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JOHN F. DASHIELL, *Editor*

On Problem-Solving

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INTRODUCTION

It is to be regretted that few psychologists take an active part in the investigation of thinking. In modest forms thinking pervades, and to a degree rules, all activities of a human being. Why, then, are we so little concerned with the study of thought processes? It seems that there are two reasons for this attitude. In the first place, many believe that thinking is not a subject matter with any specific characteristics of its own. According to this view further advance in the investigation of associations, of habits, sets, and the like will gradually reduce thought to a mere complication of such simpler facts. In the second place, we like our quantitative experimental procedures so much that a subject matter to which these procedures cannot easily be applied has little chance to arouse our interest. There are serious doubts whether in the field of thought much can be achieved by measuring.

The first of these reasons can obviously not be accepted. When a theoretical hope opens our eyes to new possibilities of research this positive influence of theory is of course entirely desirable. But no theoretical expectation, however strongly it may be ingrained in a given scientific attitude, can be allowed to exclude a subject matter from impartial inspection. How, indeed, will theoretical expectations ever be tested if we provide no independent knowledge of the facts to which these expectations refer and with which they must be confronted in a test?

I do not think that the second reason has greater force than the first. To be sure, we do like to deal with topics for which a ready-made and elegant technique of investigation is available. To penetrate into a field in which standardized procedures are of little avail is not a task by which we feel attracted. But, surely, our personal weaknesses ought not to masquerade as good reasons. If given methods do not fit an unfamiliar subject matter we are under obligation to devise others which give us at least a first acquaintance with the new territory. Precisely this is being done in social psychology, another field in which experimental procedures have not proved particularly successful.

It will be objected that any new endeavor which makes little use of quantitative techniques must necessarily lack the precision of science in a strict sense. Let us not be impressed by this argument. It is familiar to all students of human nature. When an action is not to our taste we tend at once to discover that results will not live up to a certain ideal, and that therefore we ought not to act at all. But in any branch of knowledge, just as in politics, in economic and in social issues, our conduct will almost invariably be defined as right only by its *direction;* we have little commerce with perfection. To point to the imperfections of first steps is under these circumstances almost a symptom of inertia. How do the critics plan ever to conquer difficult new ground if the absence of paved roads seems to them sufficient reason for keeping out entirely?

I wish to express my gratitude to Professor E. G. Boring who was kind enough to make the library facilities of the Department of Psychology at Harvard University available to me. Without this help I could not have done the present translation. I am also indebted to Professor W. Köhler who compared the translation with the original, and made suggestions for the improvement of the English version.

LYNNE S. LEES

In view of so much hesitation any courageous attempt to throw light upon the nature of thinking must be welcomed. It is only such attempts which can gradually make us familiar with the specific problems and with the technical possibilities of this field. In 1935 Karl Duncker published his book *Zur Psychologie des produktiven Denkens* which, it seems to me, has penetrated farther into its subject than most other enterprises with a similar program. But few psychologists and hardly any philosophers in this country are acquainted with this important investigation. Moreover, for years the original has been virtually unobtainable. It is therefore most fortunate that after Duncker's death one of his students decided to prepare an English translation. Those who know the German text will be able to realize the difficulty of Mrs. Lees' task; the skill with which she has overcome this difficulty seems to me admirable.

This is not an easy book. The author did not write it when the hard labors of research lay far behind him so that he could serenely look upon his findings from a distance. Rather, every page seems to show him in the midst of his untiring struggle for clearness. Thus to our interest in his analysis as such is added a more particular attraction which few books have.

Duncker's work is not easy for a further reason. He hated any compromise with vague terms which give an appearance of knowledge while they actually hide problems. He simply would not let go until he knew the very anatomy of a concept. I also have the impression that once he had started on his journey he was not satisfied until he had visited all provinces of thought. Thus in some chapters we find the author far within philosophy because the ramifications of insight would not become clear without that transgression. In other chapters he studies the obstacles to productive thought rather than its course when it succeeds. There are sections from which all teachers of mathematics could greatly profit, and many pages in which the investigation of thought elucidates problems of perception.

Nobody who reads this book will be able to deny that fruitful research is possible in the realm of thought. Moreover, in several points Duncker confirms the work of others. Clearly, observation in this field is largely independent of theoretical preconceptions. To one conclusion or another the reader with a training in psychology may be inclined to object. If that happens he will also like to prove his point—and will thus find himself working on thought processes. It *is* time that we turn to essentials.

In Duncker himself this was the greatest intellectual virtue: He was forever impatient of little things and happy only when he found a way that led to fundamentals. The best we can do in remembering our friend is to give his work as an example which others may follow.

WOLFGANG KÖHLER

FOREWORD

To study productive thinking where it is most conspicuous in great achievements is certainly a temptation, and without a doubt, important information about the genesis of productive thought could be found in biographical material. But although a thunderstorm is the most striking example of electrical discharge, its laws are better investigated in little sparks within the laboratory. To study in simple, convenient forms what is complicated and difficult of access is the method of experimental science; to lose in this simplification just the essential aspects, is its notorious danger. Experimental psychology, more than all other sciences, is continually faced with this danger. I hope to have succeeded in simplifying my subject without altering its essential nature.

The restriction of the investigation to practical and mathematical problems in thinking is entirely intentional. This restriction was adopted because such material is more accessible, more suitable for experimentation. I believe, however—and the reader may agree with me at the end of the book—that essential features of problem-solving are independent of the specific thought-material. But there is still another restriction for which I must account to the reader. In my analysis, all facts are intentionally omitted which have no immediate significance for the *problem of finding,* of εὑρίσκειν—be they ever so essential parts of the complete psychological inventory of a solution-process. According to the modern psychology of needs and affective states, every thought-problem which the subject actually makes his own has at its base, as its source of energy, a system of tensions of the type of needs (Lewin). In the course of a solution-process, this tension-system undergoes all kinds of alterations until, where possible, it finds complete release in a final solution. The fate of this tension-system, as shown in rising and subsiding interest, in experiences of success and of failure, in digressions, substitute behavior, resignation and anger, or in pride of performance and elevated level of aspiration—all these and similar aspects of a solution-process have with a certain ruthless consistency been left untouched in the following analysis. For, they are irrelevant to the specific question of finding: In what way can a meaningful solution be found at all? And in addition, there are already many excellent papers on just these things, while the problem of finding has received scant attention in modern psychology.

Although everything in this investigation is, so to speak, under the command of this problem of finding, much may also be profitably taken from the viewpoint of quite different problems. Such is above all the discussion of *insight* and of *evidence* contained in Chapters IV and V, which may claim a certain theoretical independence. It represents an attempt partially to solve, from a new approach, the old epistemological problem of the recognition of "intrinsic necessity". This discussion continues a philosophical development which is to a degree characterized by the names of Hume, Kant, Husserl and Wertheimer.

Here I should like to express my gratitude to the Abraham Lincoln Foundation (U.S.A.), which in the year 1929, through its financial assistance, made possible comprehensive studies on the problem of mathematical talent, some of which are reported in Chapter III and chiefly in Chapter VIII.

This book is gratefully dedicated to my teachers Wolgang Köhler and Max Wertheimer.

KARL DUNCKER

Berlin, February 1935

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THE STRUCTURE AND DYNAMICS OF PROBLEM-SOLVING PROCESSES

Chapter I

The Solution of Practical Problems (I)

1. INTRODUCTION AND FORMULATION OF THE PROBLEM

A PROBLEM arises when a living creature has a goal but does not know how this goal is to be reached. Whenever one cannot go from the given situation to the desired situation simply by action, then there has to be recourse to thinking. (By action we here understand the performance of obvious operations.) Such thinking has the task of devising some action which may mediate between the existing and the desired situations. Thus the 'solution' of a practical problem must fulfill two demands: in the first place, its realization[1] must bring about the goal situation, and in the second place one must be able to arrive at it from the given situation simply through action.

The practical problem whose solution was experimentally studied in greatest detail runs as follows: Given a human being with an inoperable stomach tumor, and rays which destroy organic tissue at sufficient intensity, by what procedure can one free him of the tumor by these rays and at the same time avoid destroying the healthy tissue which surrounds it?

Such practical problems, in which one asks, "How shall I attain . . .?", are related to certain theoretical problems, in which the question is, "How, by what means, shall I comprehend . . .?" In the former case, a problem situation arises through the fact that a goal has no direct connection with the given reality; in the latter case—in theoretical problems—it arises through the fact that a proposition has no direct connection with what is given in the premises. As example in the latter field, let us take again the problem with which I experimented in greatest detail: Why is it that all six-place numbers of the type *abcabc,* for example 276276, are divisible by thirteen?

It is common to both types of problems that one seeks the ground for an anticipated consequence; in practical problems, the actual ground is sought; in theoretical problems, the logical ground.[2]

In the present investigation the question is: *How does the solution arise from the problem situation? In what ways is the solution of a problem attained?*

2. EXPERIMENTAL PROCEDURE

The experiments proceeded as follows:

[1] [Translator's note: "Realization" is used in the sense of "making real", of "actualization". The terms "embodiment" and "to embody" are used in a closely related sense, which will be clear in context. In the following, all notes of the translator will be given in parentheses. Such notes will add the German terms of the original where entirely satisfactory English terms do not seem to exist.]

[2] Other types of theoretical problems, such as: "What is the essential nature of, or the law of . . . ?" or "How are . . . related to each other?", are not investigated here.

The subjects (Ss), who were mostly students of universities or of colleges, were given various thinking problems, with the request that they think aloud. This instruction, *"Think aloud"*, is not identical with the instruction to introspect which has been common in experiments on thought-processes. While the introspecter makes himself as thinking the object of his attention, the subject who is thinking aloud remains immediately directed to the problem, so to speak allowing his activity to become verbal. When someone, while thinking, says to himself , "One ought to see if this isn't . . .", or, "It would be nice if one could show that . . .", one would hardly call this introspection; yet in such remarks something is revealed which we shall later deal with under the name of 'development of the problem'. The subject (S) was emphatically warned not to leave unspoken even the most fleeting or foolish idea. He was told that where he did not feel completely informed, he might freely question the experimenter, but that no previous specialized knowledge was necessary to solve the problems.

3. A PROTOCOL OF THE RADIATION PROBLEM

Let us begin with the radiation problem (p. 1). Usually the schematic sketch shown in Fig. 1 was given with the problem.

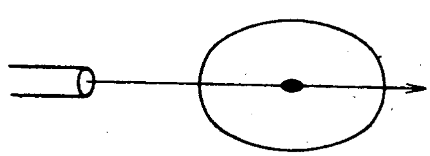


FIG. 1

Thus, it was added, somebody had visualized the situation to begin with (cross-section through the body with the tumor in the middle and the radiation apparatus on the left); but obviously this would not do.

From my records I choose that of a solution-process which was particularly rich in typical hunches and therefore also especially long and involved. The average process vacillated less and could be left to run its own course with considerably less guidance.[3]

Protocol

1. Send rays through the esophagus.
2. Desensitize the healthy tissues by means of a chemical injection.
3. Expose the tumor by operating.
4. One ought to decrease the intensity of the rays on their way; for example—would this work?—turn the rays on at full strength only after the tumor has been reached. (Experimenter: False analogy; no injection is in question.)
5. One should swallow something inorganic (which would not allow passage of the rays) to protect the healthy stomach-walls. (E: It is not merely the stomach-walls which are to be protected.)
6. Either the rays must enter the body or the tumor must come out. Perhaps one could alter the location of the tumor—but how? Through pressure? No.
7. Introduce a cannula.—(E: What, in general, does one do when, with any agent, one wishes to produce in a specific place an effect which he wishes to avoid on the way to that place?)
8. (Reply:) One neutralises the effect on the way. But that is what I have been attempting all the time.
9. Move the tumor toward the exterior. (Compare 6.) (The E repeats the problem and emphasizes, ". . . which destroy *at sufficient intensity*".)
10. The intensity ought to be variable. (Compare 4.)
11. Adaptation of the healthy tissues by previous weak application of the rays. (E: How can it be brought about that the rays destroy only the region of the tumor?)

[3] Compare the pertinent protocols in my earlier and theoretically much less developed paper, "A qualitative study of productive thinking," Ped. Sem., 1926, v. 33.

12. (Reply:) I see no more than two possibilities: either to protect the body or to make the rays harmless. (E: How could one decrease the intensity of the rays en route? [Compare 4.])

13. (Reply:) Somehow divert . . . diffuse rays . . . disperse . . . stop! Send a broad and weak bundle of rays through a lens in such a way that the tumor lies at the local point and thus receives intensive radiation.[4] (Total duration about half an hour.)

4. IMPRACTICABLE 'SOLUTIONS'

In the protocol given above, we can discern immediately that the whole process, from the original setting of the problem to the final solution, appears as a series of more or less concrete proposals. Of course, only the last one, or at least its principle, is practicable. All those preceding are in some respect inadequate to the problem, and therefore the process of solution cannot stop there. But however primitive they may be, this one thing is certain, that they cannot be discussed in terms of meaningless, blind, trial-and-error reactions. Let us take for an example the first proposal: "Send rays through the esophagus". Its clear meaning is that the rays should be guided into the stomach by some passage free from tissue. The basis of this proposal is, however, obviously an incorrect representation of the situation inasmuch as the rays are regarded as a sort of fluid, or the esophagus as offering a perfectly straight approach to the stomach, etc. Nevertheless, within the limits of this simplified concept of the situation, the proposal would actually fulfill the demands of the problem. It is therefore genuinely the solution of a problem, although not of the one which was actually presented. With the other proposals, the situation is about the same. The second presupposes that a means—for example, a chemical means—exists for making organic tissue insensitive to the rays. If such a means existed, then everything would be in order, and the solution-process would have already come to an end. The fourth proposal—that the rays be turned on at full strength only when the tumor has been reached—shows again very clearly its derivation from a false analogy, perhaps that of a syringe which is set in operation only when it has been introduced into the object. The sixth suggestion, finally, treats the body too much as analogous to a rubber ball, which can be deformed without injury. In short, it is evident that such proposals are anything but completely meaningless associations. Merely in the factual situation, they are wrecked on certain components of the situation not yet known or not yet considered by the subject.

Occasionally it is not so much the situation as the demand, whose distortion or simplification makes the proposal practically useless. In the case of the third suggestion, for example ("expose the tumor by operating"), the real reason why radiation was introduced seems to have escaped the subject. An operation is exactly what should be avoided. Similarly in the fifth proposal, the fact is forgotten that not only the healthy stomach-walls must be protected but also all parts of the healthy body which have to be penetrated by the rays.

A remark on principle may here be in order. The psychologist who is investigating, not a store of knowledge, but the genesis of a solution, is not interested

[4] This solution is closely related to the 'best' solution: *crossing of several weak bundles of rays at the tumor*, so that the intensity necessary for destruction is attained only here. Incidentally, it is quite true that the rays in question are not deflected by ordinary lenses; but this fact is of no consequence from the viewpoint of the psychology of thinking. See 4 below.

primarily in whether a proposal is actually practicable, but only in whether it is formally practicable, that is, practicable in the framework of the subject's given premises. If in planning a project an engineer relies on incorrect formulae or on non-existent material, his project can nevertheless follow from the false premises as intelligently as another from correct premises. One can be 'psychological equivalent' to the other. In short, we are interested in knowing how a solution develops out of the system of its subjective premises, and how it is fitted to this system.

5. CLASSIFICATION OF PROPOSALS

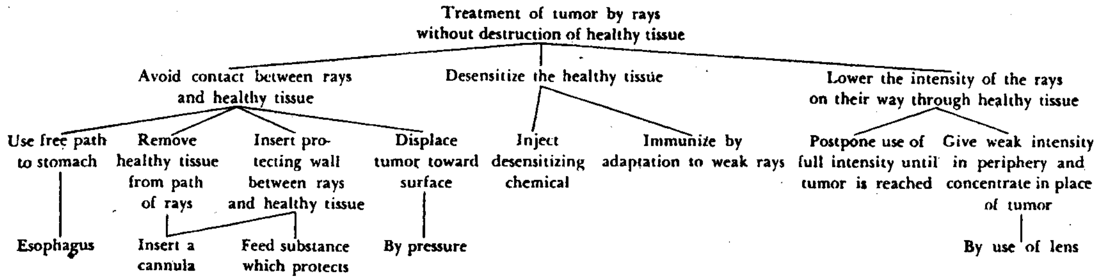
If one compares the various tentative solutions in the protocol with one another, they fall naturally into certain groups. Proposals 1, 3, 5, 6, 7 and 9 have clearly in common the attempt to *avoid contact between the rays and the healthy tissue.* This goal is attained in quite different ways: in 1 by re-directing the rays over a path naturally free from tissue; in 3 by the removal of the healthy tissue from the original path of the rays by operation; in 5 by interposing a protective wall (which may already have been tacitly implied in 1 and 3); in 6 by translocating the tumor towards the exterior; and in 7, finally, by a combination of 3 and 5. In proposals 2 and 11, the problem is quite differently attacked: the accompanying destruction of healthy tissue is here to be avoided by the *desensitizing or immunizing of this tissue.* A third method is used in 4, perhaps in 8, in 10 and 13: *the reduction of radiation intensity on the way.* As one can see, the process of solution shifts noticeably back and forth between these three methods of approach.

In the interests of clarity, the relationships described are presented graphically on the next page.

6. FUNCTIONAL VALUE AND UNDERSTANDING

In this classification, the tentative solutions are grouped according to the manner in which they try to solve the problem, i.e., according to their "by-means-of-which", their "functional value". Consider the proposal to send rays through the esophagus. The S says nothing at all about avoiding contact, or about a free passage. Nevertheless, the solution-character of the esophagus in this context is due to no other characteristic than that of being a tissue-free path to the stomach. It functions as the embodiment solely of this property (not of the property of being a muscular pipe, or of lying behind the windpipe, or the like). In short, in the context of this problem, the "by-means-of-which", the "functional value" of the esophagus is: a free path to the stomach. The proposals: "direct the rays by a natural approach", "expose by operation", "translocate the tumor toward the exterior", "protective wall", and "cannula" all embody the functional value: no contact between rays and healthy tissue. The functional value of the solution, "concentration of diffuse rays in the tumor", is the characteristic: "less intensity on the way, great intensity in the tumor". The functional value of the lens is the quality: "medium to concentrate rays", and so forth.

The functional value of a solution is indispensable for the understanding of its being a solution. It is exactly what is called the sense, the principle or the point of the solution. The subordinated, more specialized characteristics and properties of a solution embody this principle, apply it to the particular circum-



stances of the situation. For example, the esophagus is in this way an application of the principle: "free passage to the stomach", to the particular circumstances of the human body. To understand the solution as a solution is just the same as to comprehend the solution as embodying its functional value. When someone is asked, "Why is such-and-such a solution?", he necessarily has recourse to the functional value. In all my experiments, aside from two or three unmistakable exceptions, when the E asked about a proposal: "In what way is this a solution of the problem?", the S responded promptly with a statement of its functional value. (In spontaneous statements of the Ss, the functional value was frequently left unmentioned as being too obvious.)

Incidentally, the realization of its functional value mediates understanding of a solution even where there is nothing but an 'unintelligible' (though sufficiently general) relation between the functional value and the demand which it fulfills. Blowing on a weakly glimmering fire, for example, undoubtedly solves the problem of rekindling the fire because in this way fresh oxygen is supplied. In other words, the increase of the oxygen supply is the immediate functional value of blowing on the fire. But why combination with oxygen produces warmth and flame is ultimately not intelligible. Even if the whole of chemistry should be successfully and without a gap derived from the principles of atomic physics, these principles are not in themselves altogether intelligible, i.e., ultimately they must be "accepted as mere facts". (See details in Chap. IV, especially page 47f.) Thus, intelligibility frequently means no more than participation in, or derivability from, sufficiently elementary and universal causal relationships. But even if these general laws are not in themselves intelligible, reducibility to such general laws actually mediates a certain type of understanding.

To the same degree to which a solution is understood, it can be transposed, which means that under altered conditions it may be changed correspondingly in such a way as to preserve its functional value. For, one can transpose a solution only when one has grasped its functional value, its general principle, i.e., the invariants from which, by introduction of changed conditions, the corresponding variations of the solution follow each time.

An example: When, seen from the standpoint of a spectator, someone makes a detour around some obstacle, and yet acts from his own point of view in terms of nothing but, say, "now three yards to the left, then two yards straight ahead, then to the right . . ."—these properties of the solution would certainly satisfy the concrete circumstances of the special situation here and now. But so long as the person in question has not grasped the functional value, the general structure: "detour around an obstacle", he must necessarily fail when meeting a new obstacle which is differently located and of different shape. For to different obstacles correspond different final forms of the solution; but the structure, "detour around an obstacle", remains always the same. Whoever has grasped this structure is able to transpose a detour properly.

7. MEANINGLESS ERRORS AS A SYMPTOM OF DEFICIENT UNDERSTANDING

A solution conceived without functional understanding often betrays itself through nonsensical errors. A good example is supplied by experiments with another thinking problem.

The problem was worded as follows: "You know what a pendulum is, and that a pendulum plays an important rôle in a clock. Now, in order for a clock to go accurately, the swings of the pendulum must be strictly regular. The duration of a pendulum's swing depends, among other things, on its length, and this of course in turn on the temperature. Warming produces expansion and cooling produces contraction, although to a different degree in different materials. Thus every temperature-change would change the length of the pendulum. But the clock should go with absolute regularity. How can this be brought about? —By the way, the length of a pendulum is defined solely by the shortest distance between the point of suspension and the center of gravity. We are concerned only with this length; for the rest, the pendulum may have any appearance at all."

The customary solution of this pendulum problem in actual practice is reproduced in Figure 2. At first this solution

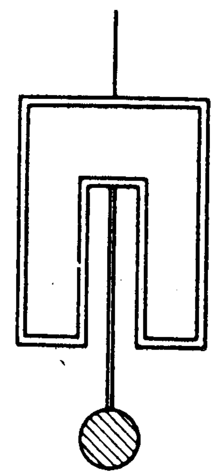


FIG. 2

will be entirely 'unintelligible' to many a reader.

Let him watch now what takes place when the solution suddenly becomes clear to him. Its functional value is that every expansion in one direction is compensated by an equally great expansion in the opposite direction.

The bars a and a' (see Fig. 3) can expand only downwards; b and b', on the other hand, only upwards, since they are fastened below. The bars b and b' are meant to raise the strip of metal to which c is fastened by exactly as much as a and c together expand downwards. To this end, b and b' must of course be constructed of a material with a greater coefficient of expansion than a and a' and c.

Only when Figure 3 is grasped as the

embodiment of this functional value, is it understood as the solution.

Among the many Ss to whom I gave the pendulum problem, there were two who were already vaguely familiar with a pendulum-model, and simply reconstructed it from memory. One was fortunate and did it correctly, while the

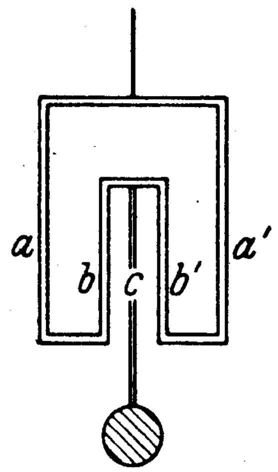


Fig. 3

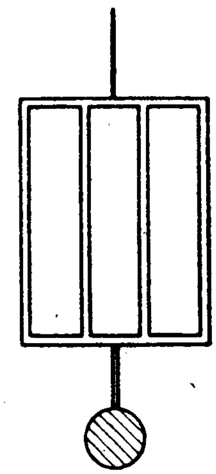


Fig. 4

other drew "just four or five bars like this, from which the weight hung below". (Fig. 4) It is evident that this is a completely meaningless construction, despite all external resemblance to Fig. 3, and devoid of any functional understanding (as the S clearly realized and expressed himself). Compare with this the solutions of the problem contained in Figure 5, a-g, which, in spite of all external differences, embody the identical functional value and at the same time represent completely new constructions.

In all of them there is compensation in the sense of Figure 3; thus we deal with appropriate transpositions of Figure 3. It is worth mentioning that one S drew the model of Figure 5a, believing that it was the compensation-pendulum dimly familiar from experience. Here it is clear that the reconstruction can have taken place only via the common functional value. Nothing in their form is common to the two pendulums.

"Good" and "stupid" errors in Köhler's[5] sense can be clearly distinguished as follows: In the case of good, intelligent errors, at least the general functional value of the situation is correctly outlined, only the specific manner of its realization is not adequate. For example, an ape stands a box on its corner under the goal object, which hangs high above, because in this way the box comes closer—to be sure, at the price of its stability. In the case of stupid errors, on the other hand, the outward form of an earlier, or an imitated solution is blindly reproduced without functional understanding. For example, an ape jumps into the air from a box—but the goal object is hanging at quite a different spot.

8. THE PROCESS OF SOLUTION AS DEVELOPMENT OF THE PROBLEM

It may already have become clear that the relationship between superordinate and subordinate properties of a solution has *genetic* significance. *The final form*

[5] Köhler, (20), p. 194, 217.

of an individual solution is, in general, not reached by a single step from the original setting of the problem; on the contrary, the principle, the functional value of the solution, typically arises first, and the final form of the solution in question develops only as this principle becomes successively more and more concrete. In other words, the general or "essential" properties of a solution reached, although certainly wrong, arises only as a solution of this new, re-formulated problem. From this same reformulation of the problem there arises, at the end of the whole process, the practicable solution, "concentration of diffuse rays in the tumor". With the other proposals in the protocol, the case is similar: the solution-properties found at first, the functional values, *always serve as pro-*

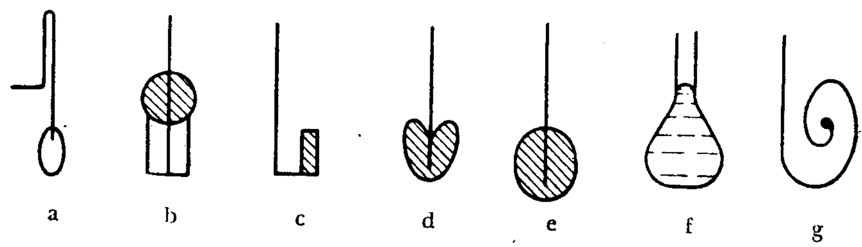


FIG. 5, a-g

genetically precede the specific properties; the latter are developed out of the former. The classification given on page 5 presents, thus, a sort of *"family tree"* of the solution of the radiation problem.

The finding of a general property of a solution means each time a *reformulation of the original problem.* Consider for example the fourth proposal in the protocol above. Here it is clearly evident that at first there exists only the very general functional value of the solution: "one must decrease the intensity of the radiation on the way". But the decisive re-formulation of the original problem is thereby accomplished. No longer, as at the beginning, does the S seek simply a "means to apply rays to the tumor without also destroying healthy tissue", but already—over and above this—a means to decrease the intensity of the radiation on the way. The formulation of the problem has thus been made sharper, more specific—and the proposal not to turn the rays on at full strength until the tumor has been *ductive reformulations of the original problem.*[6]

We can accordingly describe a process

[6] O. Selz, in his experiments on problem-solving, has already found similar transformations of problems. (Selz, (36), p. 87; (37), p. 41.) Selz defines transformation of the task as the "exchange of the original goal for a more specific one," or, more explicitly, "the substitution for the task of another task, through whose solution the original problem is also to be solved." I give two examples from his experimental repertoire. The first runs as follows: "Give two species of electoral franchise." The first thing which occurs to one S is "that he is familiar with several classifications of electoral franchise; and now he sets himself the specific task of reproducing one of these classifications." Another S realizes "that there exists a contrast more extreme than that in the electoral franchises of North and South Germany; and now he tries to define more closely that extreme contrast; thus the more familiar definition by location again precedes the less familiar definition by content." (36), p. 65; for the protocol itself, see p. 40.)

A second example: In connection with the task of finding "a concept co-ordinated with 'railway platform,'" a transformation of the task attained by many Ss is to produce "another part of the concrete spatial whole which includes 'railway platform'" (which leads, for example, to the solution, 'railroad track'); cf. (37), p. 142.

Selz calls transformations which are severally ordered to whole types of problems, "methods of solution".

of solution either as development of the solution or as development of the problem. Every solution-principle found in the process, which is itself not yet ripe for concrete realization, and which therefor fulfills only the first of the two demands given on page 1, functions from then on as reformulation, as sharpening of the original setting of the problem. *It is therefore meaningful to say that what is really done in any solution of problems consists in formulating the problem more productively.*

To sum up: *The final form of a solution is typically attained by way of mediating phases of the process, of which each one, in retrospect, possesses the character of a solution, and, in prospect, that of a problem.*

At the same time it is evident that, generally speaking, a process of solution penetrates only by degrees into the more specific circumstances and possibilities of the given situation. In the phase, "avoiding contact between rays and healthy tissue", for example, there is still very little reference to the concrete individuality of the situation. The rays function for the time being as "active agent", the tumor as "the place to be influenced", and the healthy tissue as "surrounding region which must be protected". In the next phase, "redirection of the rays over a tissue-free path to the stomach", at least the possibility of such a displacement of the rays is already made use of. In the search for a free pathway, the situation is then subjected to an even more precise inspection; as a consequence, such a specific component of the situation as the esophagus enters the solution-process and is used in a sensible manner.

To widen our horizon, let us here demonstrate with a mathematical example how a solution-process typically arrives at the final solution by way of mediation, problem- or solution-phases. The original problem is to prove that there is an infinite number of prime numbers (to find "something from which follows that there exists . . ."). A step which is quite decisive, although subjectively hardly noted, consists in the solution-phase: "I must prove that for any prime number p there exists a greater one". This reformulation of the problem sounds quite banal and insignificant. Nevertheless I had Ss who never hit on it. And without this step, the final solution cannot be reached.[7]

A further solution-phase would run as follows: "To prove the existence of such a prime number, I must try to construct it." With one of my Ss, I could follow clearly the way in which, to this phase, a further one attached itself as a mere explication: "One must therefore construct a number greater than p which cannot be represented as a product". From here on, clearly directed to "avoiding a product", the S proceeded to construct the product of all numbers from 1 to p and to add 1—incidentally, without having realized that the resultant nummer need not be itself a prime number, but may merely contain the desired number as a fraction of itself.

9. IMPLICIT SOLUTION-PHASES

Not all phases of the various solution-processes are given in a family tree of the

[7] The solution consists in the construction of the product of all prime numbers from 1 to p and adding to it 1. The resultant number is either itself a prime number, or it is a product of prime numbers greater than p. For, with the exception of the special case of 1, a prime number less than p cannot be contained in a multiple of itself increased by 1 without a remainder. Thus in any case, a prime number greater than p exists. (Q.E.D.)

kind graphically represented on page 5: rather, only the more prominent and relatively independent among them are given. Aside from these, there exist phases which are not explicit enough and, above all, too, banal ever to appear in a protocol. In the case of the radiation problem, for instance, it is clear to all Ss from the start that, in any case, to find a solution, something must be done with the actual circumstances concerned, with the rays and the body. As modern Europeans, they do not think of looking for suitable magic formulae; nor would they anticipate that some change in another place would lead to a solution. Similarly in the case of the prime numbers problem, from the beginning there is no doubt that the solution is to be sought in the province of numbers, and not, for example, in the province of physical processes. In short, from the very first, the deliberating and searching is always confined to a province which is relatively narrow as to space and content. Thus preparation is made for the more discrete phases of a solution by certain *approximate regional demarcations,* i.e., by phases in which necessary but not yet sufficient properties of the solution are demanded. Such implicit phases of a solution do not quite fulfill even the first prerequisite of a solution mentioned on page 1.

This is valid not only for thinking, but also for attempts at solution by action (trial and error). When a layman wishes to adjust the spacing between lines on a typewriter, this much at least of the solution is known to him: "I must screw or press somewhere on the machine". He will not knock on the wall, for instance, nor does he anticipate that any change of the given colors would do. In general, one seeks to achieve mechanical effects by mechanical alterations in the critical object.

One more example, this time from animal psychology. Thorndike (39) set his experimental animals (mostly cats) problems of the following type. They had to learn to bring about the opening of their cage doors by a simple mechanical manipulation—unintelligible to them, to be sure, for they could not survey the connections—and so to escape into freedom. Part of the animals had a whole series of different cage problems to solve. In one cage they had to pull on a loop, in another to lift a bar, or press on a knob, etc. Thorndike made the very interesting observation that generally, in the course of the experiments, "the cat's general tendency to claw at loose objects within the box is strengthened and its tendency to squeeze through holes and bite bars is weakened." (p. 48) Further, "its tendency to pay attention to what it is doing gets strengthened . . ." (p. 48) It is evident that even animal 'trial and error' is for the most part already under the confining influence of certain demarcations, which, by the way, are not purely instinctive.

10. INSUFFICIENCY OF A PROTOCOL

The reader has probably received the impression that the discussions of the preceding paragraphs left the data of the protocol a long way behind. In the case of the very first proposal, for instance, that of the esophagus, there was no mention at all of "redirecting over a tissue-free path", or even of "avoiding contact". That some such thing appeared in other protocols in an analogous place naturally proves nothing about the psychological origin of just this individual proposal.

This is the place in which to say

something essential about protocols. One could formulate it thus: A protocol is relatively reliable only for what it positively contains, but not for that which it omits. For even the best-intentioned protocol is only a very scanty record of what actually happens. The reasons for this insufficiency of protocols which are based on spoken thoughts must interest us also as characteristic of a solution-process as such. Mediating phases which lead at once to their concrete final realization, and thus are not separated from the solution by clear phase-boundaries, will often not be explicitly mentioned. They blend too thoroughly with their final solutions. On the other hand, mediating phases which must persist as temporary tasks until they find their final 'application' to the situation have a better chance of being explicitly formulated. Furthermore, many superordinate phases do not appear in the protocol, because the situation does not appear to the S promising enough for them. Therefore they are at once suppressed. In other words, they are too fleeting, too provisional, too tentative, occasionally also too 'foolish', to cross the threshold of the spoken word.

In very many cases, the mediating phases are not mentioned because the S simply does not realize that he has already modified the original demand of the problem. The thing seems to him so self-evident that he does not have at all the feeling of having already taken a step forward.[8] This can go so far that the S deprives himself of freedom of movement to a dangerous degree. By substituting unawares a much narrower problem for the original, he will therefore remain in the framework of this narrower problem, just because he confuses it with the original.[9]

11. "SUGGESTION FROM BELOW"

There exist cases in which the final form of a solution is not reached from above, i.e., not by way of its functional value. This is a commonplace of 'familiar' solutions. If the final solution of a problem is familiar to the S, it certainly need no longer be constructed, but can be reproduced as a whole, as soon as the problem is stated.[10]

More interesting cases exist. We must always remember that a solution has, so to speak, two roots, one in that which is sought and one in that which is given. More precisely, *a solution arises from the claim made on that which is given by that which is sought. But these two components vary greatly in the share they have in the genesis of a solution-phase.* A property of a solution is often very definitely demanded (characterized, hinted at) before it is discovered in what is given; but sometimes it is not. An example from the radiation problem: The esophagus may be discovered because a free path to the stomach is already sought. But it may also happen that, during a relatively vague, planless inspection of what is given in the situation, one 'stumbles on the esophagus'. Then the latter—so to speak, from below—suggests its functional value: "free path to the stomach"; in other words, the concrete realization precedes the functional value. This sort of thing happens not infrequently; for the analysis of the situation is often relatively planless. Nor is this disadvanta-

[8] Such is especially the case with the demarcation of boundaries, cf. pages 9-10.

[9] Cf. pages 25-26, on "Fixation".

[10] This of course does not exclude the possibility that the solution is reproduced along with its functional value and as its realization, and that it is thus *understood*.

geous, when the point is to find new ideas. In mathematical problems, this analysis merely of the given situation, the development of consequences from the given data, plays an especially large rôle—as we shall see in Chapter III.

One more example of "suggestion from below". An attractive goal object (for example, a banana) lies out of reach before the cage of a chimpanzee. So long as the solution, "to fish for the banana with a stick", is not very familiar, something like a stick must be in the visual field as a suggesting factor.[11] The stick is not yet *sought*—as embodiment of the previously conceived functional value: 'something long and movable"—as it is in later stages; rather it must itself help to suggest this functional value.[12]

The prerequisite for such a suggestion from below is that the 'phase-distance' between what is sought for and what could give the suggestion is not too great.

The following is an example for this influence of the size of the phase distance. Right at the beginning of the radiation problem, the E can speak of "crossing", or can draw a cross, without the S's grasping what that means. (Cf. the solution by crossing a number of weak bundles of rays in the tumor.) If, on the other hand, the S is already of his own accord directed to "decreasing the intensity on the way", he will understand the suggestion sooner than if his thinking is dominated, for example, by the completely different demand for "a free path for the rays". We can formulate the general proposition that a suggestion is the sooner understood or assimilated, the closer it approaches the genealogical line already under development, and, within this line, the nearer it is to the problem-phase then in operation; in short, the more completely it is already anticipated.

This law is a special case of a more general law, which concerns not suggestions in the narrow sense, but the material of thinking in general. Selz formulated this law as "a general law of anticipation" in the following manner: "An operation succeeds the more quickly, the more the schematic anticipation of the solution approaches a complete anticipation." (37, p. 512) We shall have more to do with this law; see Chapter VI.

12. LEARNING FROM MISTAKES (CORRECTIVE PHASES)

As yet we have dealt only with the progress from the superordinate to the subordinate phases (or vice versa), in other words, with progress along a given genealogical line. That this is not the only kind of phase succession is, one should think, sufficiently indicated by the protocol given above. Here the line itself is continually changed, and one way of approach gives way to another. Such a *transition to phases in another line* takes place typically when some tentative solution does not satisfy, or when one makes no further progress in a given direction. *Another* solution, more or less clearly defined, is then looked for. For instance, the first proposal (esophagus) having been recognized as unsatisfactory, quite a radical change in direction takes place. The attempt to avoid contact is completely given up and a

[11] Cf. W. Köhler, (20), p. 37.

[12] This suggestion of the functional value from below is even the rule in problems where a number of objects are offered to begin with, with the instruction to choose from among them an appropriate tool for such and such a purpose. Especially when only few objects are concerned, thinking will tend to proceed by looking things over. i.e., it will test the given objects one after another as to their applicability, and no attempt will be made to conceive the appropriate functional value first. (See Chap. VII on experiments with such problem-situations.)

means to desensitize tissues is sought in its place. In the third proposal, however, the S has already returned to old tactics, although with a new variation. And such shifting back and forth occurs frequently.

It will be realized that, in the transition to phases in another line, the thought-process may range more or less widely. Every such transition involves a return to an earlier phase of the problem; an earlier task is set anew; a new branching off from an old point in the family tree occurs. Sometimes a S returns to the original setting of the problem, sometimes just to the immediately preceding phase. An example for the latter case: From the ingenious proposal, to apply the rays in adequate amounts by rotation of the body around the tumor as a center, a S made a prompt transition to the neighboring proposal: "One could also have the radiation apparatus rotate around the body." Another example: The S who has just realized that the proposal of the esophagus is unsatisfactory may look for another natural approach to the stomach. This would be the most "direct" transition, that is, the transition which retrogresses least. Or, renouncing the natural approach to the stomach, he looks for another method of avoiding contact. Or, again, he looks for an altogether different way to avoid the destruction of healthy tissue. Therewith, everything which can be given up at all would have been given up; a "completely different" solution would have to be sought.

In such retrogression, thinking would naturally not be taken back to precisely the point where it had been before. For the failure of a certain solution has at least the result that now one tries *"in another way."* While remaining in the framework of the old *Problemstellung,* one looks for another starting point. Or again, the original setting may itself be altered *in a definite direction,* because there is the newly added demand: From now on, that property of the unsatisfactory solution must be avoided which makes it incompatible with the given conditions. An example: The fully developed form of our radiation problem is naturally preceded by a stage in which the problem runs only as follows: Destroy the tumor with the aid of appropriate rays. The most obvious solution, which consists simply in sending a bundle of sufficiently strong rays through the body into the tumor, appears at once inadequate, since it would clearly have the result of destroying healthy tissue as well. In realization of this, *avoidance of the evil* has to be incorporated *as an additional demand* into the original form of the problem; only in this way does our form of the radiation problem arise (cure . . . without destruction of healthy tissue). One more example: In the pendulum problem, a watchman is often proposed who has the task of keeping the length constant by compensatory changes in the position of the weight. For the most part, Ss realize spontaneously that this procedure could not possibly be sufficiently precise, and that it would also incessantly interfere with the motion of the clock. Thus the problem: "compensation of the change in length of the pendulum," is enriched by the important addition: "automatically."

Such learning from errors plays as great a rôle in the solution-process as in everyday life.[13] While the simple realiza-

[13] Life is of course, among other things, a sum total of solution-processes which refer to innumerable problems, great and small. It goes without saying that of these only a small fraction emerge into consciousness. Character, so far as it is shaped by living, is of the type of a resultant solution.

tion, *that* something does not work, can lead only to some variation of the old method, the realization of *why* it does not work, the recognition of the *ground of the conflict,* results in a correspondingly definite *variation which corrects* the recognized defect.

13. FAMILY TREES OF TWO SOLUTIONS

As a supplement, there follow two family trees compiled from many combined protocols. That of the radiation problem is derived from experiments with German and American psychology students, of whom 16 were tested in individual experiments and 26 in group experiments. P_1, P_2 . . . denote successive phases of the process as they follow one another, not in chronological order, but ordered as to content. The Roman numerals and the large and small letters each denote coordinated phases. The numbers following the phases indicate how many times the phase concerned appeared in the protocols; those in parentheses how often with the E's occasional help, the rest how often spontaneously. If no number follows, the phase has been interpolated as expressing the functional value of solutions actually recorded.

The second family tree refers to the pendulum problem, and is derived from the records of six individual experiments and from 33 gathered in group experiments.

All the tentative solutions offered by the Ss are noted down in the family trees, including the foolish ones and those which are hardly comprehensible. Naturally, the records of group experiments are much poorer in mediating phases than those of individual experiments. The Ss in groups had only about ten minutes to work in; in the individual experiments, they had, where necessary, more than an hour. Moreover, the Ss in individual experiments were continually urged to think out loud, and did not themselves have to write down what occurred to them.

Family Tree of the Solution of the Radiation Problem

P_1 Destroy the tumor by means of rays without also destroying healthy tissue. 42

P_2 I No contact between rays and healthy tissue. 1

P_3 IA Redirect the rays over a path as free as possible from tissue. 2(2)

P_4 IA
- a) . . . via the esophagus or intestines. 12
- b) Swallow the radiating substance. 4
- c) Send the rays over the shortest path. 7
- d) . . . through an area less vitally important. 7[14]
- e) Send the rays from above.[15]

P_3 IB Remove healthy tissue from path of rays by operating. 17

C Place a protecting wall between rays and healthy tissue. 1

P_4 IC
- a) Introduce a cannula. 8
- b) Swallow a substance which is impermeable to the rays. 1
- c) Coat the tumor with lead, leaving an opening for the rays. 1
- d) Place an insulating plate on the surface of the body. 5
- e) Surround the rays by harmless rays. 2[16]

P_2 II Desensitize the healthy tissue on the way. 8
(or the reverse: Render the tumor more sensitive. 4)

P_3 IIA . . . by chemical injection. 10

B . . . by previous immunizing application of radiation. 2
Or: Aren't pathological tissues perhaps more sensitive? 6(1)
Or: Apply rays to which pathological tissues are more sensitive. 9

P_2III Less intensity of radiation on the way. 6(6)

[14] These two suggestions came from another group experiment, in which by accident the ellipse in the accompanying sketch was made especially narrow. (See 3.)

[15] This S took literally the accompanying sketch of a cross-section.

[16] Proposals P_4 IC *b* and *c* clearly depend on a partially false setting of the problem. Not only the healthy part of the stomach, or the second half of the path, must be protected. Proposal *d* comes from a completely false conception of the task. Proposal *e* involves the application of a false model: the rays are treated as though they were poison which must not come in contact with the body.

P_3 IIIA Have the rays less dense on the way. 2(2)

P_4 IIIA
- a) By means of a focussing lens. 5(2)
- b) By crossing several weak bundles of rays. 2(2)
- c) By rotation of the body, or of the radiation apparatus, around the tumor. 1

B Decrease intensity on the way by opposing forces (for example, by other rays falling on the first at right angles). 12

C Weaken intensity on the way by interference. 3

D Use rays which work only at a definite distance? 3

E Give the rays full strength only when the tumor is reached. 1

P_2 IV Cure without rays. 1[17]

P_2 V Do not mind having the healthy tissue destroyed (it would be the lesser evil). 1

Family Tree of the Solution of the Pendulum Problem

P_1 Prevent irregular working of the clock which would follow alteration in the length of the pendulum caused by temperature-change. 39

P_2 I Constant temperature. 11

P_3 IA Protected place (prevent temperature-changes from reaching the clock). 3(4)

P_4 IA
- a) Clock in cellar. 1
- b) . . . Not on outer walls. 1
- c) . . . In a narrow space protected from draught. 1
- d) . . . In a vacuum. 4
- e) . . . In a hermetically sealed container. 5
- f) . . . In steam. 1
- g) . . . In a coating of ice. 1

P_3 IB Compensate the changes in temperature.

P_4 IB
- a) Someone to regulate the temperature by appropriate warming or cooling of the room. 1
- b) . . . By appropriately changing the location of the clock. 1
- c) Automatic introduction of compensating influences on the temperature (expansion into cold water). 1[18]

P_2 II Avoid the alterations of length caused by actual temperature-changes. 1

P_3 IIA Use insensitive material. 10(1)

P_4 IIA
- a) . . . Wood. 3
- b) . . . Platinum. 1
- c). . . A coating of appropriate varnish. 1

P_3 IIB Compensate the changes in length. 4(5)

P_4 IIB
- a) Someone to regulate (change location of) weight on the shaft. 10
- b) Combine simultaneous contractions (or expansions) in opposite directions. (Fig. 5A, h–p)
- c) . . . Simultaneous expansion and contraction. (Fig. 5A, q–s)
- d) Have the weight lifted by expansion of the shaft. (Fig. 5, t)
- e) The point of suspension can move up and down on the shaft (or chain). (Fig. 5A, u,v)
- f) The point of suspension shifts in the same direction as the center of gravity. 2[19]
- g) . . . In the opposite direction. . . .[20]
- h) Leaving leeway for change in length (as with railroad tracks). 1[21]

P_3 IIC Block the change in length.

P_4 IIC
- a) . . . By having it blocked by a spatial barrier. 1[22]
- b) . . . By keeping it restricted within a fixed framework. 2[23]

P_2III Prevent the effect which change in length would have on working of clock.

P_3IIIA A counter-weight above the point of suspension. 3

B Compensating electric or magnetic influences. 1

C Compensating change in the force of gravity by change in location of the clock. 1

D Compensating change of the clockwork. 1

E Proportional change of the whole system. 2

F A metal which retains its center of gravity despite displacement. 1[24]

G The weight to be very heavy, the shaft very light.[25]

H Change in the weight compensates change in length. 1[26]

14. DISCUSSION OF N. R. F. MAIER'S VIEWS

From N. R. F. Maier, the American psychologist, we have had in recent years several reports on admirable experiments

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[17] This and the next proposal are attempts to escape from the original setting of the problem.

[18] This proposal is a particularly crass example of a solution which is 'single-track', i.e., leaves all other factors out of consideration. ("Operation successful, patient dead.")

[19] This is a crude error in thinking and an unfortunate attempt toward the proposal in *e*. For the displacement of the point of suspension in "absolute" space naturally has no effect on the length of the pendulum.

[20] This is the solution of a problem mistakenly confused with the original, namely, to keep the weight in a constant location.

[21] Not wholly clear.

[22] See Footnote 8, page 11.

[23] And the framework?

[24] Indeed a vain wish!

[25] Completely confused.

[26] But how?

in which the problem-solving of men and of rats was investigated. (26, 27, 28.) Maier belongs to the few psychologists who are now concerned with precisely the questions which are raised in the present chapter. On the basis of his experiments, Maier arrived at the following main statements about productive thinking:

1. Thinking consists in *combining* parts of previous experiences *in new patterns.*

2. Such a new combination occurs always under the influence of a definite *'direction'.*

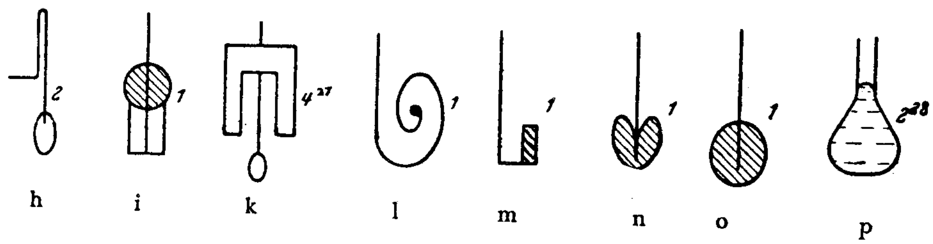

FIG. 5a, h-p

"The parts . . . must be combined in a certain manner, and a direction or way the problem is attacked seems to be a factor which determines the nature of the combination." (27), p. 143.

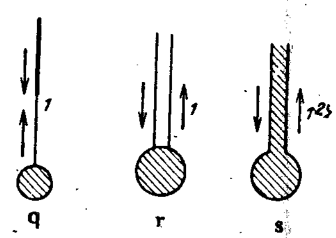

FIG. 5a, q-s

These theses derive principally from the following experimental procedure: Maier divides the solution of a problem in several parts, and gives these to his Ss as aids. He finds that these parts of the solution are of hardly any help, if the 'direction' is not also given.

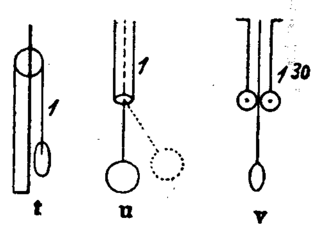

FIG. 5a, t-v

Now when one closely examines what Maier calls 'direction', it becomes evident that *'direction' is nothing but the earliest phase of the solution, i.e., the reformulation of the problem as it initiates the solution-process concerned. The other 'aids' contain later (subordinate) properties of the solution.* The reader may convince himself that this is the correct interpretation of Maier's views by considering the following example: "There are several angles or directions, that one may take from the same starting point. . . . If two doctors wished to solve the yellow-fever problem they might see the difficulty altogether differently. One might believe that the cure depended upon making the human immune to the germ [direction 1], the other might think that the germ must be kept from the hu-

[27] This solution was familiar in all four instances. In the case of the two preceding solutions, the Ss concerned remarked that "something of this sort" was known to them.

[28] Containing mercury (or instead of this: a column of mercury lifts a pendulum-weight from without).

[29] Expansion and contraction are here supposed to "hinder each other".

[30] Here the pendulum swings perpendicular to the plane of the drawings. This proposal of course makes the mistake that the "wheels" could not move along the pendulum-shaft, which is fixed nowhere else.

man [direction 2]. The first might experiment with serums, the other might seek to learn what carried the germ and hence seek a way to remove the carrier." (27), p. 137.

From the comparison of Maier's concepts with those of the present investigation, it follows that:

1. There is no need for a special concept of a 'direction' which combines elements. 'Direction' is entirely of the type of a *problem*, or, more exactly, of the reformulation of a problem and of a mediating phase in the solution-process. Let the yellow-fever problem begin or end with one of the two 'directions'; then the problem- or solution-character of the 'direction' involved becomes clearly evident. In short, what Maier calls 'direction' is an 'organizing principle' in no other sense than is what he calls 'problem'.

2. There exists as little fundamental difference between 'direction' and 'the elements to be combined' as between 'direction' and 'problem'.[31] For these elements combine with one another with only apparent simultaneity. In reality, they usually follow upon one another in a sequence in which the each element possesses problem-character (thus, 'direction'-character) with respect to the following, and solution-character with respect to the preceding elements. In other words, the combination is carried out in the form of a succession in which more particular realization-aspects or means-aspects of the solution develop from functional values or goal-aspects. Incidentally, the same follows from Maier's rat experiments.

I have spent so much time in the reinterpretation of Maier's concepts for the reason that material of a given type should not for long be expressed by different concepts.[32]

[31] That these elements need not under all circumstances be "parts of *past* experiences" is shown at least by all those solutions into which newly offered objects enter according to their suitable properties.

[32] For the same reason, Claparède's concept of *hypothesis* (2) appears to me superfluous. For Claparède's "hypothèse" is nothing but a solution or a preliminary, provisional phase of a solution.