

SOUTH CHINA UNIVERSITY OF TECHNOLOGY

SCUT_GUGUGU

TEMPLATE



0 error(s), 0 warning(s)

Last build at September 28, 2019

Contents

1	Graph Theory	4
1.1	Shortest Path	4
1.1.1	Dijkstra	4
1.1.2	SPFA	4
1.2	Johnson	5
1.3	Network Flow	5
1.3.1	ISAP	5
1.3.2	HLPP	6
1.3.3	Dinic	8
1.3.4	MCMF	9
1.4	Tree Related	10
1.4.1	Union Set	10
1.4.2	Kruskal	10
1.4.3	Prim	10
1.4.4	Tree Divide and Conquer	11
1.5	LCA	12
1.5.1	Tree Decomposition LCA	12
1.5.2	Tarjan LCA	13
1.6	Tarjan	14
1.6.1	SCC	14
1.6.2	BCC	14
1.7	Cactus	16
1.7.1	Circle-Square Tree	16
2	Data Structures	20
2.1	Basic Structures	20
2.1.1	RMQ	20
2.1.2	Divide Blocks	21
2.2	Stack Structures	21
2.2.1	Cartesian Tree	21
2.3	Sequence Structures	22
2.3.1	Segment Tree	22
2.3.2	LiChao Tree	23
2.3.3	Splay Tree	24
2.4	Persistent Data Structures	26
2.4.1	Chairman Tree	26
2.4.2	Persistent Trie	26
2.5	Tree Structures	27
2.5.1	Tree Decomposition	27
2.5.2	Link-Cut Tree	29

3	String	32
3.1	Basics	32
3.1.1	Hash	32
3.1.2	KMP && exKMP	32
3.1.3	AC Automaton	33
3.1.4	Minimum String	35
3.2	Suffix Related	35
3.2.1	Suffix Array	35
3.2.2	Suffix Automaton	36
3.3	Palindrome Related	38
3.3.1	Manacher	38
3.3.2	Palindromic Automaton	39
4	Math	40
4.1	Algebra	40
4.1.1	FFT	40
4.1.2	NTT	41
4.1.3	MTT	42
4.1.4	FWT	43
4.1.5	FFT Divide and Conquer	44
4.1.6	Linear Basis	45
4.1.7	Lagrange Polynomial	46
4.1.8	BM Alogrithm	47
4.2	Math Theory	49
4.2.1	Inverse	49
4.2.2	Lucas	49
4.2.3	CRT && exCRT	50
4.2.4	BSGS	51
4.2.5	Miller-Rabin && PollardRho	52
4.2.6	$\varphi(n)$	53
4.2.7	Euler Sieve	53
4.2.8	DuJiao Sieve	54
4.2.9	Möbius Inversion	55
5	Geometry	57
5.1	Commonly Definition and Functions	57
5.1.1	Const and Functions	57
5.1.2	Point Definition	57
5.1.3	Line Definition	57
5.1.4	Get Area	58
5.1.5	Get Circumference	58
5.1.6	Anticlockwise Sort	59
5.2	Convex Hull	59
5.2.1	Get Convex Hull	59

5.2.2	Point in Convex Hull	59
5.3	Minkowski Sum	60
5.4	Rotating Calipers	60
5.4.1	The Diameter of Convex Hull	60
5.4.2	The Min Distance Between two Convex Hull	60
5.5	Half Plane Intersection	61
5.6	Min Circle Cover	61
5.7	Circle Union Area	62
5.8	Simpson Integrate	64
6	Conclusion	65
6.1	Game theory	65
6.1.1	Fibonacci's Game / Zeckendorf's theory	65
7	Others	66
7.1	Sample	66
7.1.1	vimrc	66
7.1.2	Check	66
7.1.3	FastIO	66
7.1.4	Java BigInt	67
7.2	Offline Algorithm	69
7.2.1	CDQ Divide and Conquer	69
7.2.2	Mo's Algorithm	69
7.2.3	Mo's Algorithm On Tree	70
7.3	Randomized Algorithm	72
7.3.1	Simulated Annealing	72
7.4	Other Method	73
7.4.1	Enumerate Subset	73
7.4.2	Enumerate $\lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$	73
7.4.3	Find Primitive Root Modulo N	73
7.5	Formula	73
7.5.1	Euler's Theorem	73
7.5.2	Möbius Inversion Formula	73
7.5.3	Math Theory Tips	74
7.5.4	Sieve Tips	74
7.6	Convolution Tips	75
7.6.1	FWT Tips	75
7.7	The Number of Integer Point on a Circle	75

1 Graph Theory

1.1 Shortest Path

1.1.1 Dijkstra

```

1  typedef long long LL;
2  const int MAXN = ;
3  const int MAXM = ;
4  const LL DINF = ;
5  typedef pair<LL, int> P;
6  struct Edge {
7      int to, nxt;
8      LL w;
9  }e[MAXM];
10 int head[MAXN], ecnt;
11 LL d[MAXN];
12 priority_queue<P, vector<P>, greater<P> > q;
13 inline void addEdge(int x, int y, LL w) {
14     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
15 }
16 void dijkstra(int st, int n) {
17     for(int i = 0; i <= n; i++) d[i] = DINF;
18     d[st] = 0;
19     q.push(make_pair(0, st));
20     while(!q.empty()) {
21         P x = q.top(); q.pop();
22         int u = x.second;
23         if(d[u] != x.first) continue;
24         for(int i = head[u], v; i; i = e[i].nxt) {
25             v = e[i].to;
26             if(d[v] > d[u] + e[i].w) {
27                 d[v] = d[u] + e[i].w;
28                 q.push(make_pair(d[v], v));
29             }
30         }
31     }
32 }

```

1.1.2 SPFA

```

1  struct Edge {
2      int to, nxt;
3      LL w;
4  }e[MAXE];
5  int head[MAXN], ecnt;
6  LL d[MAXN];
7  bool exist[MAXN];
8  queue<int> q;
9  inline void addEdge(int x, int y, LL w) {
10     e[++ecnt] = (Edge) {y, head[x], w}; head[x] = ecnt;
11 }
12 void SPFA(int st) {
13     memset(d, 0x3f, sizeof(d));
14     d[st] = 0;
15     q.push(st);
16     exist[st] = 1;
17     while(!q.empty()) {

```

```

18     int u = q.front(); q.pop();
19     exist[u] = 0;
20     for(int i = head[u], v; i; i = e[i].nxt) {
21         v = e[i].to;
22         if(d[v] > d[u] + e[i].w) {
23             d[v] = d[u] + e[i].w;
24             //pre[v] = u;
25             if(!exist[v]) {
26                 q.push(v);
27                 exist[v] = 1;
28             }
29         }
30     }
31 }
32 }

```

1.2 Johnson

```

1 void johnson() {
2     //建图中, Edge 需要 from, w1, w2, 去掉 w;
3     spfa(1);
4     for(int u = 1; u <= n; u++)
5         for(int i = head[u]; i; i = e[i].nxt)
6             e[i].w2 = e[i].w1 + d[e[i].from] - d[e[i].to];
7     dijkstra(s, n);
8 }

```

1.3 Network Flow

1.3.1 ISAP

```

1 namespace NWF {
2     struct Edge{
3         int to, nxt; LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], pre[MAXN], num[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(num, 0, (tot + 1) * sizeof(int));
11        memset(head, 0, (tot + 1) * sizeof(int));
12    }
13    inline void addEdge(int u, int v, LL f) {
14        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
15        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
16    }
17    void bfs() {
18        memset(dis, 0, (tot + 1) * sizeof(int));
19        q.push(T);
20        dis[T] = 1;
21        while(!q.empty()) {
22            int u = q.front(), v; q.pop();
23            num[dis[u]]++;
24            for(int i = cur[u] = head[u]; i; i = e[i].nxt) {
25                if(!dis[v = e[i].to]) {
26                    dis[v] = dis[u] + 1;

```

```

27         q.push(v);
28     }
29 }
30 }
31 }
32 LL augment() {
33     LL flow = INF;
34     for(int i = S; i != T; i = e[cur[i]].to)
35         flow = min(flow, e[cur[i]].f);
36     for(int i = S; i != T; i = e[cur[i]].to) {
37         e[cur[i]].f -= flow;
38         e[cur[i] ^ 1].f += flow;
39     }
40     return flow;
41 }
42 LL isap() {
43     bfs();
44     int u = S, v;
45     LL flow = 0;
46     while(dis[S] <= tot) {
47         if(u == T) {
48             flow += augment();
49             u = S;
50         }
51         bool fg = 0;
52         for(int i = cur[u]; i; i = e[i].nxt) {
53             if(e[i].f && dis[u] > dis[v = e[i].to]) {
54                 pre[v] = u;
55                 cur[u] = i;
56                 u = v;
57                 fg = 1;
58                 break;
59             }
60         }
61         if(fg) continue;
62         if(!--num[dis[u]]) break;
63         int maxDis = tot;
64         for(int i = head[u]; i; i = e[i].nxt) {
65             if(e[i].f && maxDis > dis[v = e[i].to]) {
66                 maxDis = dis[v];
67                 cur[u] = i;
68             }
69         }
70         num[dis[u] = maxDis + 1]++;
71         if(u != S) u = pre[u];
72     }
73     return flow;
74 }
75 }

```

1.3.2 HLPP

```

1 namespace NWF{
2     struct Edge{
3         int to,nxt;LL f;
4     }e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], dis[MAXN], num[MAXN];
7     LL sumf[MAXN];
8     queue<int> q;

```

```

9      list<int> dep[MAXN];
10     void init(int _S,int _T,int _tot){
11         ecnt = 1;S = _S;T = _T;tot = _tot;
12         memset(num, 0, (tot + 1) * sizeof(int));
13         memset(head, 0, (tot + 1) * sizeof(int));
14         memset(sumf, 0, (tot + 1) * sizeof(LL));
15     }
16     void addEdge(int u,int v,LL f){
17         e[++ecnt] = (Edge) {v, head[u], f};head[u] = ecnt;
18         e[++ecnt] = (Edge) {u, head[v], 0};head[v] = ecnt;
19     }
20     void bfs(){
21         memset(dis, 0, (tot + 1) * sizeof(int));
22         q.push(T); dis[T] = 1;
23         while(!q.empty()){
24             int u=q.front(), v; q.pop();
25             for(int i = head[u]; i; i = e[i].nxt)
26                 if(!dis[v = e[i].to]){
27                     dis[v] = dis[u] + 1;
28                     q.push(v);
29                 }
30         }
31     }
32     LL hlpp(){
33         bfs();
34         dis[S] = tot + 1;
35         for(int i = 1;i <= tot; ++i)num[dis[i]]++;
36         for(int i = tot + 1; ~i; --i)dep[i].clear();
37         int maxd = dis[S];LL f;
38         dep[maxd].push_back(S);sumf[S] = INF;
39         for(;;){
40             while(maxd && dep[maxd].empty())maxd--;
41             if(!maxd)break;
42             int u = dep[maxd].back(), v;dep[maxd].pop_back();
43             int minDis = tot + 1;
44             for(int i = head[u]; i; i = e[i].nxt)
45                 if(e[i].f){
46                     if(dis[u] > dis[v = e[i].to]){
47                         f = min(sumf[u], e[i].f);
48                         e[i].f -= f;e[i^1].f += f;
49                         if(sumf[u] != INF) sumf[u] -= f;
50                         if(sumf[v] != INF) sumf[v] += f;
51                         if(v!=S && v!=T && sumf[v] == f){
52                             maxd = max(maxd, dis[v]);
53                             dep[dis[v]].push_back(v);
54                         }
55                     }
56                     if(!sumf[u])break;
57                     }else minDis=min(minDis, dis[v] + 1);
58             }
59             if(sumf[u]){
60                 if(!--num[dis[u]]){
61                     for(int i = dis[u];i <= maxd;++i){
62                         while(!dep[i].empty()){
63                             --num[i];
64                             dis[dep[i].back()] = tot + 1;
65                             dep[i].pop_back();
66                         }
67                     }
68                     maxd = dis[u] - 1;dis[u] = tot + 1;
69                 }else{
68                     dis[u] = minDis;

```



```

70         if(minDis > tot)continue;
71         num[minDis]++;
72         maxd = max(maxd, minDis);
73         dep[minDis].push_back(u);
74     }
75 }
76 }
77 return sumf[T];
78 }
79 }

```

1.3.3 Dinic

```

1 namespace NWF {
2     struct Edge {
3         int to, nxt; LL f;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN], dis[MAXN];
7     queue<int> q;
8     void init(int _S, int _T, int _tot){
9         ecnt = 1; S = _S; T = _T; tot = _tot;
10        memset(head, 0, (tot + 1) * sizeof(int));
11    }
12    void addEdge(int u, int v, LL f) {
13        e[++ecnt] = (Edge) {v, head[u], f}; head[u] = ecnt;
14        e[++ecnt] = (Edge) {u, head[v], 0}; head[v] = ecnt;
15    }
16    bool bfs() {
17        memset(dis, 0, (tot + 1) * sizeof(int));
18        q.push(S); dis[S] = 1;
19        while (!q.empty()) {
20            int u = q.front(), v; q.pop();
21            for (int i = cur[u] = head[u]; i; i = e[i].nxt) {
22                if (e[i].f && !dis[v = e[i].to]) {
23                    q.push(v);
24                    dis[v] = dis[u] + 1;
25                }
26            }
27        }
28        return dis[T];
29    }
30    LL dfs(int u, LL maxf) {
31        if (u == T) return maxf;
32        LL sumf = maxf;
33        for (int &i = cur[u]; i; i = e[i].nxt) {
34            if (e[i].f && dis[e[i].to] > dis[u]) {
35                LL tmpf = dfs(e[i].to, min(sumf, e[i].f));
36                e[i].f -= tmpf; e[i ^ 1].f += tmpf;
37                sumf -= tmpf;
38                if (!sumf) return maxf;
39            }
40        }
41        return maxf - sumf;
42    }
43    LL dinic() {
44        LL ret = 0;
45        while (bfs()) ret += dfs(S, INF);
46        return ret;
47    }

```

48 }

1.3.4 MCMF

```

1 namespace NWF{
2     struct Edge {
3         int to, nxt; LL f, c;
4     } e[MAXM << 1];
5     int S, T, tot;
6     int ecnt, head[MAXN], cur[MAXN]; LL dis[MAXN];
7     bool exist[MAXN];
8     queue<int> q;
9     void init(int _S, int _T, int _tot){
10         ecnt = 1; S = _S; T = _T; tot = _tot;
11         memset(head, 0, (tot + 1) * sizeof(int));
12     }
13     void addEdge(int u, int v, LL f, LL c) {
14         e[++ecnt] = (Edge) {v, head[u], f, c}; head[u] = ecnt;
15         e[++ecnt] = (Edge) {u, head[v], 0, -c}; head[v] = ecnt;
16     }
17     bool spfa() {
18         for(int i = 0; i <= tot; ++i){
19             dis[i] = INF; exist[i] = cur[i] = 0;
20         }
21         q.push(S); dis[S] = 0; exist[S] = 1;
22         while(!q.empty()) {
23             int u = q.front(), v; q.pop(); exist[u] = 0;
24             for(int i = head[u]; i; i = e[i].nxt) {
25                 if(e[i].f && dis[v = e[i].to] > dis[u] + e[i].c) {
26                     dis[v] = dis[u] + e[i].c;
27                     cur[v] = i;
28                     if(!exist[v]) {
29                         q.push(v);
30                         exist[v] = 1;
31                     }
32                 }
33             }
34         }
35         return dis[T] != INF;
36     }
37     LL mcmf() {
38         LL cost = 0;
39         while(spfa()) {
40             LL flow = INF;
41             for(int i = T; i != S; i = e[cur[i] ^ 1].to)
42                 flow = min(flow, e[cur[i]].f);
43             for(int i = T; i != S; i = e[cur[i] ^ 1].to) {
44                 e[cur[i]].f -= flow;
45                 e[cur[i] ^ 1].f += flow;
46             }
47             cost += flow * dis[T];
48         }
49         return cost;
50     }
51 }

```

1.4 Tree Related

1.4.1 Union Set

```

1  int fa[MAXN], rnk[MAXN];
2  int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
3  bool same(int x, int y){ return Find(x) == Find(y); }
4  void unite(int x, int y)
5  {
6      x = Find(x);
7      y = Find(y);
8      if(x == y) return;
9      if(rnk[x] < rnk[y]) {
10         fa[x] = y;
11     }
12     else {
13         fa[y] = x;
14         if(rnk[x] == rnk[y]) rnk[x]++;
15     }
16 }

```

1.4.2 Kruskal

```

1  namespace MST{
2      struct Edge{
3          int u,v; LL w;
4          bool operator < (const Edge& x) const { return w < x.w; }
5      }e[MAXM];
6      int ecnt, fa[MAXN];
7      void addEdge(int u, int v, LL w) {
8          e[++ecnt] = (Edge){v, u, w}; headp[u] = ecnt;
9      }
10     int Find(int x) { return x == fa[x] ? x : fa[x] = Find(fa[x]); }
11     LL kruskal(int n) {
12         sort(e + 1, e + ecnt + 1);
13         for(int i = 1; i <= n; i++) fa[i] = i;
14         LL sum = 0;
15         for (int i = 1; i <= ecnt; i++){
16             int fu = Find(e[i].u), fv = Find(e[i].v);
17             if(fu != fv){
18                 fa[fu] = fv;
19                 sum += e[i].w;
20             }
21         }
22         return sum;
23     }
24 }

```

1.4.3 Prim

```

1  namespace MST {
2      struct Edge{
3          int to,nxt; LL w;
4      }e[MAXM];
5      int ecnt, head[MAXN], vis[MAXN]; // pre[MAXN];
6      LL dis[MAXN];
7      void addEdge(int u, int v, LL w){

```

```

8      e[++ecnt] = (Edge){v, head[u], w}; head[u] = ecnt;
9      e[++ecnt] = (Edge){u, head[v], w}; head[v] = ecnt;
10     }
11     LL Prim(int n){
12         for (int i = 1; i <= n; i++){
13             //pre[i] = 0;
14             vis[i] = 0;
15             dis[i] = INF;
16         }
17         vis[1] = 1;
18         LL sum = 0;
19         for (int i = head[1]; i; i = e[i].nxt)
20             dis[e[i].to] = min(dis[e[i].to], e[i].w);
21         for (int j = 1; j < n; j++){
22             int u; LL minDis = INF;
23             for (int i = 1; i <= n; ++i)
24                 if (!vis[i] && dis[i] < minDis){
25                     minDis = dis[i];
26                     u = i;
27                 }
28             if (minDis == INF) return -1;
29             vis[u] = 1;
30             sum += minDis;
31             for (int i = head[u]; i; i = e[i].nxt)
32                 if (!vis[v = e[i].to] && e[i].w < dis[v]){
33                     //pre[u] = v;
34                     dis[v] = e[i].w;
35                 }
36         }
37         return sum;
38     }
39 }

```

1.4.4 Tree Divide and Conquer

```

1 struct Edge {
2     int to, nxt, w;
3 }e[MAXM];
4 int head[MAXN], ecnt;
5 int sz[MAXN];
6 int d[MAXN], t[5], ans;
7 bool vis[MAXN];
8 inline void add_edge(int u, int v, int w) {
9     e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
10    e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
11 }
12 int getsz(int x, int fa) {
13     sz[x] = 1;
14     for(int i = head[x]; i; i = e[i].nxt) {
15         int y = e[i].to;
16         if(vis[y] || y == fa) continue;
17         sz[x] += getsz(y, x);
18     }
19     return sz[x];
20 }
21 int getrt(int x) {
22     int tot = getsz(x, 0) >> 1;
23     while(1) {
24         int u = -1;
25         for(int i = head[x]; i; i = e[i].nxt) {

```

```

26         int y = e[i].to;
27         if(vis[y] || sz[y] > sz[x]) continue;
28         if(u == -1 || sz[y] > sz[u]) u = y;
29     }
30     if(~u && sz[u] > tot) x = u;
31     else break;
32 }
33 return x;
34 }
35 void getdep(int x, int fa) {
36     t[d[x]]++;
37     for(int i = head[x]; i; i = e[i].nxt) {
38         int y = e[i].to;
39         if(vis[y] || y == fa) continue;
40         d[y] = (d[x] + e[i].w) % 3;
41         getdep(y, x);
42     }
43 }
44 int cal(int x, int v) {
45     t[0] = t[1] = t[2] = 0;
46     d[x] = v % 3;
47     getdep(x, 0);
48     return t[0] * t[0] + t[1] * t[2] * 2;
49 }
50 void solve(int x) {
51     vis[x] = 1;
52     ans += cal(x, 0);
53     for(int i = head[x]; i; i = e[i].nxt) {
54         int y = e[i].to;
55         if(vis[y]) continue;
56         ans -= cal(y, e[i].w);
57         solve(getrt(y));
58     }
59 }
60 int main() {
61     solve(getrt(1));
62 }

```

1.5 LCA

1.5.1 Tree Decomposition LCA

```

1  int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
2  struct Edge {
3      int to, nxt;
4  }e[MAXN << 1];
5  int head[MAXN], ecnt;
6  inline void add_edge(int x, int y) {
7      e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
8  }
9  void dfs1(int x) {
10     sz[x] = 1; son[x] = 0;
11     for(int i = head[x]; i; i = e[i].nxt) {
12         int v = e[i].to;
13         if(v == fa[x]) continue;
14         fa[v] = x;
15         dep[v] = dep[x] + 1;
16         dfs1(v);
17         sz[x] += sz[v];

```

```

18     if(sz[v] > sz[son[x]]) son[x] = v;
19 }
20 }
21 void dfs2(int x) {
22     B[num[x]] = A[x];
23     if(son[x]) {
24         top[son[x]] = top[x];
25         num[son[x]] = ++totw;
26         dfs2(son[x]);
27     }
28     for(int i = head[x]; i; i = e[i].nxt) {
29         int v = e[i].to;
30         if(v == fa[x] || v == son[x]) continue;
31         top[v] = v;
32         num[v] = ++totw;
33         dfs2(v);
34     }
35 }
36 int lca(int u, int v) {
37     if(u == v) return u;
38     while(top[u] != top[v]) {
39         if(dep[top[u]] > dep[top[v]]) swap(u, v);
40         v = fa[top[v]];
41     }
42     if(dep[u] > dep[v]) swap(u, v);
43     return u;
44 }
45 inline void init() {
46     memset(head, 0, sizeof(head)); ecnt = 0;
47     fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
48 }
49 inline void pre() {
50     dfs1(1); dfs2(1);
51 }

```

1.5.2 Tarjan LCA

```

1  vector< pair<int,int> > G[MAXN],ask[MAXN];
2  int fa[MAXN], ans[MAXN], vis[MAXN],dis[MAXN];
3  int Find(int x){
4      return x == fa[x] ? x : fa[x] = Find(fa[x]);
5  }
6  void init(int n){
7      memset(ans, 0, sizeof ans);
8      memset(vis, 0, sizeof vis);
9      for(int i = 0; i <= n; i++){
10         G[i].clear();
11         ask[i].clear();
12     }
13 }
14 void LCA(int u){
15     int v;
16     fa[u] = u;
17     vis[u] = true;
18     for(auto it : ask[u])
19         if(vis[v = it.first])
20             ans[it.second] = dis[u] + dis[v] - 2 * dis[Find(it.first)];
21     for(auto it : G[u])
22         if(!vis[v = it.first]){
23             dis[v] = dis[u] + it.second;

```

```

24     LCA(v);
25     fa[v] = u;
26 }
27 }

```

1.6 Tarjan

1.6.1 SCC

```

1 namespace SCC{
2     vector<int> G[MAXN];
3     int dfs_clock, scc_cn, dfn[MAXN], low[MAXN], sccno[MAXN];
4     stack<int> S;
5     void addEdge(int u, int v) {
6         G[u].push_back(v);
7     }
8     void tarjan(int u) {
9         dfn[u] = low[u] = ++dfs_clock;
10        S.push(u);
11        for(auto v : G[u]) {
12            if(!dfn[v]) {
13                tarjan(v);
14                low[u] = min(low[u], low[v]);
15            }else if(!sccno[v]) {
16                low[u] = min(low[u], dfn[v]);
17            }
18        }
19        if(dfn[u] == low[u]) {
20            scc_cnt++;
21            for(;;) {
22                int v = S.top(); S.pop();
23                sccno[v] = scc_cnt;
24                if(v == u) break;
25            }
26        }
27    }
28    void findSCC(int n) {
29        for(int i = 1; i <= n; i++)
30            if(!dfn[i]) tarjan(i);
31    }
32    void init(int n){
33        dfs_clock = scc_cnt = 0;
34        for(int i = 0; i <= n; ++i){
35            dfn[i] = low[i] = sccno[i] = 0;
36            G[i].clear();
37        }
38    }
39 }

```

1.6.2 BCC

```

1 namespace BCC{
2     struct Edge {
3         int to, nxt;
4     }e[MAXM << 1];
5     int ecnt, head[MAXN];
6     int dfs_clock, dfn[MAXN], low[MAXN];
7

```

```

8  int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
9  vector<int> vbcc[MAXN];
10 stack<int> vS;
11
12 int ebcc_cnt, ebccno[MAXN];
13 stack<int> eS;
14
15 inline void addEdge(int u, int v) {
16     e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
17     e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
18 }
19 inline void init(int n) {
20     ecnt = 1;
21     dfs_clock = 0;
22     vbcc_cnt = 0;
23     ebcc_cnt = 0;
24     for(int i = 1; i <= n; ++i){
25         head[i] = dfn[i] = low[i] = 0;
26         is_vertex[i] = 0;
27         vbccno[i] = 0;
28         ebccno[i] = 0;
29     }
30     while(!vS.empty()) vS.pop();
31 }
32 //root's edge = -1;
33 void tarjan(int u, int edge) {
34     dfn[u] = low[u] = ++dfs_clock;
35     int ch = 0;
36     vS.push(u);
37     eS.push(u);
38     for(int i = head[u], v; i; i = e[i].nxt) {
39         if(!dfn[v = e[i].to]) {
40             tarjan(v, i ^ 1);
41             low[u] = min(low[u], low[v]);
42             if(low[v] >= dfn[u]) {
43                 ++ch;
44                 if(edge > 0 || ch > 1) is_vertex[u] = 1;
45                 vbcc[++vbcc_cnt].clear();
46                 vbcc[vbcc_cnt].push_back(u);
47                 for(int x;;){
48                     x = vS.top(); vS.pop();
49                     vbcc[vbcc_cnt].push_back(x);
50                     vbccno[x] = vbcc_cnt;
51                     if(x == v) break;
52                 }
53             }
54             if(low[v] > dfn[u]) {
55                 // i && i ^ 1 is bridge
56             }
57         }
58         else if(dfn[v] < dfn[u] && i != edge)
59             low[u] = min(low[u], dfn[v]);
60     }
61     if(dfn[u] == low[u]) {
62         ebcc_cnt++;
63         for(int v;;) {
64             v = eS.top(); eS.pop();
65             ebccno[v] = ebcc_cnt;
66             if(v == u) break;
67         }
68     }

```



```

69     }
70     void findBCC(int n){
71         for(int i = 1; i <= n; i++){
72             if(!dfn[i]) tarjan(i, -1);
73
74             //findBridge
75             for(int u = 1; u <= n; u++) {
76                 for(int i = head[u], v; i; i = e[i].nxt)
77                     if(ebccno[u] != ebccno[v = e[i].to]) {
78                         //is bridge
79                     }
80             }
81         }
82     }

```

1.7 Cactus

1.7.1 Circle-Square Tree

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef pair<int, int> P;
4  const int MAXN = 2e4 + 5;
5  const int S = 15;
6  namespace Tree {
7      struct Edge {
8          int to, nxt, w;
9      }e[MAXN << 1];
10     int ecnt, head[MAXN];
11     int rt, isrt[MAXN], fa[MAXN][S + 3];
12     int sz[MAXN];
13     inline void addEdge(int u, int v, int w) {
14         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
15         fa[v][0] = u;
16     }
17 }
18 int n, m, Q;
19 namespace BCC {
20     struct Edge {
21         int to, nxt, w;
22     }e[MAXN << 1];
23     int ecnt, head[MAXN];
24     int dfs_clock, dfn[MAXN], low[MAXN];
25     int is_vertex[MAXN], vbcc_cnt, vbccno[MAXN];
26     vector<P> vbcc[MAXN];
27     stack<P> vs;
28     int tag[MAXN];
29     inline void addEdge(int u, int v, int w) {
30         e[++ecnt] = (Edge) {v, head[u], w}; head[u] = ecnt;
31         e[++ecnt] = (Edge) {u, head[v], w}; head[v] = ecnt;
32     }
33     inline void init(int n) {
34         ecnt = 1;
35         dfs_clock = 0;
36         vbcc_cnt = 0;
37         for(int i = 0; i <= 2 * n; i++){
38             head[i] = dfn[i] = low[i] = 0;
39             vbccno[i] = 0;
40             tag[i] = 0;

```

```

41     }
42     while(!vs.empty()) vs.pop();
43 }
44 //root's edge = -1;
45 void tarjan(int u, int edge) {
46     dfn[u] = low[u] = ++dfs_clock;
47     vs.push(P(u, e[edge ^ 1].w));
48     for(int i = head[u], v; i; i = e[i].nxt) {
49         if(!dfn[v = e[i].to]) {
50             tarjan(v, i ^ 1);
51             low[u] = min(low[u], low[v]);
52             if(low[v] >= dfn[u]) {
53                 if(vs.top().first == v) {
54                     Tree::addEdge(u, v, vs.top().second);
55                     vs.pop();
56                     continue;
57                 }
58                 vbcc[++vbcc_cnt].clear();
59                 vbcc[vbcc_cnt].push_back(P(u, 0));
60                 Tree::isrt[u] = 1;
61                 int &sz = Tree::sz[n + vbcc_cnt];
62                 tag[vs.top().first] = n + vbcc_cnt;
63                 //Tree::addEdge(u, rt, 0);
64                 for(P x;;) {
65                     x = vs.top(); vs.pop();
66                     sz += x.second;
67                     //Tree::addEdge(rt, x.first, sz);
68                     vbcc[vbcc_cnt].push_back(x);
69                     vbccno[x.first] = vbcc_cnt;
70                     if(x.first == v) break;
71                 }
72             }
73         }
74         else if(dfn[v] < dfn[u] && i != edge)
75             low[u] = min(low[u], dfn[v]);
76     }
77     for(int i = head[u], v; i; i = e[i].nxt) {
78         if(tag[v = e[i].to]) {
79             int r = tag[v]; Tree::sz[r] += e[i].w;
80             tag[v] = 0;
81         }
82     }
83 }
84 void findBCC(int n) {
85     for(int i = 1; i <= n; i++)
86         if(!dfn[i]) tarjan(i, -1);
87 }
88 }
89 namespace Tree {
90     int dis[MAXN], dep[MAXN], len[MAXN];
91     inline void init(int n) {
92         BCC::init(n);
93         rt = n;
94         ecnt = 1;
95         for(int i = 0; i <= 2 * n; i++) {
96             head[i] = 0;
97             fa[i][0] = isrt[i] = dis[i] = dep[i] = len[i] = 0;
98         }
99     }
100     void dfs(int x) {
101         for(int i = head[x], y; i; i = e[i].nxt) {

```

```

102         if(!dep[y = e[i].to]) {
103             dep[y] = dep[x] + 1;
104             dis[y] = dis[x] + e[i].w;
105             dfs(y);
106         }
107     }
108 }
109 void pre() {
110     for(int k = 1; k <= BCC::vbcc_cnt; k++) {
111         rt++;
112         vector<P> &E = BCC::vbcc[k];
113         addEdge(E[0].first, rt, 0);
114         int cnt = 0;
115         for(int i = E.size() - 1; i >= 1; i--) {
116             cnt += E[i].second;
117             len[E[i].first] = cnt;
118             addEdge(rt, E[i].first, min(cnt, sz[rt] - cnt));
119         }
120     }
121     for(int k = 1; k <= S; k++) {
122         for(int i = 1; i <= rt; i++) {
123             fa[i][k] = fa[fa[i][k - 1]][k - 1];
124         }
125     }
126     dep[1] = 1;
127     dfs(1);
128 }
129 int up(int x, int d) {
130     for(int i = S; i >= 0; i--) {
131         if(dep[fa[x][i]] >= d) x = fa[x][i];
132     }
133     return x;
134 }
135 int lca(int u, int v) {
136     if(dep[u] > dep[v]) swap(u, v);
137     v = up(v, dep[u]);
138     if(u == v) return u;
139     for(int i = S; i >= 0; i--) {
140         if(fa[u][i] != fa[v][i]) {
141             u = fa[u][i], v = fa[v][i];
142         }
143     }
144     return fa[u][0];
145 }
146 int query(int u, int v) {
147     int l = lca(u, v);
148     if(l <= n) return dis[u] + dis[v] - 2 * dis[l];
149     int x = up(u, dep[l] + 1), y = up(v, dep[l] + 1);
150     int res = dis[u] - dis[x] + dis[v] - dis[y];
151     int tmp = abs(len[x] - len[y]);
152     return res + min(tmp, sz[l] - tmp);
153 }
154 }
155
156 int main() {
157     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
158     using namespace Tree;
159     cin >> n >> m >> Q;
160     init(n);
161     for(int i = 1, u, v, w; i <= m; i++) {
162         cin >> u >> v >> w;

```

```
163     BCC::addEdge(u, v, w);
164 }
165 BCC::findBCC(n);
166 pre();
167 int u, v;
168 while(Q--) {
169     cin >> u >> v;
170     cout << query(u, v) << endl;
171 }
172 return 0;
173 }
```

2 Data Structures

2.1 Basic Structures

2.1.1 RMQ

```

1 struct RMQ {
2     int d[MAXN][S + 2];
3     inline void init(int *a, int n) {
4         for(int i = 1; i <= n; i++) d[i][0] = a[i];
5         for(int k = 1; (1 << k) <= n; k++)
6             for(int i = 1; i + (1 << k) - 1 <= n; i++)
7                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
8     }
9     inline int query(int l, int r) {
10        if(l > r) swap(l, r);
11        int k = 0;
12        while((1 << (k + 1)) <= r - l + 1) k++;
13        return min(d[l][k], d[r - (1 << k) + 1][k]);
14    }
15 }rmq;
16 const int MAXM = 2e5 + 5, MAXN = 3e6 + 5, S = 22;
17 const LL INF = 1e18;
18 #define belong(x) (x / S + 1)
19 #define pos(x) (x % S + 1)
20 int Log[MAXN], sz;
21 struct RMQ {
22     LL a[MAXN];
23     LL d[MAXM][S + 2];
24     LL pre[MAXM][S + 2], aft[MAXM][S + 2];
25     inline void init(int n) {
26         sz = n / S + 1;
27         Log[0] = -1; for(int i = 1; i <= n; i++) Log[i] = Log[i / 2] + 1;
28         for(int i = 1; i <= sz; i++) {
29             pre[i][0] = aft[i][S + 1] = INF;
30         }
31         for(int i = 1; i <= n; i++) {
32             pre[belong(i)][pos(i)] = min(pre[belong(i)][pos(i) - 1], a[i]);
33         }
34         for(int i = n; i >= 1; i--) {
35             aft[belong(i)][pos(i)] = min(aft[belong(i)][pos(i) + 1], a[i]);
36         }
37         for(int i = 1; i <= sz; i++) {
38             d[i][0] = aft[i][1];
39         }
40         for(int k = 1; k <= S; k++)
41             for(int i = 1; i + (1 << k) <= sz; i++)
42                 d[i][k] = min(d[i][k - 1], d[i + (1 << (k - 1))][k - 1]);
43     }
44     inline LL ask(int l, int r) {
45         assert(l <= r);
46         LL res = INF;
47         if(belong(l) == belong(r)) {
48             for(int i = l; i <= r; i++) res = min(res, a[i]);
49             return res;
50         }
51         res = min(aft[belong(l)][pos(l)], pre[belong(r)][pos(r)]);
52         int k = Log[belong(r) - belong(l) - 1];
53         if(~k) {

```

```

54         res = min(res, d[belong(l) + 1][k]);
55         res = min(res, d[belong(r) - (1 << k)][k]);
56     }
57     return res;
58 }
59 }rmq;

```

2.1.2 Divide Blocks

```

1  int belong[MAXN], l[MAXN], r[MAXN];
2  int sz, num;
3  void build(int n) {
4      sz = sqrt(n);
5      num = n / sz; if(n % sz) num++;
6      for(int i = 1; i <= num; i++) {
7          l[i] = (i - 1) * sz + 1;
8          r[i] = i * sz;
9      }
10     r[num] = n;
11     for(int i = 1; i <= n; i++) {
12         belong[i] = (i - 1) / sz + 1;
13     }
14 }

```

2.2 Stack Structures

2.2.1 Cartesian Tree

```

1  struct CartesianTree{
2      int rt, fa[MAXN], ls[MAXN], rs[MAXN];
3      int top, st[MAXN];
4      int cnt[MAXN];
5      void build(LL *a, int n) {
6          top = rt = 0;
7          for(int i = 1; i <= n; i++) {
8              ls[i] = rs[i] = fa[i] = 0;
9              while(top && a[st[top]] > a[i]) ls[i] = st[top--];
10             fa[i] = st[top];
11             if(ls[i]) fa[ls[i]] = i;
12             if(fa[i]) rs[fa[i]] = i; else rt = i;
13             st[++top] = i;
14         }
15     }
16     void dfs(int x) {
17         cnt[x] = 1;
18         if(ls[x]) {dfs(ls[x]); cnt[x] += cnt[ls[x]];}
19         if(rs[x]) {dfs(rs[x]); cnt[x] += cnt[rs[x]];}
20     }
21     LL getAns(LL *a, int n) {
22         //dfs(rt);
23         //—————
24         return res;
25     }
26 }T;

```

2.3 Sequence Structures

2.3.1 Segment Tree

```

1  #define Ls(x) (x << 1)
2
3  #define Rs(x) (x << 1 | 1)
4  struct Tree {
5      int l, r, lazy;
6      LL sum, mx;
7  } tree[MAXN << 2];
8  int A[MAXN];
9  void push_up(int x) {
10     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
11     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
12 }
13 void push_down(int x) {
14     if(tree[x].lazy) {
15         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
16         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
17         tree[Ls(x)].mx += tree[x].lazy;
18         tree[Rs(x)].mx += tree[x].lazy;
19         tree[Ls(x)].lazy += tree[x].lazy;
20         tree[Rs(x)].lazy += tree[x].lazy;
21         tree[x].lazy = 0;
22     }
23 }
24 void build(int x, int L, int R) {
25     tree[x].lazy = 0;
26     tree[x].l = L; tree[x].r = R;
27     if(L == R) {
28         tree[x].sum = A[L];
29         tree[x].mx = A[L];
30
31         return;
32     }
33     int mid = (L + R) >> 1;
34     build(Ls(x), L, mid);
35     build(Rs(x), mid + 1, R);
36     push_up(x);
37 }
38 void update(int x, int L, int R, LL val) {
39     if(tree[x].l >= L && tree[x].r <= R) {
40         tree[x].lazy += val;
41         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
42         tree[x].mx += val;
43         return;
44     }
45     push_down(x);
46     int mid = (tree[x].l + tree[x].r) >> 1;
47     if(L <= mid) update(Ls(x), L, R, val);
48     if(R > mid) update(Rs(x), L, R, val);
49     push_up(x);
50 }
51 LL query(int x, int L, int R) {
52     if(tree[x].l >= L && tree[x].r <= R)
53         return tree[x].sum;
54     push_down(x);
55     int mid = (tree[x].l + tree[x].r) >> 1;
56     LL res = 0;

```

```

57     if(L <= mid) res += query(Ls(x), L, R);
58     if(R > mid) res += query(Rs(x), L, R);
59
60     return res;
61 }
62 LL query2(int x, int L, int R) {
63     if(tree[x].l >= L && tree[x].r <= R)
64         return tree[x].mx;
65     push_down(x);
66     int mid = (tree[x].l + tree[x].r) >> 1;
67     LL res = -INF;
68     if(L <= mid) res = max(res, query2(Ls(x), L, R));
69     if(R > mid) res = max(res, query2(Rs(x), L, R));
70     return res;
71 }

```

2.3.2 LiChao Tree

```

1  const double eps = 1e-12;
2  namespace LiT{
3      const int MLIMIT = 40000;
4      typedef double LD;
5      struct line{LD k,b;int l,r,id;} T[MAXN << 2];
6      //inline LD calc(line &a,int pos) {return a.k*vec[pos]+a.b;}
7      inline LD calc(line &a,int pos) {return a.k*pos+a.b;}
8      inline double cross(line &a,line &b) {
9          if(b.k == a.k) return -1e9;
10         return (double)(a.b-b.b)/(b.k-a.k);
11     }
12     void build(int v, int l, int r) {
13         T[v].k = 0;T[v].b = -1e18;
14         T[v].l = 0;T[v].r = MLIMIT;
15         T[v].id = 0;
16         if(l == r)return;
17         int mid = (l+r)>>1;
18         build(v<<1,l,mid);
19         build(v<<1|1,mid+1,r);
20     }
21     void ins(int v,int l,int r, line k) {
22         if(k.l <= l && r <= k.r) {
23             LD fl = calc(k, l), fr = calc(k, r);
24             LD gl = calc(T[v], l), gr = calc(T[v], r);
25             if(fl - gl > eps && fr - gr > eps) T[v] = k;
26             else if(fl - gl > eps || fr - gr > eps) {
27                 int mid = (l+r)>>1;
28                 if(calc(k, mid) - calc(T[v], mid) > eps) swap(k, T[v]);
29                 //if(vec[mid] - cross(k, T[v]) > eps)
30                 if(mid - cross(k, T[v]) > eps)
31                     ins(v<<1, l, mid, k);else ins(v<<1|1, mid+1, r, k);
32             }
33             return;
34         }
35         int mid=(l+r)>>1;
36         if(k.l <= mid) ins(v<<1, l, mid, k);
37         if(mid < k.r) ins(v<<1|1, mid+1, r, k);
38     }
39     LD ans;int ansid;
40     void que(int v, int l, int r, int x) {
41         LD tmp = calc(T[v], x);
42         if(tmp > ans || (tmp == ans && T[v].id < ansid)) {

```



```

43         ans = tmp;
44         ansid = T[v].id;
45     }
46     if(l == r) return;
47     int mid = (l+r)>>1;
48     if(x <= mid) que(v<<1,l,mid,x);else que(v<<1|1,mid+1,r,x);
49 }
50 };
51 //左闭右闭

```

2.3.3 Splay Tree

```

1 namespace splay{
2     int n, m, sz, rt;
3     int val[MAXN], id[MAXN];
4     int tr[MAXN][2], size[MAXN], fa[MAXN], rev[MAXN], s[MAXN], lazy[MAXN];
5     void push_up(int x) {
6         int l = tr[x][0], r = tr[x][1];
7         s[x] = max(val[x], max(s[l], s[r]));
8         size[x] = size[l] + size[r] + 1;
9     }
10    void push_down(int x) {
11        int l = tr[x][0], r = tr[x][1];
12        if(lazy[x]) {
13            if(l) {
14                lazy[l] += lazy[x];
15                s[l] += lazy[x];
16                val[l] += lazy[x];
17            }
18            if(r) {
19                lazy[r] += lazy[x];
20                s[r] += lazy[x];
21                val[r] += lazy[x];
22            }
23            lazy[x] = 0;
24        }
25        if(rev[x]) {
26            rev[x] = 0;
27            rev[l] ^= 1; rev[r] ^= 1;
28            swap(tr[x][0], tr[x][1]);
29        }
30    }
31    void rotate(int x, int &k) {
32        int y = fa[x];
33        int z = fa[y];
34        int l, r;
35        if(tr[y][0] == x) l = 0;
36        else l = 1;
37        r = l ^ 1;
38        if(y == k) k = x;
39        else {
40            if(tr[z][0] == y) tr[z][0] = x;
41            else tr[z][1] = x;
42        }
43        fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
44        tr[y][l] = tr[x][r]; tr[x][r] = y;
45        push_up(y); push_up(x);
46    }
47    void splay(int x, int &k) {
48        int y, z;

```

```

49     while(x != k) {
50         y = fa[x];
51         z = fa[y];
52         if(y != k) {
53             if((tr[y][0] == x) ^ (tr[z][0] == y)) rotate(x, k);
54
55             else rotate(y, k);
56         }
57         rotate(x, k);
58     }
59 }
60 int find(int x, int rank) {
61     push_down(x);
62
63     int l = tr[x][0], r = tr[x][1];
64     if(size[l] + 1 == rank) return x;
65     else if(size[l] >= rank) return find(l, rank);
66     else return find(r, rank - size[l] - 1);
67 }
68 void update(int l, int r, int v) {
69     int x = find(rt, l), y = find(rt, r + 2);
70     splay(x, rt); splay(y, tr[x][1]);
71     int z = tr[y][0];
72     lazy[z] += v;
73     val[z] += v;
74     s[z] += v;
75 }
76 void reverse(int l, int r) {
77     int x = find(rt, l), y = find(rt, r + 2);
78     splay(x, rt); splay(y, tr[x][1]);
79     int z = tr[y][0];
80     rev[z] ^= 1;
81 }
82 void query(int l, int r) {
83     int x = find(rt, l), y = find(rt, r + 2);
84     splay(x, rt); splay(y, tr[x][1]);
85     int z = tr[y][0];
86     printf("%d\n", s[z]);
87 }
88 void build(int l, int r, int f) {
89     if(l > r) return;
90     int now = id[l], last = id[f];
91     if(l == r) {
92         fa[now] = last; size[now] = 1;
93         if(l < f) tr[last][0] = now;
94         else tr[last][1] = now;
95         return;
96     }
97     int mid = (l + r) >> 1; now = id[mid];
98     build(l, mid - 1, mid); build(mid + 1, r, mid);
99     fa[now] = last;
100    push_up(now);
101    if(mid < f) tr[last][0] = now;
102    else tr[last][1] = now;
103 }
104 void init() {
105     s[0] = -INF;
106     scanf("%d%d", &n, &m);
107     for(int i = 1; i <= n + 2; i++) id[i] = ++sz;
108     build(1, n + 2, 0); rt = (n + 3) >> 1;
109 }

```

110 }

2.4 Persistent Data Structures

2.4.1 Chairman Tree

```

1
2 struct Node {
3     int l, r;
4
5     LL sum;
6 }t[MAXN * 40];
7 int cnt, n;
8 int rt[MAXN];
9 void update(int pre, int &x, int l, int r, int v) {
10     x = ++cnt; t[x] = t[pre]; t[x].sum++;
11     if(l == r) return;
12     int mid = (l + r) >> 1;
13     if(v <= mid) update(t[pre].l, t[x].l, l, mid, v);
14     else update(t[pre].r, t[x].r, mid + 1, r, v);
15 }
16 int query(int x, int y, int l, int r, int v) {
17     if(l == r) return l;
18     int mid = (l + r) >> 1;
19     int sum = t[t[y].l].sum - t[t[x].l].sum;
20     if(sum >= v) return query(t[x].l, t[y].l, l, mid, v);
21     else return query(t[x].r, t[y].r, mid + 1, r, v - sum);
22 }

```

2.4.2 Persistent Trie

```

1 //区间异或最值查询
2 const int N=5e4+10;
3 int t[N];
4 int ch[N*32][2],val[N*32];
5 int cnt;
6 void init(){
7     mem(ch,0);
8     mem(val,0);
9     cnt=1;
10 }
11 int add(int root,int x){
12     int newroot=cnt++,ret=newroot;
13     for(int i=30;i>=0;i--){
14         ch[newroot][0]=ch[root][0];
15         ch[newroot][1]=ch[root][1];
16         int now=(x>>i)&1;
17         root=ch[root][now];
18
19         ch[newroot][now]=cnt++;
20         newroot=ch[newroot][now];
21         val[newroot]=val[root]+1;
22     }
23
24     return ret;
25 }
26 int query(int lt,int rt,int x){
27     int ans=0;

```

```

28     for(int i=30;i>=0;i--){
29         int now=(x>>i)&1;
30         if(val[ch[rt][now^1]]-val[ch[lr][now^1]]){
31             ans+=(1<<i);
32             rt=ch[rt][now^1];
33             lr=ch[lr][now^1];
34         } else{
35             rt=ch[rt][now];
36             lr=ch[lr][now];
37         }
38     }
39     return ans;
40 }

```

2.5 Tree Structures

2.5.1 Tree Decomposition

```

1  int sz[MAXN], dep[MAXN], top[MAXN], fa[MAXN], son[MAXN], num[MAXN], totw;
2  struct Edge {
3      int to, nxt;
4  }e[MAXN << 1];
5  int head[MAXN], ecnt;
6  int n, m, Q;
7  #define Ls(x) (x << 1)
8  #define Rs(x) (x << 1 | 1)
9  struct Tree {
10     int l, r, lazy;
11     LL sum, mx;
12 }tree[MAXN << 2];
13 int A[MAXN], B[MAXN];
14 void push_up(int x) {
15     tree[x].sum = tree[Ls(x)].sum + tree[Rs(x)].sum;
16     tree[x].mx = max(tree[Ls(x)].mx, tree[Rs(x)].mx);
17 }
18 void push_down(int x) {
19     if(tree[x].lazy) {
20         tree[Ls(x)].sum += tree[x].lazy * (tree[Ls(x)].r - tree[Ls(x)].l + 1);
21         tree[Rs(x)].sum += tree[x].lazy * (tree[Rs(x)].r - tree[Rs(x)].l + 1);
22         tree[Ls(x)].mx += tree[x].lazy;
23         tree[Rs(x)].mx += tree[x].lazy;
24         tree[Ls(x)].lazy += tree[x].lazy;
25         tree[Rs(x)].lazy += tree[x].lazy;
26         tree[x].lazy = 0;
27     }
28 }
29 void build(int x, int L, int R) {
30     tree[x].lazy = 0;
31     tree[x].l = L; tree[x].r = R;
32     if(L == R) {
33         tree[x].sum = B[L];
34         tree[x].mx = B[L];
35         return;
36     }
37     int mid = (L + R) >> 1;
38     build(Ls(x), L, mid);
39     build(Rs(x), mid + 1, R);
40     push_up(x);
41 }

```

```

42 void update(int x, int L, int R, LL val) {
43     if(tree[x].l >= L && tree[x].r <= R) {
44         tree[x].lazy += val;
45         tree[x].sum += val * (tree[x].r - tree[x].l + 1);
46         tree[x].mx += val;
47         return;
48     }
49     push_down(x);
50     int mid = (tree[x].l + tree[x].r) >> 1;
51     if(L <= mid) update(Ls(x), L, R, val);
52     if(R > mid) update(Rs(x), L, R, val);
53     push_up(x);
54 }
55 LL query(int x, int L, int R) {
56     if(tree[x].l >= L && tree[x].r <= R)
57         return tree[x].sum;
58     push_down(x);
59     int mid = (tree[x].l + tree[x].r) >> 1;
60     LL res = 0;
61     if(L <= mid) res += query(Ls(x), L, R);
62     if(R > mid) res += query(Rs(x), L, R);
63     return res;
64 }
65 LL query2(int x, int L, int R) {
66     if(tree[x].l >= L && tree[x].r <= R)
67         return tree[x].mx;
68     push_down(x);
69     int mid = (tree[x].l + tree[x].r) >> 1;
70     LL res = -INF;
71     if(L <= mid) res = max(res, query2(Ls(x), L, R));
72     if(R > mid) res = max(res, query2(Rs(x), L, R));
73     return res;
74 }
75 inline void add_edge(int x, int y) {
76     e[++ecnt] = (Edge) {y, head[x]}; head[x] = ecnt;
77 }
78 void dfs1(int x) {
79     sz[x] = 1; son[x] = 0;
80     for(int i = head[x]; i; i = e[i].nxt) {
81         int v = e[i].to;
82         if(v == fa[x]) continue;
83         fa[v] = x;
84         dep[v] = dep[x] + 1;
85         dfs1(v);
86         sz[x] += sz[v];
87         if(sz[v] > sz[son[x]]) son[x] = v;
88     }
89 }
90 void dfs2(int x) {
91     B[num[x]] = A[x];
92     if(son[x]) {
93         top[son[x]] = top[x];
94         num[son[x]] = ++totw;
95         dfs2(son[x]);
96     }
97     for(int i = head[x]; i; i = e[i].nxt) {
98         int v = e[i].to;
99         if(v == fa[x] || v == son[x]) continue;
100         top[v] = v;
101         num[v] = ++totw;
102         dfs2(v);

```

```

103     }
104 }
105 void up(int a, int b, int c) {
106     int f1 = top[a], f2 = top[b];
107     while(f1 != f2) {
108         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
109         update(1, num[f1], num[a], c);
110         a = fa[f1];
111         f1 = top[a];
112     }
113     if(dep[a] > dep[b]) swap(a, b);
114     update(1, num[a], num[b], c);
115 }
116 int qsum(int a, int b) {
117     if(a == b) return query(1, num[a], num[a]);
118     int f1 = top[a], f2 = top[b];
119     int res = 0;
120     while(f1 != f2) {
121         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
122         res += query(1, num[f1], num[a]);
123         a = fa[f1];
124         f1 = top[a];
125     }
126     if(dep[a] > dep[b]) swap(a, b);
127     res += query(1, num[a], num[b]);
128     return res;
129 }
130 int qmax(int a, int b) {
131     if(a == b) return query2(1, num[a], num[a]);
132     int f1 = top[a], f2 = top[b];
133     int res = -1000000000;
134     while(f1 != f2) {
135         if(dep[f1] < dep[f2]) { swap(a, b); swap(f1, f2); }
136         res = max(res, query2(1, num[f1], num[a]));
137         a = fa[f1];
138         f1 = top[a];
139     }
140     if(dep[a] > dep[b]) swap(a, b);
141     res = max(res, query2(1, num[a], num[b]));
142     return res;
143 }
144 inline void init() {
145     memset(head, 0, sizeof(head)); ecnt = 0;
146     fa[1] = 0; dep[1] = 1; top[1] = 1; num[1] = 1; totw = 1;
147 }
148 inline void pre() {
149     dfs1(1); dfs2(1); build(1, 1, totw);
150 }

```

2.5.2 Link-Cut Tree

```

1 namespace LCT {
2     int fa[MAXN], rev[MAXN], tr[MAXN][2];
3     int s[MAXN], val[MAXN];
4     void push_up(int x) {
5         int l = tr[x][0], r = tr[x][1];
6         s[x] = s[l] + s[r] + val[x];
7     }
8     void Rev(int x) {
9         rev[x] ^= 1; swap(tr[x][0], tr[x][1]);

```

```

10 }
11 void push_down(int x) {
12     if(!rev[x]) return;
13     int l = tr[x][0], r = tr[x][1];
14     rev[x] = 0;
15     if(l) Rev(l); if(r) Rev(r);
16 }
17 bool isroot(int x) {
18     return tr[fa[x]][0] != x && tr[fa[x]][1] != x;
19 }
20 void pre(int x) {
21     if(!isroot(x)) pre(fa[x]);
22     push_down(x);
23 }
24 void rotate(int x) {
25     int y = fa[x]; int z = fa[y];
26     int l = tr[y][1] == x;
27     int r = l ^ 1;
28     if(!isroot(y)) tr[z][tr[z][1] == y] = x;
29     fa[x] = z; fa[y] = x; fa[tr[x][r]] = y;
30     tr[y][l] = tr[x][r]; tr[x][r] = y;
31     push_up(y);
32 }
33 void splay(int x) {
34     pre(x);
35     int y, z;
36     while(!isroot(x)) {
37         y = fa[x]; z = fa[y];
38         if(!isroot(y)) {
39             if((tr[z][0] == y) == (tr[y][0] == x)) rotate(y);
40             else rotate(x);
41         }
42         rotate(x);
43     }
44     push_up(x);
45 }
46 void access(int x) {
47     int y = 0;
48     while(x) {
49         splay(x); tr[x][1] = y;
50         push_up(x);
51         y = x; x = fa[x];
52     }
53 }
54 void makeroot(int x) {
55     access(x); splay(x); Rev(x);
56 }
57 void lnk(int x, int y) {
58     makeroot(x); fa[x] = y;
59 }
60 void cut(int x, int y) {
61     makeroot(x); access(y); splay(y);
62     tr[y][0] = fa[x] = 0; push_up(y);
63 }
64 void update(int x, int y) {
65     makeroot(x); val[x] = y; push_up(x);
66 }
67 int query(int x, int y) {
68     makeroot(x); access(y); splay(y);
69     return s[y];
70 }

```

```
71     bool check(int x, int y) {  
72         int tmp = y;  
73         makeroot(x); access(y); splay(x);  
74         while(!isroot(y)) y = fa[y];  
75         splay(tmp);  
76         return x == y;  
77     }  
78 }
```


3 String

3.1 Basics

3.1.1 Hash

```

1  const LL p1 = 201, p2 = 301, mod1 = 1200000319, mod2 = 2147483647;
2  struct Hash {
3      LL a, b;
4      void append(Hash pre, int v) {
5          a = (pre.a * p1 + v) % mod1;
6          b = (pre.b * p2 + v) % mod2;
7      }
8      void init(string S) {
9          a = b = 0;
10         for(int i = 0; i < S.size(); i++) append(*this, S[i]);
11     }
12     bool operator == (const Hash &x) const {
13         return a == x.a && b == x.b;
14     }
15     bool operator < (const Hash &x) const {
16         return a < x.a || (a == x.a && b < x.b);
17     }
18 };

```

3.1.2 KMP && exKMP

```

1  namespace KMP {
2      int fa[MAXN];
3      void get_fail(char* t, int tn) {
4          fa[0] = -1;
5          int i = 0, j = -1;
6          while(i < tn) {
7              if (j == -1 || t[i] == t[j]) {
8                  ++i; ++j;
9                  fa[i] = t[i] != t[j] ? j : fa[j];
10             }else{
11                 j = fa[j];
12             }
13         }
14     }
15     void kmp(char* s, int sn, char* t, int tn) {
16         int i = 0, j = 0;
17         while(i < sn) {
18             if (j == -1 || s[i] == t[j]) {
19                 i++; j++;
20                 if(j == tn) {
21                     }
22             }else j = fa[j];
23         }
24     }
25 }
26 namespace exKMP {
27     int nxt[MAXN], ext[MAXN];
28     void get_nxt(char* t, int tn) {
29         int j = 0, mx = 0;
30         nxt[0] = tn;
31         for(int i = 1; i < tn; i++) {

```

```

32         if(i >= mx || i + nxt[i - j] >= mx) {
33             if(i > mx) mx = i;
34             while(mx < tn && t[mx] == t[mx - i]) mx++;
35             nxt[i] = mx - i;
36             j = i;
37         }else nxt[i] = nxt[i - j];
38     }
39 }
40 void exkmp(char *s, int sn, char *t, int tn) {
41     int j = 0, mx = 0;
42     for(int i = 0; i < sn; i++) {
43         if(i >= mx || i + nxt[i - j] >= mx) {
44             if(i > mx) mx = i;
45             while(mx < sn && mx - i < tn && s[mx] == t[mx - i]) mx++;
46             ext[i] = mx - i;
47             j = i;
48         }else ext[i] = nxt[i - j];
49     }
50 }
51 }

```

3.1.3 AC Automaton

```

1 namespace AC {
2     int ch[MAXN][sigma_size], last[MAXN];
3     int val[MAXN], f[MAXN], sz;
4     inline void init() { sz = 1; memset(ch[0], 0, sizeof(ch[0])); }
5     inline int idx(char c) { return c - 'a'; }
6     void insert(string s, int v) {
7         int u = 0;
8         for(int i = 0; i < s.size(); i++) {
9             int c = idx(s[i]);
10            if(!ch[u][c]) {
11                memset(ch[sz], 0, sizeof(ch[sz]));
12                val[sz] = 0;
13                ch[u][c] = sz++;
14            }
15            u = ch[u][c];
16        }
17        val[u] = v;
18    }
19    void get_fail() {
20        queue<int> q;
21        f[0] = 0;
22        for(int c = 0; c < sigma_size; c++) {
23            int u = ch[0][c];
24            if(u) { f[u] = 0; q.push(u); last[u] = 0; }
25        }
26        while(!q.empty()) {
27            int r = q.front(); q.pop();
28            for(int c = 0; c < sigma_size; c++) {
29                int u = ch[r][c];
30                if(!u) { ch[r][c] = ch[f[r]][c]; continue; }
31                q.push(u);
32                int v = f[r];
33                while(v && !ch[v][c]) v = f[v];
34                f[u] = ch[v][c];
35                last[u] = val[f[u]] ? f[u] : last[f[u]];
36            }
37        }

```

```

38     }
39     inline void solve(int j) {
40         if(j) {
41             ans += val[j];
42             solve(last[j]);
43         }
44     }
45     void find(string T) {
46         int j = 0;
47         for(int i = 0; i < T.size(); i++) {
48             int c = idx(T[i]);
49             j = ch[j][c];
50             if(val[j]) solve(j);
51             else if(last[j]) solve(last[j]);
52         }
53     }
54 }
55 namespace AC {
56     int root, tcnt;
57     int ch[MAXN][sigma_size], fa[MAXN];
58     inline int newnode() {
59         fa[++tcnt] = 0;
60         for(int i = 0; i < sigma_size; ++i) ch[tcnt][i] = 0;
61         return tcnt;
62     }
63     inline void init() {
64         tcnt = -1;
65         root = newnode();
66     }
67     inline int idx(char c) { return c - 'a'; }
68     void extend(char *s, int sn) {
69         int cur = root;
70         for(int i = 0, c; i < sn; i++) {
71             if(!ch[cur][c = idx(s[i])])
72                 ch[cur][c] = newnode();
73             cur = ch[cur][c];
74         }
75     }
76     int q[MAXN], qh, qt;
77     void get_fail() {
78         qh = 1; qt = 0;
79         fa[root] = 0;
80         for(int c = 0, now; c < sigma_size; c++)
81             if((now = ch[root][c]) != 0)
82                 q[++qt] = now;
83         while(qh <= qt) {
84             int cur = q[qh++];
85             for(int c = 0, now; c < sigma_size; c++)
86                 if((now = ch[cur][c]) != 0) {
87                     fa[now] = ch[fa[cur]][c];
88                     q[++qt] = now;
89                 }else
90                     ch[cur][c] = ch[fa[cur]][c];
91         }
92     }
93     //统计模板串出现次数，每个模板串只计算一次
94     //     int cur = root, ans = 0;
95     //     for(int i = 0; i < sn; ++i) {
96     //         cur = ch[cur][idx(s[i])];
97     //         for(int j = cur; j && cnt[j] != -1; j = fa[j]) {
98     //             ans += cnt[j];

```

```

99 //          cnt[j] = -1;
100 //      }
101 //  }
102
103 }

```

3.1.4 Minimum String

```

1 namespace minstring{
2     int getmin(char *s, int sn) {
3         int i = 0, j = 1, k = 0, t;
4         while(i < sn && j < sn && k < sn) {
5             t = s[(i + k) % sn] - s[(j + k) % sn];
6             if(!t) k++;
7             else {
8                 if(t > 0) i += k + 1; else j += k + 1;
9                 if(i == j) j++;
10                k = 0;
11            }
12        }
13        return i < j ? i : j;
14    }
15 }

```

3.2 Suffix Related

3.2.1 Suffix Array

```

1 namespace SA {
2     char s[MAXN];
3     int sa[MAXN], rank[MAXN], height[MAXN];
4     int t[MAXN], t2[MAXN], c[MAXN], n;
5     void clear() { n = 0; memset(sa, 0, sizeof(sa)); }
6     void build(int m) {
7         int *x = t, *y = t2;
8         for(int i = 0; i < m; i++) c[i] = 0;
9         for(int i = 0; i < n; i++) c[x[i]] = s[i]++;
10        for(int i = 1; i < m; i++) c[i] += c[i - 1];
11        for(int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
12        for(int k = 1; k <= n; k <= 1) {
13            int p = 0;
14            for(int i = n - k; i < n; i++) y[p++] = i;
15            for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
16            for(int i = 0; i < m; i++) c[i] = 0;
17            for(int i = 0; i < n; i++) c[x[y[i]]]++;
18            for(int i = 1; i < m; i++) c[i] += c[i - 1];
19            for(int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
20            swap(x, y);
21            p = 1; x[sa[0]] = 0;
22            for(int i = 1; i < n; i++)
23                x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k]
24                ? p - 1 : p++;
25            if(p >= n) break;
26            m = p;
27        }
28        void buildHeight() {
29            int k = 0;

```

```

30     for(int i = 0; i < n; i++) rank[sa[i]] = i;
31     for(int i = 0; i < n; i++) {
32         if(k) k--;
33         int j = sa[rank[i] - 1];
34         while(s[i + k] == s[j + k]) k++;
35         height[rank[i]] = k;
36     }
37 }
38 void init() {
39     n = strlen(s) + 1;
40     build('z' + 1);
41     buildHeight();
42 }
43 }

```

3.2.2 Suffix Automaton

```

1 namespace SAM{
2     int scnt, root, last;
3     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
4     int sc[MAXN<<1], tmp1[MAXN<<1], minl[MAXN<<1];
5
6     int newnode(int _len, int q = 0) {
7         fa[++scnt] = fa[q]; len[scnt] = _len;
8         sc[scnt] = 0; tmp1[scnt] = 0; minl[scnt] = INF;
9         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];
10        return scnt;
11    }
12    void init() {
13        scnt = 0;
14        root = last = newnode(0);
15    }
16    void extend(int c) {
17        int p = last, np = newnode(len[p] + 1);
18        for(; p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
19        if(!p) fa[np] = root;
20        else{
21            int q = ch[p][c];
22            if(len[p] + 1 == len[q]) fa[np] = q;
23            else{
24                int nq = newnode(len[p] + 1, q);
25                fa[np] = fa[q] = nq;
26                for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
27            }
28        }
29        last = np;
30    }
31    int c[MAXN], rs[MAXN << 1];
32    void radix_sort(int n){
33        for(int i = 0; i <= n; i++) c[i] = 0;
34        for(int i = 1; i <= scnt; i++) c[len[i]]++;
35        for(int i = 1; i <= n; i++) c[i] += c[i-1];
36        for(int i = scnt; i >= 1; i--) rs[c[len[i]]--] = i;
37    }
38    void go(){
39        scanf("%s", s);
40        int n = strlen(s);
41        for(int i = 0; i < n; ++i)
42            extend(s[i] - 'a');
43        radix_sort(n);

```

```

44 //以下sc集合意义不同
45 { //每个节点对应的位置之后有多少个不同子串
46     for(int i = scnt; i >= 1; i--) {
47         int S = 0;
48         for(int j = 0; j < 26; j++)
49             S += sc[ ch[rs[i]][j] ];
50         sc[rs[i]] = S + 1;
51     }
52 }
53 { //right集合大小
54     int cur = root;
55     for(int i = 0; i < n; ++i) {
56         cur = ch[cur][s[i] - 'a'];
57         sc[cur]++;
58     }
59     for(int i = scnt; i >= 1; --i) {
60         sc[ fa[rs[i]] ] += sc[rs[i]];
61     }
62 }
63 //公共子串
64 //tpl, 当前字符串: 在状态cur, 与模板串的最长公共后缀
65 //minl, 多个字符串: 在状态cur, 与模板串的最长公共后缀
66 //注意: 在状态cur匹配成功时, cur的祖先状态与字符串的最长公共后缀
67 for(; ~scanf("%s", s);) {
68     int cur = root, Blen = 0;
69     for(int i = 0; i <= scnt; i++)
70         tpl[i] = 0;
71     n = strlen(s);
72     for(int i = 0, x; i < n; i++) {
73         x = s[i] - 'a';
74         if(ch[cur][x]) {
75             ++Blen;
76             cur = ch[cur][x];
77         } else {
78             for(; cur && ch[cur][x] == 0; cur = fa[cur]);
79             if(cur) {
80                 Blen = len[cur] + 1;
81                 cur = ch[cur][x];
82             } else {
83                 cur = root; Blen = 0;
84             }
85         }
86         tpl[cur] = max(tpl[cur], Blen);
87     }
88     for(int i = scnt; i > 0; --i) {
89         if( tpl[ fa[rs[i]] ] < tpl[ rs[i] ] )
90             tpl[ fa[rs[i]] ] = len[ fa[rs[i]] ];
91         minl[ rs[i] ] = min(minl[ rs[i] ], tpl[ rs[i] ]);
92     }
93 }
94 }
95 }
96 namespace exSAM{
97     int scnt, root;
98     int fa[MAXN<<1], len[MAXN<<1], ch[MAXN<<1][26];
99     int sc[MAXN<<1], tpl[MAXN<<1], minl[MAXN<<1];
100
101     int newnode(int _len, int q = 0) {
102         fa[++scnt] = fa[q]; len[scnt] = _len;
103         sc[scnt] = 0; tpl[scnt] = 0; minl[scnt] = INF;
104         for(int i = 0; i < 26; i++) ch[scnt][i] = ch[q][i];

```

```

105     return scnt;
106 }
107 void init() {
108     scnt = 0;
109     root = newnode(0);
110 }
111 int work(int p, int c){
112     int q = ch[p][c];
113     int nq = newnode(len[p] + 1, q);
114     fa[q] = nq;
115     for(; p && ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
116     return nq;
117 }
118 int extend(int p, int c) {
119     if (ch[p][c]){
120         int q = ch[p][c];
121         if (len[p] + 1 == len[q]) return q;
122         return work(p, c);
123     }
124     int np = newnode(len[p] + 1);
125     for(; p && ch[p][c] == 0; p = fa[p]) ch[p][c] = np;
126     if (!p) fa[np] = root;
127     else{
128         int q = ch[p][c];
129         if (len[p] + 1 == len[q]) fa[np] = q;
130         else fa[np] = work(p, c);
131     }
132     return np;
133 }
134 void solve() {
135     int n; scanf("%d",&n);
136     for(int i = 1; i <= n; i++) {
137         scanf("%s", s);
138         int sn = strlen(s);
139         int last = root;
140         for(int j = 0; j < sn; ++j)
141             last = extend(last, s[j] - 'a');
142     }
143 }
144 }

```

3.3 Palindrome Related

3.3.1 Manacher

```

1 namespace Manacher {
2     char S[MAXN << 1];
3     int scnt, ans;
4     int p[MAXN << 1]; //p[i] - 1
5     void init(char *s0, int sn0) {
6         S[0] = '$'; S[1] = '#';
7         for(int i = 0; i < sn0; i++) {
8             S[2 * i + 2] = s0[i];
9             S[2 * i + 3] = '#';
10        }
11        scnt = sn0 * 2 + 2;
12        S[scnt] = '&';
13    }
14    void manacher() {

```

```

15     int id = 0, mx = 0;
16     for(int i = 1; i < scnt; i++) {
17         p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
18         while(S[i + p[i]] == S[i - p[i]]) p[i]++;
19         if(i + p[i] > mx) {
20             mx = i + p[i];
21             id = i;
22         }
23     }
24 }
25 }

```

3.3.2 Palindromic Automaton

```

1 namespace PAM {
2     int scnt, S[MAXN];
3     int pcnt, last, len[MAXN], fail[MAXN], ch[MAXN][26];
4     int cnt[MAXN]; //节点i表示的本质不同的串的个数(调用count())
5     int num[MAXN]; //以节点i表示的最长回文串的最右端点为回文串结尾的回文串个数
6     int newnode(int _len) {
7         len[pcnt] = _len;
8         cnt[pcnt] = num[pcnt] = 0;
9         for(int i = 0; i < 26; i++) ch[pcnt][i] = 0;
10        return pcnt++;
11    }
12    inline void init() {
13        S[scnt = 0] = -1;
14        pcnt = 0; newnode(0); newnode(-1);
15        fail[0] = 1; last = 0;
16    }
17    int getfail(int x) {
18        while(S[scnt - len[x] - 1] != S[scnt]) x = fail[x];
19        return x;
20    }
21    void extend(int c) {
22        S[++scnt] = c;
23        int cur = getfail(last);
24        if(!ch[cur][c]) {
25            int now = newnode(len[cur] + 2);
26            fail[now] = ch[getfail(fail[cur])][c];
27            ch[cur][c] = now;
28            num[now] = num[fail[now]] + 1;
29        }
30        last = ch[cur][c];
31        cnt[last]++;
32    }
33    void count() {
34        for(int i = pcnt - 1; i >= 0; i--) cnt[fail[i]] += cnt[i];
35    }
36 };

```


4 Math

4.1 Algebra

4.1.1 FFT

```

1 //不预处理精度
2 const double pi = acos(-1.0);
3 const int MAXN = 300003;
4 struct comp {
5     double x, y;
6     comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
7     comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
8     comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
9         y * a.x}; }
10 };
11 int rev[MAXN], T;
12 comp tmp;
13 void fft(comp *a, int r) {
14     if(r == -1) for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
15     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17         comp step = (comp) {cos(pi / mid), r * sin(pi / mid)};
18         for(int j = 0; j < T; j += i) {
19             comp cur = (comp) {1, 0};
20             for(int k = j; k < j + mid; k++, cur = cur * step) {
21                 tmp = a[k + mid] * cur;
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27     if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
28 }
29 comp A[MAXN];
30 void init(int n) {
31     for(T = 1; T <= n; T <= 1);
32     for(int i = 1; i < T; i++) {
33         if(i & 1) rev[i] = (rev[i] >> 1) >> 1 ^ (T >> 1);
34         else rev[i] = rev[i] >> 1;
35         //A[i] = (comp) {0, 0};
36     }
37 }
38 //预处理精度
39 int rev[MAXN], T;
40 comp Sin[MAXN], tmp;
41 void fft(comp *a, int r) {
42     if(r == -1) {
43         for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
44         for(int i = 0; i < T; i++) a[i] = a[i] * a[i];
45     }
46     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
47     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
48         for(int j = 0; j < T; j += i) {
49             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
50                 tmp = a[k + mid] * Sin[cur];
51                 a[k + mid] = a[k] - tmp;
52                 a[k] = a[k] + tmp;
53             }
54         }
55     }
56 }

```

```

53     }
54 }
55 if(r == -1) for(int i = 0; i < T; i++) a[i].y = (int)(a[i].y / T / 2 + 0.5);
56 }
57 comp A[MAXN];
58 void init(int n) {
59     for(T = 1; T <= n; T <= 1);
60     for(int i = 0; i < T; i++) {
61         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
62         else rev[i] = rev[i >> 1] >> 1;
63         //A[i] = (comp) {0, 0};
64     }
65     for(int i = 0; i < (T >> 1); i++) {
66         Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
67     }
68 }
69 int main() {
70     scanf("%d%d", &n, &m);
71     init(n + m);
72     for(int i = 0; i <= n; i++) scanf("%lf", &A[i].x);
73     for(int i = 0; i <= m; i++) scanf("%lf", &A[i].y);
74     fft(A, 1);
75     fft(A, -1);
76     for(int i = 0; i <= n + m; i++) printf("%d%c", (int)(A[i].y), i == n + m ? '\n' : ' ');
77     return 0;
78 }

```

4.1.2 NTT

4.常用NTT模数:

以下模数的共同 $g = 3189$

$p = r \times 2^k + 1$	k	g
104857601	22	3
167772161	25	3
469762049	26	3
95009857	21	7
998244353	23	3
1004535809	21	3
2013265921	27	31
2281701377	27	3
3221225473	30	5

```

1  const int MAXN = 300005, G = 3, mod = 998244353; //or (479LL<<21) + 1
2  int rev[MAXN], T;
3  LL qpow(LL x, LL y) {
4      LL res = 1;
5      while(y) {
6          if(y & 1) res = res * x % mod;
7          x = x * x % mod;
8          y >>= 1;
9      }
10     return res;
11 }
12 LL A[MAXN], B[MAXN];
13 void ntt(LL *a, int r) {

```

```

14 if(r == -1) for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
15 for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
16 for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
17     LL gn = qpow(G, (mod - 1) / i);
18     if(r == -1) gn = qpow(gn, mod - 2);
19     for(int j = 0; j < T; j += i) {
20         LL cur = 1, tmp;
21         for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
22             tmp = a[k + mid] * cur % mod;
23             a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;
24             a[k] = (a[k] + tmp) % mod;
25         }
26     }
27 }
28 if(r == -1) {
29     LL inv = qpow(T, mod - 2);
30     for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
31 }
32 }
33 void init(int n) {
34     for(T = 1; T <= n; T <= 1);
35     for(int i = 0; i < T; i++) {
36         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
37         else rev[i] = rev[i >> 1] >> 1;
38     }
39 }

```

4.1.3 MTT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long LL;
4 const double pi = acos(-1.0);
5 const int MAXN = 300003;
6 struct comp {
7     double x, y;
8     comp operator + (const comp& a) const { return (comp) {x + a.x, y + a.y}; }
9     comp operator - (const comp& a) const { return (comp) {x - a.x, y - a.y}; }
10    comp operator * (const comp& a) const { return (comp) {x * a.x - y * a.y, x * a.y +
11        y * a.x}; }
12 };
13 #define conj(a) ((comp){a.x, -a.y})
14 int rev[MAXN], T;
15 comp Sin[MAXN], tmp;
16 void fft(comp *a, int r) {
17     for(int i = 1; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
18     for(int i = 2, mid = 1, s = (T >> 1); i <= T; mid = i, i <= 1, s >= 1) {
19         for(int j = 0; j < T; j += i) {
20             for(int k = j, cur = 0; k < j + mid; k++, cur += s) {
21                 tmp = a[k + mid] * Sin[cur];
22                 a[k + mid] = a[k] - tmp;
23                 a[k] = a[k] + tmp;
24             }
25         }
26     }
27 }
28 void init(int n) {
29     for(T = 1; T <= n; T <= 1);
30     for(int i = 0; i < T; i++) {
31         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
32         else rev[i] = rev[i >> 1] >> 1;
33     }
34 }

```

```

31     else rev[i] = rev[i >> 1] >> 1;
32 }
33 for(int i = 0; i < (T >> 1); i++) {
34     Sin[i] = (comp) {cos(2 * pi * i / T), sin(2 * pi * i / T)};
35 }
36 }
37 int n, m, mod;
38 void mtt(int *x, int *y) {
39     for(int i = 0; i < T; i++) (x[i] += mod) %= mod, (y[i] += mod) %= mod;
40     static comp a[MAXN], b[MAXN];
41     static comp dfta[MAXN], dftb[MAXN], dftc[MAXN], dftd[MAXN];
42     for(int i = 0; i < T; i++) {
43         a[i] = {x[i] & 0x7fff, x[i] >> 15};
44         b[i] = {y[i] & 0x7fff, y[i] >> 15};
45     }
46     fft(a, 1); fft(b, 1);
47     for(int i = 0; i < T; i++) {
48         int j = (T - i) & (T - 1);
49         static comp da, db, dc, dd;
50         da = (a[i] + conj(a[j])) * (comp){0.5, 0};
51         db = (a[i] - conj(a[j])) * (comp){0, -0.5};
52         dc = (b[i] + conj(b[j])) * (comp){0.5, 0};
53         dd = (b[i] - conj(b[j])) * (comp){0, -0.5};
54         dfta[i] = da * dc;
55         dftb[i] = da * dd;
56         dftc[i] = db * dc;
57         dftd[i] = db * dd;
58     }
59     for(int i = 0; i < T; i++) {
60         a[i] = dfta[i] + dftb[i] * (comp) {0, 1};
61         b[i] = dftc[i] + dftd[i] * (comp) {0, 1};
62     }
63     for(int i = 0; i < (T >> 1); i++) Sin[i].y = -Sin[i].y;
64     fft(a, -1); fft(b, -1);
65     for(int i = 0; i < T; i++) {
66         static int da, db, dc, dd;
67         da = (LL)(a[i].x / T + 0.5) % mod;
68         db = (LL)(a[i].y / T + 0.5) % mod;
69         dc = (LL)(b[i].x / T + 0.5) % mod;
70         dd = (LL)(b[i].y / T + 0.5) % mod;
71         x[i] = ((da + ((LL)(db + dc) << 15) + ((LL)dd << 30)) % mod + mod) % mod;
72     }
73 }
74 int main() {
75     static int a[MAXN], b[MAXN];
76     scanf("%d%d%d", &n, &m, &mod);
77     for(int i = 0; i <= n; i++) scanf("%d", a + i);
78     for(int i = 0; i <= m; i++) scanf("%d", b + i);
79     init(n + m);
80     mtt(a, b);
81     for(int i = 0; i <= n + m; i++) printf("%d%c", a[i], i == n + m ? '\n' : ' ');
82     return 0;
83 }

```

4.1.4 FWT

```

1 void FWT(LL *a, int n) {
2     for(int i = 2; i <= n; i <= 1) {
3         for(int j = 0; j < n; j += i) {
4             for(int d = 0, w = i >> 1; d < w; d++){

```

```

5         LL u = a[j + d], v = a[j + d + w];
6         //xor: a[j + d] = u + v, a[j + d + w] = u - v;
7         //and: a[j + d] = u + v;
8         //or : a[j + d + w] = u + v;
9     }
10 }
11 }
12 }
13 void UFWT(LL *a, int n) {
14     for(int i = 2; i <= n; i <= 1) {
15         for(int j = 0; j < n; j += i) {
16             for(int d = 0, w = i >> 1; d < w; d++) {
17                 LL u = a[j + d], v = a[j + d + w];
18                 //xor: a[j + d] = (u + v) / 2, a[j + d + w] = (u - v) / 2;
19                 //and: a[j + d] = u - v;
20                 //or : a[j + d + w] = v - u;
21             }
22         }
23     }
24 }
25 void solve(int n) {
26     FWT(a, n); FWT(b, n);
27     for(int i = 0; i < n; i++) a[i] = a[i] * b[i];
28     UFWT(a, n);
29 }

```

4.1.5 FFT Divide and Conquer

$$f_i = \sum_{j=1}^{i-1} f_j \cdot g_{i-j}$$

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 typedef long long LL;
5 const int MAXN = 300005, G = 3, mod = 998244353;
6 namespace NTT {
7     LL A[MAXN], B[MAXN]
8     int rev[MAXN], T;
9     LL qpow(LL x, LL y) {
10         LL res = 1;
11         while(y) {
12             if(y & 1) res = res * x % mod;
13             x = x * x % mod;
14             y >>= 1;
15         }
16         return res;
17     }
18 void ntt(LL *a, int r) {
19     for(int i = 0; i < T; i++) if(rev[i] > i) swap(a[rev[i]], a[i]);
20     for(int i = 2, mid = 1; i <= T; mid = i, i <= 1) {
21         LL gn = qpow(G, (mod - 1) / i);
22         if(r == -1) gn = qpow(gn, mod - 2);
23         for(int j = 0; j < T; j += i) {
24             LL cur = 1, tmp;
25             for(int k = j; k < j + mid; k++, cur = cur * gn % mod) {
26                 tmp = a[k + mid] * cur % mod;
27                 a[k + mid] = ((a[k] - tmp) % mod + mod) % mod;

```

```

28         a[k] = (a[k] + tmp) % mod;
29     }
30 }
31 }
32 if(r == -1) {
33     LL inv = qpow(T, mod - 2);
34     for(int i = 0; i < T; i++) a[i] = a[i] * inv % mod;
35 }
36 }
37 void init(int n) {
38     for(T = 1; T <= n; T <= 1);
39     for(int i = 0; i < T; i++) {
40         if(i & 1) rev[i] = (rev[i >> 1] >> 1) ^ (T >> 1);
41         else rev[i] = rev[i >> 1] >> 1;
42     }
43 }
44 }
45 LL f[MAXN], g[MAXN];
46 using namespace NTT;
47 void solve(int l, int r) {
48     if(l == r) return;
49     int mid = (l + r) >> 1;
50     solve(l, mid);
51     init(r - l);
52     for(int i = 0; i < T; i++) A[i] = B[i] = 0;
53     for(int i = 0; i <= mid - l; i++) A[i] = f[i + l];
54     for(int i = 0; i <= r - l; i++) B[i] = g[i];
55     ntt(A, 1); ntt(B, 1);
56     for(int i = 0; i < T; i++) A[i] = A[i] * B[i] % mod;
57     ntt(A, -1);
58     for(int i = mid + 1; i <= r; i++) f[i] = (f[i] + A[i - l]) % mod;
59     solve(mid + 1, r);
60 }
61 int main() {
62     int n; scanf("%d", &n);
63     for(int i = 1; i < n; i++) scanf("%lld", g + i);
64     f[0] = 1;
65     solve(0, n - 1);
66     for(int i = 0; i < n; i++) printf("%lld%c", f[i], i == n - 1 ? '\n' : ' ');
67     return 0;
68 }

```

4.1.6 Linear Basis

```

1 //dynamic
2 const int D = 60;
3 struct Basis {
4     vector<int> ind;
5     vector<LL> base;
6     Basis() {
7         ind.resize(D, -1);
8         base.resize(D);
9     }
10    bool update(LL x, int id) {
11        for(int i = 0; i < D; i++) if(~ind[i] && x >> i & 1) {
12            x ^= base[i];
13        }
14        if(!x) return 1;
15        int pos = __builtin_ctzll(x);
16        ind[pos] = id;

```

```

17     base[pos] = x;
18     return 0;
19 }
20 };
21 //array
22 int Gauss(int n, int m) {
23     int num = 1;
24     for(int x = 1; x <= n && x <= m; x++) {
25         int t = 0;
26         for(int j = x; j <= m; j++) if(g[j][x]) { t = j; break; }
27         if(t) {
28             swap(g[x], g[t]);
29             for(int i = x + 1; i <= n; i++) {
30                 if(g[i][x]) {
31                     for(int k = 1; k <= m; k++) g[i][k] ^= g[x][k];
32                 }
33             }
34             num++;
35         }
36     }
37     return --num;
38 }
39 //long long
40 int Gauss() {
41     int num = 1;
42     for(int k = 61; k >= 0; k--) {
43         int t = 0;
44         for(int j = num; j <= cnt; j++) if((A[j] >> k) & 1) { t = j; break; }
45         if(t) {
46             swap(A[t], A[num]);
47             for(int j = num + 1; j <= cnt; j++) if((A[j] >> k) & 1) A[j] ^= A[num];
48             num++;
49         }
50     }
51     return --num;
52 }

```

4.1.7 Lagrange Polynomial

$$L(x) = \sum_{i=0}^n y_i \prod_{j=0, j \neq i}^n \frac{x - x_j}{x_i - x_j}$$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long LL;
4  typedef pair<int, int> P;
5  const int MAXN = 3005, mod = 998244353;
6  int exgcd(int a, int b, int &x, int &y) {
7      int d = a;
8      if(b != 0) {
9          d = exgcd(b, a % b, y, x);
10         y -= (a / b) * x;
11     }
12     else {
13         x = 1; y = 0;
14     }
15     return d;
16 }

```

```

17 int inv(int a) {
18     int x, y;
19     exgcd(a, mod, x, y);
20     return (x % mod + mod) % mod;
21 }
22 struct Lagrange {
23     int n, a[MAXN][2];
24     void init() {
25         for(int i = 0; i <= n; i++) a[i][0] = a[i][1] = 0;
26         n = 0;
27         a[0][1] = 1;
28     }
29     int query(int x, int q = 0) {
30         int res = 0;
31         for(int i = n; i >= 0; i--) res = ((LL)res * x + a[i][q]) % mod;
32         return res;
33     }
34     void update(int x, int y) {
35         a[n][0] = 0;
36         int v = (LL)(y - query(x) + mod) % mod * inv(query(x, 1)) % mod;
37         for(int i = 0; i <= n; i++) a[i][0] = (a[i][0] + (LL)a[i][1] * v) % mod;
38         a[++n][1] = 0;
39         for(int i = n; i; i--) a[i][1] = (a[i - 1][1] + (LL)a[i][1] * (mod - x)) % mod;
40         a[0][1] = (LL)a[0][1] * (mod - x) % mod;
41     }
42 }p;
43 int main() {
44     ios::sync_with_stdio(0); cin.tie(0); cout.precision(6); cout << fixed;
45     int Q;
46     cin >> Q;
47     int op, x, y;
48     p.n = 0;
49     p.init();
50     while(Q--) {
51         cin >> op >> x;
52         if(op == 1) {
53             cin >> y;
54             p.update(x, y);
55         }
56         else cout << p.query(x) << endl;
57     }
58     return 0;
59 }

```

4.1.8 BM Alogrithm

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  #define rep(i,a,n) for (int i=a;i<n;i++)
4  #define per(i,a,n) for (int i=n-1;i>=a;i--)
5  #define pb push_back
6  #define mp make_pair
7  #define all(x) (x).begin(),(x).end()
8  #define fi first
9  #define se second
10 #define SZ(x) ((int)(x).size())
11 typedef vector<int> VI;
12 typedef long long ll;
13 typedef pair<int,int> PII;
14 const ll mod=1000000007;

```



```

15 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b;b>>=1){if(b&1)res=res*a%mod;
    a=a*a%mod;}return res;}
16 // head
17 namespace linear_seq {
18     const int N=10010;
19     ll res[N],base[N],_c[N],_md[N];
20
21     vector<int> Md;
22     void mul(ll *a,ll *b,int k) {
23         rep(i,0,k+k) _c[i]=0;
24         rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
25         for (int i=k+k-1;i>=k;i--) if (_c[i])
26             rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
27         rep(i,0,k) a[i]=_c[i];
28     }
29     int solve(ll n,VI a,VI b) { // a 系数 b 初值 b[n+1]=a[0]*b[n]+...
30         // printf("%d\n",SZ(b));
31         ll ans=0,pnt=0;
32         int k=SZ(a);
33         assert(SZ(a)==SZ(b));
34         rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
35         Md.clear();
36         rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
37         rep(i,0,k) res[i]=base[i]=0;
38         res[0]=1;
39         while ((1ll<pnt)<=n) pnt++;
40         for (int p=pnt;p>=0;p--) {
41             mul(res,res,k);
42             if ((n>p)&1) {
43                 for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
44                 rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
45             }
46         }
47         rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
48         if (ans<0) ans+=mod;
49         return ans;
50     }
51     VI BM(VI s) {
52         VI C(1,1),B(1,1);
53         int L=0,m=1,b=1;
54         rep(n,0,SZ(s)) {
55             ll d=0;
56             rep(i,0,L+1) d=(d+(ll)C[i]*s[n-i])%mod;
57             if (d==0) ++m;
58             else if (2*L<=n) {
59                 VI T=C;
60                 ll c=mod-d*powmod(b,mod-2)%mod;
61                 while (SZ(C)<SZ(B)+m) C.pb(0);
62                 rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
63                 L=n+1-L; B=T; b=d; m=1;
64             } else {
65                 ll c=mod-d*powmod(b,mod-2)%mod;
66                 while (SZ(C)<SZ(B)+m) C.pb(0);
67                 rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
68                 ++m;
69             }
70         }
71         return C;
72     }
73     int gao(VI a,ll n) {
74         VI c=BM(a);

```

```

75     c.erase(c.begin());
76     rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
77     return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
78 }
79 };
80
81 int main() {
82     while (~scanf("%d",&n)) {
83         vector<int>v;
84         v.push_back(1);
85         v.push_back(2);
86         v.push_back(4);
87         v.push_back(7);
88         v.push_back(13);
89         v.push_back(24);
90         //VI{1,2,4,7,13,24}
91         printf("%d\n",linear_seq::gao(v,n-1));
92     }
93 }

```

4.2 Math Theory

4.2.1 Inverse

```

1  //O(logn) 求n的逆元
2  const int mod = 1e6 + 3;
3  int exgcd(int a, int b, int &x, int &y) {
4      int d = a;
5      if(b != 0) {
6          d = exgcd(b, a % b, y, x);
7          y -= (a / b) * x;
8      }
9      else {
10         x = 1; y = 0;
11     }
12     return d;
13 }
14 int inverse(int a) {
15     int x, y;
16     exgcd(a, mod, x, y);
17     return (x % mod + mod) % mod;
18 }
19 int inverse(int a) { return qpow(a, mod - 2); }
20 //O(n) 求1~n的逆元
21 int inv[MAXN];
22 void init() {
23     inv[0] = inv[1] = 1;
24     for(int i = 2; i < MAXN; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
25     mod;
26 }

```

4.2.2 Lucas

```

1  //mod很小可以预处理逆元的情况
2  void init() {
3      fac[0] = 1;
4      for(int i = 1; i < mod; i++) fac[i] = (long long)fac[i - 1] * i % mod;
5      inv[0] = inv[1] = 1;

```

```

6   for(int i = 2; i < mod; i++) inv[i] = (long long)(mod - mod / i) * inv[mod % i] %
    mod;
7   for(int i = 1; i < mod; i++) inv[i] = (long long)inv[i] * inv[i - 1] % mod;
8 }
9 int C(int a, int b) {
10  if(b > a) return 0;
11  if(a < mod) return (long long)fac[a] * inv[b] % mod * inv[a - b] % mod;
12  return (long long)C(a / mod, b / mod) * C(a % mod, b % mod) % mod;
13 }
14 //mod过大不能预处理逆元的情况
15 LL qpow(LL x, LL y) {
16  LL res = 1;
17  while(y) {
18      if(y & 1) res = res * x % mod;
19      x = x * x % mod;
20      y >>= 1;
21  }
22  return res;
23 }
24 LL C(LL a, LL b) {
25  if(b > a) return 0;
26  if(b > a - b) b = a - b;
27  LL s1 = 1, s2 = 1;
28  for(LL i = 0; i < b; i++) {
29      s1 = s1 * (a - i) % mod;
30      s2 = s2 * (i + 1) % mod;
31  }
32  return s1 * qpow(s2, mod - 2) % mod;
33 }
34 LL lucas(LL a, LL b) {
35  if(a < mod) return C(a, b);
36  return lucas(a / mod, b / mod) * C(a % mod, b % mod);
37 }

```

4.2.3 CRT && exCRT

$$x \equiv a_i \pmod{m_i}$$

```

1 namespace CRT {
2   LL m[MAXN], a[MAXN];
3   LL exgcd(LL _a, LL _b, LL &x, LL &y) {
4       if(!_b) {
5           x = 1; y = 0;
6           return _a;
7       }
8       LL d = exgcd(_b, _a % _b, y, x);
9       y -= (_a / _b) * x;
10      return d;
11  }
12  LL crt(int n) {
13      LL M = 1, tmp, res = 0, x, y;
14      for(int i = 1; i <= n; i++) M *= m[i];
15      for(int i = 1; i <= n; i++) {
16          tmp = M / m[i];
17          exgcd(tmp, m[i], x, y);
18          x = (x + m[i]) % m[i];
19          res = (a[i] * x % M * tmp % M + res) % M;
20      }
21  }
22 }

```

```

21     return res;
22 }
23 }
24 namespace EXCRT {
25     LL m[MAXN], a[MAXN];
26     LL exgcd(LL _a, LL _b, LL &x, LL &y) {
27         if(!_b) {
28             x = 1; y = 0;
29             return _a;
30         }
31         LL d = exgcd(_b, _a % _b, y, x);
32         y -= (_a / _b) * x;
33         return d;
34     }
35     LL excrt(int n) {
36         LL M = m[1], A = a[1], x, y, d, tmp;
37         for(int i = 2; i <= n; i++) {
38             d = exgcd(M, m[i], x, y);
39             if((A - a[i]) % d) return -1; //No solution
40             tmp = M / d; M *= m[i] / d;
41             y = (A - a[i]) / d % M * y % M;
42             y = (y + tmp) % tmp;
43             A = (m[i] % M * y % M + a[i]) % M;
44             A = (A + M) % M;
45         }
46         return A;
47     }
48     LL inv(LL _a, LL _b) {
49         LL x, y;
50         exgcd(_a, _b, x, y);
51         return (x % _b + _b) % _b;
52     }
53     LL excrt(int n) {
54         LL M = m[1], A = a[1], x, y, d, c, tmp;
55         for(int i = 2; i <= n; i++) {
56             d = exgcd(M, m[i], x, y);
57             c = a[i] - A;
58             if(c % d) return -1;
59             c = (c % m[i] + m[i]) % m[i];
60             M /= d; m[i] /= d;
61             c = c / d * inv(M % m[i], m[i]) % m[i];
62             tmp = M;
63             M *= m[i] * d;
64             A = (c * tmp % M * d % M + A) % M;
65         }
66         return A;
67     }
68 }

```

4.2.4 BSGS

```

1  const int MOD = 76543;
2  int hs[MOD + 5], head[MOD + 5], nxt[MOD + 5], id[MOD + 5], ecnt;
3  void insert(int x, int y) {
4      int k = x % MOD;
5      hs[ecnt] = x, id[ecnt] = y, nxt[ecnt] = head[k], head[k] = ecnt++;
6  }
7  int find(int x) {
8      int k = x % MOD;
9      for(int i = head[k]; i; i = nxt[i])

```

```

10         if(hs[i] == x)
11             return id[i];
12     return -1;
13 }
14 int BSGS(int a, int b, int c){
15     memset(head, 0, sizeof head); ecnt = 1;
16     if(b == 1) return 0;
17     int m = sqrt(c * 1.0), j;
18     LL x = 1, p = 1;
19     for(int i = 0; i < m; i++, p = p * a % c)
20         insert(p * b % c, i);
21     for(LL i = m; ; i += m){
22         if((j = find(x = x * p % c)) != -1) return i - j;
23         if(i > c) break;
24     }
25     return -1;
26 }

```

4.2.5 Miller-Rabin & PollardRho

```

1 LL ksc(LL a, LL n, LL mod){
2     LL ret=0;
3     for(;n>=1){
4         if(n&1){ret+=a;if(ret>=mod)ret-=mod;}
5         a<<=1;if(a>=mod)a-=mod;
6     }
7     return ret;
8 }
9 LL ksm(LL a, LL n, LL mod){
10    LL ret = 1;
11    for(;n>=1){
12        if(n&1)ret=ksc(ret,a,mod);
13        a=ksc(a,a,mod);
14    }
15    return ret;
16 }
17 int millerRabin(LL n){
18     if(n<2 || (n!=2 && !(n&1)))return 0;
19     LL d=n-1;for(;!(d&1);d>>=1);
20     for(int i=0;i<20;++i){
21         LL a=rand()%(n-1)+1;
22         LL t=d,m=ksm(a,d,n);
23         for(;t!=n-1 && m!=1 && m!=n-1;m=ksc(m,m,n),t<<=1);
24         if(m!=n-1 && !(t&1)) return 0;
25     }
26     return 1;
27 }
28 LL cnt,fact[100];
29 LL gcd(LL a,LL b){return !b?a:gcd(b,a%b);}
30 LL pollardRho(LL n, int a){
31     LL x=rand()%n,y=x,d=1,k=0,i=1;
32     while(d==1){
33         ++k;
34         x=ksc(x,x,n)+a;if(x>=n)x-=n;
35         d=gcd(x>y?x-y:y-x,n);
36         if(k==i){y=x;i<<=1;}
37     }
38     if(d==n)return pollardRho(n,a+1);
39     return d;
40 }

```

```

41 void findfac(LL n){
42     if(millerRabin(n)){fact[++cnt]=n;return;}
43     LL p=pollardRho(n,rand()%(n-1)+1);
44     findfac(p);
45     findfac(n/p);
46 }

```

4.2.6 $\varphi(n)$

```

1  int phi(int x) {
2      int res = x;
3      for(int i = 2; i * i <= x; i++) {
4          if(x % i == 0) {
5              res = res / i * (i - 1);
6              while(x % i == 0) x /= i;
7          }
8      }
9      if(x > 1) res = res / x * (x - 1);
10     return res;
11 }

```

4.2.7 Euler Sieve

```

1  int prime[MAXN], cnt, phi[MAXN], mu[MAXN];
2  bool isp[MAXN];
3
4  int min_pow[MAXN]; //最小质因子最高次幂
5  int min_sum[MAXN]; //1+p+p^2+...+p^k
6  int div_sum[MAXN]; //约数和
7
8  int min_index[MAXN]; //最小质因子的指数
9  int div_num[MAXN]; //约数个数
10 void Euler(int n) {
11     mu[1] = phi[1] = div_num[1] = div_sum[1] = 1;
12     for(int i = 2; i <= n; i++) {
13         if(!isp[i]) {
14             prime[++cnt] = min_pow[i] = i;
15             phi[i] = i - 1;
16             mu[i] = -1;
17             min_index[i] = 1; div_num[i] = 2;
18             div_sum[i] = min_sum[i] = i + 1;
19         }
20         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
21             isp[i * prime[j]] = 1;
22             if(i % prime[j] == 0) {
23                 phi[i * prime[j]] = phi[i] * prime[j];
24                 mu[i * prime[j]] = 0;
25
26                 min_index[i * prime[j]] = min_index[i] + 1;
27                 div_num[i * prime[j]] = div_num[i] / (min_index[i] + 1) * (min_index[i] *
prime[j] + 1);
28
29                 min_sum[i * prime[j]] = min_sum[i] + min_pow[i] * prime[j];
30                 div_sum[i * prime[j]] = div_sum[i] / min_sum[i] * min_sum[i * prime[j]];
31                 min_pow[i * prime[j]] = min_pow[i] * prime[j];
32                 break;
33             }
34             phi[i * prime[j]] = phi[i] * (prime[j] - 1);

```

```

35     mu[i * prime[j]] = -mu[i];
36
37     div_num[i * prime[j]] = div_num[i] << 1;
38     min_index[i * prime[j]] = 1;
39
40     div_sum[i * prime[j]] = div_sum[i] * (prime[j] + 1);
41     min_pow[i * prime[j]] = prime[j];
42     min_sum[i * prime[j]] = prime[j] + 1;
43 }
44 }
45 }

```

4.2.8 DuJiao Sieve

$$\sum_{i=1}^n \varphi(i)$$

```

1  vector<int> prime;
2  int phi[MAXN], P[MAXN];
3  bool isp[MAXN];
4  unordered_map<LL, int> mp;
5  void Euler(int n) {
6      phi[1] = 1;
7      for(int i = 2; i <= n; i++) {
8          if(!isp[i]) {
9              prime.push_back(i);
10             phi[i] = i - 1;
11         }
12         for(auto x : prime) {
13             if(i * x > n) break;
14             isp[i * x] = 1;
15             if(i % x == 0) {
16                 phi[i * x] = phi[i] * x;
17                 break;
18             }
19             phi[i * x] = phi[i] * (x - 1);
20         }
21     }
22     for(int i = 1; i <= n; i++) P[i] = (P[i - 1] + phi[i]) % mod;
23 }
24 LL cal(LL n) {
25     if(n < MAXN) return P[n];
26     if(mp.count(n)) return mp[n];
27     LL res = 0;
28     for(LL i = 2, last; i <= n; i = last + 1) {
29         last = n / (n / i);
30         res += (last - i + 1) % mod * cal(n / i) % mod;
31         res %= mod;
32     }
33     mp[n] = ((__int128)n * (n + 1) / 2 % mod + mod - res) % mod;
34     return mp[n];
35 }

```

$$\sum_{i=1}^n \mu(i)$$

```

1  LL cal(LL n) {
2      if(n < MAXN) return M[n];

```

```

3  if(mp.count(n)) return mp[n];
4  LL res = 0;
5  for(LL i = 2, last; i <= n; i = last + 1) {
6      last = n / (n / i);
7      res += (last - i + 1) * cal(n / i);
8  }
9  mp[n] = 1 - res;
10 return 1 - res;
11 }

```

4.2.9 Möbius Inversion

$$\sum_i^n \sum_j^m lcm(i, j) \pmod{p}$$

```

1  int mu[MAXN], prime[MAXN], sum[MAXN], cnt;
2  bool isp[MAXN];
3  void getmu(int n) {
4      mu[1] = 1;
5      for(int i = 2; i <= n; i++) {
6          if(!isp[i]) {
7              mu[i] = -1;
8              prime[++cnt] = i;
9          }
10         for(int j = 1; j <= cnt && i * prime[j] <= n; j++) {
11             isp[i * prime[j]] = 1;
12             if(i % prime[j] == 0) {
13                 mu[i * prime[j]] = 0;
14                 break;
15             }
16             mu[i * prime[j]] = -mu[i];
17         }
18     }
19 }
20 ll n, m, ans;
21 ll query(ll x, ll y) { return (x * (x + 1) / 2 % mod) * (y * (y + 1) / 2 % mod) % mod; }
22 ll F(ll x, ll y) {
23     ll res = 0, last;
24     for(ll i = 1; i <= min(x, y); i = last + 1) {
25         last = min(x / (x / i), y / (y / i));
26         res = (res + (sum[last] - sum[i - 1]) * query(x / i, y / i) % mod) % mod;
27     }
28     return res;
29 }
30 int main() {
31     cin >> n >> m;
32     getmu(min(n, m));
33     for(ll i = 1; i <= min(n, m); i++) sum[i] = (sum[i - 1] + (i * i * mu[i]) % mod) % mod;
34     ll last;
35     for(ll d = 1; d <= min(n, m); d = last + 1) {
36         last = min(n / (n / d), m / (m / d));
37         ans = (ans + (last - d + 1) * (d + last) / 2 % mod * F(n / d, m / d) % mod) % mod;
38     }
39     ans = (ans + mod) % mod;
40     cout << ans << endl;
41     return 0;

```


42 }

5 Geometry

5.1 Commonly Definition and Functions

5.1.1 Const and Functions

```

1 namespace CG{
2     #define Point Vector
3     const double pi=acos(-1.0);
4     const double inf=1e100;
5     const double eps=1e-9;
6     template <typename T> inline T Abs(T x){return x>0?x:-x;}
7     template <typename T> inline bool operator == (T x,T y){return Abs(x-y)<eps;}
8     int sgn(double x){
9         if (Abs(x)<eps) return 0;
10        if (x>0) return 1;
11        else return -1;
12    }
13 }

```

5.1.2 Point Definition

```

1 namespace CG{
2     struct Point{
3         double x,y;
4         Point(double x=0,double y=0):x(x),y(y){}
5     };
6     Vector operator + (const Vector a,const Vector b){return Vector(a.x+b.x,a.y+b.y);}
7     Vector operator - (const Vector a,const Vector b){return Vector(a.x-b.x,a.y-b.y);}
8     Vector operator * (const Vector a,const double k){return Vector(a.x*k,a.y*k);}
9     Vector operator / (const Vector a,const double k){return Vector(a.x/k,a.y/k);}
10    bool operator < (const Vector a,const Vector b) {return a.x==b.x?a.y<b.y:a.x<b.x;}
11    bool operator == (const Vector a,const Vector b) {return a.x==b.x && a.y==b.y;}
12    double Dot(const Vector a,const Vector b){return a.x*b.x+a.y*b.y;}
13    double Cross(const Vector a,const Vector b){return a.x*b.y-a.y*b.x;}
14    double mult_Cross(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.
y-c.y)-(b.x-c.x)*(a.y-c.y);}
15    double mult_Dot(const Vector a,const Vector b,const Vector c){return (a.x-c.x)*(b.x-
c.x)+(a.y-c.y)*(b.y-c.y);}
16    double Norm(const Vector a){return sqrt(Dot(a,a));}
17    double Angle(const Vector a,const Vector b){return acos(Dot(a,b)/Norm(a)/Norm(b));}
18    Vector Rotate(const Vector a,const double theta){return Vector(a.x*cos(theta)-a.y*
sin(theta),a.x*sin(theta)+a.y*cos(theta));}
19    boolToLeftTest(const Vector a,const Vector b){return Cross(a,b)<0;}
20    double DisPP(const Vector a,const Vector b){return sqrt((a.x-b.x)*(a.x-b.x)+(a.y-b.y
)*(a.y-b.y));}
21 }

```

5.1.3 Line Definition

```

1 namespace CG{
2     struct Line{
3         Point p0,v,p1;
4         double t,theta;
5         Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),v(_v),t(_t){p1=p0+v*t; theta=
atan2(v.y,v.x);}

```

```

6      // Line(Point _p0=0,Point _v=0,double _t=1):p0(_p0),p1(_v){v=(p1-p0)/t; theta=
      atan2(v.y,v.x);}
7  };
8  bool operator < (const Line n,const Line m) {return n.theta<m.theta;}
9  Point GetIntersection(const Line n,const Line m){return n.p0+n.v*Cross(m.v,(n.p0-m.
  p0))/Cross(n.v,m.v);}
10 bool OnLine(const Vector a,const Line l){return Cross(l.p0-a,l.p1-a)==0;}
11 bool OnSegment(const Point a,const Line l){return sgn(Cross(l.p0-a,l.p1-a))==0 &&
  sgn(Dot(l.p0-a,l.p1-a))<0;}
12 double DisPL(const Point a,const Line l){return Abs(Cross(l.p1-l.p0,a-l.p0)/Norm(l.
  p1-l.p0));}
13 double DisPS(const Point a,const Line l){
14     if (l.p0==l.p1) return Norm(a-l.p0);
15     Vector v1=l.p1-l.p0,v2=a-l.p0,v3=a-l.p1;
16     if (sgn(Dot(v1,v2))<0) return Norm(v2);
17     if (sgn(Dot(v1,v3))>0) return Norm(v3);
18     return DisPL(a,l);
19 }
20 Point GetProjection(const Point a,const Line l){
21     Vector v=l.p1-l.p0;
22     return l.p0+v*(Dot(v,a-l.p0)/Dot(v,v));
23 }
24 bool SegmentIntersection(const Line n,const Line m,bool p){
25     double c1=Cross(n.p1-n.p0,m.p1-m.p0);
26     double c2=Cross(n.p1-n.p0,m.p1-n.p0);
27     double c3=Cross(m.p1-m.p0,n.p0-m.p0);
28     double c4=Cross(m.p1-m.p0,n.p1-m.p0);
29     if (p){
30         if (!sgn(c1) || !sgn(c2) || !sgn(c3) || !sgn(c4)){
31             return OnSegment(n.p0,m) || OnSegment(n.p1,m) || OnSegment(m.p0,n) ||
  OnSegment(m.p0,m);
32         }
33     }
34     }
35     return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);
36 }
37 }

```

5.1.4 Get Area

```

1 namespace CG{
2     double GetArea(Point *p,int n){
3         double area=Cross(p[n],p[1]);
4         for (int i=2;i<=n;i++) area+=0.5*Cross(p[i-1],p[i]);
5         return Abs(area);
6     }
7 }

```

5.1.5 Get Circumference

```

1 namespace CG{
2     double GetCircumference(Point *p,int n){
3         double Circumference=DisPP(p[n],p[1]);
4         for (int i=2;i<=n;i++) Circumference+=DisPP(p[i-1],p[i]);
5         return Circumference;
6     }
7 }

```

5.1.6 Anticlockwise Sort

```

1 namespace CG{
2     \p
3     void clockwise_sort(Point *p,int n){
4         for(int i=0;i<n-2;i++){
5             double tmp = mult_Cross(p[i+1],p[i+2],p[i]);
6             if(tmp>0) return;
7             else if(tmp<0){
8                 reverse(p,p+n);
9                 return;
10            }
11        }
12    }
13 }

```

5.2 Convex Hull

5.2.1 Get Convex Hull

```

1 namespace CG{
2     Point p[MAXN],s[MAXN];
3     int ConvexHull(Point *p,int n,Point *s){
4         sort(p,p+n,cmp); //x从小到大,y从小到大;
5         int m=0;
6         for (int i=0;i<n;i++){
7             for (;m>=2 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
8             s[++m]=p[i];
9         }
10        int k=m;
11        for (int i=n-2;i;i--){
12            for (;m>=k+1 && Cross(s[m-1]-s[m-2],p[i]-s[m-1])<=0;m--);
13            s[++m]=p[i];
14        }
15        return m-1;
16    }
17 }

```

5.2.2 Point in Convex Hull

```

1 namespace CG{
2     bool PointInConvexHull(Point A){
3         int l=1,r=tot-2,mid;
4         while(l<=r){
5             mid=(l+r)>>1;
6             double a1=Cross(p[mid]-p[0],A-p[0]);
7             double a2=Cross(p[mid+1]-p[0],A-p[0]);
8             if(a1>=0 && a2<=0){
9                 if(Cross(p[mid+1]-p[mid],A-p[mid])>=0) return true;
10                return false;
11            }
12            else if(a1<0) r=mid-1;
13            else l=mid+1;
14        }
15        return false;
16    }
17 }

```

5.3 Minkowski Sum

```

1 namespace CG{
2     void Minkowski(Point *C1,int n,Point *C2,int m){
3         for(int i=1;i<=n;i++) s1[i]=C1[i]-C1[i-1];
4         for(int i=1;i<=m;i++) s2[i]=C2[i]-C2[i-1];
5         A[tot=1]=C1[1]+C2[1];
6         int p1=1,p2=1;
7         while (p1<=n && p2<=m) ++tot,A[tot]=A[tot-1]+(s1[p1]*s2[p2]>=0?s1[p1++]:s2[p2
++]);
8         while (p1<=n) ++tot,A[tot]=A[tot-1]+s1[p1++];
9         while (p2<=m) ++tot,A[tot]=A[tot-1]+s2[p2++];
10        tot=ConvexHull(A,tot);
11    }
12 }

```

5.4 Rotating Calipers

5.4.1 The Diameter of Convex Hull

```

1 namespace CG{
2     double RotatingCalipers(Point *p,int n){
3         double dis=0;
4         for(int i=0,j=2;i<n;++i){
5             while (abs(Cross(p[i+1]-p[i],p[j]-p[i]))<abs(Cross(p[i+1]-p[i],p[j+1]-p[i])))
6             ) j=(j+1)%n;
7             dis=max(dis,max(DisPP(p[j],p[i]),DisPP(p[j],p[i+1])));
8         }
9         return dis;
10    }
11 }

```

5.4.2 The Min Distance Between two Convex Hull

```

1 namespace CG{
2     ///点c到线段ab的最短距离
3     double GetDist(Point a,Point b,Point c){
4         if(dis(a,b)<esp) return dis(b,c); ///a,b是同一个点
5         if(mult_Dot(b,c,a)<-esp) return dis(a,c); ///投影
6         if(mult_Dot(a,c,b)<-esp) return dis(b,c);
7         return fabs(mult_Cross(b,c,a)/dis(a,b));
8     }
9
10    ///求一条线段ab的两端点到另外一条线段bc的距离，反过来一样，共4种情况
11    double MinDist(Point a,Point b,Point c,Point d){
12        return min(min(GetDist(a,b,c),GetDist(a,b,d)),min(GetDist(c,d,a),GetDist(c,d,b))
13    );
14    }
15    double RotatingCalipers(Point *p,int n,Point *q,int m){
16        int yminP = 0,ymaxQ=0;
17        for(int i=1;i<n;i++){ ///找到点集p组成的凸包的左下角
18            if(p[i].y<p[yminP].y || (p[i].y==p[yminP].y)&&(p[i].x<p[yminP].x)) yminP = i;
19        }
20        for(int i=1;i<m;i++){ ///找到点集q组成的凸包的右上角
21            if(q[i].y>q[ymaxQ].y || (q[i].y==q[ymaxQ].y)&&(q[i].x>q[ymaxQ].x)) ymaxQ = i;
22        }
23        double ans = DisPP(p[yminP],q[ymaxQ]); ///距离(yminP,ymaxQ)维护为当前最小值。
24    }
25 }

```

```

23     for(int i=0;i<n;i++){
24         double tmp;
25         while(tmp=(mult_Cross(q[ymaxQ+1],p[yminP],p[yminP+1])-mult_Cross(q[ymaxQ],p[
yminP],p[yminP+1]))>esp)
26             ymaxQ = (ymaxQ+1)%m;
27         if(tmp<=-esp) ans = min(ans,GetDist(p[yminP],p[yminP+1],q[ymaxQ]));
28         else ans=min(ans,MinDist(p[yminP],p[yminP+1],q[ymaxQ],q[ymaxQ+1]));
29         yminP = (yminP+1)%n;
30     }
31     return ans;
32 }
33 }

```

5.5 Half Plane Intersection

```

1 namespace CG{
2     void HalfPlaneIntersection(Line l[],int n){
3         deque <Point> p;
4         sort(l+1,l+1+n);
5         deque <Line> q;
6         q.push_back(l[1]);
7         for (int i=2;i<=n;i++){
8             for (;!p.empty() && !ToLeftTest(p.back()-l[i].p0,l[i].v);q.pop_back(),p.
pop_back());
9             for (;!p.empty() && !ToLeftTest(p.front()-l[i].p0,l[i].v);q.pop_front(),p.
pop_front());
10            if (sgn(Cross(l[i].v,q.back().v))==0)
11                if (ToLeftTest(l[i].p0-q.back().p0),q.back().v){
12                    q.pop_back();
13                    if (!p.empty()) p.pop_back();
14                }
15            if (!q.empty()) p.push_back(GetIntersection(q.back(),l[i]));
16            q.push_back(l[i]);
17        }
18        for (;!p.empty() && !ToLeftTest(p.back()-q.front().p0,q.front().v);q.pop_back(),
p.pop_back());
19        p.push_back(GetIntersection(q.back(),q.front()));
20        double area=0.5*Cross(p.back(),p.front()); Point last=p.front();
21        for (p.pop_front();!p.empty();last=p.front(),p.pop_front()) area+=0.5*Cross(last
,p.front());
22        printf("%.1f",Abs(area));
23    }
24 }

```

5.6 Min Circle Cover

```

1 namespace CG{
2     Point GetCircleCenter(const Point a,const Point b,const Point c){
3         Point p=(a+b)/2.0,q=(a+c)/2.0;
4         Vector v=Rotate(b-a,pi/2.0),w=Rotate(c-a,pi/2.0);
5         if (sgn(Norm(Cross(v,w)))==0){
6             if (sgn(Norm(a-b)+Norm(b-c)-Norm(a-c))==0) return (a+c)/2;
7             if (sgn(Norm(b-a)+Norm(a-c)-Norm(b-c))==0) return (b+c)/2;
8             if (sgn(Norm(a-c)+Norm(c-b)-Norm(a-b))==0) return (a+c)/2;
9         }
10        return GetIntersection(Line(p,v),Line(q,w));
11    }
12    void MinCircleCover(Point p[],int n){

```

```

13     random_shuffle(p+1,p+1+n);
14     Point c=p[1];
15     double r=0;
16     for (int i=2;i<=n;i++)
17         if (sgn(Norm(c-p[i])-r)>0){
18             c=p[i],r=0;
19             for (int j=1;j<i;j++)
20                 if (sgn(Norm(c-p[j])-r)>0){
21                     c=(p[i]+p[j])/2.0;
22                     r=Norm(c-p[i]);
23                     for (int k=1;k<j;k++)
24                         if (sgn(Norm(c-p[k])-r)>0){
25                             c=GetCircleCenter(p[i],p[j],p[k]);
26                             r=Norm(c-p[i]);
27                         }
28                 }
29     }
30     printf("%.10f\n%.10f %.10f",r,c.x,c.y);
31 }
32 }

```

5.7 Circle Union Area

```

1 //k次覆盖
2 //圆并去重后s[0]
3 typedef pair<double, int> P;
4 const double pi = acos(-1.0);
5 const int MAXN = 10003;
6 P arc[MAXN << 1];
7 int acnt, cnt;
8 double s[1003];
9 bool del[1003];
10 void add(double st, double en) {
11     if(st < -pi) {
12         add(st + 2 * pi, pi);
13         add(-pi, en);
14         return;
15     }
16     if(en > pi) {
17         add(st, pi);
18         add(-pi, en - 2 * pi);
19         return;
20     }
21     arc[++acnt] = P(st, 1);
22     arc[++acnt] = P(en, -1);
23 }
24 double F(double x) {
25     return (x - sin(x)) / 2;
26 }
27 struct Node {
28     int x, y, r;
29     Node(int _x = 0, int _y = 0, int _r = 0):x(_x), y(_y), r(_r) {}
30     bool operator == (const Node& t) {
31         return x == t.x && y == t.y && r == t.r;
32     }
33     inline void read() {
34         scanf("%d%d%d", &x, &y, &r);
35     }
36 }a[1003];

```

```

37 int main() {
38     int n;
39     scanf("%d", &n);
40     for(int i = 1; i <= n; i++) a[i].read();
41     /*
42     //去重
43     int nn = 0;
44     for(int i = 1; i <= n; i++) {
45         bool same = 0;
46         for(int j = 1; j < i; j++) {
47             if(a[i] == a[j]) {
48                 same = 1; break;
49             }
50         }
51         if(!same) a[++nn] = a[i];
52     }
53     n = nn;
54     //去包含
55     for(int i = 1; i <= n; i++) {
56         for(int j = 1; j <= n; j++) if(i != j) {
57             if(hypot(a[i].x - a[j].x, a[i].y - a[j].y) < (double)(a[i].r - a[j].r)) del[
58                 j] = 1;
59         }
60     }
61     nn = 0;
62     for(int i = 1; i <= n; i++) if(!del[i]) {
63         a[++nn] = a[i];
64     }
65     n = nn;
66     /*
67     for(int i = 1; i <= n; i++) {
68         acnt = 0;
69         for(int j = 1; j <= n; j++) if(i != j) {
70             int dis = (a[i].x - a[j].x) * (a[i].x - a[j].x) + (a[i].y - a[j].y) * (a[i].
71             y - a[j].y);
72             if(a[j].r > a[i].r && dis <= (a[j].r - a[i].r) * (a[j].r - a[i].r)) add(-pi,
73             pi);
74             else if(dis > (a[i].r - a[j].r) * (a[i].r - a[j].r) && dis < (a[i].r + a[j].
75             r) * (a[i].r + a[j].r)){
76                 double c = sqrt(dis);
77                 double angle = acos((a[i].r * a[i].r + c * c - a[j].r * a[j].r) / (2 * a
78                 [i].r * c));
79                 double k = atan2(a[j].y - a[i].y, a[j].x - a[i].x);
80                 add(k - angle, k + angle);
81             }
82         }
83         arc[++acnt] = P(pi, -1);
84         sort(arc + 1, arc + acnt + 1);
85         cnt = 0;
86         double last = -pi;
87         for(int j = 1; j <= acnt; j++) {
88             s[cnt] += F(arc[j].first - last) * a[i].r * a[i].r; //扇形 - 三角形
89             double xa = a[i].x + a[i].r * cos(last);
90             double ya = a[i].y + a[i].r * sin(last);
91             last = arc[j].first;
92             double xb = a[i].x + a[i].r * cos(last);
93             double yb = a[i].y + a[i].r * sin(last);
94             s[cnt] += (xa * yb - xb * ya) / 2; //到圆心的三角形面积
95             cnt += arc[j].second;
96         }
97     }
98 }

```



```
93 //printf("%.3f\n", s[0]);
94 for (int i = 0; i < n; i++) {
95     printf("[%d] = %.3f\n", i + 1, s[i] - s[i + 1]);
96 }
97 return 0;
98 }
```

5.8 Simpson Integrate

```
1 double Simpson(double l, double r){
2     return (r-l)*(F(l)+4*F((l+r)/2)+F(r))/6;
3 }
4 double Integrate(double l, double r, double S){
5     double mid=(l+r)/2;
6     double A=Simpson(l, mid);
7     double B=Simpson(mid, r);
8     if(A+B-S<eps) return S;
9     return Integrate(l, mid, A)+Integrate(mid, r, B);
10 }
```

6 Conclusion

6.1 Game theory

6.1.1 Fibonacci's Game / Zeckendorf's theory

Fibonacci's Game (斐波那契博弈)

有一堆个数为 n 的石子，游戏双方轮流取石子，满足：

1. 先手不能在第一次把所有的石子取完；
2. 之后每次可以取的石子数介于 1 到对手刚取的石子数的 2 倍之间（包含 1 和对手刚取的石子数的 2 倍）。

结论：必败点是斐波那契数

齐肯多夫定理：任何正整数可以表示为若干个不连续的 Fibonacci 数之和

7 Others

7.1 Sample

7.1.1 vimrc

```

1 set cindent
2 set number
3 set mouse=a
4 set tabstop=4
5 set shiftwidth=4
6 syntax on
7 inoremap { {}<left>
8 map <F9> :w<CR> :! g++ % -o %< -Wall --std=c++14 -g && ./%< <CR>

```

7.1.2 Check

```

1 while true; do
2     ./data > in
3     ./tmp < in > out
4     ./std < in > ans
5     diff out ans
6     if [ $? -ne 0 ] ; then exit; fi
7     echo Passed
8 done

```

7.1.3 FastIO

```

1 namespace IO {
2     const int MB = 1048576;
3     const int RMAX = 16 * MB;
4     const int WMAX = 16 * MB;
5     #define getchar() *(rp++)
6     #define putchar(x) (*(wp++) = (x))
7     char rb[RMAX], *rp = rb, wb[WMAX], *wp = wb;
8     inline void init() {
9         fread(rb, sizeof(char), RMAX, stdin);
10    }
11    template <class _T> inline void read(_T &a) {
12        _a = 0; register bool _f = 0; register int _c = getchar();
13        while (_c < '0' || _c > '9') _f != _c == '-', _c = getchar();
14        while (_c >= '0' && _c <= '9') _a = _a * 10 + (_c ^ '0'), _c = getchar();
15        _a = _f ? -_a : _a;
16    }
17    template <class _T> inline void write(_T _a) {
18        static char buf[20], *top = buf;
19        if (_a) {
20            while (_a) {
21                register _T tm = _a / 10;
22                *(++top) = char(_a - tm * 10) | '0';
23                _a = tm;
24            }
25            while (top != buf) putchar(*(top--));
26        }
27        else putchar('0');
28    }
29 }

```

```

29     void output() {
30         fwrite(wb, sizeof(char), wp - wb, stdout);
31     }
32 }

```

7.1.4 Java BigNum

```

1  import java.math.*;
2  import java.util.*;
3  import java.lang.*;
4
5  public class Main{
6      public static void main(String []args){}
7  }
8  //IO
9  Scanner in = new Scanner(System.in);
10 while(in.hasNext()){} //EOF
11 //fast-IO
12 public static void main(String argv[]) throws IOException{
13     StreamTokenizer cin = new StreamTokenizer(new BufferedReader(new InputStreamReader(
14         System.in)));
15     PrintWriter cout = new PrintWriter(new OutputStreamWriter(System.out));
16     while(cin.nextToken() != StreamTokenizer.TT_EOF) ;//EOF
17     cin.nextToken();int n = (int)cin.nval;String s = cin.sval;
18     cout.println( Type );cout.flush();
19     cin.ordinaryChar('/');
20     BufferedReader br = new BufferedReader(new InputStreamReader(System.in));
21     br.ready();//EOF
22     while ((valueString=bf.readLine())!=null);
23     br.close();
24     //true fast-IO
25     static class InputReader {
26         public BufferedReader reader;
27         public StringTokenizer tokenizer;
28
29         public InputReader(InputStream stream) {
30             reader = new BufferedReader(new InputStreamReader(stream), 32768);
31             tokenizer = null;
32         }
33
34         public String next() {
35             while (tokenizer == null || !tokenizer.hasMoreTokens()) {
36                 try {
37                     tokenizer = new StringTokenizer(reader.readLine());
38                 } catch (IOException e) {
39                     throw new RuntimeException(e);
40                 }
41             }
42             return tokenizer.nextToken();
43         }
44
45         public int nextInt() {
46             return Integer.parseInt(next());
47         }
48     }
49 }
50 //类 Number
51 //doubleValue()
52 //intValue()

```

```

53 //longValue()
54 //shortValue()
55 //类 BigDecimal
56 //ROUND_CEILING 接近正无穷大的舍入模式。
57 //ROUND_FLOOR 接近负无穷大的舍入模式。
58 //ROUND_DOWN 接近零的舍入模式
59 //ROUND_HALF_UP 四舍五入 >=0.5向上舍入
60 //ROUND_HALF_DOWN 四舍五入 >0.5向上舍入
61 //BigDecimal(BigInteger val)
62 //BigDecimal(BigInteger unscaledVal, int scale)
63 //BigDecimal(char[] in, int offset, int len, MathContext mc)
64 //BigDecimal(double val, MathContext mc)不建议
65 //BigDecimal(int val, MathContext mc)
66 //BigDecimal(long val, MathContext mc)
67 //BigDecimal(String val, MathContext mc)
68 //abs()
69 //add(BigDecimal augend, MathContext mc)
70 //compareTo(BigDecimal val)
71 //divide(BigDecimal divisor, MathContext mc)
72 //divideToIntegralValue(BigDecimal divisor, MathContext mc)
73 //max(BigDecimal val)
74 //min(BigDecimal val)
75 //multiply(BigDecimal multiplicand, MathContext mc)
76 //negate() 其值为  $(-this)$ , 其标度为  $this.scale()$ 
77 //pow(int n)
78 //remainder(BigDecimal divisor) 返回其值为  $(this \% divisor)$  的 BigDecimal
79 //round(MathContext mc) 返回根据 MathContext 设置进行舍入后的 BigDecimal。
80 //scaleByPowerOfTen(int n) 返回其数值等于  $(this * 10^n)$  的 BigDecimal。
81 //subtract(BigDecimal subtrahend, MathContext mc)
82 //setScale(int newScale, RoundingMode roundingMode)
83 //toString()
84 //ulp() 返回此 BigDecimal 的 ulp (最后一位的单位) 的大小
85 //String s = b.stripTrailingZeros().toPlainString(); 让 bigdecimal 不用科学计数法显示
86 //类 BigInteger
87 //parseInt
88 //BigInteger zero = BigInteger.valueOf(0);
89 //BigInteger a = in.nextBigInteger();
90 //abs()
91 //and(BigInteger val) 返回其值为  $(this \& val)$ 
92 //or(BigInteger val) 返回其值为  $(this | val)$ 
93 //andNot(BigInteger val) 返回其值为  $(this \& \sim val)$ 
94 //compareTo(BigInteger val)
95 //add(BigInteger val)
96 //divide(BigInteger val)
97 //BigInteger[] divideAndRemainder(BigInteger val) 返回包含  $(this / val)$  后跟  $(this \% val)$  的两个 BigInteger 的数组。
98 //equals(Object x)
99 //gcd(BigInteger val)
100 //isProbablePrime(int certainty) e.g.: a.isProbablePrime(4)
101 //max(BigInteger val) min(BigInteger val)
102 //mod(BigInteger m)
103 //modInverse(BigInteger m) 返回其值为  $(this^{-1} \bmod m)$ 
104 //modPow(BigInteger exponent, BigInteger m) 返回其值为  $(this^{exponent} \bmod m)$ 
105 //multiply(BigInteger val)
106 //not() 返回其值为  $(\sim this)$ 
107 //shiftLeft(int n) 返回其值为  $(this \ll n)$ 
108 //shiftRight(int n) 返回其值为  $(this \gg n)$ 
109 //toString()
110 //valueOf(long val)
111 //xor(BigInteger val) 返回其值为  $(this \wedge val)$ 
112 //other

```

```
113 //Arrays.sort(array);
```

7.2 Offline Algorithm

7.2.1 CDQ Divide and Conquer

```

1 struct Node {
2     int x, y, z, ans;
3     Node() {}
4     Node(int _x, int _y, int _z):x(_x), y(_y), z(_z) {}
5     bool operator < (const Node &b) const {
6         if(y == b.y) {
7             if(z == b.z) return x < b.x;
8             return z < b.z;
9         }
10        return y < b.y;
11    }
12 }A[MAXN], B[MAXN], C[MAXN];
13 int bit[MAXN];
14 void add(int k, int v) {
15     for(; k <= m; k += k & -k) bit[k] = max(bit[k], v);
16 }
17 void clear(int k) {
18     for(; k <= m; k += k & -k) bit[k] = 0;
19 }
20 int sum(int k) {
21     int res = 0;
22     for(; k; k -= k & -k) res = max(res, bit[k]);
23     return res;
24 }
25 void solve(int l, int r) {
26     if(l == r) {
27         B[l] = A[l];
28         return;
29     }
30     int mid = (l + r) >> 1;
31     solve(l, mid);
32     for(int i = mid + 1; i <= r; i++) B[i] = A[i];
33     //sort(B + l, B + mid + 1);
34     sort(B + mid + 1, B + r + 1);
35     int L = l;
36     for(int R = mid + 1; R <= r; R++) {
37         while(L <= mid && B[L].y < B[R].y) add(B[L].z, B[L].ans), L++;
38         A[B[R].x].ans = max(A[B[R].x].ans, sum(B[R].z - 1) + 1);
39         B[R].ans = A[B[R].x].ans;
40     }
41     for(int i = l; i <= L; i++) clear(B[i].z);
42     solve(mid + 1, r);
43     L = l;
44     int p = l, q = mid + 1;
45     while(p <= mid || q <= r) {
46         if(q > r || (p <= mid && B[p].y <= B[q].y)) C[L++] = B[p++];
47         else C[L++] = B[q++];
48     }
49     for(int i = l; i <= r; i++) B[i] = C[i];
50 }

```

7.2.2 Mo' s Algorithm

```

1 struct Node{
2     int l, r, t, id;
3     bool operator < (const Node& a) const {
4         if(l / sz == a.l / sz) {
5             if(r == a.r) return t < a.t;
6             return r < a.r;
7         }
8         return l / sz < a.l / sz;
9     }
10 }q[MAXN];
11 void solve() {
12     while (t < q[i].t) addTime(t++, 1);
13     while (t > q[i].t) addTime(--t, -1);
14     while(L < q[i].l) add(L++, -1);
15     while(L > q[i].l) add(--L, 1);
16     while(R < q[i].r) add(++R, 1);
17     while(R > q[i].r) add(R--, -1);
18 }

```

7.2.3 Mo's Algorithm On Tree

```

1 struct Edge {
2     int to, nxt;
3 }e[MAXN << 1];
4 int head[MAXN], ecnt;
5 int stack[MAXN], top, belong[MAXN], cnt, sz;
6 struct Node {
7     int l, r, id, ti;
8     bool operator < (const Node &x) const {
9         return belong[l] < belong[x.l] || (belong[l] == belong[x.l] && belong[r] <
10         belong[x.r]) || (belong[l] == belong[x.l] && belong[r] == belong[x.r] && ti < x.ti);
11 }
12 }q[MAXN];
13 struct Node2 {
14     int l, r, ti;
15 }qq[MAXN];
16 int n, m, Q, Q0, Q1;
17 int V[MAXN], W[MAXN], C[MAXN];
18 int fa[MAXN][S + 3], dep[MAXN];
19 long long ans[MAXN], tans;
20 int vis[MAXN], cur[MAXN];
21 long long sum[MAXN];
22 int l, r, tm;
23 inline int read() {
24     int x = 0; char ch = getchar(); bool fg = 0;
25     while(ch < '0' || ch > '9') { if(ch == '-') fg = 1; ch = getchar(); }
26     while(ch >= '0' && ch <= '9') { x = x * 10 + ch - '0'; ch = getchar(); }
27     return fg ? -x : x;
28 }
29 inline void add_edge(int u, int v) {
30     e[++ecnt] = (Edge) {v, head[u]}; head[u] = ecnt;
31     e[++ecnt] = (Edge) {u, head[v]}; head[v] = ecnt;
32 }
33 void dfs(int u, int f) {
34     fa[u][0] = f;
35     dep[u] = dep[f] + 1;
36     int bot = top;
37     for(int i = head[u]; i; i = e[i].nxt) {
38         int v = e[i].to;
39         if(v == f) continue;

```

```

39     dfs(v, u);
40     if(top - bot >= sz) {
41         cnt++;
42         while(top != bot) belong[stack[top--]] = cnt;
43     }
44 }
45 stack[++top] = u;
46 }
47 void G(int &u, int step) {
48     for(int i = 0; i < S; i++) if((1 << i) & step) u = fa[u][i];
49 }
50 int lca(int u, int v) {
51     if(dep[u] > dep[v]) swap(u, v);
52     G(v, dep[v] - dep[u]);
53     if(u == v) return u;
54     for(int i = S; i >= 0; i--) if(fa[u][i] != fa[v][i]) {
55         u = fa[u][i]; v = fa[v][i];
56     }
57     return fa[u][0];
58 }
59 inline void modify(int u) {
60     tans -= V[C[u]] * sum[cur[C[u]]];
61     cur[C[u]] += vis[u];
62     vis[u] = -vis[u];
63     tans += V[C[u]] * sum[cur[C[u]]];
64 }
65 inline void update(int u, int v) {
66     if(u == v) return;
67     if(dep[u] > dep[v]) swap(u, v);
68     while(dep[v] > dep[u]) {
69         modify(v);
70         v = fa[v][0];
71     }
72     while(u != v) {
73         modify(u); modify(v);
74         u = fa[u][0]; v = fa[v][0];
75     }
76 }
77 inline void upd(int t) {
78     if(vis[q[t].l] == -1) {
79         modify(q[t].l);
80         swap(C[q[t].l], q[t].r);
81         modify(q[t].l);
82     }
83     else swap(C[q[t].l], q[t].r);
84 }
85 inline void moveto(int u, int v) {
86     update(l, u); update(r, v);
87     l = u; r = v;
88 }
89 int main() {
90     n = read(); m = read(); Q = read();
91     sz = (int)pow(n, 2.0 / 3.0);
92     for(int i = 1; i <= m; i++) V[i] = read();
93     for(int i = 1; i <= n; i++) W[i] = read();
94     for(int i = 1, u, v; i < n; i++) {
95         u = read(); v = read();
96         add_edge(u, v);
97     }
98     for(int i = 1; i <= n; i++) {
99         C[i] = read();

```



```

100     vis[i] = 1;
101     sum[i] = sum[i - 1] + W[i];
102 }
103 for(int i = 1, tp; i <= Q; i++) {
104     tp = read();
105     if(tp) {
106         ++Q1;
107         q[Q1].l = read(); q[Q1].r = read();
108         q[Q1].id = Q1;
109         q[Q1].ti = i;
110     }
111     else {
112         ++Q0;
113         qq[Q0].l = read(); qq[Q0].r = read();
114         qq[Q0].ti = i;
115     }
116 }
117 dfs(1, 0);
118 while(top) belong[stack[top--]] = cnt;
119 sort(q + 1, q + Q1 + 1);
120 for(int k = 1; k <= S; k++) {
121     for(int i = 1; i <= n; i++) {
122         fa[i][k] = fa[fa[i][k - 1]][k - 1];
123     }
124 }
125 for(int i = 1; i <= Q1; i++) {
126     if(belong[q[i].l] > belong[q[i].r]) swap(q[i].l, q[i].r);
127     moveto(q[i].l, q[i].r);
128     int lc = lca(l, r);
129     modify(lc);
130     while(qq[tm + 1].ti < q[i].ti && tm < Q0) upd(++tm);
131     while(qq[tm].ti > q[i].ti) upd(tm--);
132     ans[q[i].id] = tans;
133     modify(lc);
134 }
135 for(int i = 1; i <= Q1; i++) printf("%lld\n", ans[i]);
136 return 0;
137 }

```

7.3 Randomized Algorithm

7.3.1 Simulated Annealing

```

1 void solve() {
2     while(T > eps) {
3         double alpha = ((rand() % 30001) / 15000.0) * pi;
4         double theta = ((rand() % 10001) / 10000.0) * pi;
5         tmp.x = cur.x + T * sin(theta) * cos(alpha);
6         tmp.y = cur.y + T * sin(theta) * sin(alpha);
7         tmp.z = cur.z + T * cos(theta);
8         tmp.dis = cal(tmp);
9         if(tmp.dis < cur.dis || (tmp.dis * 0.999 < cur.dis && (rand() & 7) == 7)) cur =
tmp;
10         //if(exp((cur.d - tmp.d) / T) > ((double)rand() / RAND_MAX)) cur = tmp;
11
12         T *= 0.999;
13     }
14 }

```

7.4 Other Method

7.4.1 Enumerate Subset

```

1 for(int i = 0; i < (1 << k); i++) {
2     for(int j = i; ; --j &= i) {
3         // work();
4         if(j == 0) break;
5     }
6 }

```

7.4.2 Enumerate $\lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$

```

1 int cal(int n, int m) {
2     if(n > m) swap(n, m);
3     int res = 0, last;
4     for(int i = 1; i <= n; i = last + 1) {
5         last = min(n / (n / i), m / (m / i));
6         res += (n / i) * (m / i) * (sum(last) - sum(i - 1));
7     }
8     return res;
9 }

```

7.4.3 Find Primitive Root Modulo N

```

1 for i in range(1,mod):
2     if 3 ** i % mod == 1:
3         if i == mod - 1:
4             print("yes")
5             break
6     print("no")

```

7.5 Formula

7.5.1 Euler's Theorem

$$a^b \equiv \begin{cases} a^{b \% \varphi(p)} & \gcd(a, p) = 1 \\ a^b & \gcd(a, p) \neq 1, b < \varphi(p) \\ a^{b \% \varphi(p) + \varphi(p)} & \gcd(a, p) \neq 1, b \geq \varphi(p) \end{cases} \pmod{p}$$

7.5.2 Möbius Inversion Formula

Dirichlet Convolution is $(f \times g)(N) = \sum_{d|N} f(d) * g(\frac{N}{d})$

Theorem:

$$\begin{cases} f = g \times 1 \\ g = f \times \mu \end{cases}$$

7.5.3 Math Theory Tips

$$\varphi(nm) = \varphi(n) \cdot \varphi(m) \cdot \frac{\gcd(n, m)}{\varphi(\gcd(n, m))} \quad (1)$$

$$\begin{cases} id(n) = \sum_{d|n} \varphi(d) \\ e(n) = \sum_{d|n} \mu(d) \end{cases} \quad (2)$$

$$\begin{cases} \sum_i^n \sum_j^m \gcd(i, j) = \sum_d^{\max(n, m)} \varphi(d) * \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \\ \sum_i^n \sum_j^m e(\gcd(i, j)) = \sum_d^{\min(n, m)} \mu(d) * \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor \\ \sum_{i=1}^n |\mu(i)| = \sum_{i=1}^{\lfloor \sqrt{n} \rfloor} \mu(i) * \lfloor \frac{n}{i * i} \rfloor \end{cases} \quad (3)$$

$$\begin{cases} sum(x, y) = \sum_i^x \sum_j^y i * j = \frac{x * (x + 1)}{2} * \frac{y * (y + 1)}{2} \\ F(x, y) = \sum_{i=1}^{\min(x, y)} i^2 * \mu(i) * sum(\lfloor \frac{x}{i} \rfloor, \lfloor \frac{y}{i} \rfloor) \\ \sum_i^n \sum_j^m lcm(i, j) = \sum_{i=1}^{\min(n, m)} d * F(\lfloor \frac{n}{i} \rfloor, \lfloor \frac{y}{i} \rfloor) \end{cases} \quad (4)$$

7.5.4 Sieve Tips

$$\varphi(n) = \sum_{i=1}^n [(n, i) = 1] \cdot i = \frac{n * \varphi(n) + [n = 1]}{2} \quad (5)$$

$$\begin{cases} id = \varphi \times 1 \\ \frac{n \cdot (n + 1)}{2} = \sum_{i=1}^n i = \sum_{i=1}^n \sum_{d|i} \varphi(d) = \sum_{\frac{i}{d}=1}^n \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} \varphi(d) = \sum_{i=1}^n \phi(\lfloor \frac{n}{i} \rfloor) \end{cases} \quad (6)$$

$$\begin{cases} e = \mu \times 1 \\ 1 = \sum_{i=1}^n [i = 1] = \sum_{i=1}^n \sum_{d|i} \mu(d) = \sum_{i=1}^n \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} \mu(d) = \sum_{i=1}^n M(\lfloor \frac{n}{i} \rfloor) \end{cases} \quad (7)$$

$$\begin{cases} id^2 = (id \cdot \varphi) \times id \\ \phi'(n) = \sum_{i=1}^n i \cdot \varphi(i) \\ \frac{n \cdot (n + 1) \cdot (2n + 1)}{6} = \sum_{i=1}^n i^2 = \sum_{i=1}^n \sum_{d|i} d \cdot \varphi(d) \cdot \frac{i}{d} = \sum_{\frac{i}{d}=1}^n \frac{i}{d} \sum_{d=1}^{\lfloor \frac{n}{i} \rfloor} d \cdot \varphi(d) = \sum_{i=1}^n i \cdot \phi'(\lfloor \frac{n}{i} \rfloor) \end{cases} \quad (8)$$

7.6 Convolution Tips

7.6.1 FWT Tips

$$\left\{ \begin{array}{l} C_k = \sum_{i \oplus j = k} A_i * B_j \\ DWT(A)_i = \sum_j^n A_j * f_{i,j} \\ DWT(C)_i = DWT(A)_i * DWT(B)_i \\ f_{i,j} \cdot f_{i,k} = f_{i,j \oplus k} \\ f_{i,j} = [i \text{ and } j == i] \quad (and) \\ f_{i,j} = [i \text{ and } j == j] \quad (or) \\ f_{i,j} = (-1)^{[i \text{ and } j]} \quad (xor) \end{array} \right.$$

7.7 The Number of Integer Point on a Circle

Set $r = const$ is the radius of the circle.

$$r^2 = p_1^{a_1} + p_2^{a_2} + \dots + p_m^{a_m} = \sum_{i=1}^m p_i^{a_i}$$

Define

$$\chi(n) = \begin{cases} 1 & n \% 4 = 1 \\ -1 & n \% 4 = 3 \\ 0 & n \% 2 = 0 \end{cases}$$

By the way, $\chi(n)$ is a multiplicative function.

Define

$$\Gamma(p_i, a_i) = \sum_{j=0}^{a_i} \chi(p_i^j) = \begin{cases} 1 & p_i = 2 \quad || \quad (p_i \% 4 = 3 \quad \&\& \quad a_i \% 2 = 0) \\ 0 & p_i \% 4 = 3 \quad \&\& \quad a_i \% 2 = 1 \\ a_i + 1 & p_i \% 4 = 1 \end{cases}$$

Define cnt is the number of integer point on circle

$$cnt(r) = 4 \prod_{i=1}^m \sum_{j=0}^{a_i} \chi(p_i^j) = 4 \prod_{i=1}^m \Gamma(p_i, a_i) = 4 \sum_{k|r^2} \chi(k)$$

Define CNT is the number of integer point in circle

$$CNT(r) = 1 + \sum_{i=1}^{r^2} cnt(i) = 1 + \sum_{i=1}^{r^2} \left\lfloor \frac{r^2}{i} \right\rfloor \chi(i)$$