

SOUTH CHINA UNIVERSITY OF TECHNOLOGY

SCUT\_GUGUGU

# TEMPLATE



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# 1 Graph Theory

## 1.1 Shortest Path

### 1.1.1 Dijkstra

```

1  typedef long long LL;
2  const int MAXN = ;
3  const int MAXM = ;
4  const LL DINF = ;
5  typedef pair<LL, int> P;
6  struct Edge {
7      int to, nxt;
8      LL w;
9  }e[MAXM];
10 int head[MAXN], ecnt;
11 LL d[MAXN];
12 priority_queue<P, vector<P>, greater<P> > q;
13 inline void addEdge(int x, int y, LL w) {
14     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
15 }
16 void dijkstra(int st, int n) {
17     for(int i = 0; i <= n; i++) d[i] = DINF;
18     d[st] = 0;
19     q.push(make_pair(0, st));
20     while(!q.empty()) {
21         P x = q.top(); q.pop();
22         int u = x.second;
23         if(d[u] != x.first) continue;
24         for(int i = head[u], v; i; i = e[i].nxt) {
25             v = e[i].to;
26             if(d[v] > d[u] + e[i].w) {
27                 d[v] = d[u] + e[i].w;
28                 q.push(make_pair(d[v], v));
29             }
30         }
31     }
32 }

```

### 1.1.2 SPFA

```

1  struct Edge {
2      int to, nxt;
3      LL w;
4  }e[MAXE];
5  int head[MAXN], ecnt;
6  LL d[MAXN];
7  bool exist[MAXN];
8  queue<int> q;
9  inline void addEdge(int x, int y, LL w) {
10     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
11 }
12 void SPFA(int st) {
13     memset(d, 0x3f, sizeof(d));
14     d[st] = 0;
15     q.push(st);
16     exist[st] = 1;
17     while(!q.empty()) {

```

```

18     int u = q.front(); q.pop();
19     exist[u] = 0;
20     for(int i = head[u], v; i; i = e[i].nxt) {
21         v = e[i].to;
22         if(d[v] > d[u] + e[i].w) {
23             d[v] = d[u] + e[i].w;
24             //pre[v] = u;
25             if(!exist[v]) {
26                 q.push(v);
27                 exist[v] = 1;
28             }
29         }
30     }
31 }
32 }

```

### 1.1.3 Johnson

```

1 void johnson() {
2     //建图中, Edge需要from, w1, w2, 去掉w;
3     spfa(1);
4     for(int u = 1; u <= n; u++)
5         for(int i = head[u]; i; i = e[i].nxt)
6             e[i].w2 = e[i].w1 + d[e[i].from] - d[e[i].to];
7     dijkstra(s, n);
8 }

```

### 1.1.4 K Shortest Path (A\*)

```

1 //可重复走同一条边 利用了反向边表示, ecnt初始化为1
2 //调用 dijkstra(ed, n)跑反向图, 注意if(i & 1){松弛}
3 int shrt[MAXN];
4 LL A_star(int st, int ed, int k, int n) {
5     if(d[st] == d[0]) return -1;
6     if(st == ed) k++;
7     q.push(make_pair(d[st], st));
8     while(!q.empty()) {
9         P x = q.top(); q.pop();
10        int u = x.second;
11        LL xd = x.first;
12        ++shrt[u];
13        if(u == ed) {
14            if(shrt[ed] == k) return xd;
15        }
16        for(int i = head[u], v; i; i = e[i].nxt)
17            if(!(i & 1)) {
18                v = e[i].to;
19                if(shrt[v] < k) q.push(make_pair(xd - d[u] + e[i].w + d[v], v));
20            }
21    }
22    return -1;
23 }

```

### 1.1.5 K Shortest Path (Protractable Heap)

```

1 //可重复走同一条边
2 typedef double LD;

```

```

3  const int MAXN = ;
4  const int MAXM = ;
5  const int MAXLT = MAXM * 20;
6  const LD DINF = ;
7  const LD eps = ;
8
9  namespace LT{
10     int tcnt;
11     int ls[MAXLT], rs[MAXLT], dis[MAXLT];
12     int to[MAXLT];
13     LD val[MAXLT];
14
15     inline int newnode(LD w, int _to) {
16         ++tcnt;
17         ls[tcnt] = rs[tcnt] = 0; dis[tcnt] = 1;
18         val[tcnt] = w; to[tcnt] = _to;
19         return tcnt;
20     }
21     inline int copynode(int id) {
22         ++tcnt;
23         ls[tcnt] = ls[id]; rs[tcnt] = rs[id]; dis[tcnt] = dis[id];
24         val[tcnt] = val[id]; to[tcnt] = to[id];
25         return tcnt;
26     }
27     void push_up(int x) {
28         if(dis[ls[x]] < dis[rs[x]]) swap(ls[x], rs[x]);
29         dis[x] = dis[rs[x]] + 1;
30     }
31     int merge(int x, int y) {
32         if(!x || !y) return x^y;
33         if(val[x] - val[y] > eps) swap(x, y);
34         int p = copynode(x);
35         rs[p] = merge(rs[p], y);
36         push_up(p);
37         return p;
38     }
39 }
40 int rt[MAXN];
41
42 typedef pair<LD, int> P;
43 struct Edge {
44     int to, nxt;
45     LD w;
46 }e[MAXM];
47 int head[MAXN], ecnt;
48 int stan, sta[MAXN], fa[MAXN];
49 int vis[MAXN], cov[MAXM];
50 LD d[MAXN];
51 priority_queue<P, vector<P>, greater<P> > q;
52 inline void addEdge(int x, int y, LD w) {
53     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
54 }
55 void init(int n, int m) {
56     ecnt = 1; stan = 0;
57     for(int i = 1; i <= n; i++)
58         head[i] = cov[i] = fa[i] = 0;
59     for(int i = 1; i <= m; i++) {
60         int u, v; LD w;
61         scanf("%d%d%lf", &u, &v, &w);
62         addEdge(u, v, w);
63         addEdge(v, u, w);

```



```

64     }
65 }
66 void dijkstra(int st, int n) {
67     for(int i = 0; i <= n; i++) {d[i] = DINF; vis[i] = 0;}
68     while(!q.empty()) q.pop();
69     d[st] = 0; q.push(make_pair(0, st));
70     while(!q.empty()) {
71         P x = q.top(); q.pop();
72         int u = x.second;
73         if(vis[u]) continue;
74         vis[u] = 1;
75         for(int i = head[u], v; i; i = e[i].nxt)
76             if(i & 1) {
77                 v = e[i].to;
78                 if(d[v] - (d[u] + e[i].w) > eps) {
79                     d[v] = d[u] + e[i].w;
80                     q.push(make_pair(d[v], v));
81                 }
82             }
83     }
84 }
85 void buildT(int u) {
86     sta[++stan] = u;
87     vis[u] = 1;
88     for(int i = head[u], v; i; i = e[i].nxt)
89         if(i & 1) {
90             v = e[i].to;
91             if(fabs(d[v] - (d[u] + e[i].w)) < eps && !vis[v]) {
92                 fa[v] = u; cov[i^1] = 1; buildT(v);
93             }
94         }
95 }
96 void buildH(int st, int n) {
97     buildT(st);
98     for(int i = 2, u, v; i <= ecnt; i += 2) {
99         if(!cov[i]) {
100             u = e[i^1].to; v = e[i].to;
101             if(fabs(d[u] - d[0]) < eps || fabs(d[v] - d[0]) < eps) continue;
102             rt[u] = LT::merge(rt[u], LT::newnode(d[v]+e[i].w-d[u], v));
103         }
104     }
105     for(int i = 2, u; i <= n; i++)
106         if(fa[u = sta[i]]) rt[u] = LT::merge(rt[u], rt[fa[u]]);
107 }
108 //求前k短路径,其和不超过W,问最大k int ans = 0;
109 //求第k短路径,LD ans = 0;
110 void getKth(int st, int ed, LD W) {
111     while(!q.empty()) q.pop();
112     //最短路要记入答案,注意d[st]-W<eps与W-d[st]<eps
113     //!if(d[st] - W > eps) return;
114     //!else {W -= d[st]; ++ans;}
115     //注意st和ed相同
116     //!if(st == ed) k++;
117     //!if(d[st] == d[0]) {ans = -1; return;}
118     //!if(--k == 0) {ans = d[st]; return;}
119
120     int u = rt[st], v;
121     if(u) q.push(make_pair(LT::val[u], u));
122     while (!q.empty()) {
123         u = q.top().second; LD cur = q.top().first;
124         q.pop();

```

```

125     //!if (cur + d[st] - W > eps) break;
126     //!else {W -= cur + d[st]; ++ans;}
127     //!if (--k == 0) {ans = cur + d[st]; break;}
128     v = LT::ls[u];
129     if (v) q.push(make_pair(cur - LT::val[u] + LT::val[v], v));
130     v = LT::rs[u];
131     if (v) q.push(make_pair(cur - LT::val[u] + LT::val[v], v));
132     v = rt[LT::to[u]];
133     if (v) q.push(make_pair(cur + LT::val[v], v));
134 }
135 }
136 void sol() {
137     int n, m, st = , ed = ;
138     //LD W; int k;
139     init(n, m);
140     dijkstra(ed, n);
141     for(int i = 0; i <= n; i++) vis[i] = 0;
142     buildH(ed, n);
143     getKth(st, ed, W);
144     printf("%d\n", ans);
145 }

```

## 1.2 Network Flow

### 1.2.1 ISAP

```

1 namespace NWF {
2     struct Edge{
3         int to, nxt; LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], pre[MAXN], num[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(num, 0, (tot + 1) * sizeof(int));
11        memset(head, 0, (tot + 1) * sizeof(int));
12    }
13    inline void addEdge(int u, int v, LL f) {
14        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
15        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
16    }
17    void bfs() {
18        memset(dis, 0, (tot + 1) * sizeof(int));
19        q.push(T);
20        dis[T] = 1;
21        while(!q.empty()) {
22            int u = q.front(), v; q.pop();
23            num[dis[u]]++;
24            for(int i = cur[u] = head[u]; i; i = e[i].nxt) {
25                if(!dis[v = e[i].to]) {
26                    dis[v] = dis[u] + 1;
27                    q.push(v);
28                }
29            }
30        }
31    }
32    LL augment() {
33        LL flow = INF;

```

```

34     for(int i = S; i != T; i = e[cur[i]].to)
35         flow = min(flow, e[cur[i]].f);
36     for(int i = S; i != T; i = e[cur[i]].to) {
37         e[cur[i]].f -= flow;
38         e[cur[i] ^ 1].f += flow;
39     }
40     return flow;
41 }
42 LL isap() {
43     bfs();
44     int u = S, v;
45     LL flow = 0;
46     while(dis[S] <= tot) {
47         if(u == T) {
48             flow += augment();
49             u = S;
50         }
51         bool fg = 0;
52         for(int i = cur[u]; i; i = e[i].nxt) {
53             if(e[i].f && dis[u] > dis[v = e[i].to]) {
54                 pre[v] = u;
55                 cur[u] = i;
56                 u = v;
57                 fg = 1;
58                 break;
59             }
60         }
61         if(fg) continue;
62         if(!--num[dis[u]]) break;
63         int maxDis = tot;
64         for(int i = head[u]; i; i = e[i].nxt) {
65             if(e[i].f && maxDis > dis[v = e[i].to]) {
66                 maxDis = dis[v];
67                 cur[u] = i;
68             }
69         }
70         num[dis[u] = maxDis + 1]++;
71         if(u != S) u = pre[u];
72     }
73     return flow;
74 }
75 }

```

### 1.2.2 HLPP

```

1 namespace NWF{
2     struct Edge{
3         int to,nxt;LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], dis[MAXN], num[MAXN];
7     LL sumf[MAXN];
8     queue<int> q;
9     list<int> dep[MAXN];
10    void init(int _S,int _T,int _tot){
11        ecnt = 1;S = _S;T = _T;tot = _tot;
12        memset(num, 0, (tot + 1) * sizeof(int));
13        memset(head, 0, (tot + 1) * sizeof(int));
14        memset(sumf, 0, (tot + 1) * sizeof(LL));
15    }

```

```

16 void addEdge(int u,int v,LL f){
17     e[++ecnt] = (Edge) {v, head[u], f};head[u] = ecnt;
18     e[++ecnt] = (Edge) {u, head[v], 0};head[v] = ecnt;
19 }
20 void bfs(){
21     memset(dis, 0, (tot + 1) * sizeof(int));
22     q.push(T); dis[T] = 1;
23     while(!q.empty()){
24         int u=q.front(), v; q.pop();
25         for(int i = head[u]; i; i = e[i].nxt)
26             if(!dis[v = e[i].to]){
27                 dis[v] = dis[u] + 1;
28                 q.push(v);
29             }
30     }
31 }
32 LL h1pp(){
33     bfs();
34     dis[S] = tot + 1;
35     for(int i = 1;i <= tot; ++i)num[dis[i]]++;
36     for(int i = tot + 1; ~i; --i)dep[i].clear();
37     int maxd = dis[S];LL f;
38     dep[maxd].push_back(S);sumf[S] = INF;
39     for(;;){
40         while(maxd && dep[maxd].empty())maxd--;
41         if(!maxd)break;
42         int u = dep[maxd].back(), v;dep[maxd].pop_back();
43         int minDis = tot + 1;
44         for(int i = head[u]; i;i = e[i].nxt)
45             if(e[i].f){
46                 if(dis[u] > dis[v = e[i].to]){
47                     f = min(sumf[u], e[i].f);
48                     e[i].f -= f;e[i^1].f += f;
49                     if(sumf[u] != INF) sumf[u] -= f;
50                     if(sumf[v] != INF) sumf[v] += f;
51                     if(v!=S && v!=T && sumf[v] == f){
52                         maxd = max(maxd, dis[v]);
53                         dep[dis[v]].push_back(v);
54                     }
55                     if(!sumf[u])break;
56                 }else minDis=min(minDis, dis[v] + 1);
57             }
58         if(sumf[u]){
59             if(!--num[dis[u]]){
60                 for(int i = dis[u];i <= maxd;++i){
61                     while(!dep[i].empty()){
62                         --num[i];
63                         dis[dep[i].back()] = tot + 1;
64                         dep[i].pop_back();
65                     }
66                 }
67                 maxd = dis[u] - 1;dis[u] = tot + 1;
68             }else{
69                 dis[u] = minDis;
70                 if(minDis > tot)continue;
71                 num[minDis]++;
72                 maxd = max(maxd, minDis);
73                 dep[minDis].push_back(u);
74             }
75         }
76     }

```

```

77     return sumf[T];
78 }
79 }

```

### 1.2.3 Dinic

```

1 namespace NWF {
2     struct Edge {
3         int to, nxt; LL f;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(head, 0, (tot + 1) * sizeof(int));
11    }
12    void addEdge(int u, int v, LL f) {
13        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
14        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
15    }
16    bool bfs() {
17        memset(dis, 0, (tot + 1) * sizeof(int));
18        q.push(S); dis[S] = 1;
19        while (!q.empty()) {
20            int u = q.front(), v; q.pop();
21            for (int i = cur[u] = head[u]; i; i = e[i].nxt) {
22                if (e[i].f && !dis[v = e[i].to]) {
23                    q.push(v);
24                    dis[v] = dis[u] + 1;
25                }
26            }
27        }
28        return dis[T];
29    }
30    LL dfs(int u, LL maxf) {
31        if (u == T) return maxf;
32        LL sumf = maxf;
33        for (int &i = cur[u]; i; i = e[i].nxt) {
34            if (e[i].f && dis[e[i].to] > dis[u]) {
35                LL tmpf = dfs(e[i].to, min(sumf, e[i].f));
36                e[i].f -= tmpf; e[i ^ 1].f += tmpf;
37                sumf -= tmpf;
38                if (!sumf) return maxf;
39            }
40        }
41        return maxf - sumf;
42    }
43    LL dinic() {
44        LL ret = 0;
45        while (bfs()) ret += dfs(S, INF);
46        return ret;
47    }
48 }

```

### 1.2.4 MCMF

```

1 namespace NWF{

```

```

2   struct Edge {
3       int to, nxt; LL f, c;
4   } e[MAXM << 1];
5   int S, T, tot;
6   int ecnt, head[MAXN], cur[MAXN]; LL dis[MAXN];
7   bool exist[MAXN];
8   queue<int> q;
9   void init(int _S, int _T, int _tot){
10       ecnt = 1; S = _S; T = _T; tot = _tot;
11       memset(head, 0, (tot + 1) * sizeof(int));
12   }
13   void addEdge(int u, int v, LL f, LL c) {
14       e[++ecnt] = (Edge) {v, head[u], f, c}; head[u] = ecnt;
15       e[++ecnt] = (Edge) {u, head[v], 0, -c}; head[v] = ecnt;
16   }
17   bool spfa() {
18       for(int i = 0; i <= tot; ++i){
19           dis[i] = INF; cur[i] = exist[i] = 0;
20       }
21       q.push(S); dis[S] = 0; exist[S] = 1;
22       while(!q.empty()) {
23           int u = q.front(), v; q.pop(); exist[u] = 0;
24           for(int i = head[u]; i; i = e[i].nxt) {
25               if(e[i].f && dis[v = e[i].to] > dis[u] + e[i].c) {
26                   dis[v] = dis[u] + e[i].c;
27                   cur[v] = i;
28                   if(!exist[v]) {
29                       q.push(v);
30                       exist[v] = 1;
31                   }
32               }
33           }
34       }
35       return dis[T] != INF;
36   }
37   LL mcmf() {
38       LL cost = 0;
39       //while(spfa() && dis[T] < 0) { //最小费用可行流
40       while(spfa()) {
41           LL flow = INF;
42           for(int i = T; i != S; i = e[cur[i] ^ 1].to)
43               flow = min(flow, e[cur[i]].f);
44           for(int i = T; i != S; i = e[cur[i] ^ 1].to) {
45               e[cur[i]].f -= flow;
46               e[cur[i] ^ 1].f += flow;
47           }
48           cost += flow * dis[T];
49       }
50       return cost;
51   }
52 }

```

## 1.3 Tree Related

### 1.3.1 Union Set

```

1   int fa[MAXN], rnk[MAXN];
2   int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
3   bool same(int x, int y){ return Find(x) == Find(y); }

```

```

4 void unite(int x, int y)
5 {
6     x = Find(x);
7     y = Find(y);
8     if(x == y) return;
9     if(rnk[x] < rnk[y]) {
10         fa[x] = y;
11     }
12     else {
13         fa[y] = x;
14         if(rnk[x] == rnk[y]) rnk[x]++;
15     }
16 }

```

### 1.3.2 Kruskal

```

1 namespace MST{
2     struct Edge{
3         int u,v; LL w;
4         bool operator < (const Edge& x) const { return w < x.w; }
5     }e[MAXM];
6     int ecnt, fa[MAXN];
7     void addEdge(int u, int v, LL w) {
8         e[++ecnt] = (Edge){v, u, w}; //headp[u] = ecnt;
9     }
10    int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
11    LL kruskal(int n) {
12        sort(e + 1, e + ecnt + 1);
13        for(int i = 1; i <= n; i++) fa[i] = i;
14        LL sum = 0;
15        for (int i = 1; i <= ecnt; i++){
16            int fu = Find(e[i].u), fv = Find(e[i].v);
17            if(fu != fv){
18                fa[fu] = fv;
19                sum += e[i].w;
20            }
21        }
22        return sum;
23    }
24 }

```

### 1.3.3 Prim

```

1 namespace MST {
2     struct Edge{
3         int to,nxt; LL w;
4     }e[MAXM];
5     int ecnt, head[MAXN], vis[MAXN]; // pre[MAXN];
6     LL dis[MAXN];
7     void addEdge(int u, int v, LL w){
8         e[++ecnt] = (Edge){v, head[u], w}; head[u] = ecnt;
9         e[++ecnt] = (Edge){u, head[v], w}; head[v] = ecnt;
10    }
11    LL Prim(int n){
12        for (int i = 1; i <= n; i++){
13            //pre[i] = 0;
14            vis[i] = 0;
15            dis[i] = INF;

```

```

16     }
17     vis[1] = 1;
18     LL sum = 0;
19     for (int i = head[1]; i; i = e[i].nxt)
20         dis[e[i].to] = min(dis[e[i].to], e[i].w);
21     for (int j = 1; j < n; j++){
22         int u; LL minDis = INF;
23         for (int i = 1; i <= n; ++i)
24             if (!vis[i] && dis[i] < minDis){
25                 minDis = dis[i];
26                 u = i;
27             }
28         if (minDis == INF) return -1;
29         vis[u] = 1;
30         sum += minDis;
31         for (int i = head[u], v; i; i = e[i].nxt)
32             if (!vis[v = e[i].to] && e[i].w < dis[v]){
33                 //pre[u] = v;
34                 dis[v] = e[i].w;
35             }
36     }
37     return sum;
38 }
39 }

```

### 1.3.4 Spanning Tree Calculation

关联矩阵  $B:n*m$  的矩阵, 其中  $ek=(vi,vj)$ ,  $Bik$  和  $Bjk$  一个为 1 一个为 -1, 第  $k$  列其他元素为 0 度数矩阵  $D:n*n$  的矩阵, 其中  $i!=j$  时,  $D[i][j]=0$ ;  $i=j$  时,  $D[i][j]=vi$  的度邻接矩阵  $A:n*n$  的矩阵,  $vi,vj$  有边相连, 为 1, 否则为 0 Kirchhoff 矩阵:  $B*BT = D - A$  生成树个数: Kirchhoff 矩阵  $n-1$  阶主子式的行列式值构造 Kirchhoff 矩阵, 调用  $\det(n)$

```

1 LL a[MAXN][MAXN];
2 void getSTC(int n, int m) {
3     for(int i = 1; i <= n; i++) {
4         for(int j = 1; j <= n; j++)
5             a[i][j] = a[j][i] = 0;
6     }
7     for(int i = 1, u, v; i <= m; i++) {
8         scanf("%d%d", &u, &v);
9         if(u == v) continue;
10        a[u][v] = --a[v][u];
11    }
12    for(int i = 1; i <= n; i++) {
13        int t = 0;
14        for(int j = 1; j <= n; j++)
15            t += a[i][j];
16        a[i][i] = -t;
17    }
18    LL ans = det(n);
19 }

```

### 1.3.5 Minimum Spanning Tree Calculation

```

1 typedef long long LL;
2 const int MAXN = ;
3 const int MAXM = ;
4 int sum, ans1, ans2=1, Mod=;

```



```

5  int fa1[MAXN],fa2[MAXN];
6  bool vis[MAXN];
7  struct Edge{int v,u,val;}e[MAXM];
8  bool cmp(Edge A,Edge B){return A.val<B.val;}
9  int getfa1(int *fa,int x){return fa[x]==x?x:getfa1(fa,fa[x]);}
10 int getfa2(int *fa,int x){return fa[x]==x?x:getfa2(fa,fa[x]);}
11 void dfs(int tot,int l,int r){
12     if(tot==0){++sum;return;}
13     for(int i=l,fx,fy;i<r;++i)
14         if(!vis[i]){
15             vis[i]=true;
16             fx=getfa2(fa2,e[i].u);fy=getfa2(fa2,e[i].v);
17             if(fx!=fy){
18                 fa2[fx]=fy;
19                 dfs(tot-1,i+1,r);
20                 fa2[fx]=fx;
21             }
22             vis[i]=false;
23         }
24 }
25 void sol(){
26     int n,m;
27     scanf("%d %d",&n,&m);
28     for(int i=1;i<=n;++i)fa1[i]=fa2[i]=i;
29     for(int i=1;i<=m;++i)
30         scanf("%d %d %d",&e[i].v,&e[i].u,&e[i].val);
31     std::sort(e+1,e+m+1,cmp);e[m+1].val=-1;
32     for(int i=1,j=1,fx,fy,tot;i<=m;++i){
33         for(;e[i].val==e[j].val;++j);
34         tot=0;
35         for(int k=i;k<j;++k){
36             fx=getfa1(fa1,e[k].u);fy=getfa1(fa1,e[k].v);
37             if(fx!=fy){++tot;ans1++;fa1[fx]=fy;}
38         }
39         if(!tot)continue;
40         sum=0;dfs(tot,i,j);
41         (ans2*=sum)%=Mod;
42         for(int k=i;k<j;++k){
43             fx=getfa1(fa2,e[k].u);fy=getfa1(fa2,e[k].v);
44             if(fx!=fy)fa2[fx]=fy;
45         }
46     }
47     if(ans1!=n-1)puts("0");else printf("%d",ans2);
48 }

```

### 1.3.6 Steiner Tree

```

1  const int MAXH = 128;
2  const int MAXW = 128;
3  const int MAXST = 1256;
4  namespace SteinerTree{
5      const int dx[4] = {0,0,1,-1};
6      const int dy[4] = {1,-1,0,0};
7      int n, m, k, stn;
8      bool vis[MAXH][MAXW];
9      LL G[MAXH][MAXW],f[MAXST][MAXH][MAXW];
10     queue<pair<int, int>> q;
11     struct PRE{int s, x, y;} pre[MAXST][MAXH][MAXW];
12
13     void spfa(int st) {

```

```

14 while(!q.empty()) {
15     int vx = q.front().first,vy = q.front().second;
16     q.pop();
17     vis[vx][vy] = 0;
18     for (int i = 0, ux,uy; i < 4; ++i) {
19         ux = vx + dx[i];
20         uy = vy + dy[i];
21         if (ux==0||uy==0||ux==n+1||uy==m+1)continue;
22         if (f[st][vx][vy]+G[ux][uy]<f[st][ux][uy]) {
23             f[st][ux][uy] = f[st][vx][vy] + G[ux][uy];
24             pre[st][ux][uy] = (PRE) {st, vx, vy};
25             if (!vis[ux][uy]) {
26                 vis[ux][uy]=1;
27                 q.push(make_pair(ux,uy));
28             }
29         }
30     }
31 }
32 }
33 LL sum = 0;
34 void init() {
35     k = 0;
36     for(int i = 1; i <= n; ++i)
37         for(int j = 1; j <= m; ++j) {
38             scanf("%d", &G[i][j]);
39             if(G[i][j] == 0) k++;
40         }
41     stn = 1<<k;
42     for (int st = 0; st < stn; ++st)
43         for (int i = 1; i <= n; ++i)
44             for (int j = 1; j <= m; ++j)
45                 f[st][i][j] = INF;
46     int tk = 0;
47     for(int i = 1; i <= n; ++i)
48         for(int j = 1; j <= m; ++j) {
49             vis[i][j] = 0;
50             if(G[i][j] == 0) {f[1<<tk][i][j] = 0; tk++;}
51         }
52 }
53 void dfs(int st, int x, int y) {
54     vis[x][y] = 1;
55     PRE tmp = pre[st][x][y];
56     if (tmp.x == 0 && tmp.y == 0) return ;
57     dfs(tmp.s, tmp.x, tmp.y);
58     if (tmp.x == x && tmp.y == y) dfs(st - tmp.s, tmp.x, tmp.y);
59 }
60 void sol(int _n, int _m) {
61     n = _n; m = _m;
62     init();
63     for (int st = 0; st < stn; ++st) {
64         for (int i = 1; i <= n; ++i)
65             for (int j = 1; j <= m; ++j) {
66                 for (int s = st&(st-1); s; s = st&(s-1))
67                     if(f[st-s][i][j]+f[s][i][j]-G[i][j] < f[st][i][j]){
68                         f[st][i][j] = f[st-s][i][j]+f[s][i][j]-G[i][j];
69                         pre[st][i][j] = (PRE) {s, i, j};
70                     }
71                 if (f[st][i][j]!=INF) {
72                     q.push(make_pair(i,j));
73                     vis[i][j]=1;
74                 }
75             }
76     }
77 }

```

```

75     }
76     spfa(st);
77 }
78 int ansx, ansy, fg = 0;
79 for(int i = 1; i <= n && !fg; ++i)
80     for(int j = 1; j <= m; ++j)
81         if(!G[i][j]) {ansx = i; ansy = j; fg = 1; break;}
82 printf("%d\n", f[stn-1][ansx][ansy]);
83 memset(vis, 0, sizeof vis);
84 dfs(stn-1, ansx, ansy);
85 for(int i = 1; i <= n; i++, puts("")) {
86     for(int j = 1; j <= m; j++) {
87         if(G[i][j] == 0) putchar('x');
88         else if(vis[i][j]) putchar('o');
89         else putchar(' ');
90     }
91 }
92 }
93 }

```

### 1.3.7 Tree Divide and Conquer

```

1 struct Edge {
2     int to, nxt, w;
3 }e[MAXM];
4 int head[MAXN], ecnt;
5 int sz[MAXN];
6 int d[MAXN], t[5], ans;
7 bool vis[MAXN];
8 inline void add_edge(int u, int v, int w) {
9     e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
10    e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
11 }
12 int getsz(int x, int fa) {
13     sz[x] = 1;
14     for(int i = head[x]; i; i = e[i].nxt) {
15         int y = e[i].to;
16         if(vis[y] || y == fa) continue;
17         sz[x] += getsz(y, x);
18     }
19     return sz[x];
20 }
21 int getrt(int x) {
22     int tot = getsz(x, 0) >> 1;
23     while(1) {
24         int u = -1;
25         for(int i = head[x]; i; i = e[i].nxt) {
26             int y = e[i].to;
27             if(vis[y] || sz[y] > sz[x]) continue;
28             if(u == -1 || sz[y] > sz[u]) u = y;
29         }
30         if(~u && sz[u] > tot) x = u;
31         else break;
32     }
33     return x;
34 }
35 void getdep(int x, int fa) {
36     t[d[x]]++;
37     for(int i = head[x]; i; i = e[i].nxt) {
38         int y = e[i].to;

```

```

39     if(vis[y] || y == fa) continue;
40     d[y] = (d[x] + e[i].w) % 3;
41     getdep(y, x);
42 }
43 }
44 int cal(int x, int v) {
45     t[0] = t[1] = t[2] = 0;
46     d[x] = v % 3;
47     getdep(x, 0);
48     return t[0] * t[0] + t[1] * t[2] * 2;
49 }
50 void solve(int x) {
51     vis[x] = 1;
52     ans += cal(x, 0);
53     for(int i = head[x]; i; i = e[i].nxt) {
54         int y = e[i].to;
55         if(vis[y]) continue;
56         ans -= cal(y, e[i].w);
57         solve(getrt(y));
58     }
59 }
60 int main() {
61     solve(getrt(1));
62 }

```

### 1.3.8 Dominator Tree

```

1  #define LL long long
2  #define FILE "dagch"
3  using namespace std;
4
5  const int N = 200010;
6  struct Node{int to,next;}E[N<<1];
7  int n,m,q,head[N],tot,dfn[N],clo,rev[N],fa[N],semi[N],Ans[N];
8  vector<int>G[N];
9  struct Union_Merge_Set{
10     int fa[N],Mi[N];
11     inline void init(){
12         for(int i=0;i<=n;++i)
13             fa[i]=Mi[i]=semi[i]=i;
14     }
15     inline int find(int x){
16         if(x==fa[x])return x;
17         int fx=fa[x],y=find(fa[x]);
18         if(dfn[semi[Mi[fx]]]<dfn[semi[Mi[x]]])Mi[x]=Mi[fx];
19         return fa[x]=y;
20     }
21 }uset;
22
23 inline void tarjan(int u) {
24     rev[dfn[u] = ++tarjan_time] = u;
25     for(auto v : G[u])
26         if(!dfn[v]) {
27             fa[v] = u;
28             tarjan(v);
29         }
30 }
31 inline void get_semi() {
32     for(int i = tarjan_time; i >= 2; i--) {
33         int u = rev[i], tsemi = n;

```

```

34     for(auto v : rG[u]) {
35         if(!dfn[v]) continue;
36         if(dfn[v] < dfn[u]) tsemi = min(tsemi, dfn[x]);
37         else{
38             uset.find(x);
39             tsemi = min(tsemi, dfn[semi[uset.Mi[x]]]);
40         }
41     }
42     uset.fa[y] = fa[y];
43     semi[y] = rev[tsemi];
44     Ans[rev[tsemi]]++;
45 }
46 }
47
48 inline void solve() {
49     scanf("%d %d %d", &n, &m, &q);
50     fa[1]=1;
51     for(int i = 1, u, v; i <= m; ++i){
52         scanf("%d%d", &u,&v);
53         link(v,u);
54         G[u].push_back(v);
55     }
56     for(int i = 1; i <= n; i++)
57         if(G[i].size())
58             sort(G[i].begin(), G[i].end());
59
60     uset.init();
61
62     tarjan(1);
63     build();
64     for(int i=1;i<=q;++i)
65         printf("%d ",Ans[gi()]);
66     printf("\n");
67     for(int i=0;i<=n;++i){
68         G[i].clear();head[i]=0;
69         Ans[i]=semi[i]=fa[i]=0;
70     }
71     clo=tot=0;
72 }
73
74 int main() {
75     int T; scanf("%d", &T);
76     while(T--) solve();
77     return 0;
78 }

```

## 1.4 LCA

### 1.4.1 Tree Decomposition LCA

见树链剖分

### 1.4.2 Tarjan LCA

```

1  vector< pair<int,int> > G[MAXN],ask[MAXN];
2  int fa[MAXN], ans[MAXN], vis[MAXN] ,dis[MAXN];
3  int Find(int x){
4      return x == fa[x] ? x : fa[x] = Find(fa[x]);
5  }

```

```

6 void init(int n){
7     memset(ans, 0, sizeof ans);
8     memset(vis, 0, sizeof vis);
9     for(int i = 0; i <= n; i++){
10         G[i].clear();
11         ask[i].clear();
12     }
13 }
14 void LCA(int u){
15     int v;
16     fa[u] = u;
17     vis[u] = true;
18     for(auto it : ask[u])
19         if(vis[v = it.first])
20             ans[it.second] = dis[u] + dis[v] - 2 * dis[Find(it.first)];
21     for(auto it : G[u])
22         if(!vis[v = it.first]){
23             dis[v] = dis[u] + it.second;
24             LCA(v);
25             fa[v] = u;
26         }
27 }

```

## 1.5 Tarjan

### 1.5.1 SCC

```

1 namespace SCC{
2     vector<int> G[MAXN];
3     int dfs_clock, scc_cn, dfn[MAXN], low[MAXN], sccno[MAXN];
4     stack<int> S;
5     void addEdge(int u, int v) {
6         G[u].push_back(v);
7     }
8     void tarjan(int u) {
9         dfn[u] = low[u] = ++dfs_clock;
10        S.push(u);
11        for(auto v : G[u]) {
12            if(!dfn[v]) {
13                tarjan(v);
14                low[u] = min(low[u], low[v]);
15            } else if(!sccno[v]) {
16                low[u] = min(low[u], dfn[v]);
17            }
18        }
19        if(dfn[u] == low[u]) {
20            scc_cnt++;
21            for(;;) {
22                int v = S.top(); S.pop();
23                sccno[v] = scc_cnt;
24                if(v == u) break;
25            }
26        }
27    }
28    void findSCC(int n) {
29        for(int i = 1; i <= n; i++)
30            if(!dfn[i]) tarjan(i);
31    }
32    void init(int n){

```

```

33     dfs_clock = scc_cnt = 0;
34     for(int i = 0; i <= n; ++i){
35         dfn[i] = low[i] = sccno[i] = 0;
36         G[i].clear();
37     }
38 }
39 }

```

### 1.5.2 BCC

```

1  namespace BCC{
2      struct Edge {
3          int to, nxt;
4      }e[MAXM << 1];
5      int ecnt, head[MAXN];
6      int dfs_clock, dfn[MAXN], low[MAXN];
7
8      int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
9      vector<int> vbcc[MAXN];
10     stack<int> vS;
11
12     int ebcc_cnt, ebccno[MAXN];
13     stack<int> eS;
14
15     inline void addEdge(int u, int v) {
16         e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
17         e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
18     }
19     inline void init(int n) {
20         ecnt = 1;
21         dfs_clock = 0;
22         vbcc_cnt = 0;
23         ebcc_cnt = 0;
24         for(int i = 1; i <= n; ++i){
25             head[i] = dfn[i] = low[i] = 0;
26             is_vertex[i] = 0;
27             vbccno[i] = 0;
28             ebccno[i] = 0;
29         }
30         while(!vS.empty()) vS.pop();
31     }
32     //root's edge = -1;
33     void tarjan(int u, int edge) {
34         dfn[u] = low[u] = ++dfs_clock;
35         int ch = 0;
36         vS.push(u);
37         eS.push(u);
38         for(int i = head[u], v; i; i = e[i].nxt) {
39             if(!dfn[v = e[i].to]) {
40                 tarjan(v, i ^ 1);
41                 low[u] = min(low[u], low[v]);
42                 if(low[v] >= dfn[u]) {
43                     ++ch;
44                     if(edge > 0 || ch > 1) is_vertex[u] = 1;
45                     vbcc[++vbcc_cnt].clear();
46                     vbcc[vbcc_cnt].push_back(u);
47                     for(int x;;){
48                         x = vS.top(); vS.pop();
49                         vbcc[vbcc_cnt].push_back(x);
50                         vbccno[x] = vbcc_cnt;

```

```

51         if(x == v)break;
52     }
53 }
54 if(low[v] > dfn[u]) {
55     // i && i ^ 1 is bridge
56 }
57 }
58 else if(dfn[v] < dfn[u] && i != edge)
59     low[u] = min(low[u], dfn[v]);
60 }
61 if(dfn[u] == low[u]) {
62     ebcc_cnt++;
63     for(int v;;) {
64         v = eS.top(); eS.pop();
65         ebccno[v] = ebcc_cnt;
66         if(v == u) break;
67     }
68 }
69 }
70 void findBCC(int n){
71     for(int i = 1; i <= n; i++)
72         if(!dfn[i]) tarjan(i, -1);
73
74     //findBridge
75     for(int u = 1; u <= n; u++) {
76         for(int i = head[u], v; i; i = e[i].nxt)
77             if(ebccno[u] != ebccno[v = e[i].to]) {
78                 //is bridge
79             }
80     }
81 }
82 }

```

## 1.6 Cactus

### 1.6.1 Circle-Square Tree

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef pair<int, int> P;
4  const int MAXN = 2e4 + 5;
5  const int S = 15;
6  namespace Tree {
7      struct Edge {
8          int to, nxt, w;
9      }e[MAXN << 1];
10     int ecnt, head[MAXN];
11     int rt, isrt[MAXN], fa[MAXN][S + 3];
12     int sz[MAXN];
13     inline void addEdge(int u, int v, int w) {
14         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
15         fa[v][0] = u;
16     }
17 }
18 int n, m, Q;
19 namespace BCC {
20     struct Edge {
21         int to, nxt, w;
22     }e[MAXN << 1];

```



```

23 int ecnt, head[MAXN];
24 int dfs_clock, dfn[MAXN], low[MAXN];
25 int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
26 vector<P> vbcc[MAXN];
27 stack<P> vs;
28 int tag[MAXN];
29 inline void addEdge(int u, int v, int w) {
30     e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
31     e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
32 }
33 inline void init(int n) {
34     ecnt = 1;
35     dfs_clock = 0;
36     vbcc_cnt = 0;
37     for(int i = 0; i <= 2 * n; i++){
38         head[i] = dfn[i] = low[i] = 0;
39         vbccno[i] = 0;
40         tag[i] = 0;
41     }
42     while(!vs.empty()) vs.pop();
43 }
44 //root's edge = -1;
45 void tarjan(int u, int edge) {
46     dfn[u] = low[u] = ++dfs_clock;
47     vs.push(P(u, e[edge ^ 1].w));
48     for(int i = head[u], v; i; i = e[i].nxt) {
49         if(!dfn[v = e[i].to]) {
50             tarjan(v, i ^ 1);
51             low[u] = min(low[u], low[v]);
52             if(low[v] >= dfn[u]) {
53                 if(vs.top().first == v) {
54                     Tree::addEdge(u, v, vs.top().second);
55                     vs.pop();
56                     continue;
57                 }
58                 vbcc[++vbcc_cnt].clear();
59                 vbcc[vbcc_cnt].push_back(P(u, 0));
60                 Tree::isrt[u] = 1;
61                 int &sz = Tree::sz[n + vbcc_cnt];
62                 tag[vs.top().first] = n + vbcc_cnt;
63                 //Tree::addEdge(u, rt, 0);
64                 for(P x;;) {
65                     x = vs.top(); vs.pop();
66                     sz += x.second;
67                     //Tree::addEdge(rt, x.first, sz);
68                     vbcc[vbcc_cnt].push_back(x);
69                     vbccno[x.first] = vbcc_cnt;
70                     if(x.first == v) break;
71                 }
72             }
73         }
74         else if(dfn[v] < dfn[u] && i != edge)
75             low[u] = min(low[u], dfn[v]);
76     }
77     for(int i = head[u], v; i; i = e[i].nxt) {
78         if(tag[v = e[i].to]) {
79             int r = tag[v]; Tree::sz[r] += e[i].w;
80             tag[v] = 0;
81         }
82     }
83 }

```

```

84 void findBCC(int n) {
85     for(int i = 1; i <= n; i++)
86         if(!dfn[i]) tarjan(i, -1);
87 }
88 }
89 namespace Tree {
90     int dis[MAXN], dep[MAXN], len[MAXN];
91     inline void init(int n) {
92         BCC::init(n);
93         rt = n;
94         ecnt = 1;
95         for(int i = 0; i <= 2 * n; i++) {
96             head[i] = 0;
97             fa[i][0] = isrt[i] = dis[i] = dep[i] = len[i] = 0;
98         }
99     }
100 void dfs(int x) {
101     for(int i = head[x], y; i; i = e[i].nxt) {
102         if(!dep[y = e[i].to]) {
103             dep[y] = dep[x] + 1;
104             dis[y] = dis[x] + e[i].w;
105             dfs(y);
106         }
107     }
108 }
109 void pre() {
110     for(int k = 1; k <= BCC::vbcc_cnt; k++) {
111         rt++;
112         vector<P> &E = BCC::vbcc[k];
113         addEdge(E[0].first, rt, 0);
114         int cnt = 0;
115         for(int i = E.size() - 1; i >= 1; i--) {
116             cnt += E[i].second;
117             len[E[i].first] = cnt;
118             addEdge(rt, E[i].first, min(cnt, sz[rt] - cnt));
119         }
120     }
121     for(int k = 1; k <= S; k++) {
122         for(int i = 1; i <= rt; i++) {
123             fa[i][k] = fa[fa[i][k - 1]][k - 1];
124         }
125     }
126     dep[1] = 1;
127     dfs(1);
128 }
129 int up(int x, int d) {
130     for(int i = S; i >= 0; i--) {
131         if(dep[fa[x][i]] >= d) x = fa[x][i];
132     }
133     return x;
134 }
135 int lca(int u, int v) {
136     if(dep[u] > dep[v]) swap(u, v);
137     v = up(v, dep[u]);
138     if(u == v) return u;
139     for(int i = S; i >= 0; i--) {
140         if(fa[u][i] != fa[v][i]) {
141             u = fa[u][i], v = fa[v][i];
142         }
143     }
144     return fa[u][0];

```

```

145     }
146     int query(int u, int v) {
147         int l = lca(u, v);
148         if(l <= n) return dis[u] + dis[v] - 2 * dis[l];
149         int x = up(u, dep[l] + 1), y = up(v, dep[l] + 1);
150         int res = dis[u] - dis[x] + dis[v] - dis[y];
151         int tmp = abs(len[x] - len[y]);
152         return res + min(tmp, sz[l] - tmp);
153     }
154 }
155
156 int main() {
157     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
158     using namespace Tree;
159     cin >> n >> m >> Q;
160     init(n);
161     for(int i = 1, u, v, w; i <= m; i++) {
162         cin >> u >> v >> w;
163         BCC::addEdge(u, v, w);
164     }
165     BCC::findBCC(n);
166     pre();
167     int u, v;
168     while(Q--) {
169         cin >> u >> v;
170         cout << query(u, v) << endl;
171     }
172     return 0;
173 }

```

## 2 Data Structures

### 2.1 Basic Structures

#### 2.1.1 RMQ

```

1 struct RMQ {
2     int d[MAXN][S + 2];
3     inline void init(int *a, int n) {
4         for(int i = 1; i <= n; i++) d[i][0] = a[i];
5         for(int k = 1; (1 << k) <= n; k++)
6             for(int i = 1; i + (1 << k) - 1 <= n; i++)
7                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
8     }
9     inline int query(int l, int r) {
10        if(l > r) swap(l, r);
11        int k = 0;
12        while((1 << (k + 1)) <= r - l + 1) k++;
13        return min(d[l][k], d[r - (1 << k) + 1][k]);
14    }
15 }rmq;
16 const int MAXM = 2e5 + 5, MAXN = 3e6 + 5, S = 22;
17 const LL INF = 1e18;
18 #define belong(x) (x / S + 1)
19 #define pos(x) (x % S + 1)
20 int Log[MAXN], sz;
21 struct RMQ {
22     LL a[MAXN];
23     LL d[MAXM][S + 2];
24     LL pre[MAXM][S + 2], aft[MAXM][S + 2];
25     inline void init(int n) {
26         sz = n / S + 1;
27         Log[0] = -1; for(int i = 1; i <= n; i++) Log[i] = Log[i / 2] + 1;
28         for(int i = 1; i <= sz; i++) {
29             pre[i][0] = aft[i][S + 1] = INF;
30         }
31         for(int i = 1; i <= n; i++) {
32             pre[belong(i)][pos(i)] = min(pre[belong(i)][pos(i) - 1], a[i]);
33         }
34         for(int i = n; i >= 1; i--) {
35             aft[belong(i)][pos(i)] = min(aft[belong(i)][pos(i) + 1], a[i]);
36         }
37         for(int i = 1; i <= sz; i++) {
38             d[i][0] = aft[i][1];
39         }
40         for(int k = 1; k <= S; k++)
41             for(int i = 1; i + (1 << k) <= sz; i++)
42                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
43     }
44     inline LL ask(int l, int r) {
45         assert(l <= r);
46         LL res = INF;
47         if(belong(l) == belong(r)) {
48             for(int i = l; i <= r; i++) res = min(res, a[i]);
49             return res;
50         }
51         res = min(aft[belong(l)][pos(l)], pre[belong(r)][pos(r)]);
52         int k = Log[belong(r) - belong(l) - 1];
53         if(~k) {

```

```

54         res = min(res, d[belong(l) + 1][k]);
55         res = min(res, d[belong(r) - (1 << k)][k]);
56     }
57     return res;
58 }
59 }rmq;

```

### 2.1.2 Divide Blocks

```

1  int belong[MAXN], l[MAXN], r[MAXN];
2  int sz, num;
3  void build(int n) {
4      sz = sqrt(n);
5      num = n / sz; if(n % sz) num++;
6      for(int i = 1; i <= num; i++) {
7          l[i] = (i - 1) * sz + 1;
8          r[i] = i * sz;
9      }
10     r[num] = n;
11     for(int i = 1; i <= n; i++) {
12         belong[i] = (i - 1) / sz + 1;
13     }
14 }

```

## 2.2 Heap Structures

### 2.2.1 Leftist Tree

```

1  const int MAXN = ;
2  namespace LeftistTree{
3      int ls[MAXN], rs[MAXN];
4      int dis[MAXN];
5      int fg[MAXN], sfa[MAXN], rt[MAXN]; //利用 rt 得到堆根节点
6      int val[MAXN];
7
8      void push_down(int x) {};
9      void push_up(int x) {
10         if(dis[ls[x]] < dis[rs[x]]) swap(ls[x], rs[x]);
11         dis[x] = dis[rs[x]] + 1;
12     }
13     int merge(int x, int y) {
14         if(!x || !y) return x^y;
15         if(val[x] > val[y] || (val[x] == val[y] && x > y)) swap(x, y);
16         push_down(x);
17         rs[x] = merge(rs[x], y);
18         push_up(x);
19         return x;
20     }
21     int getSfa(int x) {return sfa[x] == x ? x : sfa[x] = getSfa(sfa[x]);}
22     int uni(int x, int y) { //返回合并后的根
23         if(!x || !y) return x^y;
24         if(fg[x] || fg[y]) return;
25         x = getSfa(x); y = getSfa(y);
26         if(x == y) return;
27         int z = merge(x, y);
28         return sfa[x] = sfa[y] = sfa[z] = z;
29     }
30     void uni2(int a, int b) {

```

```

31 //val[a] >= 1;
32 int c = merge(ls[a], rs[a]);
33 ls[a] = rs[a] = dis[a] = 0;
34 int a1 = merge(c, a);
35 //val[b] >= 1;
36 c = merge(ls[b], rs[b]);
37 ls[b] = rs[b] = dis[b] = 0;
38 int b1 = merge(c, b);
39 c = merge(a1, b1);
40 sfa[a] = sfa[b] = sfa[c] = c;
41 printf("%d\n", val[c])
42 }
43 int pop(int x) { //返回堆顶值,也可以用于返回根
44 if(!x || fg[x]) return -1;
45 x = getSfa(x); fg[x] = 1;
46 push_down(x); //在删除堆顶时要下传标记
47 int y = merge(ls[x], rs[x]);
48 sfa[x] = sfa[y] = y;
49 return val[x];
50 }
51 void init(int n) {
52 for(int i = 1; i <= n; i++) {
53 sfa[i] = i;
54 ls[i] = rs[i] = dis[i] = fg[i] = 0;
55 }
56 }
57 }
58 //可持久化版本见k短路

```

## 2.3 Sequence Structures

### 2.3.1 Cartesian Tree

```

1 struct CartesianTree{
2 int rt, fa[MAXN], ls[MAXN], rs[MAXN];
3 int top, st[MAXN];
4 int cnt[MAXN];
5 void build(LL *a, int n) {
6 top = rt = 0;
7 for(int i = 1; i <= n; i++) {
8 ls[i] = rs[i] = fa[i] = 0;
9 while(top && a[st[top]] > a[i]) ls[i] = st[top--];
10 fa[i] = st[top];
11 if(ls[i]) fa[ls[i]] = i;
12 if(fa[i]) rs[fa[i]] = i; else rt = i;
13 st[++top] = i;
14 }
15 }
16 void dfs(int x) {
17 cnt[x] = 1;
18 if(ls[x]) {dfs(ls[x]); cnt[x] += cnt[ls[x]];}
19 if(rs[x]) {dfs(rs[x]); cnt[x] += cnt[rs[x]];}
20 }
21 LL getAns(LL *a, int n) {
22 //dfs(rt);
23 //_____
24 return res;
25 }
26 }T;

```

### 2.3.2 TreeArray

```

1 //树状数组上二分
2 int BS(int x) {
3     int res = 0;
4     for (int i = 1 << 18; i; i >>= 1)
5         if ((res | i) <= Tn && T[res | i] <= x)
6             x -= T[res | i];
7     return res;
8 }

```

### 2.3.3 Segment Tree

```

1 #define Ls(x) (x << 1)
2
3 #define Rs(x) (x << 1 | 1)
4 struct Tree {
5     int l, r, lazy;
6     LL sum, mx;
7 }tree[MAXN << 2];
8 int A[MAXN];
9 void push_up(int x) {
10     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
11     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
12 }
13 void push_down(int x) {
14     if(tree[x].lazy) {
15         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
16         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
17         tree[Ls(x)].mx += tree[x].lazy;
18         tree[Rs(x)].mx += tree[x].lazy;
19         tree[Ls(x)].lazy += tree[x].lazy;
20         tree[Rs(x)].lazy += tree[x].lazy;
21         tree[x].lazy = 0;
22     }
23 }
24 void build(int x, int L, int R) {
25     tree[x].lazy = 0;
26     tree[x].l = L; tree[x].r = R;
27     if(L == R) {
28         tree[x].sum = A[L];
29         tree[x].mx = A[L];
30
31         return;
32     }
33     int mid = (L + R) >> 1;
34     build(Ls(x), L, mid);
35     build(Rs(x), mid + 1, R);
36     push_up(x);
37 }
38 void update(int x, int L, int R, LL val) {
39     if(tree[x].l >= L && tree[x].r <= R) {
40         tree[x].lazy += val;
41         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
42         tree[x].mx += val;
43         return;
44     }
45     push_down(x);
46     int mid = (tree[x].l + tree[x].r) >> 1;

```

```

47     if(L <= mid) update(Ls(x), L, R, val);
48     if(R > mid) update(Rs(x), L, R, val);
49     push_up(x);
50 }
51 LL query(int x, int L, int R) {
52     if(tree[x].l >= L && tree[x].r <= R)
53         return tree[x].sum;
54     push_down(x);
55     int mid = (tree[x].l + tree[x].r) >> 1;
56     LL res = 0;
57     if(L <= mid) res += query(Ls(x), L, R);
58     if(R > mid) res += query(Rs(x), L, R);
59
60     return res;
61 }
62 LL query2(int x, int L, int R) {
63     if(tree[x].l >= L && tree[x].r <= R)
64         return tree[x].mx;
65     push_down(x);
66     int mid = (tree[x].l + tree[x].r) >> 1;
67     LL res = -INF;
68     if(L <= mid) res = max(res, query2(Ls(x), L, R));
69     if(R > mid) res = max(res, query2(Rs(x), L, R));
70     return res;
71 }

```

### 2.3.4 LiChao Tree

```

1  const double eps = 1e-12;
2  namespace LiT{
3      const int MLIMIT = 40000;
4      typedef double LD;
5      struct line{LD k,b;int l,r,id;} T[MAXN << 2];
6      //inline LD calc(line &a,int pos) {return a.k*vec[pos]+a.b;}
7      inline LD calc(line &a,int pos) {return a.k*pos+a.b;}
8      inline double cross(line &a,line &b) {
9          if(b.k == a.k) return -1e9;
10         return (double)(a.b-b.b)/(b.k-a.k);
11     }
12     void build(int v, int l, int r) {
13         T[v].k = 0;T[v].b = -1e18;
14         T[v].l = 0;T[v].r = MLIMIT;
15         T[v].id = 0;
16         if(l == r)return;
17         int mid = (l+r)>>1;
18         build(v<<1,l,mid);
19         build(v<<1|1,mid+1,r);
20     }
21     void ins(int v,int l,int r, line k) {
22         if(k.l <= l && r <= k.r) {
23             LD fl = calc(k, l), fr = calc(k, r);
24             LD gl = calc(T[v], l), gr = calc(T[v], r);
25             if(fl - gl > eps && fr - gr > eps) T[v] = k;
26             else if(fl - gl > eps || fr - gr > eps) {
27                 int mid = (l+r)>>1;
28                 if(calc(k, mid) - calc(T[v], mid) > eps) swap(k, T[v]);
29                 //if(vec[mid] - cross(k, T[v]) > eps)
30                 if(mid - cross(k, T[v]) > eps)
31                     ins(v<<1, l, mid, k);else ins(v<<1|1, mid+1, r, k);
32             }
33         }
34     }

```



```

33         return;
34     }
35     int mid=(l+r)>>1;
36     if(k.l <= mid) ins(v<<1, l, mid, k);
37     if(mid < k.r) ins(v<<1|1, mid+1, r, k);
38 }
39 LD ans;int ansid;
40 void que(int v, int l, int r, int x) {
41     LD tmp = calc(T[v], x);
42     if(tmp > ans || (tmp == ans && T[v].id < ansid)) {
43         ans = tmp;
44         ansid = T[v].id;
45     }
46     if(l == r) return;
47     int mid = (l+r)>>1;
48     if(x <= mid) que(v<<1,l,mid,x);else que(v<<1|1,mid+1,r,x);
49 }
50 };
51 //左闭右闭

```

### 2.3.5 Splay Tree

```

1 namespace splay{
2     int n, m, sz, rt;
3     int val[MAXN], id[MAXN];
4     int tr[MAXN][2], size[MAXN], fa[MAXN], rev[MAXN], s[MAXN], lazy[MAXN];
5     void push_up(int x) {
6         int l = tr[x][0], r = tr[x][1];
7         s[x] = max(val[x], max(s[l], s[r]));
8         size[x] = size[l] + size[r] + 1;
9     }
10    void push_down(int x) {
11        int l = tr[x][0], r = tr[x][1];
12        if(lazy[x]) {
13            if(l) {
14                lazy[l] += lazy[x];
15                s[l] += lazy[x];
16                val[l] += lazy[x];
17            }
18            if(r) {
19                lazy[r] += lazy[x];
20                s[r] += lazy[x];
21                val[r] += lazy[x];
22            }
23            lazy[x] = 0;
24        }
25        if(rev[x]) {
26            rev[x] = 0;
27            rev[l] ^= 1; rev[r] ^= 1;
28            swap(tr[x][0], tr[x][1]);
29        }
30    }
31    void rotate(int x, int &k) {
32        int y = fa[x];
33        int z = fa[y];
34        int l, r;
35        if(tr[y][0] == x) l = 0;
36        else l = 1;
37        r = l ^ 1;
38        if(y == k) k = x;

```

```

39     else {
40         if(tr[z][0] == y) tr[z][0] = x;
41         else tr[z][1] = x;
42     }
43     fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
44     tr[y][l] = tr[x][r]; tr[x][r] = y;
45     push_up(y); push_up(x);
46 }
47 void splay(int x, int &k) {
48     int y, z;
49     while(x != k) {
50         y = fa[x];
51         z = fa[y];
52         if(y != k) {
53             if((tr[y][0] == x) ^ (tr[z][0] == y)) rotate(x, k);
54
55             else rotate(y, k);
56         }
57         rotate(x, k);
58     }
59 }
60 int find(int x, int rank) {
61     push_down(x);
62
63     int l = tr[x][0], r = tr[x][1];
64     if(size[l] + 1 == rank) return x;
65     else if(size[l] >= rank) return find(l, rank);
66     else return find(r, rank - size[l] - 1);
67 }
68 void update(int l, int r, int v) {
69     int x = find(rt, l), y = find(rt, r + 2);
70     splay(x, rt); splay(y, tr[x][1]);
71     int z = tr[y][0];
72     lazy[z] += v;
73     val[z] += v;
74     s[z] += v;
75 }
76 void reverse(int l, int r) {
77     int x = find(rt, l), y = find(rt, r + 2);
78     splay(x, rt); splay(y, tr[x][1]);
79     int z = tr[y][0];
80     rev[z] ^= 1;
81 }
82 void query(int l, int r) {
83     int x = find(rt, l), y = find(rt, r + 2);
84     splay(x, rt); splay(y, tr[x][1]);
85     int z = tr[y][0];
86     printf("%d\n", s[z]);
87 }
88 void build(int l, int r, int f) {
89     if(l > r) return;
90     int now = id[l], last = id[f];
91     if(l == r) {
92         fa[now] = last; size[now] = 1;
93         if(l < f) tr[last][0] = now;
94         else tr[last][1] = now;
95         return;
96     }
97     int mid = (l + r) >> 1; now = id[mid];
98     build(l, mid - 1, mid); build(mid + 1, r, mid);
99     fa[now] = last;

```

```

100     push_up(now);
101     if(mid < f) tr[last][0] = now;
102     else tr[last][1] = now;
103 }
104 void init() {
105     s[0] = -INF;
106     scanf("%d%d", &n, &m);
107     for(int i = 1; i <= n + 2; i++) id[i] = ++sz;
108     build(1, n + 2, 0); rt = (n + 3) >> 1;
109 }
110 }
111 namespace splay{
112     //内存回收池见fhq_treap
113     int tcnt, root;
114     int sz[MAXN];
115     int tr[MAXN][2], fa[MAXN];
116     int val[MAXN];
117     int newnode(int w) {
118         ++tcnt;
119         sz[tcnt] = 1;
120         fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
121         //val[tcnt] = w;
122         return tcnt;
123     }
124     void push_up(int v) {
125         int l = tr[v][0], r = tr[v][1];
126         sz[v] = sz[l] + 1 + sz[r];
127     }
128     void push_down(int v) {
129         if(!v) return;
130     }
131     void init() {
132         tcnt = 2;
133         tr[root = fa[1] = 2][0] = 1;
134         sz[1] = 1; sz[2] = 2;
135         //val[1] = -INF; val[2] = INF; //权值平衡树
136         //val[1] = val[2] = 0; //位置平衡树
137         //1,2为哨兵节点,根据题意也可设置为n+1,n+2或1,n+1
138     }
139     void rotate(int x) {
140         int y = fa[x], z = fa[y];
141         push_down(y); push_down(x);
142         int lr = tr[y][1] == x;
143         if(z) tr[z][tr[z][1] == y] = x;
144         fa[x] = z;
145         fa[tr[y][lr]] = tr[x][lr^1] = y;
146         fa[tr[x][lr^1] = y] = x;
147         push_up(y); push_up(x);
148     }
149     void splay(int x, int k) {
150         for(int y, z; (y = fa[x]) != k; rotate(x)) {
151             if((z = fa[y]) != k) {
152                 if((tr[y][0] == x) ^ (tr[z][0] == y))
153                     rotate(x); else rotate(y);
154             }
155         }
156         if(!k) root = x;
157     }
158     int find(int x, int rank) {
159         push_down(x);
160         int l = tr[x][0], r = tr[x][1];

```

```

161     if(sz[l] + 1 == rank) return x;
162     if(sz[l] >= rank) return find(l, rank);
163     return find(r, rank - sz[l] - 1);
164 }
165 int build(int l, int r) {
166     if(l > r) return 0;
167     if(l == r) {
168         int num; scanf("%d", &num);
169         return newnode(num);
170     }
171     int mid = (l + r) >> 1;
172     int ls = build(l, mid-1);
173     int num; scanf("%d", &num);
174     int v = newnode(num);
175     int rs = build(mid+1, r);
176     if(ls) fa[ls] = v;
177     tr[v][0] = ls;
178     if(rs) fa[rs] = v;
179     tr[v][1] = rs;
180     push_up(v);
181     return v;
182 }
183 void insert(int pos, ...) {
184     int x = find(root, pos+1), y = find(root, pos+2);
185     splay(x, 0); splay(y, x);
186     //int z = newnode(w); //插入一个节点
187     //int z = build(1, n); //插入n个节点
188     fa[tr[y][0] = z] = y;
189     splay(z, 0);
190 }
191 void modifyOrQuery(int l, int r, int v) {
192     int x = find(root, l), y = find(root, r + 2);
193     splay(x, 0); splay(y, x);
194     int z = tr[y][0];
195     if(!z) return;
196     //标记对本身无效, 处理时将z点重新计算
197     splay(z, 0);
198 }
199 void display(int v) {
200     if(!v) return;
201     push_down(v);
202     display(tr[v][0]);
203     if(val[v]) printf("%d ", val[v]);
204     display(tr[v][1]);
205 }
206 /*int findValue(int v) {
207     int res = root;
208     for(int cur = root; cur; res = cur, cur = tr[cur][val[cur] <= v]);
209     return res;
210 }
211 void insert(int w) {
212     int y = findValue(w);
213     int z = newnode(w);
214     fa[tr[y][val[y] <= w] = z] = y;
215     splay(z, 0);
216 }*/
217 /*void split(int v) { //splay维护区间[l, r], 区间分裂为[l, k-1], [k, k], [k+1, r];
218     //ump查看标号是否出现, mp维护子区间左端点
219     if(ump.find(v) == ump.end()) {
220         auto it = mp.upper_bound(v); --it;
221         int z = it->second;

```

```

222     splay(z, 0);
223     int pos = sz[tr[z][0]];
224     int x = find(root, pos), y = find(root, pos+rc[z]-lc[z]+2);
225     splay(x, 0); splay(y, x);
226     z = tr[y][0];
227     if(lc[z] != v) {
228         tr[z][0] = newnode(lc[z], v-1);
229         fa[tr[z][0]] = z;
230         mp[lc[z]] = tr[z][0];
231     }
232     if(rc[z] != v) {
233         tr[z][1] = newnode(v+1, rc[z]);
234         fa[tr[z][1]] = z;
235         mp[v+1] = tr[z][1];
236     }
237     lc[z] = rc[z] = v;
238     splay(z, 0);
239     mp[v] = z;
240     ump[v] = z;
241 }
242 }*/
243
244 }

```

### 2.3.6 Scapegoat Tree

```

1 struct ScapegoatTree{
2     int Tsn; queue<int> q;
3     int val[MAXM], ext[MAXM];
4     int sz[MAXM], tsz[MAXM];
5     int fa[MAXM], tr[MAXM][2];
6     int root;
7     double alp;
8     void init() {
9         root = 0;
10        alp = 0.7;
11    }
12    int newnode(int x) {
13        if(q.empty()) q.push(++Tsn);
14        int tcnt = q.front(); q.pop();
15        val[tcnt] = x; ext[tcnt] = 1;
16        fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
17        sz[tcnt] = tsz[tcnt] = 0;
18        return tcnt;
19    }
20    void push_up(int v) {
21        sz[v] = ext[v]; tsz[v] = 1;
22        if(tr[v][0]) {
23            sz[v] += sz[tr[v][0]];
24            tsz[v] += tsz[tr[v][0]];
25        }
26        if(tr[v][1]) {
27            sz[v] += sz[tr[v][1]];
28            tsz[v] += tsz[tr[v][1]];
29        }
30    }
31    bool isBad(int v) {
32        return (double(tsz[ tr[v][0] ]) > double(tsz[v]) * alp) ||
33            (double(tsz[ tr[v][1] ]) > double(tsz[v]) * alp) ||
34            (sz[v] * 2 < tsz[v]);

```

```

35     }
36     vector<int> vec;
37     void rRecycle(int v) {
38         if(tr[v][0]) rRecycle(tr[v][0]);
39         if(ext[v]) vec.push_back(v); else q.push(v);
40         if(tr[v][1]) rRecycle(tr[v][1]);
41     }
42     int rBuild(int l, int r) {
43         int mid = (l + r) >> 1, v = vec[mid];
44         tr[v][0] = (l <= mid-1) ? rBuild(l, mid - 1) : 0;
45         if(tr[v][0]) fa[tr[v][0]] = v;
46         tr[v][1] = (mid+1 <= r) ? rBuild(mid + 1, r) : 0;
47         if(tr[v][1]) fa[tr[v][1]] = v;
48         push_up(v);
49         return v;
50     }
51     void rebuild(int x) {
52         int v = 0;
53         for(;x; x= fa[x]) {
54             push_up(x);
55             if(isBad(x)) v = x;
56         }
57         if(v && isBad(v)){
58             vec.clear();
59             int u = fa[v], lr = tr[u][1] == v;
60             rRecycle(v);
61             if(vec.size()) v = rBuild(0, vec.size() - 1); else v = 0;
62             if(u == 0) fa[root = v] = 0;
63             else{
64                 tr[u][lr] = v;
65                 if(v) fa[v] = u;
66             }
67         }
68     }
69     void ins(int x) {
70         int p = root, q = root;
71         for(;p && val[p] != x; q = p, p = tr[p][ val[p] < x]) ;
72         if(!q) {
73             p = root = newnode(x);
74         }else if(p) {
75             ext[p]++;
76         }else{
77             fa[p = tr[q][val[q] < x] = newnode(x) ] = q;
78         }
79         rebuild(p);
80     }
81     void del(int x) {
82         int p = root;
83         for(;p && val[p] != x; p = tr[p][ val[p] < x]);
84         if(p && ext[p]){
85             --ext[p];
86             rebuild(p);
87         }
88     }
89     int get_rank(int x) {
90         int ret = 0;
91         for(int p = root;p;) {
92             if(val[p] < x) {
93                 ret += sz[tr[p][0]] + ext[p];
94                 p = tr[p][1];
95             }else p = tr[p][0];

```

```

96     }
97     return ret + 1;
98 }
99 int get_Kth(int p, int k) {
100     if(sz[tr[p][0]] >= k) return get_Kth(tr[p][0], k);
101     k -= sz[tr[p][0]];
102     if(ext[p] >= k) return val[p];
103     k -= ext[p];
104     return get_Kth(tr[p][1], k);
105 }
106 int pre(int x) {
107     int id = get_rank(x);
108     return get_Kth(root, id - 1);
109 }
110 int nxt(int x) {
111     int id = get_rank(x + 1);
112     return get_Kth(root, id);
113 }
114 void display(int v) {
115     if(tr[v][0]) display(tr[v][0]);
116     cerr<<val[v]<<" ";
117     if(tr[v][1]) display(tr[v][1]);
118 }
119 }T;

```

### 2.3.7 FHQ Treap

```

1 namespace fhq_treap{
2     int Tsz; queue<int> q; //内存回收池
3     int tcnt, root;
4     //int rt[MAXN]; //可持久化时使用rt,维护版本号(int &root), 空间开大
5     int sz[MAXN], rnd[MAXN];
6     int tr[MAXN][2];
7     //int fa[MAXN]; //维护fa时除了在pushup更新v节点左右孩子父节点信息还要在split和merge结
8     //束时维护root的fa信息, fa[root]=0;
9     int val[MAXN], rev[MAXN];
10    void init() {
11        srand(time(0));
12        Tsz = tcnt = root = 0;
13    }
14    int newnode(int v) {
15        if(q.empty()) q.push(++Tsz);
16        tcnt = q.front(); q.pop();
17        sz[tcnt] = 1;
18        rnd[tcnt] = rand();
19        tr[tcnt][0] = tr[tcnt][1] = 0;
20        //val[tcnt] = v;
21        return tcnt;
22    }
23    /*int copynode(int id) {
24        //++tcnt; //获取一个新的节点编号
25        sz[tcnt] = sz[id];
26        rnd[tcnt] = rnd[id];
27        tr[tcnt][0] = tr[id][0];
28        tr[tcnt][1] = tr[id][1];
29        //val[tcnt] = val[id];
30        return tcnt;
31    }*/
32    void push_up(int v) {
33        int l = tr[v][0], r = tr[v][1];

```

```

33     sz[v] = sz[l] + 1 + sz[r];
34 }
35 void push_down(int v) {
36     if(!v) return;
37     int l = tr[v][0], r = tr[v][1];
38     //if(l) ;
39     //if(r) ;
40     //swap时候注意交换 tr[v][0]和tr[v][1],而不是l和r;
41 }
42 /* //可持久化在push_down是要新建节点,否则历史版本有可能被多次下传,以rev为例
43 void push_down(int v) {
44     if(!v || !rev[v]) return;
45     int &l = tr[v][0], &r = tr[v][1];
46     if(l) {
47         l = copynode(l);
48         rev[l]^=1;
49         swap(tr[l][0], tr[l][1]);
50     };
51     if(r) {
52         r = copynode(r);
53         rev[r]^=1;
54         swap(tr[r][0], tr[r][1]);
55     };
56     rev[v] = 0;
57 }*/
58 void split(int v,int k,int &x,int &y) {
59     if(!v) {x=y=0;return;}
60     push_down(v);
61     //v = copynode(v); //可持久化时复制节点
62     /*if(k > sz[tr[v][0]]) {
63         x = v;
64         split(tr[v][1], k-sz[tr[v][0]]-1, tr[v][1], y);
65     }else{
66         y = v;
67         split(tr[v][0], k, x, tr[v][0]);
68     }*/
69     if(val[v] <= k) {
70         x = v;
71         split(tr[v][1], k, tr[v][1], y);
72     }else{
73         y = v;
74         split(tr[v][0], k, x, tr[v][0]);
75     }
76     push_up(v);
77 }
78 int merge(int x,int y) { //x堆所有值均小于y堆
79     if(!x || !y) return x|y;
80     push_down(x); push_down(y);
81     if(rnd[x]<rnd[y]){
82         //x = copynode(x); //可持久化时复制节点,可不写
83         tr[x][1] = merge(tr[x][1],y);
84         push_up(x);
85         return x;
86     }else{
87         //y = copynode(y); //可持久化时复制节点,可不写
88         tr[y][0] = merge(x,tr[y][0]);
89         push_up(y);
90         return y;
91     }
92 }
93 void insert(int k) {

```



```

94     int x,y;
95     split(root,k,x,y);
96     root = merge(merge(x,newnode(k)),y);
97 }
98 void recycle(int v) { //回收一颗treap上所有节点
99     if(!v) return;
100    q.push(v);
101    recycle(tr[v][0]); recycle(tr[v][1]);
102 }
103 void erase(int k) {
104     int x,y,z;
105     split(root,k,x,y);
106     split(x,k-1,x,z);
107     z = merge(tr[z][0],tr[z][1]);
108     root = merge(x,merge(z,y));
109 }
110 void krank(int k) {
111     int x,y;
112     split(root,k-1,x,y);
113     printf("%d\n",sz[x]+1);
114     root = merge(x,y);
115 }
116 int find(int v,int k) {
117     if(sz[tr[v][0]]==k-1) return val[v];
118     if(sz[tr[v][0]]>=k) return find(tr[v][0],k);
119     return find(tr[v][1],k-sz[tr[v][0]]-1);
120 }
121 void pre(int k) {
122     int x,y;
123     split(root,k-1,x,y);
124     printf("%d\n",find(x,sz[x]));
125     root=merge(x,y);
126 }
127 void nxt(int k){
128     int x,y;
129     split(root,k,x,y);
130     printf("%d\n",find(y,1));
131     root=merge(x,y);
132 }
133 void reverse(int l,int r){
134     int x,y,z;
135     split(root, r, x, y);
136     split(x, l-1, x, z);
137     //rev[z] ^= 1;标记对本身无效,处理时将z点重新计算
138     root = merge(merge(x,z),y);
139 }
140 /*int getRank(int S) {
141     int res = sz[tr[S][0]]+1;
142     for (; fa[S]; S = fa[S])
143         if (tr[fa[S]][1] == S) res += sz[tr[fa[S]][0]] + 1;
144     return res;
145 }*/
146 void display(int v) {
147     if(!v) return;
148     push_down(v);
149     display(tr[v][0]);
150     printf("%d ",val[v]);
151     display(tr[v][1]);
152 }
153 }

```

154 //一种可持久化平衡树的替代(非强制在线),由历史版本向当前版本连边,在dfs遍历中利用权值树状数组,普通平衡树等获取答案

## 2.4 Persistent Data Structures

### 2.4.1 Chairman Tree

```

1 struct Node {
2     int l, r;
3     LL sum;
4 }t[MAXN * 40];
5 int cnt, n;
6 int rt[MAXN];
7 void update(int pre, int &x, int l, int r, int v) {
8     x = ++cnt; t[x] = t[pre]; t[x].sum++;
9     if(l == r) return;
10    int mid = (l + r) >> 1;
11    if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
12    else update(t[pre].r, t[x].r, mid + 1, r, v);
13 }
14 int query(int x, int y, int l, int r, int v) {
15     if(l == r) return l;
16     int mid = (l + r) >> 1;
17     int sum = t[t[y].l].sum - t[t[x].l].sum;
18     if(sum >= v) return query(t[x].l, t[y].l, l, mid, v);
19     else return query(t[x].r, t[y].r, mid + 1, r, v - sum);
20 }

```

### 2.4.2 Unite Chairman Tree

```

1 //Q x到y路径第k大
2 //L link(x, y)
3 #include <bits/stdc++.h>
4 using namespace std;
5 typedef int LL;
6 const int MAXN = 8e4 + 5;
7 const int S = 18;
8 struct Node {
9     int l, r;
10    LL sum;
11 }t[MAXN * 800]; //2 * log^2(n)
12 int n, m, Q;
13 int cnt;
14 int rt[MAXN], sz[MAXN];
15 void update(int pre, int &x, int l, int r, int v) {
16     x = ++cnt; t[x] = t[pre]; t[x].sum++;
17     if(l == r) return;
18     int mid = (l + r) >> 1;
19     if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
20     else update(t[pre].r, t[x].r, mid + 1, r, v);
21 }
22 int query(int x, int y, int z, int w, int l, int r, int v) {
23     if(l == r) return l;
24     int mid = (l + r) >> 1;
25     int sum = t[t[x].l].sum + t[t[y].l].sum - t[t[z].l].sum - t[t[w].l].sum;
26     if(sum >= v) return query(t[x].l, t[y].l, t[z].l, t[w].l, l, mid, v);
27     return query(t[x].r, t[y].r, t[z].r, t[w].r, mid + 1, r, v - sum);
28 }

```

```

29 int fa[MAXN][S + 3], dep[MAXN];
30 int val[MAXN];
31 vector<int> G[MAXN];
32 inline void addEdge(int x, int y) {
33     G[x].push_back(y);
34     G[y].push_back(x);
35 }
36 inline void upd(int x) {
37     update(rt[fa[x][0]], rt[x], 1, n, val[x]);
38     for(int i = 1; i <= S; i++) fa[x][i] = fa[fa[x][i - 1]][i - 1];
39 }
40 inline void Go(int &x, int step) {
41     for(int i = S; i >= 0; i--) if(step >> i & 1) x = fa[x][i];
42 }
43 int lca(int x, int y) {
44     if(dep[x] < dep[y]) swap(x, y);
45     Go(x, dep[x] - dep[y]);
46     if(x == y) return x;
47     for(int i = S; i >= 0; i--) if(fa[x][i] != fa[y][i]) {
48         x = fa[x][i], y = fa[y][i];
49     }
50     return fa[x][0];
51 }
52 int get_rt(int x) {
53     for(int i = S; i >= 0; i--) if(fa[x][i]) x = fa[x][i];
54     return x;
55 }
56 void dfs(int x, int f) {
57     fa[x][0] = f;
58     dep[x] = dep[f] + 1;
59     upd(x);
60     for(auto y : G[x]) {
61         if(y == f) continue;
62         dfs(y, x);
63     }
64 }
65 void unite(int x, int y) {
66     int rx = get_rt(x), ry = get_rt(y);
67     if(sz[rx] > sz[ry]) swap(x, y), swap(rx, ry);
68     addEdge(x, y);
69     dfs(x, y);
70     sz[ry] += cnt - rt[x] + 1;
71 }
72 void init() {
73     cnt = 0;
74     for(int i = 1; i <= n; i++) {
75         rt[i] = sz[i] = 0;
76         G[i].clear();
77         dep[i] = 0;
78     }
79 }
80 int main() {
81     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
82     int T; cin >> T;
83     while(T--) {
84         cin >> n >> m >> Q;
85         init();
86         vector<int> b;
87         map<int, int> mp;
88         for(int i = 1; i <= n; i++) {
89             cin >> val[i];

```

```

90         b.push_back(val[i]);
91     }
92     sort(b.begin(), b.end());
93     b.erase(unique(b.begin(), b.end()), b.end());
94     for(int i = 1, tmp; i <= n; i++) {
95         tmp = val[i];
96         val[i] = lower_bound(b.begin(), b.end(), val[i]) - b.begin() + 1;
97         mp[val[i]] = tmp;
98     }
99     for(int i = 1, u, v; i <= m; i++) {
100         cin >> u >> v;
101         addEdge(u, v);
102     }
103     for(int i = 1; i <= n; i++) if(!dep[i]) {
104         dep[i] = 1;
105         dfs(i, 0);
106         sz[i] = cnt - rt[i] + 1;
107     }
108     char s[3]; int x, y, z, k, ans = 0;
109     while(Q--) {
110         cin >> s >> x >> y;
111         x ^= ans; y ^= ans;
112         if(s[0] == 'Q') {
113             cin >> k; k ^= ans;
114             z = lca(x, y);
115             ans = query(rt[x], rt[y], rt[z], rt[fa[z][0]], 1, n, k);
116             ans = mp[ans];
117             cout << ans << endl;
118         }
119         else {
120             unite(x, y);
121         }
122     }
123 }
124 return 0;
125 }

```

### 2.4.3 Persistent Trie

```

1 //区间异或最值查询
2 const int N=5e4+10;
3 int t[N];
4 int ch[N*32][2],val[N*32];
5 int cnt;
6 void init(){
7     mem(ch,0);
8     mem(val,0);
9     cnt=1;
10 }
11 int add(int root,int x){
12     int newroot=cnt++,ret=newroot;
13     for(int i=30;i>=0;i--){
14         ch[newroot][0]=ch[root][0];
15         ch[newroot][1]=ch[root][1];
16         int now=(x>>i)&1;
17         root=ch[root][now];
18
19         ch[newroot][now]=cnt++;
20         newroot=ch[newroot][now];
21         val[newroot]=val[root]+1;

```

```

22     }
23
24     return ret;
25 }
26 int query(int lt,int rt,int x){
27     int ans=0;
28     for(int i=30;i>=0;i--){
29         int now=(x>>i)&1;
30         if(val[ch[rt][now^1]]-val[ch[lt][now^1]]){
31             ans|=(1<<i);
32             rt=ch[rt][now^1];
33             lt=ch[lt][now^1];
34         } else{
35             rt=ch[rt][now];
36             lt=ch[lt][now];
37         }
38     }
39     return ans;
40 }

```

#### 2.4.4 SGT in BBST

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int MAXN = 1e5;
4  const int MAXM = 2e7;
5  const int LM = 0;
6  const int RM = 70005;
7  namespace T{
8      int Tsz; queue<int> q;
9      int ls[MAXM], rs[MAXM], val[MAXM];
10     int Tan, Ta[105], Tbn, Tb[105];
11     int newnode() {
12         if(q.empty()) q.push(++Tsz);
13         int x = q.front(); q.pop();
14         ls[x] = rs[x] = val[x] = 0;
15         return x;
16     }
17     void insert(int &x, int l, int r, int k, int f) {
18         if(!x) x = newnode();
19         val[x] += f;
20         if(l == r) return;
21         int mid = (l + r) >> 1;
22         if(k <= mid) {
23             insert(ls[x], l, mid, k, f);
24         } else{
25             insert(rs[x], mid+1, r, k, f);
26         }
27     }
28     int query(int l, int r, int y) {
29         if(l == r) return l;
30         int mid = (l + r) >> 1;
31         int sum = 0;
32         for(int i = 0; i <= Tan; i++) sum -= val[ls[Ta[i]]];
33         for(int i = 0; i <= Tbn; i++) sum += val[ls[Tb[i]]];
34
35         if(y <= sum) {
36             for(int i = 0; i <= Tan; i++) Ta[i] = ls[Ta[i]];
37             for(int i = 0; i <= Tbn; i++) Tb[i] = ls[Tb[i]];
38             return query(l, mid, y);

```

```

39     }
40     for(int i = 0; i <= Tan; i++) Ta[i] = rs[Ta[i]];
41     for(int i = 0; i <= Tbn; i++) Tb[i] = rs[Tb[i]];
42     return query(mid+1, r, y - sum);
43 }
44 void recycle(int v) {
45     if(!v) return;
46     q.push(v);
47     recycle(ls[v]);
48     recycle(rs[v]);
49 }
50 };
51 namespace TT{
52     int Tsn;
53     int val[MAXN], Trt[MAXN];
54     int sz[MAXN];
55     int fa[MAXN], tr[MAXN][2];
56
57     int root;
58     double alp;
59
60     int newnode(int x) {
61         int tcnt = ++Tsn;
62         val[tcnt] = x; Trt[tcnt] = 0;
63         fa[tcnt] = tr[tcnt][0] = tr[tcnt][1] = 0;
64         sz[tcnt] = 0;
65         return tcnt;
66     }
67     bool isBad(int v) {
68         return (double(sz[ tr[v][0] ]) > double(sz[v]) * alp) ||
69             (double(sz[ tr[v][1] ]) > double(sz[v]) * alp);
70     }
71     vector<int> vec;
72     void rRecycle(int v) {
73         if(tr[v][0]) rRecycle(tr[v][0]);
74         vec.push_back(v);
75         T::recycle(Trt[v]);
76         if(tr[v][1]) rRecycle(tr[v][1]);
77     }
78     int rBuild(int l, int r) {
79         int mid = (l + r) >> 1, v = vec[mid];
80         Trt[v] = 0; sz[v] = r - l + 1;
81         for(int i = l; i <= mid; i++) {
82             T::insert(Trt[v], LM, RM, val[vec[i]], 1);
83         }
84         tr[v][0] = (l <= mid-1) ? rBuild(l, mid - 1) : 0;
85         if(tr[v][0]) fa[tr[v][0]] = v;
86         tr[v][1] = (mid+1 <= r) ? rBuild(mid + 1, r) : 0;
87         if(tr[v][1]) fa[tr[v][1]] = v;
88         return v;
89     }
90     void rebuild(int v) {
91         if(isBad(v)) {
92             vec.clear();
93             int u = fa[v], lr = tr[u][1] == v;
94             rRecycle(v);
95             if(vec.size()) v = rBuild(0, vec.size() - 1); else v = 0;
96             if(u == 0) fa[root = v] = 0;
97             else{
98                 tr[u][lr] = v;
99                 if(v) fa[v] = u;

```

```

100     }
101 }
102 }
103 int find(int x, int k) {
104     int l = tr[x][0], r = tr[x][1];
105     if(sz[l] + 1 == k) return x;
106     if(sz[l] >= k) return find(l, k);
107     return find(r, k - sz[l] - 1);
108 }
109 void ins(int x, int y) {
110     int v = find(root, x);
111     int p = tr[v][1], q = v;
112     if(p) {
113         for(; p; q = p, p = tr[p][0]);
114         fa[p = tr[q][0] = newnode(y)] = q;
115     } else {
116         fa[p = tr[q][1] = newnode(y)] = q;
117     }
118     int fg = 0;
119     T::insert(Trt[p], LM, RM, y, 1);
120     sz[p] = 1;
121     for(; fa[p]; p = fa[p]) {
122         if(tr[fa[p]][0] == p)
123             T::insert(Trt[fa[p]], LM, RM, y, 1);
124         sz[fa[p]]++;
125         if(isBad(fa[p])) fg = fa[p];
126     }
127     rebuild(fg);
128 }
129 void upd(int x, int y) {
130     int p = find(root, x+1);
131     int ty = val[p]; val[p] = y;
132     T::insert(Trt[p], LM, RM, ty, -1);
133     T::insert(Trt[p], LM, RM, y, 1);
134     for(; fa[p]; p = fa[p]) {
135         if(tr[fa[p]][0] == p) {
136             T::insert(Trt[fa[p]], LM, RM, ty, -1);
137             T::insert(Trt[fa[p]], LM, RM, y, 1);
138         }
139     }
140 }
141 int que(int x, int y, int z) {
142     x = find(root, x);
143     T::Ta[T::Tan = 0] = Trt[x];
144     for(; fa[x]; x = fa[x])
145         if(tr[fa[x]][1] == x) {
146             T::Ta[++T::Tan] = Trt[fa[x]];
147         }
148     y = find(root, y+1);
149     T::Tb[T::Tbn = 0] = Trt[y];
150     for(; fa[y]; y = fa[y]) {
151         if(tr[fa[y]][1] == y) {
152             T::Tb[++T::Tbn] = Trt[fa[y]];
153         }
154     }
155     return T::query(LM, RM, z);
156 }
157 void init(int n) {
158     alp = 0.7;
159     vec.clear();
160     vec.push_back(newnode(RM));

```

```

161     for(int i = 1, a; i <= n; i++) {
162         scanf("%d", &a);
163         vec.push_back(newnode(a));
164     }
165     root = rBuild(0,vec.size() - 1);
166     fa[root] = 0;
167 }
168 void display(int v) {
169     if(tr[v][0]) display(tr[v][0]);
170     cerr<<val[v]<<" ";
171     if(tr[v][1]) display(tr[v][1]);
172 }
173 };

```

## 2.5 Tree Structures

### 2.5.1 dsu on tree

```

1  const int MAXN = 1e5 + 7;
2  vector<int> G[MAXN];
3  int bgison, dfs_clock, sz[MAXN], st[MAXN], bt[MAXN], et[MAXN];
4  int fg[MAXN], col[MAXN];
5  long long ans[MAXN];
6  void dfs1(int u, int fa) {
7      sz[u] = 1;
8      st[bt[u] = ++dfs_clock] = u;
9      for(auto v : G[u])
10         if(v != fa) {
11             dfs1(v, u);
12             sz[u] += sz[v];
13         }
14         et[u] = dfs_clock;
15     }
16     int maxx = 0;
17     void dfs2(int u, int fa, int keep) {
18         int mx = -1, bigson = -1;
19         for(auto &v : G[u])
20             if(v != fa) {
21                 if(sz[v] > mx)
22                     mx = sz[v], bigson = v;
23             }
24         for(auto &v : G[u])
25             if(v != fa && v != bigson)
26                 dfs2(v,u,0);
27         if(bigson != -1) {
28             dfs2(bigson, u, 1);
29             ans[u] = ans[bigson];
30             for(int &v : G[u])
31                 if(v != fa && v != bigson)
32                     for(int i = bt[v]; i <= et[v]; i++) {
33                         ++fg[col[st[i]]];
34                         if(fg[col[st[i]]] > maxx) maxx=fg[col[st[i]]], ans[u] = 0;
35                         if(fg[col[st[i]]] == maxx) ans[u] += col[st[i]];
36                     }
37         }
38         ++fg[col[u]];
39         if(fg[col[u]] > maxx) maxx = fg[col[u]], ans[u] = 0;
40         if(fg[col[u]] == maxx) ans[u] += col[u];
41         if(keep == 0) {

```



```

42     maxx = 0;
43     for(int i = bt[u]; i <= et[u]; i++)
44         fg[col[st[i]]] = 0;
45 }
46 }

```

### 2.5.2 Vitural Tree

```

1  const int MAXN = ;
2  const int MAXM = ;
3  const LL INF = ;
4  const int S = 19;
5  int ecnt, head[MAXN];
6  struct Edge{int to, nxt; LL w;} e[MAXM], ve[MAXM];
7  inline void addEdgeT(int x, int y, LL w) {
8      e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
9  }
10 int dep[MAXN], dfn_time, dfn[MAXN], fa[MAXN][S+1];
11 LL dis[MAXN][S+1];
12 void dfs(int v, int _fa) {
13     dfn[v] = ++dfn_time;
14     dep[v] = dep[_fa] + 1;
15     fa[v][0] = _fa;
16     for(int i = 1; i <= S; i++) {
17         fa[v][i] = fa[fa[v][i-1]][i-1];
18         dis[v][i] = min(dis[v][i-1], dis[fa[v][i-1]][i-1]);
19     }
20     for(int i = head[v], u; i; i = e[i].nxt)
21         if((u = e[i].to) != _fa) {
22             dis[u][0] = e[i].w;
23             dfs(u, v);
24         }
25 }
26 int getLca(int u, int v) {
27     if(dep[u] < dep[v]) swap(u, v);
28     for(int i = S; i >= 0; i--)
29         if(dep[fa[u][i]] >= dep[v]) u = fa[u][i];
30     if(u == v) return u;
31     for(int i = S; i >= 0; i--)
32         if(fa[u][i] != fa[v][i])
33             u = fa[u][i], v = fa[v][i];
34     return fa[u][0];
35 }
36 LL getDis(int u, int v) {
37     if(dep[u] < dep[v]) swap(u, v);
38     LL res = INF;
39     for(int i = S; i >= 0; i--)
40         if(dep[fa[u][i]] >= dep[v]) {
41             res = min(res, dis[u][i]);
42             u = fa[u][i];
43         }
44     return res;
45 }
46 namespace VituralTree{
47     int hn, h[MAXN];
48     int vecnt, vhead[MAXN];
49     Edge ve[MAXM];
50     int top, st[MAXN], cln, cl[MAXN];
51     int fgn, fg[MAXN]; //利用 fg[i]==fgn 判断是否为当前有效点
52     void addEdgeVT(int x, int y, LL w) {

```

```

53     ve[++vecnt] = (Edge) {y, vhead[x], w}; vhead[x] = vecnt;
54 }
55 inline void link(int u, int v) {
56     LL w = getDis(u, v);
57     addEdgeVT(u, v, w);
58     addEdgeVT(v, u, w);
59 }
60 inline bool cmp(int a, int b) {return dfn[a] < dfn[b];}
61 void build() {
62     ++fgn;
63     for(int i = 1; i <= hn; i++) fg[h[i]] = fgn;
64     sort(h + 1, h + hn + 1, cmp);
65     cl[cln = 1] = st[top = 1] = 1;
66     for(int i = 1; i <= hn; i++) {
67         int rem = getLca(st[top], h[i]);
68         if(rem == st[top]) {
69             if(rem != st[top]) cl[++cln] = st[++top] = h[i];
70             continue;
71         }
72         while(top > 1 && dep[st[top - 1]] >= dep[rem]) {
73             link(st[top - 1], st[top]); top--;
74         }
75         if(dep[st[top]] > dep[rem]) {
76             link(rem, st[top]), top--;
77         }
78         if(rem != st[top]) cl[++cln] = st[++top] = rem;
79         if(h[i] != st[top]) cl[++cln] = st[++top] = h[i];
80     }
81     while(top > 1) {
82         link(st[top - 1], st[top]), top--;
83     }
84 }
85 void clear() {
86     vecnt = 0;
87     for(;cln; --cln) vhead[cl[cln]] = 0;
88 }
89 void sol() {
90     build();
91     //注意一号节点可能在虚树外面
92     clear();
93 }
94 }

```

### 2.5.3 Tree Decomposition

```

1  int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
2  struct Edge {
3      int to, nxt;
4  }e[MAXN << 1];
5  int head[MAXN], ecnt;
6  int n, m, Q;
7  #define Ls(x) (x << 1)
8  #define Rs(x) (x << 1 | 1)
9  struct Tree {
10     int l, r, lazy;
11     LL sum, mx;
12 }tree[MAXN << 2];
13 int A[MAXN], B[MAXN];
14 void push_up(int x) {
15     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;

```

```

16     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
17 }
18 void push_down(int x) {
19     if(tree[x].lazy) {
20         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
21         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
22         tree[Ls(x)].mx += tree[x].lazy;
23         tree[Rs(x)].mx += tree[x].lazy;
24         tree[Ls(x)].lazy += tree[x].lazy;
25         tree[Rs(x)].lazy += tree[x].lazy;
26         tree[x].lazy = 0;
27     }
28 }
29 void build(int x, int L, int R) {
30     tree[x].lazy = 0;
31     tree[x].l = L; tree[x].r = R;
32     if(L == R) {
33         tree[x].sum = B[L];
34         tree[x].mx = B[L];
35         return;
36     }
37     int mid = (L + R) >> 1;
38     build(Ls(x), L, mid);
39     build(Rs(x), mid + 1, R);
40     push_up(x);
41 }
42 void update(int x, int L, int R, LL val) {
43     if(tree[x].l >= L && tree[x].r <= R) {
44         tree[x].lazy += val;
45         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
46         tree[x].mx += val;
47         return;
48     }
49     push_down(x);
50     int mid = (tree[x].l + tree[x].r) >> 1;
51     if(L <= mid) update(Ls(x), L, R, val);
52     if(R > mid) update(Rs(x), L, R, val);
53     push_up(x);
54 }
55 LL query(int x, int L, int R) {
56     if(tree[x].l >= L && tree[x].r <= R)
57         return tree[x].sum;
58     push_down(x);
59     int mid = (tree[x].l + tree[x].r) >> 1;
60     LL res = 0;
61     if(L <= mid) res += query(Ls(x), L, R);
62     if(R > mid) res += query(Rs(x), L, R);
63     return res;
64 }
65 LL query2(int x, int L, int R) {
66     if(tree[x].l >= L && tree[x].r <= R)
67         return tree[x].mx;
68     push_down(x);
69     int mid = (tree[x].l + tree[x].r) >> 1;
70     LL res = -INF;
71     if(L <= mid) res = max(res, query2(Ls(x), L, R));
72     if(R > mid) res = max(res, query2(Rs(x), L, R));
73     return res;
74 }
75 inline void add_edge(int x, int y) {
76     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;

```

```

77 }
78 void dfs1(int x) {
79     sz[x] = 1; son[x] = 0;
80     for(int i = head[x]; i; i = e[i].nxt) {
81         int v = e[i].to;
82         if(v == fa[x]) continue;
83         fa[v] = x;
84         dep[v] = dep[x] + 1;
85         dfs1(v);
86         sz[x] += sz[v];
87         if(sz[v] > sz[son[x]]) son[x] = v;
88     }
89 }
90 void dfs2(int x) {
91     B[num[x]] = A[x];
92     if(son[x]) {
93         top[son[x]] = top[x];
94         num[son[x]] = ++totw;
95         dfs2(son[x]);
96     }
97     for(int i = head[x]; i; i = e[i].nxt) {
98         int v = e[i].to;
99         if(v == fa[x] || v == son[x]) continue;
100         top[v] = v;
101         num[v] = ++totw;
102         dfs2(v);
103     }
104 }
105 void up(int a, int b, int c) {
106     int f1 = top[a], f2 = top[b];
107     while(f1 != f2) {
108         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
109         update(1, num[f1], num[a], c);
110         a = fa[f1];
111         f1 = top[a];
112     }
113     if(dep[a] > dep[b]) swap(a, b);
114     update(1, num[a], num[b], c);
115 }
116 int qsum(int a, int b) {
117     if(a == b) return query(1, num[a], num[a]);
118     int f1 = top[a], f2 = top[b];
119     int res = 0;
120     while(f1 != f2) {
121         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
122         res += query(1, num[f1], num[a]);
123         a = fa[f1];
124         f1 = top[a];
125     }
126     if(dep[a] > dep[b]) swap(a, b);
127     res += query(1, num[a], num[b]);
128     return res;
129 }
130 int qmax(int a, int b) {
131     if(a == b) return query2(1, num[a], num[a]);
132     int f1 = top[a], f2 = top[b];
133     int res = -1000000000;
134     while(f1 != f2) {
135         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
136         res = max(res, query2(1, num[f1], num[a]));
137         a = fa[f1];

```

```

138     f1 = top[a];
139 }
140 if(dep[a] > dep[b]) swap(a, b);
141 res = max(res, query2(1, num[a], num[b]));
142 return res;
143 }
144 inline void init() {
145     memset(head, 0, sizeof(head)); ecnt = 0;
146     fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
147 }
148 inline void pre() {
149     dfs1(1); dfs2(1); build(1, 1, totw);
150 }
151 //-----
152 const int MAXN = ;
153 const int INF = ;
154 int A[MAXN], B[MAXN], C[MAXN];
155 struct SGT{
156     int sL, sR;
157     LL sW, mx[MAXN << 2];
158     void push_down(int v) {
159     }
160     void push_up(int v) {
161         mx[v] = max(mx[v << 1], mx[v<<1|1]);
162     }
163     void build(int v, int l, int r) {
164         if(l == r) {
165             mx[v] = B[l];
166             return;
167         }
168         int mid = (l + r) >> 1;
169         build(v << 1, l, mid);
170         build(v<<1|1, mid+1, r);
171         push_up(v);
172     }
173     void upd(int v, int l, int r) {
174         if(sL <= l && r <= sR) {
175             //sum[v] = sW *(r - l + 1);
176             mx[v] = sW;
177             return;
178         }
179         push_down(v);
180         int mid = (l + r) >> 1;
181         if(sL <= mid) upd(v << 1, l, mid);
182         if(mid < sR) upd(v<<1|1, mid+1, r);
183         push_up(v);
184     }
185     void qmax(int v, int l, int r) {
186         if(sL <= l && r <= sR) {
187             sW = max(sW, mx[v]);
188             return;
189         }
190         push_down(v);
191         int mid = (l + r) >> 1;
192         if(sL <= mid) qmax(v << 1, l, mid);
193         if(mid < sR) qmax(v<<1|1, mid+1, r);
194     }
195 }T;
196 namespace TD{
197     struct Edge {int to, nxt;}e[MAXN << 1];
198     int ecnt, head[MAXN];

```

```

199 inline void addEdge(int x, int y) {
200     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
201     e[++ecnt] = (Edge) {x, head[y]}; head[y] = ecnt;
202 }
203 int fa[MAXN], sz[MAXN], dep[MAXN], son[MAXN], top[MAXN];
204 int dfn_time, dfn[MAXN], rnk[MAXN];
205 void dfs1(int u) {
206     sz[u] = 1; son[u] = -1;
207     for(int i = head[u], v; i; i = e[i].nxt) {
208         v = e[i].to;
209         if(v == fa[u]) continue;
210         fa[v] = u;
211         dep[v] = dep[u] + 1;
212         dfs1(v);
213         sz[u] += sz[v];
214         if(son[u] == -1 || sz[v] > sz[son[u]]) son[u] = v;
215     }
216 }
217 void dfs2(int u) {
218     dfn[u] = ++dfn_time;
219     //rnk[dfn_time] = u;
220     B[dfn[u]] = A[u];
221     if(son[u] == -1) return;
222     top[son[u]] = top[u];
223     dfs2(son[u]);
224     for(int i = head[u], v; i; i = e[i].nxt) {
225         v = e[i].to;
226         if(v == fa[u] || v == son[u]) continue;
227         top[v] = v;
228         dfs2(v);
229     }
230 }
231 inline void init(int n) {
232     ecnt = 1;
233     for(int i = 0; i <= n; i++) head[i] = 0;
234 }
235 inline void pre(int rt) {
236     fa[rt] = -1; dep[rt] = 1;
237     top[rt] = rt; dfn_time = 0;
238     dfs1(rt); dfs2(rt);
239     // edge -> point
240     //for(int i = 2; i <= ecnt; i += 2) {
241     //    if(e[i].to == fa[e[i^1].to]) {
242     //        C[i / 2] = e[i^1].to;
243     //    } else {
244     //        C[i / 2] = e[i].to;
245     //    }
246     //    B[dfn[C[i / 2]]] = e[i].w;
247     //}
248     T.build(1, 1, dfn_time);
249 }
250 int lca(int u, int v) {
251     while(top[u] != top[v]) {
252         if(dep[top[u]] > dep[top[v]]) swap(u, v);
253         v = fa[top[v]];
254     }
255     if(dep[u] > dep[v]) swap(u, v);
256     return u;
257 }
258 void upd(int a, int b, int c) {
259     int ta = top[a], tb = top[b];

```

```

260     while(ta != tb) {
261         if(dep[ta] < dep[tb]) { swap(a, b); swap(ta, tb); }
262         T.sL = dfn[ta]; T.sR = dfn[a]; T.sW = c;
263         T.upd(1, 1, dfn_time);
264         a = fa[ta]; ta = top[a];
265     }
266     if(dep[a] > dep[b]) swap(a, b); // point
267     //if(a == b) return; // edge
268     //if(dep[a] > dep[b]) swap(a, b); // edge
269     //a = son[a]; // edge
270     T.sL = dfn[a]; T.sR = dfn[b]; T.sW = c;
271     T.upd(1, 1, dfn_time);
272 }
273 //更新子树, 由于dfs2中有if(son[u] == -1) return; 小心使用end[u];
274 void upd2(int a, int c) {
275     T.sL = dfn[a]; T.sR = dfn[a] + sz[a] - 1; T.sW = c;
276     T.upd(1, 1, dfn_time);
277 }
278 int qmax(int a, int b) {
279     int ta = top[a], tb = top[b]; int res = - INF;
280     while(ta != tb) {
281         if(dep[ta] < dep[tb]) { swap(a, b); swap(ta, tb); }
282         T.sL = dfn[ta]; T.sR = dfn[a]; T.sW = - INF;
283         T.qmax(1, 1, dfn_time); res = max(T.sW, res);
284         a = fa[ta]; ta = top[a];
285     }
286     if(dep[a] > dep[b]) swap(a, b); // point
287     //if(a == b) return res; // edge
288     //if(dep[a] > dep[b]) swap(a, b); // edge
289     //a = son[a]; // edge
290     T.sL = dfn[a]; T.sR = dfn[b]; T.sW = -INF;
291     T.qmax(1, 1, dfn_time); res = max(T.sW, res);
292     return res;
293 }
294 }

```

#### 2.5.4 Link-Cut Tree

```

1 namespace LCT {
2     int tcnt; //动态开点
3     int fa[MAXN], tr[MAXN][2], rev[MAXN];
4     //int val[MAXN], sval[MAXN];
5     //void clear(int id) { //或者改为int newnode() {}
6     //     fa[id] = tr[id][0] = tr[id][1] = 0;
7     //     rev[id] = 0; //sz[id] = 1;
8     //}
9     void Rev(int x) {
10         rev[x] ^= 1; swap(tr[x][0], tr[x][1]);
11     }
12     void push_up(int x) {
13         //int l = tr[x][0], r = tr[x][1];
14         //sval[x] = sval[l] + val[x] + sval[r];
15     }
16     void push_down(int x) {
17         int l = tr[x][0], r = tr[x][1];
18         if(rev[x]) {
19             if(l) Rev(l);
20             if(r) Rev(r);
21             rev[x] = 0;
22         }

```

```

23 }
24 bool isroot(int x) {
25     return tr[fa[x]][0] != x && tr[fa[x]][1] != x;
26 }
27 void pre(int x) {
28     if(!isroot(x)) pre(fa[x]);
29     push_down(x);
30 }
31 void rotate(int x) {
32     int y = fa[x], z = fa[y], lr = tr[y][1] == x;
33     if(!isroot(y)) tr[z][tr[z][1] == y] = x;
34     fa[x] = z;
35     fa[tr[y][lr]] = tr[x][lr^1] = y;
36     fa[tr[x][lr^1] = y] = x;
37     push_up(y);
38 }
39 inline void splay(int x) {
40     pre(x);
41     for (int y, z; !isroot(x); rotate(x)) {
42         y = fa[x]; z = fa[y];
43         if (!isroot(y)) rotate((tr[z][0] == y) ^ (tr[y][0] == x) ? x : y);
44     }
45     push_up(x);
46 }
47 inline int access(int x) {
48     int y = 0;
49     for (; x; y = x, x = fa[x]) {
50         splay(x);
51         //sz2[x] += sz[tr[x][1]] - sz[y]; //subtree
52         tr[x][1] = y;
53         push_up(x);
54     }
55     return y; //不求LCA不必
56 }
57 inline void makeroot(int x) {
58     access(x); splay(x); Rev(x);
59 }
60 inline int findroot(int x) {
61     access(x); splay(x);
62     for(; tr[x][0]; x = tr[x][0]) push_down(x);
63     splay(x);
64     return x;
65 }
66 inline void lnk(int x, int y) {
67     makeroot(x);
68     if(findroot(y) != x) fa[x] = y;
69     //sz2[y] += sz[x]; //subtree
70 }
71 inline void cut(int x, int y) {
72     makeroot(x); //access(y); splay(x);
73     if(findroot(y) == x && fa[y] == x && !tr[y][0]){
74         fa[y] = tr[x][1] = 0;
75         push_up(x);
76     }
77 }
78 inline void cut(int y) { //有根树断开与父节点连边
79     access(y); splay(y);
80     fa[tr[y][0]] = 0;
81     tr[y][0] = 0;
82     push_up(y);
83 }

```



```

84 inline int lca(int u, int v) {
85     access(u);
86     return access(v);
87 }
88 void split(int x, int y) {
89     makeroot(x); access(y); splay(y);
90 }
91 //维护节点或者维护路径
92 //例如：染色（注意tag_rev）、tag_add的区间信息、splay维护连续端最远位置
93 void upd(int x, int y) {
94     makeroot(x); val[x] = y; push_up(x);
95 }
96 int que(int x, int y) {
97     split(x, y);
98     //return sval[y];
99 }
100 //维护边权(y -> eid -> x)，需要初始化 vcnt, ecnt, 可用map(注意双向维护)维护eid信息
101 //注意 if(x == y) continue;
102 struct LCTEdge{int u, v; int w;} e[MAXN];
103 void addEdge(int eid) { //e[eid = ++ecnt] = (Edge){x, y, w};
104     lnk(e[eid].u, vcnt + eid);
105     lnk(vcnt + eid, e[eid].v);
106 }
107 void delEdge(int eid) {
108     cut(e[eid].u, vcnt + eid);
109     cut(vcnt + eid, e[eid].v);
110 }
111 //维护边双连通分量
112 //并查集,所有的fa[x]改为Find(fa[x]),public调用函数使用前Find(x), 需要保证只在shrink进行合并
113 //维护边双时, 节点自身信息在unite中维护, 路径等信息在push_up和push_down中维护
114 int Rt[MAXN];
115 int findroot(int x) {return Rt[x] == x ? x : Rt[x] = findroot(Rt[x]);}
116 void unite_dfs(int x) {
117     push_down(x);
118     if (tr[x][0]) unite_dfs(tr[x][0]), unite(tr[x][0], x);
119     if (tr[x][1]) unite_dfs(tr[x][1]), unite(tr[x][1], x);
120 }
121 void shrink(int x, int y) {
122     split(x, y);
123     unite_dfs(y);
124     int z = Find(y);
125     fa[z] = fa[y]; tr[z][0] = tr[z][1] = 0;
126     push_up(z);
127 }
128 void addEdge(int x, int y) {
129     x = Find(x); y = Find(y);
130     if(findroot(x) != findroot(y)) {
131         lnk(x, y); Rt[findroot(x)] = Rt[findroot(y)];
132     }else shrink(x, y);
133 }
134 };
135 //未连成树、初始化节点时, 需要调用LCT::push_up 维护节点其余信息
136 //维护的信息要有 可减性, 如子树结点数, 子树权值和, 但不能直接维护子树最大最小值, 因为在
    将一条虚边变成实边时要排除原先虚边的贡献。
137 //新建一个附加值存储虚子树的贡献, 在统计时将其加入本结点答案, 在改变边的虚实时及时维护。
138 //其余部分同普通 LCT, 在统计子树信息时一定要将其作为根节点。
139 //如果维护的信息没有可减性, 如维护区间最值, 可以对每个结点开一个平衡树维护结点的虚子树中的
    最值。
140 if(x == y) continue;
141 if(Find(x) != Find(y)) {

```

```

142     unite(x, y);
143     addEdge(mp[{x, y}]);
144 }else{
145     int eid = que(x, y);
146     int id = mp[{x, y}];
147     if(val[eid] > e[id].w) {
148         delEdge(eid - vcnt);
149         addEdge(id);
150     }
151 }

```

### 2.5.5 Divide Combine Tree

1.  $[i, i+1]$  构造依赖, 利用线段树辅助建图, 利用 tarjan 求 scc, 利用 rmq 求最左最右边界 2. 定义  $(i, j)$  为一个好二元组, 当且仅当  $a[i]-a[j]=1$  这样的两项的二元组在  $[l, r]$  中恰好有  $r-l$  个线段树维护  $val + l = r$ , 其中  $val$  是区间  $[l, r]$  中好二元组的个数离线 3. 析合树维护  $mx - mn = r - l \Leftrightarrow fx = (mx - mn) - (r - l)$

```

1 namespace DCT{
2     struct RMQ {
3         int lg[MAXN], mn[MAXN][S+1], mx[MAXN][S+1];
4         inline void init(int *a, int n) {
5             for (int i = 2; i <= n; i++) lg[i] = lg[i >> 1] + 1;
6             for (int i = 1; i <= n; i++) mn[i][0] = mx[i][0] = a[i];
7             for (int k = 1; (1 << k) <= n; k++)
8                 for (int i = 1; i + (1 << k) - 1 <= n; i++) {
9                     mn[i][k] = min(mn[i][k - 1], mn[i + (1 << (k - 1))][k - 1]);
10                    mx[i][k] = max(mx[i][k - 1], mx[i + (1 << (k - 1))][k - 1]);
11                }
12        }
13        inline int Min(int l, int r) {
14            int len = lg[r - l + 1];
15            return min(mn[l][len], mn[r - (1 << len) + 1][len]);
16        }
17        inline int Max(int l, int r) {
18            int len = lg[r - l + 1];
19            return max(mx[l][len], mx[r - (1 << len) + 1][len]);
20        }
21    } D;
22
23    struct SEG {
24        int setL, setR, setW;
25        int mn[MAXN << 2], tag[MAXN << 2];
26
27        inline void pushup(int x) {
28            mn[x] = min(mn[x << 1], mn[x << 1 | 1]);
29        }
30        inline void pushdown(int x) {
31            if(!tag[x]) return;
32            mn[x << 1] += tag[x]; mn[x << 1 | 1] += tag[x];
33            tag[x << 1] += tag[x]; tag[x << 1 | 1] += tag[x]; tag[x] = 0;
34        }
35        void init(int x, int l, int r) {
36            mn[x] = tag[x] = 0;
37            if (l == r) return;
38            int mid = (l + r) >> 1;
39            init(x << 1, l, mid);
40            init(x << 1 | 1, mid + 1, r);
41        }
42        void upt(int x, int l, int r) {
43            if (setL <= l && r <= setR) {

```

```

44         tag[x] += setW; mn[x] += setW;
45         return;
46     }
47     pushdown(x);
48     int mid = (l + r) >> 1;
49     if (setL <= mid) upt(x << 1, l, mid);
50     if (mid < setR) upt(x << 1 | 1, mid+1, r);
51     pushup(x);
52 }
53 int que(int x, int l, int r) {
54     if (l == r) return l;
55     pushdown(x);
56     int mid = (l+r)>>1;
57     if (!mn[x << 1]) return que(x << 1, l, mid);
58     return que(x << 1 | 1, mid+1, r);
59 }
60 } T;
61
62 int tpmn, stmn[MAXN], tpmx, stmx[MAXN], tpk, stk[MAXN];
63 int ncnt, type[MAXN<<1], L[MAXN<<1], R[MAXN<<1], M[MAXN<<1];
64 int dep[MAXN<<1], fa[MAXN<<1][S+1], C[MAXN<<1];
65 int id[MAXN << 1];
66 int newnode(int _type, int _L, int _R, int _M = 0) {
67     ++ncnt; type[ncnt] = _type;
68     L[ncnt] = _L; R[ncnt] = _R; M[ncnt] = _M;
69     C[ncnt] = 0;
70     return ncnt;
71 }
72
73 inline bool judge(int l, int r) {
74     return D.Max(l, r) - D.Min(l, r) == r - l;
75 }
76
77 int ecnt, head[MAXN << 1];
78 struct Edge{int to, nxt;} e[MAXN<<1];
79 inline void addEdge(int x, int y) {
80     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
81     fa[y][0] = x; C[x]++;
82 }
83 void dfs(int u) {
84     for(int j = 0; j < S; j++) fa[u][j+1] = fa[fa[u][j]][j];
85     for(int i = head[u]; i; i = e[i].nxt) {
86         dep[e[i].to] = dep[u] + 1;
87         dfs(e[i].to);
88     }
89 }
90
91 inline void init(int n) {
92     ecnt = 0;
93     for(int i = 0; i <= n; i++) head[i] = 0;
94 }
95 void buildT(int *a, int n) {
96     init(n);
97     D.init(a, n);
98     T.init(1, 1, n);
99     tpmn = tpmx = tpk = 0;
100    stmn[0] = stmx[0] = stk[0] = 0;
101    for (int i = 1; i <= n; i++) {
102        for (; tpmn && a[i] <= a[stmn[tpmn]]; --tpmn) {
103            T.setL = stmn[tpmn - 1] + 1; T.setR = stmn[tpmn]; T.setW = a[stmn[tpmn]
]];
```

```

104         T.upt(1, 1, n);
105     }
106     T.setL = stmn[tpmn] + 1; T.setR = i; T.setW = -a[i];
107     T.upt(1, 1, n);
108     stmn[++tpmn] = i;
109
110     for (; tpmx && a[i] >= a[stmx[tpmx]]; --tpmx) {
111         T.setL = stmx[tpmx - 1] + 1; T.setR = stmx[tpmx]; T.setW = -a[stmx[tpmx
112     ]];
113         T.upt(1, 1, n);
114     }
115     T.setL = stmx[tpmx] + 1; T.setR = i; T.setW = a[i];
116     T.upt(1, 1, n);
117     stmx[++tpmx] = i;
118
119     int Li = T.que(1, 1, n), np = id[i] = newnode(0, i, i), nq, nw;
120     while (tpk && L[nq = stk[tpk]] >= Li) {
121         if (type[nq] && judge(M[nq], i)) {
122             R[nq] = i;
123             addEdge(nq, np);
124             np = nq; tpk--;
125         } else if (judge(L[nq], i)) {
126             nw = newnode(1, L[nq], i, L[np]);
127             addEdge(nw, nq); addEdge(nw, np);
128             np = nw; tpk--;
129         } else {
130             nw = newnode(0, -1, i);
131             addEdge(nw, np);
132             do {
133                 addEdge(nw, nq);
134                 nq = stk[--tpk];
135             } while (tpk && !judge(L[nq], i));
136             addEdge(nw, nq);
137             L[nw] = L[nq]; R[nw] = i;
138             np = nw; --tpk;
139         }
140     }
141     stk[++tpk] = np;
142     T.setL = 1; T.setR = i; T.setW = -1;
143     T.upt(1, 1, n);
144 }
145 assert(tpk == 1);
146 dfs(stk[tpk]);
147 }
148 void lca(int u, int v, int &aL, int &bR) {
149     if(u == v) {
150         aL = L[u]; bR = R[v];
151         return;
152     }
153     if(dep[u] > dep[v]) swap(u, v);
154     for(int i = S; i >= 0; i--)
155         if(dep[fa[v][i]] >= dep[u]) v = fa[v][i];
156     assert(u != v);
157     for(int i = S; i >= 0; i--)
158         if(fa[u][i] != fa[v][i]) {
159             u = fa[u][i]; v = fa[v][i];
160         }
161     if(type[fa[u][0]]) {
162         aL = min(L[v], L[u]);
163         bR = max(R[v], R[u]);
164     } else {

```

```
164         aL = L[fa[u][0]];
165         bR = R[fa[u][0]];
166     }
167 }
168 };
```

## 3 String

### 3.1 Basics

#### 3.1.1 Hash

```

1  const LL p1 = 201, p2 = 301, mod1 = 1200000319, mod2 = 2147483647;
2  struct Hash {
3      LL a, b;
4      void append(Hash pre, int v) {
5          a = (pre.a * p1 + v) % mod1;
6          b = (pre.b * p2 + v) % mod2;
7      }
8      void init(string S) {
9          a = b = 0;
10         for(int i = 0; i < S.size(); i++) append(*this, S[i]);
11     }
12     bool operator == (const Hash &x) const {
13         return a == x.a && b == x.b;
14     }
15     bool operator < (const Hash &x) const {
16         return a < x.a || (a == x.a && b < x.b);
17     }
18 };

```

#### 3.1.2 Minimum String

```

1  namespace minstring{
2      int getmin(char *s, int sn) {
3          int i = 0, j = 1, k = 0, t;
4          while(i < sn && j < sn && k < sn) {
5              t = s[(i + k) % sn] - s[(j + k) % sn];
6              if(!t) k++;
7              else {
8                  if(t > 0) i += k + 1; else j += k + 1;
9                  if(i == j) j++;
10                 k = 0;
11             }
12         }
13         return i < j ? i : j;
14     }
15 }

```

### 3.2 String Matching

#### 3.2.1 Bitset Match

```

1  namespace BitMatch{
2      const int S = 26;
3      bitset<MAXN> bs[S], ret;
4      char s[MAXN];
5      inline int idx(char c) { return c - 'a'; }
6      inline void init() {
7          for(int i = 0; i < 26; i++) bs[i].reset();
8          scanf("%s", s);
9          int sn = strlen(s);

```

```

10     for(int i = 0; i < sn; i++) bs[idx(s[i])].set(i);
11 }
12 void modify(int p, char ch) {
13     bs[idx(s[p])].reset(p);
14     s[p] = ch;
15     bs[idx(s[p])].set(p);
16 }
17 int match(char *t, int tn) { //返回 t 串在 s 串中出现的次数
18     ret = bs[idx(t[0])];
19     for(int i = 1; i < tn; i++) {
20         ret <<= 1;
21         ret &= bs[idx(t[i])];
22     }
23     return ret.count();
24 }
25 }

```

### 3.2.2 KMP && exKMP

```

1 判断循环子串的充要条件:  $i/(i-fa[i]) > 1$  &&  $i\%(i-fa[i])==0$  (i是长度)且去除下面fa的优化
2 namespace KMP {
3     int fa[MAXN];
4     void get_fail(char* t, int tn) {
5         fa[0] = -1;
6         int i = 0, j = -1;
7         while(i < tn) {
8             if (j == -1 || t[i] == t[j]) {
9                 ++i; ++j;
10                fa[i] = t[i] != t[j] ? j : fa[j];
11            } else {
12                j = fa[j];
13            }
14        }
15    }
16    void kmp(char* s, int sn, char* t, int tn) {
17        int i = 0, j = 0;
18        while(i < sn) {
19            if (j == -1 || s[i] == t[j]) {
20                i++; j++;
21                if(j == tn) {
22                }
23            } else j = fa[j];
24        }
25    }
26 }
27 namespace exKMP {
28     int nxt[MAXN], ext[MAXN];
29     void get_nxt(char* t, int tn) {
30         int j = 0, mx = 0;
31         nxt[0] = tn;
32         for(int i = 1; i < tn; i++) {
33             if(i >= mx || i + nxt[i - j] >= mx) {
34                 if(i > mx) mx = i;
35                 while(mx < tn && t[mx] == t[mx - i]) mx++;
36                 nxt[i] = mx - i;
37                 j = i;
38             } else nxt[i] = nxt[i - j];
39         }
40     }
41     void exkmp(char *s, int sn, char *t, int tn) {

```

```

42     int j = 0, mx = 0;
43     for(int i = 0; i < sn; i++) {
44         if(i >= mx || i + nxt[i - j] >= mx) {
45             if(i > mx) mx = i;
46             while(mx < sn && mx - i < tn && s[mx] == t[mx - i]) mx++;
47             ext[i] = mx - i;
48             j = i;
49         } else ext[i] = nxt[i - j];
50     }
51 }
52 }

```

### 3.2.3 AC Automaton

```

1  namespace AC {
2      int ch[MAXN][sigma_size], last[MAXN];
3      int val[MAXN], f[MAXN], sz;
4      inline void init() { sz = 1; memset(ch[0], 0, sizeof(ch[0])); }
5      inline int idx(char c) { return c - 'a'; }
6      void insert(string s, int v) {
7          int u = 0;
8          for(int i = 0; i < s.size(); i++) {
9              int c = idx(s[i]);
10             if(!ch[u][c]) {
11                 memset(ch[sz], 0, sizeof(ch[sz]));
12                 val[sz] = 0;
13                 ch[u][c] = sz++;
14             }
15             u = ch[u][c];
16         }
17         val[u] = v;
18     }
19     void get_fail() {
20         queue<int> q;
21         f[0] = 0;
22         for(int c = 0; c < sigma_size; c++) {
23             int u = ch[0][c];
24             if(u) { f[u] = 0; q.push(u); last[u] = 0; }
25         }
26         while(!q.empty()) {
27             int r = q.front(); q.pop();
28             for(int c = 0; c < sigma_size; c++) {
29                 int u = ch[r][c];
30                 if(!u) { ch[r][c] = ch[f[r]][c]; continue; }
31                 q.push(u);
32                 int v = f[r];
33                 while(v && !ch[v][c]) v = f[v];
34                 f[u] = ch[v][c];
35                 last[u] = val[f[u]] ? f[u] : last[f[u]];
36             }
37         }
38     }
39     inline void solve(int j) {
40         if(j) {
41             ans += val[j];
42             solve(last[j]);
43         }
44     }
45     void find(string T) {
46         int j = 0;

```



```

47     for(int i = 0; i < T.size(); i++) {
48         int c = idx(T[i]);
49         j = ch[j][c];
50         if(val[j]) solve(j);
51         else if(last[j]) solve(last[j]);
52     }
53 }
54 }
55 namespace AC {
56     int root, tcnt;
57     int ch[MAXN][sigma_size], fa[MAXN];
58     inline int newnode() {
59         fa[++tcnt] = 0;
60         for(int i = 0; i < sigma_size; ++i) ch[tcnt][i] = 0;
61         return tcnt;
62     }
63     inline void init() {
64         tcnt = -1;
65         root = newnode();
66     }
67     inline int idx(char c) { return c - 'a'; }
68     void extend(char *s, int sn) {
69         int cur = root;
70         for(int i = 0, c; i < sn; i++) {
71             if(!ch[cur][c = idx(s[i])])
72                 ch[cur][c] = newnode();
73             cur = ch[cur][c];
74         }
75     }
76     int q[MAXN], qh, qt;
77     void get_fail() {
78         qh = 1; qt = 0;
79         fa[root] = 0;
80         for(int c = 0, now; c < sigma_size; c++)
81             if((now = ch[root][c]) != 0)
82                 q[++qt] = now;
83         while(qh <= qt) {
84             int cur = q[qh++];
85             for(int c = 0, now; c < sigma_size; c++)
86                 if((now = ch[cur][c]) != 0) {
87                     fa[now] = ch[fa[cur]][c];
88                     q[++qt] = now;
89                 }else
90                     ch[cur][c] = ch[fa[cur]][c];
91         }
92     }
93     //统计模板串出现次数，每个模板串只计算一次
94     // int cur = root, ans = 0;
95     // for(int i = 0; i < sn; ++i) {
96     //     cur = ch[cur][idx(s[i])];
97     //     for(int j = cur; j && cnt[j] != -1; j = fa[j]) {
98     //         ans += cnt[j];
99     //         cnt[j] = -1;
100    //     }
101    // }
102 }
103 }

```

### 3.3 Suffix Related

#### 3.3.1 Suffix Array

```

1 namespace SA {
2     char s[MAXN];
3     int sa[MAXN], rank[MAXN], height[MAXN];
4     int t[MAXN], t2[MAXN], c[MAXN], n;
5     void clear() { n = 0; memset(sa, 0, sizeof(sa)); }
6     void build(int m) {
7         int *x = t, *y = t2;
8         for(int i = 0; i < m; i++) c[i] = 0;
9         for(int i = 0; i < n; i++) c[x[i] = s[i]]++;
10        for(int i = 1; i < m; i++) c[i] += c[i - 1];
11        for(int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
12        for(int k = 1; k <= n; k <= 1) {
13            int p = 0;
14            for(int i = n - k; i < n; i++) y[p++] = i;
15            for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
16            for(int i = 0; i < m; i++) c[i] = 0;
17            for(int i = 0; i < n; i++) c[x[y[i]]]++;
18            for(int i = 1; i < m; i++) c[i] += c[i - 1];
19            for(int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
20            swap(x, y);
21            p = 1; x[sa[0]] = 0;
22            for(int i = 1; i < n; i++)
23                x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k]
? p - 1 : p++;
24            if(p >= n) break;
25            m = p;
26        }
27    }
28    void buildHeight() {
29        int k = 0;
30        for(int i = 0; i < n; i++) rank[sa[i]] = i;
31        for(int i = 0; i < n; i++) {
32            if(k) k--;
33            int j = sa[rank[i] - 1];
34            while(s[i + k] == s[j + k]) k++;
35            height[rank[i]] = k;
36        }
37    }
38    void init() {
39        n = strlen(s) + 1;
40        build('z' + 1);
41        buildHeight();
42    }
43 }

```

#### 3.3.2 Suffix Automaton

```

1 namespace SAM{
2     int scnt, root, last;
3     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
4     int sc[MAXN<<1], tmp1[MAXN<<1], minl[MAXN<<1];
5
6     int newnode(int _len, int q = 0) {
7         fa[++scnt] = fa[q]; len[scnt] = _len;
8         sc[scnt] = 0; tmp1[scnt] = 0; minl[scnt] = INF;

```

```

9         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
10        return scnt;
11    }
12    void init() {
13        scnt = 0;
14        root = last = newnode(0);
15    }
16    void extend(int c) {
17        int p = last, np = newnode(len[p] + 1);
18        for(; p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
19        if(!p) fa[np] = root;
20        else{
21            int q = ch[p][c];
22            if(len[p] + 1 == len[q]) fa[np] = q;
23            else{
24                int nq = newnode(len[p] + 1, q);
25                fa[np] = fa[q] = nq;
26                for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
27            }
28        }
29        last = np;
30    }
31    int c[MAXN], rs[MAXN << 1];
32    void radix_sort(int n){
33        for(int i = 0; i <= n; i++) c[i] = 0;
34        for(int i = 1; i <= scnt; i++) c[len[i]]++;
35        for(int i = 1; i <= n; i++) c[i] += c[i-1];
36        for(int i = scnt; i >= 1; i--) rs[c[len[i]]--] = i;
37    }
38    void go(){
39        scanf("%s",s);
40        int n = strlen(s);
41        for(int i = 0; i < n; ++i)
42            extend(s[i] - 'a');
43        radix_sort(n);
44        //以下sc集合意义不同
45        { //每个节点对应的位置之后有多少个不同子串
46            for(int i = scnt; i >= 1; i--) {
47                int S = 0;
48                for(int j = 0; j < 26; j++)
49                    S += sc[ ch[rs[i]][j] ];
50                sc[rs[i]] = S + 1;
51            }
52        }
53        { //right集合大小
54            int cur = root;
55            for(int i = 0; i < n; ++i) {
56                cur = ch[cur][s[i] - 'a'];
57                sc[cur]++;
58            }
59            for(int i = scnt; i >= 1; --i) {
60                sc[ fa[rs[i]] ] += sc[rs[i]];
61            }
62        }
63        //公共子串
64        //tmpl, 当前字符串: 在状态cur, 与模板串的最长公共后缀
65        //minl, 多个字符串: 在状态cur, 与模板串的最长公共后缀
66        //注意: 在状态cur匹配成功时, cur的祖先状态与字符串的最长公共后缀
67        for(; ~scanf("%s",s);) {
68            int cur = root, Blen = 0;
69            for(int i = 0; i <= scnt; i++)

```

```

70         tmp1[i] = 0;
71         n = strlen(s);
72         for(int i = 0, x; i < n; i++) {
73             x = s[i] - 'a';
74             if(ch[cur][x]) {
75                 ++Blen;
76                 cur = ch[cur][x];
77             }else{
78                 for(;cur && ch[cur][x] == 0; cur = fa[cur]);
79                 if(cur) {
80                     Blen = len[cur] + 1;
81                     cur = ch[cur][x];
82                 }else{
83                     cur = root; Blen = 0;
84                 }
85             }
86             tmp1[cur] = max(tmp1[cur], Blen);
87         }
88         for(int i = scnt; i ; --i) {
89             if( tmp1[ fa[rs[i]] ] < tmp1[ rs[i] ])
90                 tmp1[ fa[rs[i]] ] = len[ fa[rs[i]] ];
91             minl[ rs[i] ] = min(minl[ rs[i] ], tmp1[ rs[i] ]);
92         }
93     }
94 }
95
96 namespace exSAM{
97     int scnt, root;
98     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
99     int sc[MAXN<<1], tmp1[MAXN<<1], minl[MAXN<<1];
100
101     int newnode(int _len, int q = 0) {
102         fa[++scnt] = fa[q]; len[scnt] = _len;
103         sc[scnt] = 0; tmp1[scnt] = 0; minl[scnt] = INF;
104         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
105         return scnt;
106     }
107     void init() {
108         scnt = 0;
109         root = newnode(0);
110     }
111     int work(int p, int c){
112         int q = ch[p][c];
113         int nq = newnode(len[p] + 1, q);
114         fa[q] = nq;
115         for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
116         return nq;
117     }
118     int extend(int p, int c) {
119         if (ch[p][c]){
120             int q = ch[p][c];
121             if (len[p] + 1 == len[q]) return q;
122             return work(p, c);
123         }
124         int np = newnode(len[p] + 1);
125         for(;p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
126         if (!p) fa[np] = root;
127         else{
128             int q = ch[p][c];
129             if (len[p] + 1 == len[q]) fa[np] = q;
130             else fa[np] = work(p, c);

```

```

131     }
132     return np;
133 }
134 void solve() {
135     int n; scanf("%d",&n);
136     for(int i = 1; i <= n; i++) {
137         scanf("%s", s);
138         int sn = strlen(s);
139         int last = root;
140         for(int j = 0; j < sn; ++j)
141             last = extend(last, s[j] - 'a');
142     }
143 }
144 }

```

### 3.4 Palindrome Related

#### 3.4.1 Manacher

```

1 namespace Manacher {
2     char S[MAXN << 1];
3     int scnt, ans;
4     int p[MAXN << 1]; //p[i] - 1
5     void init(char *s0, int sn0) {
6         S[0] = '$'; S[1] = '#';
7         for(int i = 0; i < sn0; i++) {
8             S[2 * i + 2] = s0[i];
9             S[2 * i + 3] = '#';
10        }
11        scnt = sn0 * 2 + 2;
12        S[scnt] = '&';
13    }
14    void manacher() {
15        int id = 0, mx = 0;
16        for(int i = 1; i < scnt; i++) {
17            p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
18            while(S[i + p[i]] == S[i - p[i]]) p[i]++;
19            if(i + p[i] > mx) {
20                mx = i + p[i];
21                id = i;
22            }
23        }
24    }
25 }

```

#### 3.4.2 Palindromic Automaton

```

1 namespace PAM {
2     int scnt, S[MAXN];
3     int pcnt, last, len[MAXN], fail[MAXN], ch[MAXN][26];
4     int cnt[MAXN]; //节点i表示的本质不同的串的个数(调用count())
5     int num[MAXN]; //以节点i表示的最长回文串的最右端点为回文串结尾的回文串个数
6     int newnode(int _len) {
7         len[pcnt] = _len;
8         cnt[pcnt] = num[pcnt] = 0;
9         for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
10        return pcnt++;
11    }

```

```

12 inline void init() {
13     S[scnt = 0] = -1;
14     pcnt = 0; newnode(0); newnode(-1);
15     fail[0] = 1; last = 0;
16 }
17 int getfail(int x) {
18     while(S[scnt - len[x] - 1] != S[scnt]) x = fail[x];
19     return x;
20 }
21 void extend(int c) {
22     S[++scnt] = c;
23     int cur = getfail(last);
24     if(!ch[cur][c]) {
25         int now = newnode(len[cur] + 2);
26         fail[now] = ch[getfail(fail[cur])][c];
27         ch[cur][c] = now;
28         num[now] = num[fail[now]] + 1;
29     }
30     last = ch[cur][c];
31     cnt[last]++;
32 }
33 void count() {
34     for(int i = pcnt - 1; i >= 0; i--) cnt[fail[i]] += cnt[i];
35 }
36 };

```

### 3.5 Substring Automaton

```

1 for(int j = 0; j < 26; j++)
2     ch[n][j] = ch[n+1][j] = n + 1;
3 for(int i = n; i >= 1; i--) {
4     for(int j = 0; j < 26; j++)
5         ch[i-1][j] = ch[i][j];
6     ch[i-1][s[i] - 'a'] = i;
7 }

```

## 4 Math

### 4.1 Algebra

#### 4.1.1 FFT

```

1  //不预处理精度
2  const double pi = acos(-1.0);
3  const int MAXN = 300003;
4  struct comp {
5      double x, y;
6      comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
7      comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
8      comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
9      y * a.x}; }
10 };
11 int rev[MAXN], T;
12 comp tmp;
13 void fft(comp *a, int r) {
14     if(r == -1) for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
15     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17         comp step = (comp) {cos(pi / mid), r * sin(pi / mid)};
18         for(int j = 0; j < T; j += i) {
19             comp cur = (comp) {1, 0};
20             for(int k = j; k < j + mid; k++, cur = cur * step) {
21                 tmp = a[k + mid] * cur;
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27     if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
28 }
29 comp A[MAXN];
30 void init(int n) {
31     //A[0] = (comp) {0, 0};
32     for(T = 1; T <= n; T <= 1);
33     for(int i = 1; i < T; i++) {
34         if(i & 1) rev[i] = (rev[i] >> 1) >> 1 ^ (T >> 1);
35         else rev[i] = rev[i] >> 1;
36     }
37     //A[i] = (comp) {0, 0};
38 }
39 //预处理精度
40 int rev[MAXN], T;
41 comp Sin[MAXN], tmp;
42 void fft(comp *a, int r) {
43     if(r == -1) {
44         for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
45         for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
46     }
47     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
48     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
49         for(int j = 0; j < T; j += i) {
50             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
51                 tmp = a[k + mid] * Sin[cur];
52                 a[k + mid] = a[k] - tmp;
53                 a[k] = a[k] + tmp;

```

```

53     }
54 }
55 }
56 if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
57 }
58 comp A[MAXN];
59 void init(int n) {
60     for(T = 1; T <= n; T <= 1);
61     for(int i = 0; i < T; i++) {
62         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
63         else rev[i] = rev[i >> 1] >> 1;
64         //A[i] = (comp) {0, 0};
65     }
66     for(int i = 0; i < (T >> 1); i++) {
67         Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
68     }
69 }
70 int main() {
71     scanf("%d%d", &n, &m);
72     init(n + m);
73     for(int i = 0; i <= n; i++) scanf("%lf", &A[i].x);
74     for(int i = 0; i <= m; i++) scanf("%lf", &A[i].y);
75     fft(A, 1);
76     fft(A, -1);
77     for(int i = 0; i <= n + m; i++) printf("%d%c", (int)(A[i].y), i == n + m ? '\n' : ' ');
78     return 0;
79 }

```

#### 4.1.2 NTT

4.常用NTT模数:

以下模数的共同 $g = 3189$

$p = r \times 2^k + 1$	$k$	$g$
104857601	22	3
167772161	25	3
469762049	26	3
950009857	21	7
998244353	23	3
1004535809	21	3
2013265921	27	31
2281701377	27	3
3221225473	30	5

```

1  const int MAXN = 300005, G = 3, mod = 998244353; //or (479LL<<21) + 1
2  int rev[MAXN], T;
3  LL qpow(LL x, LL y) {
4      LL res = 1;
5      while(y) {
6          if(y & 1) res = res * x % mod;
7          x = x * x % mod;
8          y >>= 1;
9      }
10     return res;
11 }
12 LL A[MAXN], B[MAXN];

```



```

13 void ntt(LL *a, int r) {
14     if(r == -1) for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
15     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17         LL gn = qpow(G, (mod - 1) / i);
18         if(r == -1) gn = qpow(gn, mod - 2);
19         for(int j = 0; j < T; j += i) {
20             LL cur = 1, tmp;
21             for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
22                 tmp = a[k + mid] * cur % mod;
23                 a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
24                 a[k] = (a[k] + tmp) % mod;
25             }
26         }
27     }
28     if(r == -1) {
29         LL inv = qpow(T, mod - 2);
30         for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
31     }
32 }
33 void init(int n) {
34     for(T = 1; T <= n; T <= 1);
35     for(int i = 0; i < T; i++) {
36         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
37         else rev[i] = rev[i >> 1] >> 1;
38     }
39 }

```

#### 4.1.3 MTT

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  const double pi = acos(-1.0);
5  const int MAXN = 300003;
6  struct comp {
7      double x, y;
8      comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
9      comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
10     comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
11         y * a.x}; }
12 };
13 #define conj(a) ((comp){a.x, -a.y})
14 int rev[MAXN], T;
15 comp Sin[MAXN], tmp;
16 void fft(comp *a, int r) {
17     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
18     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
19         for(int j = 0; j < T; j += i) {
20             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
21                 tmp = a[k + mid] * Sin[cur];
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27 }
28 void init(int n) {
29     for(T = 1; T <= n; T <= 1);
30     for(int i = 0; i < T; i++) {

```

```

30     if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
31     else rev[i] = rev[i >> 1] >> 1;
32 }
33 for(int i = 0; i < (T >> 1); i++) {
34     Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
35 }
36 }
37 int n, m, mod;
38 void mtt(int *x, int *y) {
39     for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
40     static comp a[MAXN], b[MAXN];
41     static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
42     for(int i = 0; i < T; i++) {
43         a[i] = {x[i] & 0x7fff, x[i] >> 15};
44         b[i] = {y[i] & 0x7fff, y[i] >> 15};
45     }
46     fft(a, 1); fft(b, 1);
47     for(int i = 0; i < T; i++) {
48         int j = (T - i) & (T - 1);
49         static comp da, db, dc, dd;
50         da = (a[i] + conj(a[j])) * (comp){0.5, 0};
51         db = (a[i] - conj(a[j])) * (comp){0, -0.5};
52         dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
53         dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
54         dfta[j] = da * dc;
55         dftb[j] = da * dd;
56         dftc[j] = db * dc;
57         dftd[j] = db * dd;
58     }
59     for(int i = 0; i < T; i++) {
60         a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
61         b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
62     }
63     //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
64     fft(a, -1); fft(b, -1);
65     for(int i = 0; i < T; i++) {
66         static int da, db, dc, dd;
67         da = (LL)(a[i].x / T + 0.5) % mod;
68         db = (LL)(a[i].y / T + 0.5) % mod;
69         dc = (LL)(b[i].x / T + 0.5) % mod;
70         dd = (LL)(b[i].y / T + 0.5) % mod;
71         x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
72     }
73 }
74 int main() {
75     static int a[MAXN], b[MAXN];
76     scanf("%d%d%d", &n, &m, &mod);
77     for(int i = 0; i <= n; i++) scanf("%d", a + i);
78     for(int i = 0; i <= m; i++) scanf("%d", b + i);
79     init(n + m);
80     mtt(a, b);
81     for(int i = 0; i <= n + m; i++) printf("%d%c", a[i], i == n + m ? '\n' : ' ');
82     return 0;
83 }

```

#### 4.1.4 FWT

```

1 void FWT(LL *a, int n) {
2     for(int i = 2; i <= n; i <= 1) {
3         for(int j = 0; j < n; j += i) {

```

```

4         for(int d = 0, w = i >> 1; d < w; d++){
5             LL u = a[j + d], v = a[j + d + w];
6             //xor: a[j + d] = u + v, a[j + d + w] = u - v;
7             //and: a[j + d] = u + v;
8             //or : a[j + d + w] = u + v;
9         }
10    }
11 }
12 }
13 void UFWT(LL *a, int n) {
14     for(int i = 2; i <= n; i <= 1) {
15         for(int j = 0; j < n; j += i) {
16             for(int d = 0, w = i >> 1; d < w; d++) {
17                 LL u = a[j + d], v = a[j + d + w];
18                 //xor: a[j + d] = (u + v) / 2, a[j + d + w] = (u - v) / 2;
19                 //and: a[j + d] = u - v;
20                 //or : a[j + d + w] = v - u;
21             }
22         }
23     }
24 }
25 void solve(int n) {
26     FWT(a, n); FWT(b, n);
27     for(int i = 0; i < n; i++) a[i] = a[i] * b[i];
28     UFWT(a, n);
29 }

```

#### 4.1.5 FFT Divide and Conquer

$$f_i = \sum_{j=1}^{i-1} f_j \cdot g_{i-j}$$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  const int MAXN = 300005, G = 3, mod = 998244353;
5  namespace NTT {
6      LL A[MAXN], B[MAXN]
7      int rev[MAXN], T;
8      LL qpow(LL x, LL y) {
9          LL res = 1;
10         while(y) {
11             if(y & 1) res = res * x % mod;
12             x = x * x % mod;
13             y >>= 1;
14         }
15         return res;
16     }
17     void ntt(LL *a, int r) {
18         for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
19         for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
20             LL gn = qpow(G, (mod - 1) / i);
21             if(r == -1) gn = qpow(gn, mod - 2);
22             for(int j = 0; j < T; j += i) {
23                 LL cur = 1, tmp;
24                 for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
25                     tmp = a[k + mid] * cur % mod;
26                     a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;

```

```

27         a[k] = (a[k] + tmp) % mod;
28     }
29 }
30 }
31 if(r == -1) {
32     LL inv = qpow(T, mod - 2);
33     for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
34 }
35 }
36 void init(int n) {
37     for(T = 1; T <= n; T <= 1);
38     for(int i = 0; i < T; i++) {
39         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
40         else rev[i] = rev[i >> 1] >> 1;
41     }
42 }
43 }
44 LL f[MAXN], g[MAXN];
45 using namespace NTT;
46 void solve(int l, int r) {
47     if(l == r) return;
48     int mid = (l + r) >> 1;
49     solve(l, mid);
50     init(r - l);
51     for(int i = 0; i < T; i++) A[i] = B[i] = 0;
52     for(int i = 0; i <= mid - 1; i++) A[i] = f[i + 1];
53     for(int i = 0; i <= r - 1; i++) B[i] = g[i];
54     ntt(A, 1); ntt(B, 1);
55     for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
56     ntt(A, -1);
57     for(int i = mid + 1; i <= r; i++) f[i] = (f[i] + A[i - 1]) % mod;
58     solve(mid + 1, r);
59 }
60 int main() {
61     int n; scanf("%d", &n);
62     for(int i = 1; i < n; i++) scanf("%lld", g + i);
63     f[0] = 1;
64     solve(0, n - 1);
65     for(int i = 0; i < n; i++) printf("%lld%c", f[i], i == n - 1 ? '\n' : ' ');
66     return 0;
67 }

```

#### 4.1.6 Linear Basis

```

1 //dynamic
2 const int D = 60;
3 struct Basis {
4     vector<int> ind;
5     vector<LL> base;
6     Basis() {
7         ind.resize(D, -1);
8         base.resize(D);
9     }
10    bool update(LL x, int id) {
11        for(int i = 0; i < D; i++) if(~ind[i] && x >> i & 1) {
12            x ^= base[i];
13        }
14        if(!x) return 1;
15        int pos = __builtin_ctzll(x);
16        ind[pos] = id;

```

```

17     base[pos] = x;
18     return 0;
19 }
20 };
21 //array
22 int Gauss(int n, int m) {
23     int num = 1;
24     for(int x = 1; x <= n && x <= m; x++) {
25         int t = 0;
26         for(int j = x; j <= m; j++) if(g[j][x]) { t = j; break; }
27         if(t) {
28             swap(g[x], g[t]);
29             for(int i = x + 1; i <= n; i++) {
30                 if(g[i][x]) {
31                     for(int k = 1; k <= m; k++) g[i][k] ^= g[x][k];
32                 }
33             }
34             num++;
35         }
36     }
37     return --num;
38 }
39 //long long
40 int Gauss() {
41     int num = 1;
42     for(int k = 61; k >= 0; k--) {
43         int t = 0;
44         for(int j = num; j <= cnt; j++) if((A[j] >> k) & 1) { t = j; break; }
45         if(t) {
46             swap(A[t], A[num]);
47             for(int j = num + 1; j <= cnt; j++) if((A[j] >> k) & 1) A[j] ^= A[num];
48             num++;
49         }
50     }
51     return --num;
52 }
53 //det
54 LL det(int n){
55     LL ret = 1;
56     for(int i = 1; i < n; i++){
57         for(int j = i + 1; j < n; j++)
58             while(a[j][i]){
59                 LL t = (LL)a[i][i] / a[j][i];
60                 for(int k = i; k < n; k++) {
61                     a[i][k] = (a[i][k]-a[j][k] * t);
62                     swap(a[i][k], a[j][k]);
63                 }
64                 ret = -ret;
65             }
66         if(a[i][i] == 0)return 0;
67         ret = ret * a[i][i];
68     }
69     if(ret<0)ret = -ret;
70     return ret;
71 }

```

## 4.1.7 Polynomial

Inverse:

$$A_x * B'_x \equiv 1 \pmod{x^{\frac{n}{2}}}$$

$$A_x * B_x \equiv 1 \pmod{x^n}$$

$$B_x \equiv 2 \cdot B'_x - A_x \cdot B'^2_x \pmod{x^n}$$

Division:

$$A_r(x) = x^n A\left(\frac{1}{x}\right) \Rightarrow A_r(x)[i] = A(x)[n - i]$$

$$A(x) = B(x) * Q(x) + R(x)$$

$$Q_r(x) = A_r(x) * B_r^{-1}(x)$$

$$R(x) = A(x) - B(x) * Q(x)$$

```

1 //NTT模数
2 #include <bits/stdc++.h>
3 using namespace std;
4 typedef long long LL;
5 const int MAXN = 300005, G = 3, mod = 998244353;
6 namespace NTT {
7     int rev[MAXN], T;
8     LL qpow(LL x, LL y) {
9         LL res = 1;
10        while(y) {
11            if(y & 1) res = res * x % mod;
12            x = x * x % mod;
13            y >>= 1;
14        }
15        return res;
16    }
17    void ntt(LL *a, int r) {
18        for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
19        for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
20            LL gn = qpow(G, (mod - 1) / i);
21            if(r == -1) gn = qpow(gn, mod - 2);
22            for(int j = 0; j < T; j += i) {
23                LL cur = 1, tmp;
24                for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
25                    tmp = a[k + mid] * cur % mod;
26                    a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
27                    a[k] = (a[k] + tmp) % mod;
28                }
29            }
30        }
31        if(r == -1) {
32            LL inv = qpow(T, mod - 2);
33            for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
34        }
35    }
36    void init(int n) {
37        for(T = 1; T <= n; T <= 1);
38        for(int i = 0; i < T; i++) {
39            if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
40            else rev[i] = rev[i >> 1] >> 1;

```

```

41     }
42 }
43 }
44 namespace poly {
45 using namespace NTT;
46 void mul(LL *a, LL *b, LL *c, int n, int m) {
47     init(n + m);
48     static LL x[MAXN], y[MAXN];
49     for(int i = 0; i < T; i++) {
50         x[i] = i <= n ? a[i] : 0;
51         y[i] = i <= m ? b[i] : 0;
52     }
53     ntt(x, 1); ntt(y, 1);
54     for(int i = 0; i < T; i++) c[i] = x[i] * y[i] % mod;
55     ntt(c, -1);
56 }
57 void poly_inv(LL *a, LL *b, int n) {
58     if(n == 1) {
59         b[0] = qpow(a[0], mod - 2);
60         return;
61     }
62     static LL c[MAXN], d[MAXN];
63     memset(c, 0, n * sizeof(LL));
64     poly_inv(a, c, n >> 1);
65     for(int i = 0; i < n; i++) {
66         d[i] = a[i];
67     }
68     init(n);
69     ntt(c, 1); ntt(d, 1);
70     for(int i = 0; i < T; i++) b[i] = c[i] * (2 + mod - d[i] * c[i] % mod) % mod;
71     ntt(b, -1);
72     for(int i = n; i < T; i++) b[i] = 0;
73 }
74 void inv(LL *a, LL *b, int n) {//A must be different from B}
75     init(n);
76     poly_inv(a, b, T);
77 }
78 //A_x = B_x * Q_x + R_x;
79 void div(LL *A, LL *B, LL *Q, LL *R, int n, int m) {
80     static LL f[MAXN], g[MAXN], inv_g[MAXN];
81     for(int i = 0; i <= n; i++) f[n - i] = A[i];
82     for(int i = 0; i <= m; i++) g[m - i] = B[i];
83     //inv(G_r)
84     for(int i = n - m + 1; i <= m; i++) g[i] = 0;
85     inv(g, inv_g, n - m);
86     //Q
87     mul(f, inv_g, f, n, n - m);
88     for(int i = 0; i <= n - m; i++) Q[i] = f[n - m - i];
89     //R
90     mul(Q, B, f, n - m, m);
91     for(int i = 0; i < m; i++) R[i] = (A[i] + mod - f[i]) % mod;
92 }
93 }
94 LL A[MAXN], B[MAXN];
95 LL Q[MAXN], R[MAXN];
96 int main() {
97     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
98     int n, m; cin >> n >> m;
99     for(int i = 0; i <= n; i++) cin >> A[i];
100    for(int i = 0; i <= m; i++) cin >> B[i];
101    poly::div(A, B, Q, R, n, m);

```

```

102     for(int i = 0; i <= n - m; i++) cout << Q[i] << " "; cout << endl;
103     for(int i = 0; i < m; i++) cout << R[i] << " "; cout << endl;
104     return 0;
105 }
106 //非NTT模数求逆
107 #include <bits/stdc++.h>
108 using namespace std;
109 typedef long long LL;
110 const double pi = acos(-1.0);
111 const int MAXN = 300003;
112 struct comp {
113     double x, y;
114     comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
115     comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
116     comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
117         y * a.x}; }
118 };
119 #define conj(a) ((comp){a.x, -a.y})
120 int rev[MAXN], T;
121 comp Sin[MAXN], tmp;
122 void fft(comp *a, int r) {
123     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
124     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
125         for(int j = 0; j < T; j += i) {
126             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
127                 tmp = a[k + mid] * Sin[cur];
128                 a[k + mid] = a[k] - tmp;
129                 a[k] = a[k] + tmp;
130             }
131         }
132     }
133 }
134 void init(int n) {
135     //for (T = 1; T <= n; T <= 1);
136     T = n << 1;
137     for(int i = 0; i < T; i++) {
138         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
139         else rev[i] = rev[i >> 1] >> 1;
140     }
141     for(int i = 0; i < (T >> 1); i++) {
142         Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
143     }
144 }
145 const int mod = 1e9 + 7;
146 void mtt(int *x, int *y) {
147     for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
148     static comp a[MAXN], b[MAXN];
149     static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
150     for(int i = 0; i < T; i++) {
151         a[i] = {x[i] & 0x7fff, x[i] >> 15};
152         b[i] = {y[i] & 0x7fff, y[i] >> 15};
153     }
154     fft(a, 1); fft(b, 1);
155     for(int i = 0; i < T; i++) {
156         int j = (T - i) & (T - 1);
157         static comp da, db, dc, dd;
158         da = (a[i] + conj(a[j])) * (comp){0.5, 0};
159         db = (a[i] - conj(a[j])) * (comp){0, -0.5};
160         dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
161         dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
162         dfta[j] = da * dc;

```



```

162     dftb[j] = da * dd;
163     dftc[j] = db * dc;
164     dftd[j] = db * dd;
165 }
166 for(int i = 0; i < T; i++) {
167     a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
168     b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
169 }
170 //for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
171 fft(a, -1); fft(b, -1);
172 for(int i = 0; i < T; i++) {
173     static int da, db, dc, dd;
174     da = (LL)(a[i].x / T + 0.5) % mod;
175     db = (LL)(a[i].y / T + 0.5) % mod;
176     dc = (LL)(b[i].x / T + 0.5) % mod;
177     dd = (LL)(b[i].y / T + 0.5) % mod;
178     x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
179 }
180 }
181 LL qpow(LL x, LL y) {
182     LL res = 1;
183     while(y) {
184         if(y & 1) res = res * x % mod;
185         x = x * x % mod;
186         y >>= 1;
187     }
188     return res;
189 }
190 void poly_inv(int *a, int *b, int n) {
191     if(n == 1) {
192         b[0] = qpow(a[0], mod - 2);
193         return;
194     }
195     static int c[MAXN], cc[MAXN], d[MAXN];
196     memset(c, 0, n * sizeof(int)); memset(cc, 0, n * sizeof(int)); memset(d, 0, n *
sizeof(int));
197     poly_inv(a, c, n >> 1);
198     for(int i = 0; i < n; i++) cc[i] = c[i], d[i] = a[i];
199     init(n);
200     mtt(cc, c);
201     mtt(d, cc);
202     for(int i = 0; i < T; i++) b[i] = (2 * c[i] % mod + mod - d[i]) % mod;
203     for(int i = 0; i < n; i++) b[n + i] = 0;
204 }
205 int A[MAXN], B[MAXN];
206 int main() {
207     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
208     int n; cin >> n; n--;
209     for(T = 1; T <= n; T <= 1);
210     for(int i = 0; i <= n; i++) cin >> A[i];
211     poly_inv(A, B, T);
212     for(int i = 0; i <= n; i++) cout << B[i] << " ";
213     cout << endl;
214     return 0;
215 }
216
217 //LOJ挑战多项式(备用)
218 #include <bits/stdc++.h>
219 #define rep(i, a, b) for (int i = (a); i <= (b); i++)
220 #define per(i, a, b) for (int i = (a); i >= (b); i--)
221 #define REP(i, n) for (int i = (0); i < (n); i++)

```

```

222 #define fi first
223 #define se second
224 #define mp make_pair
225 #define pb push_back
226 using namespace std;
227 typedef unsigned long long ull;
228 typedef pair<int, int> pii;
229 typedef vector<int> poly;
230 typedef long long ll;
231 struct ano {
232     char a[1 << 25], *s;
233     char b[1 << 25], *t;
234     ano() : s(a), t(b) { a[fread(a, 1, sizeof a, stdin)] = 0; }
235     ~ano() { fwrite(b, 1, t - b, stdout); }
236     operator int() {
237         int x = 0;
238         while (*s < 48) ++s;
239         while (*s > 32) x = x * 10 + *s++ - 48;
240         return x;
241     }
242     ll in() {
243         ll x = 0;
244         while (*s < 48) ++s;
245         while (*s > 32) x = x * 10 + *s++ - 48;
246         return x;
247     }
248     void out(int x, char e = ' ') {
249         static char c[12];
250         char *i = c;
251         if (!x)
252             *t++ = 48;
253         else {
254             while (x) {
255                 int y = x / 10;
256                 *i++ = x - y * 10 + 48, x = y;
257             }
258             while (i != c) *t++ = *--i;
259         }
260         *t++ = e;
261     }
262 } buf;
263 const int mod = 998244353;
264 namespace Poly {
265     const int N = (1 << 20) + 5, g = 3;
266     inline int power(int x, int p) {
267         int res = 1;
268         for (; p; p >>= 1, x = (ll)x * x % mod)
269             if (p & 1)
270                 res = (ll)res * x % mod;
271         return res;
272     }
273     inline int fix(const int x) { return x >= mod ? x - mod : x; }
274     void dft(poly &A, int n) {
275         static ull W[N << 1], *H[30], *las = W, mx = 0;
276         for (; mx < n; mx++) {
277             H[mx] = las;
278             ull w = 1, wn = power(g, (mod - 1) >> (mx + 1));
279             REP(i, 1 << n) *las++ = w, w = w * wn % mod;
280         }
281         if (A.size() != (1 << n))
282             A.resize(1 << n);

```

```

283 static ull a[N];
284 for (int i = 0, j = 0; i < (1 << n); ++i) {
285     a[i] = A[j];
286     for (int k = 1 << (n - 1); (j ^ k) < k; k >>= 1)
287         ;
288 }
289 for (int k = 0, d = 1; k < n; k++, d <= 1)
290     for (int i = 0; i < (1 << n); i += (d << 1)) {
291         ull *l = a + i, *r = a + i + d, *w = H[k], t;
292         for (int j = 0; j < d; j++, l++, r++) {
293             t = (*r) * (*w++) % mod;
294             *r = *l + mod - t, *l += t;
295         }
296     }
297     REP(i, 1 << n) A[i] = a[i] % mod;
298 }
299 void idft(poly &a, int n) {
300     a.resize(1 << n, reverse(a.begin() + 1, a.end()));
301     dft(a, n);
302     int inv = power(1 << n, mod - 2);
303     REP(i, 1 << n) a[i] = (ll)a[i] * inv % mod;
304 }
305 poly FIX(poly a) {
306     while (!a.empty() && !a.back()) a.pop_back();
307     return a;
308 }
309 poly add(poly a, poly b, int op = 0) {
310     a.resize(max(a.size(), b.size()));
311     REP(i, b.size()) a[i] = fix(op ? a[i] + mod - b[i] : a[i] + b[i]);
312     return FIX(a);
313 }
314 poly mul(poly a, poly b, int t = 1) {
315     if (t == 1 && a.size() + b.size() <= 24) {
316         poly c(a.size() + b.size(), 0);
317         REP(i, a.size()) REP(j, b.size()) c[i + j] = (c[i + j] + (ll)a[i] * b[j]) % mod;
318         return FIX(c);
319     }
320     int n = 1, aim = a.size() * t + b.size();
321     while ((1 << n) <= aim) n++;
322     dft(a, n), dft(b, n);
323     if (t == 1)
324         REP(i, 1 << n) a[i] = (ll)a[i] * b[i] % mod;
325     else
326         REP(i, 1 << n) a[i] = (ll)a[i] * a[i] % mod * b[i] % mod;
327     idft(a, n), a.resize(aim);
328     return FIX(a);
329 }
330 poly mul(poly a, int b) {
331     REP(i, a.size()) a[i] = (ll)a[i] * b % mod;
332     return FIX(a);
333 }
334
335 poly inv(poly a, int n) { // a[0] != 0
336     a.resize(n);
337     poly b;
338     if (n == 1) {
339         b.pb(power(a[0], mod - 2));
340         return b;
341     }
342     b = inv(a, n + 1 >> 1);
343     b = add(mul(b, 2), mul(b, a, 2), 1);

```

```

344     return b.resize(n), b;
345 }
346
347 poly Der(poly a) {
348     REP(i, a.size() - 1) a[i] = (ll)(i + 1) * a[i + 1] % mod;
349     return a.pop_back(), a;
350 }
351 poly Int(poly a) {
352     static int inv[N];
353     inv[1] = 1;
354     a.pb(0);
355     rep(i, 2, a.size()) inv[i] = (ll)(mod - mod / i) * inv[mod % i] % mod;
356     per(i, a.size() - 1, 1) a[i] = (ll)a[i - 1] * inv[i] % mod;
357     return a[0] = 0, a;
358 }
359 poly Ln(poly a, int n) { // a[0] = 1
360     a = mul(Der(a), inv(a, n)), a.resize(n - 1);
361     return FIX(Int(a));
362 }
363 poly Exp(poly a, int n) { // a[0] = 0
364     a.resize(n);
365     poly b, one(1, 1);
366     if (n == 1)
367         return one;
368     b = Exp(a, n + 1 >> 1);
369     b = mul(b, add(add(a, Ln(b, n), 1), one));
370     return b.resize(n), b;
371 }
372
373 poly Div(poly a, poly b) {
374     poly c;
375     int n = a.size() - 1, m = b.size() - 1;
376     if (n < m)
377         return c;
378     reverse(a.begin(), a.end());
379     a.resize(n - m + 1);
380     reverse(b.begin(), b.end());
381     b.resize(n - m + 1);
382     c = mul(a, inv(b, n - m + 1));
383     c.resize(n - m + 1);
384     return reverse(c.begin(), c.end()), c;
385 }
386 poly Mod(poly a, poly b) { return FIX(add(a, mul(Div(a, b), b), 1)); }
387 inline int chk(int x) { return power(x, (mod - 1) / 2) == 1; }
388 inline int RC() { return rand() % mod; }
389 inline pii mul(pii a, pii b, int w) {
390     return mp(((ll)a.fi * b.fi + (ll)a.se * b.se % mod * w) % mod, ((ll)a.fi * b.se + (ll)a.se * b.fi) % mod);
391 }
392 inline int Sqrt(int x) {
393     if (!chk(x))
394         return -1;
395     int a = RC();
396     while (chk(((ll)a * a - x + mod) % mod)) a = RC();
397     int w = ((ll)a * a - x + mod) % mod, p = (mod + 1) / 2;
398     pii res = mp(1, 0), t = mp(a, 1);
399     for (; p >= 1, t = mul(t, t, w))
400         if (p & 1)
401             res = mul(res, t, w);
402     assert(!res.se);
403     return min(res.fi, mod - res.fi);

```

```

404 }
405 poly Sqrt(poly a, int n) {
406     if (n == 1) {
407         poly b(1, Sqrt(a[0]));
408         return b;
409     }
410     a.resize(n);
411     poly b = Sqrt(a, n + 1 >> 1);
412     b = mul(add(b, mul(a, inv(b, n))), (mod + 1) / 2);
413     return b.resize(n), b;
414 }
415 poly fastpow(poly a, ll k, int n) {
416     a.resize(n), a = FIX(a);
417     if (!a.size())
418         return a;
419     int st = 0, base = 0;
420     while (!a[st]) ++st;
421     if (st * k >= n)
422         return a.resize(0), a;
423     REP(i, a.size() - st) a[i] = a[i + st];
424     if (st)
425         a.resize(a.size() - st);
426     base = a[0];
427     ll inv = power(base, mod - 2);
428     REP(i, a.size()) a[i] = a[i] * inv % mod;
429     a = FIX(Exp(mul(Ln(a, n), k % mod), n));
430     ;
431     if (st) {
432         reverse(a.begin(), a.end());
433         a.resize(a.size() + st * k);
434         reverse(a.begin(), a.end());
435         a.resize(n), a = FIX(a);
436     }
437     base = power(base, k);
438     REP(i, a.size()) a[i] = (ll)a[i] * base % mod;
439     return FIX(a);
440 }
441 } // namespace Poly
442 using namespace Poly;
443 int main() {
444     int n = buf + 1, K = buf;
445     poly a(n, 0), b;
446     REP(i, n) a[i] = buf;
447     b = add(a, Exp(Int(inv(Sqrt(a, n), n)), n), 1);
448     b[0] = (b[0] + 2 - a[0] + mod) % mod;
449     a = Ln(b, n);
450     a[0]++;
451     b = Der(fastpow(a, K, n));
452     b.resize(n - 1);
453     REP(i, n - 1) buf.out(b[i]);
454     return 0;
455 }

```

#### 4.1.8 Lagrange Polynomial

$$L(x) = \sum_{i=0}^n y_i \prod_{j=0, j \neq i}^n \frac{x - x_j}{x_i - x_j}$$

```

1  //O(n^2)
2  #include <bits/stdc++.h>
3  using namespace std;
4  typedef long long LL;
5  typedef pair<int, int> P;
6  const int MAXN = 3005, mod = 998244353;
7  int exgcd(int a, int b, int &x, int &y) {
8      int d = a;
9      if(b != 0) {
10         d = exgcd(b, a % b, y, x);
11         y -= (a / b) * x;
12     }
13     else {
14         x = 1; y = 0;
15     }
16     return d;
17 }
18 int inv(int a) {
19     int x, y;
20     exgcd(a, mod, x, y);
21     return (x % mod + mod) % mod;
22 }
23 struct Lagrange {
24     int n, a[MAXN][2];
25     void init() {
26         for(int i = 0; i <= n; i++) a[i][0] = a[i][1] = 0;
27         n = 0;
28         a[0][1] = 1;
29     }
30     int query(int x, int q = 0) {
31         int res = 0;
32         for(int i = n; i >= 0; i--) res = ((LL)res * x + a[i][q]) % mod;
33         return res;
34     }
35     void update(int x, int y) {
36         a[n][0] = 0;
37         int v = (LL)(y - query(x) + mod) % mod * inv(query(x, 1)) % mod;
38         for(int i = 0; i <= n; i++) a[i][0] = (a[i][0] + (LL)a[i][1] * v) % mod;
39         a[++n][1] = 0;
40         for(int i = n; i; i--) a[i][1] = (a[i - 1][1] + (LL)a[i][1] * (mod - x)) % mod;
41         a[0][1] = (LL)a[0][1] * (mod - x) % mod;
42     }
43 }p;
44 int main() {
45     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
46     int Q;
47     cin >> Q;
48     int op, x, y;
49     p.n = 0;
50     p.init();
51     while(Q--) {
52         cin >> op >> x;
53         if(op == 1) {
54             cin >> y;
55             p.update(x, y);
56         }
57         else cout << p.query(x) << endl;
58     }
59     return 0;
60 }
61 //f(x) = y, 求f(z_i)

```

```

62 #include <bits/stdc++.h>
63 using namespace std;
64 #define ll long long
65 int n, m;
66 vector<int> x, y, z;
67 namespace Poly{
68     const int P = 998244353;
69     vector<int> ans; // for Evaluate()
70     vector<vector<int>> p; // for Evaluate() & Interpolate()
71     inline int Pow(ll x, int y=P-2){ //  $x^y$ 
72         int ans=1;
73         for(; y; y>>=1, x=x*x%P) if(y&1) ans=ans*x%P;
74         return ans;
75     }
76     inline int Ge(int x){ int n=1; while(n<=x) n<<=1; return n;}
77     inline int Mod(int x){ return x<P?x:x-P;}
78     inline void NTT(vector<int> &f, int g, int n){
79         f.resize(n);
80         for(int i=0, j=0; i<n; ++i){
81             if(i>j) swap(f[i], f[j]);
82             for(int k=n>>1; (j^=k)<k; k>>=1);
83         }
84         vector<int> w(n>>1);
85         for(int i=1; i<n; i<<=1){
86             for(int j=w[0]=1, w0=(g==1?Pow(3, (P-1)/i/2):Pow(Pow(3, (P-1)/i/2))); j<i;
87             ++j) w[j]=(ll)w[j-1]*w0%P;
88             for(int j=0; j<n; j+=i<<1){
89                 for(int k=j; k<j+i; ++k){
90                     int t=(ll)f[k+i]*w[k-j]%P;
91                     f[k+i]=Mod(f[k]-t+P);
92                     f[k]=Mod(f[k]+t);
93                 }
94             }
95             if(g==1) for(int i=0, I=Pow(n); i<n; ++i) f[i]=(ll)f[i]*I%P;
96         }
97     inline vector<int> Add(const vector<int> &f, const vector<int> &g){
98         vector<int> ans=f;
99         for(unsigned i=0; i<f.size(); ++i) (ans[i]+=g[i])%=P;
100         return ans;
101     }
102     inline vector<int> Mul(const vector<int> &f, const vector<int> &g){ //  $f*g$ 
103         vector<int> F=f, G=g;
104         int p=Ge(f.size()+g.size()-2);
105         NTT(F, 1, p), NTT(G, 1, p);
106         for(int i=0; i<p; ++i) F[i]=(ll)F[i]*G[i]%P;
107         NTT(F, -1, p);
108         return F.resize(f.size()+g.size()-1), F;
109     }
110     inline vector<int> PolyInv(const vector<int> &f, int n=-1){ //  $1/f$ 
111         if(n==1) n=f.size();
112         vector<int> ans;
113         if(n==1) return ans.push_back(Pow(f[0])), ans;
114         ans=PolyInv(f, (n+1)/2);
115         vector<int> tmp(&f[0], &f[0]+n);
116         int p=Ge(n*2-2);
117         NTT(tmp, 1, p), NTT(ans, 1, p);
118         for(int i=0; i<p; ++i) ans[i]=(2-(ll)ans[i]*tmp[i]%P+P)*ans[i]%P;
119         NTT(ans, -1, p);
120         return ans.resize(n), ans;
121     }

```

```

122 inline void PolyDiv(const vector<int> &a, const vector<int> &b, vector<int> &d,
vector<int> &r){ // a=d*b+r
123     if(b.size()>a.size()) return (void)(d.clear(), r=a);
124
125     vector<int> A=a, B=b, iB;
126     int n=a.size(), m=b.size();
127     reverse(A.begin(), A.end()), reverse(B.begin(), B.end());
128     B.resize(n-m+1), iB=PolyInv(B, n-m+1);
129     d=Mul(A, iB);
130     d.resize(n-m+1), reverse(d.begin(), d.end());
131
132     r=Mul(b, d);
133     for(int i=0; i<m-1; ++i) r[i]=(P+a[i]-r[i])%P;
134     r.resize(m-1);
135 }
136 inline vector<int> Derivative(const vector<int> &a){ // a'
137     vector<int> ans;
138     ans.resize(a.size()-1);
139     for(unsigned i=1; i<a.size(); ++i) ans[i-1]=(ll)a[i]*i%P;
140     return ans;
141 }
142 void Evaluate_Interpolate_Init(int l, int r, int t, const vector<int> &a){
143     if(l==r) return p[t].clear(), p[t].push_back(P-a[l]), p[t].push_back(1);
144     int mid=(l+r)/2, k=t<<1;
145     Evaluate_Interpolate_Init(l, mid, k, a), Evaluate_Interpolate_Init(mid+1, r, k
146     ll, a);
147     p[t]=Mul(p[k], p[kll]);
148 }
149 inline void Evaluate(int l, int r, int t, const vector<int> &f, const vector<int> &a
){
150     if(r-l+1<=512){
151         for(int i=l; i<=r; ++i){
152             int x=0, j=f.size(), a1=a[i], a2=(ll)a[i]*a[i]%P, a3=(ll)a[i]*a2%P, a4=(
153             ll)a[i]*a3%P, a5=(ll)a[i]*a4%P, a6=(ll)a[i]*a5%P, a7=(ll)a[i]*a6%P, a8=(ll)a[i]*a7%P
154             ;
155             while(j>=8)
156                 x=((ll)x*a8+(ll)f[j-1]*a7+(ll)f[j-2]*a6+(ll)f[j-3]*a5+(ll)f[j-4]*a4+(ll)
157                 f[j-5]*a3+(ll)f[j-6]*a2+(ll)f[j-7]*a1+f[j-8])%P, j-=8;
158             while(j-->0) x=((ll)x*a[i]+f[j])%P;
159             ans.push_back(x);
160         }
161         return;
162     }
163     vector<int> tmp;
164     PolyDiv(f, p[t], tmp, tmp);
165     Evaluate(l, (l+r)/2, t<<1, tmp, a), Evaluate((l+r)/2+1, r, t<<1ll, tmp, a);
166 }
167 inline vector<int> Evaluate(const vector<int> &f, const vector<int> &a, int flag=-1)
168 { // f(a_i)
169     if(flag===-1) p.resize(a.size()<<2), Evaluate_Interpolate_Init(0, a.size()-1, 1,
170     a);
171     ans.clear(), Evaluate(0, a.size()-1, 1, f, a);
172     return ans;
173 }
174 vector<int> Interpolate(int l, int r, int t, const vector<int> &x, const vector<int>
&f){
175     if(l==r){
176         vector<int> ans;
177         return ans.push_back(f[l]), ans;
178     }
179     int mid=(l+r)/2, k=t<<1;

```



```

174     return Add(Mul(Interpolate(l, mid, k, x, f), p[kl1]), Mul(Interpolate(mid+1, r,
175     kl1, x, f), p[k]));
176 }
177 inline vector<int> Interpolate(const vector<int> &x, const vector<int> &y){// (x_i,
178     y_i)
179     int n=x.size();
180     p.resize(n<2), Evaluate_Interpolate_Init(0, n-1, 1, x);
181     vector<int> f=Evaluate(Derivative(p[1]), x, 0);
182     for(int i=0; i<n; ++i) f[i]=(ll)y[i]*Pow(f[i])%P;
183     return Interpolate(0, n-1, 1, x, f);
184 }
185 }
186 using namespace Poly;
187 int main() {
188     cin >> n; x.resize(n), y.resize(n);
189     for(int i=0; i<n; ++i) cin >> x[i], cin >> y[i];
190     cin >> m, z.resize(m);
191     for(int i=0; i<m; ++i) cin >> z[i];
192     x=Evaluate(Interpolate(x, y), z);
193     for(int i:x) cout << i << " ";
194     return 0;
195 }

```

#### 4.1.9 BM Alogrithm

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  #define rep(i,a,n) for (int i=a;i<n;i++)
4  #define per(i,a,n) for (int i=n-1;i>=a;i--)
5  #define pb push_back
6  #define mp make_pair
7  #define all(x) (x).begin(),(x).end()
8  #define fi first
9  #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector<int> VI;
12 typedef long long ll;
13 typedef pair<int,int> PII;
14 const ll mod=1000000007;
15 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b>=>1){if(b&1)res=res*a%mod;
16     a=a*a%mod;}return res;}
17 // head
18 namespace linear_seq {
19     const int N=10010;
20     ll res[N],base[N],_c[N],_md[N];
21
22     vector<int> Md;
23     void mul(ll *a,ll *b,int k) {
24         rep(i,0,k+k) _c[i]=0;
25         rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
26         for (int i=k+k-1;i>=k;i--) if (_c[i])
27             rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
28         rep(i,0,k) a[i]=_c[i];
29     }
30     int solve(ll n,VI a,VI b) { // a 系数 b 初值 b[n+1]=a[0]*b[n]+...
31         printf("%d\n",SZ(b));
32         ll ans=0,pnt=0;
33         int k=SZ(a);
34         assert(SZ(a)==SZ(b));
35         rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;

```

```

35 Md.clear();
36 rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
37 rep(i,0,k) res[i]=base[i]=0;
38 res[0]=1;
39 while ((1ll<<pnt)<=n) pnt++;
40 for (int p=pnt;p>=0;p--) {
41     mul(res,res,k);
42     if ((n>>p)&1) {
43         for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
44         rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
45     }
46 }
47 rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
48 if (ans<0) ans+=mod;
49 return ans;
50 }
51 VI BM(VI s) {
52     VI C(1,1),B(1,1);
53     int L=0,m=1,b=1;
54     rep(n,0,SZ(s)) {
55         ll d=0;
56         rep(i,0,L+1) d=(d+(1ll)C[i]*s[n-i])%mod;
57         if (d==0) ++m;
58         else if (2*L<=n) {
59             VI T=C;
60             ll c=mod-d*powmod(b,mod-2)%mod;
61             while (SZ(C)<SZ(B)+m) C.pb(0);
62             rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
63             L=n+1-L; B=T; b=d; m=1;
64         } else {
65             ll c=mod-d*powmod(b,mod-2)%mod;
66             while (SZ(C)<SZ(B)+m) C.pb(0);
67             rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
68             ++m;
69         }
70     }
71     return C;
72 }
73 int gao(VI a,ll n) {
74     VI c=BM(a);
75     c.erase(c.begin());
76     rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
77     return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
78 }
79 };
80
81 int main() {
82     while (~scanf("%d",&n)) {
83         vector<int>v;
84         v.push_back(1);
85         v.push_back(2);
86         v.push_back(4);
87         v.push_back(7);
88         v.push_back(13);
89         v.push_back(24);
90         //VI{1,2,4,7,13,24}
91         printf("%d\n",linear_seq:gao(v,n-1));
92     }
93 }

```

## 4.2 Math Theory

### 4.2.1 Inverse

```

1 //O(logn)求n的逆元
2 const int mod = 1e6 + 3;
3 int exgcd(int a, int b, int &x, int &y) {
4     int d = a;
5     if(b != 0) {
6         d = exgcd(b, a % b, y, x);
7         y -= (a / b) * x;
8     }
9     else {
10        x = 1; y = 0;
11    }
12    return d;
13 }
14 int inverse(int a) {
15     int x, y;
16     exgcd(a, mod, x, y);
17     return (x % mod + mod) % mod;
18 }
19 int inverse(int a) { return qpow(a, mod - 2); }
20 //O(n)求1~n的逆元
21 int inv[MAXN];
22 void init() {
23     inv[0] = inv[1] = 1;
24     for(int i = 2; i < MAXN; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
        mod;
25 }

```

### 4.2.2 Lucas

```

1 //mod很小可以预处理逆元的情况
2 void init() {
3     fac[0] = 1;
4     for(int i = 1; i < mod; i++) fac[i] = (long long)fac[i - 1] * i % mod;
5     inv[0] = inv[1] = 1;
6     for(int i = 2; i < mod; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
        mod;
7     for(int i = 1; i < mod; i++) inv[i] = (long long)inv[i] * inv[i - 1] % mod;
8 }
9 int C(int a, int b) {
10    if(b > a) return 0;
11    if(a < mod) return (long long)fac[a] * inv[b] % mod * inv[a - b] % mod;
12    return (long long)C(a / mod, b / mod) * C(a % mod, b % mod) % mod;
13 }
14 //mod过大不能预处理逆元的情况
15 LL qpow(LL x, LL y) {
16     LL res = 1;
17     while(y) {
18         if(y & 1) res = res * x % mod;
19         x = x * x % mod;
20         y >>= 1;
21     }
22     return res;
23 }
24 LL C(LL a, LL b) {
25     if(b > a) return 0;

```

```

26     if(b > a - b) b = a - b;
27     LL s1 = 1, s2 = 1;
28     for(LL i = 0; i < b; i++) {
29         s1 = s1 * (a - i) % mod;
30         s2 = s2 * (i + 1) % mod;
31     }
32     return s1 * qpow(s2, mod - 2) % mod;
33 }
34 LL lucas(LL a, LL b) {
35     if(a < mod) return C(a, b);
36     return lucas(a / mod, b / mod) * C(a % mod, b % mod);
37 }

```

### 4.2.3 CRT && exCRT

$$x \equiv a_i \pmod{m_i}$$

```

1 namespace CRT {
2     LL m[MAXN], a[MAXN];
3     LL exgcd(LL _a, LL _b, LL &x, LL &y) {
4         if(!_b) {
5             x = 1; y = 0;
6             return _a;
7         }
8         LL d = exgcd(_b, _a % _b, y, x);
9         y -= (_a / _b) * x;
10        return d;
11    }
12    LL crt(int n) {
13        LL M = 1, tmp, res = 0, x, y;
14        for(int i = 1; i <= n; i++) M *= m[i];
15        for(int i = 1; i <= n; i++) {
16            tmp = M / m[i];
17            exgcd(tmp, m[i], x, y);
18            x = (x + m[i]) % m[i];
19            res = (a[i] * x % M * tmp % M + res) % M;
20        }
21        return res;
22    }
23 }
24 namespace EXCRT {
25     LL m[MAXN], a[MAXN];
26     LL exgcd(LL _a, LL _b, LL &x, LL &y) {
27         if(!_b) {
28             x = 1; y = 0;
29             return _a;
30         }
31         LL d = exgcd(_b, _a % _b, y, x);
32         y -= (_a / _b) * x;
33         return d;
34     }
35     LL excrt(int n) {
36         LL M = m[1], A = a[1], x, y, d, tmp;
37         for(int i = 2; i <= n; i++) {
38             d = exgcd(M, m[i], x, y);
39             if((A - a[i]) % d) return -1; //No solution
40             tmp = M / d; M *= m[i] / d;
41             y = (A - a[i]) / d % M * y % M;

```

```

42     y = (y + tmp) % tmp;
43     A = (m[i] % M * y % M + a[i]) % M;
44     A = (A + M) % M;
45 }
46 return A;
47 }
48 LL inv(LL _a, LL _b) {
49     LL x, y;
50     exgcd(_a, _b, x, y);
51     return (x % _b + _b) % _b;
52 }
53 LL excrt(int n) {
54     LL M = m[1], A = a[1], x, y, d, c, tmp;
55     for(int i = 2; i <= n; i++) {
56         d = exgcd(M, m[i], x, y);
57         c = a[i] - A;
58         if(c % d) return -1;
59         c = (c % m[i] + m[i]) % m[i];
60         M /= d; m[i] /= d;
61         c = c / d * inv(M % m[i], m[i]) % m[i];
62         tmp = M;
63         M *= m[i] * d;
64         A = (c * tmp % M * d % M + A) % M;
65     }
66     return A;
67 }
68 //当 a[i] 一开始就是负数时, 转成正数 a[i]=(a[i]%m[i]+m[i])%m[i];
69 }

```

#### 4.2.4 BSGS

```

1  const int MOD = 76543;
2  int hs[MOD + 5], head[MOD + 5], nxt[MOD + 5], id[MOD + 5], ecnt;
3  void insert(int x, int y) {
4      int k = x % MOD;
5      hs[ecnt] = x, id[ecnt] = y, nxt[ecnt] = head[k], head[k] = ecnt++;
6  }
7  int find(int x) {
8      int k = x % MOD;
9      for(int i = head[k]; i; i = nxt[i])
10         if(hs[i] == x)
11             return id[i];
12     return -1;
13 }
14 int BSGS(int a, int b, int c){
15     memset(head, 0, sizeof head); ecnt = 1;
16     if(b == 1) return 0;
17     int m = sqrt(c * 1.0), j;
18     LL x = 1, p = 1;
19     for(int i = 0; i < m; i++, p = p * a % c)
20         insert(p * b % c, i);
21     for(LL i = m; ; i += m){
22         if((j = find(x = x * p % c)) != -1) return i - j;
23         if(i > c) break;
24     }
25     return -1;
26 }

```

## 4.2.5 Miller-Rabin &amp; PollardRho

```

1 LL ksc(LL a,LL n,LL mod){
2     LL ret=0;
3     for(;n>=1){
4         if(n&1){ret+=a;if(ret>=mod)ret-=mod;}
5         a<<=1;if(a==mod)a-=mod;
6     }
7     return ret;
8 }
9 LL ksm(LL a,LL n,LL mod){
10    LL ret = 1;
11    for(;n>=1){
12        if(n&1)ret=ksc(ret,a,mod);
13        a=ksc(a,a,mod);
14    }
15    return ret;
16 }
17 int millerRabin(LL n){
18     if(n<2 || (n!=2 && !(n&1)))return 0;
19     LL d=n-1;for(;!(d&1);d>>=1);
20     for(int i=0;i<20;++i){
21         LL a=rand()%(n-1)+1;
22         LL t=d,m=ksm(a,d,n);
23         for(;t!=n-1 && m!=1 && m!=n-1;m=ksc(m,m,n),t<<=1);
24         if(m!=n-1 && !(t&1)) return 0;
25     }
26     return 1;
27 }
28 LL cnt,fact[100];
29 LL gcd(LL a,LL b){return !b?a:gcd(b,a%b);}
30 LL pollardRho(LL n, int a){
31     LL x=rand()%n,y=x,d=1,k=0,i=1;
32     while(d==1){
33         ++k;
34         x=ksc(x,x,n)+a;if(x>=n)x-=n;
35         d=gcd(x>y?x-y:y-x,n);
36         if(k==i){y=x;i<<=1;}
37     }
38     if(d==n)return pollardRho(n,a+1);
39     return d;
40 }
41 void findfac(LL n){
42     if(millerRabin(n)){fact[++cnt]=n;return;}
43     LL p=pollardRho(n,rand()%(n-1)+1);
44     findfac(p);
45     findfac(n/p);
46 }

```

4.2.6  $\varphi(n)$ 

```

1 int phi(int x) {
2     int res = x;
3     for(int i = 2; i * i <= x; i++) {
4         if(x % i == 0) {
5             res = res / i * (i - 1);
6             while(x % i == 0) x /= i;
7         }
8     }

```

```

9     if(x > 1) res = res / x * (x - 1);
10    return res;
11 }

```

#### 4.2.7 Euler Sieve

```

1  int prime[MAXN], cnt, phi[MAXN], mu[MAXN];
2  bool isp[MAXN];
3
4  int min_pow[MAXN];    //最小质因子最高次幂
5  int min_sum[MAXN];    //1+p+p^2+...+p^k
6  int div_sum[MAXN];    //约数和
7
8  int min_index[MAXN];  //最小质因子的指数
9  int div_num[MAXN];    //约数个数
10 void Euler(int n) {
11     mu[1] = phi[1] = div_num[1] = div_sum[1] = 1;
12     for(int i = 2; i <= n; i++) {
13         if(!isp[i]) {
14             prime[++cnt] = min_pow[i] = i;
15             phi[i] = i - 1;
16             mu[i] = -1;
17             min_index[i] = 1; div_num[i] = 2;
18             div_sum[i] = min_sum[i] = i + 1;
19         }
20         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
21             isp[i * prime[j]] = 1;
22             if(i % prime[j] == 0) {
23                 phi[i * prime[j]] = phi[i] * prime[j];
24                 mu[i * prime[j]] = 0;
25
26                 min_index[i * prime[j]] = min_index[i] + 1;
27                 div_num[i * prime[j]] = div_num[i] / (min_index[i] + 1) * (min_index[i] *
prime[j] + 1);
28
29                 min_sum[i * prime[j]] = min_sum[i] + min_pow[i] * prime[j];
30                 div_sum[i * prime[j]] = div_sum[i] / min_sum[i] * min_sum[i * prime[j]];
31                 min_pow[i * prime[j]] = min_pow[i] * prime[j];
32                 break;
33             }
34             phi[i * prime[j]] = phi[i] * (prime[j] - 1);
35             mu[i * prime[j]] = -mu[i];
36
37             div_num[i * prime[j]] = div_num[i] << 1;
38             min_index[i * prime[j]] = 1;
39
40             div_sum[i * prime[j]] = div_sum[i] * (prime[j] + 1);
41             min_pow[i * prime[j]] = prime[j];
42             min_sum[i * prime[j]] = prime[j] + 1;
43         }
44     }
45 }

```

#### 4.2.8 DuJiao Sieve

$$\sum_{i=1}^n \varphi(i)$$

```

1  vector<int> prime;
2  int phi[MAXN], P[MAXN];
3  bool isp[MAXN];
4  unordered_map<LL, int> mp;
5  void Euler(int n) {
6      phi[1] = 1;
7      for(int i = 2; i <= n; i++) {
8          if(!isp[i]) {
9              prime.push_back(i);
10             phi[i] = i - 1;
11         }
12         for(auto x : prime) {
13             if(i * x > n) break;
14             isp[i * x] = 1;
15             if(i % x == 0) {
16                 phi[i * x] = phi[i] * x;
17                 break;
18             }
19             phi[i * x] = phi[i] * (x - 1);
20         }
21     }
22     for(int i = 1; i <= n; i++) P[i] = (P[i - 1] + phi[i]) % mod;
23 }
24 LL cal(LL n) {
25     if(n < MAXN) return P[n];
26     if(mp.count(n)) return mp[n];
27     LL res = 0;
28     for(LL i = 2, last; i <= n; i = last + 1) {
29         last = n / (n / i);
30         res += (last - i + 1) % mod * cal(n / i) % mod;
31         res %= mod;
32     }
33     mp[n] = ((__int128)n * (n + 1) / 2 % mod + mod - res) % mod;
34     return mp[n];
35 }

```

$$\sum_{i=1}^n \mu(i)$$

```

1  LL cal(LL n) {
2      if(n < MAXN) return M[n];
3      if(mp.count(n)) return mp[n];
4      LL res = 0;
5      for(LL i = 2, last; i <= n; i = last + 1) {
6          last = n / (n / i);
7          res += (last - i + 1) * cal(n / i);
8      }
9      mp[n] = 1 - res;
10     return 1 - res;
11 }

```

#### 4.2.9 Min\_25 Sieve

思路为把结果分为  $i$  为质数的和,  $i$  为合数的和,  $i=1$  的和

$g(n, j)$  表示从 1 累加到  $n$  的  $f(i)$ , 其中的  $i$  满足要么  $i$  自己是质数, 要么  $i$  的最小质因子大于第  $j$  个质数

要求  $f(p)$  ( $p$  is prime) 可被多项式表示,  $f(p^k)$  可快速计算

分别计算多项式  $x^0, x^1, x^2$  的  $g(n, j)$  和  $h(n, j)$

$h$  由欧拉筛计算, 递归起点  $g(n, 0)$



公式

$$\sum_i^n f(i)$$

$$g(n, j) = \begin{cases} g(n, j-1) & p_j^2 > n \\ g(n, j-1) - f(p_j)[g(\frac{n}{p_j}, j-1) - g(p_j-1, j-1)] & p_j^2 \leq n \end{cases}$$

其中

$$g(p_j-1, j-1) = \sum_i^{j-1} f(p_i)$$

可用 h 表示

$S(n, j)$  表示从 1 累加到  $n$  的  $f(i)$ ，同样的， $i$  满足要么  $i$  自己是质数，要么  $i$  的最小质因子大于第  $j$  个质数和  $g$  顺序相反，最后答案为  $S(n, 1)$ ，递推式为

$$S(n, j) = g(n, |P|) - \sum_{i=1}^j f(p_i) + \sum_{k \geq j} \sum_{p_k^{e+1} < n} (f(p_k^e) S(\frac{n}{p_k^e}, k+1) + f(p_k^{e+1}))$$

前一部分  $g(n, |P|) - \sum_{i=1}^j f(p_i)$  是相应的质数部分和。

例题：欧拉函数前缀和

$$\sum_{i=1}^n \varphi(i)$$

$g_{k,n}$  and  $h_{k,n}$  Count

$$\sum_{i=1}^n i^k$$

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long LL;
4 const int MAXN = 1e6 + 5, mod = 1e9 + 7;
5 const int inv2 = (mod + 1) / 2, inv6 = (mod + 1) / 6;
6 int prime[MAXN], isp[MAXN], cnt;
7 LL g[3][MAXN << 1], h[3][MAXN << 1];
8 LL w[MAXN << 1];
9 int id1[MAXN], id2[MAXN];
10 inline int MOD(LL x) { return x >= mod ? x - mod : x; }
11 //inline int MOD(LL x) { return x % mod; }
12 inline int add(LL x, LL y) { return MOD(MOD(x) + MOD(y)); }
13 void Euler(int n) {
14     for(int i = 2; i <= n; i++) {
15         if(!isp[i]) {
16             prime[++cnt] = i;
17             h[0][cnt] = h[0][cnt - 1] + 1;
18             h[1][cnt] = add(h[1][cnt - 1], i);
19             h[2][cnt] = add(h[2][cnt - 1], (LL)i * i % mod);
20         }
21         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
22             isp[i * prime[j]] = 1;
23             if(i % prime[j] == 0) {
24                 break;
25             }
26         }
27     }
28 }

```

```

27     }
28 }
29 LL n;
30 int sz, m;
31 inline int id(LL x) {
32     return x <= sz ? id1[x] : id2[n / x];
33 }
34 //f(p ^ k)
35 inline int f(int p, LL pk) {
36     return pk / p * (p - 1) % mod;
37 }
38 LL S(LL x, int y) {
39     if(x <= 1 || prime[y] > x) return 0;
40     //G(x) - H(j - 1) (first part)
41     LL res = add(add(g[1][id(x)], mod - g[0][id(x)]), mod - add(h[1][y - 1], mod - h[0][y - 1]));
42     for(int j = y, k = 1; j <= cnt && (LL)prime[j] * prime[j] <= x; j++, k = 1) {
43         for(LL pk = prime[j]; pk * prime[j] <= x; pk *= prime[j], k++) {
44             res = add(res, S(x / pk, j + 1) * f(prime[j], pk) % mod + f(prime[j], pk *
prime[j]));
45         }
46     }
47     return res;
48 }
49 int main() {
50     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
51     cin >> n;
52     sz = sqrt(n);
53     Euler(sz);
54     for(LL i = 1, last, t; i <= n; i = last + 1) {
55         last = n / (n / i);
56         w[++m] = n / i, t = n / i % mod;
57         w[m] <= sz ? id1[w[m]] = m : id2[last] = m;
58         g[0][m] = MOD(t + mod - 1);
59         g[1][m] = add(t * (t + 1) % mod * inv2 % mod, mod - 1);
60         g[2][m] = add((2 * t + 1) % mod * t * (t + 1) % mod * inv6 % mod, mod - 1);
61     }
62     for(int j = 1; j <= cnt; j++) {
63         for(int i = 1; i <= m && (LL)prime[j] * prime[j] <= w[i]; i++) {
64             g[0][i] = MOD(g[0][i] + mod - (g[0][id(w[i] / prime[j])] - h[0][j - 1]));
65             g[1][i] = MOD(g[1][i] + mod - ((LL)prime[j] * MOD(g[1][id(w[i] / prime[j])
+ mod - h[1][j - 1]) % mod));
66             g[2][i] = MOD(g[2][i] + mod - ((LL)prime[j] * prime[j] % mod * MOD(g[2][id(w
[i] / prime[j])] + mod - h[2][j - 1]) % mod));
67         }
68     }
69     //S(n, 1) + F(1);
70     LL ans = MOD(S(n, 1) + 1);
71     cout << ans << endl;
72     return 0;
73 }

```

$$\sum_{i=1}^n \mu(i)$$

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long LL;
4 const int MAXN = 1e6 + 5;
5 int prime[MAXN], isp[MAXN], cnt;
6 LL g[3][MAXN << 1], h[3][MAXN << 1];

```

```

7  LL w[MAXN << 1];
8  int id1[MAXN], id2[MAXN];
9  void Euler(int n) {
10     for(int i = 2; i <= n; i++) {
11         if(!isp[i]) {
12             prime[++cnt] = i;
13             h[0][cnt] = h[0][cnt - 1] + 1;
14         }
15         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
16             isp[i * prime[j]] = 1;
17             if(i % prime[j] == 0) {
18                 break;
19             }
20         }
21     }
22 }
23 LL a, b;
24 LL n;
25 int sz, m;
26 inline int id(LL x) {
27     return x <= sz ? id1[x] : id2[n / x];
28 }
29 //f(p ^ k)
30 inline int f(int p, int k) {
31     return k == 1 ? -1 : 0;
32 }
33 LL S(LL x, int y) {
34     if(x <= 1 || prime[y] > x) return 0;
35     //g(x) - h(j - 1)
36     LL res = -g[0][id(x)] + h[0][y - 1];
37     for(int j = y, k = 1; j <= cnt && (LL)prime[j] * prime[j] <= x; j++, k = 1) {
38         for(LL pk = prime[j]; pk * prime[j] <= x; pk *= prime[j], k++) {
39             res += S(x / pk, j + 1) * f(prime[j], k) + f(prime[j], k + 1);
40         }
41     }
42     return res;
43 }
44 LL cal(LL x) {
45     n = x;
46     m = 0;
47     sz = sqrt(n);
48     for(LL i = 1, last, t; i <= n; i = last + 1) {
49         last = n / (n / i);
50         w[++m] = n / i, t = n / i;
51         w[m] <= sz ? id1[w[m]] = m : id2[last] = m;
52         g[0][m] = t - 1;
53     }
54     for(int j = 1; j <= cnt; j++) {
55         for(int i = 1; i <= m && (LL)prime[j] * prime[j] <= w[i]; i++) {
56             g[0][i] = g[0][i] - (g[0][id(w[i] / prime[j])] - h[0][j - 1]);
57         }
58     }
59     return S(x, 1) + 1;
60 }
61 int main() {
62     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
63     cin >> a >> b;
64     Euler(sqrt(b));
65
66     //S(n, 1) + F(1);
67     cout << cal(b) - cal(a - 1) << endl;

```

```

68     return 0;
69 }

```

#### 4.2.10 Möbius Inversion

$$\sum_i^n \sum_j^m lcm(i, j) \pmod{p}$$

```

1  int mu[MAXN], prime[MAXN], sum[MAXN], cnt;
2  bool isp[MAXN];
3  void getmu(int n) {
4      mu[1] = 1;
5      for(int i = 2; i <= n; i++) {
6          if(!isp[i]) {
7              mu[i] = -1;
8              prime[++cnt] = i;
9          }
10         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
11             isp[i * prime[j]] = 1;
12             if(i % prime[j] == 0) {
13                 mu[i * prime[j]] = 0;
14                 break;
15             }
16             mu[i * prime[j]] = -mu[i];
17         }
18     }
19 }
20 ll n, m, ans;
21 ll query(ll x, ll y) { return (x * (x + 1) / 2 % mod) * (y * (y + 1) / 2 % mod) % mod; }
22 ll F(ll x, ll y) {
23     ll res = 0, last;
24     for(ll i = 1; i <= min(x, y); i = last + 1) {
25         last = min(x / (x / i), y / (y / i));
26         res = (res + (sum[last] - sum[i - 1]) * query(x / i, y / i) % mod) % mod;
27     }
28     return res;
29 }
30 int main() {
31     cin >> n >> m;
32     getmu(min(n, m));
33     for(ll i = 1; i <= min(n, m); i++) sum[i] = (sum[i - 1] + (i * i * mu[i]) % mod) % mod;
34     ll last;
35     for(ll d = 1; d <= min(n, m); d = last + 1) {
36         last = min(n / (n / d), m / (m / d));
37         ans = (ans + (last - d + 1) * (d + last) / 2 % mod * F(n / d, m / d) % mod) % mod;
38     }
39     ans = (ans + mod) % mod;
40     cout << ans << endl;
41     return 0;
42 }

```

#### 4.2.11 Primitive Root

简易版如果模  $m$  有原根，则有  $\varphi(\varphi(m))$  个原根当  $d$  遍历  $\varphi(m)$  的简化剩余系时， $g^d$  遍历完  $m$  的全部原根

```

1  const int MAXN = ;
2  LL qpow(LL a, LL b, LL mod) {
3      LL res = 1;
4      for (; b >= 1) {
5          if(b&1) res = res * a % mod;
6          a = a * a % mod;
7      }
8      return res;
9  }
10 LL phi(LL x) {
11     LL res = x;
12     for(int i = 2; i <= x / i; i++) {
13         if(x % i == 0) {
14             res = res / i * (i - 1);
15             while(x % i == 0) x /= i;
16         }
17     }
18     if(x > 1) res = res / x * (x - 1);
19     return res;
20 }
21 int has_primitive_root(int p) {
22     if (p == 4) return 1;
23     if (p % 2 == 0) p /= 2;
24     if (p % 2 == 0) return false;
25     for (int i = 2; i <= p/i; i++)
26         if (p % i == 0) {
27             while(p % i == 0) p /= i;
28             return p == 1 ? i : 0;
29         }
30     return p;
31 }
32 //int indg[MAXN];
33 int get_g_init(LL p) {
34     //p : 2 or 4 or (p**n) or (2 * p**n); else return -1;
35     if(p == 2) {puts("1"); return 1;}
36     if(p == 4) {puts("3"); return 3;}
37     if(!has_primitive_root(p)) {puts("-1"); return -1;}
38     vector<int> p_fact;
39     LL p_phi = phi(p), tp = p_phi ;
40     for(int i = 2, in = sqrt(p) + 0.5; i <= in; i++)
41         if(tp % i == 0) {
42             p_fact.push_back(p_phi / i);
43             while(tp % i == 0) tp /= i;
44         }
45     if(tp != 1) p_fact.push_back(p_phi / tp);
46     int g = 1;
47     for (bool fg = 0; !fg ;) {
48         fg = 1; g++;
49         if(qpow(g, p_phi, p) != 1) {fg = 0; continue;}
50         for(auto it : p_fact)
51             if(qpow(g, it, p) == 1) {fg = 0; break;}
52     }
53     //for(int i = 0, tg = 1; i < p_phi; i++, tg = tg * g % p) indg[tg] = i;
54     /*vector<int> fac;
55     for(int d = 1, tg = g; d < p_phi; d++, tg = tg * g % p)
56         if(__gcd((LL)d, p_phi) == 1) fac.push_back(tg);
57     sort(fac.begin(), fac.end());
58     int ed = fac.back();
59     for(auto it : fac)
60         printf("%d%c", it, it != ed ? ' ' : '\n');*/
61     return g;

```



## 5 Geometry

### 5.1 Commonly Definition and Functions

#### 5.1.1 Const and Functions

```

1 namespace CG{
2     #define Point Vector
3     const double pi=acos(-1.0);
4     const double inf=1e100;
5     const double eps=1e-9;
6     template <typename T> inline T Abs(T x){return x>0?x:-x;}
7     template <typename T> inline bool operator == (T x,T y){return Abs(x-y)<eps;}
8     int sgn(double x){
9         if (Abs(x)<eps) return 0;
10        if (x>0) return 1;
11        else return -1;
12    }
13 }

```

#### 5.1.2 Point Definition

```

1 namespace CG{
2     struct Point{
3         double x,y;
4         Point(double x=0,double y=0):x(x),y(y){}
5     };
6     Vector operator + (const Vector a,const Vector b){return Vector(a.x+b.x,a.y+b.y);}
7     Vector operator - (const Vector a,const Vector b){return Vector(a.x-b.x,a.y-b.y);}
8     Vector operator * (const Vector a,const double k){return Vector(a.x*k,a.y*k);}
9     Vector operator / (const Vector a,const double k){return Vector(a.x/k,a.y/k);}
10    bool operator < (const Vector a,const Vector b) {return a.x==b.x?a.y<b.y:a.x<b.x;}
11    bool operator == (const Vector a,const Vector b) {return a.x==b.x && a.y==b.y;}
12    double Dot(const Vector a,const Vector b){return a.x*b.x+a.y*b.y;}
13    double Cross(const Vector a,const Vector b){return a.x*b.y-a.y*b.x;}
14    double mult_Cross(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.
y-c.y)-(b.x-c.x)*(a.y-c.y);}
15    double mult_Dot(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.x-
c.x)+(a.y-c.y)*(b.y-c.y);}
16    double Norm(const Vector a){return sqrt(Dot(a,a));}
17    double Angle(const Vector a,const Vector b){return acos(Dot(a,b)/Norm(a)/Norm(b));}
18    Vector Rotate(const Vector a,const double theta){return Vector(a.x*cos(theta)-a.y*
sin(theta),a.x*sin(theta)+a.y*cos(theta));}
19    boolToLeftTest(const Vector a,const Vector b){return Cross(a,b)<0;}
20    double DisPP(const Vector a,const Vector b){return sqrt((a.x-b.x)*(a.x-b.x)+(a.y-b.y
)*(a.y-b.y));}
21 }

```

#### 5.1.3 Line Definition

```

1 namespace CG{
2     struct Line{
3         Point p0,v,p1;
4         double t,theta;
5         Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),v(_v),t(_t){p1=p0+v*t; theta=
atan2(v.y,v.x);}

```

```

6      // Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),p1(_v){v=(p1-p0)/t; theta=
      atan2(v.y,v.x);}
7  };
8  bool operator < (const Line n,const Line m) {return n.theta<m.theta;}
9  Point GetIntersection(const Line n,const Line m){return n.p0+n.v*Cross(m.v,(n.p0-m.
  p0))/Cross(n.v,m.v);}
10 bool OnLine(const Vector a,const Line l){return Cross(l.p0-a,l.p1-a)==0;}
11 bool OnSegment(const Point a,const Line l){return sgn(Cross(l.p0-a,l.p1-a))==0 &&
  sgn(Dot(l.p0-a,l.p1-a))<0;}
12 double DisPL(const Point a,const Line l){return Abs(Cross(l.p1-l.p0,a-l.p0)/Norm(l.
  p1-l.p0));}
13 double DisPS(const Point a,const Line l){
14     if (l.p0==l.p1) return Norm(a-l.p0);
15     Vector v1=l.p1-l.p0,v2=a-l.p0,v3=a-l.p1;
16     if (sgn(Dot(v1,v2))<0) return Norm(v2);
17     if (sgn(Dot(v1,v3))>0) return Norm(v3);
18     return DisPL(a,l);
19 }
20 Point GetProjection(const Point a,const Line l){
21     Vector v=l.p1-l.p0;
22     return l.p0+v*(Dot(v,a-l.p0)/Dot(v,v));
23 }
24 bool SegmentIntersection(const Line n,const Line m,bool p){
25     double c1=Cross(n.p1-n.p0,m.p1-m.p0);
26     double c2=Cross(n.p1-n.p0,m.p1-n.p0);
27     double c3=Cross(m.p1-m.p0,n.p0-m.p0);
28     double c4=Cross(m.p1-m.p0,n.p1-m.p0);
29     if (p){
30         if (!sgn(c1) || !sgn(c2) || !sgn(c3) || !sgn(c4)){
31             return OnSegment(n.p0,m) || OnSegment(n.p1,m) || OnSegment(m.p0,n) ||
  OnSegment(m.p0,m);
32         }
33     }
34     }
35     return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);
36 }
37 }

```

#### 5.1.4 Get Area

```

1 namespace CG{
2     double GetArea(Point *p,int n){
3         double area=Cross(p[n],p[1]);
4         for (int i=2;i<=n;i++) area+=0.5*Cross(p[i-1],p[i]);
5         return Abs(area);
6     }
7 }

```

#### 5.1.5 Get Circumference

```

1 namespace CG{
2     double GetCircumference(Point *p,int n){
3         double Circumference=DisPP(p[n],p[1]);
4         for (int i=2;i<=n;i++) Circumference+=DisPP(p[i-1],p[i]);
5         return Circumference;
6     }
7 }

```



### 5.1.6 Anticlockwise Sort

```

1 namespace CG{
2     \\p为一个凸包,只是不知其点集是否为逆时针
3     void clockwise_sort(Point *p,int n){
4         for(int i=0;i<n-2;i++){
5             double tmp = mult_Cross(p[i+1],p[i+2],p[i]);
6             if(tmp>0) return;
7             else if(tmp<0){
8                 reverse(p,p+n);
9                 return;
10            }
11        }
12    }
13 }

```

## 5.2 Convex Hull

### 5.2.1 Get Convex Hull

```

1 namespace CG{
2     Point p[MAXN],s[MAXN]; // both based from 0
3     int ConvexHull(Point *p,int n,Point *s){
4         sort(p,p+n,cmp); //x从小到大,y从小到大;
5         int m=0;
6         for (int i=0;i<n;i++){
7             for (;m>=2 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
8             s[m++]=p[i];
9         }
10        int k=m;
11        for (int i=n-2;i;i--){
12            for (;m>=k+1 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
13            s[m++]=p[i];
14        }
15        return m-1;
16    }
17 }

```

### 5.2.2 Point in Convex Hull

```

1 namespace CG{
2     bool PointInConvexHull(Point A){
3         int l=1,r=tot-2,mid;
4         while(l<=r){
5             mid=(l+r)>>1;
6             double a1=Cross(p[mid]-p[0],A-p[0]);
7             double a2=Cross(p[mid+1]-p[0],A-p[0]);
8             if(a1>=0 && a2<=0){
9                 if(Cross(p[mid+1]-p[mid],A-p[mid])>=0) return true;
10                return false;
11            }
12            else if(a1<0) r=mid-1;
13            else l=mid+1;
14        }
15        return false;
16    }
17 }

```

### 5.3 Minkowski Sum

```

1 namespace CG{
2     void Minkowski(Point *C1,int n,Point *C2,int m){
3         for(int i=1;i<=n;i++) s1[i]=C1[i]-C1[i-1];
4         for(int i=1;i<=m;i++) s2[i]=C2[i]-C2[i-1];
5         A[tot=1]=C1[1]+C2[1];
6         int p1=1,p2=1;
7         while (p1<=n && p2<=m) ++tot,A[tot]=A[tot-1]+(s1[p1]*s2[p2]>=0?s1[p1++]:s2[p2
8 ++]);
9         while (p1<=n) ++tot,A[tot]=A[tot-1]+s1[p1++];
10        while (p2<=m) ++tot,A[tot]=A[tot-1]+s2[p2++];
11        tot=ConvexHull(A,tot);
12    }
13 }

```

### 5.4 Rotating Calipers

#### 5.4.1 The Diameter of Convex Hull

```

1 namespace CG{
2     double RotatingCalipers(Point *p,int n){
3         double dis=0;
4         for(int i=0,j=2;i<n;++i){
5             while (abs(Cross(p[i+1]-p[i],p[j]-p[i]))<abs(Cross(p[i+1]-p[i],p[j+1]-p[i])))
6             ) j=(j+1)%n;
7             dis=max(dis,max(DisPP(p[j],p[i]),DisPP(p[j],p[i+1])));
8         }
9         return dis;
10    }
11 }

```

#### 5.4.2 The Min Distance Between two Convex Hull

```

1 namespace CG{
2     ///点c到线段ab的最短距离
3     double GetDist(Point a,Point b,Point c){
4         if(dis(a,b)<esp) return dis(b,c); ///a,b是同一个点
5         if(mult_Dot(b,c,a)<-esp) return dis(a,c); ///投影
6         if(mult_Dot(a,c,b)<-esp) return dis(b,c);
7         return fabs(mult_Cross(b,c,a)/dis(a,b));
8     }
9 }
10 ///求一条线段ab的两端点到另外一条线段bc的距离，反过来一样，共4种情况
11 double MinDist(Point a,Point b,Point c,Point d){
12     return min(min(GetDist(a,b,c),GetDist(a,b,d)),min(GetDist(c,d,a),GetDist(c,d,b))
13 );
14 }
15 double RotatingCalipers(Point *p,int n,Point *q,int m){
16     int yminP = 0,ymaxQ=0;
17     for(int i=1;i<n;i++){ ///找到点集p组成的凸包的左下角
18         if(p[i].y<p[yminP].y||p[i].y==p[yminP].y&&(p[i].x<p[yminP].x)) yminP = i;
19     }
20     for(int i=1;i<m;i++){ ///找到点集q组成的凸包的右上角
21         if(q[i].y>q[ymaxQ].y||q[i].y==q[ymaxQ].y&&(q[i].x>q[ymaxQ].x)) ymaxQ = i;
22     }
23     double ans = DisPP(p[yminP],q[ymaxQ]); ///距离(yminP,ymaxQ)维护为当前最小值。
24 }

```

```

23     for(int i=0;i<n;i++){
24         double tmp;
25         while(tmp=(mult_Cross(q[ymaxQ+1],p[yminP],p[yminP+1])-mult_Cross(q[ymaxQ],p[
yminP],p[yminP+1]))>esp)
26             ymaxQ = (ymaxQ+1)%m;
27         if(tmp<=-esp) ans = min(ans,GetDist(p[yminP],p[yminP+1],q[ymaxQ]));
28         else ans=min(ans,MinDist(p[yminP],p[yminP+1],q[ymaxQ],q[ymaxQ+1]));
29         yminP = (yminP+1)%n;
30     }
31     return ans;
32 }
33 }

```

## 5.5 Half Plane Intersection

```

1 namespace CG{
2     void HalfPlaneIntersection(Line l[],int n){
3         deque<Point> p;
4         sort(l+1,l+1+n);
5         deque<Line> q;
6         q.push_back(l[1]);
7         for (int i=2;i<=n;i++){
8             for (;!p.empty() && !ToLeftTest(p.back()-l[i].p0,l[i].v);q.pop_back(),p.
pop_back());
9             for (;!p.empty() && !ToLeftTest(p.front()-l[i].p0,l[i].v);q.pop_front(),p.
pop_front());
10            if (sgn(Cross(l[i].v,q.back().v))==0)
11                if (ToLeftTest(l[i].p0-q.back().p0),q.back().v){
12                    q.pop_back();
13                    if (!p.empty()) p.pop_back();
14                }
15            if (!q.empty()) p.push_back(GetIntersection(q.back(),l[i]));
16            q.push_back(l[i]);
17        }
18        for (;!p.empty() && !ToLeftTest(p.back()-q.front().p0,q.front().v);q.pop_back(),
p.pop_back());
19        p.push_back(GetIntersection(q.back(),q.front()));
20        if (p.size() < 3) printf("0\n");
21        else{
22            cerr << "!" << endl;
23            double area = 0.5 * Cross(p.back(), p.front());
24            Point last = p.front();
25            for (p.pop_front(); !p.empty(); last = p.front(), p.pop_front())
26                area += 0.5 * Cross(last, p.front());
27            printf("%.8lf\n", fabs(area));
28        }
29    }
30 }

```

## 5.6 Min Circle Cover

```

1 namespace CG{
2     Point GetCircleCenter(const Point a,const Point b,const Point c){
3         Point p=(a+b)/2.0,q=(a+c)/2.0;
4         Vector v=Rotate(b-a,pi/2.0),w=Rotate(c-a,pi/2.0);
5         if (sgn(Norm(Cross(v,w)))==0){
6             if (sgn(Norm(a-b)+Norm(b-c)-Norm(a-c))==0) return (a+c)/2;
7             if (sgn(Norm(b-a)+Norm(a-c)-Norm(b-c))==0) return (b+c)/2;

```

```

8         if (sgn(Norm(a-c)+Norm(c-b)-Norm(a-b))==0) return (a+c)/2;
9     }
10    return GetIntersection(Line(p,v),Line(q,w));
11 }
12 void MinCircleCover(Point p[],int n){
13     random_shuffle(p+1,p+1+n);
14     Point c=p[1];
15     double r=0;
16     for (int i=2;i<=n;i++){
17         if (sgn(Norm(c-p[i])-r)>0){
18             c=p[i],r=0;
19             for (int j=1;j<i;j++){
20                 if (sgn(Norm(c-p[j])-r)>0){
21                     c=(p[i]+p[j])/2.0;
22                     r=Norm(c-p[i]);
23                     for (int k=1;k<j;k++){
24                         if (sgn(Norm(c-p[k])-r)>0){
25                             c=GetCircleCenter(p[i],p[j],p[k]);
26                             r=Norm(c-p[i]);
27                         }
28                     }
29                 }
30             }
31             printf("%.10f\n%.10f %.10f",r,c.x,c.y);
32 }

```

## 5.7 Circle Union Area

```

1 //k次覆盖
2 //圆并去重后s[0]
3 typedef pair<double, int> P;
4 const double pi = acos(-1.0);
5 const int MAXN = 10003;
6 P arc[MAXN << 1];
7 int acnt, cnt;
8 double s[1003];
9 bool del[1003];
10 void add(double st, double en) {
11     if(st < -pi) {
12         add(st + 2 * pi, pi);
13         add(-pi, en);
14         return;
15     }
16     if(en > pi) {
17         add(st, pi);
18         add(-pi, en - 2 * pi);
19         return;
20     }
21     arc[++acnt] = P(st, 1);
22     arc[++acnt] = P(en, -1);
23 }
24 double F(double x) {
25     return (x - sin(x)) / 2;
26 }
27 struct Node {
28     int x, y, r;
29     Node(int _x = 0, int _y = 0, int _r = 0):x(_x), y(_y), r(_r) {}
30     bool operator == (const Node& t) {
31         return x == t.x && y == t.y && r == t.r;

```

```

32     }
33     inline void read() {
34         scanf("%d%d%d", &x, &y, &r);
35     }
36 }a[1003];
37 int main() {
38     int n;
39     scanf("%d", &n);
40     for(int i = 1; i <= n; i++) a[i].read();
41     /*
42     //去重
43     int nn = 0;
44     for(int i = 1; i <= n; i++) {
45         bool same = 0;
46         for(int j = 1; j < i; j++) {
47             if(a[i] == a[j]) {
48                 same = 1; break;
49             }
50         }
51         if(!same) a[++nn] = a[i];
52     }
53     n = nn;
54     //去包含
55     for(int i = 1; i <= n; i++) {
56         for(int j = 1; j <= n; j++) if(i != j) {
57             if(hypot(a[i].x - a[j].x, a[i].y - a[j].y) < (double)(a[i].r - a[j].r)) del[
j] = 1;
58         }
59     }
60     nn = 0;
61     for(int i = 1; i <= n; i++) if(!del[i]) {
62         a[++nn] = a[i];
63     }
64     n = nn;
65     */
66     for(int i = 1; i <= n; i++) {
67         acnt = 0;
68         for(int j = 1; j <= n; j++) if(i != j) {
69             int dis = (a[i].x - a[j].x) * (a[i].x - a[j].x) + (a[i].y - a[j].y) * (a[i].
y - a[j].y);
70             if(a[j].r > a[i].r && dis <= (a[j].r - a[i].r) * (a[j].r - a[i].r)) add(-pi,
pi);
71             else if(dis > (a[i].r - a[j].r) * (a[i].r - a[j].r) && dis < (a[i].r + a[j].
r) * (a[i].r + a[j].r)){
72                 double c = sqrt(dis);
73                 double angle = acos((a[i].r * a[i].r + c * c - a[j].r * a[j].r) / (2 * a
[i].r * c));
74                 double k = atan2(a[j].y - a[i].y, a[j].x - a[i].x);
75                 add(k - angle, k + angle);
76             }
77         }
78         arc[++acnt] = P(pi, -1);
79         sort(arc + 1, arc + acnt + 1);
80         cnt = 0;
81         double last = -pi;
82         for(int j = 1; j <= acnt; j++) {
83             s[cnt] += F(arc[j].first - last) * a[i].r * a[i].r; //扇形 - 三角形
84             double xa = a[i].x + a[i].r * cos(last);
85             double ya = a[i].y + a[i].r * sin(last);
86             last = arc[j].first;
87             double xb = a[i].x + a[i].r * cos(last);

```

```

88         double yb = a[i].y + a[i].r * sin(last);
89         s[cnt] += (xa * yb - xb * ya) / 2; //到圆心的三角形面积
90         cnt += arc[j].second;
91     }
92 }
93 //printf("%.3f\n", s[0]);
94 for (int i = 0; i < n; i++) {
95     printf("[%d] = %.3f\n", i + 1, s[i] - s[i + 1]);
96 }
97 return 0;
98 }

```

## 5.8 Simpson Integrate

```

1 double Simpson(double l, double r){
2     return (r-l)*(F(l)+4*F((l+r)/2)+F(r))/6;
3 }
4 double Integrate(double l, double r, double S){
5     double mid=(l+r)/2;
6     double A=Simpson(l, mid);
7     double B=Simpson(mid, r);
8     if(A+B-S<eps) return S;
9     return Integrate(l, mid, A)+Integrate(mid, r, B);
10 }

```

## 5.9 Closest Point

```

1 vector <Point> P;
2 DB CP(int l, int r) {
3     if (l == r) return DB_INF;
4     if (l + 1 == r) return DisPP(P[l], P[r]);
5     int mid = (l + r) >> 1;
6     DB d = min(CP(l, mid), CP(mid + 1, r));
7     vector <Point> tmp;
8     for (int i = l; i <= r; i++)
9         if (fabs(P[mid].x - P[i].x) < d) tmp.push_back(P[i]);
10    sort(tmp.begin(), tmp.end(), cmp);
11    for (int i = 0; i < tmp.size(); i++)
12        for (int j = i + 1; j < tmp.size() && tmp[j].y - tmp[i].y < d; j++)
13            d = min(d, DisPP(tmp[i], tmp[j]));
14    return d;
15 }

```

## 5.10 K-D Tree

```

1 #include <iostream>
2 #include <algorithm>
3 #include <stack>
4 #include <math.h>
5 using namespace std;
6 /*function of this program: build a 2d tree using the input trainingdata
7  the input is exm_set which contains a list of tuples (x,y)
8  the output is a 2d tree pointer*/
9
10
11 struct data

```

```

12 {
13     double x = 0;
14     double y = 0;
15 };
16
17 struct Tnode
18 {
19     struct data dom_elt;
20     int split;
21     struct Tnode * left;
22     struct Tnode * right;
23 };
24
25 bool cmp1(data a, data b){
26     return a.x < b.x;
27 }
28
29 bool cmp2(data a, data b){
30     return a.y < b.y;
31 }
32
33 bool equal(data a, data b){
34     if (a.x == b.x && a.y == b.y)
35     {
36         return true;
37     }
38     else{
39         return false;
40     }
41 }
42
43 void ChooseSplit(data exm_set[], int size, int &split, data &SplitChoice{
44     /*compute the variance on every dimension. Set split as the dimension that have the
45     biggest
46     variance. Then choose the instance which is the median on this split dimension.*/
47     /*compute variance on the x,y dimension.  $DX = EX^2 - (EX)^2$ */
48     double tmp1,tmp2;
49     tmp1 = tmp2 = 0;
50     for (int i = 0; i < size; ++i)
51     {
52         tmp1 += 1.0 / (double)size * exm_set[i].x * exm_set[i].x;
53         tmp2 += 1.0 / (double)size * exm_set[i].x;
54     }
55     double v1 = tmp1 - tmp2 * tmp2; //compute variance on the xdimension
56
57     tmp1 = tmp2 = 0;
58     for (int i = 0; i < size; ++i)
59     {
60         tmp1 += 1.0 / (double)size * exm_set[i].y * exm_set[i].y;
61         tmp2 += 1.0 / (double)size * exm_set[i].y;
62     }
63     double v2 = tmp1 - tmp2 * tmp2; //compute variance on the ydimension
64
65     split = v1 > v2 ? 0:1; //set the split dimension
66
67     if (split == 0)
68     {
69         sort(exm_set,exm_set + size, cmp1);
70     }
71     else{
72         sort(exm_set,exm_set + size, cmp2);

```

```

72     }
73
74     //set the split point value
75     SplitChoice.x = exm_set[size / 2].x;
76     SplitChoice.y = exm_set[size / 2].y;
77
78 }
79
80 Tnode* build_kdtree(data exm_set[], int size, Tnode* T){
81     //call function ChooseSplit to choose the split dimension and splitpoint
82     if (size == 0){
83         return NULL;
84     }
85     else{
86         int split;
87         data dom_elt;
88         ChooseSplit(exm_set, size, split, dom_elt);
89         data exm_set_right [100];
90         data exm_set_left [100];
91         int sizeleft ,sizeright;
92         sizeleft = sizeright = 0;
93
94         if (split == 0)
95         {
96             for (int i = 0; i < size; ++i)
97             {
98
99                 if (!equal(exm_set[i],dom_elt) && exm_set[i].x <=dom_elt.x)
100                 {
101                     exm_set_left[sizeleft].x = exm_set[i].x;
102                     exm_set_left[sizeleft].y = exm_set[i].y;
103                     sizeleft++;
104                 }
105                 else if (!equal(exm_set[i],dom_elt) && exm_set[i].x >dom_elt.x)
106                 {
107                     exm_set_right[sizeright].x = exm_set[i].x;
108                     exm_set_right[sizeright].y = exm_set[i].y;
109                     sizeright++;
110                 }
111             }
112         }
113         else{
114             for (int i = 0; i < size; ++i)
115             {
116
117                 if (!equal(exm_set[i],dom_elt) && exm_set[i].y <=dom_elt.y)
118                 {
119                     exm_set_left[sizeleft].x = exm_set[i].x;
120                     exm_set_left[sizeleft].y = exm_set[i].y;
121                     sizeleft++;
122                 }
123                 else if (!equal(exm_set[i],dom_elt) && exm_set[i].y >dom_elt.y)
124                 {
125                     exm_set_right[sizeright].x = exm_set[i].x;
126                     exm_set_right[sizeright].y = exm_set[i].y;
127                     sizeright++;
128                 }
129             }
130         }
131         T = new Tnode;
132         T->dom_elt.x = dom_elt.x;

```



```

133     T->dom_elt.y = dom_elt.y;
134     T->split = split;
135     T->left = build_kdtree(exm_set_left, sizeleft, T->left);
136     T->right = build_kdtree(exm_set_right, sizeright, T->right);
137     return T;
138
139 }
140 }
141
142
143 double Distance(data a, data b){
144     double tmp = (a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y);
145     return sqrt(tmp);
146 }
147
148
149 void searchNearest(Tnode * Kd, data target, data &nearestpoint, double &distance){
150
151     //1. 如果Kd是空的, 则设 dist为无穷大返回
152
153     //2. 向下搜索直到叶子结点
154
155     stack<Tnode*> search_path;
156     Tnode* pSearch = Kd;
157     data nearest;
158     double dist;
159
160     while(pSearch != NULL)
161     {
162         //pSearch加入到 search_path 中;
163         search_path.push(pSearch);
164
165         if (pSearch->split == 0)
166         {
167             if(target.x <= pSearch->dom_elt.x) /* 如果小于就进入左子树 */
168             {
169                 pSearch = pSearch->left;
170             }
171             else
172             {
173                 pSearch = pSearch->right;
174             }
175         }
176         else{
177             if(target.y <= pSearch->dom_elt.y) /* 如果小于就进入左子树 */
178             {
179                 pSearch = pSearch->left;
180             }
181             else
182             {
183                 pSearch = pSearch->right;
184             }
185         }
186     }
187     //取出 search_path 最后一个赋给 nearest
188     nearest.x = search_path.top()->dom_elt.x;
189     nearest.y = search_path.top()->dom_elt.y;
190     search_path.pop();
191
192
193     dist = Distance(nearest, target);

```

```

194 //3. 回溯搜索路径
195
196 Tnode* pBack;
197
198 while(search_path.size() != 0)
199 {
200     //取出 search_path 最后一个结点赋给 pBack
201     pBack = search_path.top();
202     search_path.pop();
203
204     if(pBack->left == NULL && pBack->right == NULL) /* 如果 pBack 为子结点 */
205     {
206
207         if( Distance(nearest, target) > Distance(pBack->dom_elt, target) )
208         {
209             nearest = pBack->dom_elt;
210             dist = Distance(pBack->dom_elt, target);
211         }
212     }
213
214     else
215     {
216
217         int s = pBack->split;
218         if (s == 0)
219         {
220             if( fabs(pBack->dom_elt.x - target.x) < dist) /* 如果 target 为中心的圆
221             (球或超球), 半径为 dist 的圆与分割超平面交, 那么就要跳到另一边的子空间去搜索 */
222             {
223                 if( Distance(nearest, target) > Distanc(pBack->dom_elt, target) )
224                 {
225                     nearest = pBack->dom_elt;
226                     dist = Distance(pBack->dom_elt, target);
227                 }
228                 if(target.x <= pBack->dom_elt.x) /* 如果 target 位于 pBack 的左子空间, 那
229                 么就要跳到右子空间去搜索 */
230                 pSearch = pBack->right;
231                 else
232                 pSearch = pBack->left; /* 如果 target 位于 pBack 的子空间, 那么就要
233                 跳到左子空间去搜索 */
234                 if(pSearch != NULL)
235                     //pSearch 加入到 search_path 中
236                     search_path.push(pSearch);
237             }
238         }
239         else {
240             if( fabs(pBack->dom_elt.y - target.y) < dist) /* 如果 target 为中心的圆
241             (球或超球), 半径为 dist 的圆与分割超平面交, 那么就要跳到另一边的子空间去搜索 */
242             {
243                 if( Distance(nearest, target) > Distanc(pBack->dom_elt, target) )
244                 {
245                     nearest = pBack->dom_elt;
246                     dist = Distance(pBack->dom_elt, target);
247                 }
248                 if(target.y <= pBack->dom_elt.y) /* 如果 target 位于 pBack 的左子空间, 那
249                 么就要跳到右子空间去搜索 */
250                 pSearch = pBack->right;
251                 else

```

```

250         pSearch = pBack->left; /* 如果 target 位于 pBack 的子空间, 那么就要
跳到左子空间去搜索 */
251         if(pSearch != NULL)
252             // pSearch 加入到 search_path 中
253             search_path.push(pSearch);
254     }
255 }
256
257 }
258 }
259
260 nearestpoint.x = nearest.x;
261 nearestpoint.y = nearest.y;
262 distance = dist;
263
264 }
265
266 int main(){
267     data exm_set[100]; //assume the max training set size is 100
268     double x,y;
269     int id = 0;
270     cout<<"Please input the training data in the form x y. One instance per line. Enter
-1 -1 to stop."<<endl;
271     while (cin>>x>>y){
272         if (x == -1)
273         {
274             break;
275         }
276         else{
277             exm_set[id].x = x;
278             exm_set[id].y = y;
279             id++;
280         }
281     }
282     struct Tnode * root = NULL;
283     root = build_kdtree(exm_set, id, root);
284
285     data nearestpoint;
286     double distance;
287     data target;
288     cout <<"Enter search point"<<endl;
289     while (cin>>target.x>>target.y)
290     {
291         searchNearest(root, target, nearestpoint, distance);
292         cout<<"The nearest distance is "<<distance<<","and the nearestpoint is "<<
nearestpoint.x<<","<<nearestpoint.y<<endl;
293         cout <<"Enter search point"<<endl;
294     }
295 }
296 }

```

## 6 Conclusion

### 6.1 Game Theory

#### 6.1.1 Bash's Game

Bash's Game 巴什博弈

有一堆个数为  $n$  的石子，游戏双方依次从中拿取，满足：

1. 每次至少取 1 个，最多取  $m$  个。

最后取光者得胜。

结论:  $n = t(m+1) + r$ , 必败态:  $r = 0$ ;

巴什博弈变种:

取一个指定集合的石头个数

取到最后一个石子输,  $n = t(m + 1) + r$ ,  $r = 1$ ;

#### 6.1.2 Wythoff's Game

Wythoff's Game (威佐夫博弈)

有两堆分别为  $(a_n, b_n)$  的石子，游戏双方依次从中拿取，满足：

1. 从任意一堆中取任意个  $> 1$ 。
2. 从两堆中取同样多个。最后取完者胜。

结论: 对于任意的局势  $(a, b)$  ( $a < b$ ), 必败点为  $(b-a) * (\sqrt{5}+1)/2 = a$ 。

#### 6.1.3 Fibonacci's Game / Zeckendorf's theory

Fibonacci's Game (斐波那契博弈)

有一堆个数为  $n$  的石子，游戏双方轮流取石子，满足：

1. 先手不能在第一次把所有的石子取完；
2. 之后每次可以取的石子数介于 1 到对手刚取的石子数的 2 倍之间 (包含 1 和对手刚取的石子数的 2 倍)。

结论: 必败点是斐波那契数

齐肯多夫定理: 任何正整数可以表示为若干个不连续的 Fibonacci 数之和

#### 6.1.4 Nim's Game / Anti-Nim's Game / K-Nim's Game / Anti-K-Nim's Game

Nim's Game (尼姆博弈)

石子的个数可以等价成某个游戏的 SG 函数。

有  $n$  堆石子，游戏双方依次从中拿取，满足：

1. 规定每次只能从一堆中取若干根，可将一堆全取走，但不可不取。

最后取完者为胜。

结论:

T 态: 所有火柴数异或和为 0

S 态: 所有火柴数异或和不为 0

必胜态:S

有  $n$  堆石子, 游戏双方依次从中拿取, 满足:

1. 规定每次只能从一堆中取若干根, 可将一堆全取走, 但不可不取.

最后取完者为败。

结论:

S0 态: 即仅有奇数个孤单堆

T0 态: 即仅有偶数个孤单堆

S1 态: 异或和大于 0, 且有 1 个充裕堆

T1 态: 不存在

S2 态: 异或和大于 0, 且有多个充裕堆

T2 态: 异或和等于 0, 且有多个充裕堆

必胜态:T0,S1,S2

必败态:S0,T2

有  $n$  堆石子, 游戏双方依次从中拿取, 满足:

1. 规定每次只能至多  $k$  堆中取若干根, 可将  $k$  堆全取走, 但不可不取.

最后取完者为胜。

结论:

对于每一堆, 把它石子的个数用二进制表示

必败态: 对所有的石子堆, 如果在任何一个二进制位上 1 的个数总是  $k+1$  的整数倍

有  $n$  堆石子, 游戏双方依次从中拿取, 满足:

1. 规定每次只能至多  $k$  堆中取若干根, 可将  $k$  堆全取走, 但不可不取

最后取完者为败。

结论:

1. 对于每一堆, 把它石子的个数用二进制表示

2. 所有的堆 (非零堆, 下同) 全是 1, 此时如果 1 堆个数模  $k+1$  的结果是 1 则必败, 否则必胜 (我们可以通过拿走 0 到  $k$  个堆来随意调整当前状态模的结果, 然后再将所有大于 1 的堆降到 1 就行了)

3. 有多于  $k$  个堆的个数大于 1。必胜

### 6.1.5 阶梯博弈

有  $n$  个阶梯呈升序排列, 每个阶梯上有若干个石子, 游戏双方轮流取石子, 满足:

1. 将一个阶梯上的石子移任意个 ( $>0$ ) 到前一个台阶。

当没有可行操作时 (所有石子都被移动到了地面, 即第 0 号台阶) 输。

结论:

奇数号台阶的 Nim 游戏

变种 1: 树上, 每个石子只能往父亲节点移动.

变种 2:

游戏双方在一个  $1*N$  的格子内挪动棋子, 刚开始在若干个位置上有棋子, 每个位置至多一个棋子

每一个选手可以进行的操作时选择一个棋子并把它向左方移动, 当然不能越过其它的棋子, 也不能超出边界。

谁不能移动谁就输了。求谁会赢?

结论:

将棋子位置按升序排列, 然后从后往前两两绑成一对, 如果个数是奇数, 那么将第一个和边界外绑定.

一对棋子的前一个和前一对棋子的后一个之间有多少个空位置对最终的结果是没有影响的。

于是我们只需要考虑同一对的两个棋子之间有多少空位, 将同一对棋子间的空位视为石子, 做 nim 游戏

两对棋子间的空格数当奇数位石子, 其他当偶数位石子, 石子相右边移动

变种 3:

山上有  $n$  个人, 每个人给出距离山顶的距离, 给出其中一个人为 king, 每次能挑选一个人向上移动, 不能越过其他人, 最后将 king 移动到山顶者获胜。问获胜者。

结论:

只要把 King 当作普通人一样处理即可。除了两种特殊情况:

1. 当 King 是第一个人时, Alice 直接胜
2. 当 King 是第二个人且一共有奇数个人时, 第一堆的大小需要减 1。

### 6.1.6 Multi-Nim

有  $n$  堆石子, 游戏双方依次从中拿取, 满足:

1. 任意一堆石子中拿任意多个石子 (不能不拿)
2. 把一堆数量不少于 2 石子分为两堆不为空的石子

最后取完者为胜。

结论:

操作一与普通的 Nim 游戏等价

操作二实际上是将一个游戏分解为两个游戏, 根据 SG 定理, 我们可以通过异或运算把两个游戏连接到一起, 作为一个后继状态

$$SG(x) \equiv \begin{cases} x-1 & (x \bmod 4 = 0) \\ x & (x \bmod 4 = 1 \text{ or } 2) \\ x+1 & (x \bmod 4 = 3) \end{cases} \quad (1)$$

Multi-SG 游戏规定, 在符合拓扑原则的前提下, 一个单一游戏的后继可以为多个单一游戏。

Multi-SG 其他规则与 SG 游戏相同。

注意在这里要分清楚后继与多个单一游戏

对于一个状态来说, 不同的划分方法会产生多个不同的后继, 而在一个后继中可能含有多个独立的游戏

一个后继状态的 SG 值即为后继状态中独立游戏的异或和

该状态的 SG 值即为后继状态的 SG 值中未出现过的最小值

### 6.1.7 Every-SG

给定一张无向图, 上面有一些棋子, 两个顶尖聪明的人在做游戏, 每人每次必须将可以移动的棋子进行移动, 不能移动的人

因为两个人都顶尖聪明, 因此当一个人知道某一个游戏一定会输的话, 它一定会尽力缩短游戏的时间, 当它知道某一个游戏一定会赢的话, 一定会尽力延长游戏的时间

对于还没有结束的单一游戏, 游戏者必须对该游戏进行一步决策;

其他规则与普通 SG 游戏相同

Every-SG 游戏与普通 SG 游戏最大的不同就是它多了一维时间

对于 SG 值为 0 的点, 我们需要知道最少需要多少步才能走到结束

对于 SG 值不为 0 的点, 我们需要知道最多需要多少步结束

这样我们用 step 变量来记录这个步数

$$step(x) \equiv \begin{cases} 0 & u \\ maxstep(v) & sg(u) \neq 0 \vee sg(v) = 0 \\ minstep(v) & sg(u) = 0 \vee u \end{cases} \quad (2)$$

### 6.1.8 树的删边游戏

给出一个有 N 个点的树, 有一个点作为树的根节点。游戏者轮流从树中删去边, 删去一条边后, 不与根节点相连的部分将被移走。谁无法移动谁输。

结论:

Colon Principle: 对于树上的某一个点, ta 的分支可以转化成以这个点为根的一根竹子, 这个竹子的长度就是 ta 各个分支的边的数量的异或和

叶子节点的 SG 值为 0; 中间节点的 SG 值为它的所有子节点的 SG 值加 1 后的异或和。

### 6.1.9 Chomp's theory?

取一个无关紧要的位置, 如果对方必胜, 则学习其策略, 我方必胜。

### 6.1.10 Other's theory?

有 n 堆石子, 游戏双方依次从中拿取, 满足:

1. 规定每次能从任意多堆中取 1 根, 不可不取。

最后取完者为胜。

结论: 如果全是偶数, 先手必败, 否者先手必胜

一个无相联通图, 有一个点作为图的根。

游戏者轮流从图中删去边, 删去一条边后, 不与根节点相连的部分将被移走。

谁无路可走谁输。

结论:

Fusion Principle: 环上的点可以融合, 且不改变图的 SG 值, 我们可以把一个带有奇数边的环等价成只有一个端点的一条边而偶数边的环等价于一个点

### 6.1.11 SG Theory

```

1 memset(mex, 0, sizeof mex);
2 for (int i = 1; i < maxN; ++i) {
3     for (int j = 1; j <= n; ++j) {
4         if (a[j] <= i)
5             mex[SG[i - a[j]]] = i;
6         for (int k = 1; i - k - a[j] > 0; ++k)
7             mex[SG[k] ^ SG[i - k - a[j]]] = 1;
8     }

```

```

9   for (int j = 0;; ++j)
10       if (mex[j] != i){
11           SG[i] = j;
12           break;
13       }
14 }

```

### 6.1.12 SJ Theory

反公平游戏 Anti-SG Game

DAG 上没有出度的点为胜利状态，其它定义与一般游戏相同。

现在的问题是解决多个反公平游戏的合并。

SJ 定理说明：先手必胜，当且仅当以下两个条件同时成立或同时不成立：

1. 合并的 SG 值为 0;
2. 所有游戏的 SG 值不超过 1。

### 6.1.13 Surreal Number Theory

## 6.2 Math

### 6.2.1 Euler's Theorem

$$a^b \equiv \begin{cases} a^{b \bmod \varphi(p)} & \gcd(a, p) = 1 \\ a^b & \gcd(a, p) \neq 1, b < \varphi(p) \\ a^{b \bmod \varphi(p) + \varphi(p)} & \gcd(a, p) \neq 1, b \geq \varphi(p) \end{cases} \pmod{p}$$

### 6.2.2 Möbius Inversion

Dirichlet Convolution is  $(f \times g)(N) = \sum_{d|N} f(d) * g(\frac{N}{d})$

Theorem:

$$\begin{cases} f = g \times 1 \\ g = f \times \mu \end{cases}$$

$$\begin{cases} id(n) = \sum_{d|n} \varphi(d) \\ e(n) = \sum_{d|n} \mu(d) \end{cases} \quad (3)$$

$$\begin{cases} \sum_i^n \sum_j^m gcd(i, j) = \sum_d^{\max(n, m)} \varphi(d) * \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \\ \sum_i^n \sum_j^m e(gcd(i, j)) = \sum_d^{\min(n, m)} \mu(d) * \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \\ \sum_{i=1}^n |\mu(i)| = \sum_{i=1}^{\lfloor \sqrt{n} \rfloor} \mu(i) * \lfloor \frac{n}{i * i} \rfloor \end{cases} \quad (4)$$



$$\left\{ \begin{array}{l} sum(x, y) = \sum_i^x \sum_j^y i * j = \frac{x * (x + 1)}{2} * \frac{y * (y + 1)}{2} \\ F(x, y) = \sum_{i=1}^{\min(x, y)} i^2 * \mu(i) * sum(\lfloor \frac{x}{i} \rfloor, \lfloor \frac{y}{i} \rfloor) \\ \sum_i^n \sum_j^m lcm(i, j) = \sum_{i=1}^{\min(n, m)} d * F(\lfloor \frac{n}{i} \rfloor, \lfloor \frac{m}{i} \rfloor) \end{array} \right. \quad (5)$$

### 6.2.3 Sieve Tips

$$\varphi(nm) = \varphi(n) \cdot \varphi(m) \cdot \frac{gcd(n, m)}{\varphi(gcd(n, m))} \quad (6)$$

$$\varphi(n) = \sum_{i=1}^n [(n, i) = 1] \cdot i = \frac{n * \varphi(n) + [n = 1]}{2} \quad (7)$$

$$\left\{ \begin{array}{l} id = \varphi \times 1 \\ \frac{n \cdot (n + 1)}{2} = \sum_{i=1}^n i = \sum_{i=1}^n \sum_{d|i} \varphi(d) = \sum_{\frac{i}{d}=1}^n \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} \varphi(d) = \sum_{i=1}^n \phi(\lfloor \frac{n}{i} \rfloor) \end{array} \right. \quad (8)$$

$$\left\{ \begin{array}{l} e = \mu \times 1 \\ 1 = \sum_{i=1}^n [i = 1] = \sum_{i=1}^n \sum_{d|i} \mu(d) = \sum_{i=1}^n \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} \mu(d) = \sum_{i=1}^n M(\lfloor \frac{n}{i} \rfloor) \end{array} \right. \quad (9)$$

$$\left\{ \begin{array}{l} id^2 = (id \cdot \varphi) \times id \\ \phi'(n) = \sum_{i=1}^n i \cdot \varphi(i) \\ \frac{n \cdot (n + 1) \cdot (2n + 1)}{6} = \sum_{i=1}^n i^2 = \sum_{i=1}^n \sum_{d|i} d \cdot \varphi(d) \cdot \frac{i}{d} = \sum_{\frac{i}{d}=1}^n \frac{i}{d} \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} d \cdot \varphi(d) = \sum_{i=1}^n i \cdot \phi'(\lfloor \frac{n}{i} \rfloor) \end{array} \right. \quad (10)$$

### 6.2.4 Newton's method

Iff

$$f'(x) \neq 0 \ \&\& \ f''(x) \text{continuous}$$

Then

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

### 6.2.5 Cantor Expansion

康托展开是一个全排列到一个自然数的双射，常用于构建哈希表时的空间压缩。康托展开的实质是计算当前排列在所有由小到大全排列中的顺序，因此是可逆的。以下称第  $x$  个全排列是都是指由小到大的顺序。

$$X = a_n(n - 1)! + a_{n-1}(n - 2)! + \cdots + a_1 \cdot 0!$$

其中  $a_i$  为排列的第  $i$  个元素值

例如, 3 5 7 4 1 2 9 6 8 展开为 98884。因为  $X=2*8!+3*7!+4*6!+2*5!+0*4!+0*3!+2*2!+0*1!+0*0!=98884$ 。

解释:

排列的第一位是 3, 比 3 小的数有两个, 以这样的数开始的排列有  $8!$  个, 因此第一项为  $2*8!$

排列的第二位是 5, 比 5 小的数有 1、2、3、4, 由于 3 已经出现, 因此共有 3 个比 5 小的数, 这样的排列有  $7!$  个, 因此第二项为  $3*7!$

以此类推, 直至  $0*0!$

用途:

显然,  $n$  位 ( $0$  到  $n-1$ ) 全排列后, 其康托展开唯一且最大约为  $n!$ , 因此可以由更小的空间来储存这些排列。由公式可将  $X$  逆推出唯一的一个排列。

逆运算:

如  $n=5, x=96$  时:

首先用  $96-1$  得到 95, 说明  $x$  之前有 95 个排列。(将此数本身减去 1)

用 95 去除  $4!$  得到 3 余 23, 说明有 3 个数比第 1 位小, 所以第一位是 4。

用 23 去除  $3!$  得到 3 余 5, 说明有 3 个数比第 2 位小, 所以是 4, 但是 4 已出现过, 因此是 5。

用 5 去除  $2!$  得到 2 余 1, 类似地, 这一位是 3。

用 1 去除  $1!$  得到 1 余 0, 这一位是 2。

最后一位只能是 1。

所以这个数是 45321。

按以上方法可以得出通用的算法。

### 6.2.6 $\sum_{i=1}^n i^k$

$$\begin{aligned}\sum_i^n i &= \frac{n(n+1)}{2} \\ \sum_i^n i^2 &= \frac{n(n+1)(2n+1)}{6} \\ \sum_i^n i^3 &= \left[\frac{n(n+1)}{2}\right]^2\end{aligned}$$

### 差分和组合数: $O(k^2)$

考虑数列  $\{1_k, 2_k, \dots, i_k\}$

相邻两项做差之后, 得到的数列的每项应该是一个  $k-1$  次关于  $i$  的多项式。

再次相邻两项做差之后, 得到的数列的每项应该是一个  $k-2$  次关于  $i$  的多项式。

如此进行  $k$  次, 得到的数列的每项应该是一个 0 次关于  $i$  的多项式, 即常数数列。

再次相邻两项做差之后, 一定会得到一个全是 0 的数列。

假设经过  $i$  次差分数列之后的数列第一项为  $r_i$  那么答案就是

$$\sum_{i=0}^k r_i \binom{n}{i+1}$$

举例来说, 考虑数列

1, 8, 27, 64, 125, 216, ...

求差分可得

7,19,37,61,91,...

12,18,24,30,...

6,6,6,...

0,0,...

每个数列的第一项为 1, 7, 12, 6。所以最终的答案即为

$$\binom{n}{1} + 7\binom{n}{2} + 12\binom{n}{3} + 6\binom{n}{4} = \frac{n^2(n+1)^2}{4}$$

拉格朗日插值: $O(k)$

### 6.2.7 Generating Function

## 生成函数

### 普通型

$$A(x) = \sum_{i=0}^{\infty} a_i x^i$$

已知  $\{a_i\}$ ,  $\{b_i\}$  的生成函数分别是  $A(x)$ ,  $B(x)$ 。

那么  $\{a_i \pm b_i\}$  的生成函数是  $A(x) \pm B(x)$ 。

值得注意的是数列  $\{a_i\}$ ,  $\{b_i\}$  的卷积的生成函数，恰好是  $C(x) = A(x)B(x)$ 。

$$c_i = \sum_j a_j b_{i-j}$$

### 关键公式

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_i x^i$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + \cdots = \sum_i x^{2i}$$

推广的二项式定理

$$(1+x)^n = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \cdots = \sum_i \binom{n}{i}x^i$$

其中有  $\beta$  函数和  $\Gamma$  函数

$$\Gamma(s) = \int_0^{+\infty} x^{s-1} e^{-x} dx$$

$$B(p, q) = \binom{p+q}{p} = \int_0^1 x^{p-1} (1-x)^{q-1} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

分别有递推公式

$$\begin{aligned}\Gamma(s+1) &= s\Gamma(s) \\ B(p, q) &= B(q, p) \\ &= \frac{q-1}{p+q-1}B(p, q-1) \\ &= \frac{p-1}{p+q-1}B(p-1, q) \\ &= \frac{(p-1)(q-1)}{(p+q-1)(p+q-2)}B(p-1, q-1)\end{aligned}$$

例子

### Fibonacci 生成函数

构造一个与  $f_n$  有关得幂级数

$$F(x) = f_0 + f_1x + f_2x^2 + \cdots + f_nx^n + \cdots$$

亦即

$$F(x) = xF(x) + x^2F(x) + f_0 + (f_1 - f_0)x$$

带入  $f_0 = 0, f_1 = 1$

$$F(x) = \frac{x}{1-x-x^2}$$

如果想继续计算通项，我们需要解方程得到

$$\frac{x}{1-x-x^2} = \frac{a}{1-\alpha x} + \frac{b}{1-\beta x}$$

解出

$$\begin{cases} \alpha = \frac{1+\sqrt{5}}{2} \\ \beta = \frac{1-\sqrt{5}}{2} \end{cases}$$

带入方程有

$$a = \frac{1}{\sqrt{5}}, b = -\frac{1}{\sqrt{5}}$$

则

$$\begin{aligned}F(x) &= \frac{a}{1-\alpha x} + \frac{b}{1-\beta x} \\ &= a(1 + \alpha x + \alpha^2 x^2 + \cdots) + b(1 + \beta x + \beta^2 x^2 + \cdots)\end{aligned}$$

蕴含通项为

$$f_n = a\alpha^n + b\beta^n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

Catalan 数

$$c_n = \sum_{i=1}^{n-1} c_i c_{n-i}$$

生成函数

$$C(x) = c_1x + c_2x^2 + \cdots$$

亦即

$$C(x) = xC(x)^2 + 1$$

解得

$$C(x) = \frac{1 - \sqrt{1 - 4x}}{2x}$$

利用推广的二项式定理展开  $\sqrt{1 - 4x}$  可得通项

$$c_n = \frac{1}{n} \binom{2n-2}{n-1}$$

## 指数型生成函数

$$A(x) = \sum_{i=0}^{\infty} \frac{a_i x^i}{i!}$$

已知  $\{a_i\}$ ,  $\{b_i\}$  的生成函数分别是  $A(x)$ ,  $B(x)$ 。

那么  $\{a_i \pm b_i\}$  的生成函数是  $A(x) \pm B(x)$ 。

值得注意的是数列  $\{a_i\}$ ,  $\{b_i\}$  的卷积的生成函数，恰好是  $C(x) = A(x)B(x)$ 。

$$c_i = \sum_j \binom{i}{j} a_j b_{i-j}$$

## 关键公式

$$\begin{aligned} e^x &= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots = \sum_i \frac{x^i}{i!} \\ e^{-x} &= 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots = \sum_i \frac{(-x)^i}{i!} \\ \frac{e^x + e^{-x}}{2} &= 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots = \sum_i \frac{x^{2i}}{(2i)!} \end{aligned}$$

### 6.2.8 Polya

设  $G$  是  $p$  个对象的一个置换群，用  $k$  种颜色去染这  $p$  个对象，若一种染色方案在群  $G$  的作用下变为另一种方案，则这两个方案当作是同一种方案，这样的不同染色方案数为：

$$\frac{1}{|G|} \times \sum (k^{C(f)}), f \in G$$

$C(f)$  为循环节， $|G|$  表示群的置换方法数

对于有  $n$  个位置的手镯，有  $n$  种旋转置换和  $n$  种翻转置换

对于旋转置换：

$$C(f_i) = \gcd(n, i)$$

i 表示一次转过 i 颗宝石, i = 0 时 c = n;

对于翻转置换:

如果 n 为偶数: 则有  $\frac{n}{2}$  个置换  $C(f) = \frac{n}{2}$ , 有  $\frac{n}{2}$  个置换  $C(f) = \frac{n}{2} + 1$

如果 n 为奇数:  $C(f) = \frac{n}{2} + 1$

### 6.2.9 FWT

$$\left\{ \begin{array}{l} C_k = \sum_{i \oplus j = k} A_i * B_j \\ DWT(A)_i = \sum_j^n A_j * f_{i,j} \\ DWT(C)_i = DWT(A)_i * DWT(B)_i \\ f_{i,j} \cdot f_{i,k} = f_{i,j \oplus k} \\ f_{i,j} = [i \text{ and } j == i] \quad (and) \\ f_{i,j} = [i \text{ and } j == j] \quad (or) \\ f_{i,j} = (-1)^{[i \text{ and } j]} \quad (xor) \end{array} \right.$$

## 6.3 Graph Theory

### 6.3.1 Matrix-Tree Theorem

基尔霍夫矩阵树定理

构造一个矩阵 A。

如果 i = j, 那么 A<sub>ij</sub> 为点 i(j) 的度数。

如果 i ≠ j, 那么 A<sub>ij</sub> 为 i 到 j 的边数的相反数。最终得到的矩阵, 删掉任意一行, 任意一列之后的矩阵行列式求值, 可得到原图中生成树的个数。

## 6.4 Geometry

### 6.4.1 The Number of Integer Point on a Circle

Set  $r = const$  is the radius of the circle.

$$r^2 = p_1^{a_1} + p_2^{a_2} + \cdots + p_m^{a_m} = \sum_{i=1}^m p_i^{a_i}$$

Define

$$\chi(n) = \begin{cases} 1 & n \% 4 = 1 \\ -1 & n \% 4 = 3 \\ 0 & n \% 2 = 0 \end{cases}$$

By the way,  $\chi(n)$  is a multiplicative function.

Define

$$\Gamma(p_i, a_i) = \sum_{j=0}^{a_i} \chi(p_i^j) = \begin{cases} 1 & p_i = 2 \quad || \quad (p_i \% 4 = 3 \quad \&\& \quad a_i \% 2 = 0) \\ 0 & p_i \% 4 = 3 \quad \&\& \quad a_i \% 2 = 1 \\ a_i + 1 & p_i \% 4 = 1 \end{cases}$$

Define cnt is the number of integer point on circle

$$cnt(r) = 4 \prod_{i=1}^m \sum_{j=0}^{a_i} \chi(p_i^j) = 4 \prod_{i=1}^m \Gamma(p_i, a_i) = 4 \sum_{k|r^2} \chi(k)$$

Define CNT is the number of integer point in circle

$$CNT(r) = 1 + \sum_{i=1}^{r^2} cnt(i) = 1 + \sum_{i=1}^{r^2} \lfloor \frac{r^2}{i} \rfloor \chi(i)$$

## 6.5 Josephus

### 6.5.1 $J(n, m)$ : The Last Surviving Person

based-0,  $m - 1$  was the first be killed.

$$J_0(n, m) = \begin{cases} 0 & n = 1; \\ (J_0(n - 1, m) + m) \% n & 1 < n < m; \\ \lfloor \frac{k((f(n', k) - n \bmod k) \bmod n')}{k-1} \rfloor \text{ where } n' = n - \lfloor \frac{n}{k} \rfloor & m \leq n \end{cases}$$

$$J_k(n, m) = (J_0(n, m) + k) \% n$$

```

1 int J0(int n, int m) {
2     if (m == 1) return n - 1;
3     int ans = 0;
4     for (int i = 2; i <= n; ) {
5         if (ans + m >= i) {
6             ans = (ans + m) % i;
7             i++;
8             continue;
9         }
10        int step = (i - 1 - ans - 1) / (m - 1);
11        if (i + step > n) {
12            ans += (n - (i - 1)) * m;
13            break;
14        }
15        i += step; ans += step * m;
16    }
17    return ans;
18 }
```

### 6.5.2 $aJ(n, 1, K)$ : Survival Time of $K$ -th Person

based-1, 1 was the first be killed.

```

1 int aJ1(int size, int be, int goal){
2     if (be > size) be = (be - 1) % size + 1;
3     if (goal % m == be % m) return (goal - be) / m + 1;
```

```
4     return (size - be) / m + 1 + get(size - (size - be) / m - 1, (be + ((size - be) / m
      + 1) * m) - size, goal > be ? goal - (goal - be) / m - 1 : goal);
5 }
6 \\ size : the size of current group;
7 \\ be : the start person of current step;
8 \\ goal : index of the asked person.
```



## 7 Others

### 7.1 Offline Algorithm

#### 7.1.1 CDQ Divide and Conquer

```

1 struct Node {
2     int x, y, z, ans;
3     Node() {}
4     Node(int _x, int _y, int _z):x(_x), y(_y), z(_z) {}
5     bool operator < (const Node &b) const {
6         if(y == b.y) {
7             if(z == b.z) return x < b.x;
8             return z < b.z;
9         }
10        return y < b.y;
11    }
12 } A[MAXN], B[MAXN], C[MAXN];
13 int bit[MAXN];
14 void add(int k, int v) {
15     for(; k <= m; k += k & -k) bit[k] = max(bit[k], v);
16 }
17 void clear(int k) {
18     for(; k <= m; k += k & -k) bit[k] = 0;
19 }
20 int sum(int k) {
21     int res = 0;
22     for(; k; k -= k & -k) res = max(res, bit[k]);
23     return res;
24 }
25 void solve(int l, int r) {
26     if(l == r) {
27         B[l] = A[l];
28         return;
29     }
30     int mid = (l + r) >> 1;
31     solve(l, mid);
32     for(int i = mid + 1; i <= r; i++) B[i] = A[i];
33     //sort(B + l, B + mid + 1);
34     sort(B + mid + 1, B + r + 1);
35     int L = l;
36     for(int R = mid + 1; R <= r; R++) {
37         while(L <= mid && B[L].y < B[R].y) add(B[L].z, B[L].ans), L++;
38         A[B[R].x].ans = max(A[B[R].x].ans, sum(B[R].z - 1) + 1);
39         B[R].ans = A[B[R].x].ans;
40     }
41     for(int i = l; i <= L; i++) clear(B[i].z);
42     solve(mid + 1, r);
43     L = l;
44     int p = l, q = mid + 1;
45     while(p <= mid || q <= r) {
46         if(q > r || (p <= mid && B[p].y <= B[q].y)) C[L++] = B[p++];
47         else C[L++] = B[q++];
48     }
49     for(int i = l; i <= r; i++) B[i] = C[i];
50 }

```

### 7.1.2 Mo' s Algorithm

```

1 struct Node{
2     int l, r, t, id;
3     bool operator < (const Node& a) const {
4         if(l / sz == a.l / sz) {
5             if(r == a.r) return t < a.t;
6             return r < a.r;
7         }
8         return l / sz < a.l / sz;
9     }
10 }q[MAXN];
11 void solve() {
12     while (t < q[i].t) addTime(t++, 1);
13     while (t > q[i].t) addTime(--t, -1);
14     while(L < q[i].l) add(L++, -1);
15     while(L > q[i].l) add(--L, 1);
16     while(R < q[i].r) add(++R, 1);
17     while(R > q[i].r) add(R--, -1);
18 }

```

### 7.1.3 Mo's Algorithm On Tree

```

1 struct Edge {
2     int to, nxt;
3 }e[MAXN << 1];
4 int head[MAXN], ecnt;
5 int stack[MAXN], top, belong[MAXN], cnt, sz;
6 struct Node {
7     int l, r, id, ti;
8     bool operator < (const Node &x) const {
9         return belong[l] < belong[x.l] || (belong[l] == belong[x.l] && belong[r] <
10         belong[x.r]) || (belong[l] == belong[x.l] && belong[r] == belong[x.r] && ti < x.ti);
11 }
12 }q[MAXN];
13 struct Node2 {
14     int l, r, ti;
15 }qq[MAXN];
16 int n, m, Q, Q0, Q1;
17 int V[MAXN], W[MAXN], C[MAXN];
18 int fa[MAXN][S + 3], dep[MAXN];
19 long long ans[MAXN], tans;
20 int vis[MAXN], cur[MAXN];
21 long long sum[MAXN];
22 int l, r, tm;
23 inline int read() {
24     int x = 0; char ch = getchar(); bool fg = 0;
25     while(ch < '0' || ch > '9') { if(ch == '-') fg = 1; ch = getchar(); }
26     while(ch >= '0' && ch <= '9') { x = x * 10 + ch - '0'; ch = getchar(); }
27     return fg ? -x : x;
28 }
29 inline void add_edge(int u, int v) {
30     e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
31     e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
32 }
33 void dfs(int u, int f) {
34     fa[u][0] = f;
35     dep[u] = dep[f] + 1;
36     int bot = top;

```

```

36     for(int i = head[u]; i; i = e[i].nxt) {
37         int v = e[i].to;
38         if(v == f) continue;
39         dfs(v, u);
40         if(top - bot >= sz) {
41             cnt++;
42             while(top != bot) belong[stack[top--]] = cnt;
43         }
44     }
45     stack[++top] = u;
46 }
47 void G(int &u, int step) {
48     for(int i = 0; i < S; i++) if((1 << i) & step) u = fa[u][i];
49 }
50 int lca(int u, int v) {
51     if(dep[u] > dep[v]) swap(u, v);
52     G(v, dep[v] - dep[u]);
53     if(u == v) return u;
54     for(int i = S; i >= 0; i--) if(fa[u][i] != fa[v][i]) {
55         u = fa[u][i]; v = fa[v][i];
56     }
57     return fa[u][0];
58 }
59 inline void modify(int u) {
60     tans -= V[C[u]] * sum[cur[C[u]]];
61     cur[C[u]] += vis[u];
62     vis[u] = -vis[u];
63     tans += V[C[u]] * sum[cur[C[u]]];
64 }
65 inline void update(int u, int v) {
66     if(u == v) return;
67     if(dep[u] > dep[v]) swap(u, v);
68     while(dep[v] > dep[u]) {
69         modify(v);
70         v = fa[v][0];
71     }
72     while(u != v) {
73         modify(u); modify(v);
74         u = fa[u][0]; v = fa[v][0];
75     }
76 }
77 inline void upd(int t) {
78     if(vis[qq[t].l] == -1) {
79         modify(qq[t].l);
80         swap(C[qq[t].l], qq[t].r);
81         modify(qq[t].l);
82     }
83     else swap(C[qq[t].l], qq[t].r);
84 }
85 inline void moveto(int u, int v) {
86     update(l, u); update(r, v);
87     l = u; r = v;
88 }
89 int main() {
90     n = read(); m = read(); Q = read();
91     sz = (int)pow(n, 2.0 / 3.0);
92     for(int i = 1; i <= m; i++) V[i] = read();
93     for(int i = 1; i <= n; i++) W[i] = read();
94     for(int i = 1, u, v; i < n; i++) {
95         u = read(); v = read();
96         add_edge(u, v);

```

```

97 }
98 for(int i = 1; i <= n; i++) {
99     C[i] = read();
100     vis[i] = 1;
101     sum[i] = sum[i - 1] + W[i];
102 }
103 for(int i = 1, tp; i <= Q; i++) {
104     tp = read();
105     if(tp) {
106         ++Q1;
107         q[Q1].l = read(); q[Q1].r = read();
108         q[Q1].id = Q1;
109         q[Q1].ti = i;
110     }
111     else {
112         ++Q0;
113         qq[Q0].l = read(); qq[Q0].r = read();
114         qq[Q0].ti = i;
115     }
116 }
117 dfs(1, 0);
118 while(top) belong[stack[top--]] = cnt;
119 sort(q + 1, q + Q1 + 1);
120 for(int k = 1; k <= S; k++) {
121     for(int i = 1; i <= n; i++) {
122         fa[i][k] = fa[fa[i][k - 1]][k - 1];
123     }
124 }
125 for(int i = 1; i <= Q1; i++) {
126     if(belong[q[i].l] > belong[q[i].r]) swap(q[i].l, q[i].r);
127     moveto(q[i].l, q[i].r);
128     int lc = lca(l, r);
129     modify(lc);
130     while(qq[tm + 1].ti < q[i].ti && tm < Q0) upd(++tm);
131     while(qq[tm].ti > q[i].ti) upd(tm--);
132     ans[q[i].id] = tans;
133     modify(lc);
134 }
135 for(int i = 1; i <= Q1; i++) printf("%lld\n", ans[i]);
136 return 0;
137 }

```

## 7.2 Randomized Algorithm

### 7.2.1 Simulated Annealing

```

1 void solve() {
2     while(T > eps) {
3         double alpha = ((rand() % 30001) / 15000.0) * pi;
4         double theta = ((rand() % 10001) / 10000.0) * pi;
5         tmp.x = cur.x + T * sin(theta) * cos(alpha);
6         tmp.y = cur.y + T * sin(theta) * sin(alpha);
7         tmp.z = cur.z + T * cos(theta);
8         tmp.dis = cal(tmp);
9         if(tmp.dis < cur.dis || (tmp.dis * 0.999 < cur.dis && (rand() & 7) == 7)) cur =
tmp;
10         //if(exp((cur.d - tmp.d) / T) > ((double)rand() / RAND_MAX)) cur = tmp;
11
12         T *= 0.999;

```

```

13     }
14 }

```

## 7.3 Other Method

### 7.3.1 Enumerate Subset

```

1  for(int i = 0; i < (1 << k); i++) {
2      for(int j = i; ; --j &= i) {
3          // work();
4          if(j == 0) break;
5      }
6  }

```

### 7.3.2 Enumerate $\lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$

```

1  int cal(int n, int m) {
2      if(n > m) swap(n, m);
3      int res = 0, last;
4      for(int i = 1; i <= n; i = last + 1) {
5          last = min(n / (n / i), m / (m / i));
6          res += (n / i) * (m / i) * (sum(last) - sum(i - 1));
7      }
8      return res;
9  }

```

### 7.3.3 Find Primitive Root Modulo N

```

1  for i in range(1,mod):
2      if 3 ** i % mod == 1:
3          if i == mod - 1:
4              print("yes")
5              break
6      print("no")

```

## 8 Samples

### 8.1 vimrc

```

1 set cindent
2 set number
3 set mouse=a
4 set tabstop=4
5 set shiftwidth=4
6 syntax on
7 inoremap { {}<left>
8 map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>
9 "ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;

1 set nocompatible
2 source $VIMRUNTIME/vimrc_example.vim
3 source $VIMRUNTIME/mswin.vim
4 se cin nu mouse=a ts=4 sw=4 ww=b,s,<,>[,]
5 syntax on
6 inoremap { {}<left>
7 map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>
8 func! AddTitle()
9     call append(0,"// Cease to struggle and you cease to live")
10    call append(1,"// Created by hjj")
11    call append(2,"#include <bits/stdc++.h>")
12    call append(3,"using namespace std;")
13    call append(4,"")
14    call append(5,"int main() {")
15    call append(6,"")
16    call append(7,"    return 0;")
17    call append(8,"}")
18 endfunc
19 map <F8> :call AddTitle()<CR>

```

### 8.2 Check

```

1 while true; do
2     ./data > in
3     ./tmp < in > out
4     ./std < in > ans
5     diff out ans
6     if [ $? -ne 0 ] ; then exit; fi
7     echo Passed
8 done

```

### 8.3 Random

```

1 mt19937_64 mt(chrono::steady_clock::now().time_since_epoch().count());
2 shuffle(per.begin(), per.end(), mt);
3 //random_shuffle(per.begin(), per.end());

```

### 8.4 FastIO

```

1 //普通情况
2 namespace IO {
3     const int MB = 1048576;
4     const int RMAX = 16 * MB;
5     const int WMAX = 16 * MB;
6     #define getchar() *(rp++)
7     #define putchar(x) (*(wp++) = (x))
8     char rb[RMAX], *rp = rb, wb[WMAX], *wp = wb;
9     inline void init() {
10         fread(rb, sizeof(char), RMAX, stdin);
11     }
12     template <class _T> inline void read(_T &a) {
13         _a = 0; bool _f = 0; int _c = getchar();
14         while (_c < '0' || _c > '9') _f |= _c == '-', _c = getchar();
15         while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getchar();
16         _a = _f ? -_a : _a;
17     }
18     template <class _T> inline void write(_T _a) {
19         static char buf[20], *top = buf;
20         if (_a) {
21             while (_a) {
22                 _T tm = _a / 10;
23                 *(++top) = char(_a - tm * 10) | '0';
24                 _a = tm;
25             }
26             while (top != buf) putchar(*(top--));
27         }
28         else putchar('0');
29     }
30     void output() {
31         fwrite(wb, sizeof(char), wp - wb, stdout);
32     }
33 }
34 //EOF结尾+分块读入
35 #define likely(x) __builtin_expect(!!(x), 1)
36 #define unlikely(x) __builtin_expect(!!(x), 0)
37 namespace IO {
38     const int MB = 1048576;
39     const int RMAX = 4 * MB;
40     const int WMAX = 4 * MB;
41     unsigned long long filesize;
42     #define putchar(x) (*(wp++) = (x))
43     char rb[RMAX], wb[WMAX], *wp = wb;
44     int rp = 0;
45     inline void init() {
46         filesize = fread(rb, sizeof(char), RMAX, stdin);
47         rp = 0;
48         wp = wb;
49     }
50     void output() {
51         fwrite(wb, sizeof(char), wp - wb, stdout);
52     }
53     inline char getCHAR(){
54         if(unlikely(rp == filesize)){
55             fwrite(wb, sizeof(char), wp - wb, stdout);
56             init();
57             if(unlikely(filesize == 0)) {
58                 //cerr << 1.0 * (clock() - st) / CLOCKS_PER_SEC << endl;
59                 exit(0);
60             }
61         }

```

```

62     return rb[rp++];
63 }
64 template <class _T> inline void read(_T &_a) {
65     _a = 0; static bool _f = 0; static int _c;
66     _f = 0; _c = getCHAR();
67     while (_c < '0' || _c > '9') _f |= _c == '-', _c = getCHAR();
68     while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getCHAR();
69     _a = _f ? -_a : _a;
70 }
71 template <class _T> inline void write(_T _a) {
72     static char buf[20], *top = buf;
73     if (_a) {
74         while (_a) {
75             _T tm = _a / 10;
76             *(++top) = char(_a - tm * 10) | '0';
77             _a = tm;
78         }
79         while (top != buf) putchar(*(top--));
80     }
81     else putchar('0');
82     putchar('\n');
83 }
84 }

```

## 8.5 Java BigNum

```

1  import java.math.*;
2  import java.util.*;
3  import java.lang.*;
4
5  public class Main{
6      public static void main(String []args){}
7  }
8  //IO
9  Scanner in = new Scanner(System.in);
10 while(in.hasNext()){} //EOF
11 //fast-IO
12 public static void main(String argv[]) throws IOException{}
13 StreamTokenizer cin = new StreamTokenizer(new BufferedReader(new InputStreamReader(
14     System.in)));
15 PrintWriter cout = new PrintWriter(new OutputStreamWriter(System.out));
16 while(cin.nextToken() != StreamTokenizer.TT_EOF) ;//EOF
17 cin.nextToken();int n = (int)cin.nval;String s = cin.sval;
18 cout.println( Type );cout.flush();
19 cin.ordinaryChar( '/' );
20
21 BufferedReader br = new BufferedReader(new InputStreamReader(System.in));
22 br.ready()//EOF
23 while ((valueString=br.readLine())!=null);
24 br.close();
25 //true fast-IO
26 static class InputReader {
27     public BufferedReader reader;
28     public StringTokenizer tokenizer;
29
30     public InputReader(InputStream stream) {
31         reader = new BufferedReader(new InputStreamReader(stream), 32768);
32         tokenizer = null;
33     }
34 }

```



```

33
34     public String next() {
35         while (tokenizer == null || !tokenizer.hasMoreTokens()) {
36             try {
37                 tokenizer = new StringTokenizer(reader.readLine());
38             } catch (IOException e) {
39                 throw new RuntimeException(e);
40             }
41         }
42         return tokenizer.nextToken();
43     }
44
45     public int nextInt() {
46         return Integer.parseInt(next());
47     }
48
49 }
50 //类 Number
51 //doubleValue()
52 //intValue()
53 //longValue()
54 //shortValue()
55 //类 BigDecimal
56 //ROUND_CEILING 接近正无穷大的舍入模式。
57 //ROUND_FLOOR 接近负无穷大的舍入模式。
58 //ROUND_DOWN 接近零的舍入模式
59 //ROUND_HALF_UP 四舍五入 >=0.5向上舍入
60 //ROUND_HALF_DOWN 四舍五入 >0.5向上舍入
61 //BigDecimal(BigInteger val)
62 //BigDecimal(BigInteger unscaledVal, int scale)
63 //BigDecimal(char[] in, int offset, int len, MathContext mc)
64 //BigDecimal(double val, MathContext mc)不建议
65 //BigDecimal(int val, MathContext mc)
66 //BigDecimal(long val, MathContext mc)
67 //BigDecimal(String val, MathContext mc)
68 //abs()
69 //add(BigDecimal augend, MathContext mc)
70 //compareTo(BigDecimal val)
71 //divide(BigDecimal divisor, MathContext mc)
72 //divideToIntegralValue(BigDecimal divisor, MathContext mc)
73 //max(BigDecimal val)
74 //min(BigDecimal val)
75 //multiply(BigDecimal multiplicand, MathContext mc)
76 //negate() 其值为 (-this), 其标度为 this.scale()
77 //pow(int n)
78 //remainder(BigDecimal divisor) 返回其值为 (this % divisor) 的 BigDecimal
79 //round(MathContext mc) 返回根据 MathContext 设置进行舍入后的 BigDecimal。
80 //scaleByPowerOfTen(int n) 返回其数值等于 (this * 10^n) 的 BigDecimal。
81 //subtract(BigDecimal subtrahend, MathContext mc)
82 //setScale(int newScale, RoundingMode roundingMode)
83 //toString()
84 //ulp() 返回此 BigDecimal 的 ulp (最后一位的单位) 的大小
85 //String s = b.stripTrailingZeros().toPlainString(); 让 bigdecimal 不用科学计数法显示
86 //类 BigInteger
87 //parseInt
88 //BigInteger zero = BigInteger.valueOf(0);
89 //BigInteger a = in.nextBigInteger();
90 //abs()
91 //and(BigInteger val) 返回其值为 (this & val)
92 //or(BigInteger val) 返回其值为 (this | val)
93 //andNot(BigInteger val) 返回其值为 (this & ~val)

```

```

94 //compareTo(BigInteger val)
95 //add(BigInteger val)
96 //divide(BigInteger val)
97 //BigInteger[] divideAndRemainder(BigInteger val) 返回包含 (this / val) 后跟 (this %
    val) 的两个 BigInteger 的数组。
98 //equals(Object x)
99 //gcd(BigInteger val)
100 //isProbablePrime(int certainty) e.g.: a.isProbablePrime(4)
101 //max(BigInteger val) min(BigInteger val)
102 //mod(BigInteger m)
103 //modInverse(BigInteger m) 返回其值为 (this-1 mod m)
104 //modPow(BigInteger exponent, BigInteger m) 返回其值为 (thisexponent mod m)
105 //multiply(BigInteger val)
106 //not() 返回其值为 (~this)
107 //shiftLeft(int n) 返回其值为 (this << n)
108 //shiftRight(int n) 返回其值为 (this >> n)
109 //toString()
110 //valueOf(long val)
111 //xor(BigInteger val) 返回其值为 (this ^ val)
112 //other
113 //Arrays.sort(array);

```

## 8.6 pb\_ds

```

1 //P.S.: 无脑正确使用 pb_ds 代替 std::set/map/priority_queue 不会变慢
2 //可持久化平衡树, 不过时间和空间都不太行
3 #include <ext/rope>
4 using namespace __gnu_cxx;
5 int a[1000];
6 rope<int> x;
7 rope<int> x(a, a + n);
8 rope<int> a(x);
9 x->at(10); x[10];
10 x->push_back(x) // 在末尾添加 x
11 x->insert(pos, x) // 在 pos 插入 x
12 x->erase(pos, x) // 从 pos 开始删除 x 个
13 x->replace(pos, x) // 从 pos 开始换成 x
14 x->substr(pos, x) // 提取 pos 开始 x 个
15
16 //树
17 //不支持低级操作 (如交换左右子树)
18 #include <ext/pb_ds/assoc_container.hpp>
19 #include <ext/pb_ds/tree_policy.hpp>
20 using namespace __gnu_pbds;
21 定义一颗红黑树
22 tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> t, other;
23 int 关键字类型
24 null_type
25 无映射 (低版本 g++ 为 null_mapped_type) (无映射为类似 set, 有映射类似 map)
26 less<int> 从小到大排序
27 rb_tree_tag 红黑树 (splay_tree_tag)
28 tree_order_statistics_node_update 结点更新 (统计子树 size, 可自写), 不写不支持 order_of_key
    以及 find_by_order
29 插入: t.insert();
30 删除: t.erase();
31 比 x 小的个数: t.order_of_key(x);
32 第 x+1 值: t.find_by_order(x);
33 前驱: t.lower_bound();
34 后继: t.upper_bound();

```

```

35 合并:t.join(other); (other和*this值域不能相交)
36 分裂:t.split(x, other); (清空other, 将t中比x小的元素移至other)
37
38 //自定义节点更新
39 template <class Node_CItr , class Node_Itr , class Cmp_Fn , class _Alloc >
40 struct my_node_update {
41     virtual Node_CItr node_begin () const = 0;
42     virtual Node_CItr node_end() const = 0;
43     typedef char metadata_type; //节点上记录的额外信息的类型
44     //以上为固定格式
45
46     //operator()的功能是将节点it的信息更新为其左右孩子的信息之和, 传入的end_it表示空节点
47     //对Node_Itr可以做的事情有: 用get_l_child, get_r_child获取左右孩子, 用两个星号获取节点信息, 用get_metadata获取节点额外信息
48     inline void operator()(Node_Itr it, Node_CItr end_it) {
49         Node_Itr l = it.get_l_child(), r = it.get_r_child();
50         int left = 0, right = 0;
51         if(l != end_it) left = l.get_metadata();
52         if(r != end_it) right = r.get_metadata();
53         const_cast<metadata_type &>(it.get_metadata()) = left + right + (*it)->second;
54         //it是node_Itr, 取*后变为iterator, 再取->second变成mapped_value
55     }
56     inline int prefix_sum(int x) {
57         int ans = 0;
58         Node_CItr it = node_begin ();
59         while(it != node_end()) {
60             Node_CItr l = it.get_l_child(), r = it.get_r_child();
61             if(Cmp_Fn()(x, (*it)->first)) it = l;
62             else {
63                 ans += (*it)->second;
64                 if(l != node_end()) ans += l.get_metadata();
65                 it = r;
66             }
67         }
68         return ans;
69     }
70     inline int interval_sum(int l, int r) {
71         return prefix_sum(r) - prefix_sum(l - 1);
72     }
73 };
74
75 tree<int, char, less<int>, rb_tree_tag, my_node_update> T; //map
76 int main() {
77     T[2] = 'a'; T[3] = 'b'; T[4] = 1;
78     cout << (char)T.interval_sum(3, 4) << endl; //c
79     return 0;
80 }
81 //堆
82 #include <ext/pb_ds/priority_queue.hpp>
83 using namespace __gnu_pbds;
84 __gnu_pbds::priority_queue<int, std::less<int>, __gnu_pbds::pairing_heap_tag> q;
85
86 template <typename Value_Type ,
87 typename Cmp_Fn = std::less<Value_Type>,
88 typename Tag = pairing_heap_tag ,
89 typename Allocator = std::allocator<char> >
90 class priority_queue
91 Tag可以是binary_heap_tag (二叉堆) binomial_heap_tag (二项堆) rc_binomial_heap_tag
92 pairing_heap_tag (配对堆) thin_heap_tag
93 用begin()和end()获取迭代器从而遍历
94 删除单个元素 void erase(point_iterator)

```

```
94 更改一个元素的值 void modify(point_iterator, const_reference)
95 合并 void join(priority_queue &other), 把other合并到*this, 并把other清空
96 push()会返回迭代器
97 五种操作: push、pop、modify、erase、join
98 • pairing_heap_tag: push和join为 $O(1)$ , 其余均摊 $O(\log n)$ 
99 • binary_heap_tag: 只支持push和pop, 均为均摊 $O(\log n)$ 
100 • binomial_heap_tag: push为均摊 $O(1)$ , 其余为 $O(\log n)$ 
101 • rc_binomial_heap_tag: push为 $O(1)$ , 其余为 $O(\log n)$ 
102 • thin_heap_tag: push为 $O(1)$ , 不支持join, 其余为 $O(\log n)$ ; 但是如果只有increase_key, modify
    均摊 $O(1)$ 
103 • 不支持不是不能用, 而是用起来很慢
104 经过实践检测得到的结论:
105 • Dijkstra算法中应用pairing_heap_tag, 速度与手写数据结构相当。
106 • binary_heap_tag在绝大多数情况下优于std::priority_queue
107 • pairing_heap_tag在绝大多数情况优于binomial_heap_tag和rc_binomial_heap_tag
108 • 只有push, pop和join操作时, binary_heap_tag速度较快
109 • 有modify操作时, 可以考虑thin_heap_tag或者pairing_heap_tag, 或手写数据结构。
110
111 //hash_table
112 #include <ext/pb_ds/assoc_container.hpp>
113 #include <ext/pb_ds/hash_policy.hpp>
114 using namespace __gnu_pbds;
115 __gnu_pbds::cc_hash_table <Key, Mapped> mp; //使用链地址法解决哈希冲突
116 __gnu_pbds::gp_hash_table <Key, Mapped> mp; //使用探测法解决哈希冲突
117 //用法和map一样
```

## 9 code

### 9.1 最大子矩阵

单调栈法:

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int MAXN = 1007;
4  int a[MAXN][MAXN], b[MAXN][MAXN];
5  pair<int, int> mp[MAXN * MAXN];
6  pair<int, int> off[MAXN][MAXN];
7  int tail;
8  pair<int, int> st[MAXN];
9  int Up[MAXN][MAXN];
10 int ans = 0;
11 void maintain(pair<int, int> v) {
12     for(; tail && st[tail].first >= v.first; tail--) {
13         ans = max(ans, st[tail].first * (v.second - st[tail-1].second-1));
14     }
15 }
16 void clear(int pos) {
17     for(; tail; tail--)
18         ans = max(ans, st[tail].first * (pos - st[tail-1].second-1));
19 }
20 int main() {
21     ios::sync_with_stdio(0); cin.tie(0);
22     int n, m;
23     cin >> n >> m;
24     for(int i = 1; i <= n; i++)
25         for(int j = 1; j <= m; j++) {
26             cin >> a[i][j];
27             mp[a[i][j]] = {i, j};
28         }
29     for(int i = 1; i <= n; i++)
30         for(int j = 1; j <= m; j++) {
31             cin >> b[i][j];
32             off[i][j] = {i - mp[b[i][j]].first, j - mp[b[i][j]].second};
33         }
34     for(int i = 1; i <= n; i++) {
35         st[0].second = 0;
36         for(int j = 1; j <= m; j++) {
37             Up[i][j] = i > 1 && off[i][j] == off[i-1][j] ? Up[i-1][j] + 1 : 1;
38             if(j > 1 && off[i][j] == off[i][j-1])
39                 maintain({Up[i][j], j}); else {clear(j); st[0].second = j-1;}
40             st[++tail] = {Up[i][j], j};
41         }
42         clear(m+1);
43     }
44     cout << ans << endl;
45     return 0;
46 }

```

悬线法:

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int MAXN = 1007;
4  int a[MAXN][MAXN], b[MAXN][MAXN];
5  pair<int, int> mp[MAXN * MAXN];

```

```

6 pair<int, int> off[MAXN][MAXN];
7 int L[MAXN][MAXN], R[MAXN][MAXN], UP[MAXN][MAXN];
8 int main() {
9     ios::sync_with_stdio(0); cin.tie(0);
10    int n, m;
11    cin >> n >> m;
12    for(int i = 1; i <= n; i++)
13        for(int j = 1; j <= m; j++) {
14            cin >> a[i][j];
15            mp[a[i][j]] = {i, j};
16        }
17    for(int i = 1; i <= n; i++)
18        for(int j = 1; j <= m; j++) {
19            cin >> b[i][j];
20            off[i][j] = {i - mp[b[i][j]].first, j - mp[b[i][j]].second};
21            L[i][j] = R[i][j] = j;
22            UP[i][j] = 1;
23        }
24    for(int i = 1; i <= n; i++)
25        for(int j = 2; j <= m; j++) {
26            if(off[i][j] == off[i][j-1]) L[i][j] = L[i][j-1];
27        }
28    for(int i = 1; i <= n; i++)
29        for(int j = m - 1; j >= 1; j--) {
30            if(off[i][j] == off[i][j+1]) R[i][j] = R[i][j+1];
31        }
32    int ans = 0;
33    for(int i = 1; i <= n; i++)
34        for(int j = 1; j <= m; j++) {
35            if(i > 1 && off[i][j] == off[i-1][j]) {
36                UP[i][j] = UP[i-1][j] + 1;
37                L[i][j] = max(L[i][j], L[i-1][j]);
38                R[i][j] = min(R[i][j], R[i-1][j]);
39            }
40            ans = max(ans, (R[i][j] - L[i][j] + 1) * UP[i][j]);
41        }
42    cout << ans << endl;
43    return 0;
44 }

```