First attempt at this was to use Z transforms, but since the $Z\{n^2 f_n\}$leads to a second order ordinary differential equation, it quickly got out of hand. Instead I looked for an approximation in the "time" (n) domain.

$$f_n = \frac{1 + (n^2 - n) f_{n-1}}{n^2 + 1}$$

$$f_n(n^2 + 1) = 1 + (n^2 - n) f_{n-1}$$

$$f_n n^2 + f_n = 1 + f_{n-1} n^2 - f_{n-1} n$$

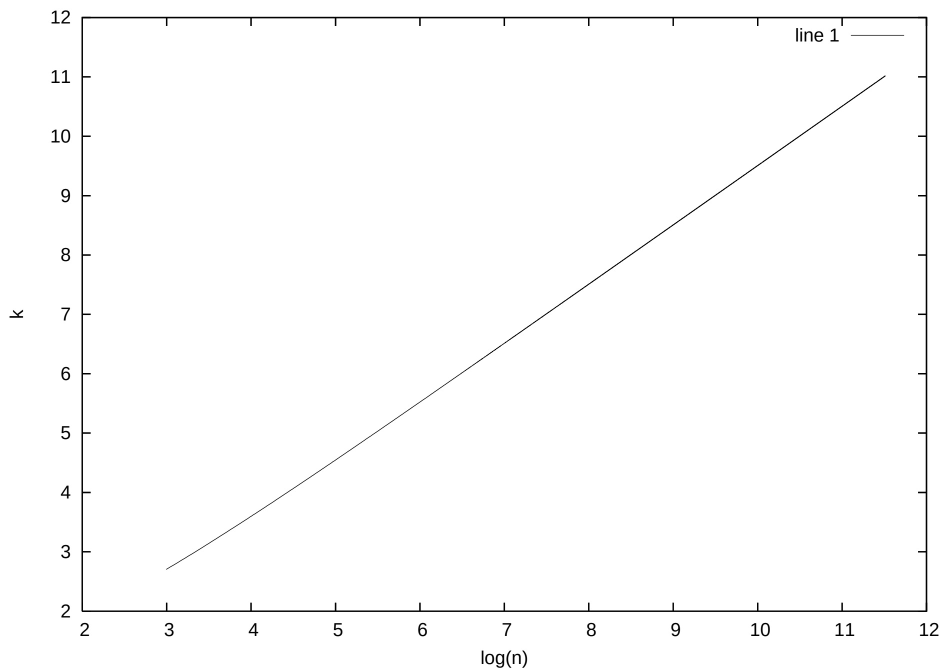
when $n \to \infty$we drop some term, namely $f_n$and 1 and are left with

$$f_n n^2 = f_{n-1} n(n-1)$$

$$\frac{f_n}{f_{n-1}} = \frac{n-1}{n}$$

Let $f_i = a$ then $f_{i+1} = a\frac{i}{i+1}$and $f_{i+2} = a\frac{i}{i+1}\frac{i+1}{i+2} = \frac{ai}{i+2}$, leading to $f_n = \frac{ai}{n} = \frac{k}{n}$, where $ai$is a constant k.

Plotting k vs log(n) gives



Looks like k is not that constant after all, whoops.

As log(n) approaches infinity, k is a linear function in $\log(n)$with a slope approaching unity and a y intercept of $-0.766501757151001...$

But seeing as $\log n \gg -.766$when $n \to \infty$, we can neglect the constant term entirely.

In conclution $g_n = \frac{-0.766+\log n}{n}$ or even simpler

$$g_n = \frac{\log n}{n}$$

Regards,
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