

Sultan Halim Random Stuff Cheatsheet (Page 25)

1. Combinatorial Design

a. Existence and Construction of

i. Balanced Incomplete Block Design

ii. Steiner Triple System (k = 3)

1. If $\lambda = 1$, then v congruent 1 or 3 mod 6 if and only if there exist a triple system

iii. Kirkman triple system

1. Steiner but the 3-block need to be partitioned into classes, each of which contains $n/3$ 3-block, and each treatment appear once in the class.
2. There is a Kirkman triple system on v treatments for all v congruent 3 mod 6.

iv. Latin squares

v. One factorisation

2. Geometry

- a. Fermat-Toricelli Point, smallest total distance to vertices if all angle less than or equal to 120 degree. Otherwise it's the vertex with angle more than 120 degree.
- b. Smallest-length pedal triangle -> Orthic triangle, pedal triangle with smallest perimeter.

3. Group Theory

- a. jika $\phi: G \rightarrow H$ group homomorphism, maka $\text{image}(\phi)$ isomorphic $G / \ker(\phi)$

4. Counting techniques

- a. Twelffold way
- i. Catat tabel
- ii. $\{n, x\}$ is Stirling number of second kind

iii. $p_x(n)$ is partition of n into x parts.

The [generating function](#) for $p(n)$ is given by^[5]

$$\sum_{n=0}^{\infty} p(n)x^n = \prod_{k=1}^{\infty} \left(\frac{1}{1-x^k} \right).$$

iv.

The twelve combinatorial objects and their enumeration formulas.

f-class	Any f	Injective f	Surjective f
f	n -sequence in X x^n	n -permutation of X x^n	composition of N with x subsets $x! \binom{n}{x}$
$f \cdot S_n$	n -multisubset of X $\binom{x+n-1}{n}$	n -subset of X $\binom{x}{n}$	composition of n with x terms $\binom{n-1}{n-x}$
$S_x \cdot f$	partition of N into $\leq x$ subsets $\sum_{k=0}^x \binom{n}{k}$	partition of N into $\leq x$ elements $[n \leq x]$	partition of N into x subsets $\left\{ \begin{matrix} n \\ x \end{matrix} \right\}$
$S_x \cdot f \cdot S_n$	partition of n into $\leq x$ parts $p_x(n+x)$	partition of n into $\leq x$ parts $[n \leq x]$	partition of n into x parts $p_x(n)$

b. General Recursion

c. GF and Exponential GF

5. Topological combinatorics

a. Sperner Lemma

- i. 2D: Given a triangle ABC, and a triangulation T of the triangle, the set S of vertices of T is colored with three colors in such a way that: (i) A, B, C are colored 1, 2, 3 respectively. Each vertex on an edge of ABC is to be colored only with one of the two colors of the ends of its edge. Result: There exists a triangle T , whose vertices are colored with the three different colors.

- ii. Multidimensional: Given n -dimensional simplex, triangulate to smaller n -dimensional simplex. (i) Vertices of the original simplex are colored with different colors. (ii) Vertices of T located on any k -dimensional subface of the large simplex are colored only with the colors in the vertices

of the original simplex that is part of the k -dimensional subface. Result -> There exists odd number of simplices from T whose vertices are colored with all $n+1$ colors.

b. Necklace Splitting Theorem

- i. Discrete splitting: The necklace has kn beads. The beads come in t different colors. There are $k \cdot a_i$ beads of each color i , where a_i is positive integer. Partition the necklace into k parts, each of which has exactly a_i beads of color i . Use at most $(k-1)t$ cuts.

6. Helly Theorem

- a. Let X_1, X_2, \dots, X_n be a finite collection of convex subsets of \mathbb{R}^d with $n > d$. If the intersection of every $d+1$ of these sets is nonempty, then the whole collection has a nonempty intersection

$f(n)$	$a_n^{(p)}$
Exponential function	
(i) Ak^n k is not a characteristic root	Bk^n
(ii) Ak^n k is a characteristic root of multiplicity m	$Bn^m k^n$
Polynomial	
(i) $\sum_{i=0}^t p_i n^i$ 1 is not a characteristic root	$\sum_{i=0}^t q_i n^i$
(ii) $\sum_{i=0}^t p_i n^i$ 1 is a characteristic root of multiplicity m	$n^m \sum_{i=0}^t q_i n^i$
Special combined function	
(i) $An^t k^n$ k is not a characteristic root	$\left(\sum_{i=0}^t q_i n^i \right) k^n$
(ii) $An^t k^n$ k is a characteristic root of multiplicity m	$n^m \left(\sum_{i=0}^t q_i n^i \right) k^n$