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Memristive Boltzmann Machine: A Hardware Accelerator for Combinatorial Optimization and Deep Learning*

Chris Kjellqvist

*Mahdi Nazm Bojnordi and Engin Ipek. Memristive boltzmann machine: A hardware accelerator for combinatorial optimization and deep learning. In Proceedings on the 22nd IEEE Symposium on High Performance Computer Architecture, Rochester, NY, March 2016.

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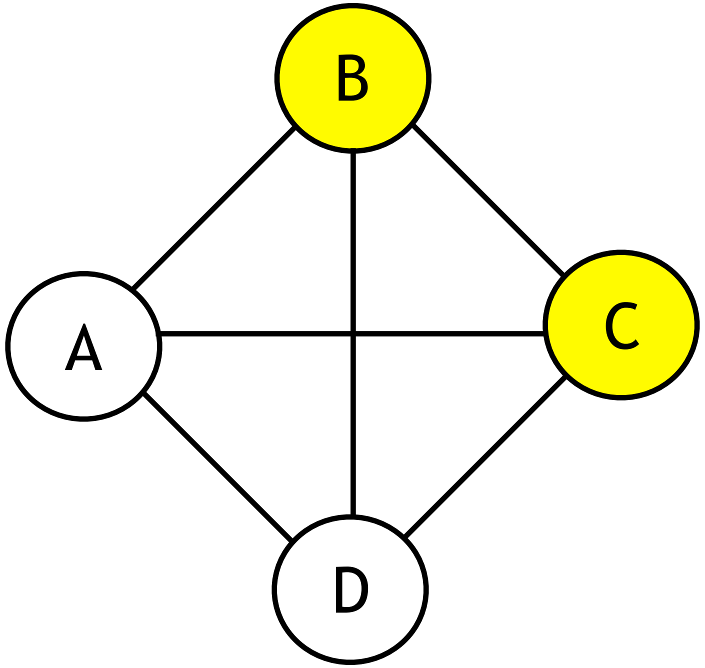
The Boltzmann Machine, Problem Mapping, and Memristors

BACKGROUND

The Boltzmann Machine

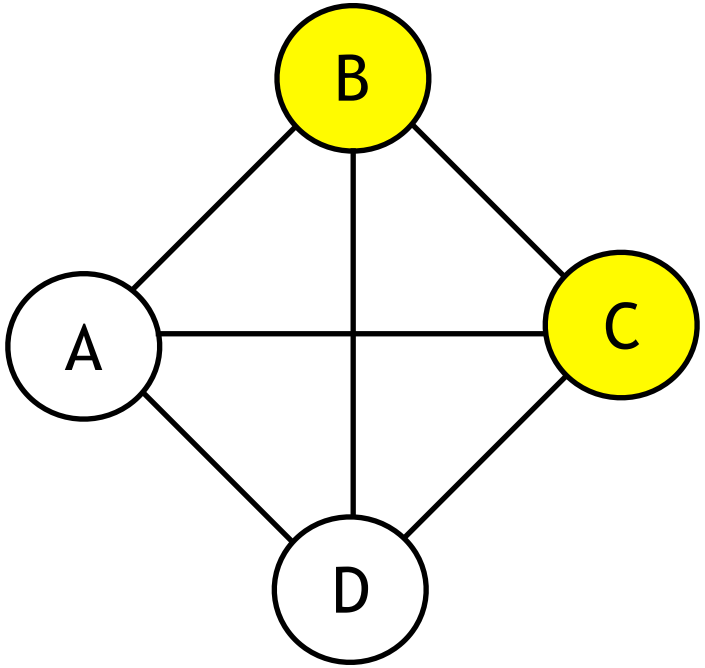
- A Fully Connected Stochastic Recurrent Neural Network capable of solving optimization problems



1) Ackley, David H; Hinton Geoffrey E; Sejnowski, Terrence J (1985), "A learning algorithm for Boltzmann machines", Cognitive Science, 9 (1): 147–169, doi:10.1207/s15516709cog0901_7

The Boltzmann Machine

- A Fully Connected Stochastic Recurrent Neural Network capable of solving optimization problems
  - Traveling Salesman



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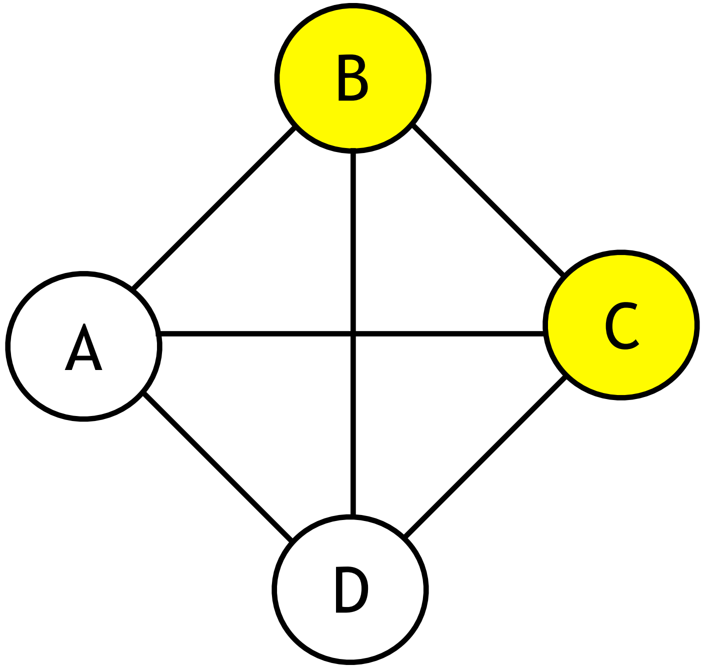
  - Max-Cut



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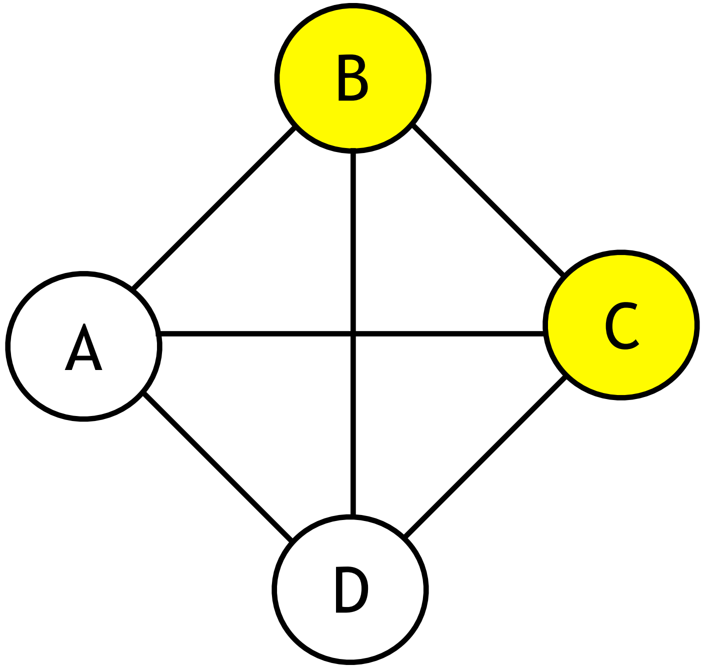
  - Maximum Satisfiability



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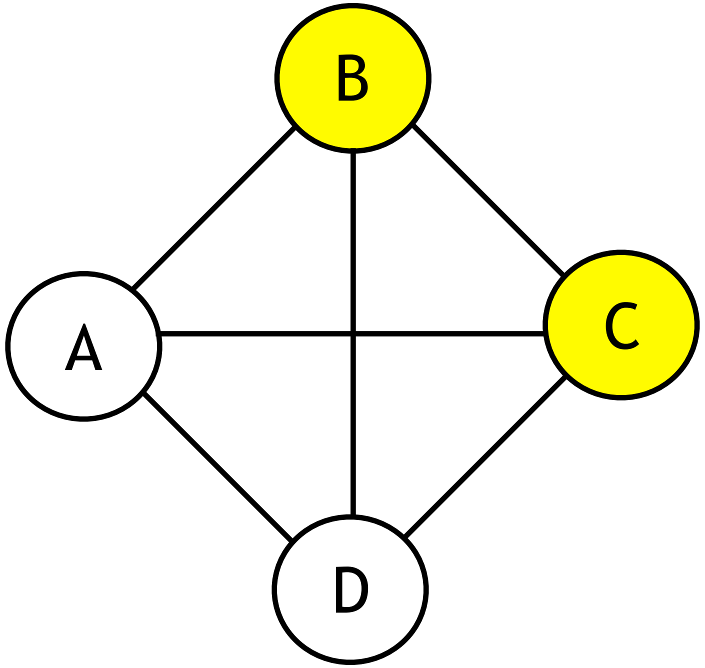
  - Deep Learning



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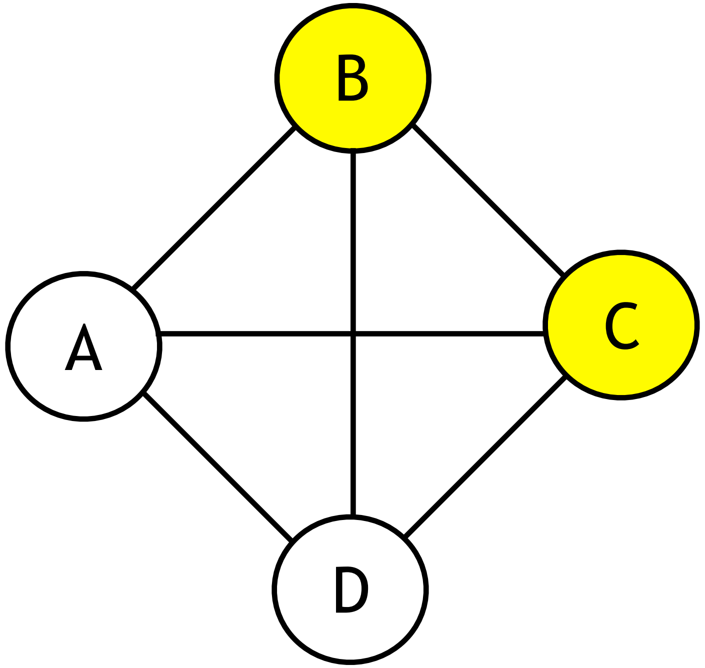
The Boltzmann Machine

- A Fully Connected Stochastic Recurrent Neural Network capable of solving optimization problems
- Nodes change state (on or off) as a function of their neighbor states and corresponding edge weights
- Goal to maximize "consensus" function



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Example Problem: Max-Cut

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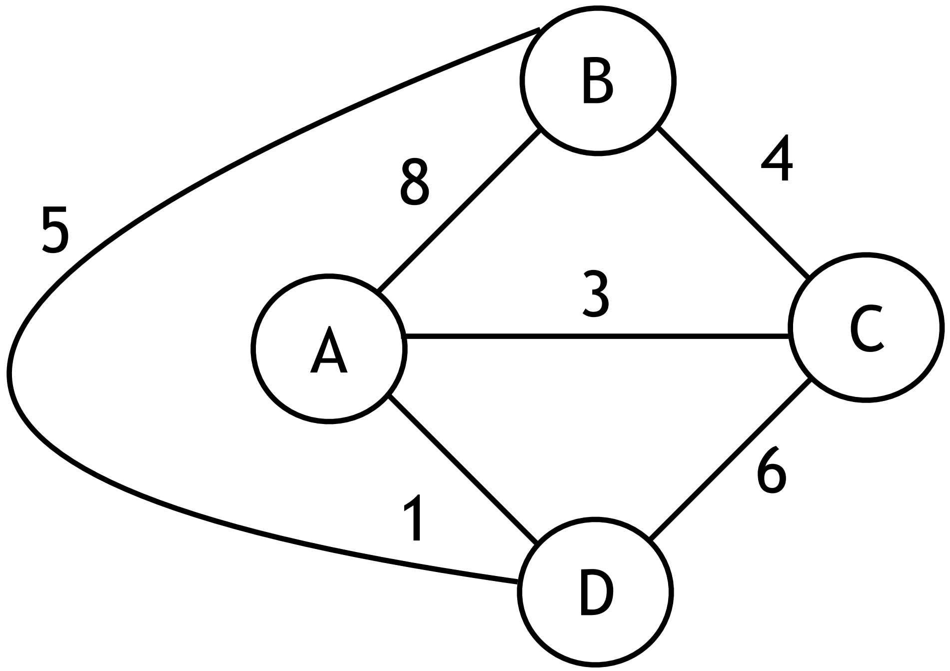

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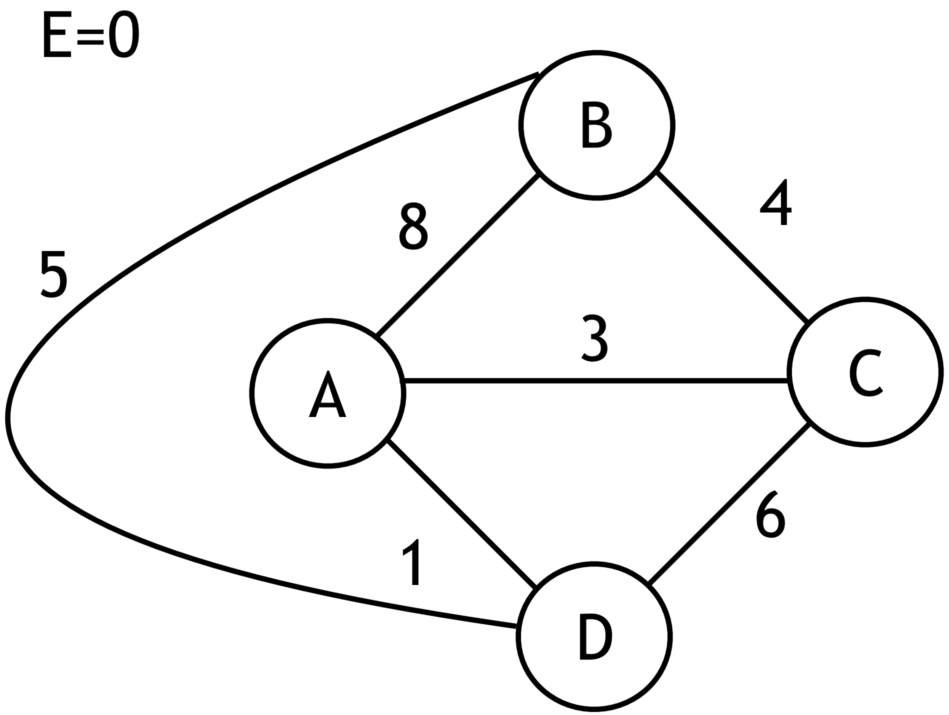

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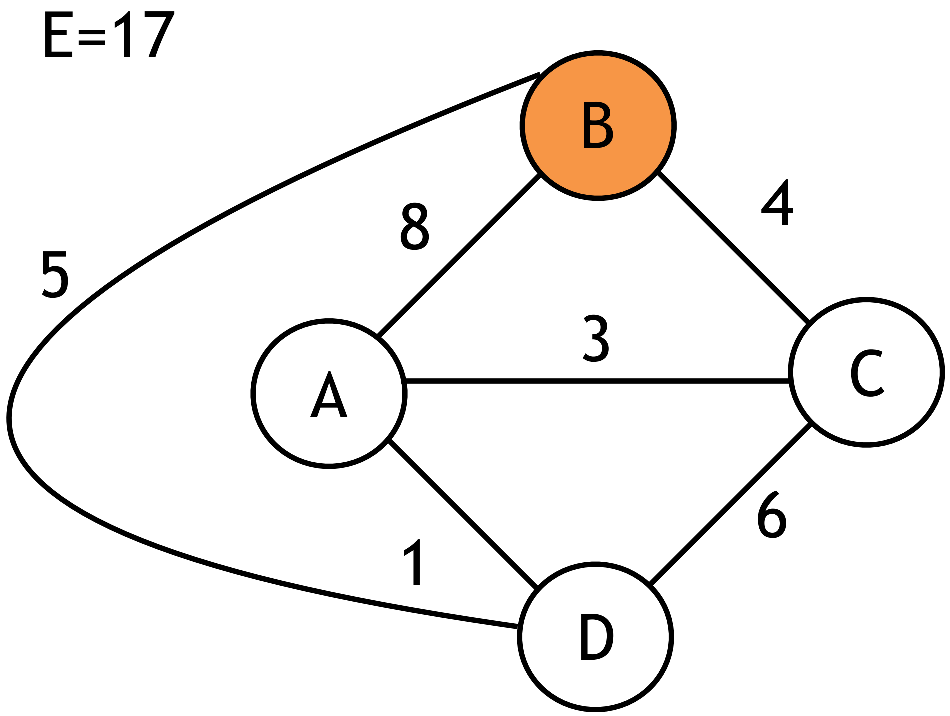

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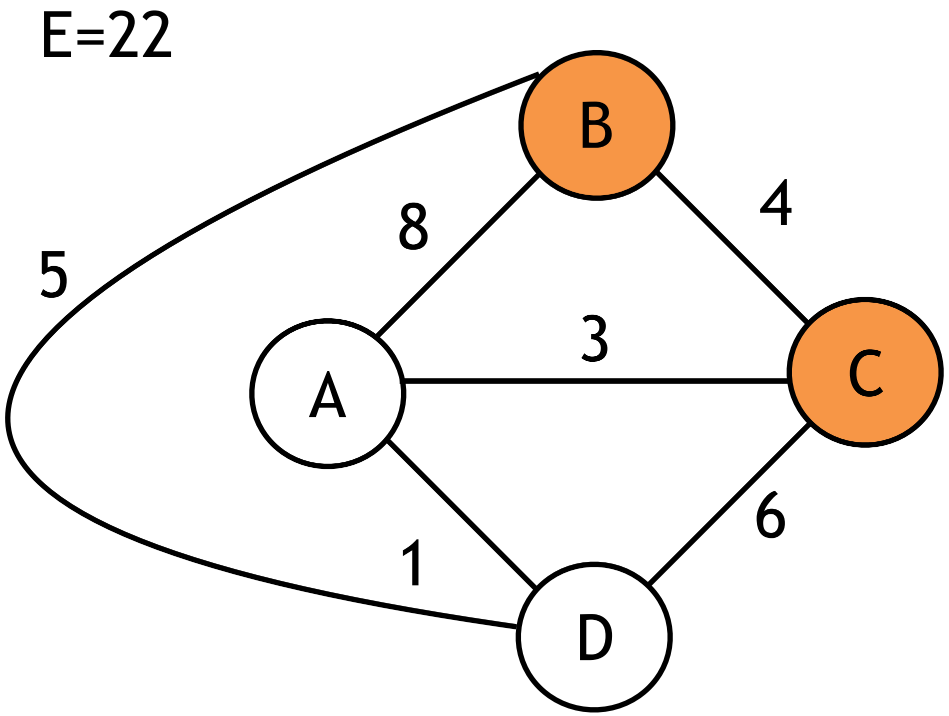

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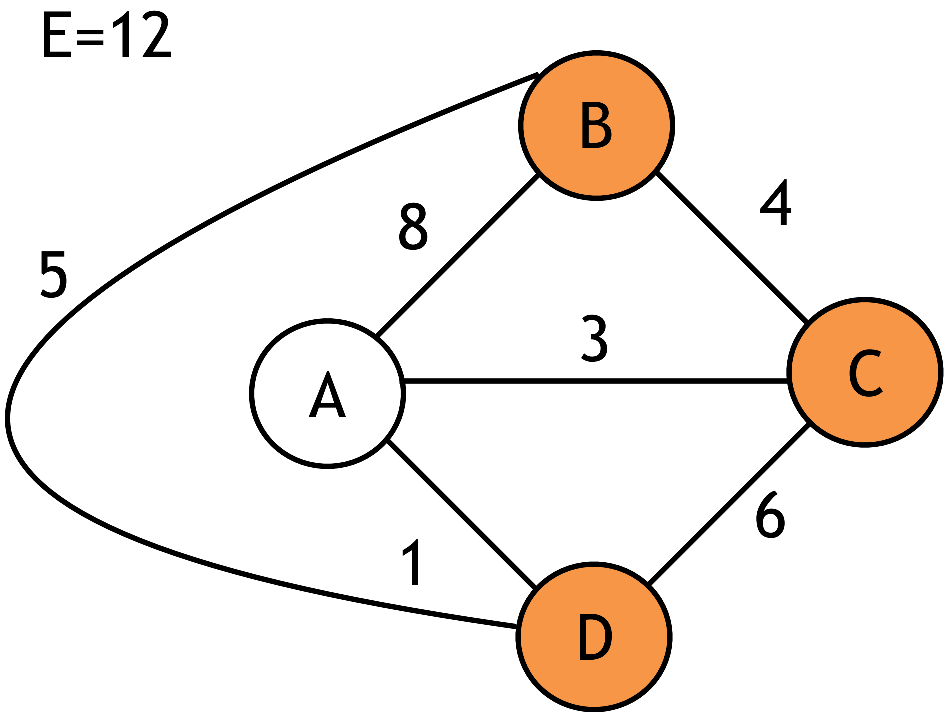

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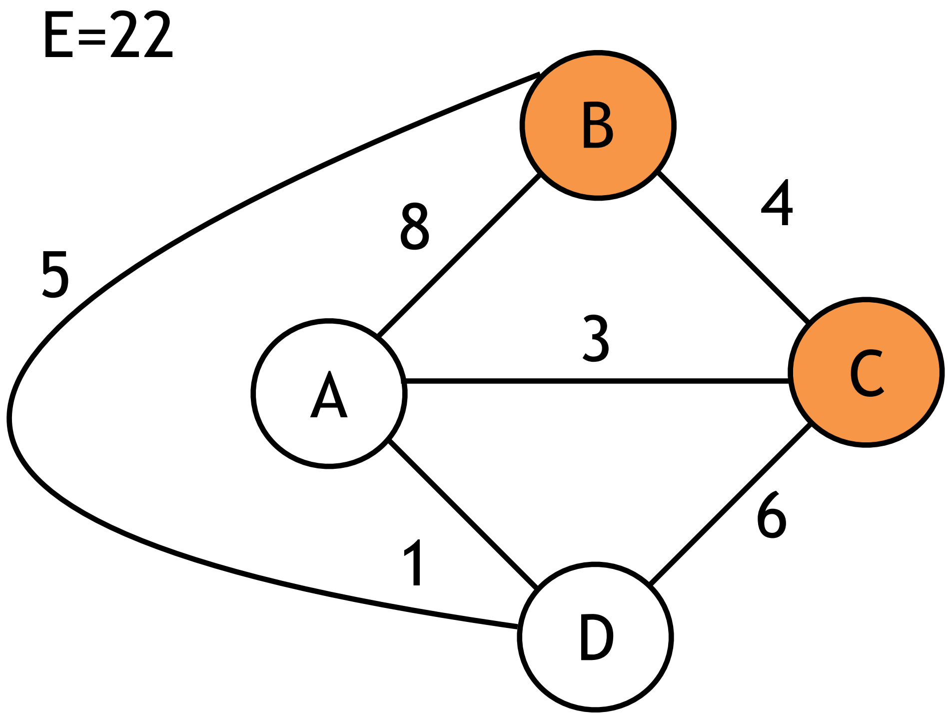
$$S = \{B, C\}$$



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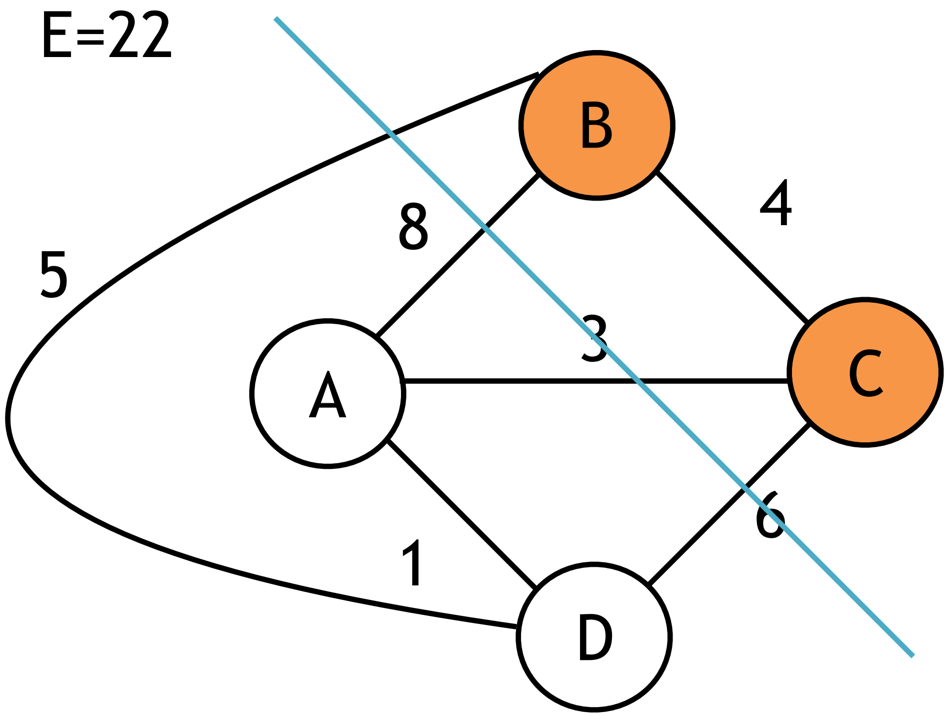

Mapping

- In order to map all graphs to the Boltzmann Machine, we add an incidence matrix ***I***
- Change edge weights to allow proper training

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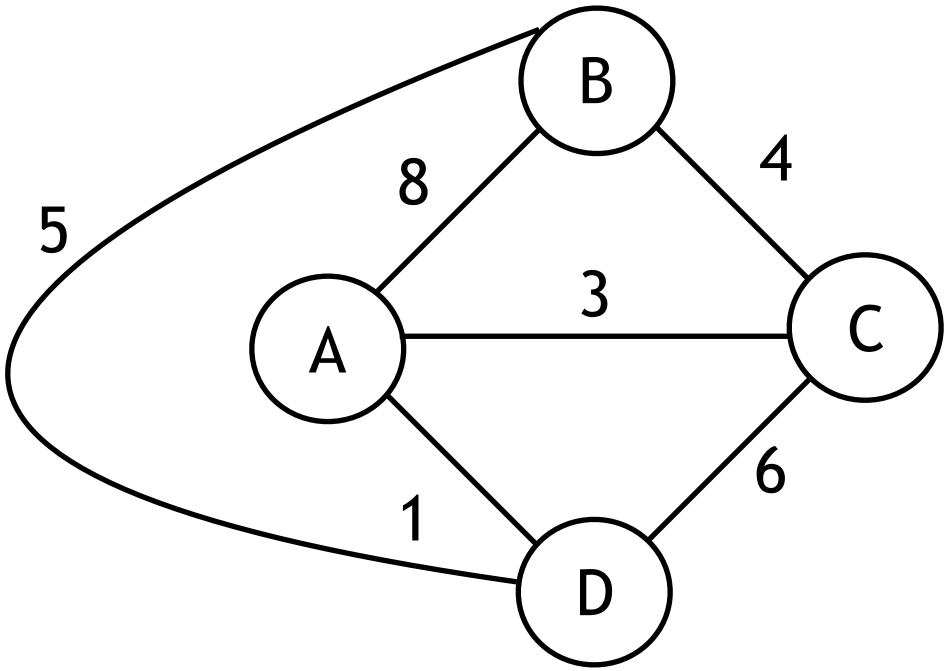

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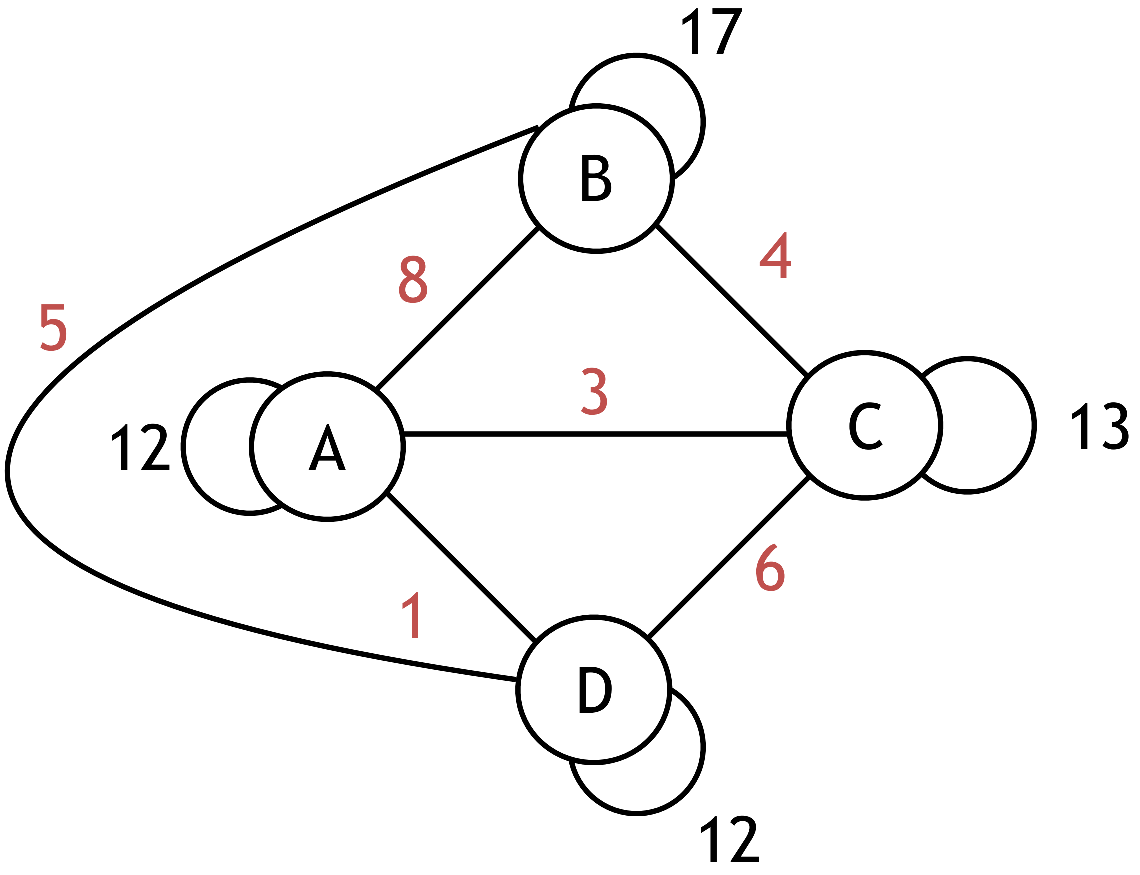

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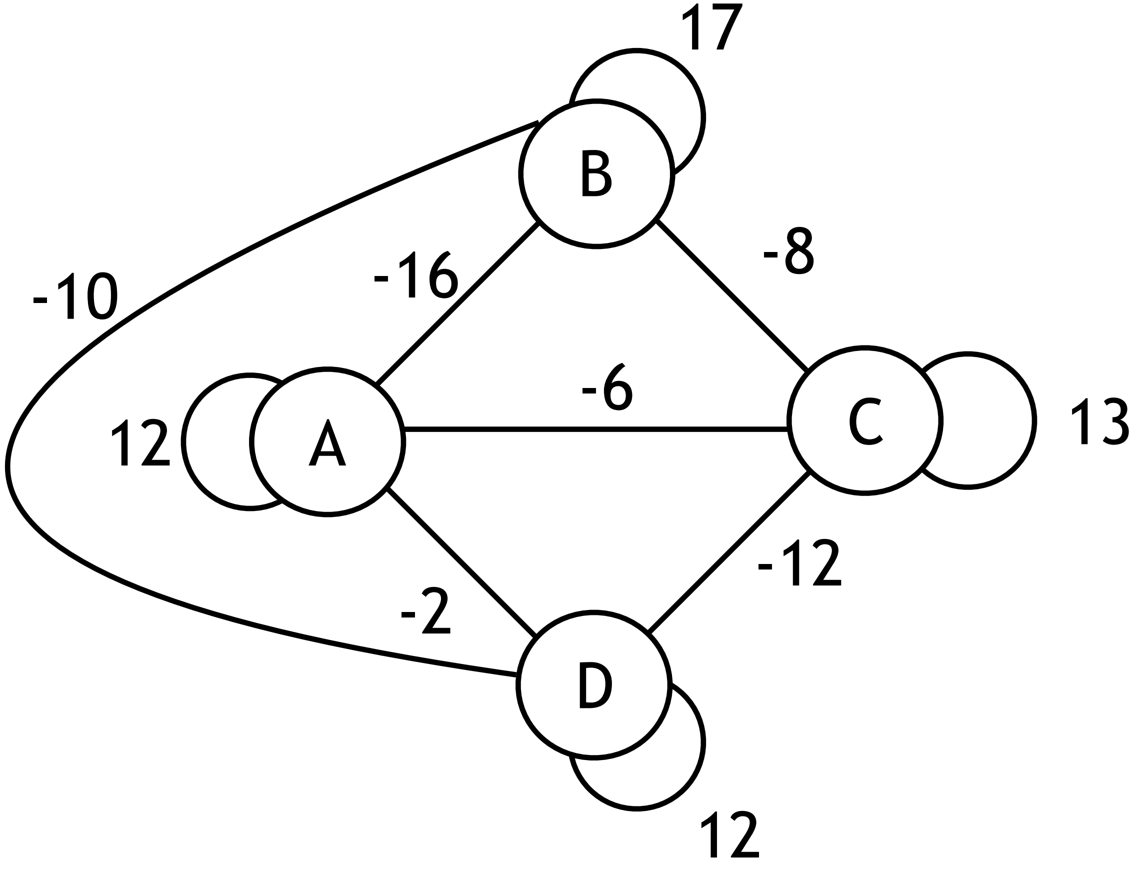

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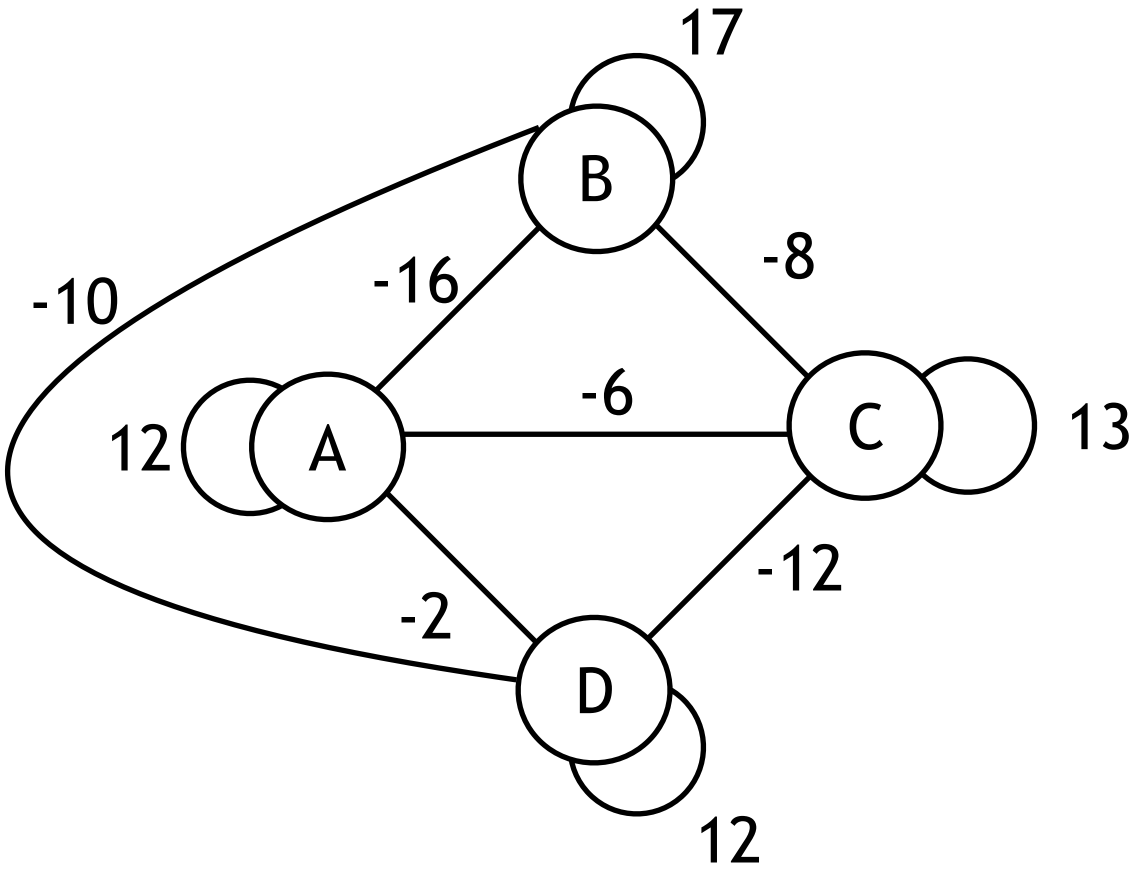
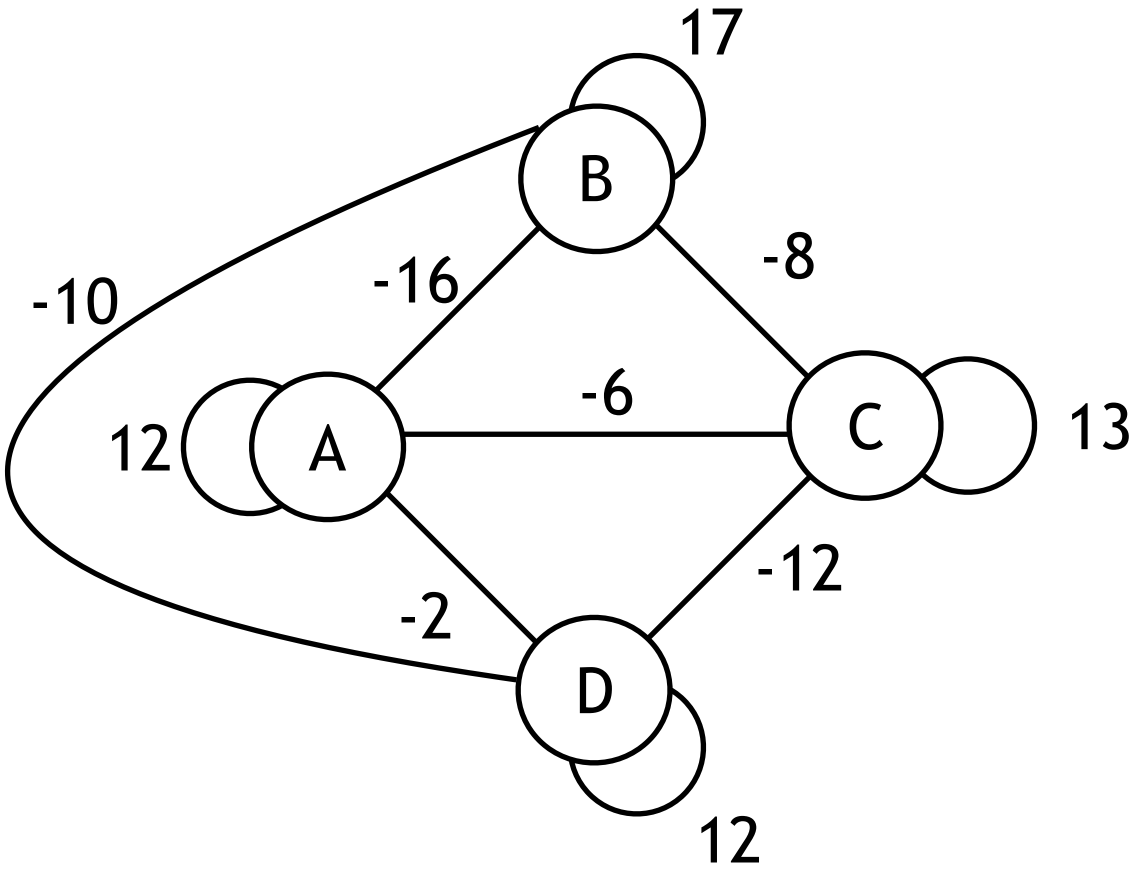


Mapping

- The graph is mapped to a Boltzmann Machine but what's the task now?



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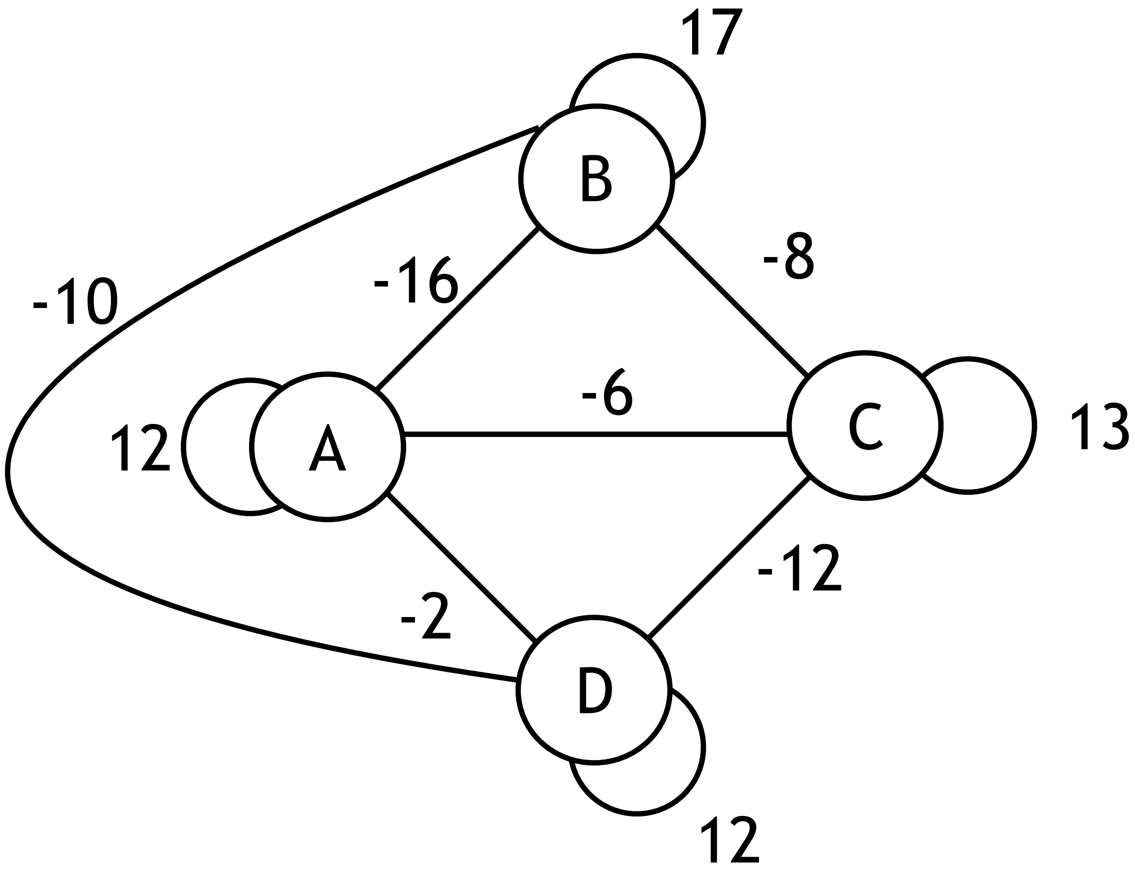

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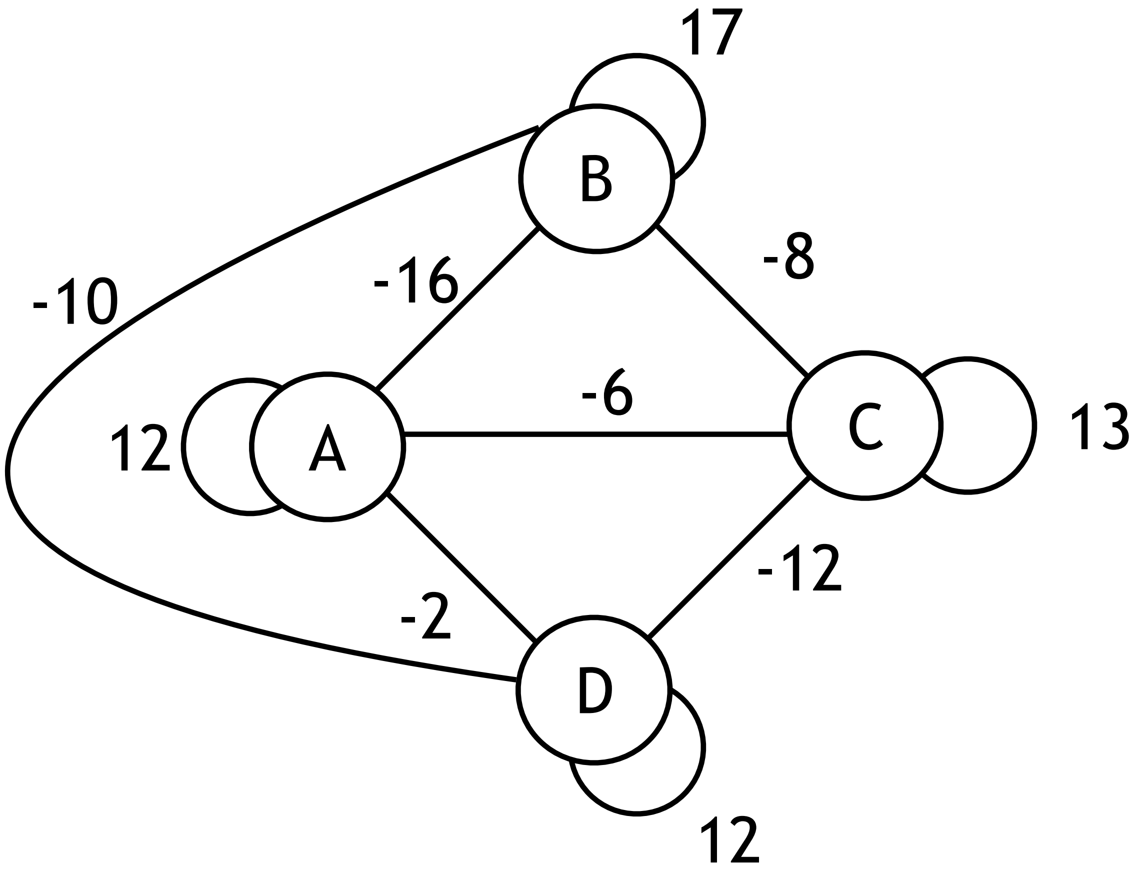
Mapping

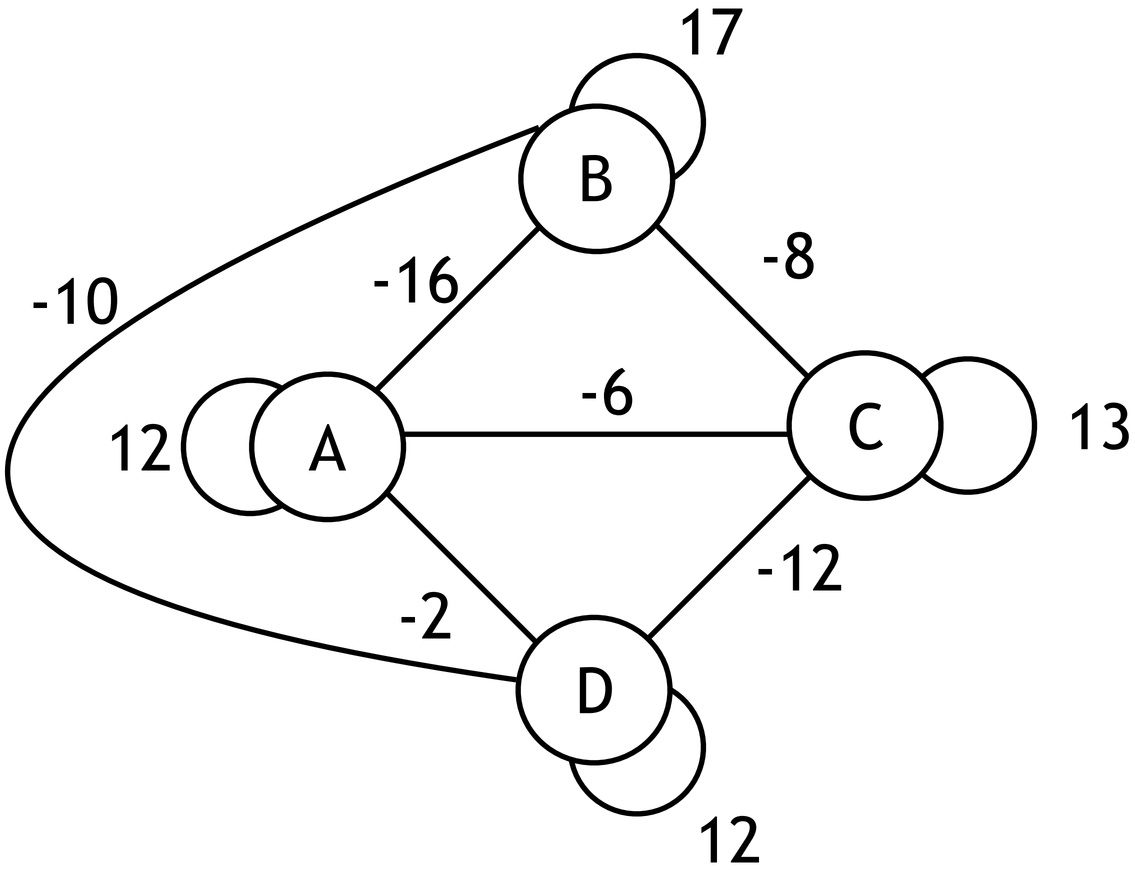
- The graph is mapped to a Boltzmann Machine but what's the task now?
- Minimize "Energy"

$$E(X) = -\frac{1}{2}\sum_{j}\sum_{i\neq j} x_j x_i w_{ij} - \sum_{j} x_j w_{jj}$$



Example

$$E(X) = -\frac{1}{2}\sum_j \sum_{i \neq j} x_j x_i w_{ij} - \sum_j x_j w_{jj}$$



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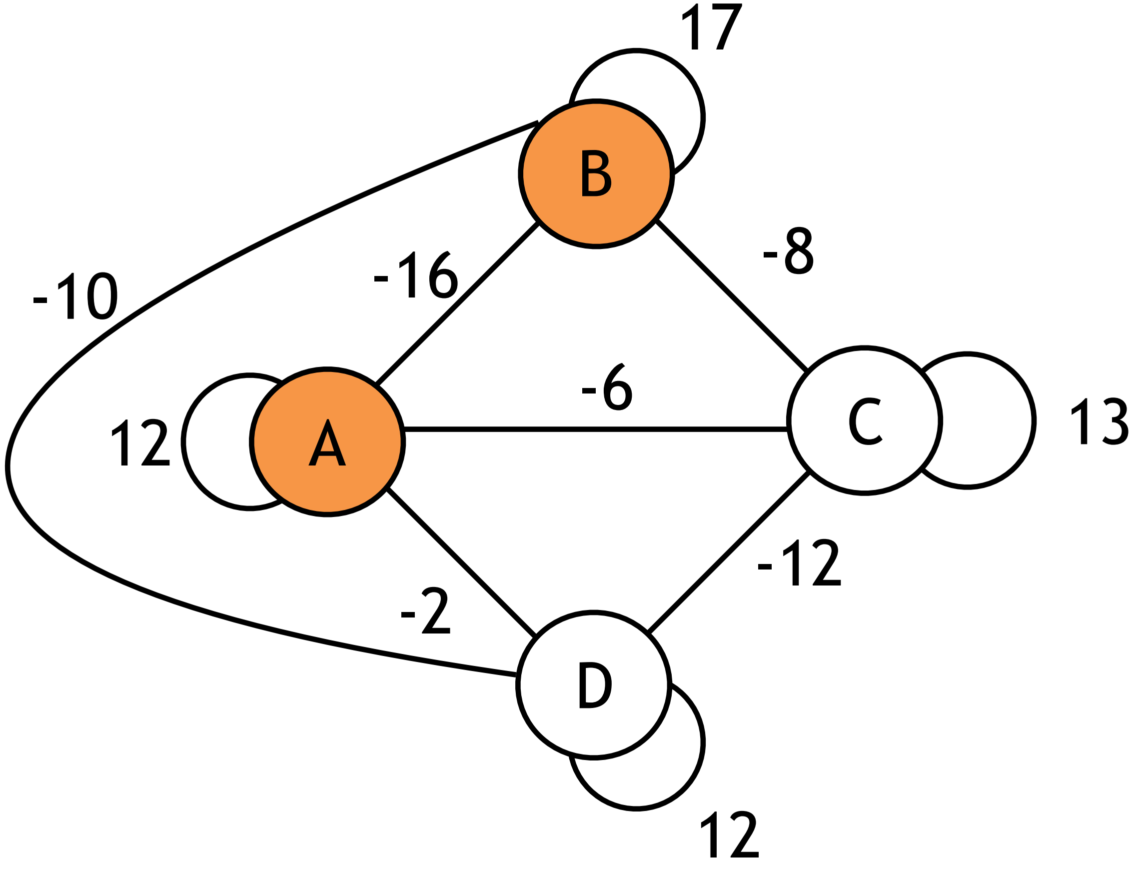
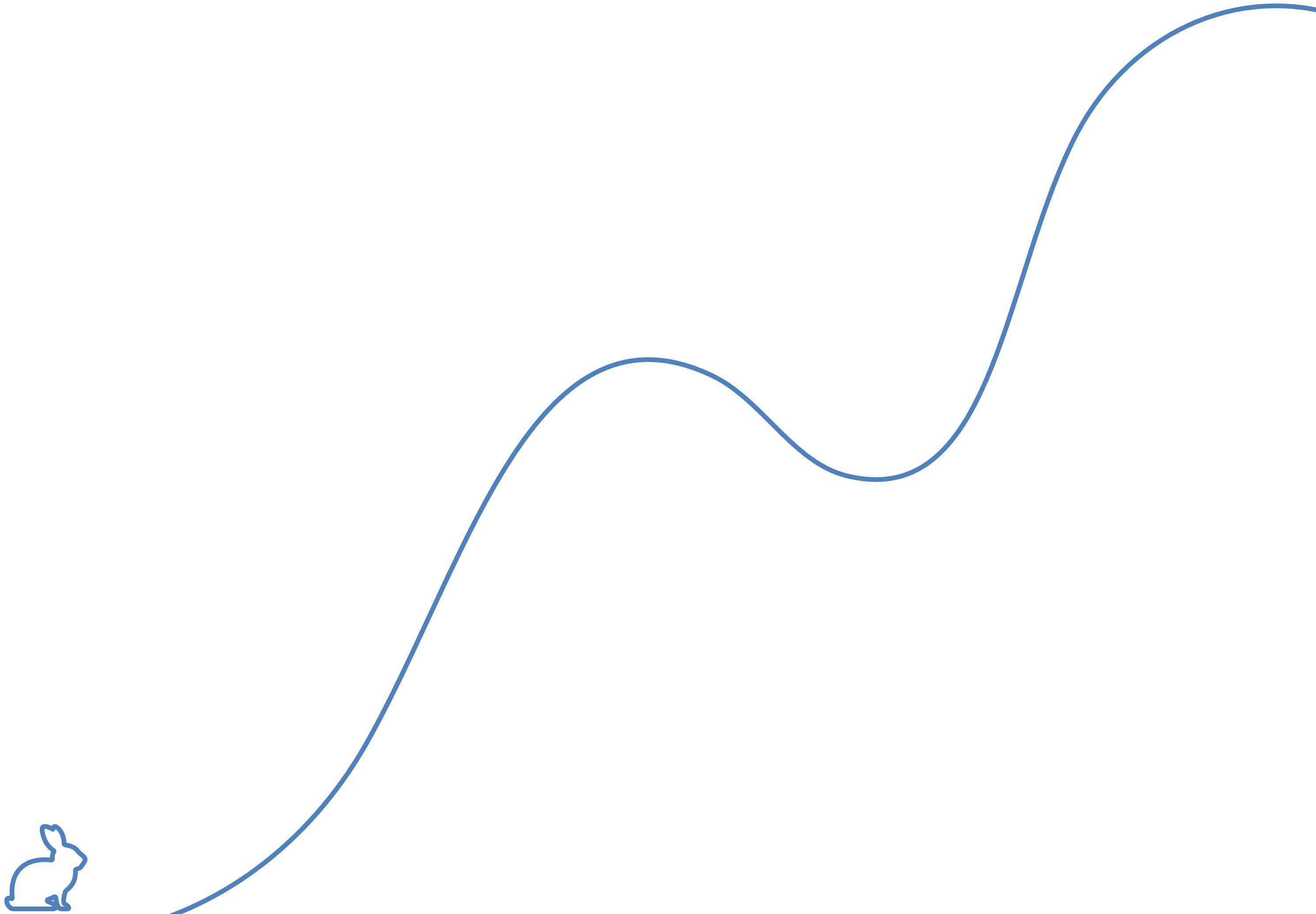
$$E(X) = -\frac{1}{2}\sum_{j}\sum_{i\neq j} x_j x_i w_{ij} - \sum_{j} x_j w_{jj}$$

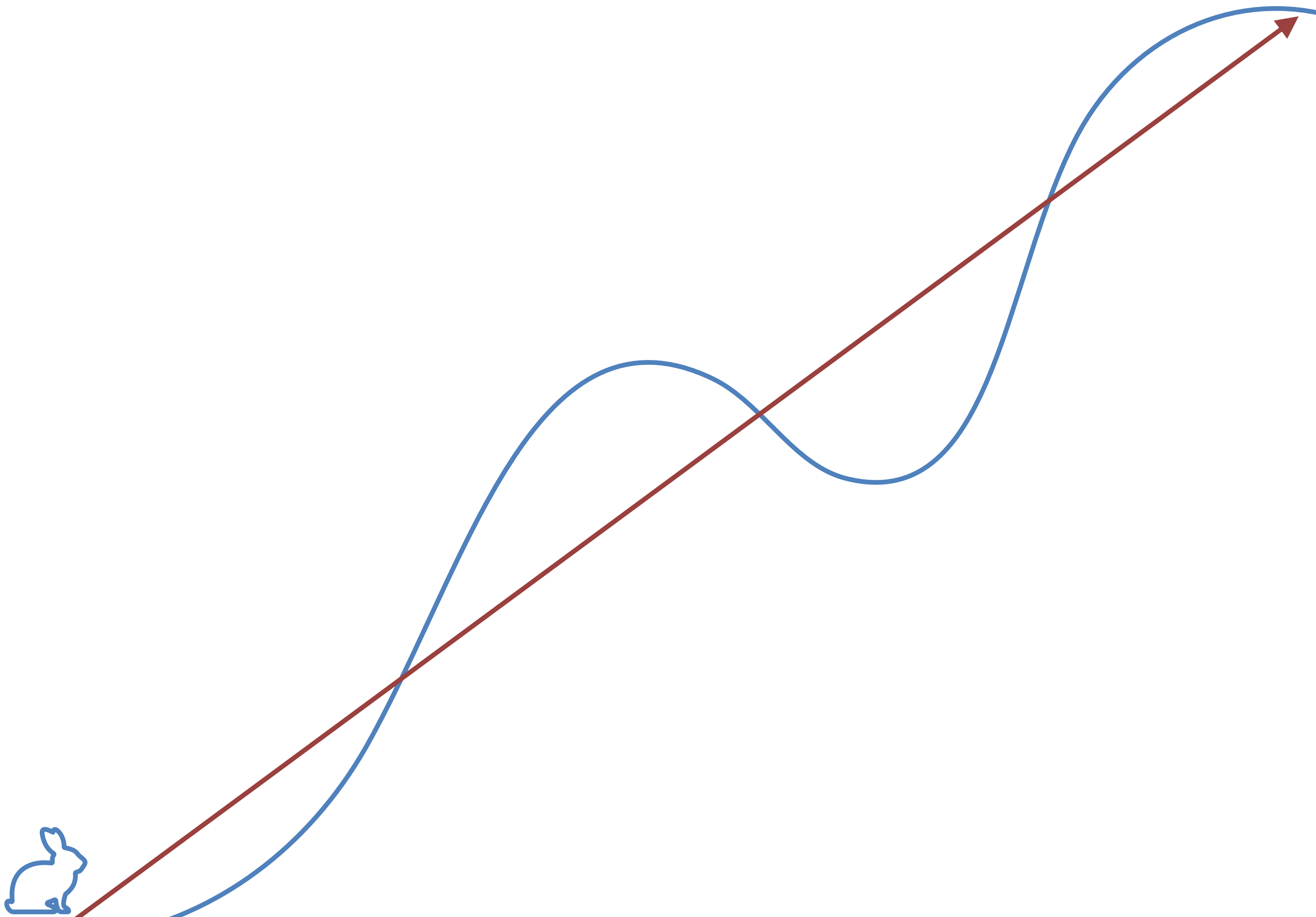
$$E(X) = -\frac{1}{2}\cdot(-16 + -16) - (12 + 17)$$

$$= 16 - 29 = -13$$



Simulated Annealing





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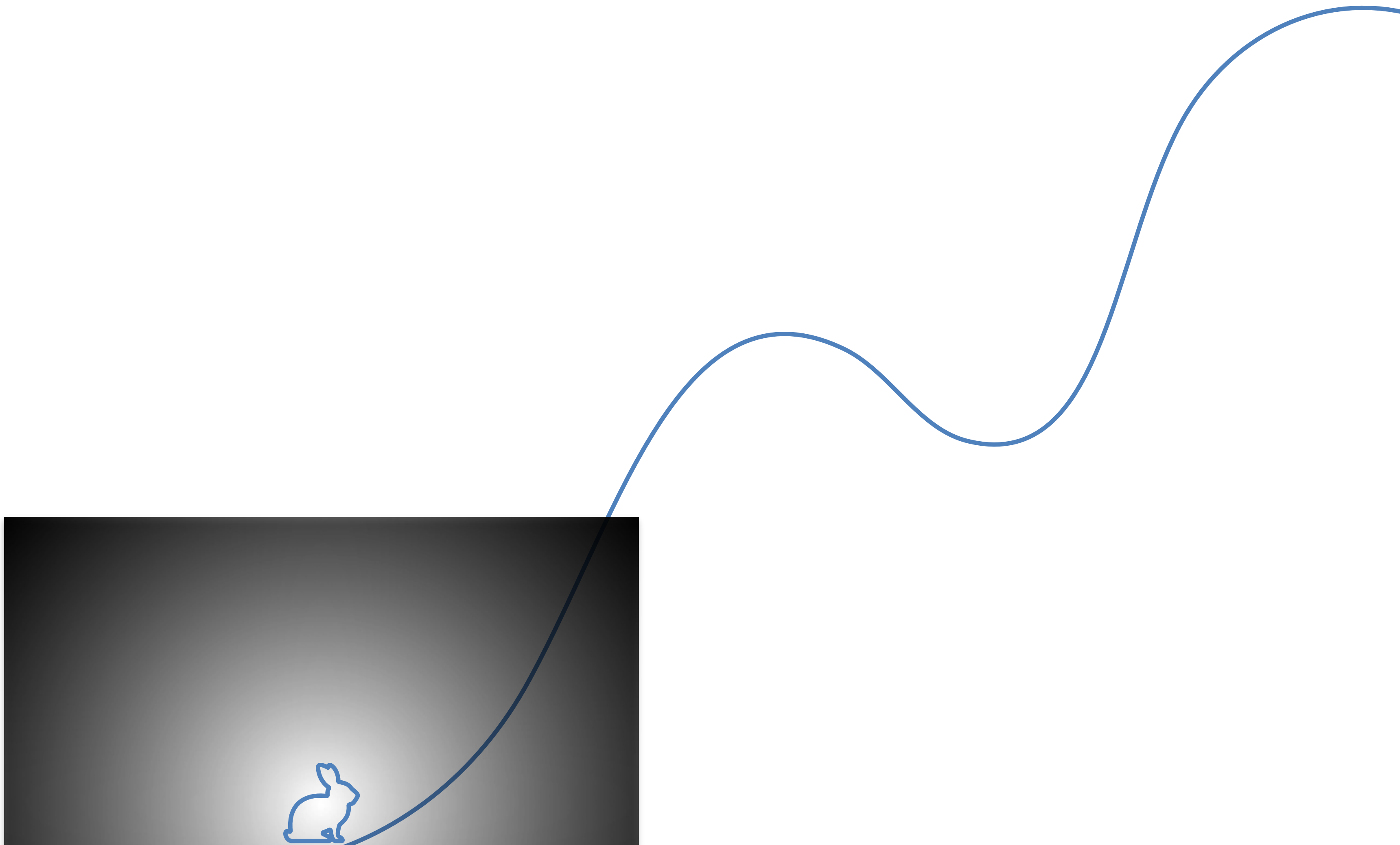

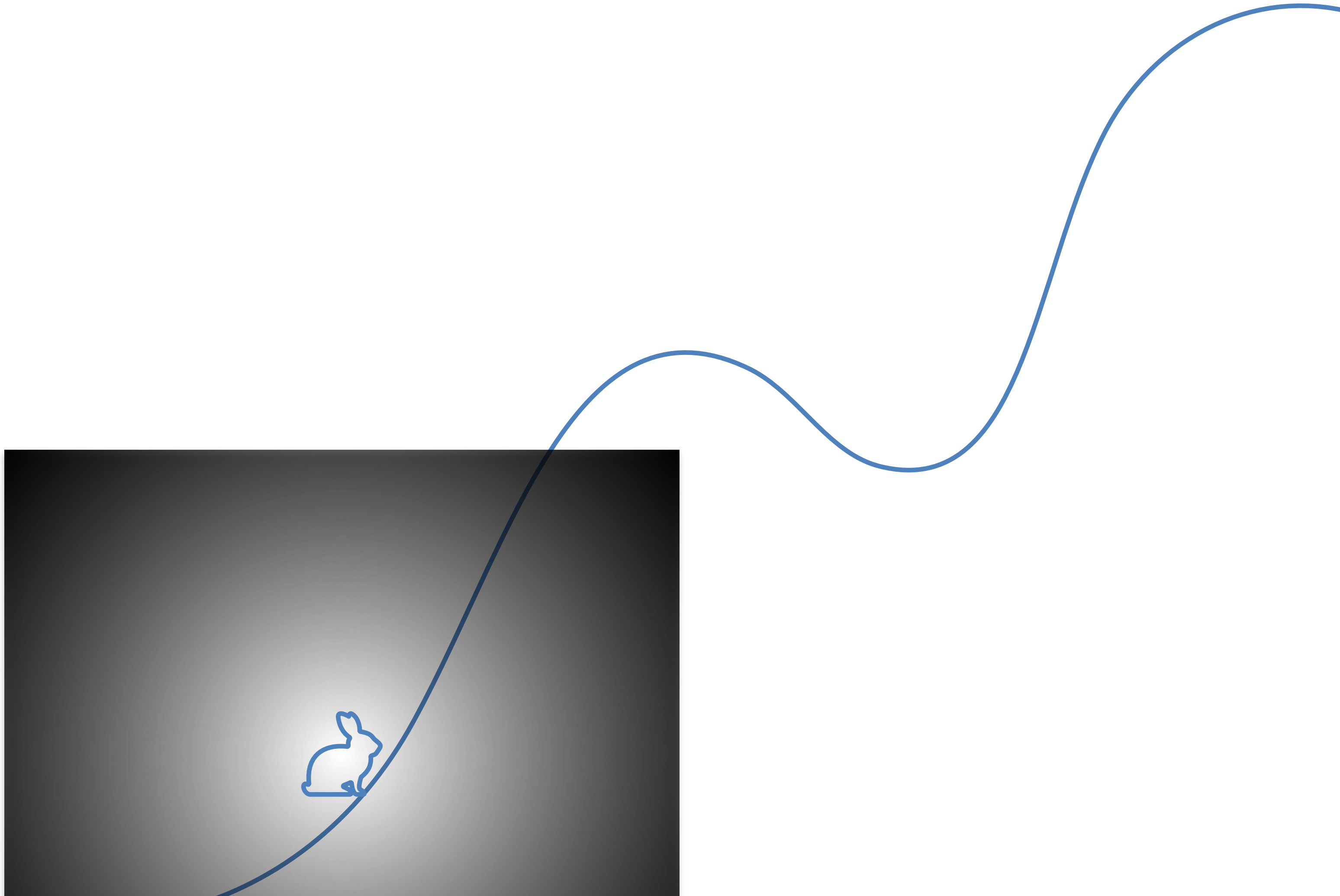

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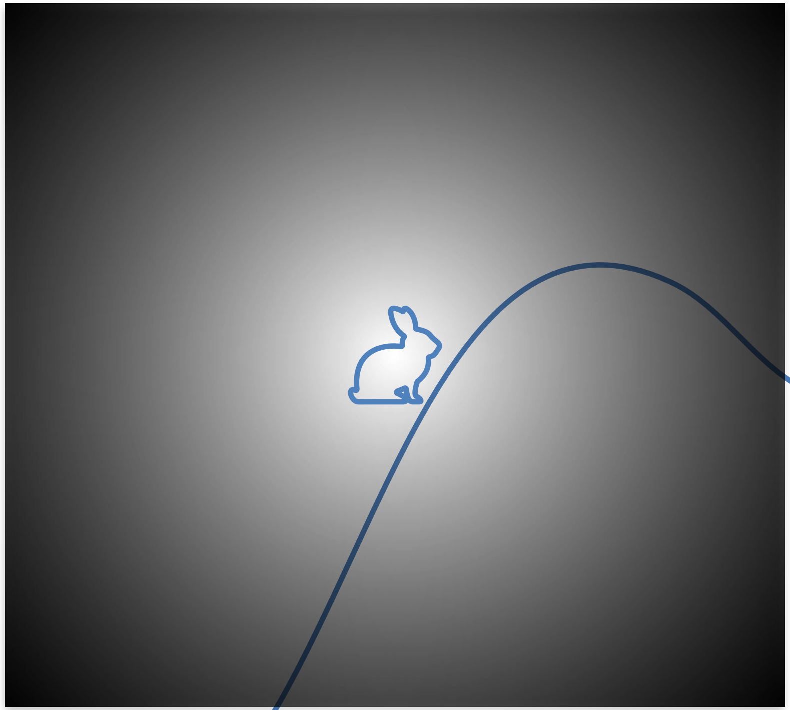

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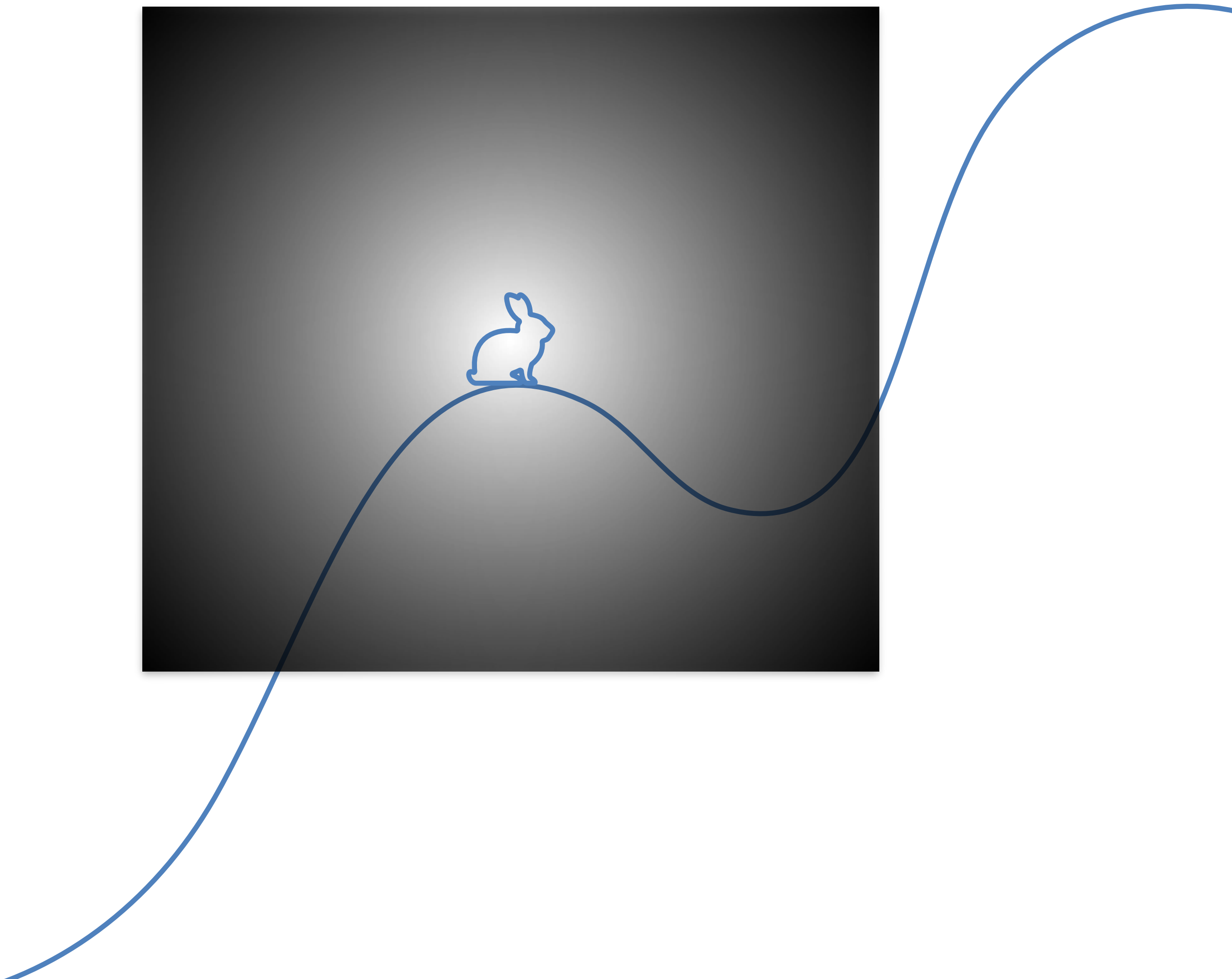
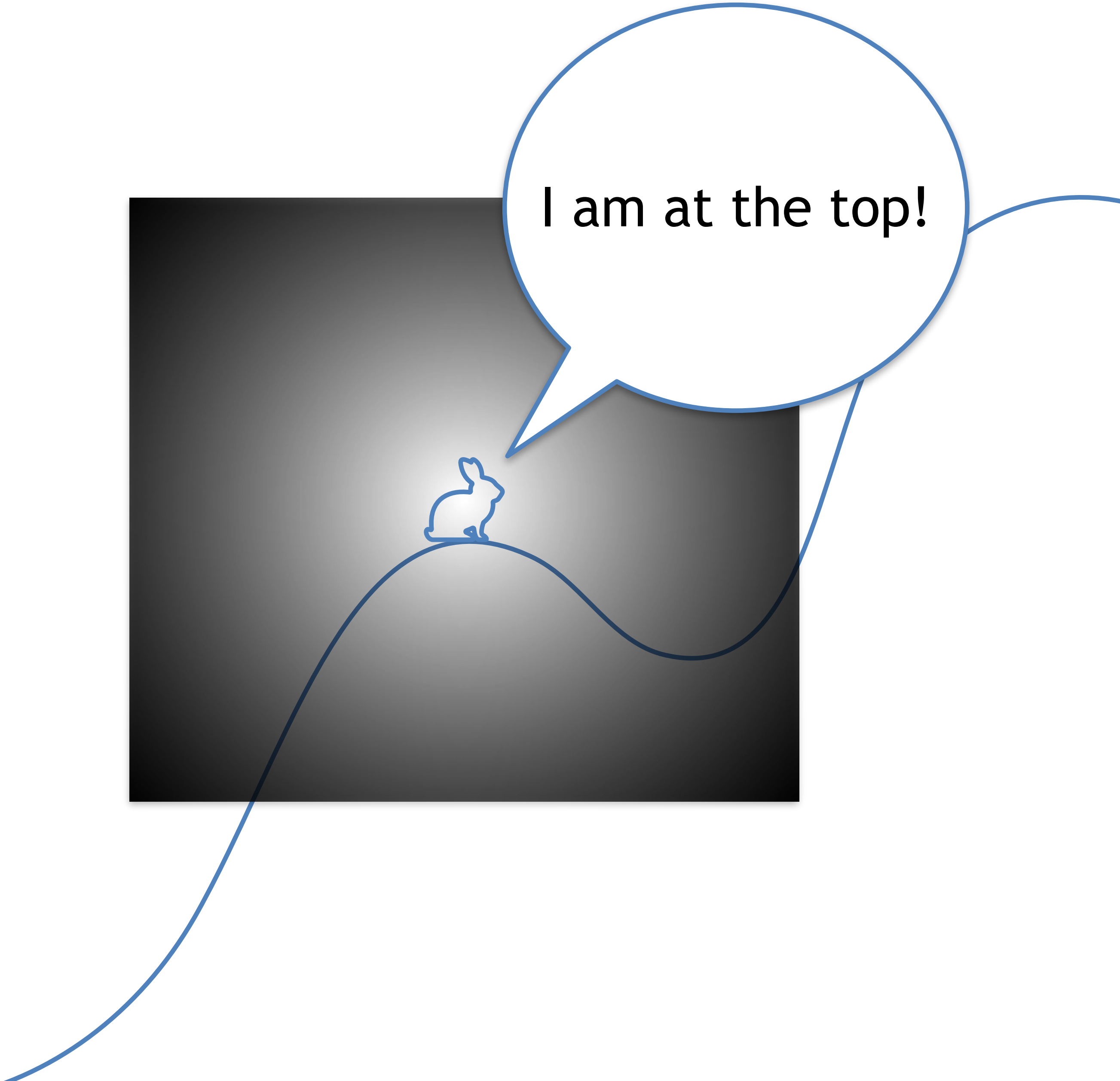
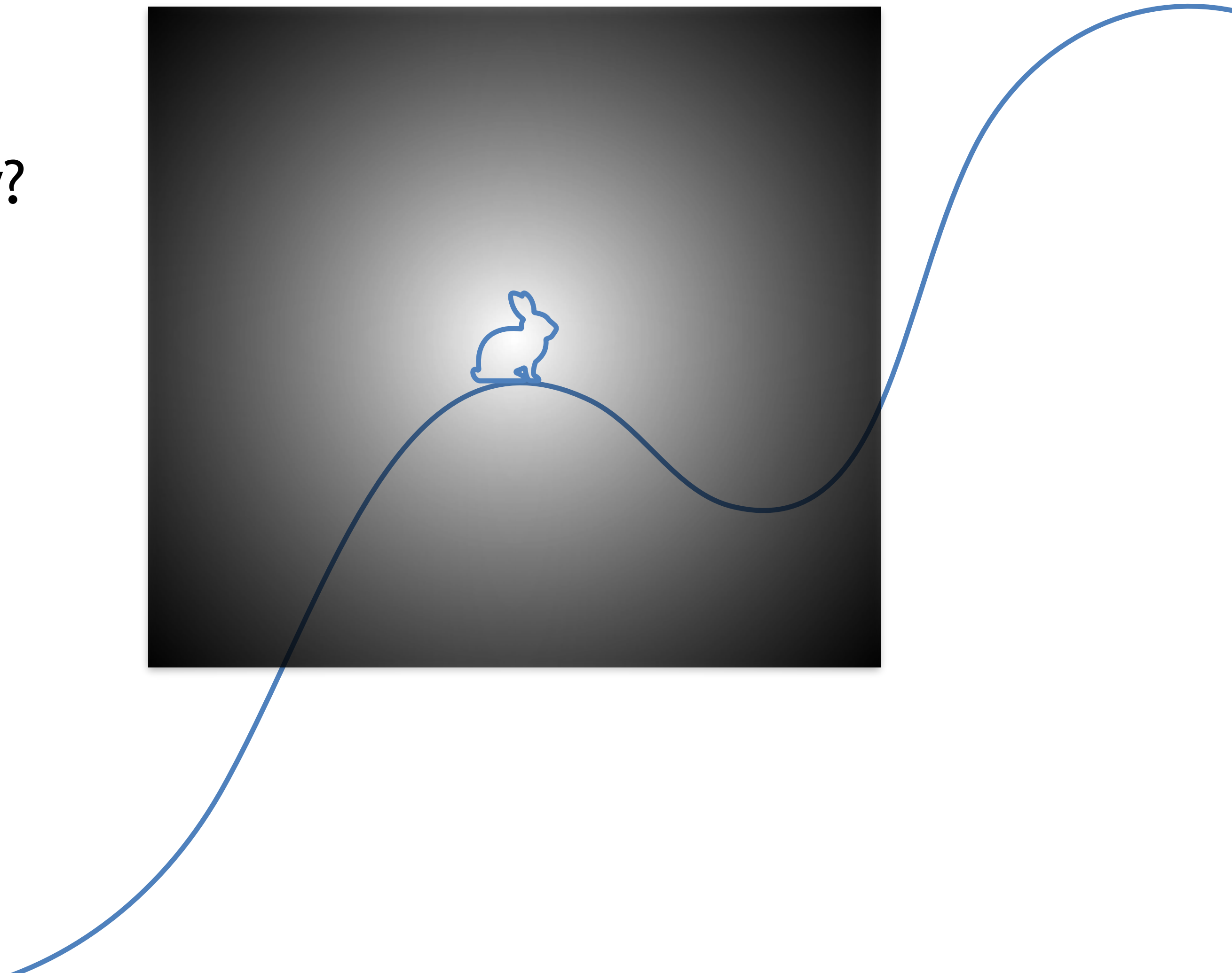
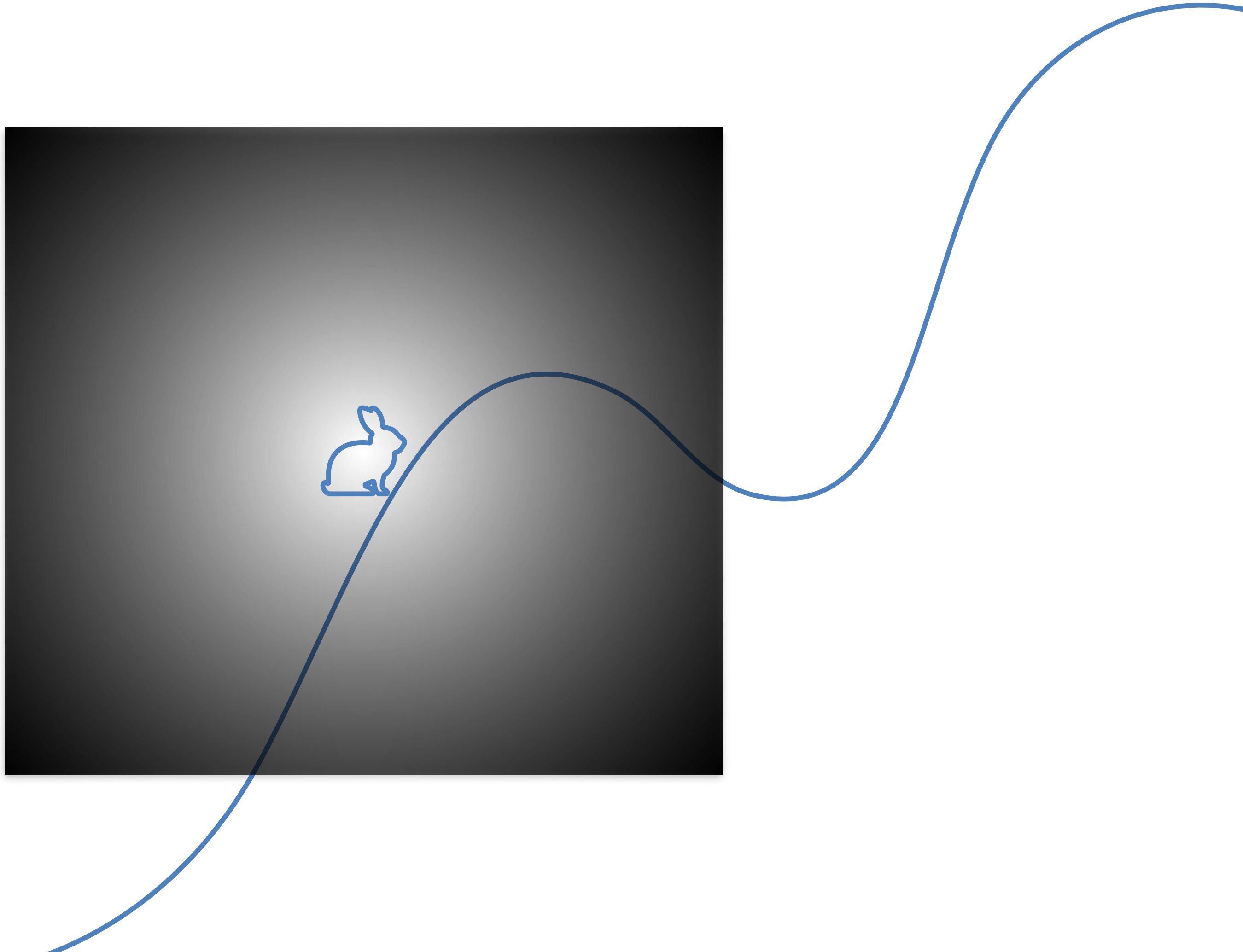



I am at the top!

What if the rabbit is feeling risky?





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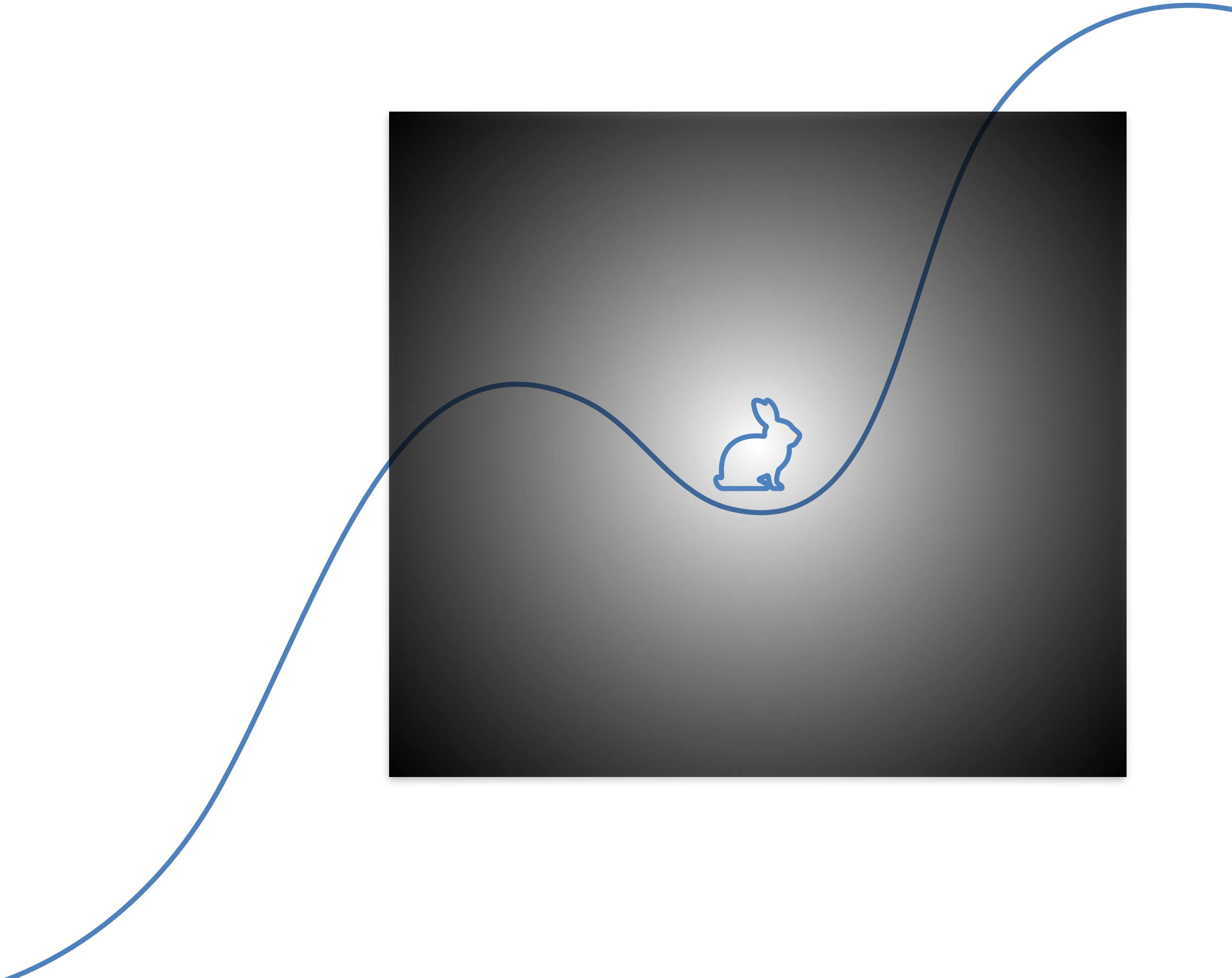

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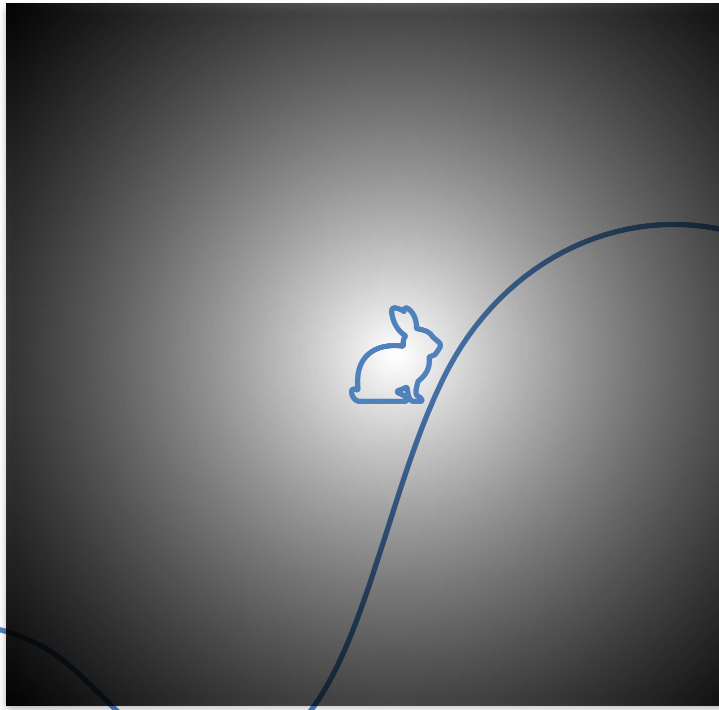

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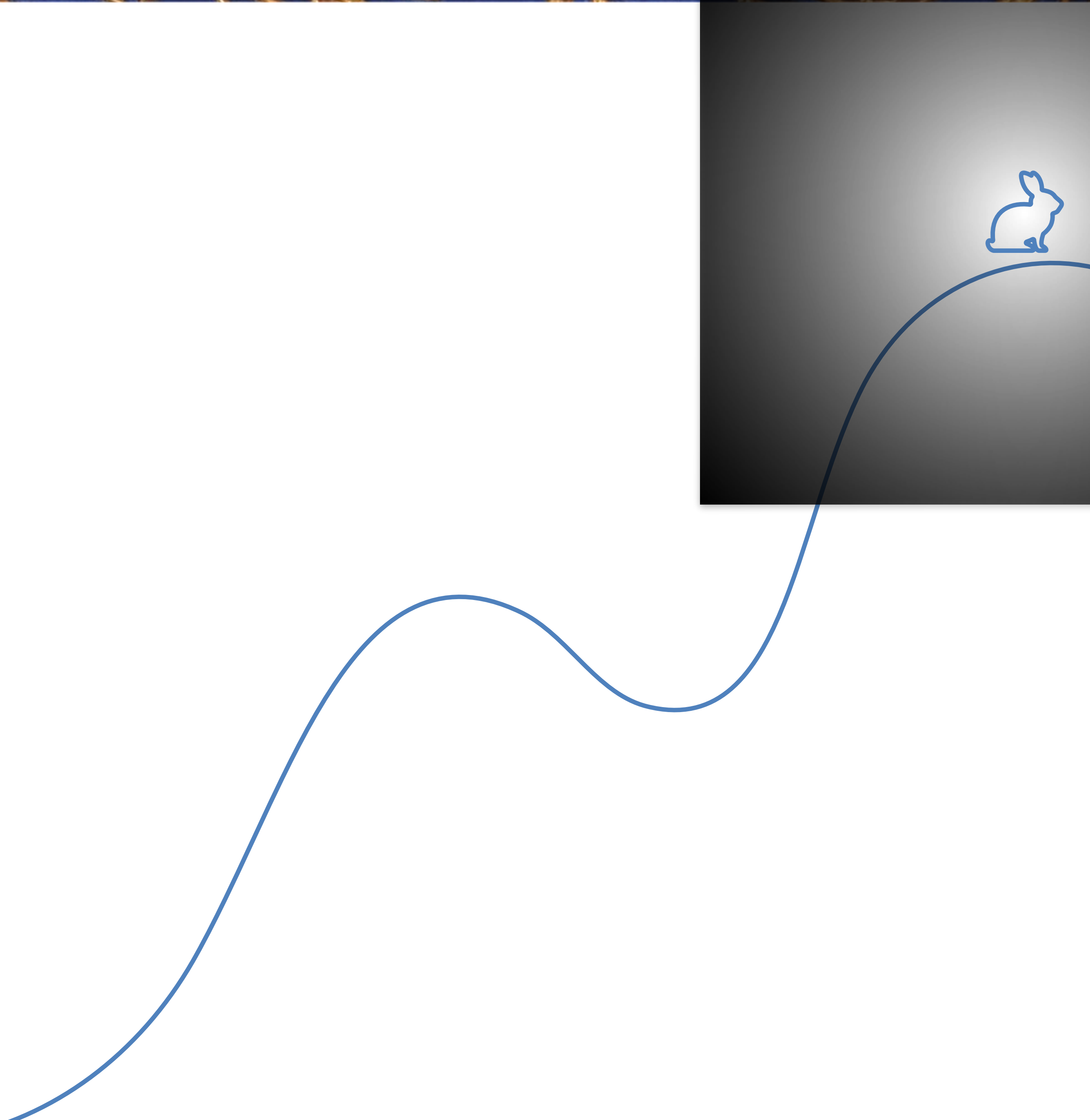
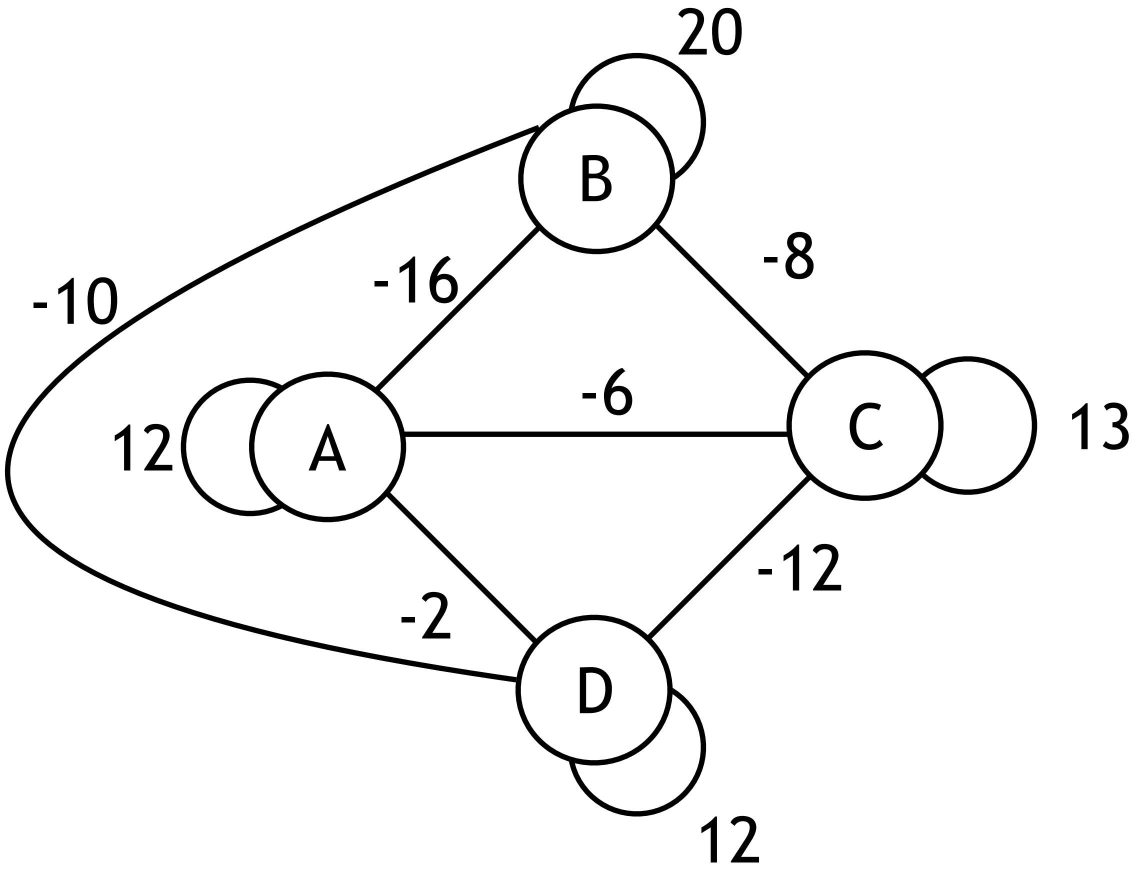


Learning the Solution

- Stochastically learn solution over time

$$\Delta E = (2x_j - 1)(\sum_{i \neq j} x_i w_{ji} + w_{jj})$$



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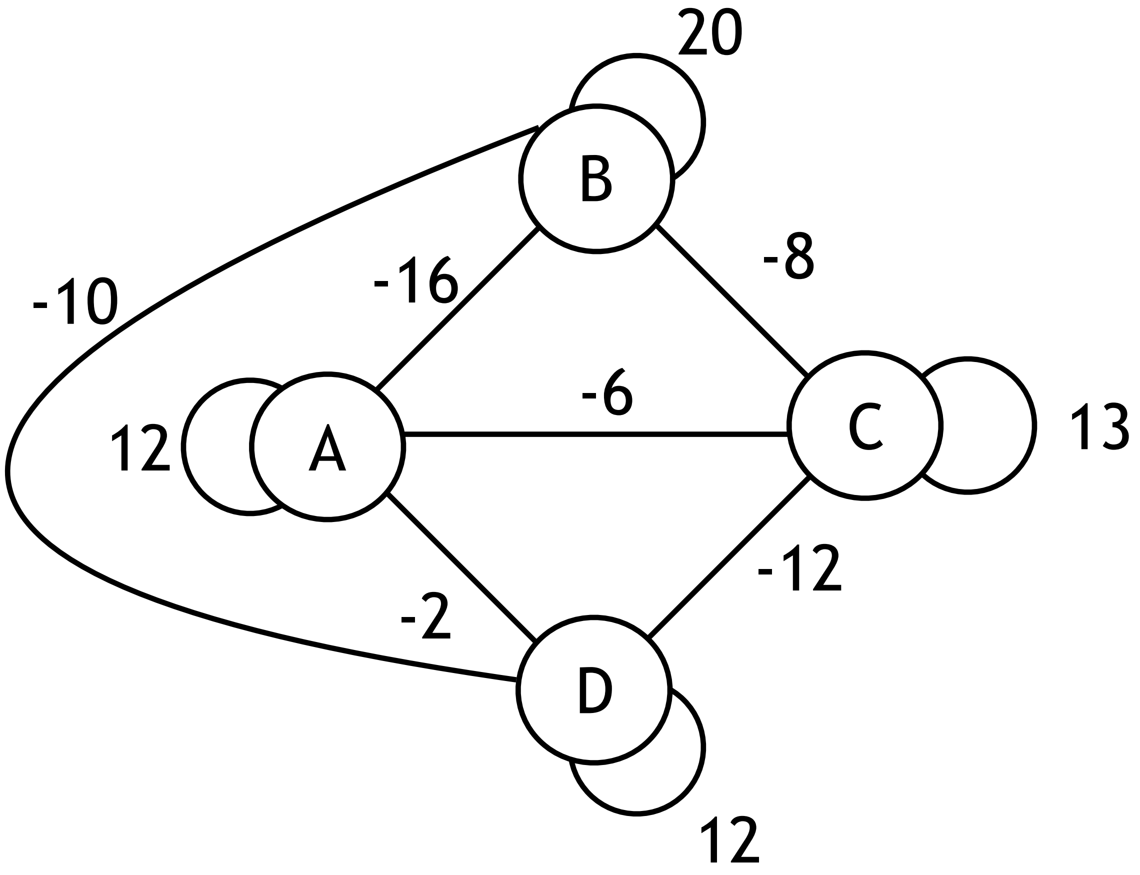
$$x^j = < x_0, x_1, \ldots, \neg x_j, \ldots, x_n >$$

$$P(x^j | x) = \frac{1}{1 + e^{\frac{\Delta E}{C}}}$$

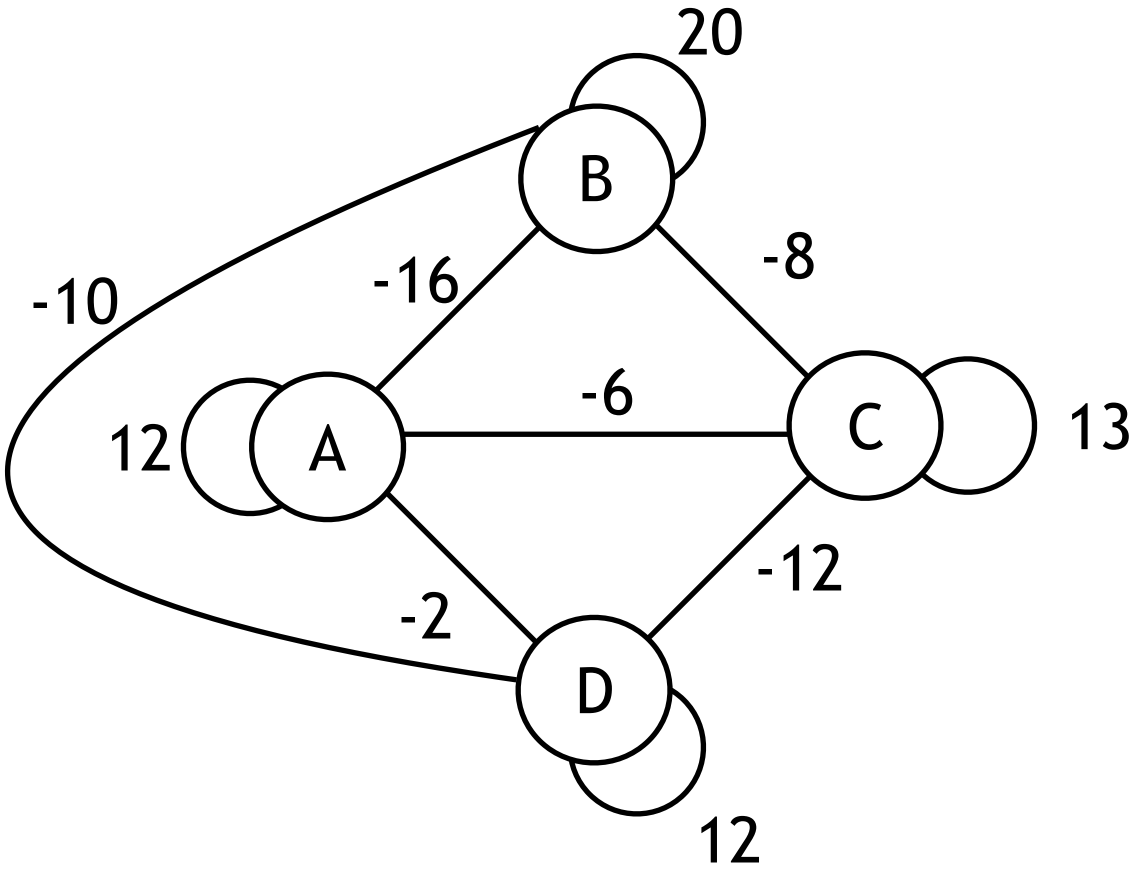
- C allows for probability control of suboptimal state changes



Learning the Solution

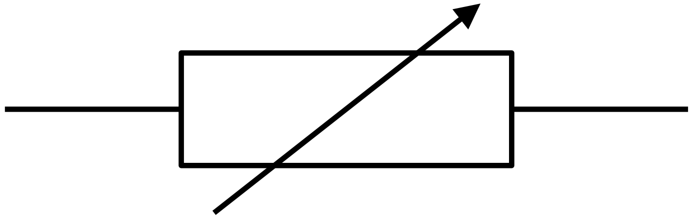
$$P(x^j \mid x) = \frac{1}{1 + e^{\frac{\Delta E}{C}}}$$

- The change in E can be continuously calculated *in situ* for each node
- C allows for simulated annealing, eventually *probably* resulting in a optimized graph



Memristors

- A component with variable conductance



Memristors

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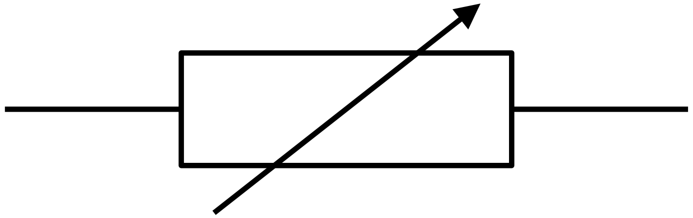
- Recall $V = I \cdot R$



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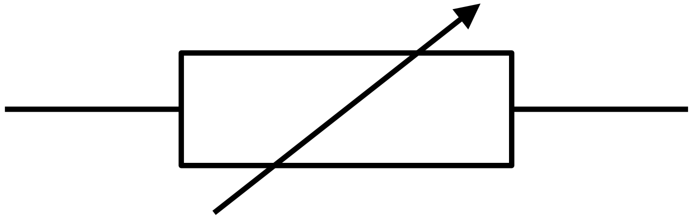
- Used to perform multiplications with the assistance of hardware to detect current



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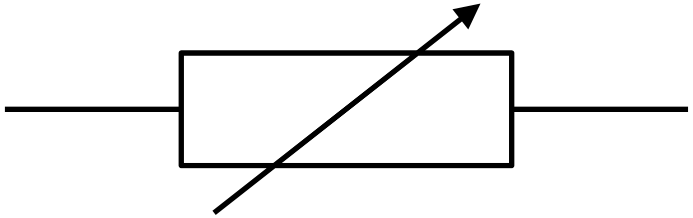
- Components of RRAM cells - memory with density of FLASH but DRAM speed



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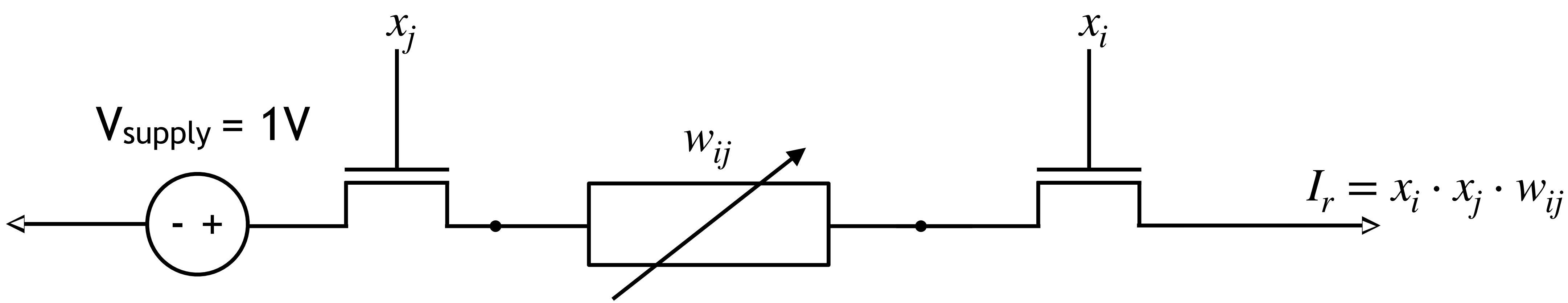
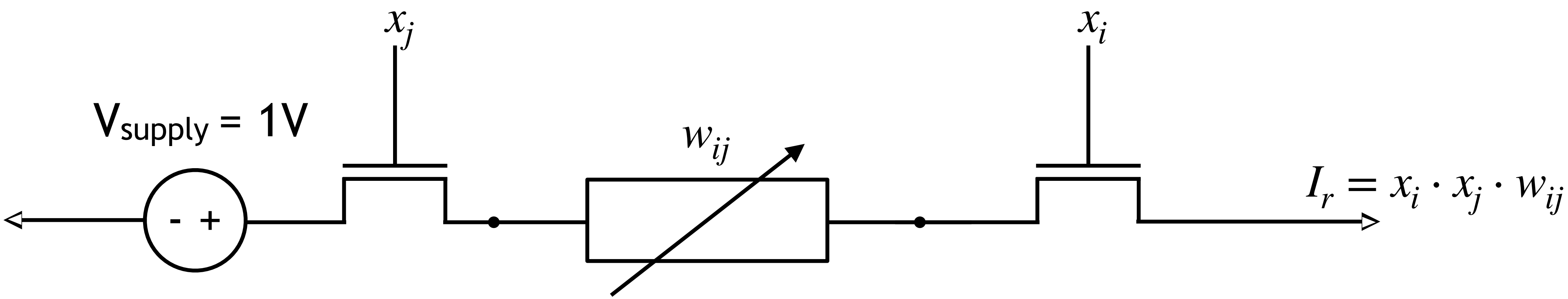
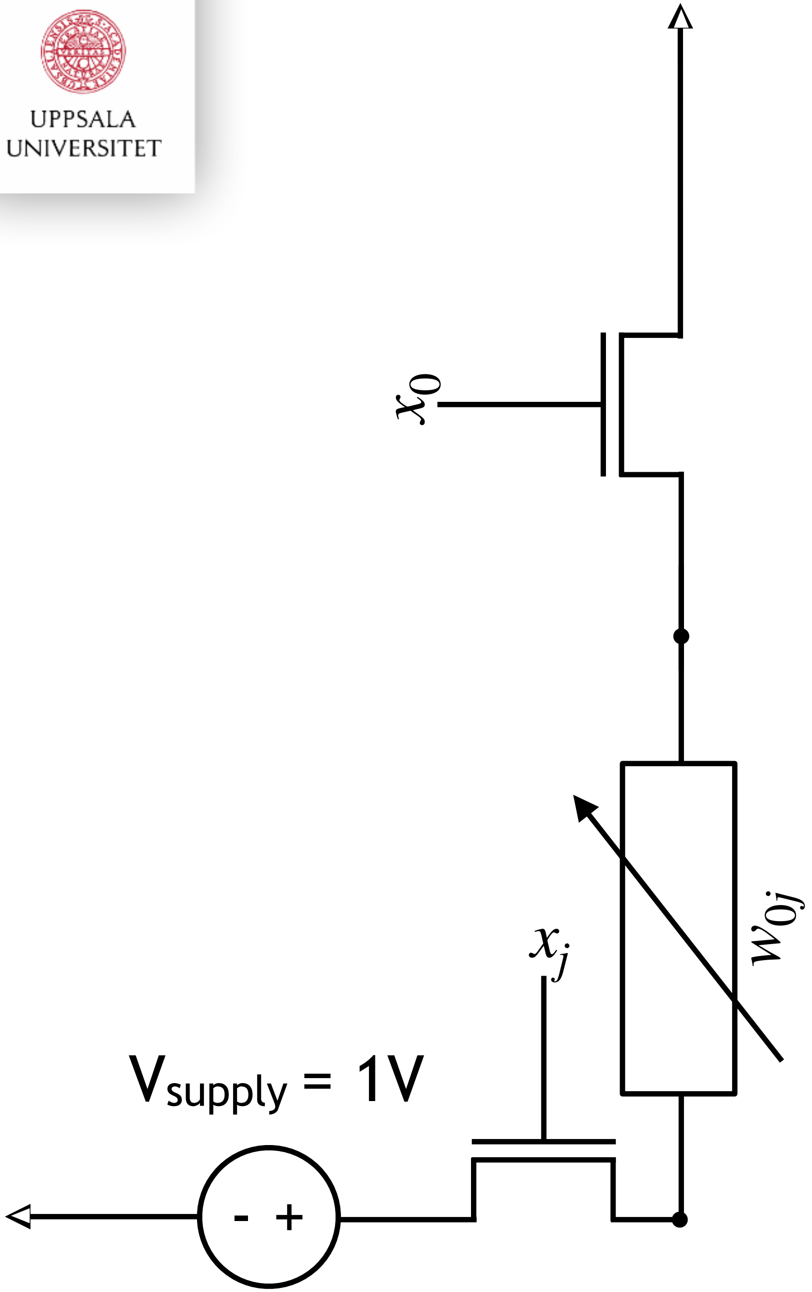
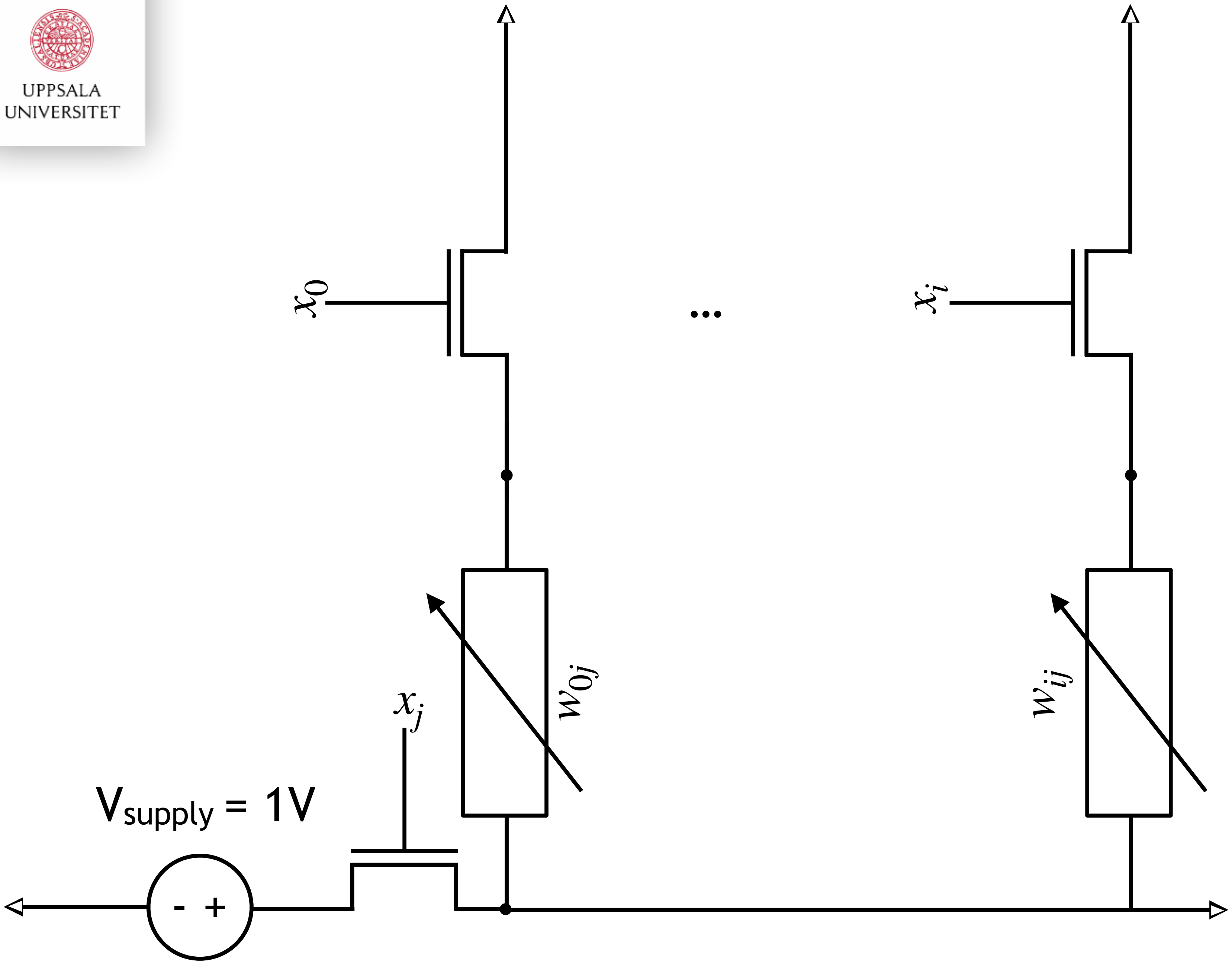
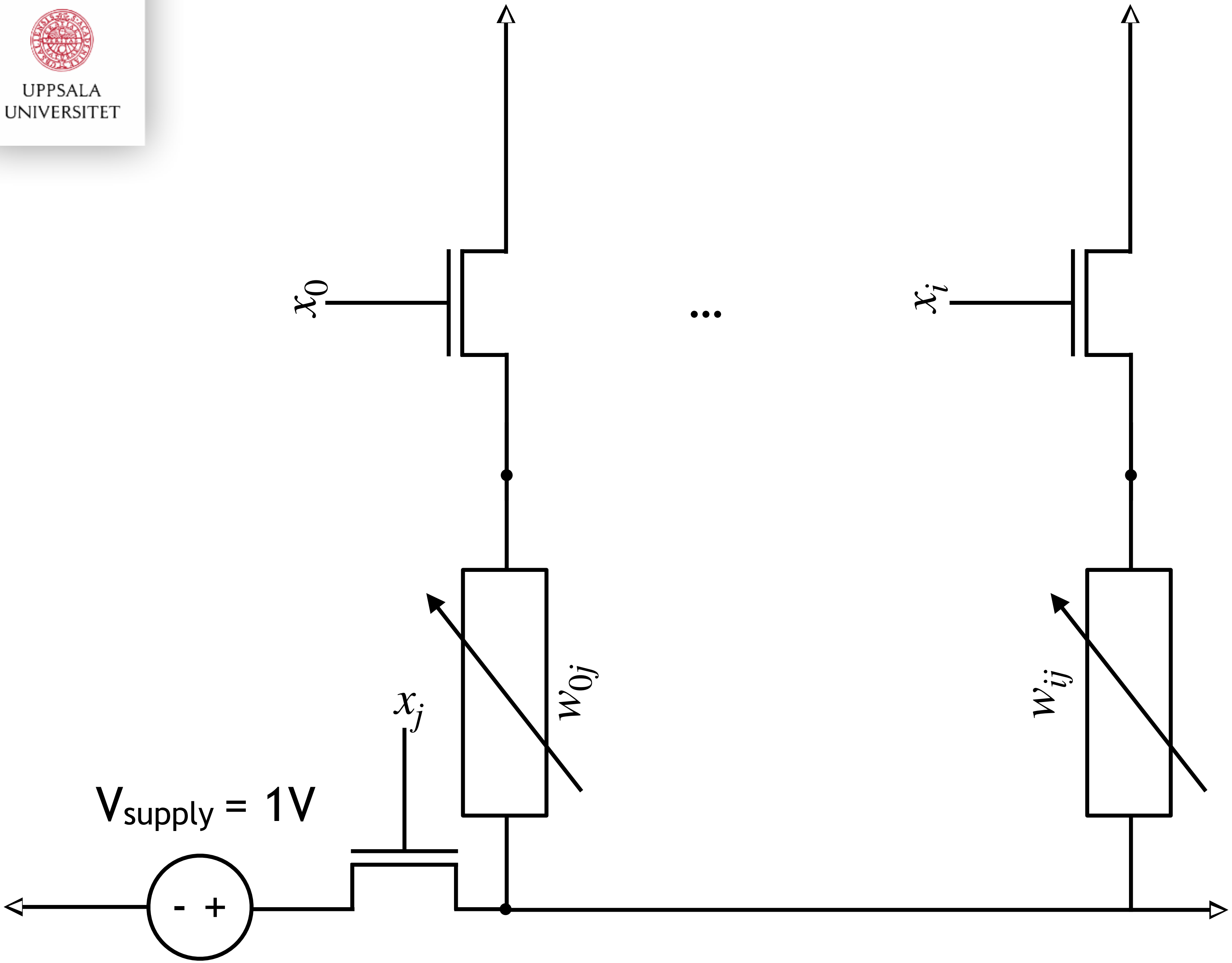



Vsupply = 1V
- +
x_j
w_{ij}
x_i
$I_r = x_i \cdot x_j \cdot w_{ij}$

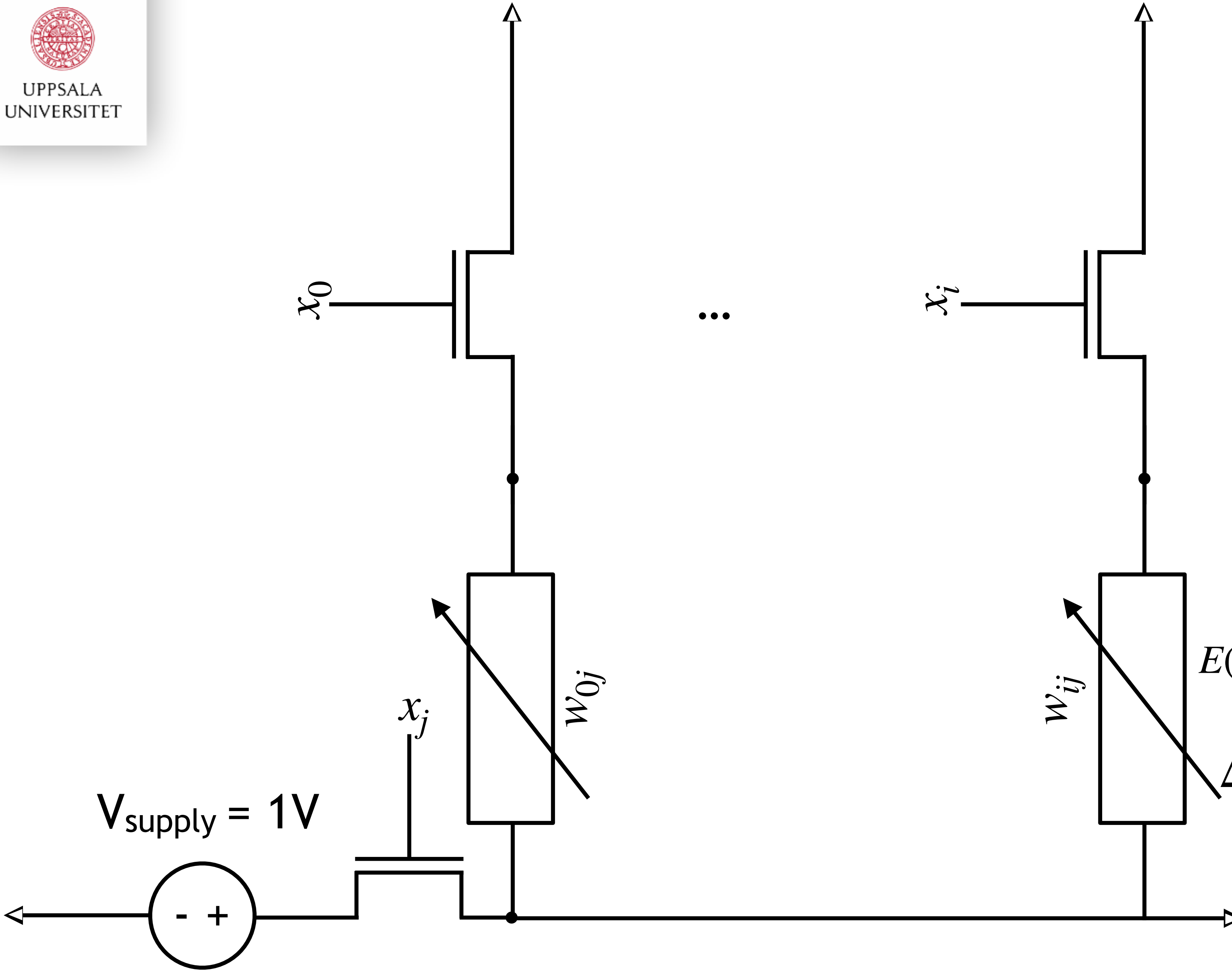

x_0
w_{0j}
x_j
V_{supply} = 1V
- +



x_0
...
x_i
w_{0j}
w_{ij}
x_j
$V_{supply} = 1V$
- +



$$I_j = \sum_{i=0} I_{ji} = \sum_{i=0} x_j x_i w_{ij}$$



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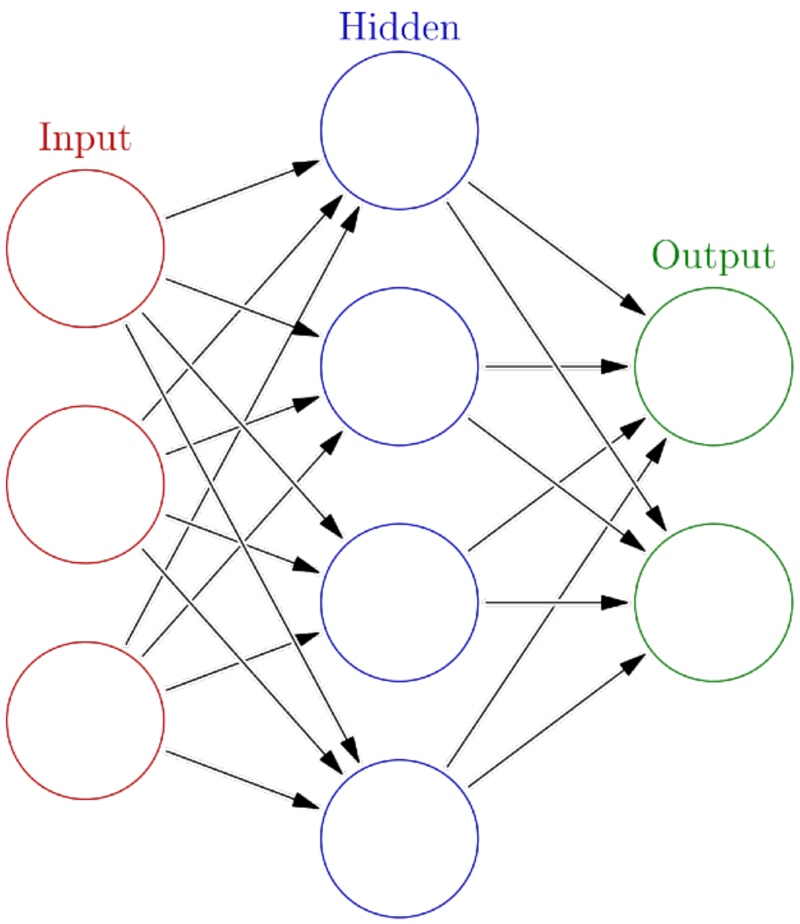
Application to Other Problems



- Different mappings for different problems
- Deep learning uses “Restricted Boltzmann Machines”
  - Combination of layered approach of modern neural nets and Boltzmann Machines

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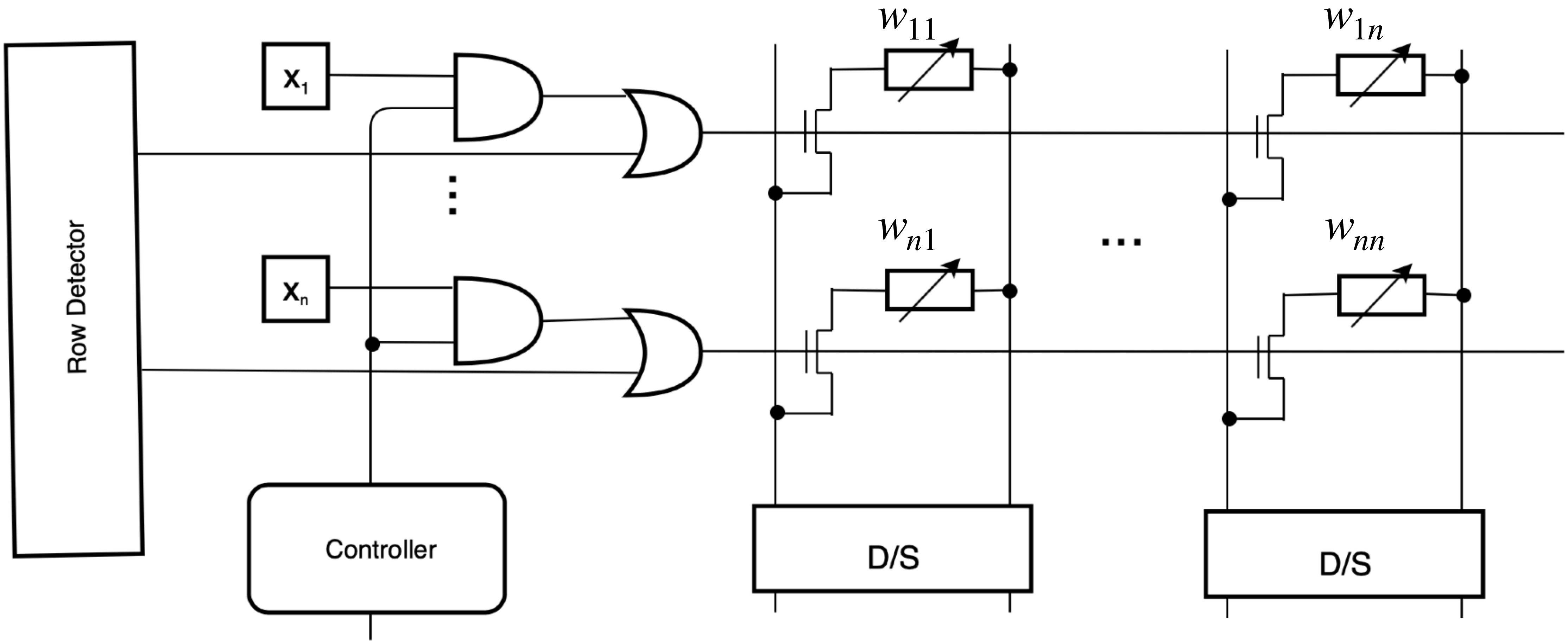
System Organization and Training

ORGANIZATION

Interface

- Interfaces through regular DDRx
- Envisioned as a modular addition to any system with applicable workload

Organization

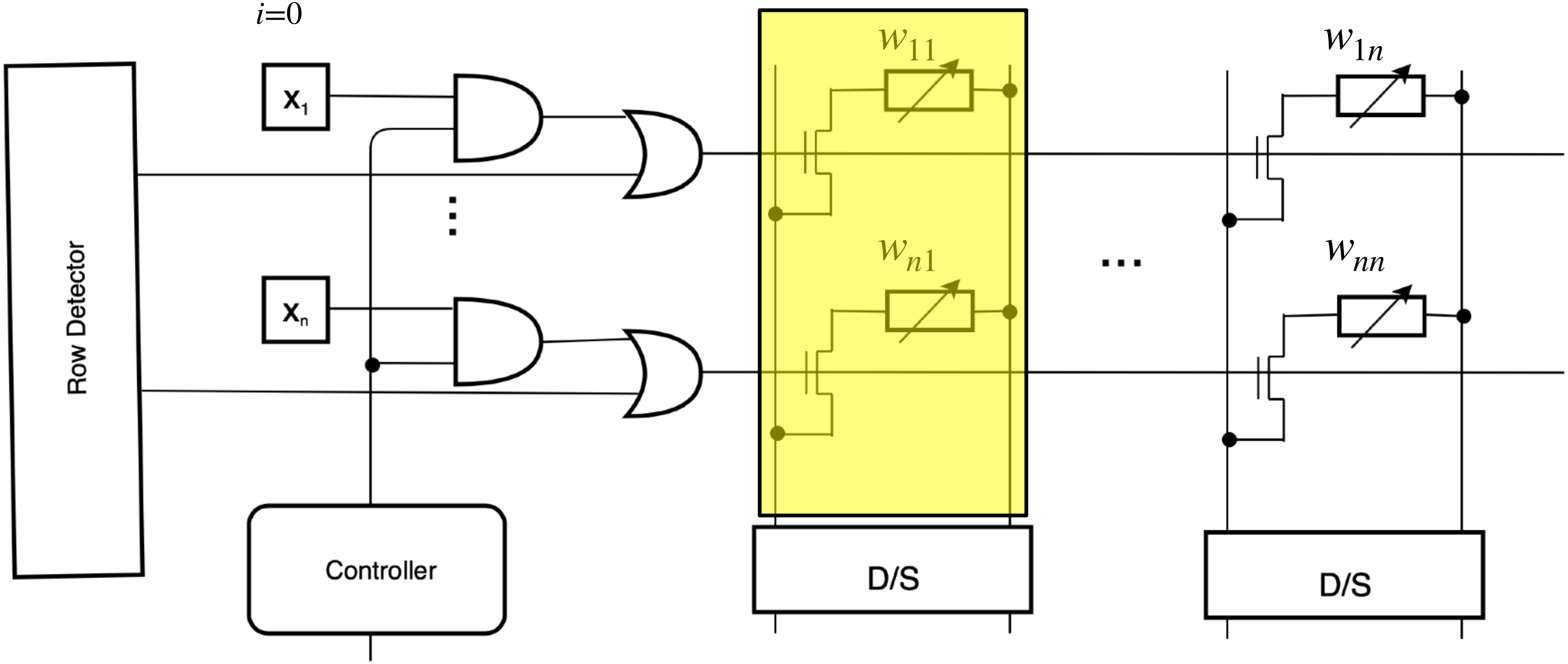


Organization

Current pulled by this column is

$$x_1 \cdot \sum_{i=0}^{n} x_i w_{ij}$$

Sensed by 'D/S' circuit



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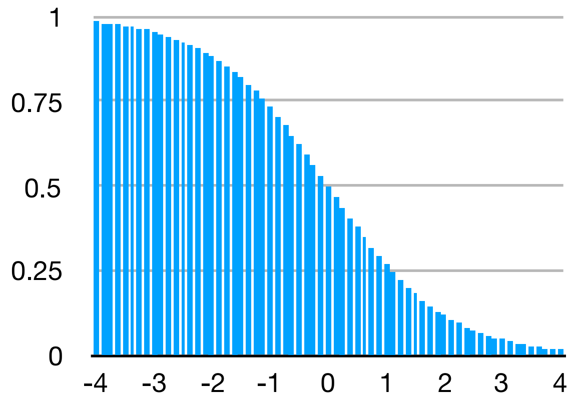
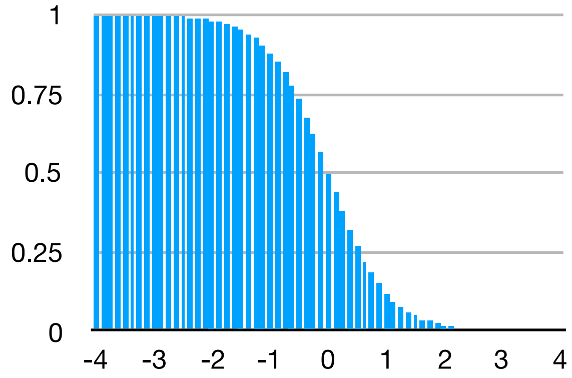
- Sigmoid function is hard to implement in hardware
- Use 64 evenly sampled points from expected range for values of ΔE [-4, 4]
- Different Cs are valid but paper uses C=1



C=1



C=.5

Notes on Sensing Circuits

- Sense amplifiers sense one bit of the current
- Tree structures to sum them in quick fashion
- A large machine must tolerate a lot of inaccuracy and noise generated by current sensing

D/S

Performance and Energy

RESULTS

Optimization Problems

- 57.75x speedup over single threaded kernel
- 6.19x over PIM (process in memory)
- Many workloads achieved closer to 100x speedup over single threaded kernel

Optimization Problems

- 57.75x speedup over single threaded kernel
- 6.19x over PIM (process in memory)
- Many workloads achieved closer to 100x speedup over single threaded kernel

- 25x less energy used than single threaded kernel
- 5.2x less energy used than PIM

Deep Learning

- Even faster… 68.79x faster than single thread
- 6.89x faster than PIM

Deep Learning

- Even faster… 68.79x faster than single thread
- 6.89x faster than PIM

- 63x less energy than single thread
- 5.3x less energy than PIM