
100     void dfs(int x) {
101         for(int i = head[x], y; i; i = e[i].nxt) {

```

```

102         if(!dep[y = e[i].to]) {
103             dep[y] = dep[x] + 1;
104             dis[y] = dis[x] + e[i].w;
105             dfs(y);
106         }
107     }
108 }
109 void pre() {
110     for(int k = 1; k <= BCC::vbcc_cnt; k++) {
111         rt++;
112         vector<P> &E = BCC::vbcc[k];
113         addEdge(E[0].first, rt, 0);
114         int cnt = 0;
115         for(int i = E.size() - 1; i >= 1; i--) {
116             cnt += E[i].second;
117             len[E[i].first] = cnt;
118             addEdge(rt, E[i].first, min(cnt, sz[rt] - cnt));
119         }
120     }
121     for(int k = 1; k <= S; k++) {
122         for(int i = 1; i <= rt; i++) {
123             fa[i][k] = fa[fa[i][k - 1]][k - 1];
124         }
125     }
126     dep[1] = 1;
127     dfs(1);
128 }
129 int up(int x, int d) {
130     for(int i = S; i >= 0; i--) {
131         if(dep[fa[x][i]] >= d) x = fa[x][i];
132     }
133     return x;
134 }
135 int lca(int u, int v) {
136     if(dep[u] > dep[v]) swap(u, v);
137     v = up(v, dep[u]);
138     if(u == v) return u;
139     for(int i = S; i >= 0; i--) {
140         if(fa[u][i] != fa[v][i]) {
141             u = fa[u][i], v = fa[v][i];
142         }
143     }
144     return fa[u][0];
145 }
146 int query(int u, int v) {
147     int l = lca(u, v);
148     if(l <= n) return dis[u] + dis[v] - 2 * dis[l];
149     int x = up(u, dep[l] + 1), y = up(v, dep[l] + 1);
150     int res = dis[u] - dis[x] + dis[v] - dis[y];
151     int tmp = abs(len[x] - len[y]);
152     return res + min(tmp, sz[l] - tmp);
153 }
154 }
155
156 int main() {
157     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
158     using namespace Tree;
159     cin >> n >> m >> Q;
160     init(n);
161     for(int i = 1, u, v, w; i <= m; i++) {
162         cin >> u >> v >> w;

```

```
163     BCC::addEdge(u, v, w);
164 }
165 BCC::findBCC(n);
166 pre();
167 int u, v;
168 while(Q--) {
169     cin >> u >> v;
170     cout << query(u, v) << endl;
171 }
172 return 0;
173 }
```

2 Data Structures

2.1 Basic Structures

2.1.1 RMQ

```

1 struct RMQ {
2     int d[MAXN][S + 2];
3     inline void init(int *a, int n) {
4         for(int i = 1; i <= n; i++) d[i][0] = a[i];
5         for(int k = 1; (1 << k) <= n; k++)
6             for(int i = 1; i + (1 << k) - 1 <= n; i++)
7                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
8     }
9     inline int query(int l, int r) {
10        if(l > r) swap(l, r);
11        int k = 0;
12        while((1 << (k + 1)) <= r - l + 1) k++;
13        return min(d[l][k], d[r - (1 << k) + 1][k]);
14    }
15 }rmq;
16 const int MAXM = 2e5 + 5, MAXN = 3e6 + 5, S = 22;
17 const LL INF = 1e18;
18 #define belong(x) (x / S + 1)
19 #define pos(x) (x % S + 1)
20 int Log[MAXN], sz;
21 struct RMQ {
22     LL a[MAXN];
23     LL d[MAXM][S + 2];
24     LL pre[MAXM][S + 2], aft[MAXM][S + 2];
25     inline void init(int n) {
26         sz = n / S + 1;
27         Log[0] = -1; for(int i = 1; i <= n; i++) Log[i] = Log[i / 2] + 1;
28         for(int i = 1; i <= sz; i++) {
29             pre[i][0] = aft[i][S + 1] = INF;
30         }
31         for(int i = 1; i <= n; i++) {
32             pre[belong(i)][pos(i)] = min(pre[belong(i)][pos(i) - 1], a[i]);
33         }
34         for(int i = n; i >= 1; i--) {
35             aft[belong(i)][pos(i)] = min(aft[belong(i)][pos(i) + 1], a[i]);
36         }
37         for(int i = 1; i <= sz; i++) {
38             d[i][0] = aft[i][1];
39         }
40         for(int k = 1; k <= S; k++)
41             for(int i = 1; i + (1 << k) <= sz; i++)
42                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
43     }
44     inline LL ask(int l, int r) {
45         assert(l <= r);
46         LL res = INF;
47         if(belong(l) == belong(r)) {
48             for(int i = l; i <= r; i++) res = min(res, a[i]);
49             return res;
50         }
51         res = min(aft[belong(l)][pos(l)], pre[belong(r)][pos(r)]);
52         int k = Log[belong(r) - belong(l) - 1];
53         if(~k) {

```

```

54         res = min(res, d[belong(l) + 1][k]);
55         res = min(res, d[belong(r) - (1 << k)][k]);
56     }
57     return res;
58 }
59 }rmq;

```

2.1.2 Divide Blocks

```

1  int belong[MAXN], l[MAXN], r[MAXN];
2  int sz, num;
3  void build(int n) {
4      sz = sqrt(n);
5      num = n / sz; if(n % sz) num++;
6      for(int i = 1; i <= num; i++) {
7          l[i] = (i - 1) * sz + 1;
8          r[i] = i * sz;
9      }
10     r[num] = n;
11     for(int i = 1; i <= n; i++) {
12         belong[i] = (i - 1) / sz + 1;
13     }
14 }

```

2.2 Stack Structures

2.2.1 Cartesian Tree

```

1  struct CartesianTree{
2      int rt, fa[MAXN], ls[MAXN], rs[MAXN];
3      int top, st[MAXN];
4      int cnt[MAXN];
5      void build(LL *a, int n) {
6          top = rt = 0;
7          for(int i = 1; i <= n; i++) {
8              ls[i] = rs[i] = fa[i] = 0;
9              while(top && a[st[top]] > a[i]) ls[i] = st[top--];
10             fa[i] = st[top];
11             if(ls[i]) fa[ls[i]] = i;
12             if(fa[i]) rs[fa[i]] = i; else rt = i;
13             st[++top] = i;
14         }
15     }
16     void dfs(int x) {
17         cnt[x] = 1;
18         if(ls[x]) {dfs(ls[x]); cnt[x] += cnt[ls[x]];}
19         if(rs[x]) {dfs(rs[x]); cnt[x] += cnt[rs[x]];}
20     }
21     LL getAns(LL *a, int n) {
22         //dfs(rt);
23         //—————
24         return res;
25     }
26 }T;

```

2.3 Sequence Structures

2.3.1 Segment Tree

```

1  #define Ls(x) (x << 1)
2
3  #define Rs(x) (x << 1 | 1)
4  struct Tree {
5      int l, r, lazy;
6      LL sum, mx;
7  } tree[MAXN << 2];
8  int A[MAXN];
9  void push_up(int x) {
10     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
11     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
12 }
13 void push_down(int x) {
14     if(tree[x].lazy) {
15         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
16         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
17         tree[Ls(x)].mx += tree[x].lazy;
18         tree[Rs(x)].mx += tree[x].lazy;
19         tree[Ls(x)].lazy += tree[x].lazy;
20         tree[Rs(x)].lazy += tree[x].lazy;
21         tree[x].lazy = 0;
22     }
23 }
24 void build(int x, int L, int R) {
25     tree[x].lazy = 0;
26     tree[x].l = L; tree[x].r = R;
27     if(L == R) {
28         tree[x].sum = A[L];
29         tree[x].mx = A[L];
30
31         return;
32     }
33     int mid = (L + R) >> 1;
34     build(Ls(x), L, mid);
35     build(Rs(x), mid + 1, R);
36     push_up(x);
37 }
38 void update(int x, int L, int R, LL val) {
39     if(tree[x].l >= L && tree[x].r <= R) {
40         tree[x].lazy += val;
41         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
42         tree[x].mx += val;
43         return;
44     }
45     push_down(x);
46     int mid = (tree[x].l + tree[x].r) >> 1;
47     if(L <= mid) update(Ls(x), L, R, val);
48     if(R > mid) update(Rs(x), L, R, val);
49     push_up(x);
50 }
51 LL query(int x, int L, int R) {
52     if(tree[x].l >= L && tree[x].r <= R)
53         return tree[x].sum;
54     push_down(x);
55     int mid = (tree[x].l + tree[x].r) >> 1;
56     LL res = 0;

```

```

57     if(L <= mid) res += query(Ls(x), L, R);
58     if(R > mid) res += query(Rs(x), L, R);
59
60     return res;
61 }
62 LL query2(int x, int L, int R) {
63     if(tree[x].l >= L && tree[x].r <= R)
64         return tree[x].mx;
65     push_down(x);
66     int mid = (tree[x].l + tree[x].r) >> 1;
67     LL res = -INF;
68     if(L <= mid) res = max(res, query2(Ls(x), L, R));
69     if(R > mid) res = max(res, query2(Rs(x), L, R));
70     return res;
71 }

```

2.3.2 LiChao Tree

```

1  const double eps = 1e-12;
2  namespace LiT{
3      const int MLIMIT = 40000;
4      typedef double LD;
5      struct line{LD k,b;int l,r,id;} T[MAXN << 2];
6      //inline LD calc(line &a,int pos) {return a.k*vec[pos]+a.b;}
7      inline LD calc(line &a,int pos) {return a.k*pos+a.b;}
8      inline double cross(line &a,line &b) {
9          if(b.k == a.k) return -1e9;
10         return (double)(a.b-b.b)/(b.k-a.k);
11     }
12     void build(int v, int l, int r) {
13         T[v].k = 0;T[v].b = -1e18;
14         T[v].l = 0;T[v].r = MLIMIT;
15         T[v].id = 0;
16         if(l == r)return;
17         int mid = (l+r)>>1;
18         build(v<<1,l,mid);
19         build(v<<1|1,mid+1,r);
20     }
21     void ins(int v,int l,int r, line k) {
22         if(k.l <= l && r <= k.r) {
23             LD fl = calc(k, l), fr = calc(k, r);
24             LD gl = calc(T[v], l), gr = calc(T[v], r);
25             if(fl - gl > eps && fr - gr > eps) T[v] = k;
26             else if(fl - gl > eps || fr - gr > eps) {
27                 int mid = (l+r)>>1;
28                 if(calc(k, mid) - calc(T[v], mid) > eps) swap(k, T[v]);
29                 //if(vec[mid] - cross(k, T[v]) > eps)
30                 if(mid - cross(k, T[v]) > eps)
31                     ins(v<<1, l, mid, k);else ins(v<<1|1, mid+1, r, k);
32             }
33             return;
34         }
35         int mid=(l+r)>>1;
36         if(k.l <= mid) ins(v<<1, l, mid, k);
37         if(mid < k.r) ins(v<<1|1, mid+1, r, k);
38     }
39     LD ans;int ansid;
40     void que(int v, int l, int r, int x) {
41         LD tmp = calc(T[v], x);
42         if(tmp > ans || (tmp == ans && T[v].id < ansid)) {

```



```

43         ans = tmp;
44         ansid = T[v].id;
45     }
46     if(l == r) return;
47     int mid = (l+r)>>1;
48     if(x <= mid) que(v<<1,l,mid,x);else que(v<<1|1,mid+1,r,x);
49 }
50 };
51 //左闭右闭

```

2.3.3 Splay Tree

```

1 namespace splay{
2     int n, m, sz, rt;
3     int val[MAXN], id[MAXN];
4     int tr[MAXN][2], size[MAXN], fa[MAXN], rev[MAXN], s[MAXN], lazy[MAXN];
5     void push_up(int x) {
6         int l = tr[x][0], r = tr[x][1];
7         s[x] = max(val[x], max(s[l], s[r]));
8         size[x] = size[l] + size[r] + 1;
9     }
10    void push_down(int x) {
11        int l = tr[x][0], r = tr[x][1];
12        if(lazy[x]) {
13            if(l) {
14                lazy[l] += lazy[x];
15                s[l] += lazy[x];
16                val[l] += lazy[x];
17            }
18            if(r) {
19                lazy[r] += lazy[x];
20                s[r] += lazy[x];
21                val[r] += lazy[x];
22            }
23            lazy[x] = 0;
24        }
25        if(rev[x]) {
26            rev[x] = 0;
27            rev[l] ^= 1; rev[r] ^= 1;
28            swap(tr[x][0], tr[x][1]);
29        }
30    }
31    void rotate(int x, int &k) {
32        int y = fa[x];
33        int z = fa[y];
34        int l, r;
35        if(tr[y][0] == x) l = 0;
36        else l = 1;
37        r = l ^ 1;
38        if(y == k) k = x;
39        else {
40            if(tr[z][0] == y) tr[z][0] = x;
41            else tr[z][1] = x;
42        }
43        fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
44        tr[y][l] = tr[x][r]; tr[x][r] = y;
45        push_up(y); push_up(x);
46    }
47    void splay(int x, int &k) {
48        int y, z;

```

```

49     while(x != k) {
50         y = fa[x];
51         z = fa[y];
52         if(y != k) {
53             if((tr[y][0] == x) ^ (tr[z][0] == y)) rotate(x, k);
54
55             else rotate(y, k);
56         }
57         rotate(x, k);
58     }
59 }
60 int find(int x, int rank) {
61     push_down(x);
62
63     int l = tr[x][0], r = tr[x][1];
64     if(size[l] + 1 == rank) return x;
65     else if(size[l] >= rank) return find(l, rank);
66     else return find(r, rank - size[l] - 1);
67 }
68 void update(int l, int r, int v) {
69     int x = find(rt, l), y = find(rt, r + 2);
70     splay(x, rt); splay(y, tr[x][1]);
71     int z = tr[y][0];
72     lazy[z] += v;
73     val[z] += v;
74     s[z] += v;
75 }
76 void reverse(int l, int r) {
77     int x = find(rt, l), y = find(rt, r + 2);
78     splay(x, rt); splay(y, tr[x][1]);
79     int z = tr[y][0];
80     rev[z] ^= 1;
81 }
82 void query(int l, int r) {
83     int x = find(rt, l), y = find(rt, r + 2);
84     splay(x, rt); splay(y, tr[x][1]);
85     int z = tr[y][0];
86     printf("%d\n", s[z]);
87 }
88 void build(int l, int r, int f) {
89     if(l > r) return;
90     int now = id[l], last = id[f];
91     if(l == r) {
92         fa[now] = last; size[now] = 1;
93         if(l < f) tr[last][0] = now;
94         else tr[last][1] = now;
95         return;
96     }
97     int mid = (l + r) >> 1; now = id[mid];
98     build(l, mid - 1, mid); build(mid + 1, r, mid);
99     fa[now] = last;
100    push_up(now);
101    if(mid < f) tr[last][0] = now;
102    else tr[last][1] = now;
103 }
104 void init() {
105     s[0] = -INF;
106     scanf("%d%d", &n, &m);
107     for(int i = 1; i <= n + 2; i++) id[i] = ++sz;
108     build(1, n + 2, 0); rt = (n + 3) >> 1;
109 }

```

110 }

2.4 Persistent Data Structures

2.4.1 Chairman Tree

```

1
2 struct Node {
3     int l, r;
4
5     LL sum;
6 }t[MAXN * 40];
7 int cnt, n;
8 int rt[MAXN];
9 void update(int pre, int &x, int l, int r, int v) {
10     x = ++cnt; t[x] = t[pre]; t[x].sum++;
11     if(l == r) return;
12     int mid = (l + r) >> 1;
13     if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
14     else update(t[pre].r, t[x].r, mid + 1, r, v);
15 }
16 int query(int x, int y, int l, int r, int v) {
17     if(l == r) return l;
18     int mid = (l + r) >> 1;
19     int sum = t[t[y].l].sum - t[t[x].l].sum;
20     if(sum >= v) return query(t[x].l, t[y].l, l, mid, v);
21     else return query(t[x].r, t[y].r, mid + 1, r, v - sum);
22 }

```

2.4.2 Persistent Trie

```

1 //区间异或最值查询
2 const int N=5e4+10;
3 int t[N];
4 int ch[N*32][2],val[N*32];
5 int cnt;
6 void init(){
7     mem(ch,0);
8     mem(val,0);
9     cnt=1;
10 }
11 int add(int root,int x){
12     int newroot=cnt++,ret=newroot;
13     for(int i=30;i>=0;i--){
14         ch[newroot][0]=ch[root][0];
15         ch[newroot][1]=ch[root][1];
16         int now=(x>>i)&1;
17         root=ch[root][now];
18
19         ch[newroot][now]=cnt++;
20         newroot=ch[newroot][now];
21         val[newroot]=val[root]+1;
22     }
23
24     return ret;
25 }
26 int query(int lt,int rt,int x){
27     int ans=0;

```

```

28     for(int i=30;i>=0;i--){
29         int now=(x>>i)&1;
30         if(val[ch[rt][now^1]]-val[ch[lr][now^1]]){
31             ans+=(1<<i);
32             rt=ch[rt][now^1];
33             lr=ch[lr][now^1];
34         } else{
35             rt=ch[rt][now];
36             lr=ch[lr][now];
37         }
38     }
39     return ans;
40 }

```

2.5 Tree Structures

2.5.1 Tree Decomposition

```

1  int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
2  struct Edge {
3      int to, nxt;
4  }e[MAXN << 1];
5  int head[MAXN], ecnt;
6  int n, m, Q;
7  #define Ls(x) (x << 1)
8  #define Rs(x) (x << 1 | 1)
9  struct Tree {
10     int l, r, lazy;
11     LL sum, mx;
12 }tree[MAXN << 2];
13 int A[MAXN], B[MAXN];
14 void push_up(int x) {
15     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
16     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
17 }
18 void push_down(int x) {
19     if(tree[x].lazy) {
20         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
21         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
22         tree[Ls(x)].mx += tree[x].lazy;
23         tree[Rs(x)].mx += tree[x].lazy;
24         tree[Ls(x)].lazy += tree[x].lazy;
25         tree[Rs(x)].lazy += tree[x].lazy;
26         tree[x].lazy = 0;
27     }
28 }
29 void build(int x, int L, int R) {
30     tree[x].lazy = 0;
31     tree[x].l = L; tree[x].r = R;
32     if(L == R) {
33         tree[x].sum = B[L];
34         tree[x].mx = B[L];
35         return;
36     }
37     int mid = (L + R) >> 1;
38     build(Ls(x), L, mid);
39     build(Rs(x), mid + 1, R);
40     push_up(x);
41 }

```

```

42 void update(int x, int L, int R, LL val) {
43     if(tree[x].l >= L && tree[x].r <= R) {
44         tree[x].lazy += val;
45         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
46         tree[x].mx += val;
47         return;
48     }
49     push_down(x);
50     int mid = (tree[x].l + tree[x].r) >> 1;
51     if(L <= mid) update(Ls(x), L, R, val);
52     if(R > mid) update(Rs(x), L, R, val);
53     push_up(x);
54 }
55 LL query(int x, int L, int R) {
56     if(tree[x].l >= L && tree[x].r <= R)
57         return tree[x].sum;
58     push_down(x);
59     int mid = (tree[x].l + tree[x].r) >> 1;
60     LL res = 0;
61     if(L <= mid) res += query(Ls(x), L, R);
62     if(R > mid) res += query(Rs(x), L, R);
63     return res;
64 }
65 LL query2(int x, int L, int R) {
66     if(tree[x].l >= L && tree[x].r <= R)
67         return tree[x].mx;
68     push_down(x);
69     int mid = (tree[x].l + tree[x].r) >> 1;
70     LL res = -INF;
71     if(L <= mid) res = max(res, query2(Ls(x), L, R));
72     if(R > mid) res = max(res, query2(Rs(x), L, R));
73     return res;
74 }
75 inline void add_edge(int x, int y) {
76     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
77 }
78 void dfs1(int x) {
79     sz[x] = 1; son[x] = 0;
80     for(int i = head[x]; i; i = e[i].nxt) {
81         int v = e[i].to;
82         if(v == fa[x]) continue;
83         fa[v] = x;
84         dep[v] = dep[x] + 1;
85         dfs1(v);
86         sz[x] += sz[v];
87         if(sz[v] > sz[son[x]]) son[x] = v;
88     }
89 }
90 void dfs2(int x) {
91     B[num[x]] = A[x];
92     if(son[x]) {
93         top[son[x]] = top[x];
94         num[son[x]] = ++totw;
95         dfs2(son[x]);
96     }
97     for(int i = head[x]; i; i = e[i].nxt) {
98         int v = e[i].to;
99         if(v == fa[x] || v == son[x]) continue;
100         top[v] = v;
101         num[v] = ++totw;
102         dfs2(v);

```

```

103     }
104 }
105 void up(int a, int b, int c) {
106     int f1 = top[a], f2 = top[b];
107     while(f1 != f2) {
108         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
109         update(1, num[f1], num[a], c);
110         a = fa[f1];
111         f1 = top[a];
112     }
113     if(dep[a] > dep[b]) swap(a, b);
114     update(1, num[a], num[b], c);
115 }
116 int qsum(int a, int b) {
117     if(a == b) return query(1, num[a], num[a]);
118     int f1 = top[a], f2 = top[b];
119     int res = 0;
120     while(f1 != f2) {
121         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
122         res += query(1, num[f1], num[a]);
123         a = fa[f1];
124         f1 = top[a];
125     }
126     if(dep[a] > dep[b]) swap(a, b);
127     res += query(1, num[a], num[b]);
128     return res;
129 }
130 int qmax(int a, int b) {
131     if(a == b) return query2(1, num[a], num[a]);
132     int f1 = top[a], f2 = top[b];
133     int res = -1000000000;
134     while(f1 != f2) {
135         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
136         res = max(res, query2(1, num[f1], num[a]));
137         a = fa[f1];
138         f1 = top[a];
139     }
140     if(dep[a] > dep[b]) swap(a, b);
141     res = max(res, query2(1, num[a], num[b]));
142     return res;
143 }
144 inline void init() {
145     memset(head, 0, sizeof(head)); ecnt = 0;
146     fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
147 }
148 inline void pre() {
149     dfs1(1); dfs2(1); build(1, 1, totw);
150 }

```

2.5.2 Link-Cut Tree

```

1 namespace LCT {
2     int fa[MAXN], rev[MAXN], tr[MAXN][2];
3     int s[MAXN], val[MAXN];
4     void push_up(int x) {
5         int l = tr[x][0], r = tr[x][1];
6         s[x] = s[l] + s[r] + val[x];
7     }
8     void Rev(int x) {
9         rev[x] ^= 1; swap(tr[x][0], tr[x][1]);

```

```

10 }
11 void push_down(int x) {
12     if(!rev[x]) return;
13     int l = tr[x][0], r = tr[x][1];
14     rev[x] = 0;
15     if(l) Rev(l); if(r) Rev(r);
16 }
17 bool isroot(int x) {
18     return tr[fa[x]][0] != x && tr[fa[x]][1] != x;
19 }
20 void pre(int x) {
21     if(!isroot(x)) pre(fa[x]);
22     push_down(x);
23 }
24 void rotate(int x) {
25     int y = fa[x]; int z = fa[y];
26     int l = tr[y][1] == x;
27     int r = l ^ 1;
28     if(!isroot(y)) tr[z][tr[z][1] == y] = x;
29     fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
30     tr[y][l] = tr[x][r]; tr[x][r] = y;
31     push_up(y);
32 }
33 void splay(int x) {
34     pre(x);
35     int y, z;
36     while(!isroot(x)) {
37         y = fa[x]; z = fa[y];
38         if(!isroot(y)) {
39             if((tr[z][0] == y) == (tr[y][0] == x)) rotate(y);
40             else rotate(x);
41         }
42         rotate(x);
43     }
44     push_up(x);
45 }
46 void access(int x) {
47     int y = 0;
48     while(x) {
49         splay(x); tr[x][1] = y;
50         push_up(x);
51         y = x; x = fa[x];
52     }
53 }
54 void makeroot(int x) {
55     access(x); splay(x); Rev(x);
56 }
57 void lnk(int x, int y) {
58     makeroot(x); fa[x] = y;
59 }
60 void cut(int x, int y) {
61     makeroot(x); access(y); splay(y);
62     tr[y][0] = fa[x] = 0; push_up(y);
63 }
64 void update(int x, int y) {
65     makeroot(x); val[x] = y; push_up(x);
66 }
67 int query(int x, int y) {
68     makeroot(x); access(y); splay(y);
69     return s[y];
70 }

```

```
71  bool check(int x, int y) {  
72      int tmp = y;  
73      makeroot(x); access(y); splay(x);  
74      while(!isroot(y)) y = fa[y];  
75      splay(tmp);  
76      return x == y;  
77  }  
78 }
```


3 String

3.1 Basics

3.1.1 Hash

```

1  const LL p1 = 201, p2 = 301, mod1 = 1200000319, mod2 = 2147483647;
2  struct Hash {
3      LL a, b;
4      void append(Hash pre, int v) {
5          a = (pre.a * p1 + v) % mod1;
6          b = (pre.b * p2 + v) % mod2;
7      }
8      void init(string S) {
9          a = b = 0;
10         for(int i = 0; i < S.size(); i++) append(*this, S[i]);
11     }
12     bool operator == (const Hash &x) const {
13         return a == x.a && b == x.b;
14     }
15     bool operator < (const Hash &x) const {
16         return a < x.a || (a == x.a && b < x.b);
17     }
18 };

```

3.1.2 KMP && exKMP

```

1  namespace KMP {
2      int fa[MAXN];
3      void get_fail(char* t, int tn) {
4          fa[0] = -1;
5          int i = 0, j = -1;
6          while(i < tn) {
7              if (j == -1 || t[i] == t[j]) {
8                  ++i; ++j;
9                  fa[i] = t[i] != t[j] ? j : fa[j];
10             }else{
11                 j = fa[j];
12             }
13         }
14     }
15     void kmp(char* s, int sn, char* t, int tn) {
16         int i = 0, j = 0;
17         while(i < sn) {
18             if (j == -1 || s[i] == t[j]) {
19                 i++; j++;
20                 if(j == tn) {
21                     }
22             }else j = fa[j];
23         }
24     }
25 }
26 namespace exKMP {
27     int nxt[MAXN], ext[MAXN];
28     void get_nxt(char* t, int tn) {
29         int j = 0, mx = 0;
30         nxt[0] = tn;
31         for(int i = 1; i < tn; i++) {

```

```

32         if(i >= mx || i + nxt[i - j] >= mx) {
33             if(i > mx) mx = i;
34             while(mx < tn && t[mx] == t[mx - i]) mx++;
35             nxt[i] = mx - i;
36             j = i;
37         }else nxt[i] = nxt[i - j];
38     }
39 }
40 void exkmp(char *s, int sn, char *t, int tn) {
41     int j = 0, mx = 0;
42     for(int i = 0; i < sn; i++) {
43         if(i >= mx || i + nxt[i - j] >= mx) {
44             if(i > mx) mx = i;
45             while(mx < sn && mx - i < tn && s[mx] == t[mx - i]) mx++;
46             ext[i] = mx - i;
47             j = i;
48         }else ext[i] = nxt[i - j];
49     }
50 }
51 }

```

3.1.3 AC Automaton

```

1 namespace AC {
2     int ch[MAXN][sigma_size], last[MAXN];
3     int val[MAXN], f[MAXN], sz;
4     inline void init() { sz = 1; memset(ch[0], 0, sizeof(ch[0])); }
5     inline int idx(char c) { return c - 'a'; }
6     void insert(string s, int v) {
7         int u = 0;
8         for(int i = 0; i < s.size(); i++) {
9             int c = idx(s[i]);
10            if(!ch[u][c]) {
11                memset(ch[sz], 0, sizeof(ch[sz]));
12                val[sz] = 0;
13                ch[u][c] = sz++;
14            }
15            u = ch[u][c];
16        }
17        val[u] = v;
18    }
19    void get_fail() {
20        queue<int> q;
21        f[0] = 0;
22        for(int c = 0; c < sigma_size; c++) {
23            int u = ch[0][c];
24            if(u) { f[u] = 0; q.push(u); last[u] = 0; }
25        }
26        while(!q.empty()) {
27            int r = q.front(); q.pop();
28            for(int c = 0; c < sigma_size; c++) {
29                int u = ch[r][c];
30                if(!u) { ch[r][c] = ch[f[r]][c]; continue; }
31                q.push(u);
32                int v = f[r];
33                while(v && !ch[v][c]) v = f[v];
34                f[u] = ch[v][c];
35                last[u] = val[f[u]] ? f[u] : last[f[u]];
36            }
37        }

```

```

38     }
39     inline void solve(int j) {
40         if(j) {
41             ans += val[j];
42             solve(last[j]);
43         }
44     }
45     void find(string T) {
46         int j = 0;
47         for(int i = 0; i < T.size(); i++) {
48             int c = idx(T[i]);
49             j = ch[j][c];
50             if(val[j]) solve(j);
51             else if(last[j]) solve(last[j]);
52         }
53     }
54 }
55 namespace AC {
56     int root, tcnt;
57     int ch[MAXN][sigma_size], fa[MAXN];
58     inline int newnode() {
59         fa[++tcnt] = 0;
60         for(int i = 0; i < sigma_size; ++i) ch[tcnt][i] = 0;
61         return tcnt;
62     }
63     inline void init() {
64         tcnt = -1;
65         root = newnode();
66     }
67     inline int idx(char c) { return c - 'a'; }
68     void extend(char *s, int sn) {
69         int cur = root;
70         for(int i = 0, c; i < sn; i++) {
71             if(!ch[cur][c = idx(s[i])])
72                 ch[cur][c] = newnode();
73             cur = ch[cur][c];
74         }
75     }
76     int q[MAXN], qh, qt;
77     void get_fail() {
78         qh = 1; qt = 0;
79         fa[root] = 0;
80         for(int c = 0, now; c < sigma_size; c++)
81             if((now = ch[root][c]) != 0)
82                 q[++qt] = now;
83         while(qh <= qt) {
84             int cur = q[qh++];
85             for(int c = 0, now; c < sigma_size; c++)
86                 if((now = ch[cur][c]) != 0) {
87                     fa[now] = ch[fa[cur]][c];
88                     q[++qt] = now;
89                 }else
90                     ch[cur][c] = ch[fa[cur]][c];
91         }
92     }
93     //统计模板串出现次数，每个模板串只计算一次
94     //     int cur = root, ans = 0;
95     //     for(int i = 0; i < sn; ++i) {
96     //         cur = ch[cur][idx(s[i])];
97     //         for(int j = cur; j && cnt[j] != -1; j = fa[j]) {
98     //             ans += cnt[j];

```

```

99 //          cnt[j] = -1;
100 //      }
101 //  }
102
103 }

```

3.1.4 Minimum String

```

1 namespace minstring{
2     int getmin(char *s, int sn) {
3         int i = 0, j = 1, k = 0, t;
4         while(i < sn && j < sn && k < sn) {
5             t = s[(i + k) % sn] - s[(j + k) % sn];
6             if(!t) k++;
7             else {
8                 if(t > 0) i += k + 1; else j += k + 1;
9                 if(i == j) j++;
10                k = 0;
11            }
12        }
13        return i < j ? i : j;
14    }
15 }

```

3.2 Suffix Related

3.2.1 Suffix Array

```

1 namespace SA {
2     char s[MAXN];
3     int sa[MAXN], rank[MAXN], height[MAXN];
4     int t[MAXN], t2[MAXN], c[MAXN], n;
5     void clear() { n = 0; memset(sa, 0, sizeof(sa)); }
6     void build(int m) {
7         int *x = t, *y = t2;
8         for(int i = 0; i < m; i++) c[i] = 0;
9         for(int i = 0; i < n; i++) c[x[i]] = s[i]++;
10        for(int i = 1; i < m; i++) c[i] += c[i - 1];
11        for(int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
12        for(int k = 1; k <= n; k <= 1) {
13            int p = 0;
14            for(int i = n - k; i < n; i++) y[p++] = i;
15            for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
16            for(int i = 0; i < m; i++) c[i] = 0;
17            for(int i = 0; i < n; i++) c[x[y[i]]]++;
18            for(int i = 1; i < m; i++) c[i] += c[i - 1];
19            for(int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
20            swap(x, y);
21            p = 1; x[sa[0]] = 0;
22            for(int i = 1; i < n; i++)
23                x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k]
24                ? p - 1 : p++;
25            if(p >= n) break;
26            m = p;
27        }
28    }
29    void buildHeight() {
30        int k = 0;

```

```

30     for(int i = 0; i < n; i++) rank[sa[i]] = i;
31     for(int i = 0; i < n; i++) {
32         if(k) k--;
33         int j = sa[rank[i] - 1];
34         while(s[i + k] == s[j + k]) k++;
35         height[rank[i]] = k;
36     }
37 }
38 void init() {
39     n = strlen(s) + 1;
40     build('z' + 1);
41     buildHeight();
42 }
43 }

```

3.2.2 Suffix Automaton

```

1 namespace SAM{
2     int scnt, root, last;
3     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
4     int sc[MAXN<<1], tmp1[MAXN<<1], minl[MAXN<<1];
5
6     int newnode(int _len, int q = 0) {
7         fa[++scnt] = fa[q]; len[scnt] = _len;
8         sc[scnt] = 0; tmp1[scnt] = 0; minl[scnt] = INF;
9         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
10        return scnt;
11    }
12    void init() {
13        scnt = 0;
14        root = last = newnode(0);
15    }
16    void extend(int c) {
17        int p = last, np = newnode(len[p] + 1);
18        for(; p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
19        if(!p) fa[np] = root;
20        else{
21            int q = ch[p][c];
22            if(len[p] + 1 == len[q]) fa[np] = q;
23            else{
24                int nq = newnode(len[p] + 1, q);
25                fa[np] = fa[q] = nq;
26                for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
27            }
28        }
29        last = np;
30    }
31    int c[MAXN], rs[MAXN << 1];
32    void radix_sort(int n){
33        for(int i = 0; i <= n; i++) c[i] = 0;
34        for(int i = 1; i <= scnt; i++) c[len[i]]++;
35        for(int i = 1; i <= n; i++) c[i] += c[i-1];
36        for(int i = scnt; i >= 1; i--) rs[c[len[i]]--] = i;
37    }
38    void go(){
39        scanf("%s", s);
40        int n = strlen(s);
41        for(int i = 0; i < n; ++i)
42            extend(s[i] - 'a');
43        radix_sort(n);

```

```

44 //以下sc集合意义不同
45 { //每个节点对应的位置之后有多少个不同子串
46     for(int i = scnt; i >= 1; i--) {
47         int S = 0;
48         for(int j = 0; j < 26; j++)
49             S += sc[ ch[rs[i]][j] ];
50         sc[rs[i]] = S + 1;
51     }
52 }
53 { //right集合大小
54     int cur = root;
55     for(int i = 0; i < n; ++i) {
56         cur = ch[cur][s[i] - 'a'];
57         sc[cur]++;
58     }
59     for(int i = scnt; i >= 1; --i) {
60         sc[ fa[rs[i]] ] += sc[rs[i]];
61     }
62 }
63 //公共子串
64 //tpl, 当前字符串: 在状态cur, 与模板串的最长公共后缀
65 //minl, 多个字符串: 在状态cur, 与模板串的最长公共后缀
66 //注意: 在状态cur匹配成功时, cur的祖先状态与字符串的最长公共后缀
67 for(; ~scanf("%s", s);) {
68     int cur = root, Blen = 0;
69     for(int i = 0; i <= scnt; i++)
70         tpl[i] = 0;
71     n = strlen(s);
72     for(int i = 0, x; i < n; i++) {
73         x = s[i] - 'a';
74         if(ch[cur][x]) {
75             ++Blen;
76             cur = ch[cur][x];
77         } else {
78             for(; cur && ch[cur][x] == 0; cur = fa[cur]);
79             if(cur) {
80                 Blen = len[cur] + 1;
81                 cur = ch[cur][x];
82             } else {
83                 cur = root; Blen = 0;
84             }
85         }
86         tpl[cur] = max(tpl[cur], Blen);
87     }
88     for(int i = scnt; i > 0; --i) {
89         if( tpl[ fa[rs[i]] ] < tpl[ rs[i] ] )
90             tpl[ fa[rs[i]] ] = len[ fa[rs[i]] ];
91         minl[ rs[i] ] = min(minl[ rs[i] ], tpl[ rs[i] ]);
92     }
93 }
94 }
95 }
96 namespace exSAM{
97     int scnt, root;
98     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
99     int sc[MAXN<<1], tpl[MAXN<<1], minl[MAXN<<1];
100
101     int newnode(int _len, int q = 0) {
102         fa[++scnt] = fa[q]; len[scnt] = _len;
103         sc[scnt] = 0; tpl[scnt] = 0; minl[scnt] = INF;
104         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];

```

```

105     return scnt;
106 }
107 void init() {
108     scnt = 0;
109     root = newnode(0);
110 }
111 int work(int p, int c){
112     int q = ch[p][c];
113     int nq = newnode(len[p] + 1, q);
114     fa[q] = nq;
115     for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
116     return nq;
117 }
118 int extend(int p, int c) {
119     if (ch[p][c]){
120         int q = ch[p][c];
121         if (len[p] + 1 == len[q]) return q;
122         return work(p, c);
123     }
124     int np = newnode(len[p] + 1);
125     for(; p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
126     if (!p) fa[np] = root;
127     else{
128         int q = ch[p][c];
129         if (len[p] + 1 == len[q]) fa[np] = q;
130         else fa[np] = work(p, c);
131     }
132     return np;
133 }
134 void solve() {
135     int n; scanf("%d",&n);
136     for(int i = 1; i <= n; i++) {
137         scanf("%s", s);
138         int sn = strlen(s);
139         int last = root;
140         for(int j = 0; j < sn; ++j)
141             last = extend(last, s[j] - 'a');
142     }
143 }
144 }

```

3.3 Palindrome Related

3.3.1 Manacher

```

1 namespace Manacher {
2     char S[MAXN << 1];
3     int scnt, ans;
4     int p[MAXN << 1]; //p[i] - 1
5     void init(char *s0, int sn0) {
6         S[0] = '$'; S[1] = '#';
7         for(int i = 0; i < sn0; i++) {
8             S[2 * i + 2] = s0[i];
9             S[2 * i + 3] = '#';
10        }
11        scnt = sn0 * 2 + 2;
12        S[scnt] = '&';
13    }
14    void manacher() {

```

```

15     int id = 0, mx = 0;
16     for(int i = 1; i < scnt; i++) {
17         p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
18         while(S[i + p[i]] == S[i - p[i]]) p[i]++;
19         if(i + p[i] > mx) {
20             mx = i + p[i];
21             id = i;
22         }
23     }
24 }
25 }

```

3.3.2 Palindromic Automaton

```

1 namespace PAM {
2     int scnt, S[MAXN];
3     int pcnt, last, len[MAXN], fail[MAXN], ch[MAXN][26];
4     int cnt[MAXN]; //节点i表示的本质不同的串的个数(调用count())
5     int num[MAXN]; //以节点i表示的最长回文串的最右端点为回文串结尾的回文串个数
6     int newnode(int _len) {
7         len[pcnt] = _len;
8         cnt[pcnt] = num[pcnt] = 0;
9         for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
10        return pcnt++;
11    }
12    inline void init() {
13        S[scnt = 0] = -1;
14        pcnt = 0; newnode(0); newnode(-1);
15        fail[0] = 1; last = 0;
16    }
17    int getfail(int x) {
18        while(S[scnt - len[x] - 1] != S[scnt]) x = fail[x];
19        return x;
20    }
21    void extend(int c) {
22        S[++scnt] = c;
23        int cur = getfail(last);
24        if(!ch[cur][c]) {
25            int now = newnode(len[cur] + 2);
26            fail[now] = ch[getfail(fail[cur])][c];
27            ch[cur][c] = now;
28            num[now] = num[fail[now]] + 1;
29        }
30        last = ch[cur][c];
31        cnt[last]++;
32    }
33    void count() {
34        for(int i = pcnt - 1; i >= 0; i--) cnt[fail[i]] += cnt[i];
35    }
36 };

```


4 Math

4.1 Algebra

4.1.1 FFT

```

1  //不预处理精度
2  const double pi = acos(-1.0);
3  const int MAXN = 300003;
4  struct comp {
5      double x, y;
6      comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
7      comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
8      comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
9      y * a.x}; }
10 };
11 int rev[MAXN], T;
12 comp tmp;
13 void fft(comp *a, int r) {
14     if(r == -1) for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
15     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17         comp step = (comp) {cos(pi / mid), r * sin(pi / mid)};
18         for(int j = 0; j < T; j += i) {
19             comp cur = (comp) {1, 0};
20             for(int k = j; k < j + mid; k++, cur = cur * step) {
21                 tmp = a[k + mid] * cur;
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27     if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
28 }
29 comp A[MAXN];
30 void init(int n) {
31     for(T = 1; T <= n; T <= 1);
32     for(int i = 1; i < T; i++) {
33         if(i & 1) rev[i] = (rev[i] >> 1) >> 1 ^ (T >> 1);
34         else rev[i] = rev[i] >> 1;
35         //A[i] = (comp) {0, 0};
36     }
37 }
38 //预处理精度
39 int rev[MAXN], T;
40 comp Sin[MAXN], tmp;
41 void fft(comp *a, int r) {
42     if(r == -1) {
43         for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
44         for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
45     }
46     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
47     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
48         for(int j = 0; j < T; j += i) {
49             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
50                 tmp = a[k + mid] * Sin[cur];
51                 a[k + mid] = a[k] - tmp;
52                 a[k] = a[k] + tmp;

```

```

53     }
54 }
55 if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
56 }
57 comp A[MAXN];
58 void init(int n) {
59     for(T = 1; T <= n; T <= 1);
60     for(int i = 0; i < T; i++) {
61         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
62         else rev[i] = rev[i >> 1] >> 1;
63         //A[i] = (comp) {0, 0};
64     }
65     for(int i = 0; i < (T >> 1); i++) {
66         Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
67     }
68 }
69 int main() {
70     scanf("%d%d", &n, &m);
71     init(n + m);
72     for(int i = 0; i <= n; i++) scanf("%lf", &A[i].x);
73     for(int i = 0; i <= m; i++) scanf("%lf", &A[i].y);
74     fft(A, 1);
75     fft(A, -1);
76     for(int i = 0; i <= n + m; i++) printf("%d%c", (int)(A[i].y), i == n + m ? '\n' : ' ');
77     return 0;
78 }

```

4.1.2 NTT

4.常用NTT模数:

以下模数的共同 $g = 3189$

$p = r \times 2^k + 1$	k	g
104857601	22	3
167772161	25	3
469762049	26	3
95009857	21	7
998244353	23	3
1004535809	21	3
2013265921	27	31
2281701377	27	3
3221225473	30	5

```

1  const int MAXN = 300005, G = 3, mod = 998244353; //or (479LL<<21) + 1
2  int rev[MAXN], T;
3  LL qpow(LL x, LL y) {
4      LL res = 1;
5      while(y) {
6          if(y & 1) res = res * x % mod;
7          x = x * x % mod;
8          y >>= 1;
9      }
10     return res;
11 }
12 LL A[MAXN], B[MAXN];
13 void ntt(LL *a, int r) {

```

```

14 if(r == -1) for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
15 for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16 for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17     LL gn = qpow(G, (mod - 1) / i);
18     if(r == -1) gn = qpow(gn, mod - 2);
19     for(int j = 0; j < T; j += i) {
20         LL cur = 1, tmp;
21         for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
22             tmp = a[k + mid] * cur % mod;
23             a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
24             a[k] = (a[k] + tmp) % mod;
25         }
26     }
27 }
28 if(r == -1) {
29     LL inv = qpow(T, mod - 2);
30     for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
31 }
32 }
33 void init(int n) {
34     for(T = 1; T <= n; T <= 1);
35     for(int i = 0; i < T; i++) {
36         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
37         else rev[i] = rev[i >> 1] >> 1;
38     }
39 }

```

4.1.3 MTT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long LL;
4 const double pi = acos(-1.0);
5 const int MAXN = 300003;
6 struct comp {
7     double x, y;
8     comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
9     comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
10    comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
11        y * a.x}; }
12 };
13 #define conj(a) ((comp){a.x, -a.y})
14 int rev[MAXN], T;
15 comp Sin[MAXN], tmp;
16 void fft(comp *a, int r) {
17     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
18     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
19         for(int j = 0; j < T; j += i) {
20             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
21                 tmp = a[k + mid] * Sin[cur];
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27 }
28 void init(int n) {
29     for(T = 1; T <= n; T <= 1);
30     for(int i = 0; i < T; i++) {
31         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
32     }
33 }

```

```

31     else rev[i] = rev[i >> 1] >> 1;
32 }
33 for(int i = 0; i < (T >> 1); i++) {
34     Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
35 }
36 }
37 int n, m, mod;
38 void mtt(int *x, int *y) {
39     for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
40     static comp a[MAXN], b[MAXN];
41     static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
42     for(int i = 0; i < T; i++) {
43         a[i] = {x[i] & 0x7fff, x[i] >> 15};
44         b[i] = {y[i] & 0x7fff, y[i] >> 15};
45     }
46     fft(a, 1); fft(b, 1);
47     for(int i = 0; i < T; i++) {
48         int j = (T - i) & (T - 1);
49         static comp da, db, dc, dd;
50         da = (a[i] + conj(a[j])) * (comp){0.5, 0};
51         db = (a[i] - conj(a[j])) * (comp){0, -0.5};
52         dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
53         dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
54         dfta[j] = da * dc;
55         dftb[j] = da * dd;
56         dftc[j] = db * dc;
57         dftd[j] = db * dd;
58     }
59     for(int i = 0; i < T; i++) {
60         a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
61         b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
62     }
63     //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
64     fft(a, -1); fft(b, -1);
65     for(int i = 0; i < T; i++) {
66         static int da, db, dc, dd;
67         da = (LL)(a[i].x / T + 0.5) % mod;
68         db = (LL)(a[i].y / T + 0.5) % mod;
69         dc = (LL)(b[i].x / T + 0.5) % mod;
70         dd = (LL)(b[i].y / T + 0.5) % mod;
71         x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
72     }
73 }
74 int main() {
75     static int a[MAXN], b[MAXN];
76     scanf("%d%d%d", &n, &m, &mod);
77     for(int i = 0; i <= n; i++) scanf("%d", a + i);
78     for(int i = 0; i <= m; i++) scanf("%d", b + i);
79     init(n + m);
80     mtt(a, b);
81     for(int i = 0; i <= n + m; i++) printf("%d%c", a[i], i == n + m ? '\n' : ' ');
82     return 0;
83 }

```

4.1.4 FWT

```

1 void FWT(LL *a, int n) {
2     for(int i = 2; i <= n; i <= 1) {
3         for(int j = 0; j < n; j += i) {
4             for(int d = 0, w = i >> 1; d < w; d++){

```

```

5         LL u = a[j + d], v = a[j + d + w];
6         //xor: a[j + d] = u + v, a[j + d + w] = u - v;
7         //and: a[j + d] = u + v;
8         //or : a[j + d + w] = u + v;
9     }
10 }
11 }
12 }
13 void UFWT(LL *a, int n) {
14     for(int i = 2; i <= n; i <= 1) {
15         for(int j = 0; j < n; j += i) {
16             for(int d = 0, w = i >> 1; d < w; d++) {
17                 LL u = a[j + d], v = a[j + d + w];
18                 //xor: a[j + d] = (u + v) / 2, a[j + d + w] = (u - v) / 2;
19                 //and: a[j + d] = u - v;
20                 //or : a[j + d + w] = v - u;
21             }
22         }
23     }
24 }
25 void solve(int n) {
26     FWT(a, n); FWT(b, n);
27     for(int i = 0; i < n; i++) a[i] = a[i] * b[i];
28     UFWT(a, n);
29 }

```

4.1.5 FFT Divide and Conquer

$$f_i = \sum_{j=1}^{i-1} f_j \cdot g_{i-j}$$

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 typedef long long LL;
5 const int MAXN = 300005, G = 3, mod = 998244353;
6 namespace NTT {
7     LL A[MAXN], B[MAXN]
8     int rev[MAXN], T;
9     LL qpow(LL x, LL y) {
10         LL res = 1;
11         while(y) {
12             if(y & 1) res = res * x % mod;
13             x = x * x % mod;
14             y >>= 1;
15         }
16         return res;
17     }
18 void ntt(LL *a, int r) {
19     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
20     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
21         LL gn = qpow(G, (mod - 1) / i);
22         if(r == -1) gn = qpow(gn, mod - 2);
23         for(int j = 0; j < T; j += i) {
24             LL cur = 1, tmp;
25             for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
26                 tmp = a[k + mid] * cur % mod;
27                 a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;

```

```

28         a[k] = (a[k] + tmp) % mod;
29     }
30 }
31 }
32 if(r == -1) {
33     LL inv = qpow(T, mod - 2);
34     for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
35 }
36 }
37 void init(int n) {
38     for(T = 1; T <= n; T <= 1);
39     for(int i = 0; i < T; i++) {
40         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
41         else rev[i] = rev[i >> 1] >> 1;
42     }
43 }
44 }
45 LL f[MAXN], g[MAXN];
46 using namespace NTT;
47 void solve(int l, int r) {
48     if(l == r) return;
49     int mid = (l + r) >> 1;
50     solve(l, mid);
51     init(r - l);
52     for(int i = 0; i < T; i++) A[i] = B[i] = 0;
53     for(int i = 0; i <= mid - l; i++) A[i] = f[i + l];
54     for(int i = 0; i <= r - l; i++) B[i] = g[i];
55     ntt(A, 1); ntt(B, 1);
56     for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
57     ntt(A, -1);
58     for(int i = mid + 1; i <= r; i++) f[i] = (f[i] + A[i - l]) % mod;
59     solve(mid + 1, r);
60 }
61 int main() {
62     int n; scanf("%d", &n);
63     for(int i = 1; i < n; i++) scanf("%lld", g + i);
64     f[0] = 1;
65     solve(0, n - 1);
66     for(int i = 0; i < n; i++) printf("%lld%c", f[i], i == n - 1 ? '\n' : ' ');
67     return 0;
68 }

```

4.1.6 Linear Basis

```

1 //dynamic
2 const int D = 60;
3 struct Basis {
4     vector<int> ind;
5     vector<LL> base;
6     Basis() {
7         ind.resize(D, -1);
8         base.resize(D);
9     }
10    bool update(LL x, int id) {
11        for(int i = 0; i < D; i++) if(~ind[i] && x >> i & 1) {
12            x ^= base[i];
13        }
14        if(!x) return 1;
15        int pos = __builtin_ctzll(x);
16        ind[pos] = id;

```

```

17     base[pos] = x;
18     return 0;
19 }
20 };
21 //array
22 int Gauss(int n, int m) {
23     int num = 1;
24     for(int x = 1; x <= n && x <= m; x++) {
25         int t = 0;
26         for(int j = x; j <= m; j++) if(g[j][x]) { t = j; break; }
27         if(t) {
28             swap(g[x], g[t]);
29             for(int i = x + 1; i <= n; i++) {
30                 if(g[i][x]) {
31                     for(int k = 1; k <= m; k++) g[i][k] ^= g[x][k];
32                 }
33             }
34             num++;
35         }
36     }
37     return --num;
38 }
39 //long long
40 int Gauss() {
41     int num = 1;
42     for(int k = 61; k >= 0; k--) {
43         int t = 0;
44         for(int j = num; j <= cnt; j++) if((A[j] >> k) & 1) { t = j; break; }
45         if(t) {
46             swap(A[t], A[num]);
47             for(int j = num + 1; j <= cnt; j++) if((A[j] >> k) & 1) A[j] ^= A[num];
48             num++;
49         }
50     }
51     return --num;
52 }

```

4.1.7 Lagrange Polynomial

$$L(x) = \sum_{i=0}^n y_i \prod_{j=0, j \neq i}^n \frac{x - x_j}{x_i - x_j}$$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  typedef pair<int, int> P;
5  const int MAXN = 3005, mod = 998244353;
6  int exgcd(int a, int b, int &x, int &y) {
7      int d = a;
8      if(b != 0) {
9          d = exgcd(b, a % b, y, x);
10         y -= (a / b) * x;
11     }
12     else {
13         x = 1; y = 0;
14     }
15     return d;
16 }

```

```

17 int inv(int a) {
18     int x, y;
19     exgcd(a, mod, x, y);
20     return (x % mod + mod) % mod;
21 }
22 struct Lagrange {
23     int n, a[MAXN][2];
24     void init() {
25         for(int i = 0; i <= n; i++) a[i][0] = a[i][1] = 0;
26         n = 0;
27         a[0][1] = 1;
28     }
29     int query(int x, int q = 0) {
30         int res = 0;
31         for(int i = n; i >= 0; i--) res = ((LL)res * x + a[i][q]) % mod;
32         return res;
33     }
34     void update(int x, int y) {
35         a[n][0] = 0;
36         int v = (LL)(y - query(x) + mod) % mod * inv(query(x, 1)) % mod;
37         for(int i = 0; i <= n; i++) a[i][0] = (a[i][0] + (LL)a[i][1] * v) % mod;
38         a[++n][1] = 0;
39         for(int i = n; i; i--) a[i][1] = (a[i - 1][1] + (LL)a[i][1] * (mod - x)) % mod;
40         a[0][1] = (LL)a[0][1] * (mod - x) % mod;
41     }
42 }p;
43 int main() {
44     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
45     int Q;
46     cin >> Q;
47     int op, x, y;
48     p.n = 0;
49     p.init();
50     while(Q--) {
51         cin >> op >> x;
52         if(op == 1) {
53             cin >> y;
54             p.update(x, y);
55         }
56         else cout << p.query(x) << endl;
57     }
58     return 0;
59 }

```

4.1.8 BM Alogrithm

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  #define rep(i,a,n) for (int i=a;i<n;i++)
4  #define per(i,a,n) for (int i=n-1;i>=a;i--)
5  #define pb push_back
6  #define mp make_pair
7  #define all(x) (x).begin(),(x).end()
8  #define fi first
9  #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector<int> VI;
12 typedef long long ll;
13 typedef pair<int,int> PII;
14 const ll mod=1000000007;

```



```

15 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b;b>>=1){if(b&1)res=res*a%mod;
    a=a*a%mod;}return res;}
16 // head
17 namespace linear_seq {
18     const int N=10010;
19     ll res[N],base[N],_c[N],_md[N];
20
21     vector<int> Md;
22     void mul(ll *a,ll *b,int k) {
23         rep(i,0,k+k) _c[i]=0;
24         rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
25         for (int i=k+k-1;i>=k;i--) if (_c[i])
26             rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
27         rep(i,0,k) a[i]=_c[i];
28     }
29     int solve(ll n,VI a,VI b) { // a 系数 b 初值 b[n+1]=a[0]*b[n]+...
30         // printf("%d\n",SZ(b));
31         ll ans=0,pnt=0;
32         int k=SZ(a);
33         assert(SZ(a)==SZ(b));
34         rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
35         Md.clear();
36         rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
37         rep(i,0,k) res[i]=base[i]=0;
38         res[0]=1;
39         while ((1ll<pnt)<=n) pnt++;
40         for (int p=pnt;p>=0;p--) {
41             mul(res,res,k);
42             if ((n>p)&1) {
43                 for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
44                 rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
45             }
46         }
47         rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
48         if (ans<0) ans+=mod;
49         return ans;
50     }
51     VI BM(VI s) {
52         VI C(1,1),B(1,1);
53         int L=0,m=1,b=1;
54         rep(n,0,SZ(s)) {
55             ll d=0;
56             rep(i,0,L+1) d=(d+(ll)C[i]*s[n-i])%mod;
57             if (d==0) ++m;
58             else if (2*L<=n) {
59                 VI T=C;
60                 ll c=mod-d*powmod(b,mod-2)%mod;
61                 while (SZ(C)<SZ(B)+m) C.pb(0);
62                 rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
63                 L=n+1-L; B=T; b=d; m=1;
64             } else {
65                 ll c=mod-d*powmod(b,mod-2)%mod;
66                 while (SZ(C)<SZ(B)+m) C.pb(0);
67                 rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
68                 ++m;
69             }
70         }
71         return C;
72     }
73     int gao(VI a,ll n) {
74         VI c=BM(a);

```

```

75     c.erase(c.begin());
76     rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
77     return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
78 }
79 };
80
81 int main() {
82     while (~scanf("%d",&n)) {
83         vector<int>v;
84         v.push_back(1);
85         v.push_back(2);
86         v.push_back(4);
87         v.push_back(7);
88         v.push_back(13);
89         v.push_back(24);
90         //VI{1,2,4,7,13,24}
91         printf("%d\n",linear_seq::gao(v,n-1));
92     }
93 }

```

4.2 Math Theory

4.2.1 Inverse

```

1  //O(logn) 求n的逆元
2  const int mod = 1e6 + 3;
3  int exgcd(int a, int b, int &x, int &y) {
4      int d = a;
5      if(b != 0) {
6          d = exgcd(b, a % b, y, x);
7          y -= (a / b) * x;
8      }
9      else {
10         x = 1; y = 0;
11     }
12     return d;
13 }
14 int inverse(int a) {
15     int x, y;
16     exgcd(a, mod, x, y);
17     return (x % mod + mod) % mod;
18 }
19 int inverse(int a) { return qpow(a, mod - 2); }
20 //O(n) 求1~n的逆元
21 int inv[MAXN];
22 void init() {
23     inv[0] = inv[1] = 1;
24     for(int i = 2; i < MAXN; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
25     mod;
26 }

```

4.2.2 Lucas

```

1  //mod很小可以预处理逆元的情况
2  void init() {
3      fac[0] = 1;
4      for(int i = 1; i < mod; i++) fac[i] = (long long)fac[i - 1] * i % mod;
5      inv[0] = inv[1] = 1;

```

```

6   for(int i = 2; i < mod; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
    mod;
7   for(int i = 1; i < mod; i++) inv[i] = (long long)inv[i] * inv[i - 1] % mod;
8 }
9 int C(int a, int b) {
10  if(b > a) return 0;
11  if(a < mod) return (long long)fac[a] * inv[b] % mod * inv[a - b] % mod;
12  return (long long)C(a / mod, b / mod) * C(a % mod, b % mod) % mod;
13 }
14 //mod过大不能预处理逆元的情况
15 LL qpow(LL x, LL y) {
16  LL res = 1;
17  while(y) {
18      if(y & 1) res = res * x % mod;
19      x = x * x % mod;
20      y >>= 1;
21  }
22  return res;
23 }
24 LL C(LL a, LL b) {
25  if(b > a) return 0;
26  if(b > a - b) b = a - b;
27  LL s1 = 1, s2 = 1;
28  for(LL i = 0; i < b; i++) {
29      s1 = s1 * (a - i) % mod;
30      s2 = s2 * (i + 1) % mod;
31  }
32  return s1 * qpow(s2, mod - 2) % mod;
33 }
34 LL lucas(LL a, LL b) {
35  if(a < mod) return C(a, b);
36  return lucas(a / mod, b / mod) * C(a % mod, b % mod);
37 }

```

4.2.3 CRT && exCRT

$$x \equiv a_i \pmod{m_i}$$

```

1 namespace CRT {
2   LL m[MAXN], a[MAXN];
3   LL exgcd(LL _a, LL _b, LL &x, LL &y) {
4       if(!_b) {
5           x = 1; y = 0;
6           return _a;
7       }
8       LL d = exgcd(_b, _a % _b, y, x);
9       y -= (_a / _b) * x;
10      return d;
11  }
12  LL crt(int n) {
13      LL M = 1, tmp, res = 0, x, y;
14      for(int i = 1; i <= n; i++) M *= m[i];
15      for(int i = 1; i <= n; i++) {
16          tmp = M / m[i];
17          exgcd(tmp, m[i], x, y);
18          x = (x + m[i]) % m[i];
19          res = (a[i] * x % M * tmp % M + res) % M;
20      }
21  }
22 }

```

```

21     return res;
22 }
23 }
24 namespace EXCRT {
25     LL m[MAXN], a[MAXN];
26     LL exgcd(LL _a, LL _b, LL &x, LL &y) {
27         if(!_b) {
28             x = 1; y = 0;
29             return _a;
30         }
31         LL d = exgcd(_b, _a % _b, y, x);
32         y -= (_a / _b) * x;
33         return d;
34     }
35     LL excrt(int n) {
36         LL M = m[1], A = a[1], x, y, d, tmp;
37         for(int i = 2; i <= n; i++) {
38             d = exgcd(M, m[i], x, y);
39             if((A - a[i]) % d) return -1; //No solution
40             tmp = M / d; M *= m[i] / d;
41             y = (A - a[i]) / d % M * y % M;
42             y = (y + tmp) % tmp;
43             A = (m[i] % M * y % M + a[i]) % M;
44             A = (A + M) % M;
45         }
46         return A;
47     }
48     LL inv(LL _a, LL _b) {
49         LL x, y;
50         exgcd(_a, _b, x, y);
51         return (x % _b + _b) % _b;
52     }
53     LL excrt(int n) {
54         LL M = m[1], A = a[1], x, y, d, c, tmp;
55         for(int i = 2; i <= n; i++) {
56             d = exgcd(M, m[i], x, y);
57             c = a[i] - A;
58             if(c % d) return -1;
59             c = (c % m[i] + m[i]) % m[i];
60             M /= d; m[i] /= d;
61             c = c / d * inv(M % m[i], m[i]) % m[i];
62             tmp = M;
63             M *= m[i] * d;
64             A = (c * tmp % M * d % M + A) % M;
65         }
66         return A;
67     }
68 }

```

4.2.4 BSGS

```

1  const int MOD = 76543;
2  int hs[MOD + 5], head[MOD + 5], nxt[MOD + 5], id[MOD + 5], ecnt;
3  void insert(int x, int y) {
4      int k = x % MOD;
5      hs[ecnt] = x, id[ecnt] = y, nxt[ecnt] = head[k], head[k] = ecnt++;
6  }
7  int find(int x) {
8      int k = x % MOD;
9      for(int i = head[k]; i; i = nxt[i])

```

```

10         if(hs[i] == x)
11             return id[i];
12     return -1;
13 }
14 int BSGS(int a, int b, int c){
15     memset(head, 0, sizeof head); ecnt = 1;
16     if(b == 1) return 0;
17     int m = sqrt(c * 1.0), j;
18     LL x = 1, p = 1;
19     for(int i = 0; i < m; i++, p = p * a % c)
20         insert(p * b % c, i);
21     for(LL i = m; ; i += m){
22         if((j = find(x = x * p % c)) != -1) return i - j;
23         if(i > c) break;
24     }
25     return -1;
26 }

```

4.2.5 Miller-Rabin & PollardRho

```

1  LL ksc(LL a, LL n, LL mod){
2      LL ret=0;
3      for(;n>=1){
4          if(n&1){ret+=a;if(ret>=mod)ret-=mod;}
5          a<<=1;if(a>=mod)a-=mod;
6      }
7      return ret;
8  }
9  LL ksm(LL a, LL n, LL mod){
10     LL ret = 1;
11     for(;n>=1){
12         if(n&1)ret=ksc(ret,a,mod);
13         a=ksc(a,a,mod);
14     }
15     return ret;
16 }
17 int millerRabin(LL n){
18     if(n<2 || (n!=2 && !(n&1)))return 0;
19     LL d=n-1;for(;!(d&1);d>>=1);
20     for(int i=0;i<20;++i){
21         LL a=rand()%(n-1)+1;
22         LL t=d,m=ksm(a,d,n);
23         for(;t!=n-1 && m!=1 && m!=n-1;m=ksc(m,m,n),t<<=1);
24         if(m!=n-1 && !(t&1)) return 0;
25     }
26     return 1;
27 }
28 LL cnt,fact[100];
29 LL gcd(LL a,LL b){return !b?a:gcd(b,a%b);}
30 LL pollardRho(LL n, int a){
31     LL x=rand()%n,y=x,d=1,k=0,i=1;
32     while(d==1){
33         ++k;
34         x=ksc(x,x,n)+a;if(x>=n)x-=n;
35         d=gcd(x>y?x-y:y-x,n);
36         if(k==i){y=x;i<<=1;}
37     }
38     if(d==n)return pollardRho(n,a+1);
39     return d;
40 }

```

```

41 void findfac(LL n){
42     if(millerRabin(n)){fact[++cnt]=n;return;}
43     LL p=pollardRho(n,rand()%(n-1)+1);
44     findfac(p);
45     findfac(n/p);
46 }

```

4.2.6 $\varphi(n)$

```

1  int phi(int x) {
2      int res = x;
3      for(int i = 2; i * i <= x; i++) {
4          if(x % i == 0) {
5              res = res / i * (i - 1);
6              while(x % i == 0) x /= i;
7          }
8      }
9      if(x > 1) res = res / x * (x - 1);
10     return res;
11 }

```

4.2.7 Euler Sieve

```

1  int prime[MAXN], cnt, phi[MAXN], mu[MAXN];
2  bool isp[MAXN];
3
4  int min_pow[MAXN];    //最小质因子最高次幂
5  int min_sum[MAXN];    //1+p+p^2+...+p^k
6  int div_sum[MAXN];    //约数和
7
8  int min_index[MAXN];  //最小质因子的指数
9  int div_num[MAXN];    //约数个数
10 void Euler(int n) {
11     mu[1] = phi[1] = div_num[1] = div_sum[1] = 1;
12     for(int i = 2; i <= n; i++) {
13         if(!isp[i]) {
14             prime[++cnt] = min_pow[i] = i;
15             phi[i] = i - 1;
16             mu[i] = -1;
17             min_index[i] = 1; div_num[i] = 2;
18             div_sum[i] = min_sum[i] = i + 1;
19         }
20         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
21             isp[i * prime[j]] = 1;
22             if(i % prime[j] == 0) {
23                 phi[i * prime[j]] = phi[i] * prime[j];
24                 mu[i * prime[j]] = 0;
25
26                 min_index[i * prime[j]] = min_index[i] + 1;
27                 div_num[i * prime[j]] = div_num[i] / (min_index[i] + 1) * (min_index[i] *
prime[j] + 1);
28
29                 min_sum[i * prime[j]] = min_sum[i] + min_pow[i] * prime[j];
30                 div_sum[i * prime[j]] = div_sum[i] / min_sum[i] * min_sum[i * prime[j]];
31                 min_pow[i * prime[j]] = min_pow[i] * prime[j];
32                 break;
33             }
34             phi[i * prime[j]] = phi[i] * (prime[j] - 1);

```

```

35     mu[i * prime[j]] = -mu[i];
36
37     div_num[i * prime[j]] = div_num[i] << 1;
38     min_index[i * prime[j]] = 1;
39
40     div_sum[i * prime[j]] = div_sum[i] * (prime[j] + 1);
41     min_pow[i * prime[j]] = prime[j];
42     min_sum[i * prime[j]] = prime[j] + 1;
43 }
44 }
45 }

```

4.2.8 DuJiao Sieve

$$\sum_{i=1}^n \varphi(i)$$

```

1  vector<int> prime;
2  int phi[MAXN], P[MAXN];
3  bool isp[MAXN];
4  unordered_map<LL, int> mp;
5  void Euler(int n) {
6      phi[1] = 1;
7      for(int i = 2; i <= n; i++) {
8          if(!isp[i]) {
9              prime.push_back(i);
10             phi[i] = i - 1;
11         }
12         for(auto x : prime) {
13             if(i * x > n) break;
14             isp[i * x] = 1;
15             if(i % x == 0) {
16                 phi[i * x] = phi[i] * x;
17                 break;
18             }
19             phi[i * x] = phi[i] * (x - 1);
20         }
21     }
22     for(int i = 1; i <= n; i++) P[i] = (P[i - 1] + phi[i]) % mod;
23 }
24 LL cal(LL n) {
25     if(n < MAXN) return P[n];
26     if(mp.count(n)) return mp[n];
27     LL res = 0;
28     for(LL i = 2, last; i <= n; i = last + 1) {
29         last = n / (n / i);
30         res += (last - i + 1) % mod * cal(n / i) % mod;
31         res %= mod;
32     }
33     mp[n] = ((__int128)n * (n + 1) / 2 % mod + mod - res) % mod;
34     return mp[n];
35 }

```

$$\sum_{i=1}^n \mu(i)$$

```

1  LL cal(LL n) {
2      if(n < MAXN) return M[n];

```

```

3  if(mp.count(n)) return mp[n];
4  LL res = 0;
5  for(LL i = 2, last; i <= n; i = last + 1) {
6      last = n / (n / i);
7      res += (last - i + 1) * cal(n / i);
8  }
9  mp[n] = 1 - res;
10 return 1 - res;
11 }

```

4.2.9 Möbius Inversion

$$\sum_i^n \sum_j^m lcm(i, j) \pmod{p}$$

```

1  int mu[MAXN], prime[MAXN], sum[MAXN], cnt;
2  bool isp[MAXN];
3  void getmu(int n) {
4      mu[1] = 1;
5      for(int i = 2; i <= n; i++) {
6          if(!isp[i]) {
7              mu[i] = -1;
8              prime[++cnt] = i;
9          }
10         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
11             isp[i * prime[j]] = 1;
12             if(i % prime[j] == 0) {
13                 mu[i * prime[j]] = 0;
14                 break;
15             }
16             mu[i * prime[j]] = -mu[i];
17         }
18     }
19 }
20 ll n, m, ans;
21 ll query(ll x, ll y) { return (x * (x + 1) / 2 % mod) * (y * (y + 1) / 2 % mod) % mod; }
22 ll F(ll x, ll y) {
23     ll res = 0, last;
24     for(ll i = 1; i <= min(x, y); i = last + 1) {
25         last = min(x / (x / i), y / (y / i));
26         res = (res + (sum[last] - sum[i - 1]) * query(x / i, y / i) % mod) % mod;
27     }
28     return res;
29 }
30 int main() {
31     cin >> n >> m;
32     getmu(min(n, m));
33     for(ll i = 1; i <= min(n, m); i++) sum[i] = (sum[i - 1] + (i * i * mu[i]) % mod) % mod;
34     ll last;
35     for(ll d = 1; d <= min(n, m); d = last + 1) {
36         last = min(n / (n / d), m / (m / d));
37         ans = (ans + (last - d + 1) * (d + last) / 2 % mod * F(n / d, m / d) % mod) % mod;
38     }
39     ans = (ans + mod) % mod;
40     cout << ans << endl;
41     return 0;

```


42 }

5 Geometry

5.1 Commonly Definition and Functions

5.1.1 Const and Functions

```

1 namespace CG{
2     #define Point Vector
3     const double pi=acos(-1.0);
4     const double inf=1e100;
5     const double eps=1e-9;
6     template <typename T> inline T Abs(T x){return x>0?x:-x;}
7     template <typename T> inline bool operator == (T x,T y){return Abs(x-y)<eps;}
8     int sgn(double x){
9         if (Abs(x)<eps) return 0;
10        if (x>0) return 1;
11        else return -1;
12    }
13 }

```

5.1.2 Point Definition

```

1 namespace CG{
2     struct Point{
3         double x,y;
4         Point(double x=0,double y=0):x(x),y(y){}
5     };
6     Vector operator + (const Vector a,const Vector b){return Vector(a.x+b.x,a.y+b.y);}
7     Vector operator - (const Vector a,const Vector b){return Vector(a.x-b.x,a.y-b.y);}
8     Vector operator * (const Vector a,const double k){return Vector(a.x*k,a.y*k);}
9     Vector operator / (const Vector a,const double k){return Vector(a.x/k,a.y/k);}
10    bool operator < (const Vector a,const Vector b) {return a.x==b.x?a.y<b.y:a.x<b.x;}
11    bool operator == (const Vector a,const Vector b) {return a.x==b.x && a.y==b.y;}
12    double Dot(const Vector a,const Vector b){return a.x*b.x+a.y*b.y;}
13    double Cross(const Vector a,const Vector b){return a.x*b.y-a.y*b.x;}
14    double mult_Cross(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.
y-c.y)-(b.x-c.x)*(a.y-c.y);}
15    double mult_Dot(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.x-
c.x)+(a.y-c.y)*(b.y-c.y);}
16    double Norm(const Vector a){return sqrt(Dot(a,a));}
17    double Angle(const Vector a,const Vector b){return acos(Dot(a,b)/Norm(a)/Norm(b));}
18    Vector Rotate(const Vector a,const double theta){return Vector(a.x*cos(theta)-a.y*
sin(theta),a.x*sin(theta)+a.y*cos(theta));}
19    boolToLeftTest(const Vector a,const Vector b){return Cross(a,b)<0;}
20    double DisPP(const Vector a,const Vector b){return sqrt((a.x-b.x)*(a.x-b.x)+(a.y-b.y
)*(a.y-b.y));}
21 }

```

5.1.3 Line Definition

```

1 namespace CG{
2     struct Line{
3         Point p0,v,p1;
4         double t,theta;
5         Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),v(_v),t(_t){p1=p0+v*t; theta=
atan2(v.y,v.x);}

```

```

6      // Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),p1(_v){v=(p1-p0)/t; theta=
      atan2(v.y,v.x);}
7  };
8  bool operator < (const Line n,const Line m) {return n.theta<m.theta;}
9  Point GetIntersection(const Line n,const Line m){return n.p0+n.v*Cross(m.v,(n.p0-m.
p0))/Cross(n.v,m.v);}
10 bool OnLine(const Vector a,const Line l){return Cross(l.p0-a,l.p1-a)==0;}
11 bool OnSegment(const Point a,const Line l){return sgn(Cross(l.p0-a,l.p1-a))==0 &&
sgn(Dot(l.p0-a,l.p1-a))<0;}
12 double DisPL(const Point a,const Line l){return Abs(Cross(l.p1-l.p0,a-l.p0)/Norm(l.
p1-l.p0));}
13 double DisPS(const Point a,const Line l){
14     if (l.p0==l.p1) return Norm(a-l.p0);
15     Vector v1=l.p1-l.p0,v2=a-l.p0,v3=a-l.p1;
16     if (sgn(Dot(v1,v2))<0) return Norm(v2);
17     if (sgn(Dot(v1,v3))>0) return Norm(v3);
18     return DisPL(a,l);
19 }
20 Point GetProjection(const Point a,const Line l){
21     Vector v=l.p1-l.p0;
22     return l.p0+v*(Dot(v,a-l.p0)/Dot(v,v));
23 }
24 bool SegmentIntersection(const Line n,const Line m,bool p){
25     double c1=Cross(n.p1-n.p0,m.p1-m.p0);
26     double c2=Cross(n.p1-n.p0,m.p1-n.p0);
27     double c3=Cross(m.p1-m.p0,n.p0-m.p0);
28     double c4=Cross(m.p1-m.p0,n.p1-m.p0);
29     if (p){
30         if (!sgn(c1) || !sgn(c2) || !sgn(c3) || !sgn(c4)){
31             return OnSegment(n.p0,m) || OnSegment(n.p1,m) || OnSegment(m.p0,n) ||
OnSegment(m.p0,m);
32         }
33     }
34     }
35     return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);
36 }
37 }

```

5.1.4 Get Area

```

1 namespace CG{
2     double GetArea(Point *p,int n){
3         double area=Cross(p[n],p[1]);
4         for (int i=2;i<=n;i++) area+=0.5*Cross(p[i-1],p[i]);
5         return Abs(area);
6     }
7 }

```

5.1.5 Get Circumference

```

1 namespace CG{
2     double GetCircumference(Point *p,int n){
3         double Circumference=DisPP(p[n],p[1]);
4         for (int i=2;i<=n;i++) Circumference+=DisPP(p[i-1],p[i]);
5         return Circumference;
6     }
7 }

```

5.1.6 Anticlockwise Sort

```

1 namespace CG{
2     \p
3     void clockwise_sort(Point *p,int n){
4         for(int i=0;i<n-2;i++){
5             double tmp = mult_Cross(p[i+1],p[i+2],p[i]);
6             if(tmp>0) return;
7             else if(tmp<0){
8                 reverse(p,p+n);
9                 return;
10            }
11        }
12    }
13 }

```

5.2 Convex Hull

5.2.1 Get Convex Hull

```

1 namespace CG{
2     Point p[MAXN],s[MAXN];
3     int ConvexHull(Point *p,int n,Point *s){
4         sort(p,p+n,cmp); //x从小到大,y从小到大;
5         int m=0;
6         for (int i=0;i<n;i++){
7             for (;m>=2 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
8             s[++m]=p[i];
9         }
10        int k=m;
11        for (int i=n-2;i;i--){
12            for (;m>=k+1 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
13            s[++m]=p[i];
14        }
15        return m-1;
16    }
17 }

```

5.2.2 Point in Convex Hull

```

1 namespace CG{
2     bool PointInConvexHull(Point A){
3         int l=1,r=tot-2,mid;
4         while(l<=r){
5             mid=(l+r)>>1;
6             double a1=Cross(p[mid]-p[0],A-p[0]);
7             double a2=Cross(p[mid+1]-p[0],A-p[0]);
8             if(a1>=0 && a2<=0){
9                 if(Cross(p[mid+1]-p[mid],A-p[mid])>=0) return true;
10                return false;
11            }
12            else if(a1<0) r=mid-1;
13            else l=mid+1;
14        }
15        return false;
16    }
17 }

```

5.3 Minkowski Sum

```

1 namespace CG{
2     void Minkowski(Point *C1,int n,Point *C2,int m){
3         for(int i=1;i<=n;i++) s1[i]=C1[i]-C1[i-1];
4         for(int i=1;i<=m;i++) s2[i]=C2[i]-C2[i-1];
5         A[tot=1]=C1[1]+C2[1];
6         int p1=1,p2=1;
7         while (p1<=n && p2<=m) ++tot,A[tot]=A[tot-1]+(s1[p1]*s2[p2]>=0?s1[p1++]:s2[p2
8 ++]);
9         while (p1<=n) ++tot,A[tot]=A[tot-1]+s1[p1++];
10        while (p2<=m) ++tot,A[tot]=A[tot-1]+s2[p2++];
11        tot=ConvexHull(A,tot);
12    }

```

5.4 Rotating Calipers

5.4.1 The Diameter of Convex Hull

```

1 namespace CG{
2     double RotatingCalipers(Point *p,int n){
3         double dis=0;
4         for(int i=0,j=2;i<n;++i){
5             while (abs(Cross(p[i+1]-p[i],p[j]-p[i]))<abs(Cross(p[i+1]-p[i],p[j+1]-p[i])))
6             ) j=(j+1)%n;
7             dis=max(dis,max(DisPP(p[j],p[i]),DisPP(p[j],p[i+1])));
8         }
9         return dis;
10    }

```

5.4.2 The Min Distance Between two Convex Hull

```

1 namespace CG{
2     ///点c到线段ab的最短距离
3     double GetDist(Point a,Point b,Point c){
4         if(dis(a,b)<esp) return dis(b,c); ///a,b是同一个点
5         if(mult_Dot(b,c,a)<-esp) return dis(a,c); ///投影
6         if(mult_Dot(a,c,b)<-esp) return dis(b,c);
7         return fabs(mult_Cross(b,c,a)/dis(a,b));
8     }
9
10    ///求一条线段ab的两端点到另外一条线段bc的距离，反过来一样，共4种情况
11    double MinDist(Point a,Point b,Point c,Point d){
12        return min(min(GetDist(a,b,c),GetDist(a,b,d)),min(GetDist(c,d,a),GetDist(c,d,b))
13    );
14    }
15    double RotatingCalipers(Point *p,int n,Point *q,int m){
16        int yminP = 0,ymaxQ=0;
17        for(int i=1;i<n;i++){ ///找到点集p组成的凸包的左下角
18            if(p[i].y<p[yminP].y || (p[i].y==p[yminP].y)&&(p[i].x<p[yminP].x)) yminP = i;
19        }
20        for(int i=1;i<m;i++){ ///找到点集q组成的凸包的右上角
21            if(q[i].y>q[ymaxQ].y || (q[i].y==q[ymaxQ].y)&&(q[i].x>q[ymaxQ].x)) ymaxQ = i;
22        }
23        double ans = DisPP(p[yminP],q[ymaxQ]); ///距离(yminP,ymaxQ)维护为当前最小值。

```

```

23     for(int i=0;i<n;i++){
24         double tmp;
25         while(tmp=(mult_Cross(q[ymaxQ+1],p[yminP],p[yminP+1])-mult_Cross(q[ymaxQ],p[
yminP],p[yminP+1]))>esp)
26             ymaxQ = (ymaxQ+1)%m;
27         if(tmp<=-esp) ans = min(ans,GetDist(p[yminP],p[yminP+1],q[ymaxQ]));
28         else ans=min(ans,MinDist(p[yminP],p[yminP+1],q[ymaxQ],q[ymaxQ+1]));
29         yminP = (yminP+1)%n;
30     }
31     return ans;
32 }
33 }

```

5.5 Half Plane Intersection

```

1 namespace CG{
2     void HalfPlaneIntersection(Line l[],int n){
3         deque <Point> p;
4         sort(l+1,l+1+n);
5         deque <Line> q;
6         q.push_back(l[1]);
7         for (int i=2;i<=n;i++){
8             for (;!p.empty() && !ToLeftTest(p.back()-l[i].p0,l[i].v);q.pop_back(),p.
pop_back());
9             for (;!p.empty() && !ToLeftTest(p.front()-l[i].p0,l[i].v);q.pop_front(),p.
pop_front());
10            if (sgn(Cross(l[i].v,q.back().v))==0)
11                if (ToLeftTest(l[i].p0-q.back().p0),q.back().v){
12                    q.pop_back();
13                    if (!p.empty()) p.pop_back();
14                }
15            if (!q.empty()) p.push_back(GetIntersection(q.back(),l[i]));
16            q.push_back(l[i]);
17        }
18        for (;!p.empty() && !ToLeftTest(p.back()-q.front().p0,q.front().v);q.pop_back(),
p.pop_back());
19        p.push_back(GetIntersection(q.back(),q.front()));
20        double area=0.5*Cross(p.back(),p.front()); Point last=p.front();
21        for (p.pop_front();!p.empty();last=p.front(),p.pop_front()) area+=0.5*Cross(last
,p.front());
22        printf("%.1f",Abs(area));
23    }
24 }

```

5.6 Min Circle Cover

```

1 namespace CG{
2     Point GetCircleCenter(const Point a,const Point b,const Point c){
3         Point p=(a+b)/2.0,q=(a+c)/2.0;
4         Vector v=Rotate(b-a,pi/2.0),w=Rotate(c-a,pi/2.0);
5         if (sgn(Norm(Cross(v,w)))==0){
6             if (sgn(Norm(a-b)+Norm(b-c)-Norm(a-c))==0) return (a+c)/2;
7             if (sgn(Norm(b-a)+Norm(a-c)-Norm(b-c))==0) return (b+c)/2;
8             if (sgn(Norm(a-c)+Norm(c-b)-Norm(a-b))==0) return (a+c)/2;
9         }
10        return GetIntersection(Line(p,v),Line(q,w));
11    }
12    void MinCircleCover(Point p[],int n){

```

```

13     random_shuffle(p+1,p+1+n);
14     Point c=p[1];
15     double r=0;
16     for (int i=2;i<=n;i++)
17         if (sgn(Norm(c-p[i])-r)>0){
18             c=p[i],r=0;
19             for (int j=1;j<i;j++)
20                 if (sgn(Norm(c-p[j])-r)>0){
21                     c=(p[i]+p[j])/2.0;
22                     r=Norm(c-p[i]);
23                     for (int k=1;k<j;k++)
24                         if (sgn(Norm(c-p[k])-r)>0){
25                             c=GetCircleCenter(p[i],p[j],p[k]);
26                             r=Norm(c-p[i]);
27                         }
28                 }
29     }
30     printf("%.10f\n%.10f %.10f",r,c.x,c.y);
31 }
32 }

```

5.7 Circle Union Area

```

1 //k次覆盖
2 //圆并去重后s[0]
3 typedef pair<double, int> P;
4 const double pi = acos(-1.0);
5 const int MAXN = 10003;
6 P arc[MAXN << 1];
7 int acnt, cnt;
8 double s[1003];
9 bool del[1003];
10 void add(double st, double en) {
11     if(st < -pi) {
12         add(st + 2 * pi, pi);
13         add(-pi, en);
14         return;
15     }
16     if(en > pi) {
17         add(st, pi);
18         add(-pi, en - 2 * pi);
19         return;
20     }
21     arc[++acnt] = P(st, 1);
22     arc[++acnt] = P(en, -1);
23 }
24 double F(double x) {
25     return (x - sin(x)) / 2;
26 }
27 struct Node {
28     int x, y, r;
29     Node(int _x = 0, int _y = 0, int _r = 0):x(_x), y(_y), r(_r) {}
30     bool operator == (const Node& t) {
31         return x == t.x && y == t.y && r == t.r;
32     }
33     inline void read() {
34         scanf("%d%d%d", &x, &y, &r);
35     }
36 }a[1003];

```

```

37 int main() {
38     int n;
39     scanf("%d", &n);
40     for(int i = 1; i <= n; i++) a[i].read();
41     /*
42     //去重
43     int nn = 0;
44     for(int i = 1; i <= n; i++) {
45         bool same = 0;
46         for(int j = 1; j < i; j++) {
47             if(a[i] == a[j]) {
48                 same = 1; break;
49             }
50         }
51         if(!same) a[++nn] = a[i];
52     }
53     n = nn;
54     //去包含
55     for(int i = 1; i <= n; i++) {
56         for(int j = 1; j <= n; j++) if(i != j) {
57             if(hypot(a[i].x - a[j].x, a[i].y - a[j].y) < (double)(a[i].r - a[j].r)) del[
58                 j] = 1;
59         }
60     }
61     nn = 0;
62     for(int i = 1; i <= n; i++) if(!del[i]) {
63         a[++nn] = a[i];
64     }
65     n = nn;
66     /*
67     for(int i = 1; i <= n; i++) {
68         acnt = 0;
69         for(int j = 1; j <= n; j++) if(i != j) {
70             int dis = (a[i].x - a[j].x) * (a[i].x - a[j].x) + (a[i].y - a[j].y) * (a[i].
71             y - a[j].y);
72             if(a[j].r > a[i].r && dis <= (a[j].r - a[i].r) * (a[j].r - a[i].r)) add(-pi,
73             pi);
74             else if(dis > (a[i].r - a[j].r) * (a[i].r - a[j].r) && dis < (a[i].r + a[j].
75             r) * (a[i].r + a[j].r)){
76                 double c = sqrt(dis);
77                 double angle = acos((a[i].r * a[i].r + c * c - a[j].r * a[j].r) / (2 * a
78                 [i].r * c));
79                 double k = atan2(a[j].y - a[i].y, a[j].x - a[i].x);
80                 add(k - angle, k + angle);
81             }
82         }
83         arc[++acnt] = P(pi, -1);
84         sort(arc + 1, arc + acnt + 1);
85         cnt = 0;
86         double last = -pi;
87         for(int j = 1; j <= acnt; j++) {
88             s[cnt] += F(arc[j].first - last) * a[i].r * a[i].r; //扇形 - 三角形
89             double xa = a[i].x + a[i].r * cos(last);
90             double ya = a[i].y + a[i].r * sin(last);
91             last = arc[j].first;
92             double xb = a[i].x + a[i].r * cos(last);
93             double yb = a[i].y + a[i].r * sin(last);
94             s[cnt] += (xa * yb - xb * ya) / 2; //到圆心的三角形面积
95             cnt += arc[j].second;
96         }
97     }
98 }

```



```
93 //printf("%.3f\n", s[0]);
94 for (int i = 0; i < n; i++) {
95     printf("[%d] = %.3f\n", i + 1, s[i] - s[i + 1]);
96 }
97 return 0;
98 }
```

5.8 Simpson Integrate

```
1 double Simpson(double l, double r){
2     return (r-l)*(F(l)+4*F((l+r)/2)+F(r))/6;
3 }
4 double Integrate(double l, double r, double S){
5     double mid=(l+r)/2;
6     double A=Simpson(l, mid);
7     double B=Simpson(mid, r);
8     if(A+B-S<eps) return S;
9     return Integrate(l, mid, A)+Integrate(mid, r, B);
10 }
```

6 Conclusion

6.1 Game theory

6.1.1 Fibonacci's Game / Zeckendorf's theory

Fibonacci's Game (斐波那契博弈)

有一堆个数为 n 的石子，游戏双方轮流取石子，满足：

1. 先手不能在第一次把所有的石子取完；
2. 之后每次可以取的石子数介于 1 到对手刚取的石子数的 2 倍之间（包含 1 和对手刚取的石子数的 2 倍）。

结论：必败点是斐波那契数

齐肯多夫定理：任何正整数可以表示为若干个不连续的 Fibonacci 数之和

7 Others

7.1 Sample

7.1.1 vimrc

```

1 set cindent
2 set number
3 set mouse=a
4 set tabstop=4
5 set shiftwidth=4
6 syntax on
7 inoremap { {}<left>
8 map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>

```

7.1.2 Check

```

1 while true; do
2     ./data > in
3     ./tmp < in > out
4     ./std < in > ans
5     diff out ans
6     if [ $? -ne 0 ] ; then exit; fi
7     echo Passed
8 done

```

7.1.3 FastIO

```

1 namespace IO {
2     const int MB = 1048576;
3     const int RMAX = 16 * MB;
4     const int WMAX = 16 * MB;
5     #define getchar() *(rp++)
6     #define putchar(x) (*(wp++) = (x))
7     char rb[RMAX], *rp = rb, wb[WMAX], *wp = wb;
8     inline void init() {
9         fread(rb, sizeof(char), RMAX, stdin);
10    }
11    template <class _T> inline void read(_T &a) {
12        _a = 0; register bool _f = 0; register int _c = getchar();
13        while (_c < '0' || _c > '9') _f != _c == '-', _c = getchar();
14        while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getchar();
15        _a = _f ? -_a : _a;
16    }
17    template <class _T> inline void write(_T _a) {
18        static char buf[20], *top = buf;
19        if (_a) {
20            while (_a) {
21                register _T tm = _a / 10;
22                *(++top) = char(_a - tm * 10) | '0';
23                _a = tm;
24            }
25            while (top != buf) putchar(*(top--));
26        }
27        else putchar('0');
28    }
29 }

```

```

29     void output() {
30         fwrite(wb, sizeof(char), wp - wb, stdout);
31     }
32 }

```

7.1.4 Java BigNum

```

1  import java.math.*;
2  import java.util.*;
3  import java.lang.*;
4
5  public class Main{
6      public static void main(String []args){}
7  }
8  //IO
9  Scanner in = new Scanner(System.in);
10 while(in.hasNext()){} //EOF
11 //fast-IO
12 public static void main(String argv[]) throws IOException{
13     StreamTokenizer cin = new StreamTokenizer(new BufferedReader(new InputStreamReader(
14         System.in)));
15     PrintWriter cout = new PrintWriter(new OutputStreamWriter(System.out));
16     while(cin.nextToken() != StreamTokenizer.TT_EOF) ;//EOF
17     cin.nextToken();int n = (int)cin.nval;String s = cin.sval;
18     cout.println( Type );cout.flush();
19     cin.ordinaryChar( '/' );
20     BufferedReader br = new BufferedReader(new InputStreamReader(System.in));
21     br.ready()//EOF
22     while ((valueString=bf.readLine())!=null);
23     br.close();
24     //true fast-IO
25     static class InputReader {
26         public BufferedReader reader;
27         public StringTokenizer tokenizer;
28
29         public InputReader(InputStream stream) {
30             reader = new BufferedReader(new InputStreamReader(stream), 32768);
31             tokenizer = null;
32         }
33
34         public String next() {
35             while (tokenizer == null || !tokenizer.hasMoreTokens()) {
36                 try {
37                     tokenizer = new StringTokenizer(reader.readLine());
38                 } catch (IOException e) {
39                     throw new RuntimeException(e);
40                 }
41             }
42             return tokenizer.nextToken();
43         }
44
45         public int nextInt() {
46             return Integer.parseInt(next());
47         }
48     }
49 }
50 //类 Number
51 //doubleValue()
52 //intValue()

```

```

53 //longValue()
54 //shortValue()
55 //类 BigDecimal
56 //ROUND_CEILING 接近正无穷大的舍入模式。
57 //ROUND_FLOOR 接近负无穷大的舍入模式。
58 //ROUND_DOWN 接近零的舍入模式
59 //ROUND_HALF_UP 四舍五入 >=0.5向上舍入
60 //ROUND_HALF_DOWN 四舍五入 >0.5向上舍入
61 //BigDecimal(BigInteger val)
62 //BigDecimal(BigInteger unscaledVal, int scale)
63 //BigDecimal(char[] in, int offset, int len, MathContext mc)
64 //BigDecimal(double val, MathContext mc)不建议
65 //BigDecimal(int val, MathContext mc)
66 //BigDecimal(long val, MathContext mc)
67 //BigDecimal(String val, MathContext mc)
68 //abs()
69 //add(BigDecimal augend, MathContext mc)
70 //compareTo(BigDecimal val)
71 //divide(BigDecimal divisor, MathContext mc)
72 //divideToIntegralValue(BigDecimal divisor, MathContext mc)
73 //max(BigDecimal val)
74 //min(BigDecimal val)
75 //multiply(BigDecimal multiplicand, MathContext mc)
76 //negate() 其值为  $(-this)$ , 其标度为  $this.scale()$ 
77 //pow(int n)
78 //remainder(BigDecimal divisor) 返回其值为  $(this \% divisor)$  的 BigDecimal
79 //round(MathContext mc) 返回根据 MathContext 设置进行舍入后的 BigDecimal。
80 //scaleByPowerOfTen(int n) 返回其数值等于  $(this * 10^n)$  的 BigDecimal。
81 //subtract(BigDecimal subtrahend, MathContext mc)
82 //setScale(int newScale, RoundingMode roundingMode)
83 //toString()
84 //ulp() 返回此 BigDecimal 的 ulp (最后一位的单位) 的大小
85 //String s = b.stripTrailingZeros().toPlainString(); 让 bigdecimal 不用科学计数法显示
86 //类 BigInteger
87 //parseInt
88 //BigInteger zero = BigInteger.valueOf(0);
89 //BigInteger a = in.nextBigInteger();
90 //abs()
91 //and(BigInteger val) 返回其值为  $(this \& val)$ 
92 //or(BigInteger val) 返回其值为  $(this | val)$ 
93 //andNot(BigInteger val) 返回其值为  $(this \& \sim val)$ 
94 //compareTo(BigInteger val)
95 //add(BigInteger val)
96 //divide(BigInteger val)
97 //BigInteger[] divideAndRemainder(BigInteger val) 返回包含  $(this / val)$  后跟  $(this \% val)$  的两个 BigInteger 的数组。
98 //equals(Object x)
99 //gcd(BigInteger val)
100 //isProbablePrime(int certainty) e.g.: a.isProbablePrime(4)
101 //max(BigInteger val) min(BigInteger val)
102 //mod(BigInteger m)
103 //modInverse(BigInteger m) 返回其值为  $(this^{-1} \bmod m)$ 
104 //modPow(BigInteger exponent, BigInteger m) 返回其值为  $(this^{exponent} \bmod m)$ 
105 //multiply(BigInteger val)
106 //not() 返回其值为  $(\sim this)$ 
107 //shiftLeft(int n) 返回其值为  $(this \ll n)$ 
108 //shiftRight(int n) 返回其值为  $(this \gg n)$ 
109 //toString()
110 //valueOf(long val)
111 //xor(BigInteger val) 返回其值为  $(this \wedge val)$ 
112 //other

```

```
113 //Arrays.sort(array);
```

7.2 Offline Algorithm

7.2.1 CDQ Divide and Conquer

```

1 struct Node {
2     int x, y, z, ans;
3     Node() {}
4     Node(int _x, int _y, int _z):x(_x), y(_y), z(_z) {}
5     bool operator < (const Node &b) const {
6         if(y == b.y) {
7             if(z == b.z) return x < b.x;
8             return z < b.z;
9         }
10        return y < b.y;
11    }
12 }A[MAXN], B[MAXN], C[MAXN];
13 int bit[MAXN];
14 void add(int k, int v) {
15     for(; k <= m; k += k & -k) bit[k] = max(bit[k], v);
16 }
17 void clear(int k) {
18     for(; k <= m; k += k & -k) bit[k] = 0;
19 }
20 int sum(int k) {
21     int res = 0;
22     for(; k; k -= k & -k) res = max(res, bit[k]);
23     return res;
24 }
25 void solve(int l, int r) {
26     if(l == r) {
27         B[l] = A[l];
28         return;
29     }
30     int mid = (l + r) >> 1;
31     solve(l, mid);
32     for(int i = mid + 1; i <= r; i++) B[i] = A[i];
33     //sort(B + l, B + mid + 1);
34     sort(B + mid + 1, B + r + 1);
35     int L = l;
36     for(int R = mid + 1; R <= r; R++) {
37         while(L <= mid && B[L].y < B[R].y) add(B[L].z, B[L].ans), L++;
38         A[B[R].x].ans = max(A[B[R].x].ans, sum(B[R].z - 1) + 1);
39         B[R].ans = A[B[R].x].ans;
40     }
41     for(int i = l; i <= L; i++) clear(B[i].z);
42     solve(mid + 1, r);
43     L = l;
44     int p = l, q = mid + 1;
45     while(p <= mid || q <= r) {
46         if(q > r || (p <= mid && B[p].y <= B[q].y)) C[L++] = B[p++];
47         else C[L++] = B[q++];
48     }
49     for(int i = l; i <= r; i++) B[i] = C[i];
50 }

```

7.2.2 Mo' s Algorithm

```

1 struct Node{
2     int l, r, t, id;
3     bool operator < (const Node& a) const {
4         if(l / sz == a.l / sz) {
5             if(r == a.r) return t < a.t;
6             return r < a.r;
7         }
8         return l / sz < a.l / sz;
9     }
10 }q[MAXN];
11 void solve() {
12     while (t < q[i].t) addTime(t++, 1);
13     while (t > q[i].t) addTime(--t, -1);
14     while(L < q[i].l) add(L++, -1);
15     while(L > q[i].l) add(--L, 1);
16     while(R < q[i].r) add(++R, 1);
17     while(R > q[i].r) add(R--, -1);
18 }

```

7.2.3 Mo's Algorithm On Tree

```

1 struct Edge {
2     int to, nxt;
3 }e[MAXN << 1];
4 int head[MAXN], ecnt;
5 int stack[MAXN], top, belong[MAXN], cnt, sz;
6 struct Node {
7     int l, r, id, ti;
8     bool operator < (const Node &x) const {
9         return belong[l] < belong[x.l] || (belong[l] == belong[x.l] && belong[r] <
10         belong[x.r]) || (belong[l] == belong[x.l] && belong[r] == belong[x.r] && ti < x.ti);
11 }
12 }q[MAXN];
13 struct Node2 {
14     int l, r, ti;
15 }qq[MAXN];
16 int n, m, Q, Q0, Q1;
17 int V[MAXN], W[MAXN], C[MAXN];
18 int fa[MAXN][S + 3], dep[MAXN];
19 long long ans[MAXN], tans;
20 int vis[MAXN], cur[MAXN];
21 long long sum[MAXN];
22 int l, r, tm;
23 inline int read() {
24     int x = 0; char ch = getchar(); bool fg = 0;
25     while(ch < '0' || ch > '9') { if(ch == '-') fg = 1; ch = getchar(); }
26     while(ch >= '0' && ch <= '9') { x = x * 10 + ch - '0'; ch = getchar(); }
27     return fg ? -x : x;
28 }
29 inline void add_edge(int u, int v) {
30     e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
31     e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
32 }
33 void dfs(int u, int f) {
34     fa[u][0] = f;
35     dep[u] = dep[f] + 1;
36     int bot = top;
37     for(int i = head[u]; i; i = e[i].nxt) {
38         int v = e[i].to;
39         if(v == f) continue;

```

```

39     dfs(v, u);
40     if(top - bot >= sz) {
41         cnt++;
42         while(top != bot) belong[stack[top--]] = cnt;
43     }
44 }
45 stack[++top] = u;
46 }
47 void G(int &u, int step) {
48     for(int i = 0; i < S; i++) if((1 << i) & step) u = fa[u][i];
49 }
50 int lca(int u, int v) {
51     if(dep[u] > dep[v]) swap(u, v);
52     G(v, dep[v] - dep[u]);
53     if(u == v) return u;
54     for(int i = S; i >= 0; i--) if(fa[u][i] != fa[v][i]) {
55         u = fa[u][i]; v = fa[v][i];
56     }
57     return fa[u][0];
58 }
59 inline void modify(int u) {
60     tans -= V[C[u]] * sum[cur[C[u]]];
61     cur[C[u]] += vis[u];
62     vis[u] = -vis[u];
63     tans += V[C[u]] * sum[cur[C[u]]];
64 }
65 inline void update(int u, int v) {
66     if(u == v) return;
67     if(dep[u] > dep[v]) swap(u, v);
68     while(dep[v] > dep[u]) {
69         modify(v);
70         v = fa[v][0];
71     }
72     while(u != v) {
73         modify(u); modify(v);
74         u = fa[u][0]; v = fa[v][0];
75     }
76 }
77 inline void upd(int t) {
78     if(vis[qq[t].l] == -1) {
79         modify(qq[t].l);
80         swap(C[qq[t].l], qq[t].r);
81         modify(qq[t].l);
82     }
83     else swap(C[qq[t].l], qq[t].r);
84 }
85 inline void moveto(int u, int v) {
86     update(l, u); update(r, v);
87     l = u; r = v;
88 }
89 int main() {
90     n = read(); m = read(); Q = read();
91     sz = (int)pow(n, 2.0 / 3.0);
92     for(int i = 1; i <= m; i++) V[i] = read();
93     for(int i = 1; i <= n; i++) W[i] = read();
94     for(int i = 1, u, v; i < n; i++) {
95         u = read(); v = read();
96         add_edge(u, v);
97     }
98     for(int i = 1; i <= n; i++) {
99         C[i] = read();

```



```

100     vis[i] = 1;
101     sum[i] = sum[i - 1] + W[i];
102 }
103 for(int i = 1, tp; i <= Q; i++) {
104     tp = read();
105     if(tp) {
106         ++Q1;
107         q[Q1].l = read(); q[Q1].r = read();
108         q[Q1].id = Q1;
109         q[Q1].ti = i;
110     }
111     else {
112         ++Q0;
113         qq[Q0].l = read(); qq[Q0].r = read();
114         qq[Q0].ti = i;
115     }
116 }
117 dfs(1, 0);
118 while(top) belong[stack[top--]] = cnt;
119 sort(q + 1, q + Q1 + 1);
120 for(int k = 1; k <= S; k++) {
121     for(int i = 1; i <= n; i++) {
122         fa[i][k] = fa[fa[i][k - 1]][k - 1];
123     }
124 }
125 for(int i = 1; i <= Q1; i++) {
126     if(belong[q[i].l] > belong[q[i].r]) swap(q[i].l, q[i].r);
127     moveto(q[i].l, q[i].r);
128     int lc = lca(l, r);
129     modify(lc);
130     while(qq[tm + 1].ti < q[i].ti && tm < Q0) upd(++tm);
131     while(qq[tm].ti > q[i].ti) upd(tm--);
132     ans[q[i].id] = tans;
133     modify(lc);
134 }
135 for(int i = 1; i <= Q1; i++) printf("%lld\n", ans[i]);
136 return 0;
137 }

```

7.3 Randomized Algorithm

7.3.1 Simulated Annealing

```

1 void solve() {
2     while(T > eps) {
3         double alpha = ((rand() % 30001) / 15000.0) * pi;
4         double theta = ((rand() % 10001) / 10000.0) * pi;
5         tmp.x = cur.x + T * sin(theta) * cos(alpha);
6         tmp.y = cur.y + T * sin(theta) * sin(alpha);
7         tmp.z = cur.z + T * cos(theta);
8         tmp.dis = cal(tmp);
9         if(tmp.dis < cur.dis || (tmp.dis * 0.999 < cur.dis && (rand() & 7) == 7)) cur =
tmp;
10         //if(exp((cur.d - tmp.d) / T) > ((double)rand() / RAND_MAX)) cur = tmp;
11
12         T *= 0.999;
13     }
14 }

```

7.4 Other Method

7.4.1 Enumerate Subset

```

1 for(int i = 0; i < (1 << k); i++) {
2     for(int j = i; ; --j &= i) {
3         // work();
4         if(j == 0) break;
5     }
6 }

```

7.4.2 Enumerate $\lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$

```

1 int cal(int n, int m) {
2     if(n > m) swap(n, m);
3     int res = 0, last;
4     for(int i = 1; i <= n; i = last + 1) {
5         last = min(n / (n / i), m / (m / i));
6         res += (n / i) * (m / i) * (sum(last) - sum(i - 1));
7     }
8     return res;
9 }

```

7.4.3 Find Primitive Root Modulo N

```

1 for i in range(1,mod):
2     if 3 ** i % mod == 1:
3         if i == mod - 1:
4             print("yes")
5             break
6     print("no")

```

7.5 Formula

7.5.1 Euler's Theorem

$$a^b \equiv \begin{cases} a^{b \% \varphi(p)} & \gcd(a, p) = 1 \\ a^b & \gcd(a, p) \neq 1, b < \varphi(p) \\ a^{b \% \varphi(p) + \varphi(p)} & \gcd(a, p) \neq 1, b \geq \varphi(p) \end{cases} \pmod{p}$$

7.5.2 Möbius Inversion Formula

Dirichlet Convolution is $(f \times g)(N) = \sum_{d|N} f(d) * g(\frac{N}{d})$

Theorem:

$$\begin{cases} f = g \times 1 \\ g = f \times \mu \end{cases}$$

7.5.3 Math Theory Tips

$$\varphi(nm) = \varphi(n) \cdot \varphi(m) \cdot \frac{\gcd(n, m)}{\varphi(\gcd(n, m))} \quad (1)$$

$$\begin{cases} id(n) = \sum_{d|n} \varphi(d) \\ e(n) = \sum_{d|n} \mu(d) \end{cases} \quad (2)$$

$$\begin{cases} \sum_i^n \sum_j^m \gcd(i, j) = \sum_d^{\max(n, m)} \varphi(d) * \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \\ \sum_i^n \sum_j^m e(\gcd(i, j)) = \sum_d^{\min(n, m)} \mu(d) * \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \\ \sum_{i=1}^n |\mu(i)| = \sum_{i=1}^{\lfloor \sqrt{n} \rfloor} \mu(i) * \lfloor \frac{n}{i * i} \rfloor \end{cases} \quad (3)$$

$$\begin{cases} sum(x, y) = \sum_i^x \sum_j^y i * j = \frac{x * (x + 1)}{2} * \frac{y * (y + 1)}{2} \\ F(x, y) = \sum_{i=1}^{\min(x, y)} i^2 * \mu(i) * sum(\lfloor \frac{x}{i} \rfloor, \lfloor \frac{y}{i} \rfloor) \\ \sum_i^n \sum_j^m lcm(i, j) = \sum_{i=1}^{\min(n, m)} d * F(\lfloor \frac{n}{i} \rfloor, \lfloor \frac{y}{i} \rfloor) \end{cases} \quad (4)$$

7.5.4 Sieve Tips

$$\varphi(n) = \sum_{i=1}^n [(n, i) = 1] \cdot i = \frac{n * \varphi(n) + [n = 1]}{2} \quad (5)$$

$$\begin{cases} id = \varphi \times 1 \\ \frac{n \cdot (n + 1)}{2} = \sum_{i=1}^n i = \sum_{i=1}^n \sum_{d|i} \varphi(d) = \sum_{\frac{i}{d}=1}^n \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} \varphi(d) = \sum_{i=1}^n \phi(\lfloor \frac{n}{i} \rfloor) \end{cases} \quad (6)$$

$$\begin{cases} e = \mu \times 1 \\ 1 = \sum_{i=1}^n [i = 1] = \sum_{i=1}^n \sum_{d|i} \mu(d) = \sum_{i=1}^n \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} \mu(d) = \sum_{i=1}^n M(\lfloor \frac{n}{i} \rfloor) \end{cases} \quad (7)$$

$$\begin{cases} id^2 = (id \cdot \varphi) \times id \\ \phi'(n) = \sum_{i=1}^n i \cdot \varphi(i) \\ \frac{n \cdot (n + 1) \cdot (2n + 1)}{6} = \sum_{i=1}^n i^2 = \sum_{i=1}^n \sum_{d|i} d \cdot \varphi(d) \cdot \frac{i}{d} = \sum_{\frac{i}{d}=1}^n \frac{i}{d} \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} d \cdot \varphi(d) = \sum_{i=1}^n i \cdot \phi'(\lfloor \frac{n}{i} \rfloor) \end{cases} \quad (8)$$

7.6 Convolution Tips

7.6.1 FWT Tips

$$\left\{ \begin{array}{l} C_k = \sum_{i \oplus j = k} A_i * B_j \\ DWT(A)_i = \sum_j^n A_j * f_{i,j} \\ DWT(C)_i = DWT(A)_i * DWT(B)_i \\ f_{i,j} \cdot f_{i,k} = f_{i,j \oplus k} \\ f_{i,j} = [i \text{ and } j == i] \quad (and) \\ f_{i,j} = [i \text{ and } j == j] \quad (or) \\ f_{i,j} = (-1)^{[i \text{ and } j]} \quad (xor) \end{array} \right.$$

7.7 The Number of Integer Point on a Circle

Set $r = const$ is the radius of the circle.

$$r^2 = p_1^{a_1} + p_2^{a_2} + \dots + p_m^{a_m} = \sum_{i=1}^m p_i^{a_i}$$

Define

$$\chi(n) = \begin{cases} 1 & n \% 4 = 1 \\ -1 & n \% 4 = 3 \\ 0 & n \% 2 = 0 \end{cases}$$

By the way, $\chi(n)$ is a multiplicative function.

Define

$$\Gamma(p_i, a_i) = \sum_{j=0}^{a_i} \chi(p_i^j) = \begin{cases} 1 & p_i = 2 \quad || \quad (p_i \% 4 = 3 \quad \&\& \quad a_i \% 2 = 0) \\ 0 & p_i \% 4 = 3 \quad \&\& \quad a_i \% 2 = 1 \\ a_i + 1 & p_i \% 4 = 1 \end{cases}$$

Define cnt is the number of integer point on circle

$$cnt(r) = 4 \prod_{i=1}^m \sum_{j=0}^{a_i} \chi(p_i^j) = 4 \prod_{i=1}^m \Gamma(p_i, a_i) = 4 \sum_{k|r^2} \chi(k)$$

Define CNT is the number of integer point in circle

$$CNT(r) = 1 + \sum_{i=1}^{r^2} cnt(i) = 1 + \sum_{i=1}^{r^2} \left\lfloor \frac{r^2}{i} \right\rfloor \chi(i)$$