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A MULTI-SATELLITE MISSION TO ILLUMINATE THE EARTH: FORMATION CONTROL BASED ON IMPULSIVE MANEUVERS

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This study examines feasibility of impulsive control to establish and maintain spacecraft formation consisting of microsatellites equipped with large sunlight reflectors. In the appropriate lighting conditions and given right attitude of the reflecting surfaces, such formations can be visible at a point of interest on Earth as pixel images in the sky. It has been shown that any formation, which is fixed with respect to the orbital reference frame, requires continuous control by onboard thrusters resulting in excessive fuel consumption. However, setting special initial conditions for each satellite that send it along a circumference in the orbital reference frame allows all "pixels" to constitute an image that rotates as a whole almost without control. Some control action is still required for all satellites to converge to the relative trajectories after launch and maintain the formation during its lifetime. It appears that aerodynamic-based control works merely for very low orbit altitudes resulting in the shorter lifetime. Therefore, we consider impulsive control with the use of a liquid-propellant propulsion system to gather the formation deployed from a single launch vehicle with nearly the same initial conditions for all satellites and to maintain the required formation geometry during the mission lifetime.

INTRODUCTION

We study the problem of small satellite formation deployment and maintenance using impulsive maneuvers. Satellites in the formation are equipped with sunlight reflectors. In the appropriate lighting conditions and given the right attitude of the reflector, a satellite can be seen from Earth as a bright star. Multiple satellites can be positioned so as to form graphic images in the sky as in Fig. 1. Preliminary feasibility study[1] showed that low-Earth Sun-synchronous orbits that pass along the terminator line are preferable for such formations.

Depending on the approach to the formation deployment, pixel images produced in the sky can have constant attitude during the demonstration as in the work[1] or rotate with orbital period as in a more recent study.[2] If the attitude of the image is fixed with respect to the orbital reference frame, it requires continuous control by onboard thrusters and leads to excessive fuel consumption. Simulation of two satellites relative dynamics along the circular Sun-synchronous orbit with 600 km altitude presented in the feasibility study[1] showed the mean propellant consumption of 2.29 kg in 30 days (for a satellite positioned at 1250 m away from the reference orbit origin) and 1.14 kg (for

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a position 625 m away). Another approach delineated in the recent study[2] is to have the image rotating with respect to a certain orbital reference frame with an orbital period. Each satellite can be appointed such initial conditions that it moves along a circumference in the orbital reference frame, so that each "pixel" in the image rotates with the same angular velocity without control in accordance with the orbital dynamics laws. However, control is required to achieve the target relative trajectories and to track it during the mission time.

Our prior study[2] analyzed the possibility of achieving a defined image configuration of the formation by decentralized aerodynamic-based control. The relative-to-the-target trajectories were reached by adjusting the attitude of the satellite reflector with respect to the incoming airflow. It allowed changing the cross-section area of satellites, so the differential drag and lift forces appeared. The results of the simulation showed that the approach can be applied for the orbit altitudes below 350 km and the reflector size significantly exceeding the minimally required one ($4 \cdot 4\, m^2$ in comparison to $2 \cdot 2\, m^2$) according to the study.[2] Since the satellites were not supposed to be equipped with thrusters their orbits could not be maintained for long periods using only aerodynamic lift force. Therefore, the ballistic lifetime for such formations is quite short and does not exceed one month considering 350 km altitude and $16\, m^2$ reflector area.

Figure 1. IAA letters above Rome

We propose to consider a different approach to control the formation in question. All satellites in the formation are firstly transferred to closed relative trajectories around the geometrical center of the formation in the orbital reference frame. These trajectories are obtained with the aid of the analytical solutions to Hill-Clohessy-Wiltshire equations.[3,4] The relative trajectories are then maintained by impulsive control with the use of cold gas thrusters. The impulsive control proposed in the paper[5] is based on a two-impulse scheme aiming to compensate the difference between the current and target orbits of a satellite. We assume the centralized control of satellite formation geometry. It implies that there is a target satellite and several chaser satellites in the formation. The chaser satellites monitor the position of the target one and control their orbital motion to achieve the required closed relative trajectories. This control approach was implemented in the recent CanX-4 and CanX-5 formation flying mission launched into orbit in June 2014.[6]

PRELIMINARY STUDY

Our preliminary study[1] was focused on designing a mission of CubeSats equipped with reflectors and coordinated in a formation to produce in the sky a graphical image, which is seen from a given point of interest (POI) on Earth. There were three main tasks to be accomplished:

1. orbit selection;
2. payload design;
3. formation control.

Let us start by defining the requirements for image demonstration. The requirements can be divided in two groups. These are single satellite visibility and formation quality during a demonstration. The requirements for the demonstration are as follows:

- the satellite should be in the direct line of sight (LOS) from the POI;
- the satellite should be lit by the Sun in order to be able to reflect the light to the ground;
- the Sun should be below maximum elevation angle during the demonstration. In this work, the limit is set to -5 degrees of elevation;
- the spacecraft must be in the darker part of the sky when it passes, so the angle between the directions from the POI to the Sun, on one hand, and the satellite on the other, should not be less than a certain value, which in this case is set to 25 degrees;
- the pixels must be clearly visible by the naked eye. In this work, this is defined by requiring the magnitude of -8.0 or brighter (the magnitude of the well-known Iridium satellite flares[7] varies between -8.0 and -9.5);
- distances between any two CubeSats in the formation must be such that they are distinguished as independent pixels. In theory, human eye resolution[8] is known to be about one arc-minute;
- the relative position error should not be greater than some established value.

For a mission requirement that a satellite shall appear above a given point on the ground at given times, it is essential to make sure that the orbit is such that the satellite is indeed observable and visible. The set of mission requirements and assumptions leaves us with a narrow range of points in time and orbital positions where the demonstration is possible. Thus, the chosen strategy is to align the orbit with the terminator line (or make the orbit cross the terminator line over the POI if there is there is just one such point). In the case of multiple POIs, if the orbit remains in accordance with the terminator, the amount of views is more stable and easily controlled by selecting the correct LTAN.

As the terminator line moves in the inertial frame in accordance with the Sun, the orbit of interest is, of course, a Sun-synchronous one (SSO).

For payload design we considered a single satellite with a large sunlight reflector orbiting the Earth as in Fig 2. In the Fig. 2 the scattering angle α is the included angle of the Sun measured from the Earth, d is the distance between reflector and POI, A_r and A_{gs} are the areas of the CubeSat reflector and the ground spot respectively. The magnitude of the reflector is usually calculated as the ratio of incident light intensity to a reference intensity:[9]

$$m = -2.5 \cdot \log\left(\frac{I}{I_{ref}}\right) \tag{1}$$

The intensity of the light at the POI is given by:[9]

$$I = \frac{I_0 A_r \rho \tau \cos(\gamma) \sin(\theta)}{4d^2 \tan(\frac{\alpha}{2})^2}, \tag{2}$$

where $I_0 = 1360\, W/m^2$ is the average intensity of solar energy at the Earth distance, ρ is the mirror reflectivity coefficient, γ is the incident angle of solar rays, θ is the elevation angle of the satellite; τ is the atmospheric transmissivity:[9]

$$\tau = 0.1283 + 0.7559 e^{-0.3878 \sec(\pi/2-\theta)} \tag{3}$$

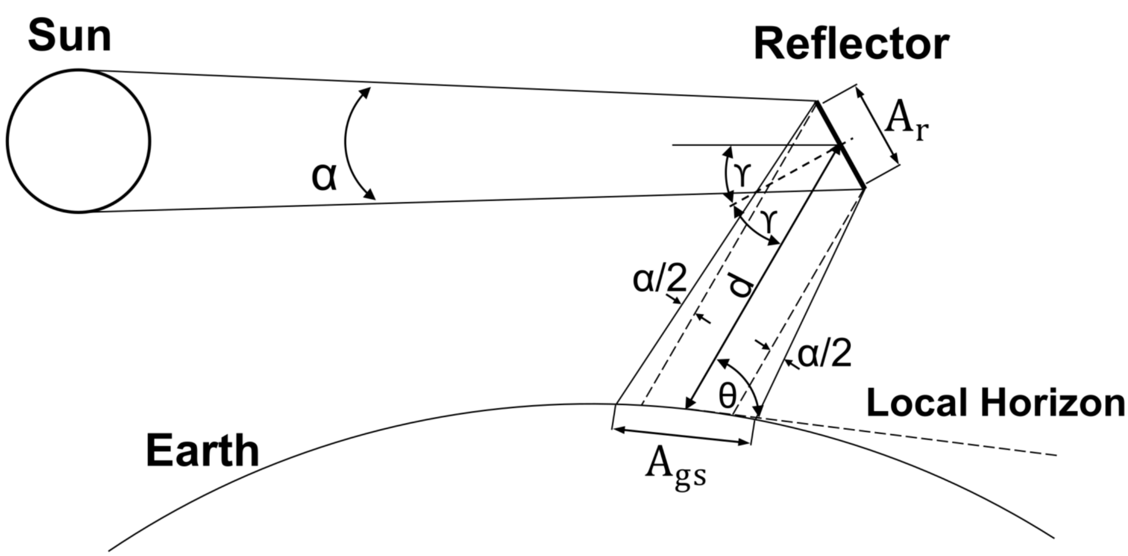

Figure 2. Reflection geometry

Thin Mylar film coated with aluminum is chosen as the reflector material because of low weight and high reflectivity coefficient ($\rho = 0.92$). The reflector, which is deployed and maintained by a rigid support structure, is assumed to be of square shape as it was also in some of the previous sail-projects.[10,11]

To estimate the size of the reflector necessary to ensure the required magnitude we used the following procedure. Given the light intensity and orbit altitude, one can express the reflector area A_r as a function of angles θ and γ. The extreme values of these two angles correspond to the maximum A_r. Hence, we obtain the characteristic linear size of the reflector of 2 m for SSO orbit at 600 km altitude. Seeing the need to pack the solar sail material and the supporting beams into the CubeSat structure along with all other satellite subsystems (including attitude control and propulsion units), we came to choose the 12U as a suitable CubeSat size for the designed formation. The total mass of each CubeSat is estimated to be 18 kg.

Let us recall now that our task is to produce a pixel image, which means that the distances between any two CubeSats in the formation must be distinguishable as independent pixels. Human eye resolution is known to be about one arc-minute, which for the specified SSO altitude yields the minimum distance between satellites in the formation of 625 m. The quality of the image is considered as acceptable when the relative positions errors do not exceed 30 m, which is well within the range of on-board GPS-module position determination. The feasibility study[1] was principally focused on maintaining a satellite formation with constant attitude in the reference frame orbiting along the terminator, which certainly requires heavy use of propulsion. In the next feasibility study[2] the differential drag control was considered to avoid significant amount of fuel consumption for formation keeping. The decentralized approach[12] for formation deployment was implemented to deploy satellite formation of required configuration. It was shown that any significant difference in the fuel consumption (if aided by the differential drag) can be achieved at rather low orbits with altitudes within 300-400 km and a reflector area far larger than is required for image demonstration purposes.

THE PROBLEM STATEMENT

The problem of the satellite formation deployment and maintenance after the separation from the launcher is considered, i.e. achieving the target relative trajectories and its maintenance during the mission is required when each satellite moves to form a specific flat image. The initial states of all satellites are determined by their simultaneous launch from the dispenser. The deployment of the satellites is carried out using a standard P-POD[13] (Poly-picosatellites orbital deployer). In this paper, we suppose a formation launched into Sun-synchronous circular orbit at 700 km altitude. It is assumed that each satellite is equipped with a cold gas thruster. Thus, the satellites can control their orbits using impulsive maneuvers.

Target Relative Trajectories

To examine relative motion between two closely orbiting satellites in the central gravity field in near circular orbits we shall employ the Hill-Clohessy-Wiltshire equations, which are extensively used in the formation flying problems.[3,4] The orbital reference frame is used, whose origin (reference point) moves along the circular orbit of radius r_0 and the mean motion n. We use the following notation to define the orbital reference frame as in Fig. 3. In the reference frame z-axis is aligned with the local vertical, y-axis coincides with the normal to the orbital plane, and x-axis completes the reference frame to the right-handed triad.

Considering the notations for the orbital reference frame the linearized equations describing

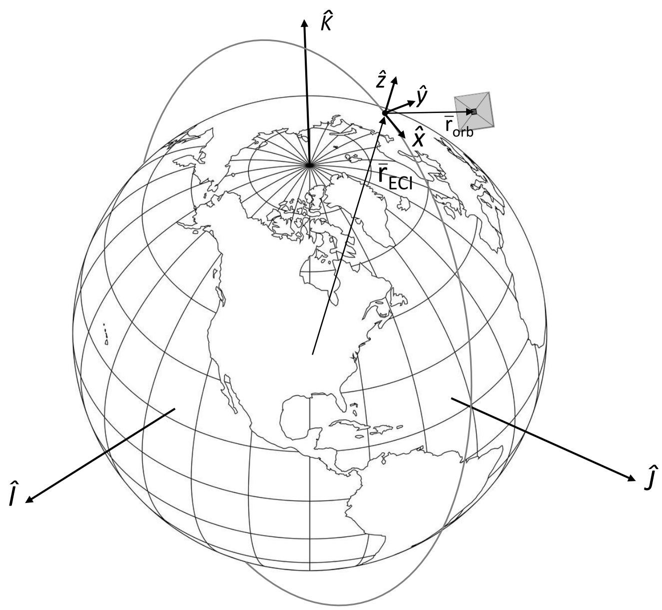

Figure 3. Orbital reference frame

spacecraft relative motion in near circular orbits can be written as follows:

$$\begin{cases} \ddot{x} + 2n\dot{z} = u_x, \\ \ddot{y} + n^2 y = u_y, \\ \ddot{z} - 2n\dot{x} - 3n^2 z = u_z, \end{cases} \tag{4}$$

where $u = \dfrac{\Delta f}{m}$. In the case of free motion, i.e. if $\Delta\ f = 0$, the exact solution to 4 has the following form:

$$\begin{cases} x(t) = -3C_1 nt + 2C_2 \cos(nt) - 2C_3 \sin(nt) + C_4; \\ y(t) = C_5 \sin(nt) + C_6 \cos(nt); \\ z(t) = 2C_1 + C_2 \sin(nt) + C_3 \cos(nt). \end{cases} \tag{5}$$

We shall set C_1 to zero to prevent constant drift of a satellite with respect to the orbital reference frame, we shall also set C_4 equal to zero since we do not require to have any constant offset along x-axis for satellites in the formation. Thus, the solution can be expressed as:

$$\begin{cases} x(t) = 2C_2 \cos(nt) - 2C_3 \sin(nt); \\ y(t) = C_5 \sin(nt) + C_6 \cos(nt); \\ z(t) = C_2 \sin(nt) + C_3 \cos(nt). \end{cases} \tag{6}$$

Let us rewrite the equations into the following form:

$$\begin{cases} x(t) = \widetilde{C}_1 \cos(nt + \alpha_0); \\ y(t) = \widetilde{C}_2 \sin(nt + \alpha_0); \\ z(t) = \dfrac{\widetilde{C}_1}{2} \sin(nt + \alpha_0). \end{cases} \tag{7}$$

In order to ensure that the relative distances remain constant for each pair of satellites we should choose the constants $\widetilde{C}_1$ and $\widetilde{C}_2$ to satisfy the relation $x^2 + y^2 + z^2 = r^2$, where r is the radius of the circumscribed circle. Let us substitute (7) into the formula:

$$\widetilde{C}_1^2 \cos^2(nt + \alpha_0) + \frac{\widetilde{C}_1^2}{4} \sin^2(nt + \alpha_0) + \widetilde{C}_2^2 \sin^2(nt + \alpha_0) = r^2. \tag{8}$$

It can be rewritten as:

$$\widetilde{C}_1^2 \cos^2(nt + \alpha_0) + \left(\frac{\widetilde{C}_1^2}{4} + \widetilde{C}_2^2 \right) \sin^2(nt + \alpha_0) = r^2. \tag{9}$$

This yields the following relation for the constants:

$$\widetilde{C}_1^2 = \frac{\widetilde{C}_1^2}{4} + \widetilde{C}_2^2 = r^2, \tag{10}$$

corresponding to $\widetilde{C}_1 = r$ and $\widetilde{C}_2 = \dfrac{\sqrt{3}}{2} r$. Thus, we can set initial conditions for a satellite by adjusting radius r of relative trajectory and the phase α.

Impulsive Control

We consider impulsive control to deploy formation of satellites for image demonstration in the sky. Relative trajectories of the chaser satellites are expressed in terms of equinoctial orbital elements differences with regards to the orbit of the target satellite:

$$\delta q_1 = -\frac{\widetilde{C}_1^2 \sin\alpha_0}{2a}, \tag{11}$$

$$\delta q_2 = -\frac{\widetilde{C}_1^2 \cos\alpha_0}{2a}, \tag{12}$$

$$\delta i = \frac{\widetilde{C}_2^2 \cos\alpha_0}{a}, \tag{13}$$

$$\delta \Omega = -\frac{\widetilde{C}_2^2 \sin\alpha_0}{a \sin i}, \tag{14}$$

$$\delta\lambda = -\delta\Omega \cos i. \tag{15}$$

where $q_1 = e \cos\omega$, $q_2 = e \sin\omega$, e - eccentricity, w - argument of periapsis, a is the orbit semi-major axis, Ω is the right ascension of the ascending node, λ is the argument of latitude, i is the orbit inclination.

We draw on the control scheme proposed by Vaddi[5] to implement satellite formation control. It implies analytical two-impulse solution for adjusting the desired orbital elements differences. The required impulses are calculated according to the formulae below.

Radial components of ΔV for both impulses are similar with only a sign difference, the first one being negative and the second – positive:

$$\Delta V_{r_{1,2}} = \mp v_o \frac{\sqrt{\delta q_1^2 + \delta q_2^2}}{2}, \quad v_o = n r_0. \tag{16}$$

Out-of-plane component of ΔV can be distributed between the first and the second impulses in any proportion, so we incorporated it all into the first impulse:

$$\Delta V_{o_1} = v_o \sqrt{\delta i^2 + \delta\Omega^2 \sin^2 i}, \tag{17}$$

$$\Delta V_{o_2} = 0. \tag{18}$$

Tangential components are zero.

Applying the impulses we can attain the closed relative trajectories of the required radius. However, whenever we need to construct a formation with the required geometry it entails setting both radius and phase of each satellite at the relative circular orbit. This can be accomplished by applying impulses at particular time moments. The first and the second impulses should be applied respectively at

$$t_1 = t_0 + \frac{2\pi - \alpha_0}{n}, \tag{19}$$

$$t_2 = t_0 + \frac{2\pi - \alpha_0 + \pi}{n}, \tag{20}$$

where t_0 is the formation establishment start time, n is the mean motion, α_0 is the angle that parametrizes the satellite's slot in the image plane.

MISSION DESIGN

Let us design a formation flying mission to demonstrate the acronym of the International Academy of Astronautics above Rome at the Gala dinner of the Conference on University Satellite Missions and CubeSat Workshop which was held on 30^{th} of January, 2020. To make the demonstration we gather the formation of thirty one CubeSats equipped with sunlight reflectors as shown in Fig. 4. Each satellite is to be regarded as a single pixel of a complex image.

The orbit of formation's central point is assumed to be circular Sun-synchronous, so that the formation would always be lit by the Sun during its lifetime. For the altitude of 700 km the inclination required for SSO is 98.2°. The RAAN of 43.5°is such that the line of nodes is perpendicular to the Earth-Sun line on the date of demonstration, 30 Jan 2020. The time of demonstration and true anomaly are chosen so that the satellite passes over Rome with 90°elevation. This should occur at 16:54:00 UTC when the satellite's true anomaly is 42.2°.

Following the procedure presented in the preliminary study section we obtain the required reflector area of 11 m^2. We estimate the size of the formation to be 6000 m, as in this case the formation

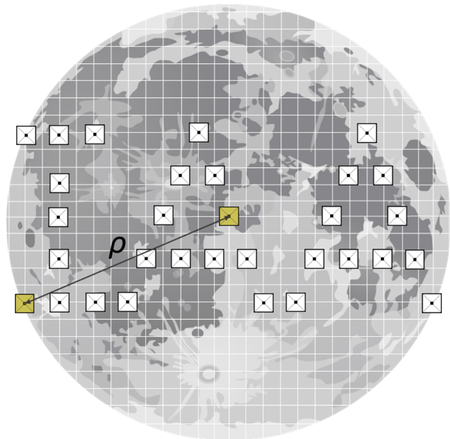

Figure 4. Designing the formation

has the angular size as that of the Moon (30 arcmin) when flying above an observer, at 90°elevation. The distance between the adjacent satellites is computed to be 460 meters. At 90°elevation the angular distances between the adjacent satellites should constitute about 2.3 arcmin, which is greater than 1 arcmin resolution limit for a human eye.[8] Before and after flying right above the observer, the formation will have smaller angular size. The angular distance between satellites will decrease to non-resolvable 1 arcmin at the elevation of 20°. This makes the demonstration readable for the total of 6 minutes. The parameters of the demonstration such as reflector magnitude at POI during the demonstration, satellite elevation and incident angle of the sunlight to the reflector are presented in Fig. 5.

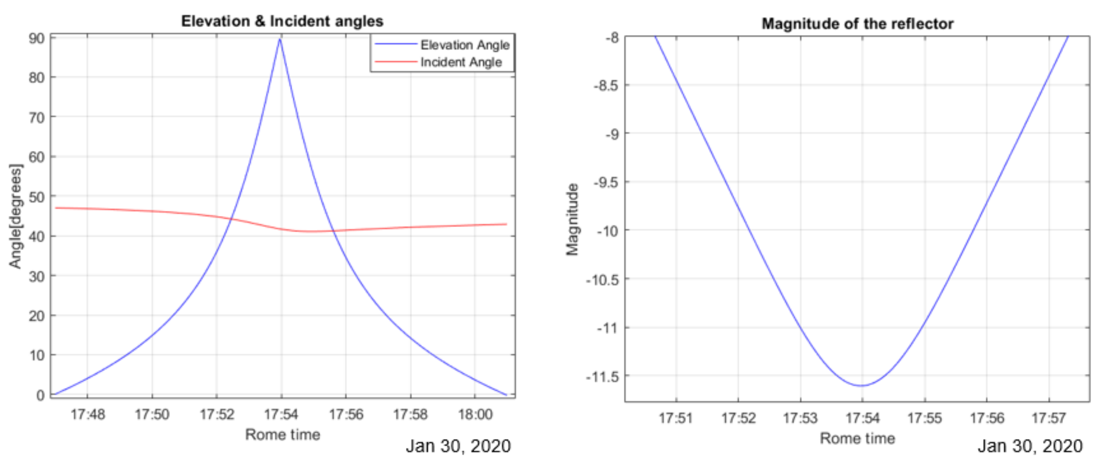

Figure 5. Parameters of demonstration

FORMATION DEPLOYMENT AND MAINTENANCE

In the numerical experiment we implement impulsive control to deploy and keep the satellite formation comprising of 31 CubeSats. The orbital dynamics model used in our simulations takes into account Earth oblateness (J2 effect). All satellites are initially positioned at the same orbit according to the mission design requirements described above, representing their state soon after their separation from a launch vehicle using P-POD. We consider centralized control approach to deploy and maintain the formation. Thus, all satellites are aware of their target relative trajectories with respect to the geometrical center of the formation where one of the satellites is positioned. This one satellite acts as a target satellite and all other are chaser satellites respectively. To establish the formation after its deployment, all chaser satellites perform two consecutive maneuvers according to (16)-(20). Simulations show that the deployment takes no longer than 1.5 orbit periods. The geometry of the formation after its deployment is presented in Fig. 6.

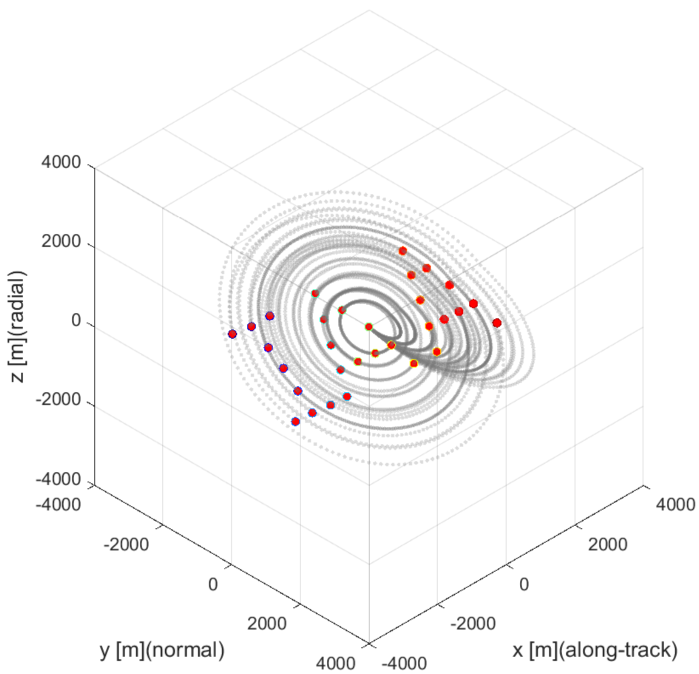

Figure 6. Satellites forming ”IAA” text when the formation is fully deployed

Let us now consider the problem of keeping the formation. It should be noted that closed relative orbits in the orbital reference frame were obtained from the linearized equations. Thus, the greater the distance from a chaser satellite to the target one is, the greater error of positioning it would experience over time. To estimate the influence of possible errors in our formation we simulate the orbital motion for the most distant chaser satellite from the formation center, and then compare the simulation results to the analytical solution given by Eq. (7). Figure 7 represents orbital dynamics of the chaser. The left graph (see Fig. 7a) depicts the chaser's relative orbit in the orbital reference frame simulated for 1 day. It can be seen that the magnitude of chaser's position vector in orbital reference frame (see Fig. 7b) is increasing significantly within one day of simulation time. Satellites' relative orbit errors must be kept within a specific range to satisfy the demonstration requirements.

Thus, for formation keeping purposes we have implemented a control algorithm, which is to

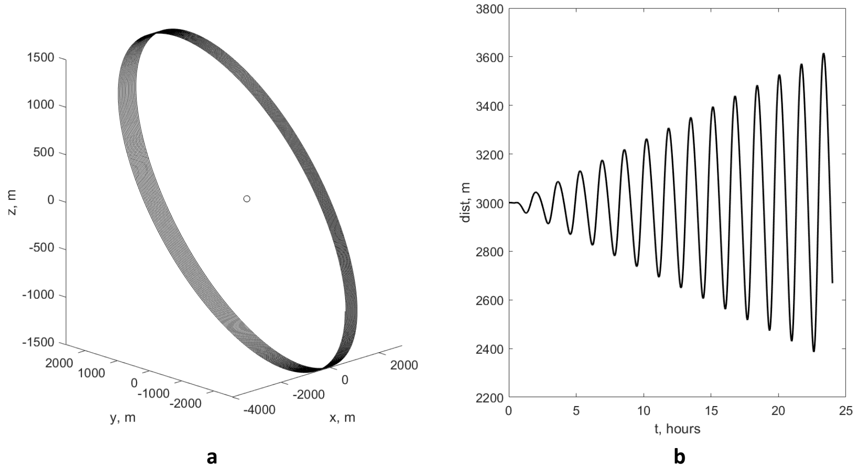

Figure 7. Chaser satellite relative orbit evolution

perform orbit corrections whenever the relative error exceeds designated limits. The correction maneuvers are defined according to the bi-impulsive transfer delineated above. Requirements for the corresponding correction burns are presented in Table 1 and calculated for a mission lifetime of one month and position 3000 meters from the center. Propellant mass is calculated for 18 kg satellite equipped with a cold gas thruster[14] with specific impulse $I_{sp} = 65$ s. The dependency on the distance is nonlinear and significantly greater amounts of propellant would be required for larger formations.

Table 1. Requirements for corrections with different error thresholds, for 30 days

Acceptable error, %	Number of corrections	Total ΔV, m/s	Propellant mass, g
5	106	1.4	40
10	61	1	29
20	31	0.9	26

CONCLUSION

- Two-impulse control was implemented to deploy and maintain small satellite formation for graphic image demonstration in the sky;
- The presented control algorithm is based on centralized approach where chaser satellites keep their relative orbits with respect to the target satellite;
- Proposed control method (as compared to the previously studied ones) has the following advantages: short time of formation deployment and reconfiguration (up to 2 orbit periods), low propellant consumption for formation keeping comparing to preliminary study results;[1]
- One of the venues to be explored is including into the model higher-order harmonics of the gravity field and other essential disturbances to quantify their effect on the results obtained in this study for J2-only perturbation;

- The proposed control strategy may be extended to be applied to reconfiguration formations geometry with the purpose of demonstrating a set of various graphic images in one mission.

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