Solution by Chris Shannon. Calgary, Canada. June 25, 2007.

Part 1

Starting with the obvious strategy: if the sum is less than x, take another random number.

Let the propability distribution function of the sum after i numbers be $a_i(x)$:

$a_0 = delta(0)$: we start at 0.

$a_1 = rect$ from 0 to 1. Uniform distribution.

$a_n = a_{n-1}$* rect(0,1), where * represents convolution, and note that they are functions of x.

Because each new random number is independent, the sum will be a convolution with the rectangle function. We repeatedly convolve with a unit rectangle representing a random value from 0 to 1, and we get

$a_2 = x$

$a_3 = \frac{x^2}{2}$

$a_4 = \frac{x^3}{6}$ and in general

$a_n = \frac{x^{n-1}}{(n-1)!}$... for 0 < x < 1.

Playing the game, we have after 0 numbers, we have a $1 - x$ chance of winning. If the first number is less than x, which happens with probability $\int_0^x a_1 dx$, we have another chance of winning of $1 - x$. In fact, as long as $a_n < x$, we have a chance of winning of $1 - x$. All these chances can be added.

Probability of winning is the sum over n, of $a_n < x * (1 - x)$

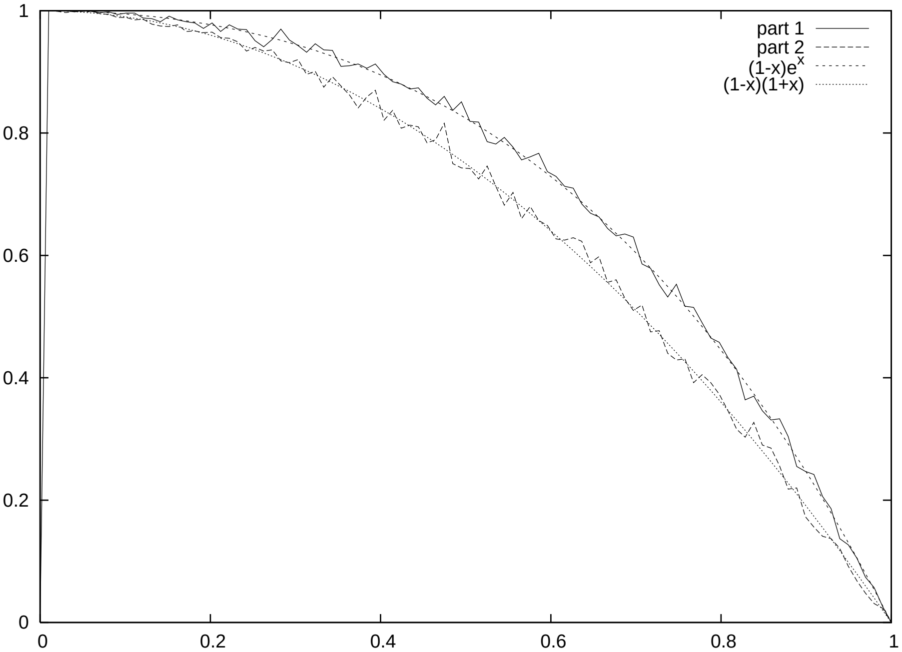
$\sum_{n=0}^{\infty} \int_0^x a_n(1 - x)dx = \sum_{n=0}^{\infty} \frac{x^n}{n!}(1 - x)$

Now we recognize the taylor series of the exponential function and our solution is written as

$e^x(1 - x)$

Part 2

First we notice that as n goes to infiinity, all values are equally likely to be passed. We turn the quesiton around and ask, "what is the probability that we will miss the interval n+x and n+1? In order for that to happen you have to land in the range from n to n+x, which occurs with probability x, and then you have to either jump before or after the interval which also occurs with probability x, so the chance we'll miss the interval is x^2. The probably of winning the game is $1 - x^2$.



These results were tested with a morte carlo simulation. x was stepped from 0 to 1 in .01 incriments, and repeated with 1000 interations each. For part 2, n was chosen to be 20.

Algorithm 1 ponderjune2007

```
close all;
n = 0;
i=1;
xlen = 100;
fr = zeros(1,xlen);
nsamp = 1000;
xs = linspace(0,1,xlen);
for x=linspace(0,1,xlen)
for b=1:nsamp
a = 0;
while (a < n + x)
a = a + rand;
if (n + x < a && a < n + 1 )
fr(i) = fr(i) + 1;
end
end
end
i = i+1;
end
plot(xs,fr / nsamp, ";part 1;");
hold on
n = 20;
i=1;
xlen = 100;
fr2 = zeros(1,xlen);
nsamp = 1000;
xs = linspace(0,1,xlen);
for x=linspace(0,1,xlen)
for b=1:nsamp
a = 0;
while (a < n + x)
a = a + rand;
if (n + x < a && a < n + 1 )
fr2(i) = fr2(i) + 1;
end
end
end
i = i+1;
end
plot(xs,fr2 / nsamp, ";part 2;");
plot(xs, (1 - xs) .* exp(xs), ";(1-x)e^x;")
plot(xs, (1 - xs) .* (1 + xs),";(1-x)(1+x);")
```