BubbleTeaM Random Stuff Cheatsheet (Page 25)

1. Combinatorial Design
   a. Existence and Construction of
      i. Balanced Incomplete Block Design
      ii. Steiner Triple System (k = 3)
         1. If lambda = 1, then v congruent 1 or 3 mod 6 if and only if there exist a triple system
      iii. Kirkman triple system
         1. Steiner but the 3-block need to be partitioned into classes, each of which contains n/3 3-block, and each treatment appear once in the class.
         2. There is a Kirkman triple system on v treatments for all v congruent 3 mod 6.
      iv. Latin squares
      v. One factorisation
2. Geometry
   a. Fermat-Toricelli Point, smallest total distance to vertices if all angle less than or equal to 120 degree. Otherwise it's the vertex with angle more than 120 degree.
   b. Smallest-length pedal triangle -> Orthic triangle, pedal triangle with smallest perimeter.
3. Group Theory
   a. jika phi: G -> H group homomorphism, maka image(phi) isomorphic G / ker(phi)
4. Counting techniques
   a. Twelvefold way
      i. Catat tabel
      ii. {n, x} is Stirling number of second kind
      iii. p_x(n) is partition of n into x parts.

The generating function for $p(n)$ is given by[5]

$$\sum_{n=0}^{\infty} p(n)x^n = \prod_{k=1}^{\infty}\left(\frac{1}{1-x^k}\right).$$

      iv.

The twelve combinatorial objects and their enumeration formulas.

f-class	Any *f*	Injective *f*	Surjective *f*
f	*n*-sequence in *X* x^n	*n*-permutation of *X* $x^{\underline{n}}$	composition of *N* with *x* subsets $x!\left\{{n \atop x}\right\}$
$f \circ S_n$	*n*-multisubset of *X* $\binom{x+n-1}{n}$	*n*-subset of *X* $\binom{x}{n}$	composition of *n* with *x* terms $\binom{n-1}{n-x}$
$S_x \circ f$	partition of *N* into ≤ *x* subsets $\sum_{k=0}^{x}\left\{{n \atop k}\right\}$	partition of *N* into ≤ *x* elements $[n \le x]$	partition of *N* into *x* subsets $\left\{{n \atop x}\right\}$
$S_x \circ f \circ S_n$	partition of *n* into ≤ *x* parts $p_x(n+x)$	partition of *n* into ≤ *x* parts 1 $[n \le x]$	partition of *n* into *x* parts $p_x(n)$

   b. General Recursion
   c. GF and Exponential GF
5. Topological combinatorics
   a. Sperner Lemma
      i. 2D: Given a triangle ABC, and a triangulation T of the triangle, the set S of vertices of T is colored with three colors in such a way that: (i) A, B, C are colored 1, 2, 3 respectively. Each vertex on an edge of ABC is to be colored only with one of the two colors of the ends of its edge. Result: There exists a triangle T, whose vertices are colored with the three different colors.
      ii. Multidimensional: Given n-dimnesional simplex, triangulate to smaller n-dimensional simplex. (i) Vertices of the original simplex are colored with differnent colors. (ii) Vertices of T located on any k-dimensional subface of the large simplex are colored only with the colors in the vertices of the original simplex that is part of the k-dimensional subface. Result -> There exists odd number of simplices from T whose vertices are colored with all n+1 colors.
   b. Necklace Splitting Theorem
      i. Discrete splitting: The necklace has kn beads. The beads come in t diferent colors. There are k * a_i beads of each color i, where a_i is positive integer. Partition the necklace into k parts, each of which has exactly a_i beads of color i. Use at most (k-1)t cuts.
6. Helly Theorem
   a. Let X_1, X_2, ..., X_n be a finite collection of convex subsets of R^d with n>d. If the intersection of every d+1 of these sets is nonempty, then the whole collection has a nonempty intersection

	$f(n)$	$a_n^{(p)}$
	<u>Exponential function</u>	
(i)	Ak^n	Bk^n
	k is not a characteristic root	
(ii)	Ak^n	$Bn^m k^n$
	k is a characteristic root of multiplicity m	
	<u>Polynomial</u>	
(i)	$\sum_{i=0}^{t} p_i n^i$	$\sum_{i=0}^{t} q_i n^i$
	1 is not a characteristic root	
(ii)	$\sum_{i=0}^{t} p_i n^i$	$n^m \sum_{i=0}^{t} q_i n^i$
	1 is a characteristic root of multiplicity m	
	<u>Special combined function</u>	
(i)	$An^t k^n$	$\left(\sum_{i=0}^{t} q_i n^i\right) k^n$
	k is not a characteristic root	
(ii)	$An^t k^n$	$n^m \left(\sum_{i=0}^{t} q_i n^i\right) k^n$
	k is a characteristic root of multiplicity m	