

THE UNIVERSITY OF MELBOURNE  
DEPARTMENT OF COMPUTING AND INFORMATION SYSTEMS  
SEMESTER 2 ASSESSMENT 2014

## COMP90046 Constraint Programming

TIME ALLOWED: 3 HOURS  
READING TIME: 15 MINUTES

**Authorized materials:** Books and calculators are not permitted.

**Instructions to Invigilators:** One 21 page script. Exam paper may leave the room.

**Instructions to students:**

This exam counts for 70% of your final grade. There are 7 pages and 8 questions for a total of 70 marks. Attempt to answer all of the questions. Values are indicated for each question and subquestion — be careful to allocate your time according to the value of each question.

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## Question 1 [6 marks]

Here is an odd little puzzle which occurred the other day at an archery meeting. The young lady who carried off the first prize scored exactly one hundred points. The scores on the target are: 16, 17, 23, 24, 39 and 40. Can you figure out how many arrows she must have used to accomplish the feat?

- (a) Write a MiniZinc model to determine, for a given set of scores and a target total score, the minimum number of arrows needed to achieve exactly that score. The data format should be

```
array[int] of int: scores;  
int: target;
```

For example the data for the problem above is

```
scores = [16,17,23,24,39,40];  
target = 100;
```

Ensure that any decision variables you define have tight upper and lower bounds. [4 marks].

- (b) Which kind of constraint solver is likely to be best at solving your model: a constraint programming (CP) solver or a mixed integer programming (MIP) solver? Briefly explain the reasons for your answer. [2 marks].

## Question 2 [8 marks]

- (a) There are at least 3 ways to represent a decision about a set of integers in MiniZinc. Briefly explain 3 methods to do so. Explain which method you would use for representing decisions for each of the following cases, and why:

- i. Choosing a set of at most 3 elements of numbers from  $\{1, \dots, 10000\}$
- ii. Choosing a subset of  $\{1, \dots, 12\}$
- iii. Choosing a subset of  $\{1, \dots, 20\}$  which adds up to 30.

[3 marks].

- (b) Explain how a multiset decision is usually represented in MiniZinc. Discuss briefly why there is no built in `multiset` type in MiniZinc. [2 marks].
- (c) Sometimes when modelling it is necessary to take the input data format and create some new data. Briefly explain three different ways in which this can be managed in MiniZinc, and contrast their advantages and disadvantages. [3 marks].

### Question 3 [7 marks]

Given  $n$  workers numbered  $1..n$  that each need to be assigned to a different task from a set of  $n$  tasks numbered  $1..n$  where the value of worker  $i$  working on task  $j$  is given by  $v[i, j]$ :

- (a) Write a MiniZinc model to find the assignment that maximizes the total value of the assignment. Make use of global constraints if you can. [4 marks].
- (b) Suppose worker 4 must work on a task numbered lower than that worked on by worker 3. Write a constraint expressing this for the model for part (a). [1 mark].
- (c) Suppose task 2 must be worked on by a worker numbered at least 2 different from the worker for task 1. Modify your model of part (a) to allow this constraint to be expressed easily. [2 marks].

### Question 4 [6 marks]

There are many counting global constraints of interest in combinatorial problems. One of the simplest is `count_eq` which enforces that  $c$  is the number of times that a position in the  $x$  array takes the value  $y$ . It is defined as

```
predicate count_eq(array[int] of var int: x, var int: y, var int: c) =  
    c = sum(i in index_set(x)) ( bool2int(x[i] == y) );
```

In this question we examine some other counting globals. The combinatorial constraint

```
common(var int: n1, var int: n2,  
        array[int] of var int: c1, array[int] of var int: c2)
```

holds if  $n1$  is the number of positions in the array  $c1$  taking a value in the array  $c2$ , and  $n2$  is the number of positions in the array  $c2$  taking a value in the array  $c1$ . For example

- `common(3,4,[1,9,1,5],[2,1,9,9,6,9])` holds since three positions in  $[1,9,1,5]$  are in the set  $\{1,2,6,9\}$  of values of  $[2,1,9,9,6,9]$ , and similarly 4 values in  $[2,1,9,9,6,9]$  are in the set  $\{1,5,9\}$ .
- `common(4,1,[1,1,1,1],[2,1,9,9,6,9])` holds

- (a) Give a predicate definition for `common`. [4 marks].
- (b) There is a relationship between  $n1$  and  $n2$  which always holds in every solution of the `common` constraint. Write down the strongest redundant constraint you can that only mentions  $n1$  and  $n2$ . [1 mark].
- (c) The combinatorial constraint

```
uses(array[int] of var int: c1, array[int] of var int: c2)
```

holds if the set of values in the array `c2` is included in the set of values in the array `c1`. Write a definition of `uses` making use of the `common` constraint. [1 mark].

## Question 5 [11 marks]

Consider the following model:

```
predicate red(var int: x) =
    let { var int: y; } in x = 4*y + 1;
predicate blue(var int: x) =
    let { var int: y; } in red(x) \ / x = 2*y;
var 0..5: a;
var 0..5: b;
constraint b = a + 2;
solve satisfy;
output [show(a), " ", show(b)];
constraint red(a) -> blue(b);
```

- (a) Give the set of solutions expected from the model. [3 marks].
- (b) Give the contexts: positive, negative or mixed for each of the expressions: `red(a)`, `blue(b)` and `red(x)` appearing in the model. [2 marks].
- (c) In fact MiniZinc complains about something in the last line of the model. Explain what the problem is, and how the model could be rewritten to avoid the problem (without changing the set of solutions intended). [2 marks].
- (d) Ignoring the problem in the previous part (or assuming its fixed) when asked for all solutions MiniZinc outputs an infinite number of solutions, all the same. Explain why this occurs, and correct the definition of the model to avoid generating an infinite set of solutions. [2 marks].
- (e) Consider the two following logically equivalent constraint expressions. Explain which is preferable and why? [2 marks].

<pre>not exists(i in 1..n)   (forall(j in 1..n-1)     (x[i,j] &lt;= x[i,j+1]));</pre>	<pre>forall(i in 1..n)   (exists(j in 1..n-1)     (x[i,j] &gt; x[i,j+1]));</pre>
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(a)

(b)

## Question 6 [14 marks]

Given a problem with for the constraints:  $x = y \times z$ ,  $z \neq 1$  and initial domains  $D(x) = 0..9$ ,  $D(y) = 0..5$ ,  $D(z) = 0..5$  and search decisions in order  $x \geq 4$ ,  $z \leq 3$ ,  $y = 3$ .

- (a) Show the result of bounds propagation for the constraints: initially, and after every addition of a new constraint by the search. Assume that at each stage in the search the propagation loop continually examines all propagators in the order shown until a fixpoint is reached. Show the domains of all variables after any change and the propagator which caused it. Recall that bounds propagators only examine and update bounds. [4 marks].
- (b) Show the result of domain propagation for the constraints: initially, and after every addition of a new constraint by the search. Assume that at each stage in the search the propagation loop continually examines all propagators in the order shown until a fixpoint is reached. Show the domains of all variables after any change and the propagator which caused it. [3 marks].
- (c) Give pseudo-code defining a bounds propagator for the constraint:  $x = \text{sign}(y)$  where  $\text{sign}(d)$  returns the sign of the integer argument: 0 if  $d = 0$ , 1 if  $d > 0$  and  $-1$  if  $d < 0$ . Make the propagator as strong as possible. Use functions  $\text{lb}(v)$  and  $\text{ub}(v)$  to access the current bounds of variable  $v$ , and functions  $\text{setlb}(v, d)$  and  $\text{setub}(v, d)$  to set the bounds of a variable. You can assume that the setting functions won't change to a weaker bound: e.g.  $\text{setlb}(x, \text{lb}(x) - 1)$  will have no effect. [3 marks].
- (d) Constraint programming solvers require that propagators are: correct and checking. Define each concept in a short paragraph, explain why each is necessary for the solver to operate correctly. [4 marks].

## Question 7 [7 marks]

Consider the following MiniZinc predicate:

```
predicate strange(array[int] of var int: s,
                  array[int] of int: d) =
  forall(i,j in index_set(s))(
    let { var bool: b1;
          var bool: b2 = not(b1); } in
    if i != j then
      (b1 -> s[i] + d[i] <= s[j])
      /\ (b2 -> s[j] + d[j] <= s[i])
    else true endif );
```

- (a) Show the conjunction of primitive constraints of the form  $b = \text{not}(b')$  and  $b \rightarrow a + d \leq c$  that result from unrolling the MiniZinc constraint `constraint strange([x,y,z],[3,1,2]);` [4 marks].
- (b) The definition of `strange` is not as simple as it could be and generates redundant constraints. Give a new MiniZinc definition of `strange` that is as efficient as possible but still has the same logical meaning. [3 marks].

## Question 8 [11 marks]

The company Splashdown delivers portable lavatories to events. Each event has a given demand for lavatories, a duration, a start date, a contract price, and a penalty price. Splashdown can choose to ignore an event earning 0, or fulfil the event contract totally earning the contract price, or fulfil it partially earning the contract price - the penalty price for each missing lavatory. Splashdown has a limited stock of lavatories, but it may choose to buy new lavatories at a cost, or subcontract out some of the lavatories required for an event, paying the daily cost of the subcontracting times the number of lavatories subcontracted times the event duration. The aim is to make the most profit for the planning period.

Data for the problem is as follows:

```
int: n;                                % days of planning period
set of int: DAY = 0..n;
int: stock;                             % initial stock of lavatories
int: m;                                 % number of events
set of int: EVENT = 1..m;
array[EVENT] of DAY: start;             % start day
array[EVENT] of int: duration;          % duration in days
array[EVENT] of int: demand;           % demand of lavatories
array[EVENT] of int: contract;         % contract price
array[EVENT] of int: penalty;          % penalty price per lavatory
int: buy_cost;                          % cost to buy new lavatory
int: daily_rent_cost;                  % cost to rent for one day
```

For example the data file:

```
n = 10;
stock = 5;
m = 8;
start = [0,0,1,2,3,3,5,7];
duration = [3,4,2,5,2,5,2,3];
demand = [4,3,4,2,5,2,3,5];
contract = [60,70,35,55,50,45,35,80];
penalty = [15,25,10,30,10,20,10,15];
buy_cost = 35;
daily_rent_cost = 5;
```

describes a situation where there is a 10 day plan, an initial stock of 5, with 8 events with varying parameters. The cost for buying a lavatory is 35, and the cost for subcontracting each lavatory is 5.

Expected output is a plan of the form:

```
sent = [4, 0, 0, 1, 4, 0, 3, 5]
rented = [0, 3, 0, 1, 0, 0, 0, 0]
bought = 0
obj = 255
```

showing the number of lavatories sent to each event, the number rented for each event, the number bought, and the total profit.

- (a) Give a MiniZinc model for the Splashdown problem. You do not need to repeat the data declarations. Ensure that each integer variable is given a tight range of possibilities. Ensure the output item returns output in the correct form. [9 marks].
- (b) Give a good search annotation for the model. Explain why you think this is likely to give good search behaviour. [2 marks].