

ABSTRACT COMONADS

FOR WHEN COMONADS AREN'T ABSTRACT ENOUGH

REVIEW

```
class Functor w => Comonad w where
```

```
extract :: w a -> a
```

```
duplicate :: w a -> w (w a)
```

```
extend :: (w a -> b) -> w a -> w b
```

SELECTION QUERIES

SELECTION QUERIES

forall a. w a -> a

SELECTION QUERIES
SELECT A SLOT
NO MATTER WHAT'S INSIDE

E.G. $\text{ix } 5$

RUNNING A
SELECTION QUERY
RETRIEVES THE VALUE FROM THAT
SLOT

EXTENDING AN SELECTION QUERY REARRANGES SLOT POSITIONS

extend (ix 2)
SELECTS THE SLOT
TWO TO THE RIGHT
IN ALL VIEWS

SHIFTING EACH SLOT INDIVIDUALLY



SHIFTING THE WHOLE STREAM

forall a. Stream a -> a

```
λ> duplicate countStream  
  (0 :> 1 :> 2 :> ...)  
:> (1 :> 2 :> 3 :> ...)  
:> (2 :> 3 :> 4 :> ...)  
:> ...
```

```
λ> ix 2 $ duplicate countStream  
2 :> 3 :> 4 :> 5 :> 6 :> ...
```

```
λ> ix 2 <$> duplicate countStream  
2 :> 3 :> 4 :> 5 :> 6 :> ...
```

NOT A SELECTION QUERY

windowedAvg :: Int -> Stream Int -> Double

```
λ> windowedAvg 3 $ duplicate countStream  
⚠ Type Error: Stream (Stream Int) != Stream Int
```

```
λ> windowedAvg 3 <$> duplicate countStream  
1.0 :> 2.0 :> 3.0 :> 4.0 :> 5.0 :> ...
```

```
λ> countStream  
0 :> 1 :> 2 :> 3 :> 4 :> ...
```

-- Running a selection

```
λ> ix 5 countStream  
5
```

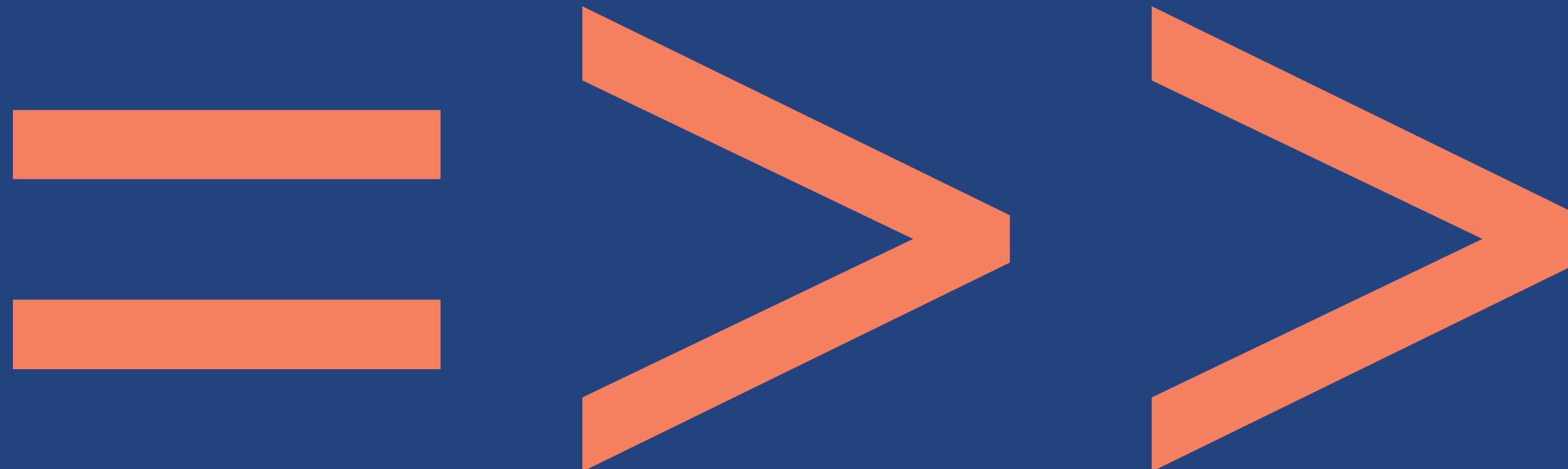
-- Extending an selection

```
λ> extend (ix 5) countStream  
5 :> 6 :> 7 :> 8 :> 9 :> ...
```

</foreshadowing>

NOTE ON NOTATION:

LXTEND



CONSTRUCTS & RUNS A CHAIN OF QUERIES

IT TAKES A STRUCTURE
ON THE LEFT

$(= >>) :: w\ a \rightarrow (w\ a \rightarrow b) \rightarrow w\ b$

AND A QUERY
ON THE RIGHT

NOTE ON NOTATION: INFIX EXTEND

```
λ> extend (ix 2) countStream  
2 :> 3 :> 4 :> 5 :> 6 :> ...
```

```
λ> countStream =>> ix 2  
2 :> 3 :> 4 :> 5 :> 6 :> ...
```

NOTATION (EXTEND)

`(=>>) :: w a -> (w a -> b) -> w b`

```
λ> countStream  
0 :> 1 :> 2 :> 3 :> 4 :> ...
```

-- =>> is left associative!

```
λ> countStream =>> ix 2 =>> ix 2  
λ> (countStream =>> ix 2) =>> ix 2  
4 :> 5 :> 6 :> 7 :> 8 :> ...
```

```
λ> countStream =>> ix 2 =>> takeS 3  
[3,4,5] :> [4,5,6] :> [5,6,7] :> [6,7,8] :> [7,8,9] :> ...
```

```
λ> countStream =>> ix 2 =>> takeS 3 & extract  
[3,4,5]
```

NOTE ON NOTATION: COMPOSITION



COMPOSES QUERIES

CO-KLEISLI COMPOSITION

```
(=>>) :: w a  
-> (w a -> b)  
-> w b
```

VS.

```
(=>=) :: (w a -> b) -- Query 1  
-> (w b -> c) -- Query 2  
-> (w a -> c) -- Composed Queries
```

```
λ> ix 2 =>= ix 2 =>= ix 2 $ countStream  
6
```

```
λ> ix 2 =>= takeS 3 $ countStream  
[2,3,4]
```

QUESTIONS?

STUFF STORED IN OUR STUFF STORED BY KEY

SO WHAT'S A
STORE?

WAREHOUSE
LABELED SHELVES WITH ITEMS
WELL STOCKED (EVERY SHELF IS OCCUPIED)
FORKLIFT IS ALWAYS AT A SHELF



WAREHOUSE
HAS AN
INVENTORY
AND A
FORKLIFT

#0	Fidget Spinners
	Books
#2	Guitars
#3	Laptops

WAREHOUSE QUERIES

POS
ASKS WHICH SHELF
THE FORKLIFT IS AT

POSW

====

1

#0	Fidget Spinners
	Books
#2	Guitars
#3	Laptops

EXTRACT
GETS THE ITEM
AT THE CURRENT SHELF

EXTRACT W

BOOKS

#0	Fidget Spinners
	Books
#2	Guitars
#3	Laptops

PEEK
GETS THE ITEM
FROM A SPECIFIC SHELF

EEK 3 W

LAPTOPS

#0	Fidget Spinners
	Books
#2	Guitars
#3	Laptops

PEEK\$
GETS THE ITEM
FROM A SHELF
RELATIVE TO CURRENT POSITION

PEEKS (+1) W

GUITARS

#0	Fidget Spinners
	Books
#2	Guitars
#3	Laptops

AS CODE:

```
data Store s a = Store (s -> a) s
```

WAREHOUSE EXAMPLE

```
inventory :: M.Map Int String
inventory =
M.fromList [ (0, "Fidget spinners")
, (1, "Books")
, (2, "Guitars")
, (3, "Laptops")
]
```

```
λ> lookup 0 inventory  
Just "Fidget spinners"
```

```
λ> lookup 42 inventory  
Nothing
```

EVERY KEY HAS A VALUE

#-2	Nothing
#-1	Nothing
#0	Fidget Spinners
	Books
#2	Guitars
#3	Laptops
#4	Nothing
#5	Nothing

```
warehouse :: Store Int (Maybe String)
warehouse =
  store (\shelf -> M.lookup shelf inventory) 1
```

```
λ> pos warehouse
```

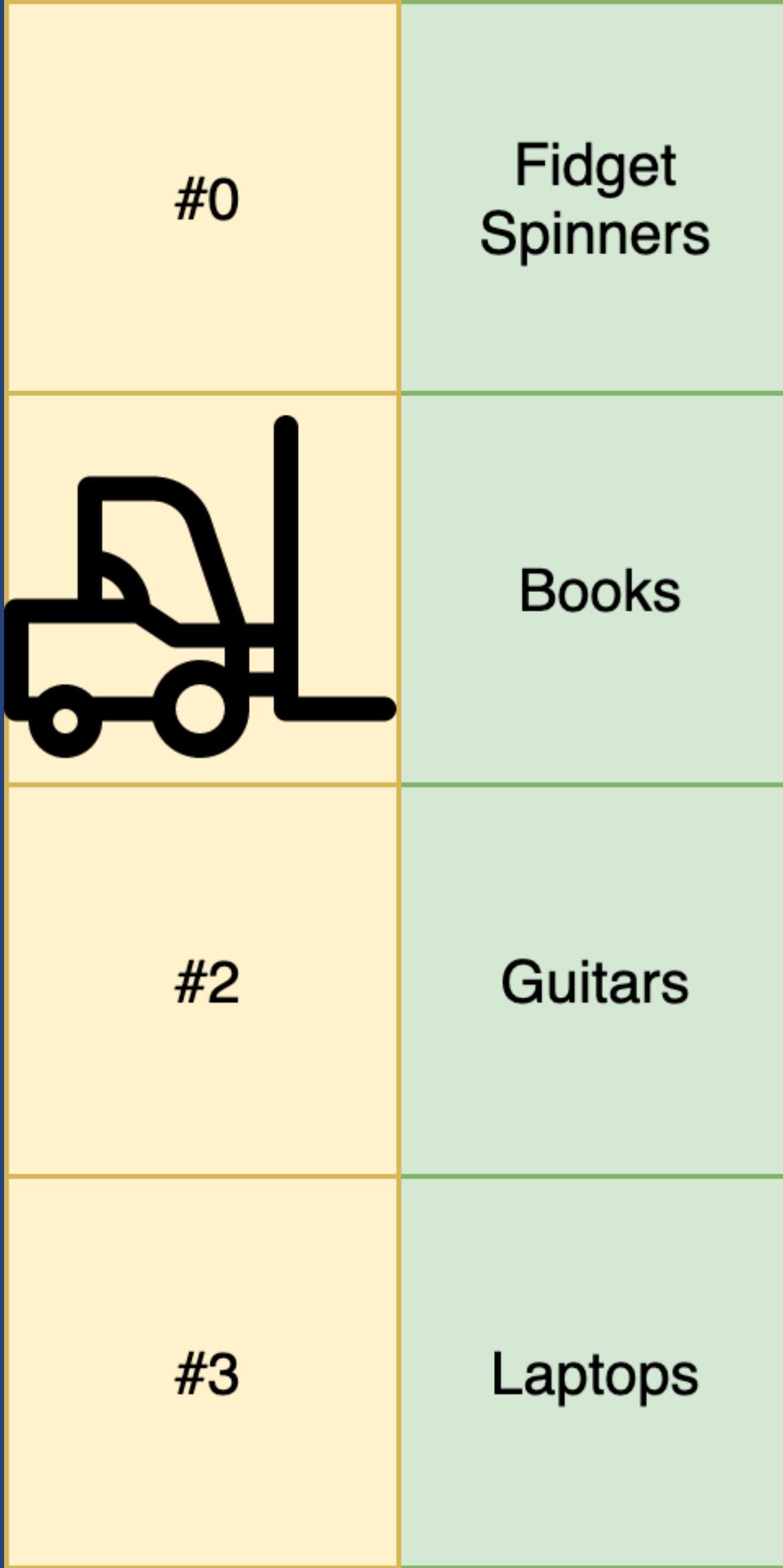
```
1
```

```
λ> peek 0 warehouse
```

```
Just "Fidget Spinners"
```

```
λ> peeks (+1) warehouse
```

```
Just "Guitars"
```



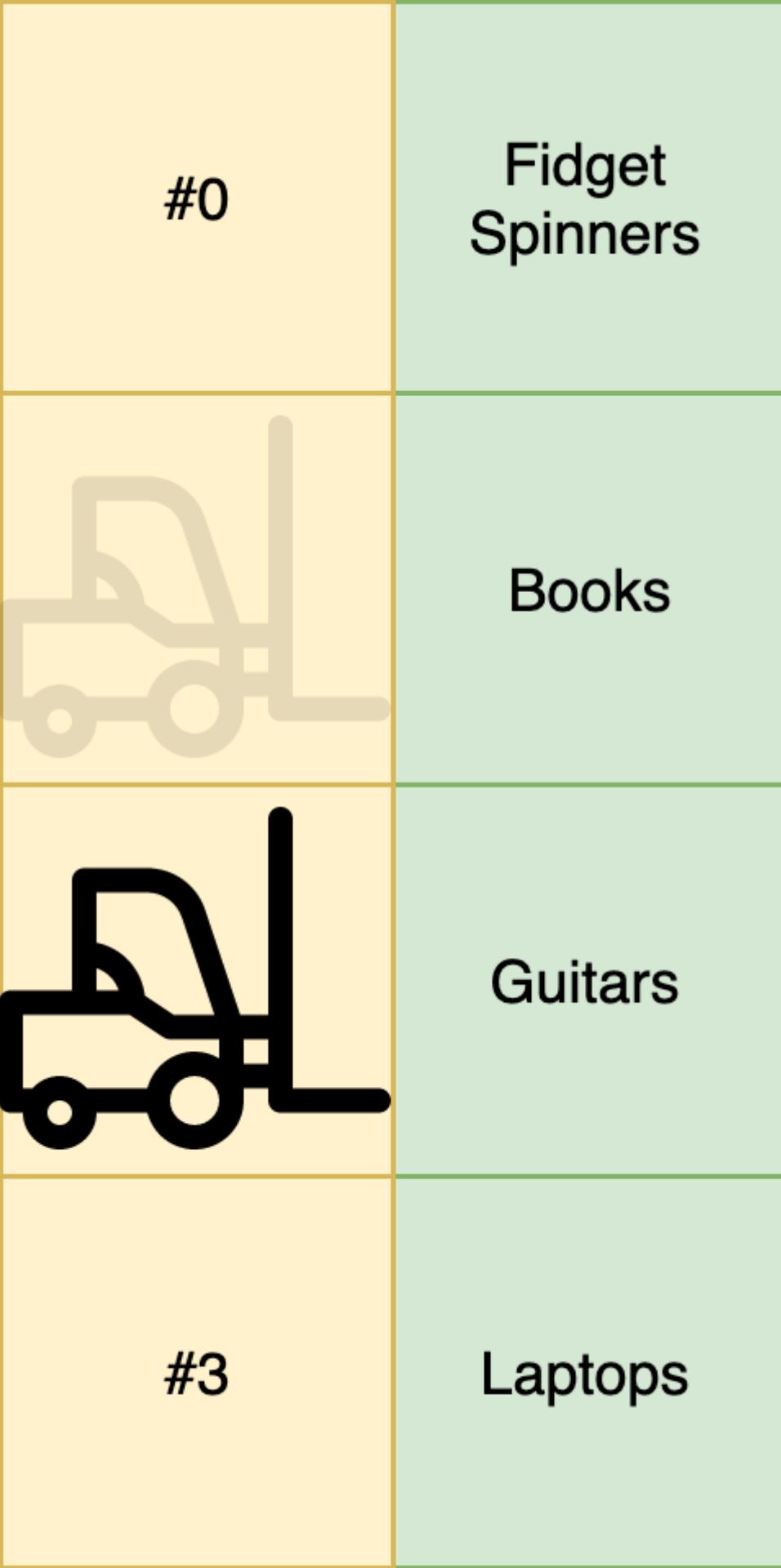
SEEK
MOVES THE FORKLIFT
TO A SPECIFIC SHELF

SEEK 3 W

#0	Fidget Spinners
#1	Books
#2	Guitars
	Laptops

SPECS
MOVES THE FORKLIFT
RELATIVE TO CURRENT POSITION

SEEKS (+1) W



STORE

A.K.A. CO-STATE

A **SLOT** FOR EVERY KEY

A **VIEW** FROM EVERY KEY

EXTRACT THE VALUE AT YOUR KEY



STORE
HAS A
GETTER
AND A
KEY

STORE QUERIES

POS

pos :: Store s a -> s

GETS THE
CURRENT KEY

EXTRACT

extract :: Store s a -> a

GETS THE SLOT
AT THE CURRENT KEY

PEEK

peek :: s -> Store s a -> a

EXTRACTS THE SLOT AT ANOTHER KEY

* SELECTION QUERY!!

```
squared :: Store Int Int
squared =
  Store (\x -> x^2) -- The getter
  10                  -- The current key
```

```
squared :: Store Int Int
squared = Store (\x -> x^2) 10
```

```
λ> pos squared
10
```

```
λ> extract squared
100 -- 10^2
```

```
λ> peek 2 squared
4 -- 2^2
```

PEEKS

peeks :: ($s \rightarrow s$) \rightarrow Store s $a \rightarrow a$

GET A SLOT AT A
RELATIVE POSITION
TO THE CURRENT KEY

```
λ> pos squared
```

```
10
```

```
λ> peek (+2) squared
```

```
144 -- (10 + 2)^2
```

RELATIVE QUERIES
LIKE PEEKS
HAVE DIFFERENT RESULTS
DEPENDING ON THE CURRENT VIEW

ABSOLUTE QUERIES
IGNORE
THE CURRENT VIEW

peek²
IS AN ABSOLUTE QUERY

STOP MUTATIONS

SEEK

seek :: s -> Store s a -> Store s a

FOCUS THE SLOT

AT THE

GIVEN KEY

SPEK

IS AN

ABSOLUTE MUTATION

STORE

```
λ> pos $ seek 5 squared  
5
```

```
λ> pos $ seek 3 squared  
3
```

```
λ> extract $ seek 3 squared  
9
```

SEEKS

seeks :: ($s \rightarrow s$) \rightarrow Store s $a \rightarrow$ Store s a

THE RELATIVE

VERSION OF

SEEK

EXPERIMENT

A photograph of a scientist wearing a white lab coat and a hairnet, looking down and through a microscope. The background is a laboratory setting with various equipment and glassware.

IT'S REALLY ANNOYING
TO PEEK AT LOTS OF NEARBY VALUES

**WHAT IF WE WANT TO
KEEP TRACK
OF THE RESULTS?**

OR EVEN
FAIL
BASED ON OUR POSITION?

**EXPERIMENT
GETS A
BUNCH
OF VALUES
RELATIVE TO YOUR POSITION**

EXPERIMENT

```
experiment :: Functor f  
=> (s -> f s)  
-> Store s a  
-> f a
```

STORE EXAMPLE: SQUARED

```
λ> experiment (\n -> [n - 10, n + 10]) squared  
[ 0      -- (10-10)^2  
, 400    -- (10+10)^2  
]
```

```
aboveZero n | n > 0      = Just n
             | otherwise = Nothing
λ> experiment aboveZero (seeks 10 squared)
Just 100
```

```
λ> experiment aboveZero (seeks (-10) squared)
Nothing
```

```
withN :: Store Int (String, Int)
withN = squared =>> experiment (\n -> (show n, n))
```

```
λ> extract withN
("10",100)
```

```
λ> peek 5 withN
("5",25)
```

STORE EXAMPLE: SQUARED

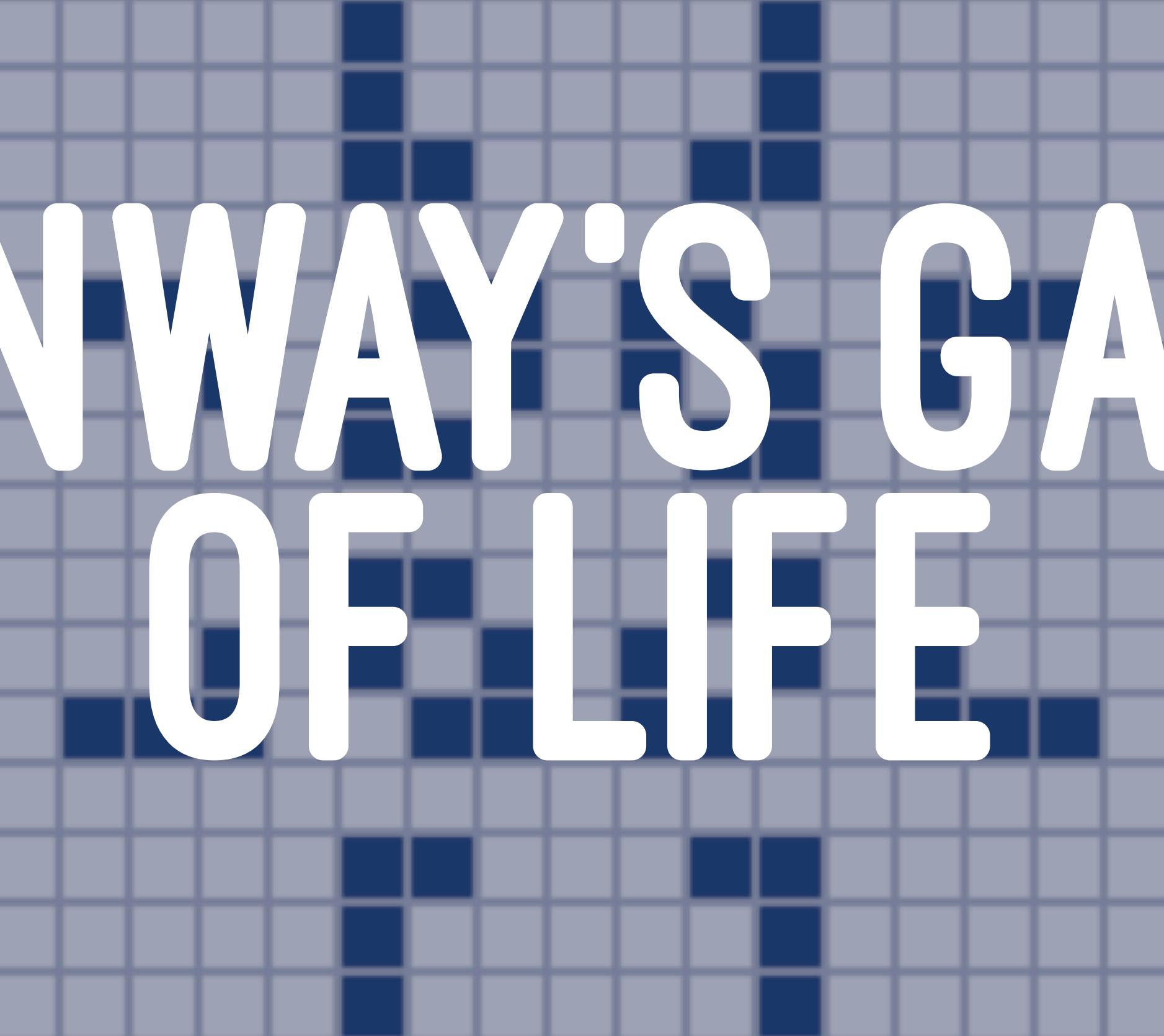
```
shifted :: Store Int (String, Int)
shifted = withN =>> peeks (+10)
```

```
λ> extract shifted
("20",400)
```

```
λ> peek 5 shifted
("15",225)
```

QUESTIONS?

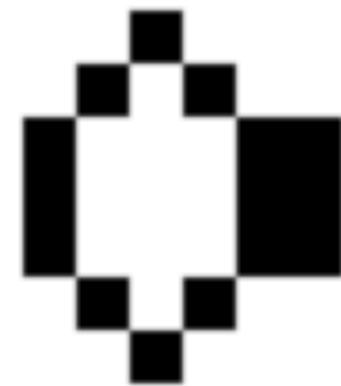
CONWAY'S GAME OF LIFE



RULES

- › LIVING CELLS WITH 2 OR 3 LIVING NEIGHBOURS STAYS ALIVE
- › DEAD CELLS WITH 3 LIVING NEIGHBOURS COME TO LIFE
- › ALL OTHER SCENARIOS CAUSE A CELL TO DIE





WE GONNA BUILD THIS:

```
λ> animateGrid startingGrid
```

```
...#.... | .#.... | ..#.... | ..... | .....
#.#.... | ..##... | ...#... | .#.#... | ...#...
.##.... | .##... | .###... | ..##... | .#.#...
..... | ..... | ..... | ..#.... | ...##...
..... | ..... | ..... | ..... | .....
```



LET'S PLAN

WE NEED TO REPRESENT A
2D GRID
OF CELLS

EACH CELL
IS EITHER
ALIVE
OR
DEAD

SO I'M THINKING
Store (Sum Int, Sum Int) Bool

(row, column) -> alive?

QUESTIONS?

```
startingGrid :: Store (Sum Int, Sum Int) Bool  
startingGrid = store checkAlive (0, 0)
```

where

```
checkAlive :: (Sum Int, Sum Int) -> Bool  
checkAlive coord = S.member coord livingCells
```

```
livingCells :: S.Set (Sum Int, Sum Int)  
livingCells = S.fromList [(1, 0), (2, 1), (0, 2), (1, 2), (2, 2)]
```

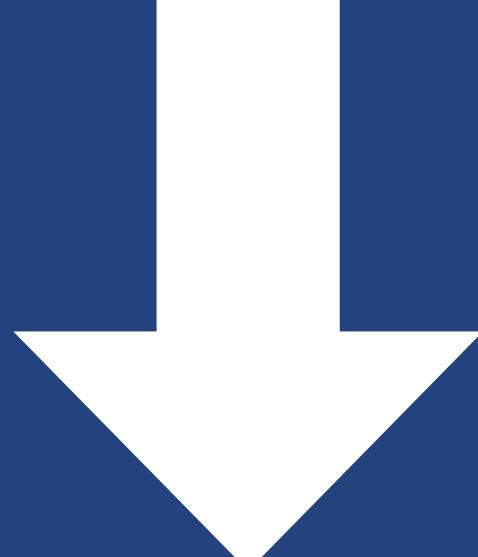
```
λ> peek (Sum 0, Sum 0) startingGrid  
False
```

```
λ> peek (Sum 1, Sum 0) startingGrid  
True
```

```
λ> putStrLn . drawGrid 3 $ startingGrid  
. .#  
#. #  
.##
```

WE WANT A
MUTATION
WHICH SIMULATES A
STEP

AS ALWAYS
LET'S BREAK IT
DOWN
INTO QUERIES



QUERY:

```
checkCellAlive :: Store (Sum Int, Sum Int) Bool  
-> Bool
```

IF WE CAN COMPUTE WHETHER
ONE CELL
IS ALIVE:
WE CAN EXTEND IT TO
ALL CELLS

惊讶表情符号 **WOW** 惊讶表情符号

OUR RECIPE NEEDS THE FOLLOWING INGREDIENTS:

1. WHETHER THE CURRENT CELL IS ALIVE OR DEAD
2. NUMBER OF LIVING NEIGHBOURS

QUERY TO CHECK THE
CURRENT CELL
ANY IDEAS?

EXTRACT

...NEXT

NUMBER OF LIVING NEIGHBOURS

```
numLivingNeighbours :: Store (Sum Int, Sum Int) Bool  
                      -> Int
```

WE NEED TO CHECK THE
CELLS NEXT DOOR

REMEMBER

EXPERIMENT?

```
experiment :: Functor f  
=> (s -> f s)  
-> Store s a  
-> f a
```

WE CAN CHECK OUR NEIGHBOURS ALL AT ONCE!

FORMULATING OUR EXPERIMENT

```
neighbourLocations :: (Sum Int, Sum Int) -> [(Sum Int, Sum Int)]
neighbourLocations location = mappend location <$> [
    (-1, -1), (-1, 0), (-1, 1)
    , (0, -1), (0, 1)
    , (1, -1), (1, 0), (1, 1)
]
```

NOW TO TEST OUR HYPOTHESIS:

```
numLivingNeighbours :: Store (Sum Int, Sum Int) Bool -> Int
numLivingNeighbours w =
  getSum . foldMap toCount . experiment neighbourLocations $ w
where
  toCount :: Bool -> Sum Int
  toCount False = Sum 0
  toCount True = Sum 1
```

ALL SET!

```
checkCellAlive :: Store (Sum Int, Sum Int) Bool -> Bool
checkCellAlive grid =
  case (extract grid, numLivingNeighbours grid) of
    (True, 2) -> True
    (_, 3) -> True
    _ -> False
```

QUESTIONS?

NOW WE EXTEND IT TO
ALL THE CELLS!

```
step :: Store (Sum Int, Sum Int) Bool  
-> Store (Sum Int, Sum Int) Bool  
step = extend checkCellAlive
```

A NOTE ON PERFORMANCE

QUESTIONS?



CHALLENGE A: IMPLEMENT COMONAD FOR STORE

```
instance Comonad (Store s) where
```

```
    extract :: Store s a -> a
```

```
    duplicate :: Store s a -> Store s (Store s a)
```

```
    extend :: (Store s a -> b) -> Store s a -> Store s b
```

CHALLENGE B: ADD A NEW RULESET FOR CONWAY'S GAME OF LIFE

END

```
instance Comonad (Store s) where  
    extract :: Store s a -> a  
    extract (Store f s) = f s
```

```
    duplicate :: Store s a -> Store s (Store s a)  
    duplicate (Store f s) =  
        Store (\s' -> Store f s') s
```

```
    extend :: (Store s a -> b) -> Store s a -> Store s b  
    extend g st = g <$> duplicate st
```

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github.com/ChrisPenner
@ChrisLPenner



BONUS

EXTEND LET'S US WRITE FUNCTION MIDDLEWARE

```
-- Precompose
normalizePath :: Store T.Text T.Text -> T.Text
normalizePath = peeks cleaned
where
  cleaned url = T.strip . T.toLower $ url

-- Postcompose
collapseSlashes :: Store T.Text T.Text -> T.Text
collapseSlashes = T.replace "//" "/" . extract

-- Wrapping transform
showTransformation :: Store T.Text T.Text -> T.Text
showTransformation w = pos w <> ": " <> extract w

badUrl :: T.Text
badUrl = " g00gLe.com//a/B//c "

urlTransformer :: Store T.Text T.Text
urlTransformer = store id "" =>> collapseSlashes =>> normalizePath =>> showTransformation
```