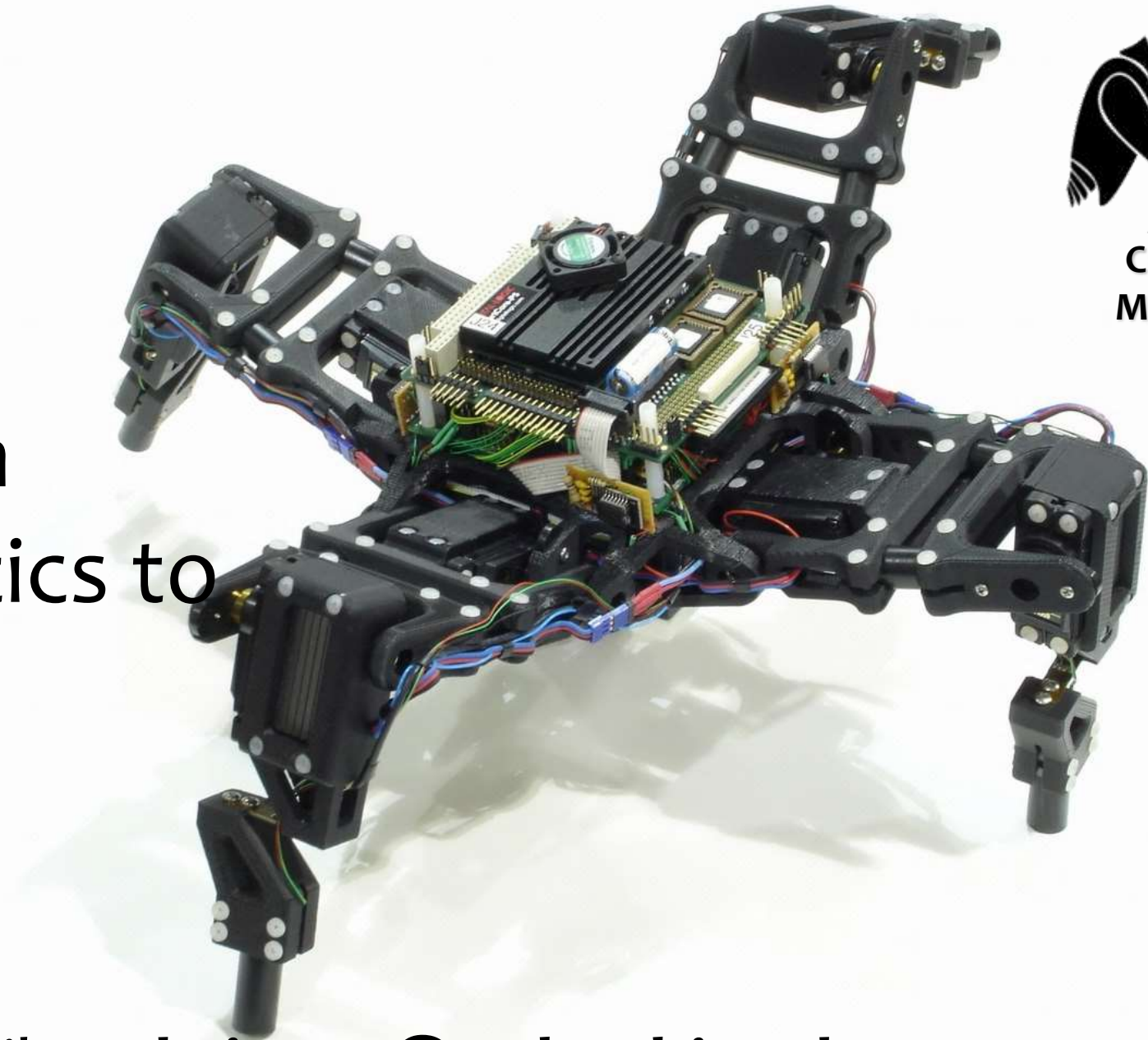


# The AI Scientist

Automating  
discovery, from  
cognitive robotics to  
computational  
biology



Creative  
Machines  
Lab

For copy of slides email **Hod.Lipson@Columbia.edu**









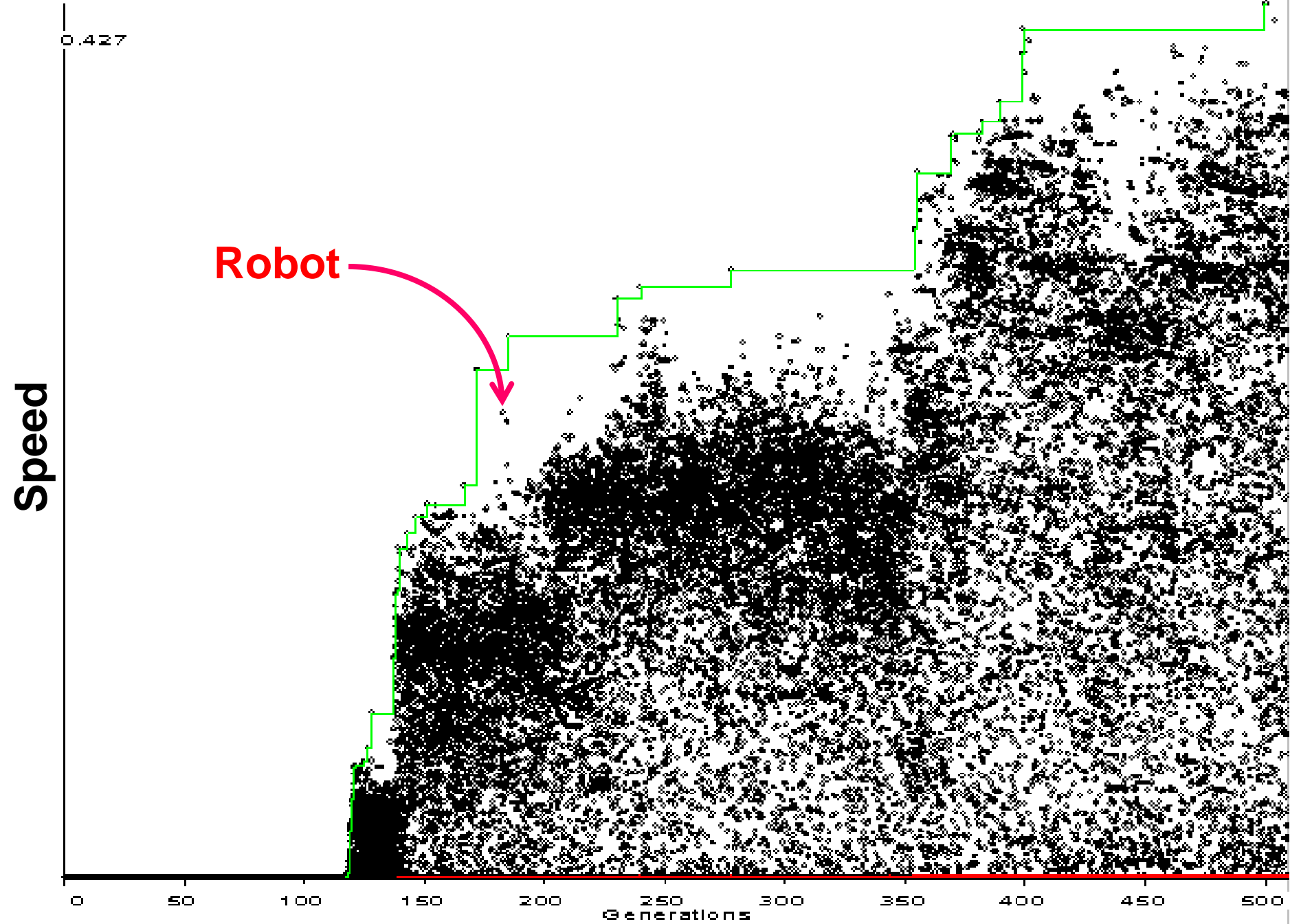
# Evolution



(







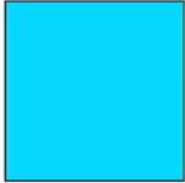




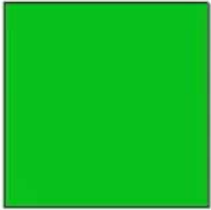




**Muscle:** contract then expand



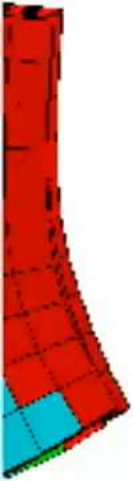
**Tissue:** soft support



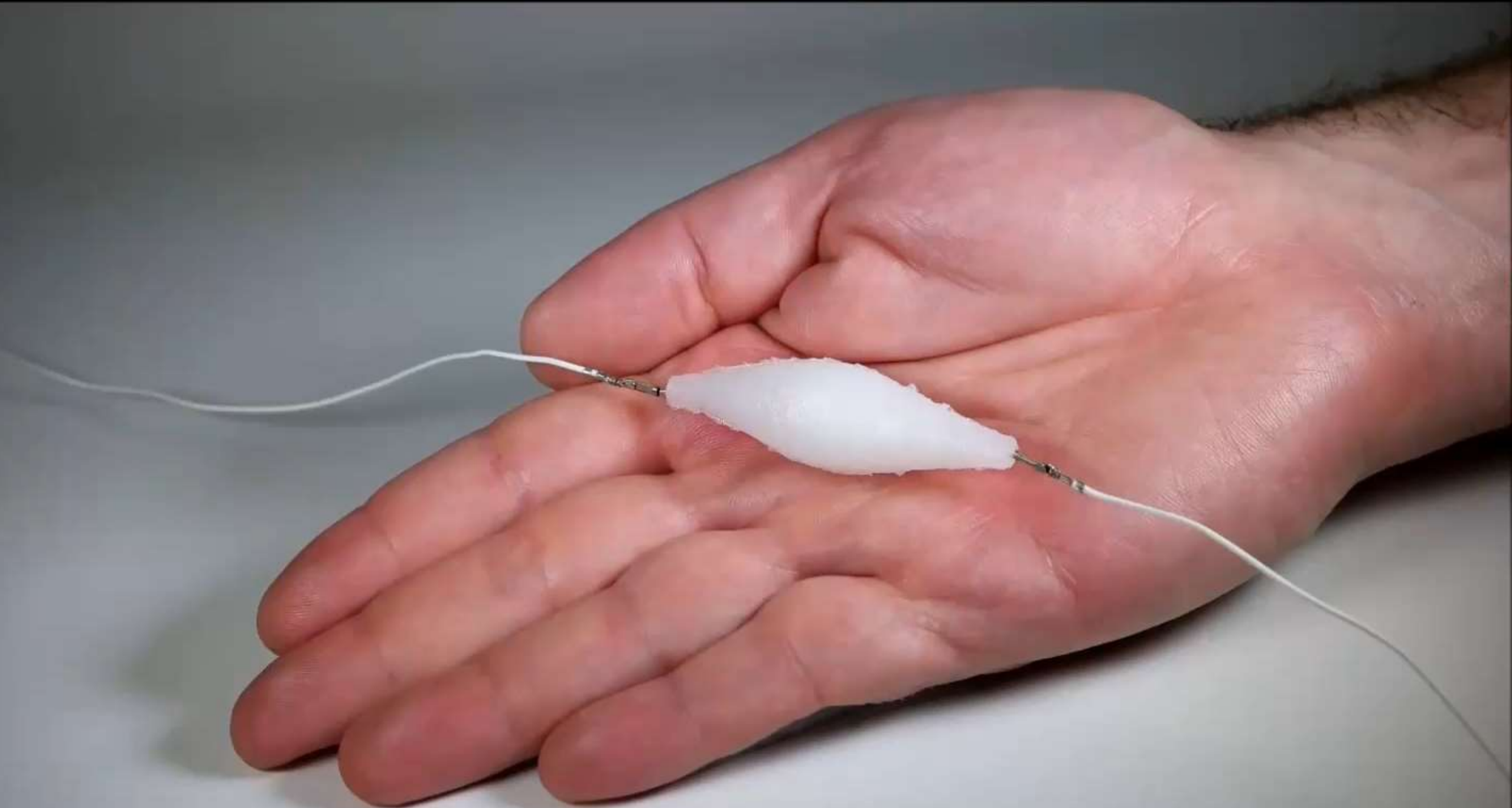
**Muscle2:** expand then contract

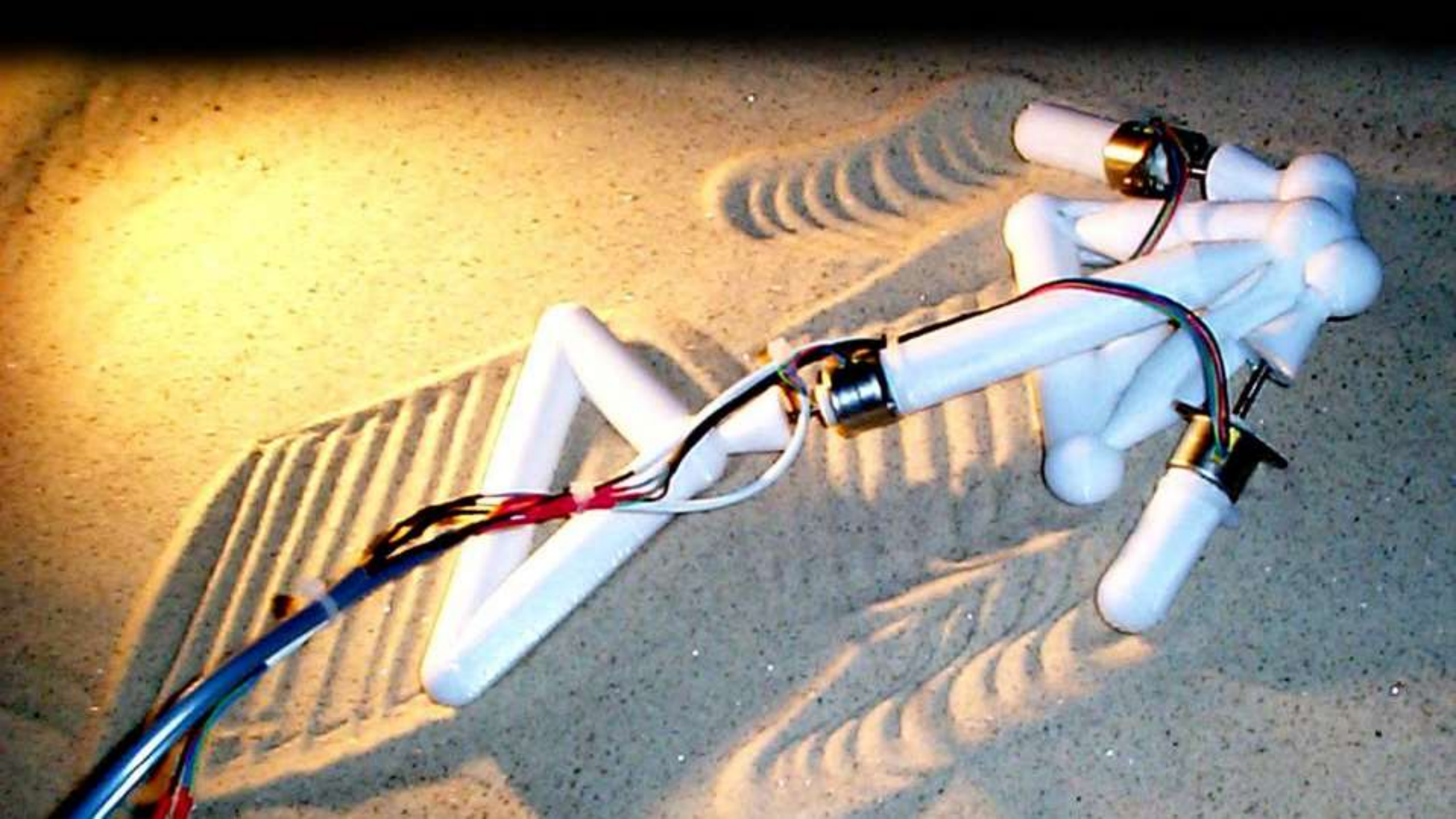


**Bone:** hard support











# The New York Times

THURSDAY, AUGUST 31, 2000

## Scientists Report They Have Made Robot That Makes Its Own Robots

By KENNETH CHANG

For the first time, computer scientists have created a robot that designs and builds other robots, almost entirely without human help.

In the short run, this advance could lead to a new industry of inexpensive robots customized for specific tasks. In the long run — decades at least — robots may one day be truly regarded as “artificial life,” able to reproduce and evolve, building improved versions of themselves.

Such durable, adaptive robots, astronomers have suggested, could someday be sent into space to explore the galaxy or search for other life.

But the quest to create artificial



Brandeis University

The “Arrow” left a trail as it crawled across a bed of sand.

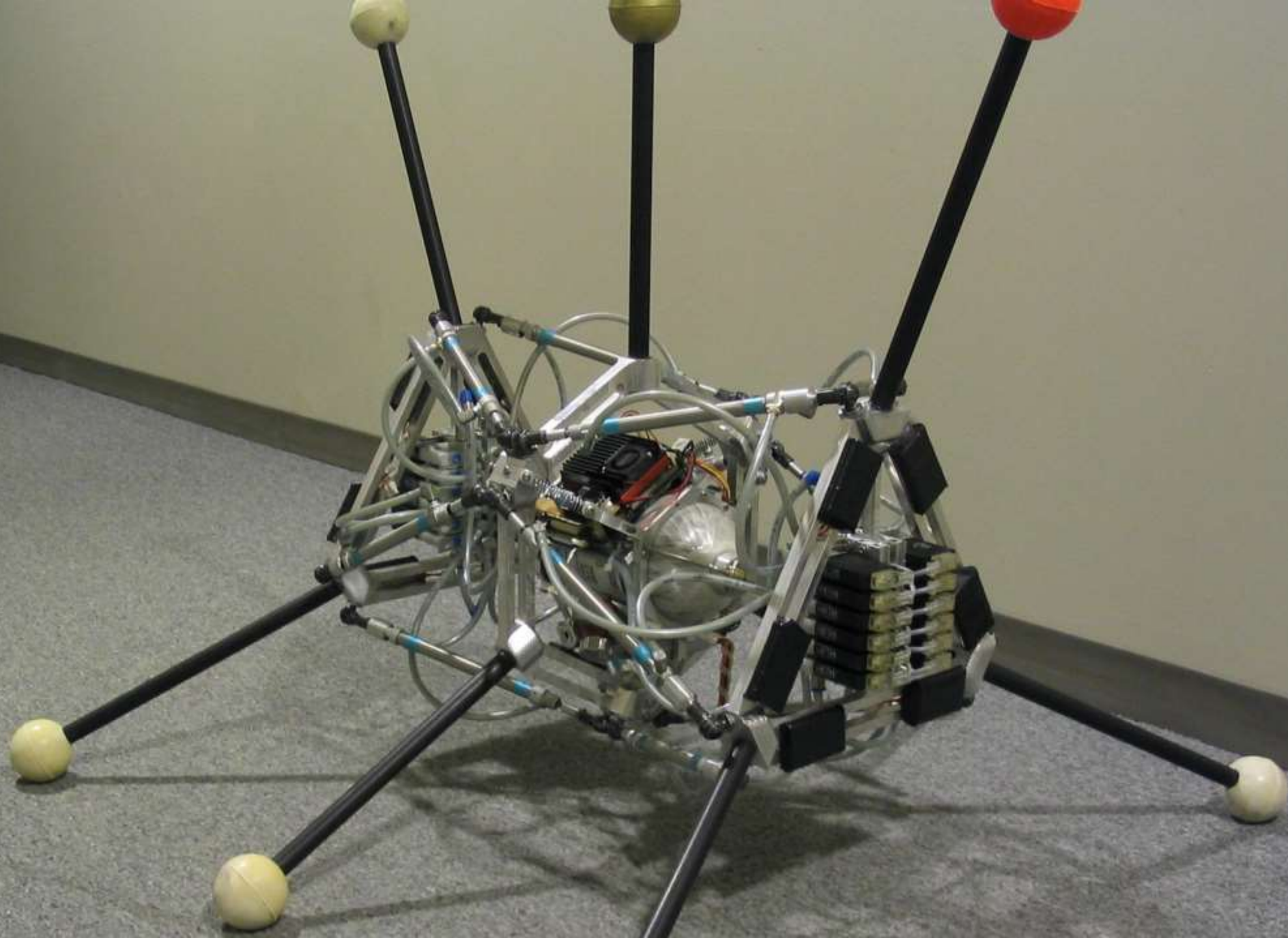
were not manufactured by humans.” Dr. Pollack and Dr. Lipson, a research scientist, report their results in today’s issue of the journal *Nature*.

“This is the first example of a robot

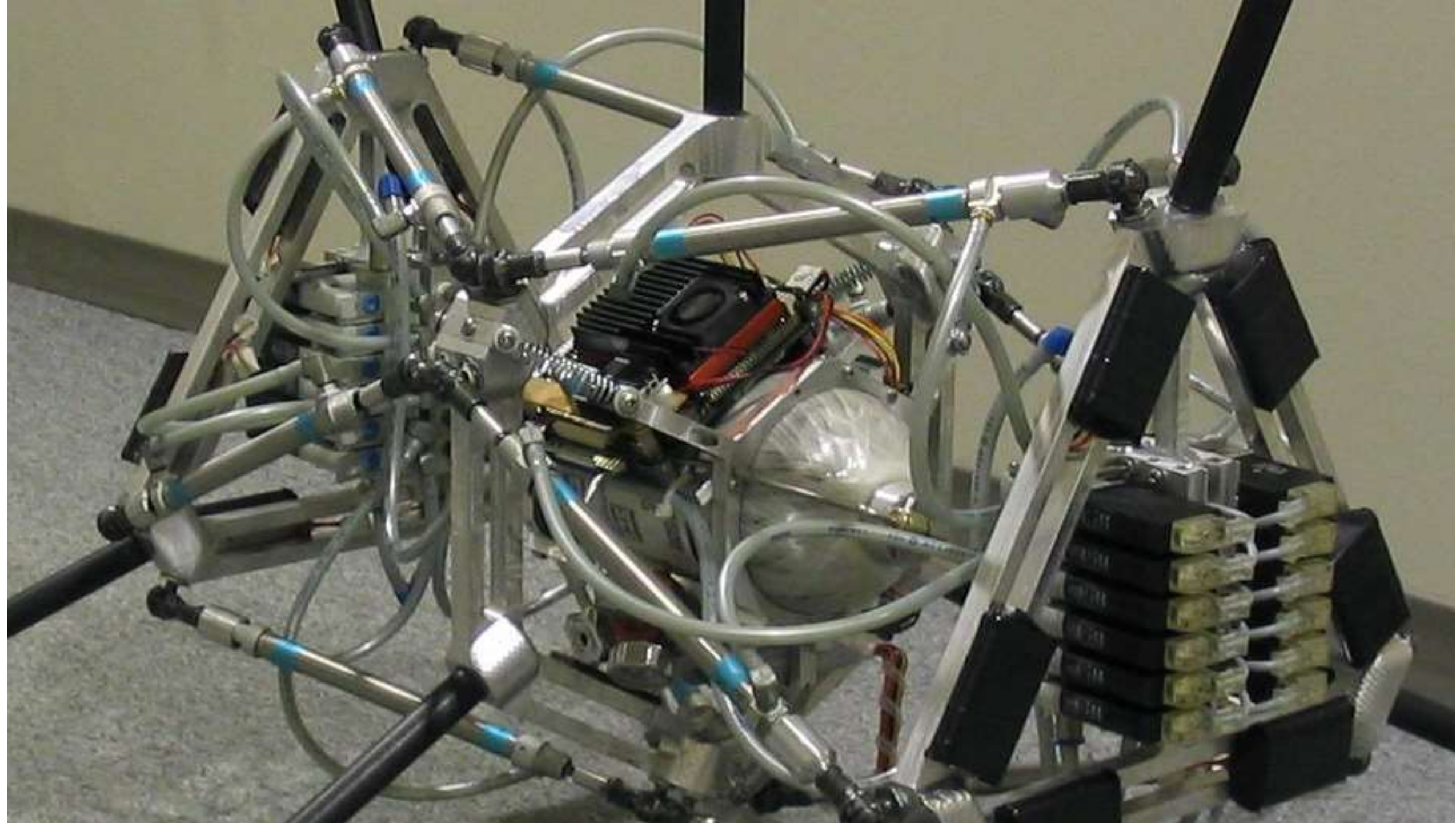




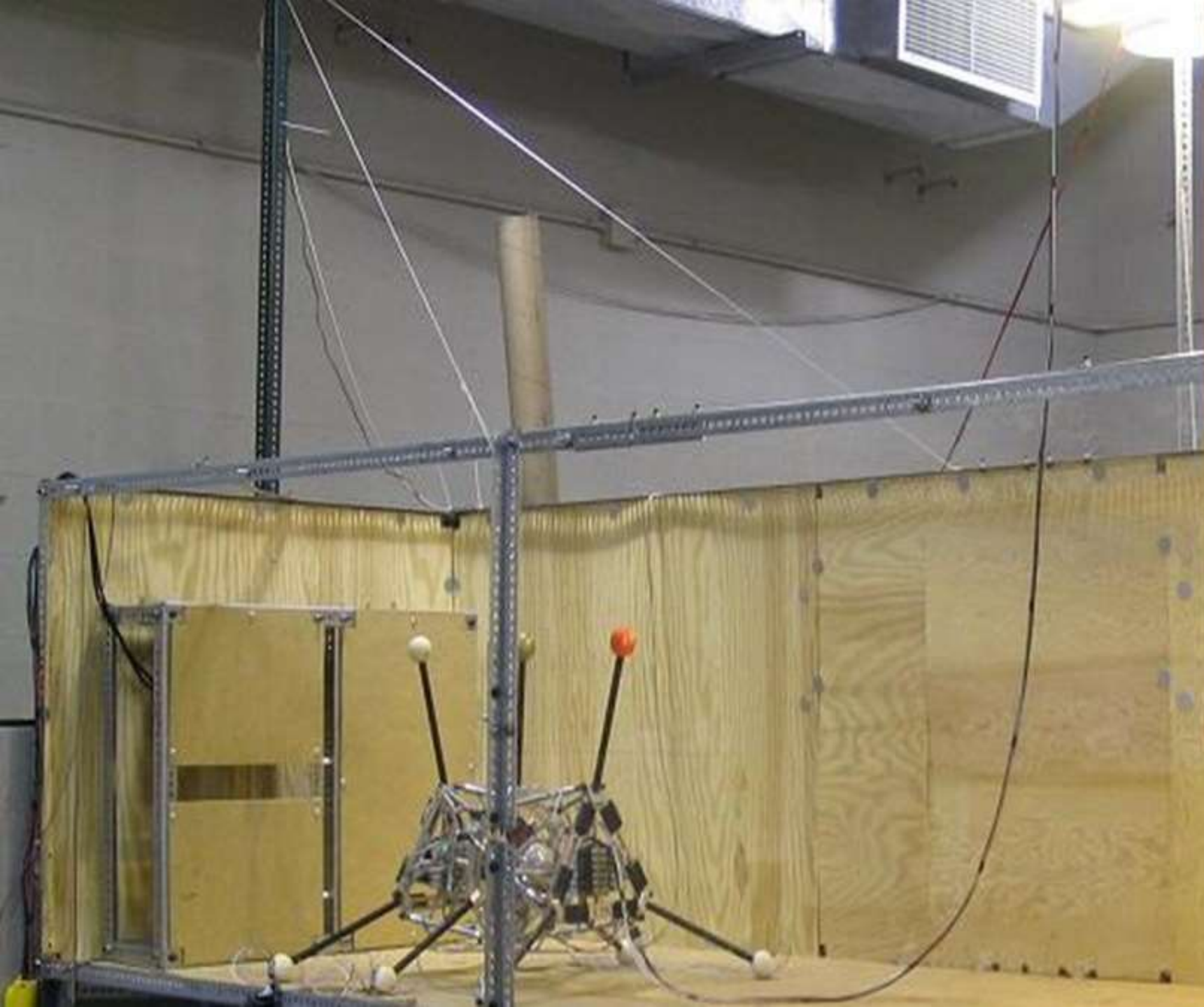










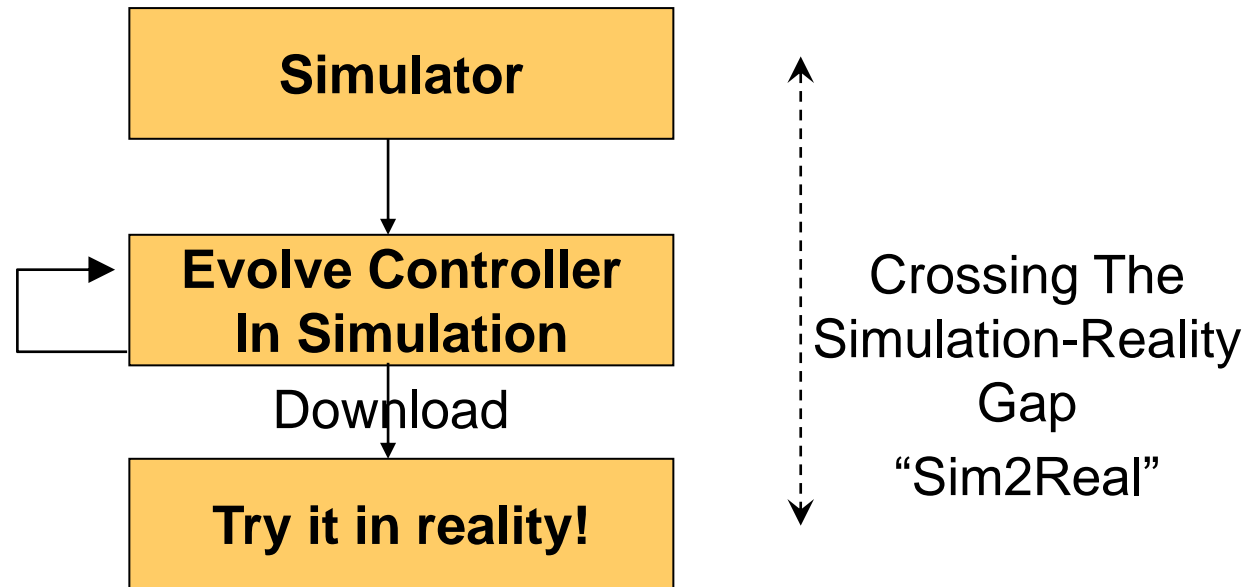




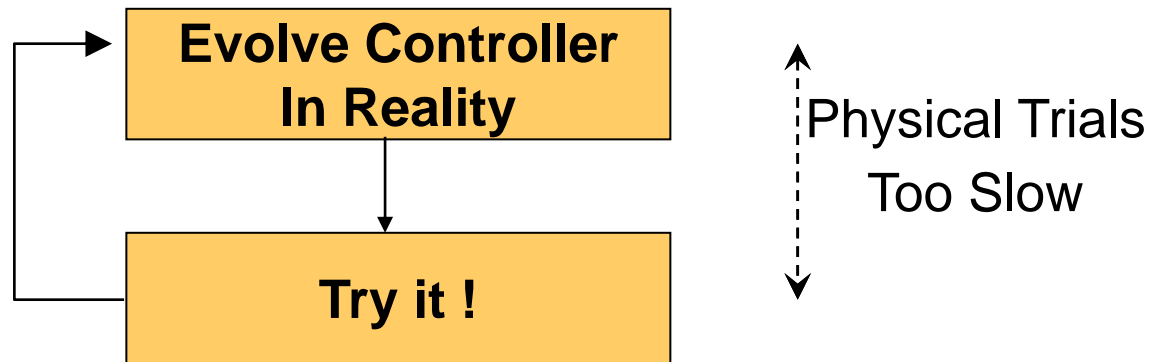


With Viktor Zykov and Josh Bongard

# Adapting in simulation



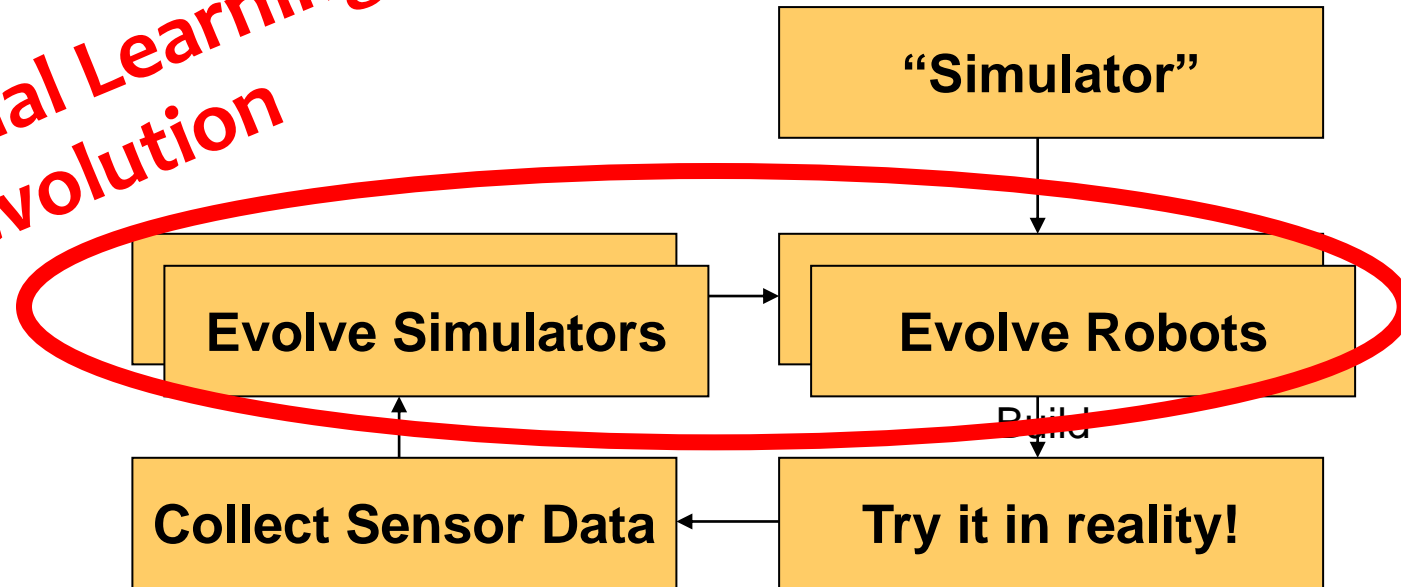
# Adapting in reality

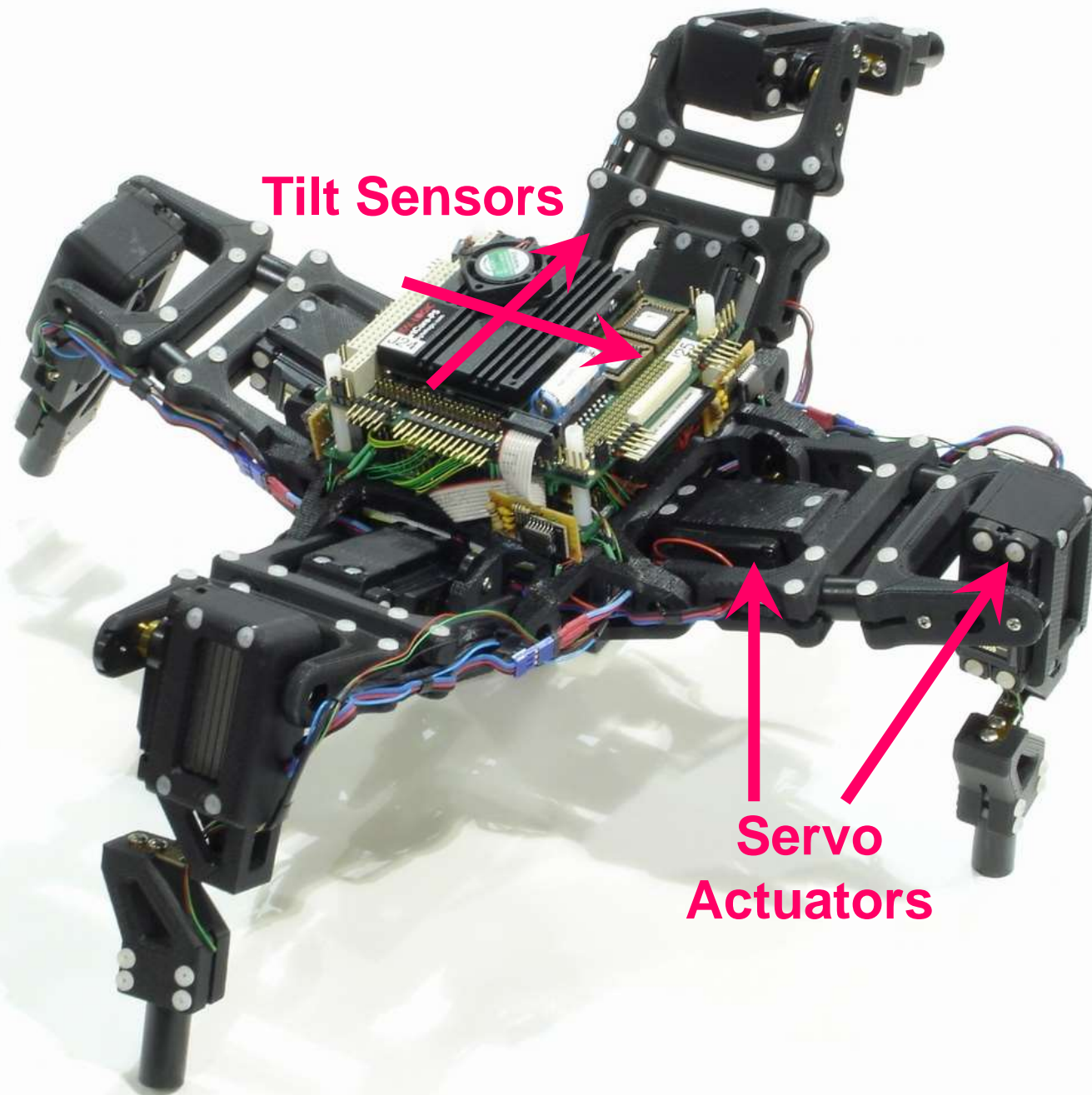




# Simulation & Reality

**Adversarial Learning  
Co-evolution**

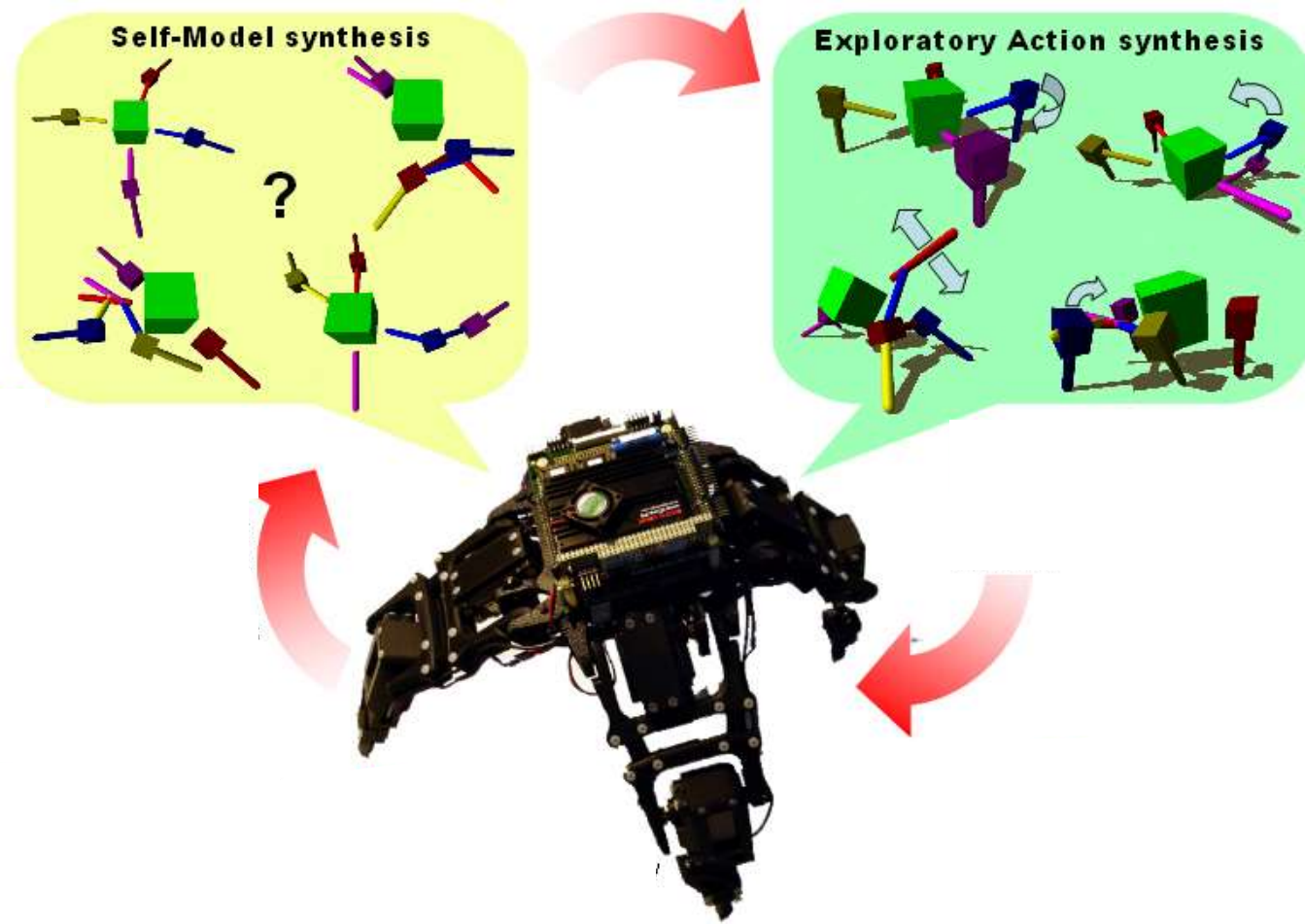


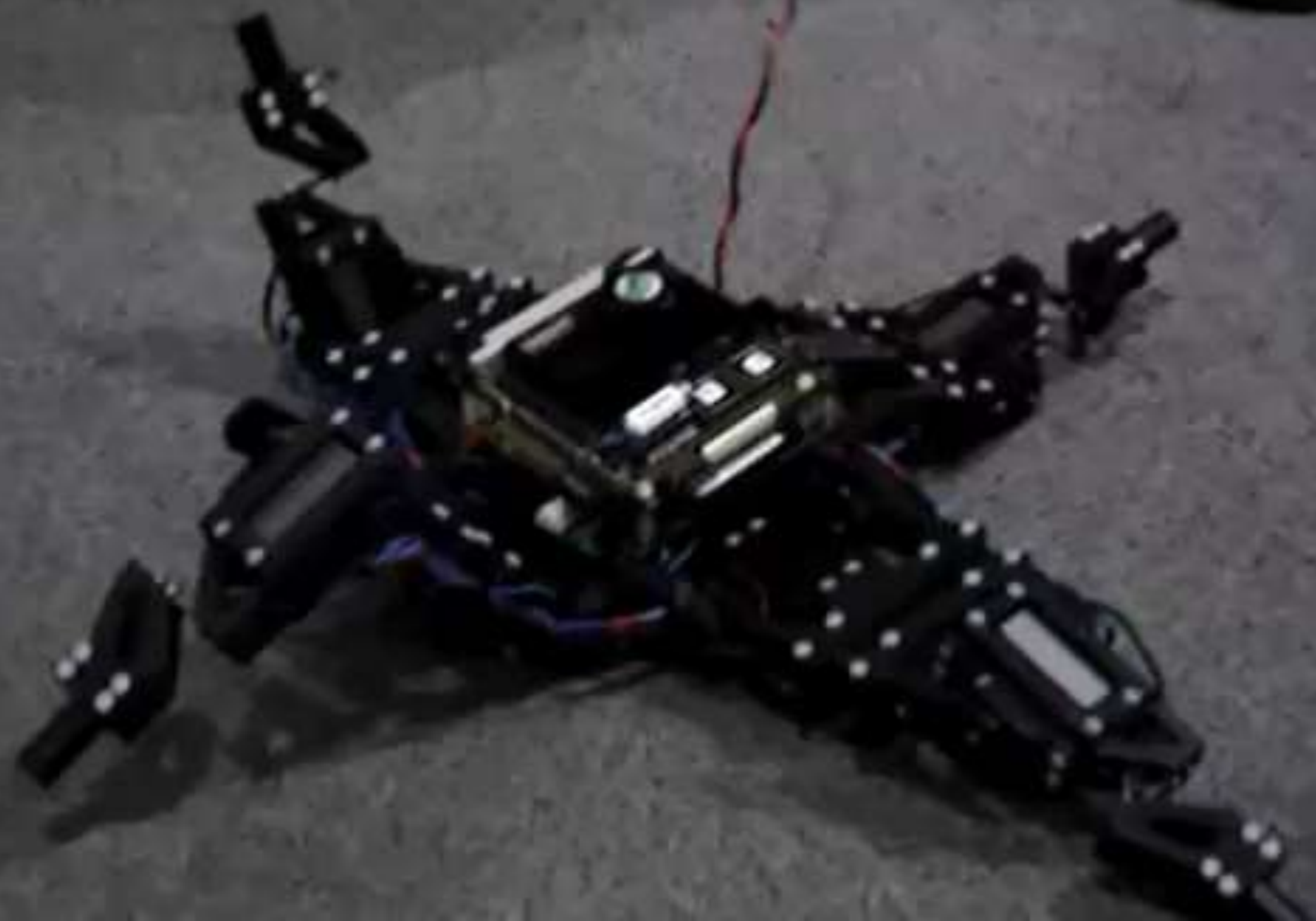


**Tilt Sensors**

**Servo  
Actuators**









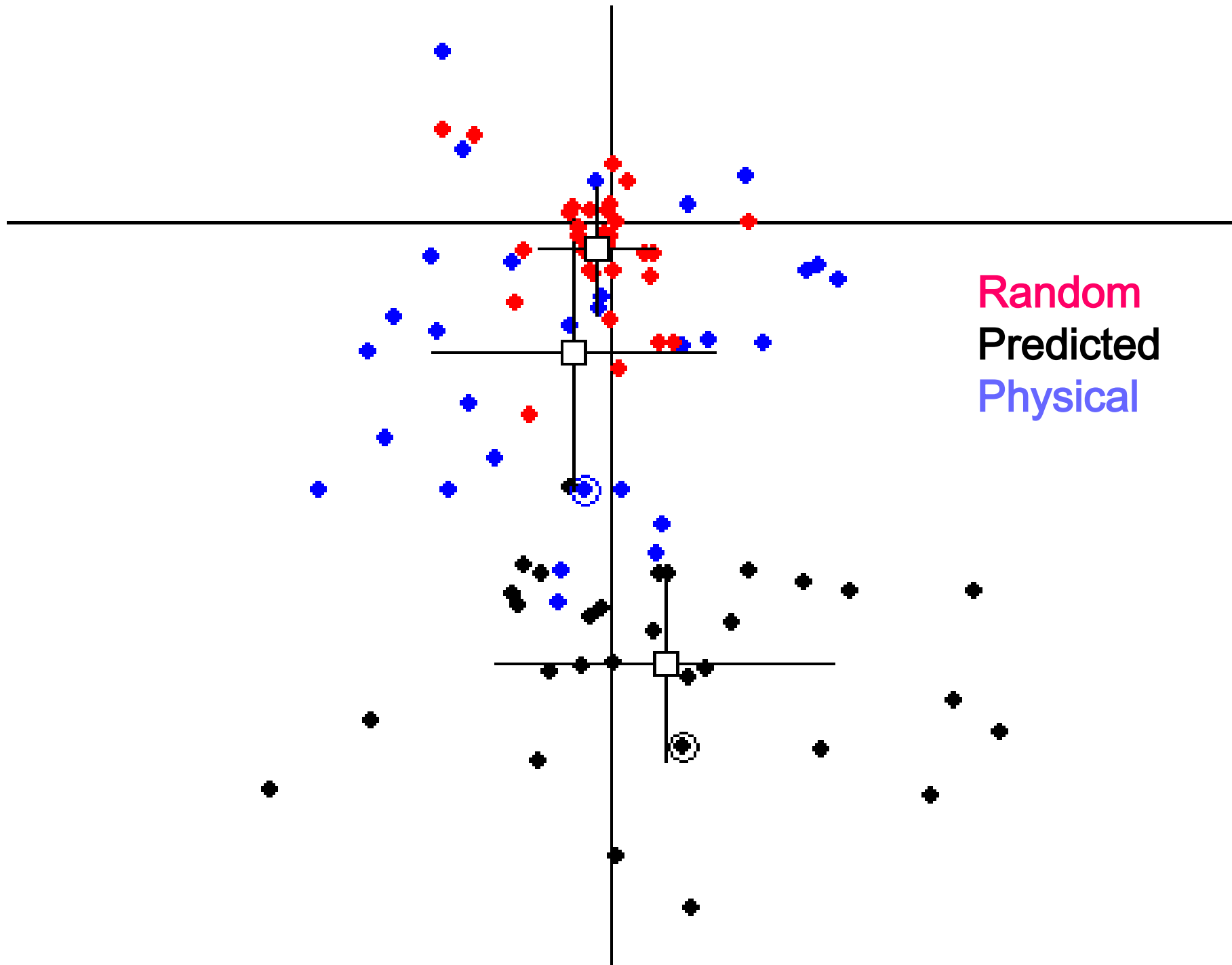
# Emergent Self-Model



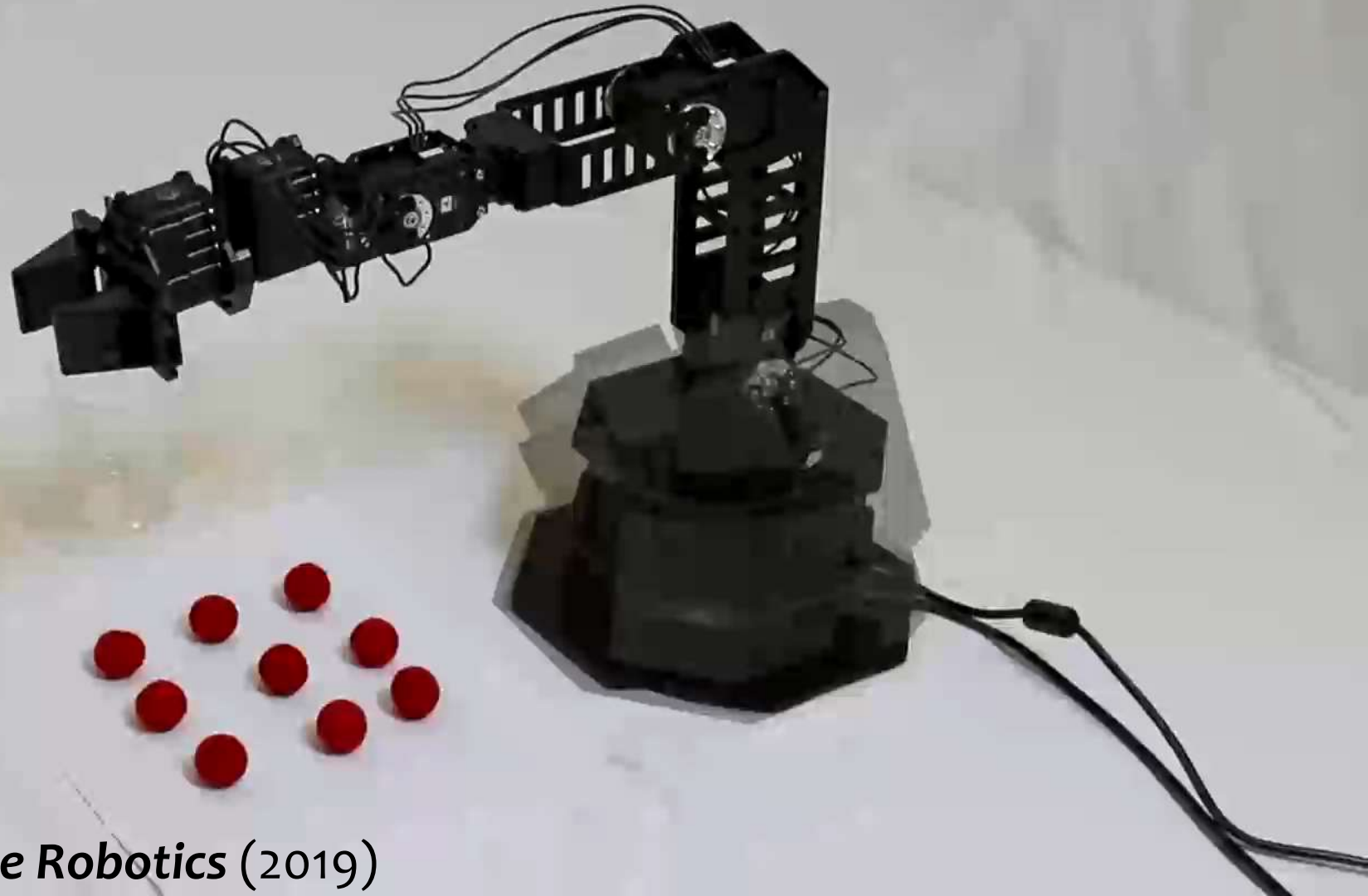
First cycle (of 16)



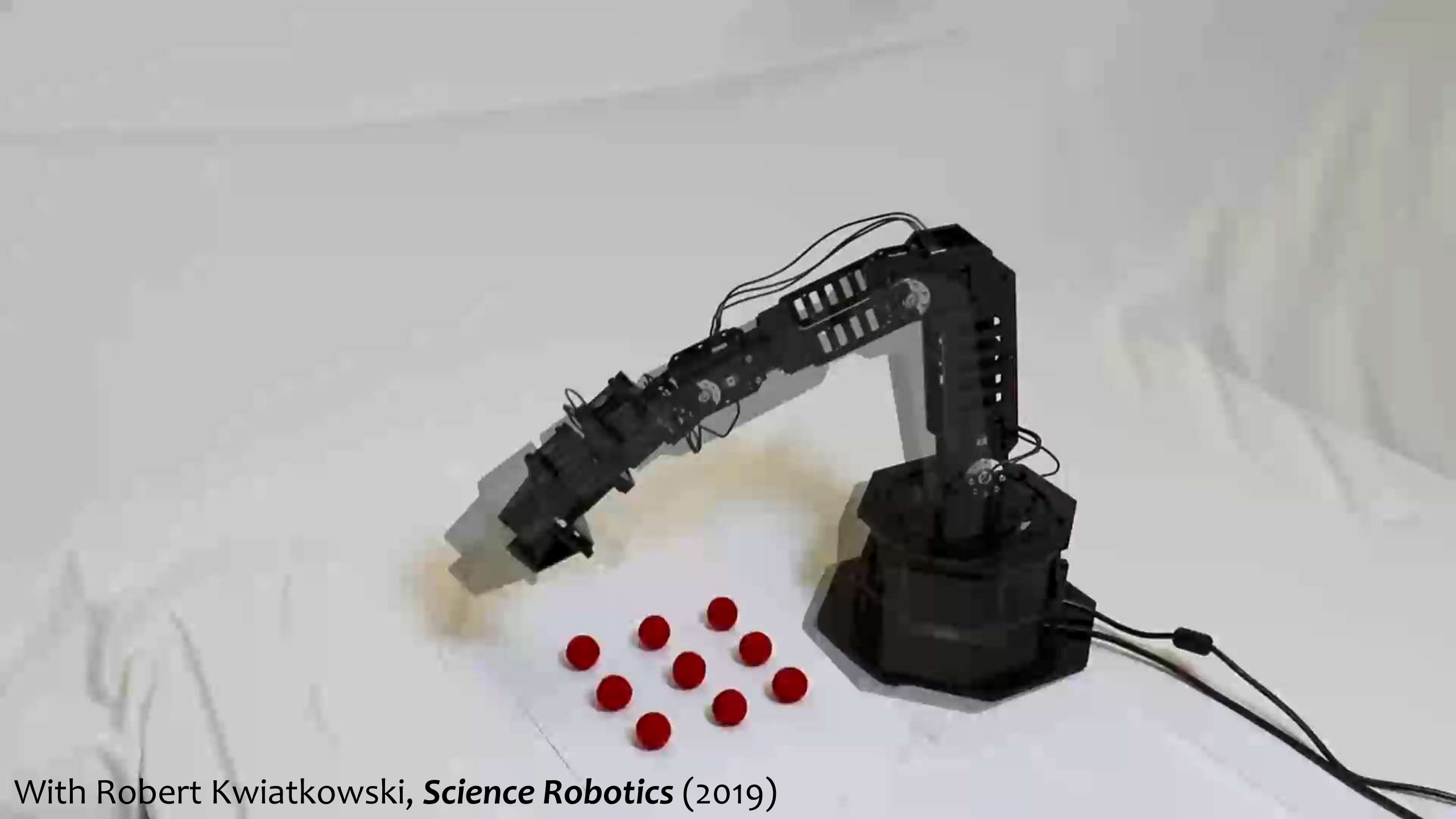
With Josh Bongard and Victor Zykov, **Science**



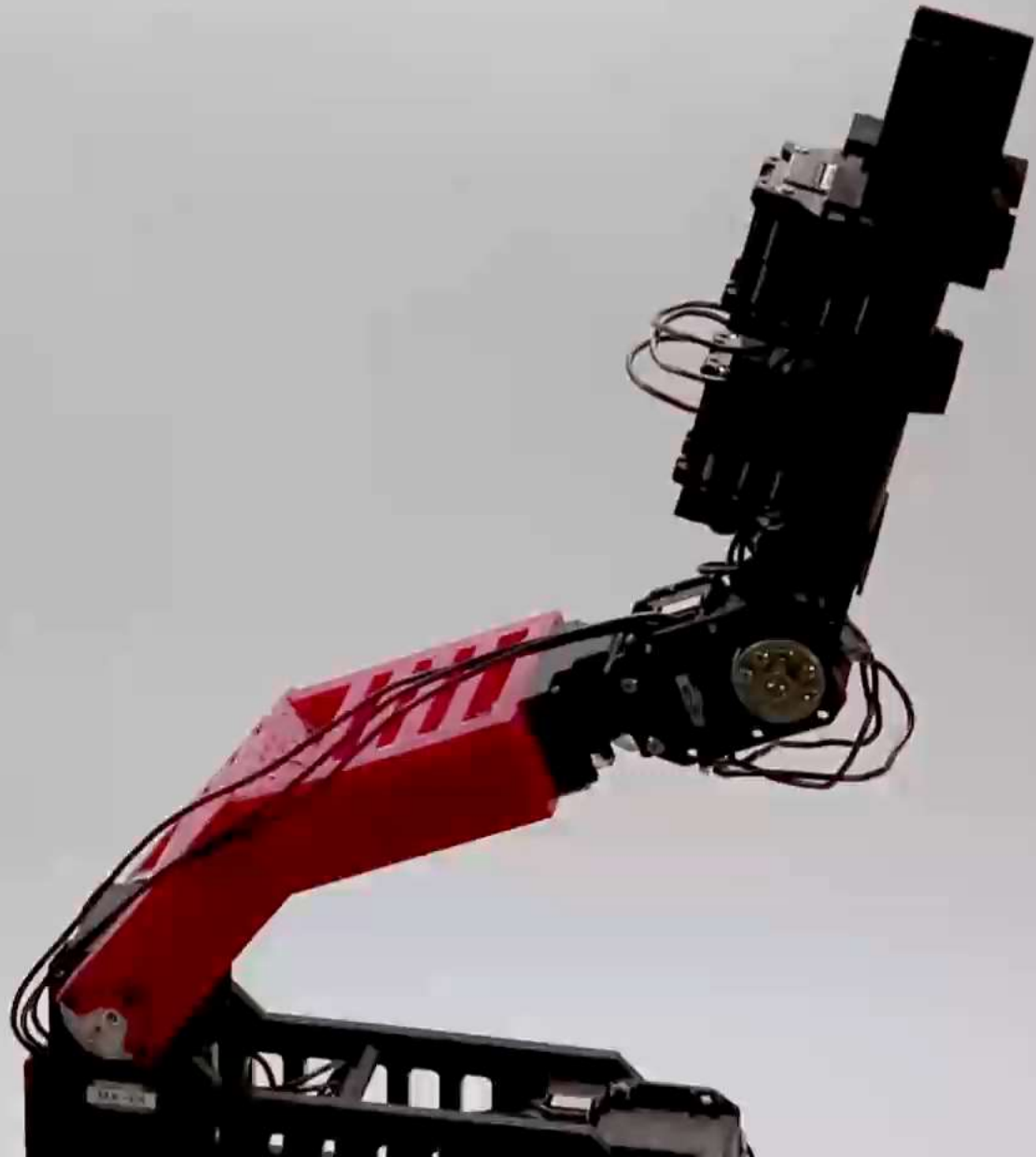




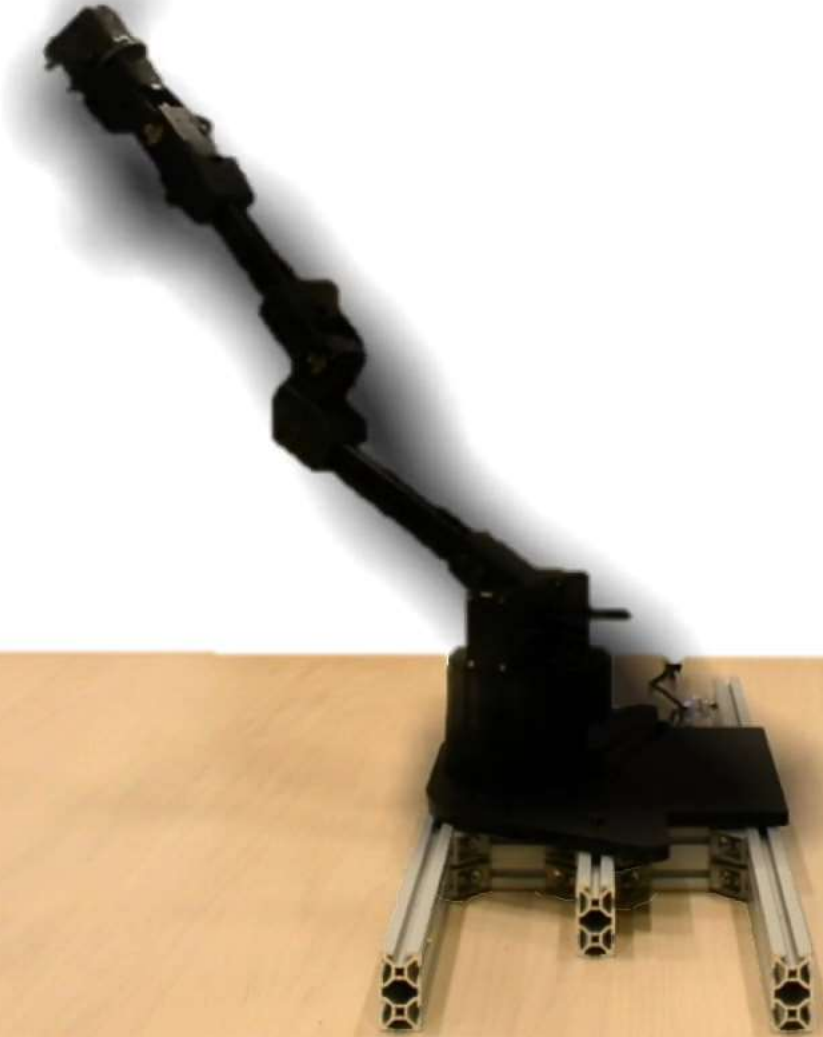
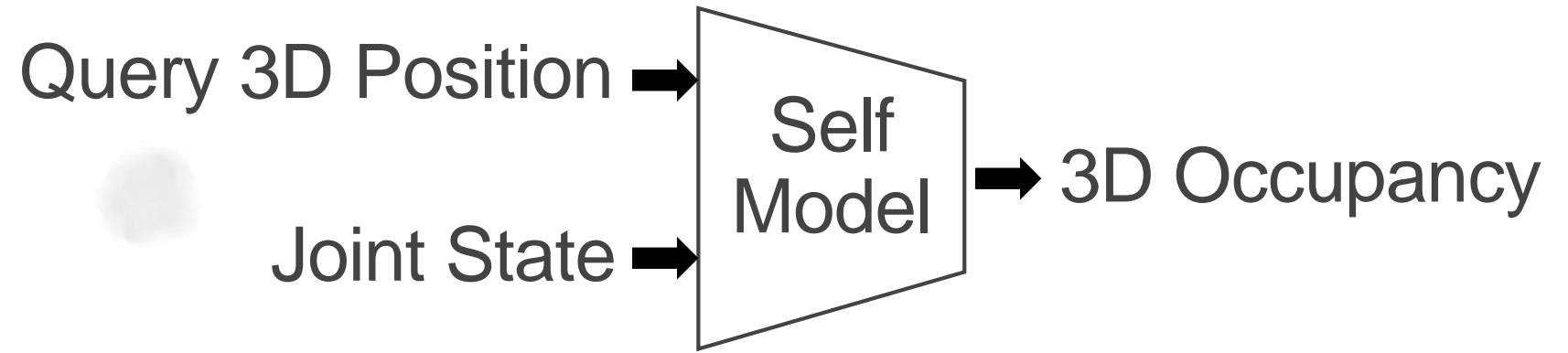
With Robert Kwiatkowski, ***Science Robotics*** (2019)



With Robert Kwiatkowski, *Science Robotics* (2019)





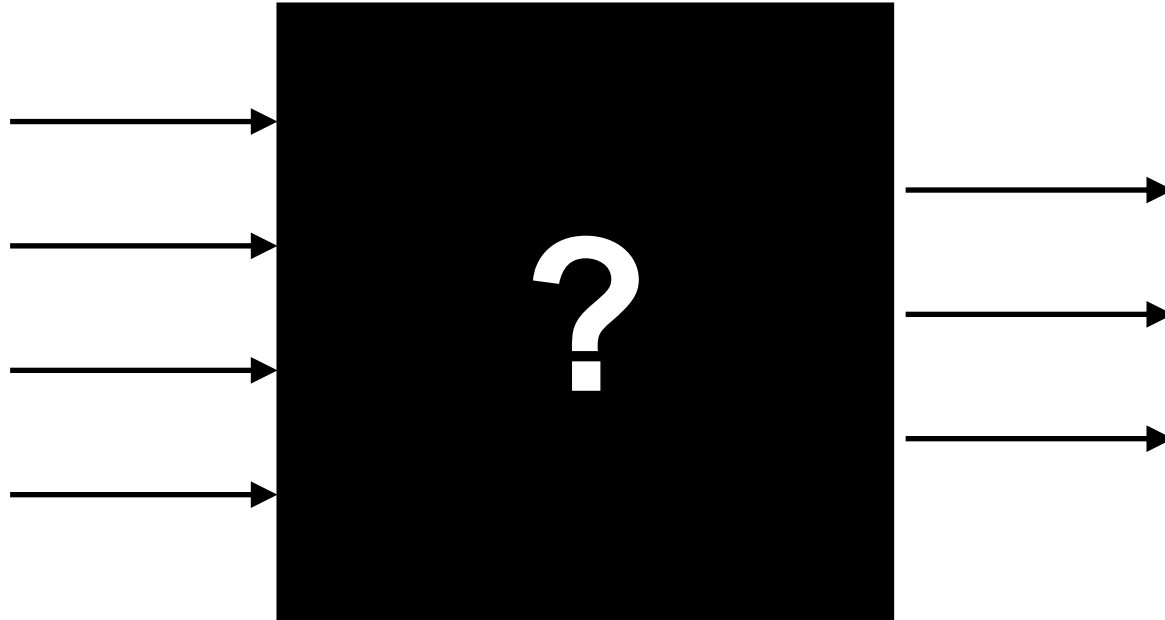








# System Identification

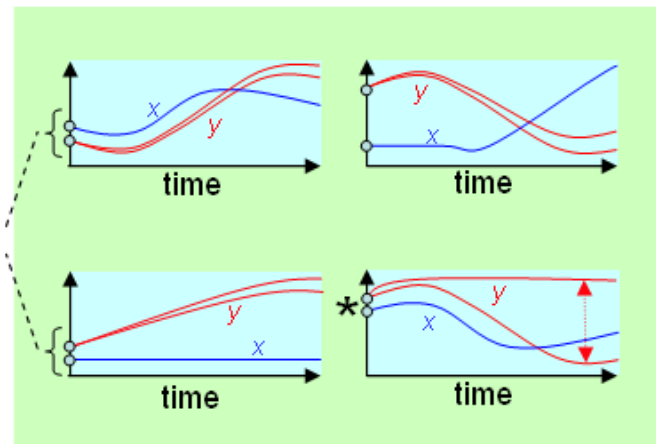


## Candidate models

$$\begin{cases} \frac{dx}{dt} = -2y^2 + \log x \\ \frac{dy}{dt} = -x + \frac{y}{6} \end{cases} \quad ? \quad \begin{cases} \frac{dx}{dt} = -\sqrt{y} + \frac{x}{5} \\ \frac{dy}{dt} = -\sin y \end{cases}$$
$$\begin{cases} \frac{dx}{dt} = -3\frac{y+1}{y-1} \\ \frac{dy}{dt} = -\frac{x^2}{x^2+1} \end{cases} \quad \begin{cases} \frac{dx}{dt} = -y^{1.8} + \log x \\ \frac{dy}{dt} = -x + \frac{y}{4x} \end{cases}$$

## Candidate tests

Candidate  
Initial  
conditions

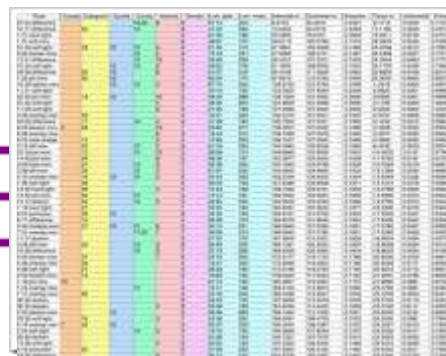


**c**

Inference Process



Outputs  
(sensors)



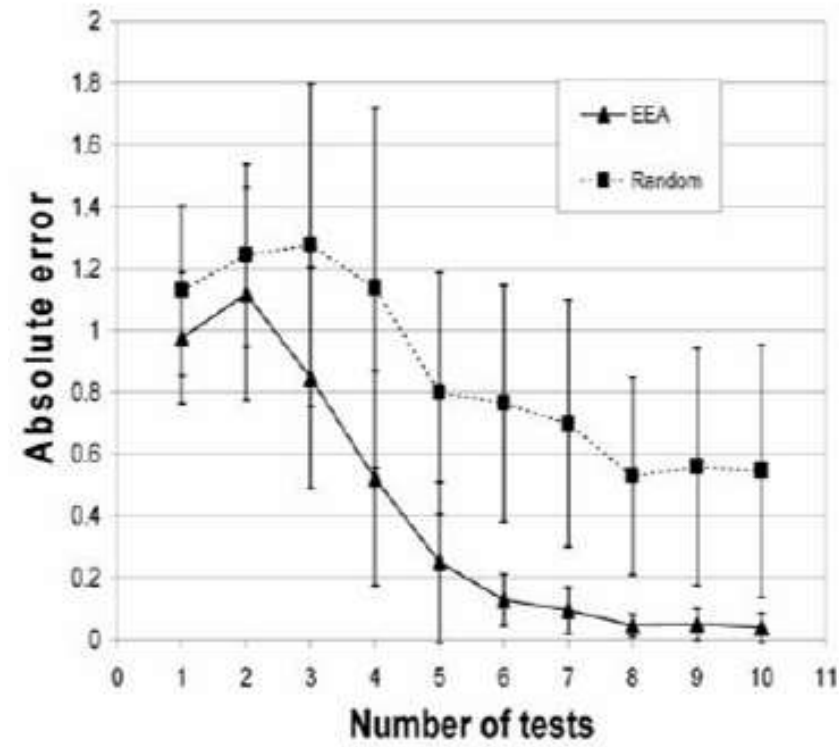
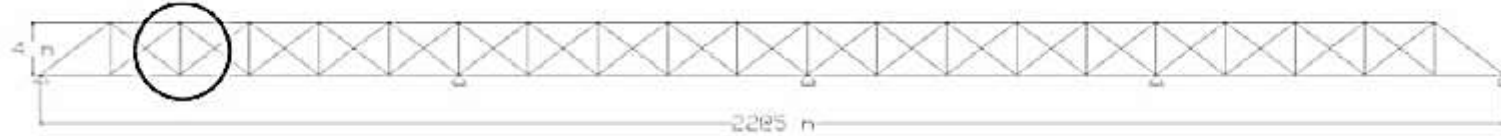
Initial  
Conditions  
(actuators)

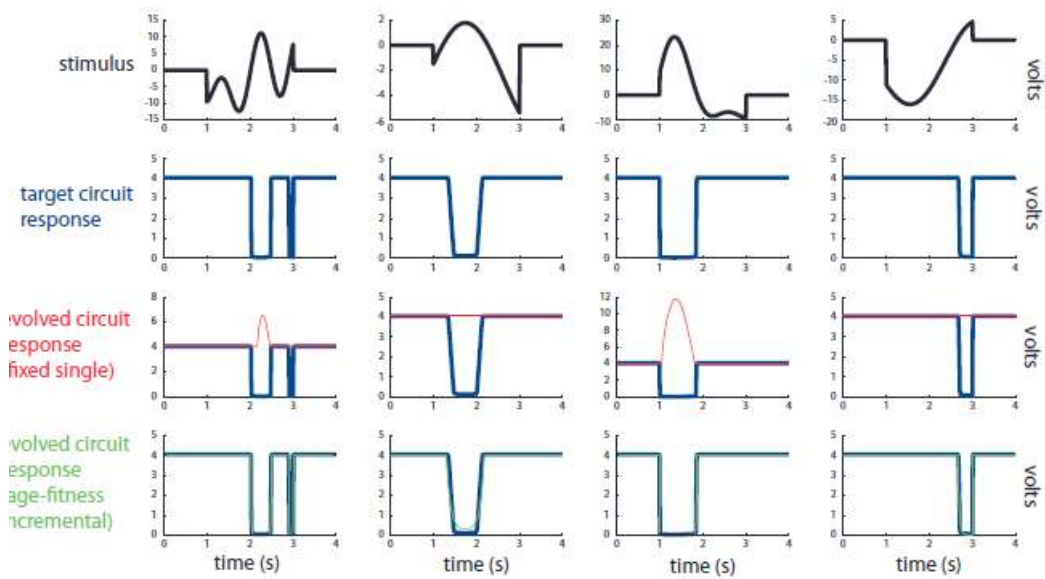
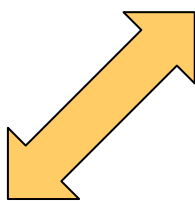
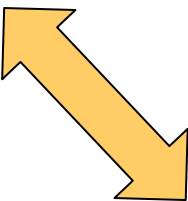
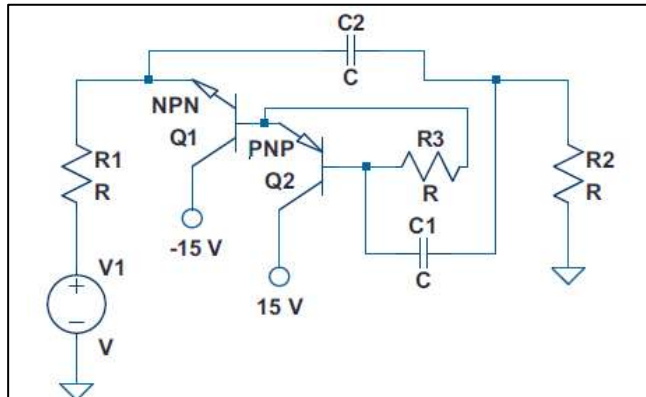
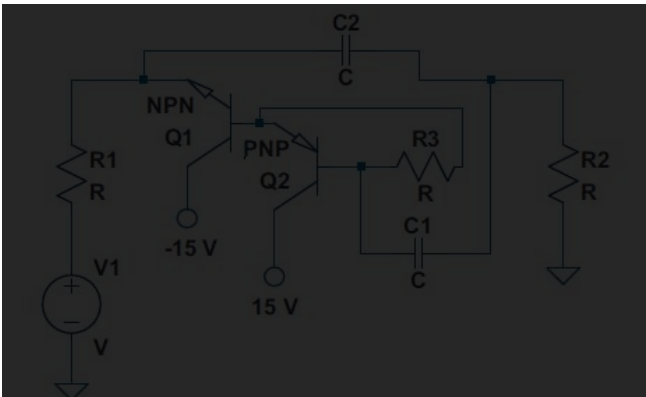






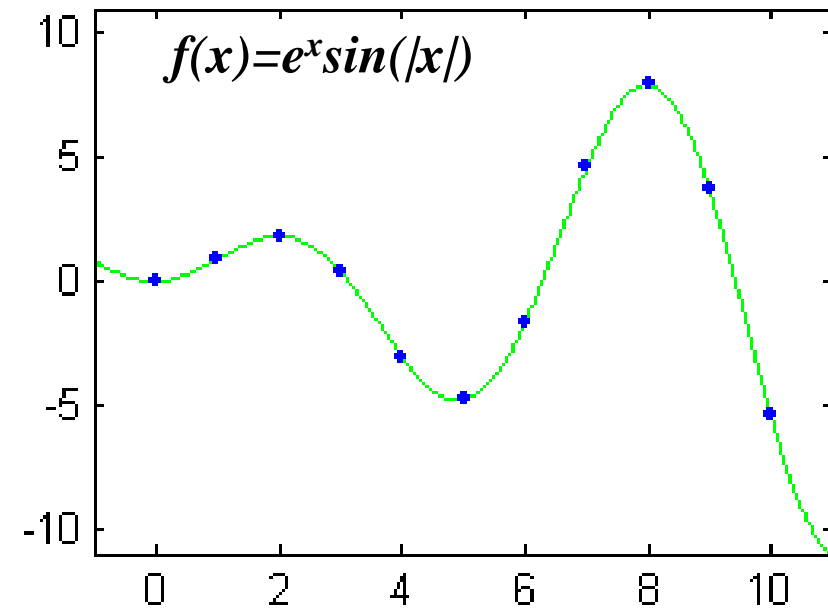
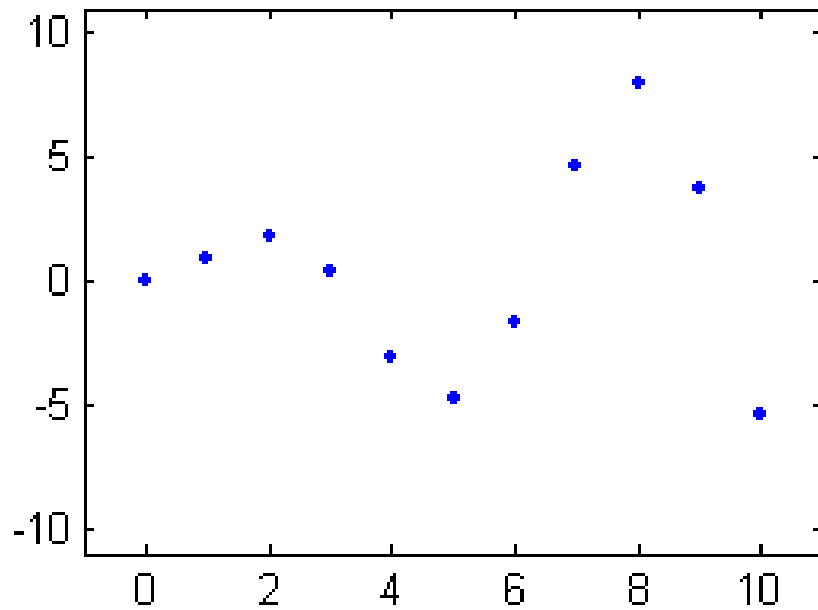
# Structural Damage Diagnosis



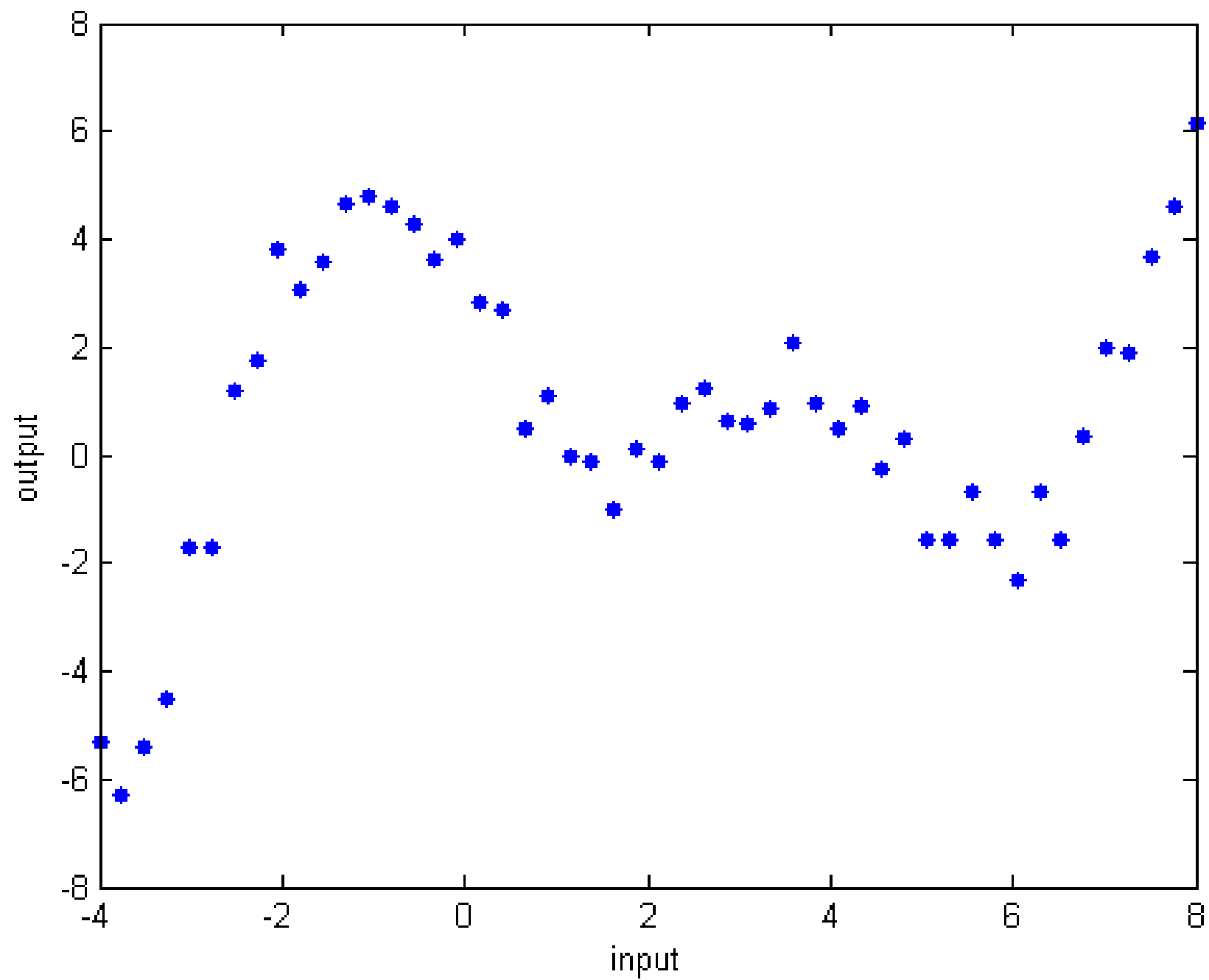


# Symbolic Regression

**What function describes this data?**

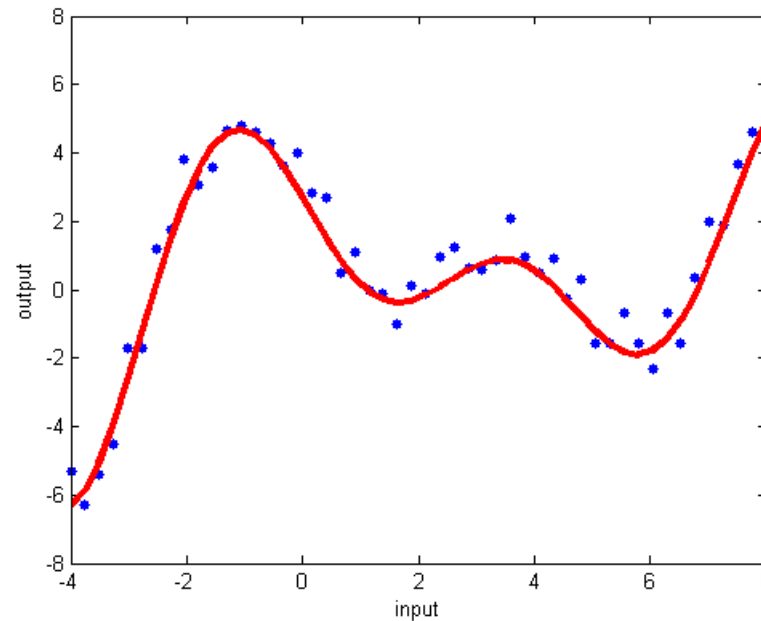
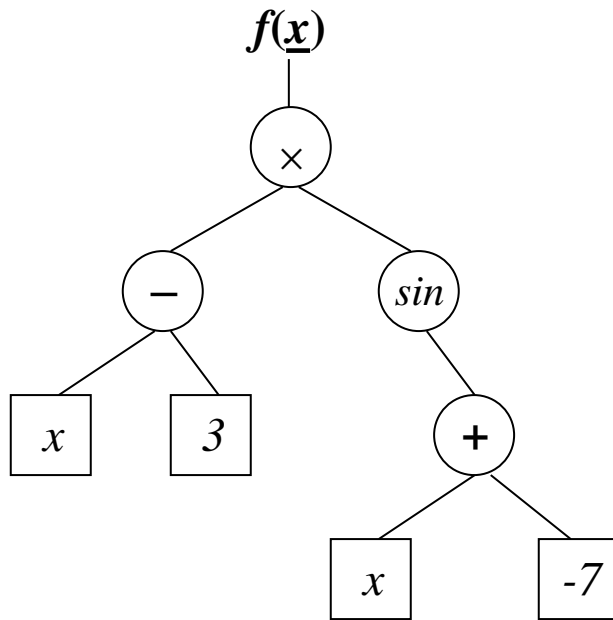






# Encoding Equations

**Building Blocks:** + - \* / sin cos exp log ... etc

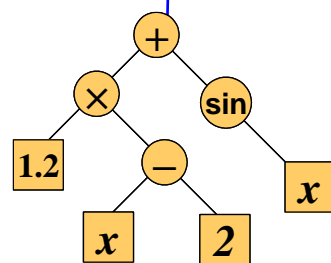
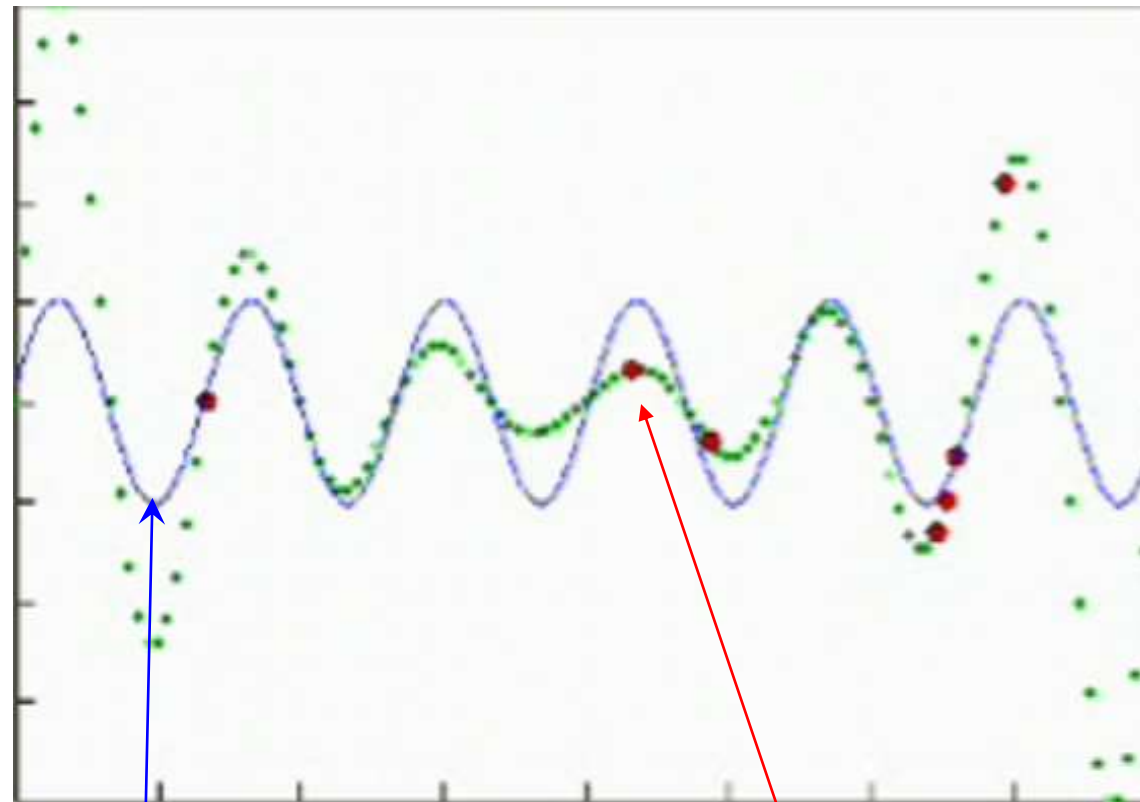


$\sin(x_2)$

$x_1 * \sin(x_2)$

$(x_1 - 3) * \sin(x_2)$

$(x_1 - 3) * \sin(-7 + x_2)$



**Models:** Expression trees  
Subject to mutation and selection

**Experiments:** Data-points  
Subject to mutation and selection

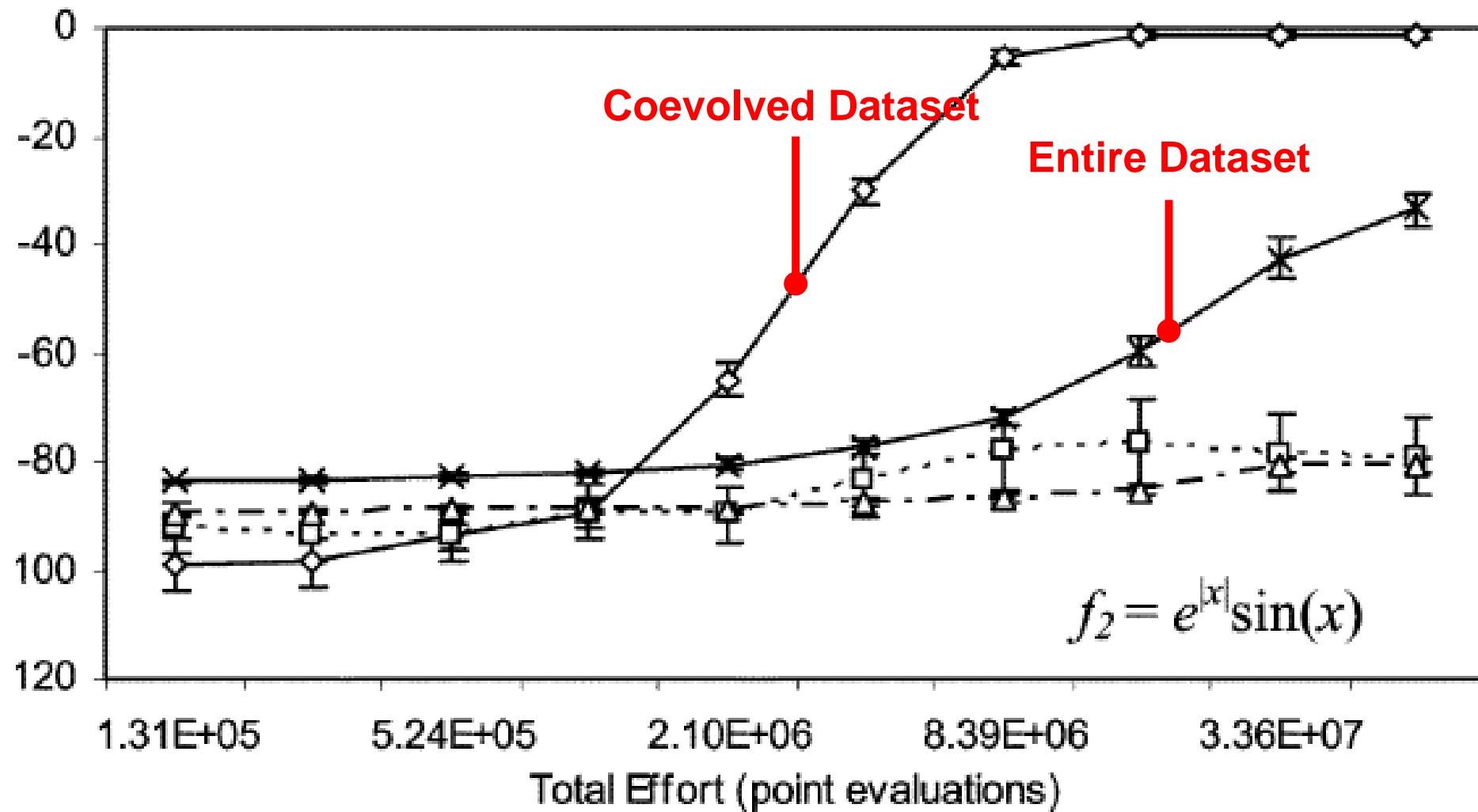
{const, +, -, \*, /, sin, cos, exp, log, abs}



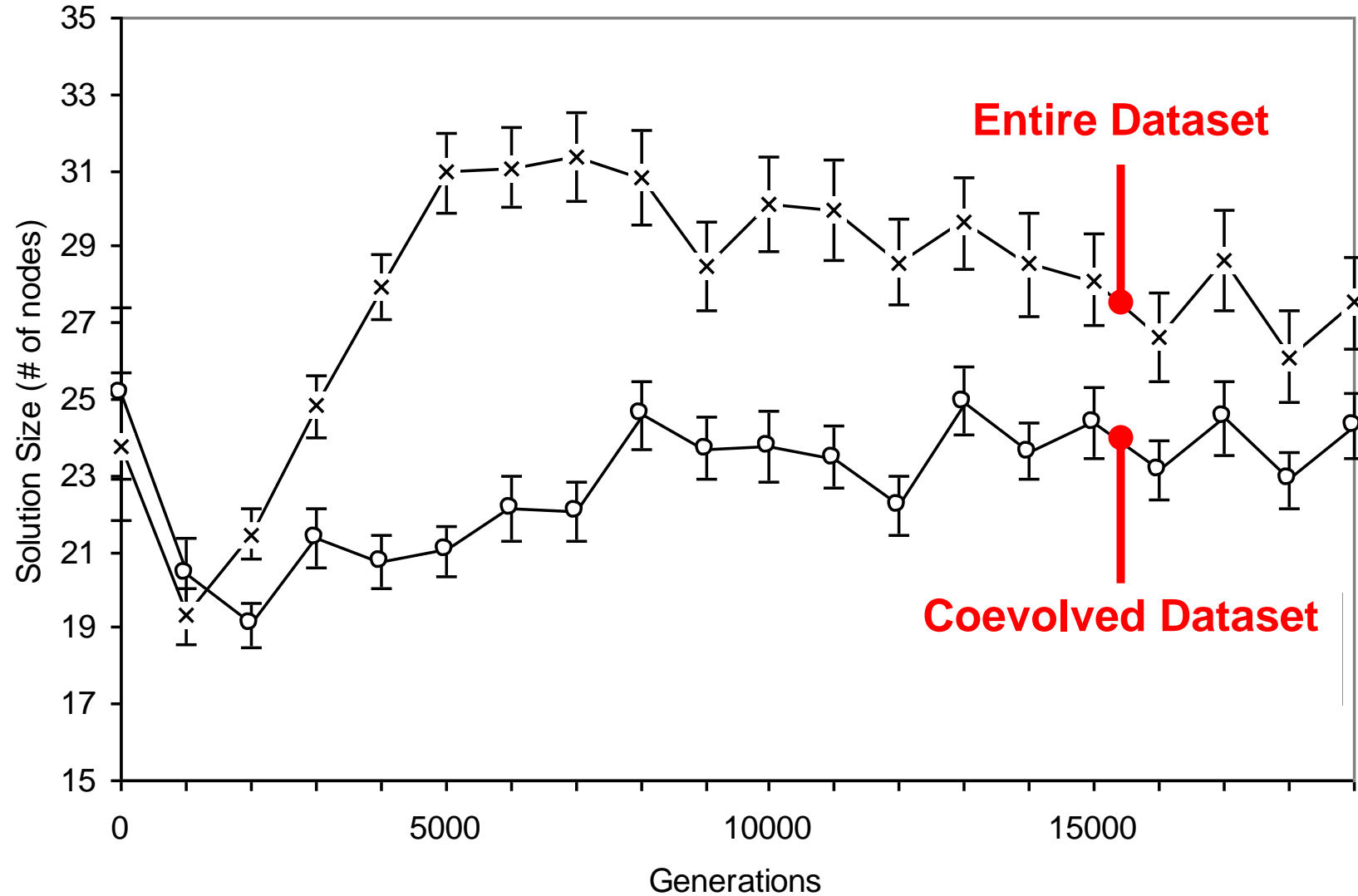
Item	Value	Units
Running Time		
Solver Generations		
Fitness Model Generations		
Local Fitness Calculations		
Model Fitness Evaluations		
Total FPEE Evaluations		

```
((0:48:04) -----
(15:48:26) OverlaidSummary: Symbolic Reasoner v2.1
(15:48:36) by:
(15:48:36) Michael Schmidt, Odr57@cornell.edu
(15:48:36) Rod Laporte, Rod.Laporte@cornell.edu
(15:48:36) Cornell University (2006)
(15:48:36) -----
(15:48:36) Seed=03263025
```

# Solution Accuracy



# Solution Complexity





A close-up, angled view of a digital stock market display. The screen is black with green and red numbers indicating price changes. The numbers are arranged in columns, with some larger numbers like '150000' and '120000' visible. The perspective is from a low angle, looking up at the screen.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

# Semi-empirical mass formula

Modeling the binding energy of an atomic nucleus

**Inferred Formula:**

$$E_B = 14.83 - 13.43A + 12.39A^{0.64} + \frac{0.39Z^2}{A^{0.26}} + \frac{17.29(N-Z)^2}{A} \longrightarrow R^2 = 0.99944$$

**Weizsäcker's Formula:**

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z) \longrightarrow R^2 = 0.999915$$

$$\delta(A, Z) = \begin{cases} +\delta_0 & Z, N \text{ even} \\ 0 & A \text{ odd} \\ -\delta_0 & Z, N \text{ odd} \end{cases} \quad \delta_0 = \frac{a_P}{A^{1/2}}$$



# Systems of Differential Equations

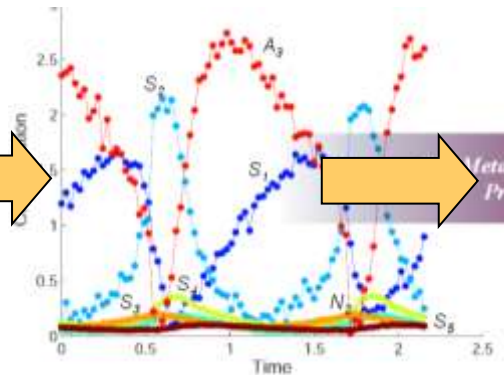
- Regress on derivative

<i>State Variables</i>				<i>Derivatives</i>		
<u>time</u>	<u><math>x_1</math></u>	<u><math>x_2</math></u>	...	<u><math>dx_1/dt</math></u>	<u><math>dx_2/dt</math></u>	...
0	3.4	-1.7	...	-2.0	8.0	...
0.1	3.2	-0.9	...	-1.0	8.0	...
0.2	3.1	-0.1	...	-4.0	1.3	...
0.3	2.7	1.2	...	-5.7	1.9	...
...	...	...	...	...	...	...

# Inferring Biological Networks

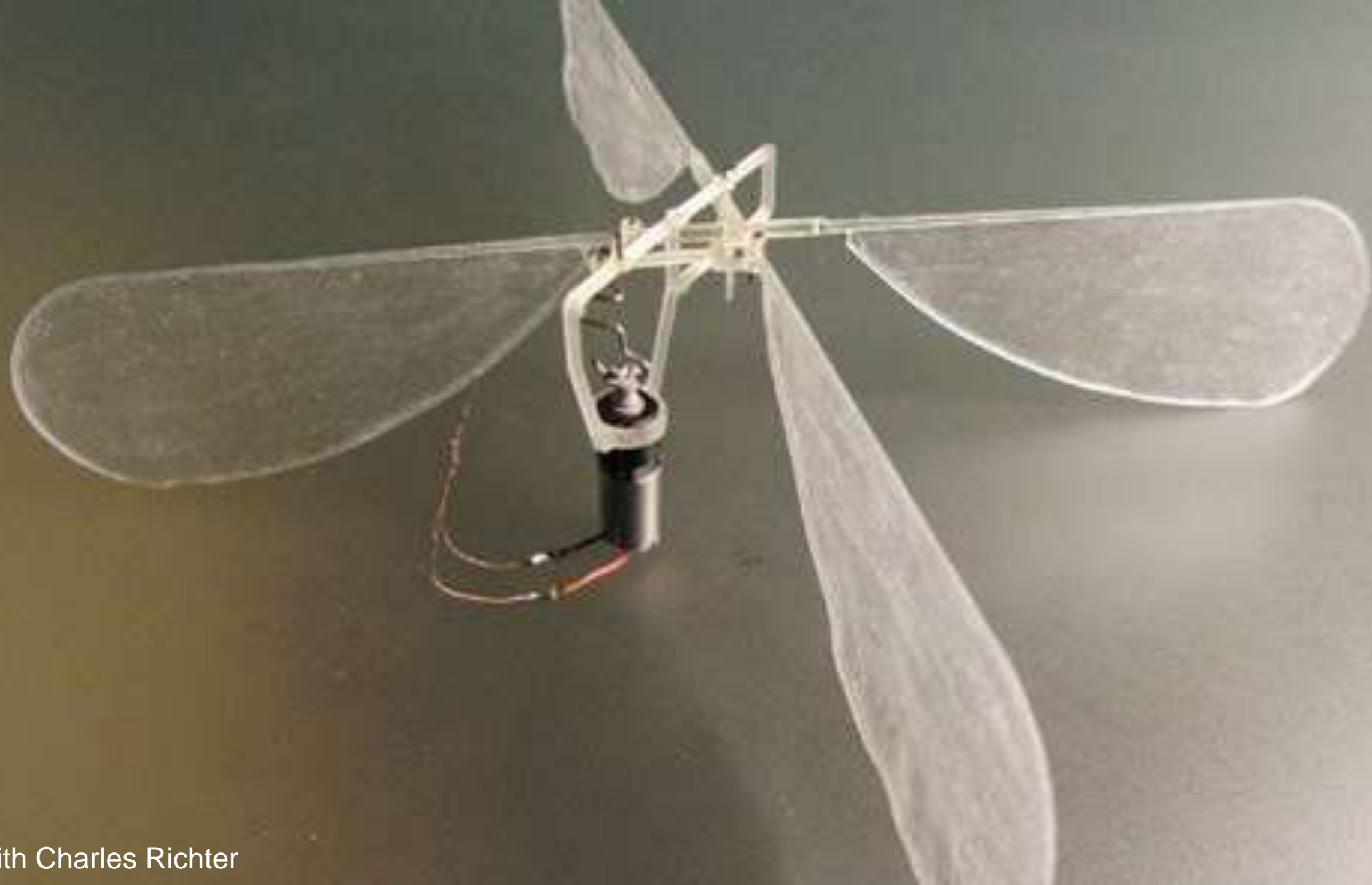
$$\begin{aligned}
 \frac{dS_1}{dt} &= 2.5 - \underbrace{100 \left( \frac{S_1 * A_3}{1 + 13.6769 * A_3^4} \right)}_{-v_1} \\
 \frac{dS_2}{dt} &= 200 \underbrace{\left( \frac{S_1 * A_3}{1 + 13.6769 * A_3^4} \right)}_{2*v_1} - \underbrace{6 * S_2 * N_1}_{-v_2} - \underbrace{12 * S_2 * N_1}_{-v_6} \\
 \frac{dS_3}{dt} &= \underbrace{6 * S_2 * N_1}_{v_2} + \underbrace{16 * S_3 * A_2}_{v_3} \\
 \frac{dS_4}{dt} &= \underbrace{16 * A_2 * S_3}_{v_3} - \underbrace{100 * N_2 * S_4}_{-v_4} \\
 \frac{dN_2}{dt} &= \underbrace{6 * S_2 * N_1}_{v_2} - \underbrace{100 * N_2 * S_4}_{-v_4} \\
 \frac{dA_3}{dt} &= -200 \underbrace{\left( \frac{S_1 * A_3}{1 + 13.6769 * A_3^4} \right)}_{-2*v_1} + \underbrace{32 * A_2 * S_3}_{2*v_3} - \underbrace{1.28 * A_3}_{-v_5} \\
 \frac{dS_5}{dt} &= \underbrace{-1.3 * S_5}_{\phi J}
 \end{aligned}$$

Original Equations



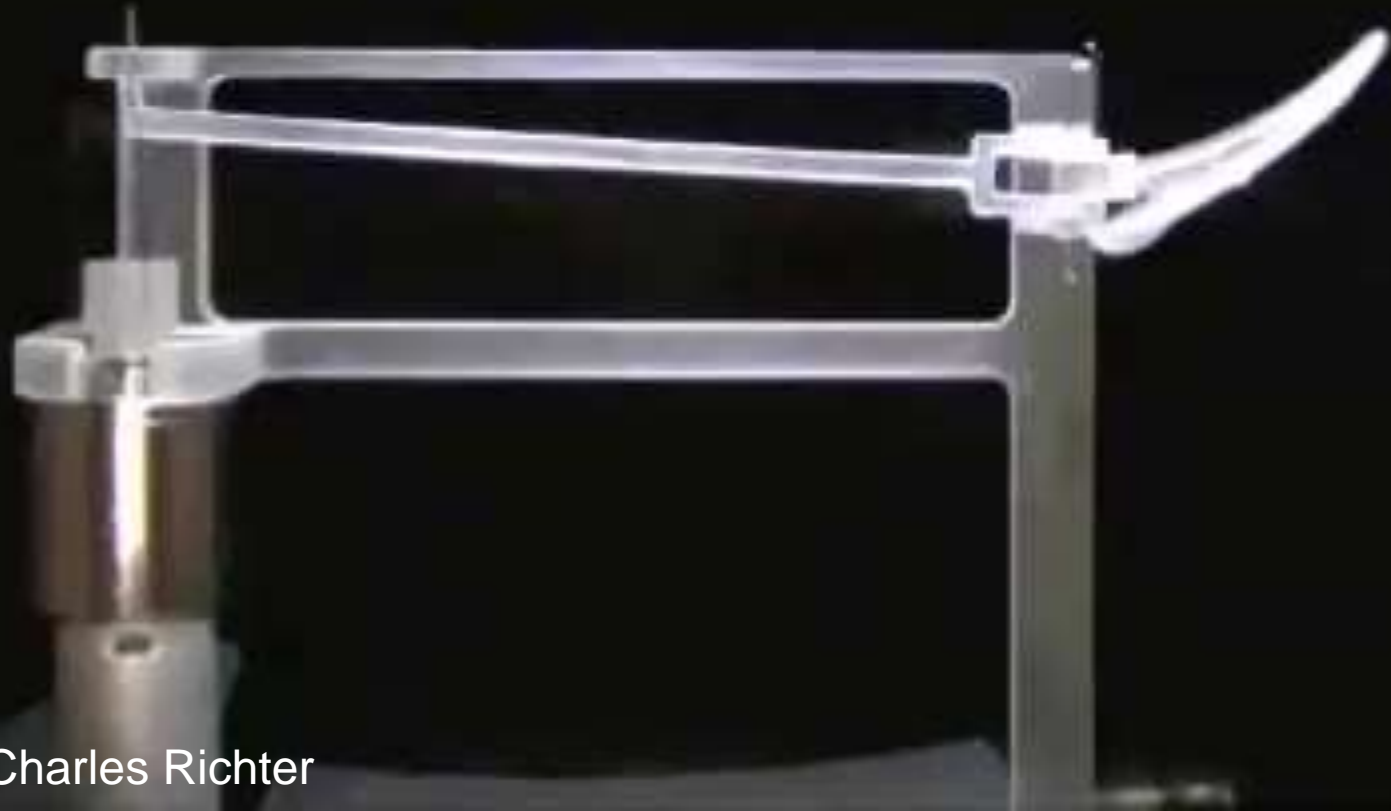
$$\begin{aligned}
 \frac{dS_1}{dt} &= \underbrace{2.42114}_{J_0} - \underbrace{99.2721 \left( \frac{S_1 * A_3}{1 + 13.5956 * A_3^4} \right)}_{-v_1} \\
 \frac{dS_2}{dt} &= \underbrace{199.935 \left( \frac{S_1 * A_3}{1 + 13.6734 * A_3^4} \right)}_{2*v_1} - \underbrace{5.99475 * S_2 * N_1}_{-v_2} - \underbrace{11.9895 * S_2 * N_1}_{-v_6} \\
 \frac{dS_3}{dt} &= \underbrace{5.99857 * S_2 * N_1}_{v_2} + \underbrace{15.99606 * S_3 * A_2}_{v_3} - \underbrace{0.01286 * S_3}_{\text{extraneous}} \\
 \frac{dS_4}{dt} &= \underbrace{15.997 * A_2 * S_3}_{v_3} - \underbrace{100.015 * N_2 * S_4}_{-v_4} \\
 \frac{dN_2}{dt} &= \underbrace{5.99857 * S_2 * N_1}_{v_2} - \underbrace{99.9963 * N_2 * S_4}_{-v_4} \\
 \frac{dA_3}{dt} &= -197.781 \underbrace{\left( \frac{S_1 * A_3}{1 + 13.2633 * A_3^4} \right)}_{-2*v_1} + \underbrace{31.9682 * A_2 * S_3}_{2*v_3} - \underbrace{1.29659 * A_3}_{-v_5} \\
 \frac{dS_5}{dt} &= \underbrace{-1.29626 * S_5}_{\phi J}
 \end{aligned}$$

Inferred Equations

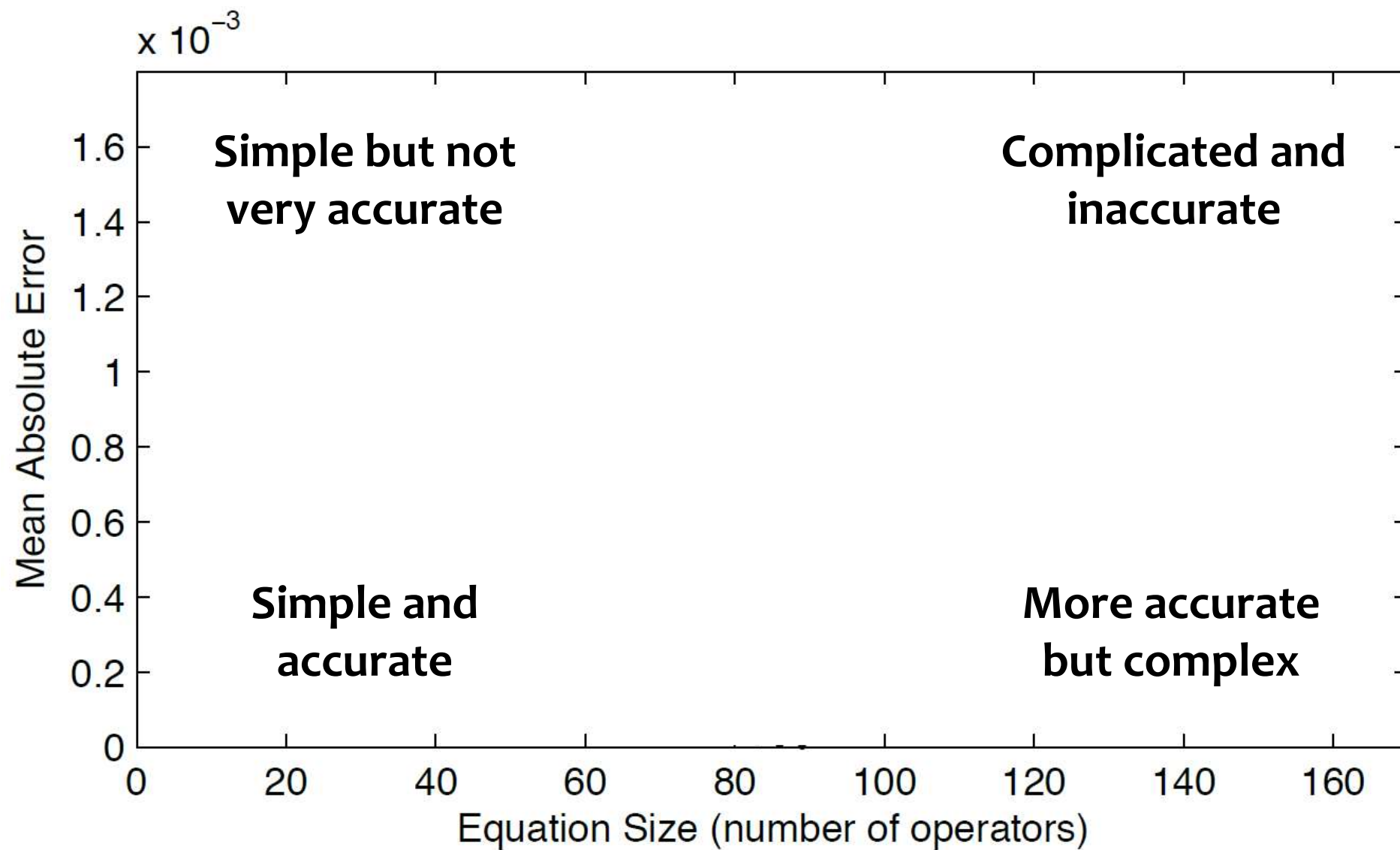


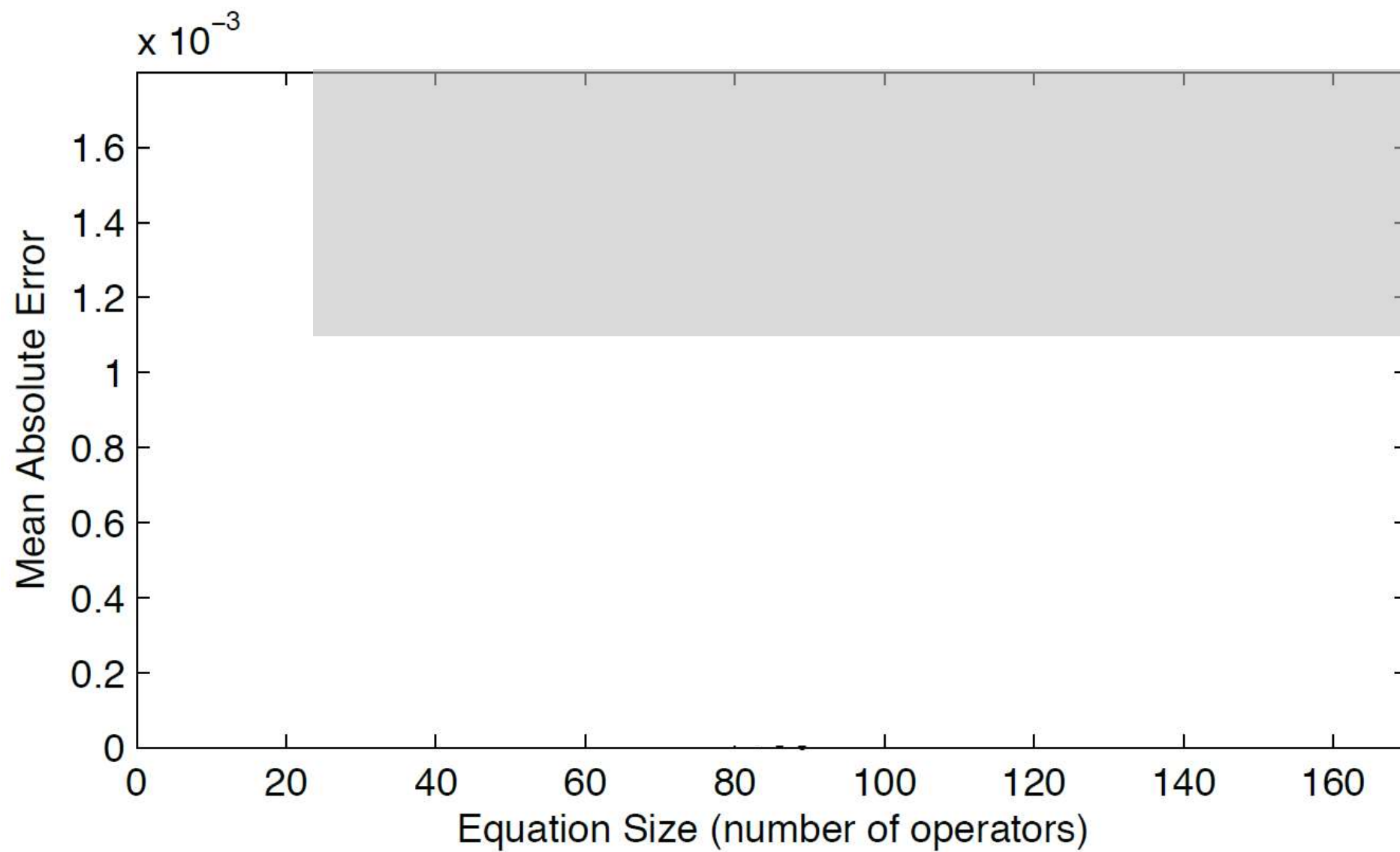
With Charles Richter





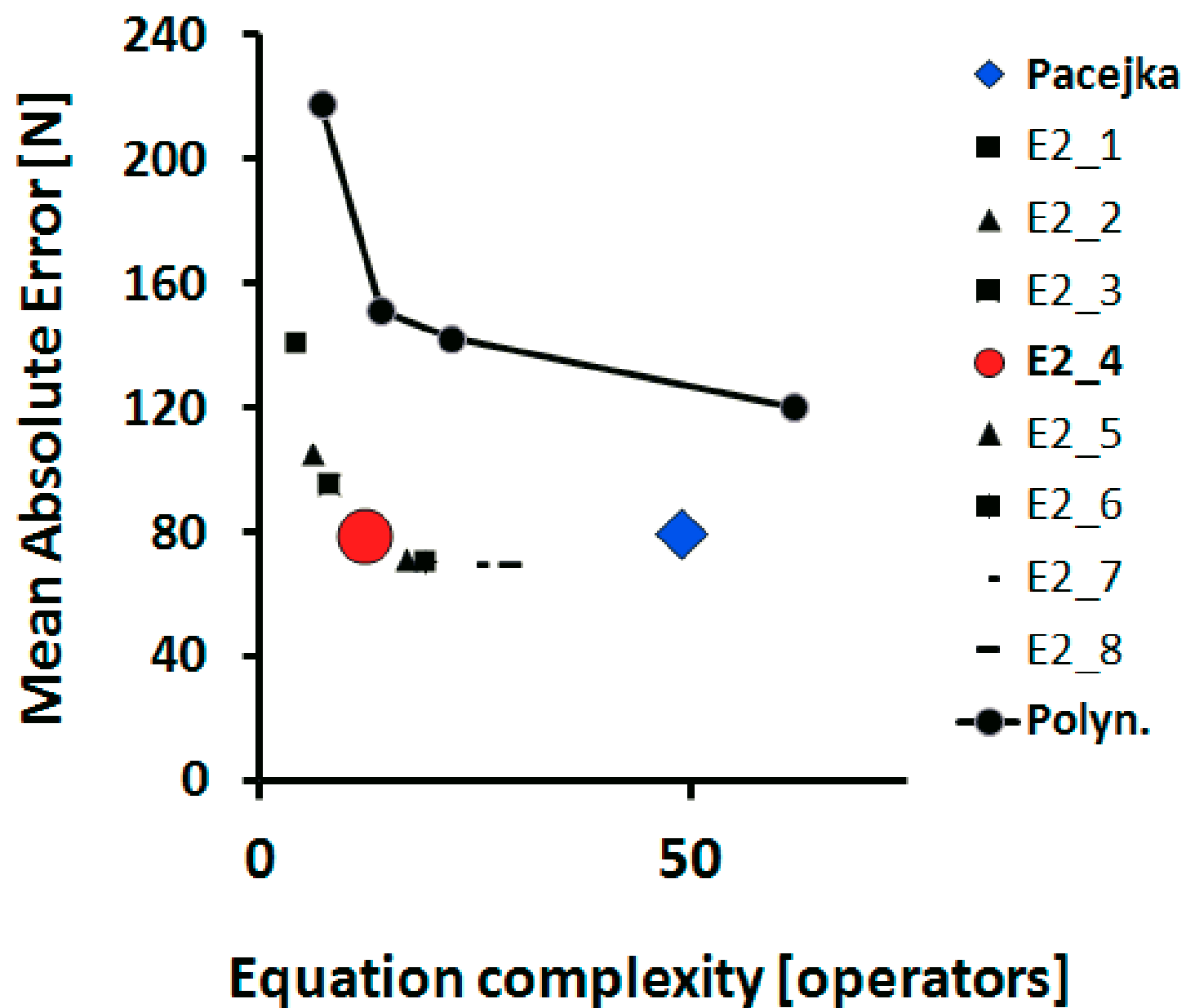
With Charles Richter



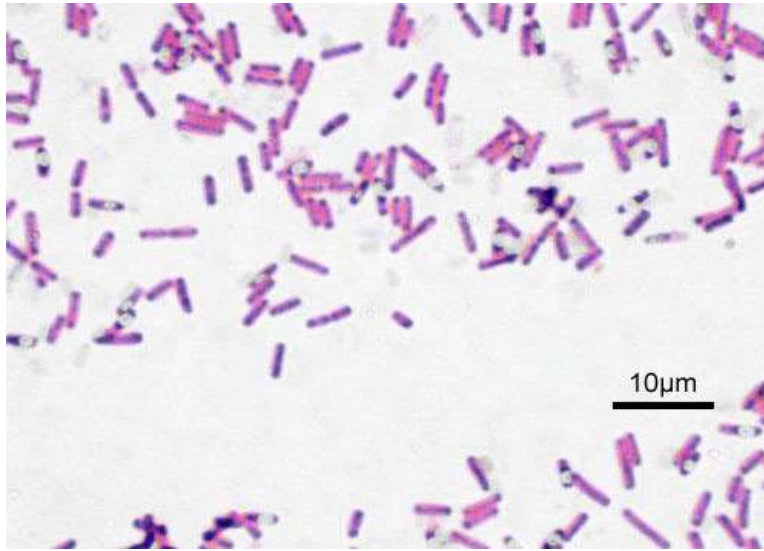
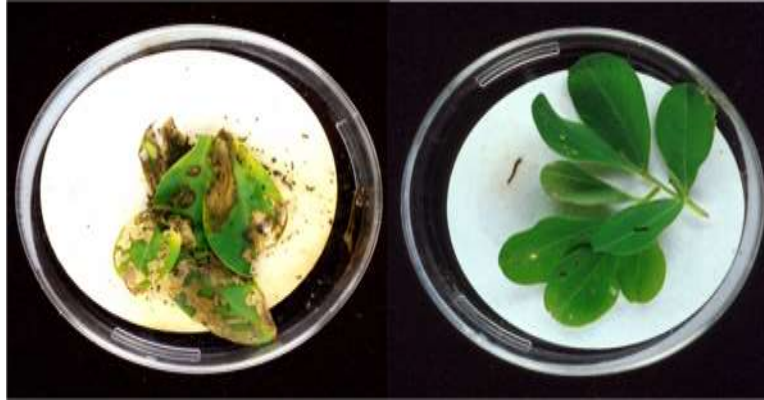




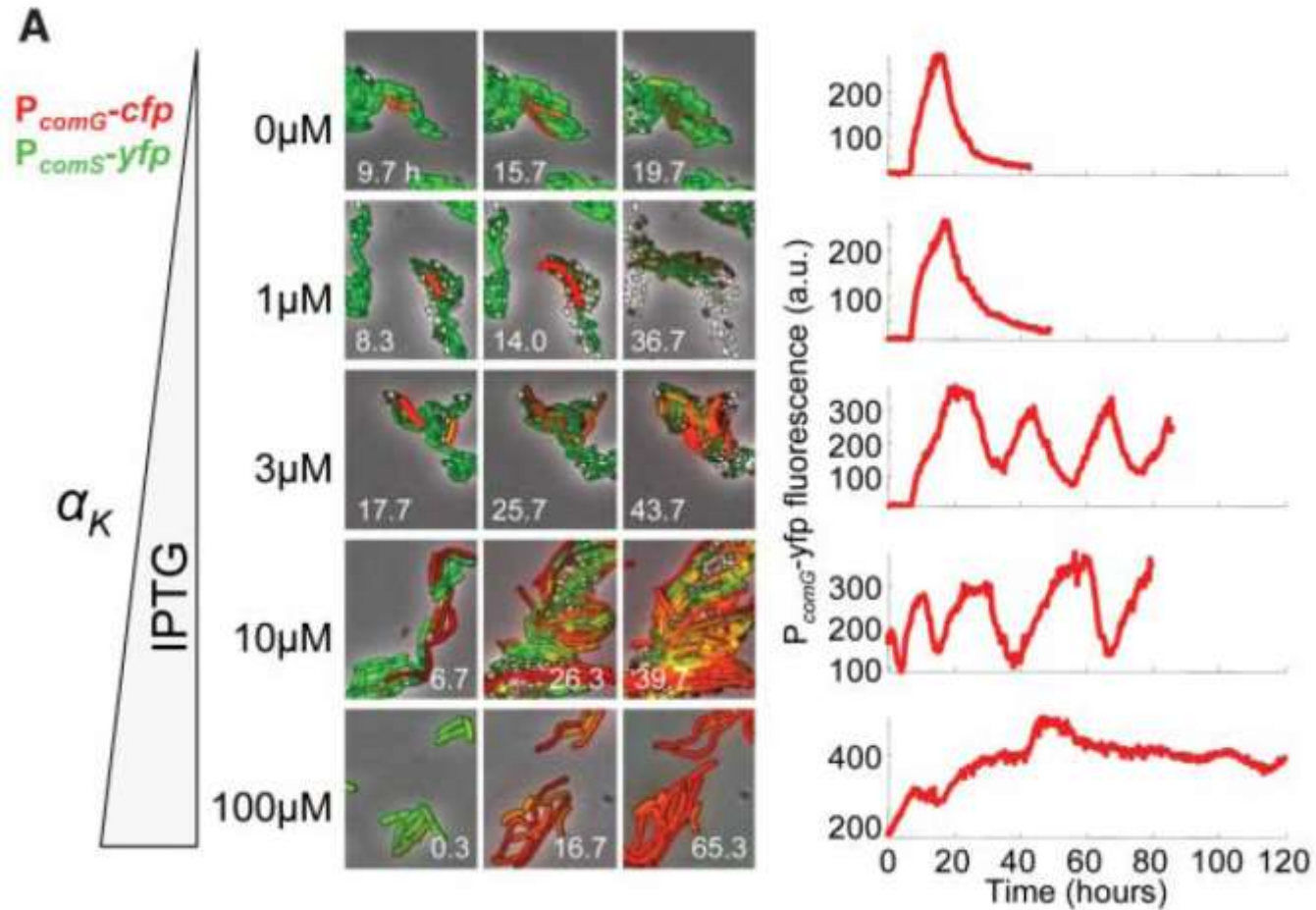




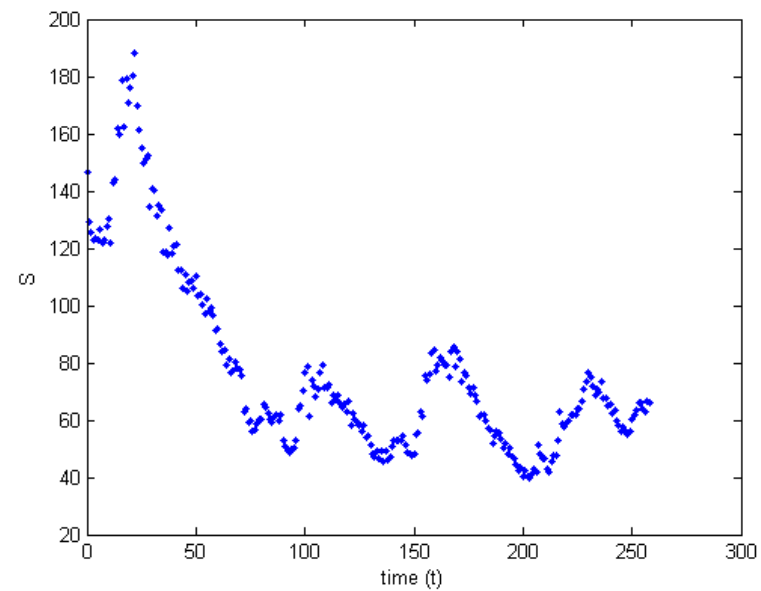
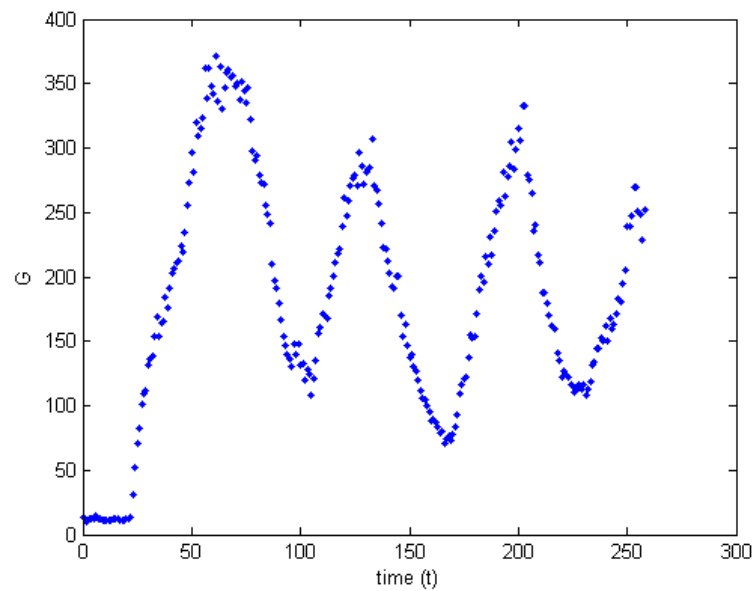
# *Bacillus Subtilis*



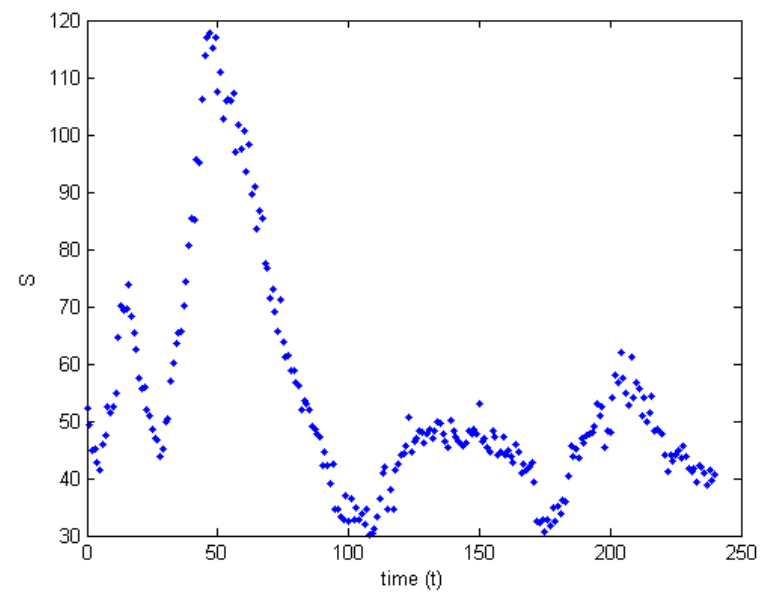
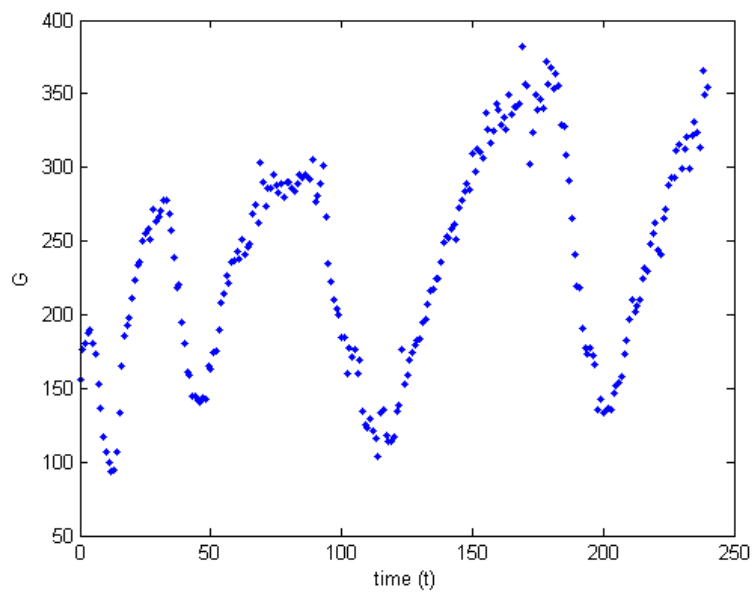
# Wet Data, Unknown System



**Cell #1**



**Cell #2**

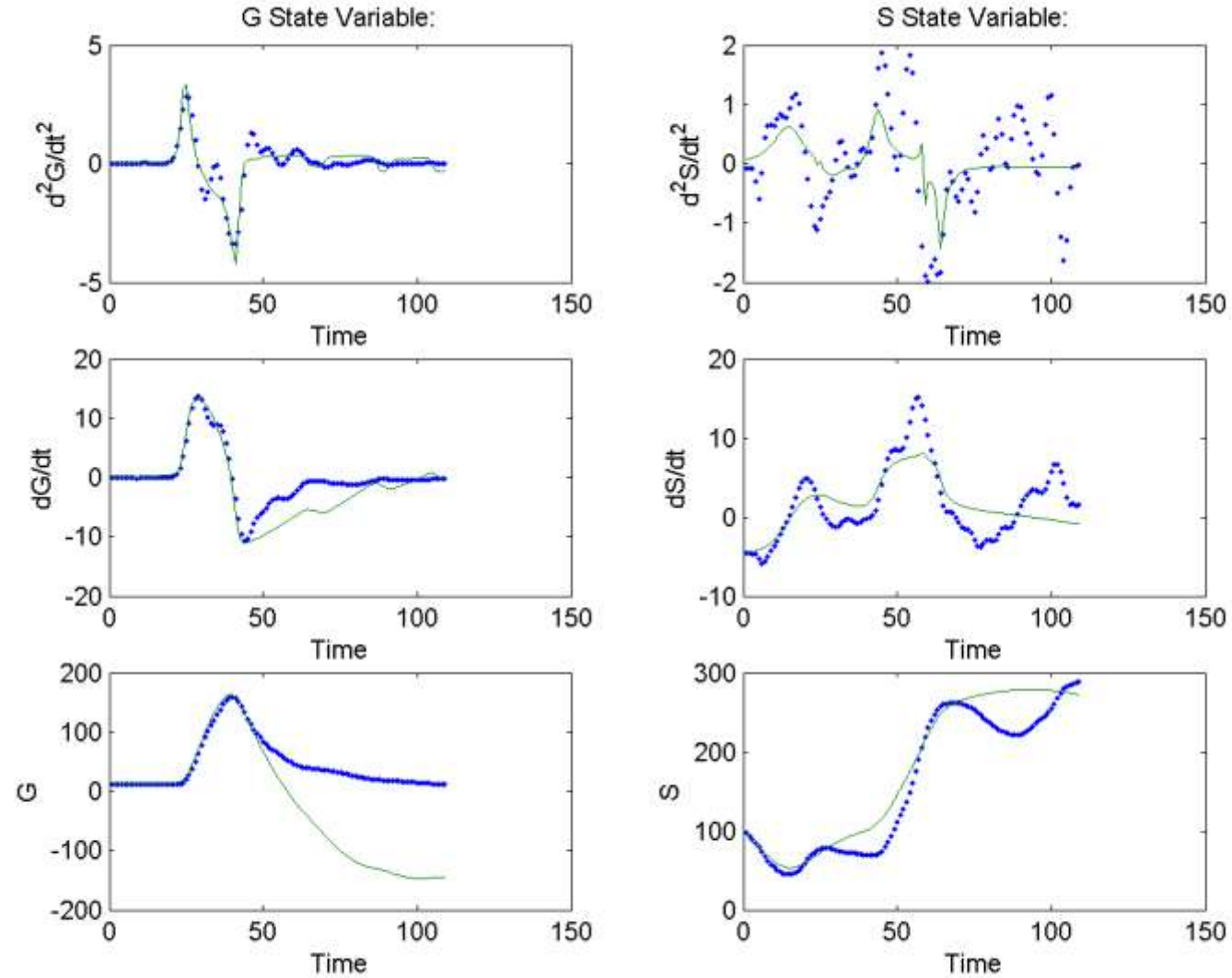


**Cell #3-60 ...**



$$\frac{d^2 G}{dt^2} = 0.329708 - \frac{0.494562 \left( G - \frac{dG/dt}{(dG/dt)^2} \right)}{19.75 + 2 dG + e^{\frac{dG}{dt}}}$$

$$\frac{d^2 S}{dt^2} = -0.0949 + \frac{0.511803}{-36.951 + 4 dG + G} + \frac{13.1334 - 1.46 dG}{-1.42633 (18.96 - dG) + S}$$



Blue Dots = data points, Green Line = regressed fit

Symbolic Regression Inferred *Time-Delay* Model:

$$\frac{dK}{dt} = a_K + \frac{b_K + c_K S}{K}$$
$$\frac{dS}{dt} = a_S + \frac{b_S + c_S K}{S}$$

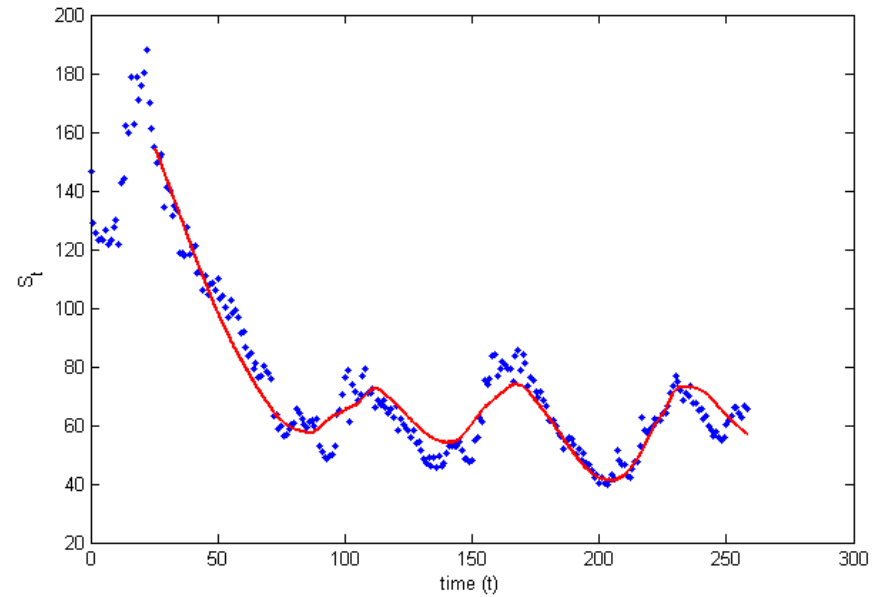
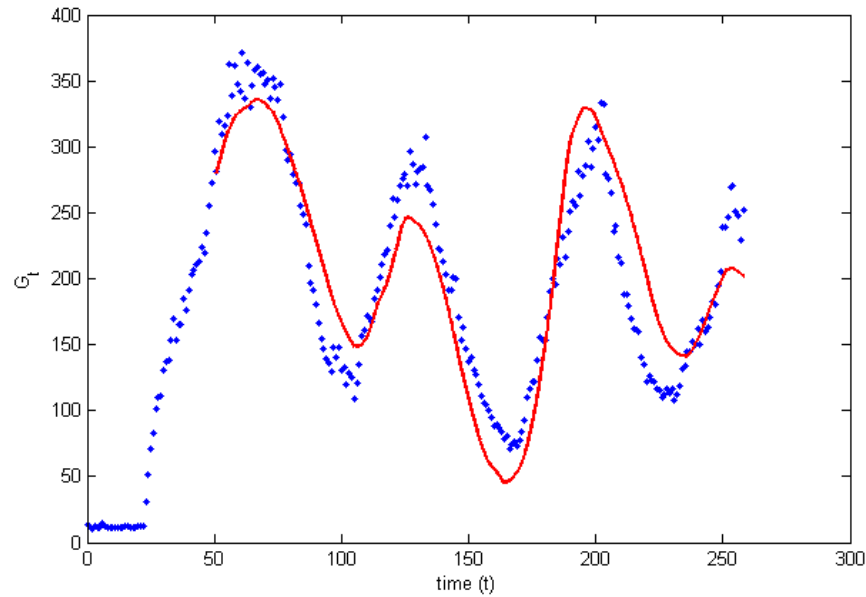
Biologist's Inferred Model: Gurol Suel, et. al., Science

$$\frac{dK}{dt} = \alpha_k + \frac{\beta_K K^n}{k_0^n + K^n} - \frac{\delta_K K}{1 + K / \Gamma_K + S / \Gamma_S} - \lambda_K K$$
$$\frac{dS}{dt} = \alpha_s + \frac{\beta_S}{1 + (K / k_1)^p} - \frac{\delta_k S}{1 + K / \Gamma_K + S / \Gamma_S} - \lambda_S S$$

# Withheld Test Set #1 Fit

$$\frac{dG_t}{dt} = \frac{1582.0 + 17.3214 \cdot S_{t-51}}{G_{t-18}} - 16.7423$$

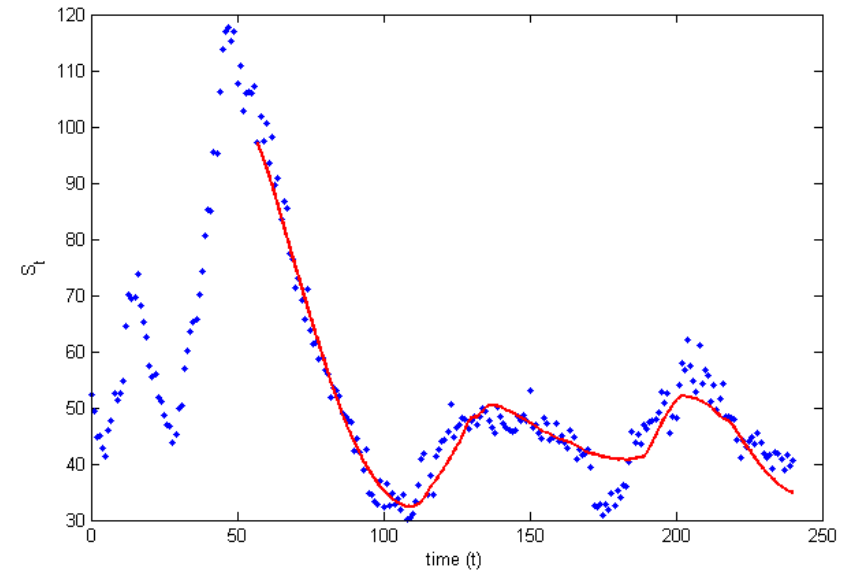
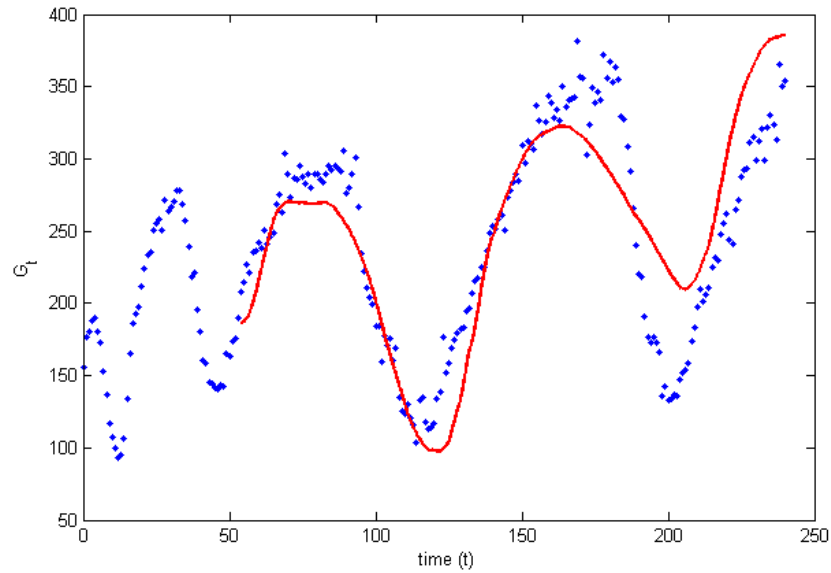
$$\frac{dS_t}{dt} = \frac{114.922 + 0.3019 \cdot G_{t-25}}{S_{t-15}} - 3.05$$



# Withheld Test Set #2 Fit

$$\frac{dG_t}{dt} = \frac{3526.92 - 21.312 \cdot S_{t-54}}{G_{t-17}} - 10.1355$$

$$\frac{dS_t}{dt} = \frac{132.271 - 0.0178 \cdot G_{t-57}}{S_{t-18}} - 2.9693$$

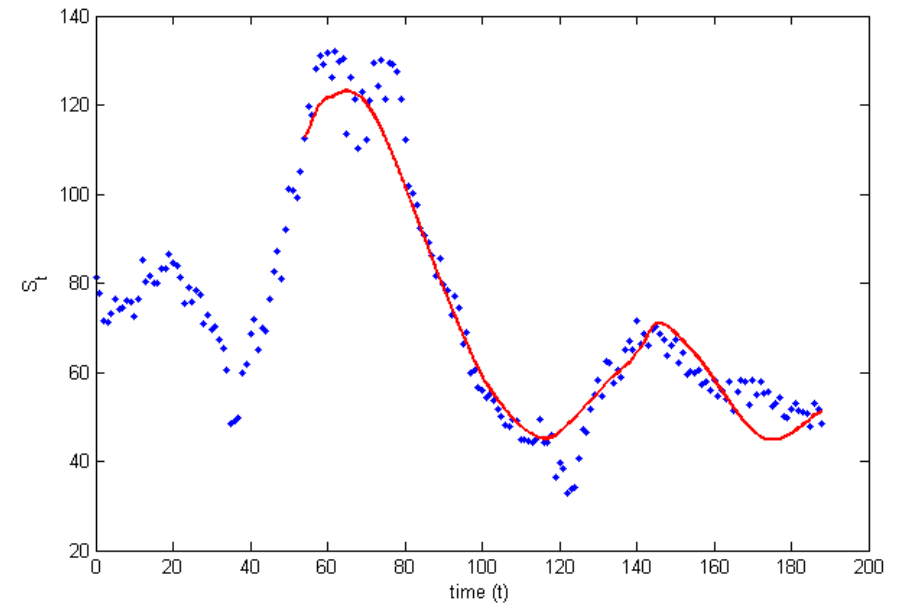
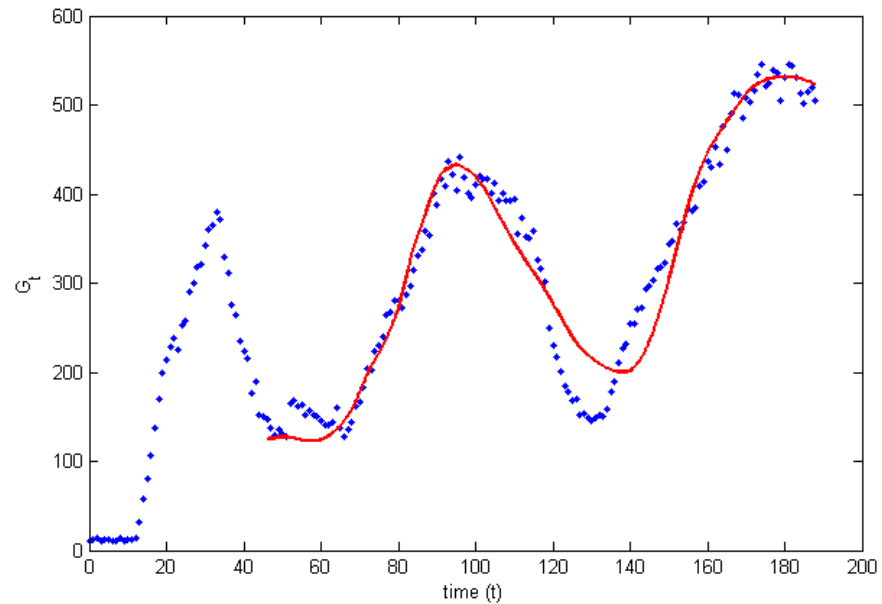




# Withheld Test Set #3 Fit

$$\frac{dG_t}{dt} = \frac{5057.1 - 39.7452 \cdot S_{t-46}}{G_{t-21}} - 6.4406$$

$$\frac{dS_t}{dt} = \frac{295.426 - 0.2965 \cdot G_{t-54}}{S_{t-20}} - 3.871$$



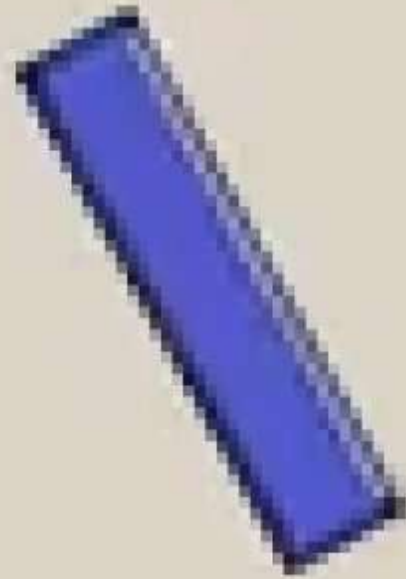
# Searching for meaning



# Correlations

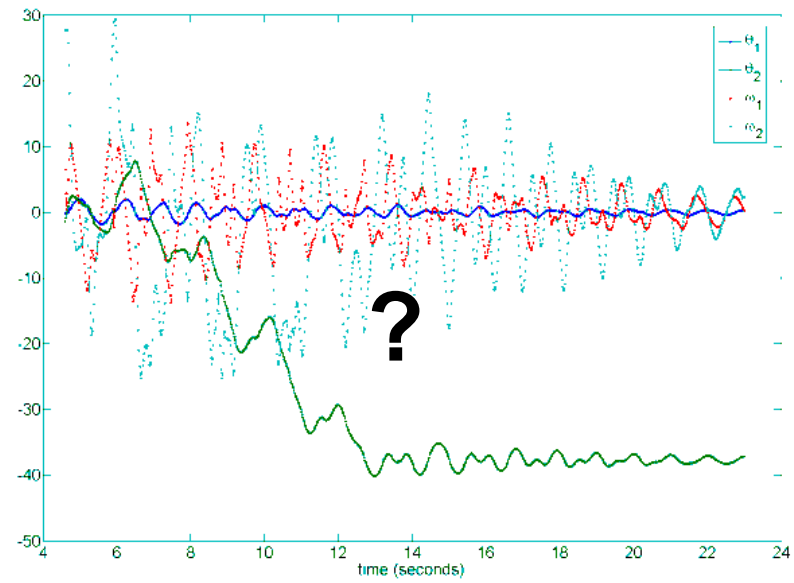


$$\theta_t, \omega_t$$



$$f(\theta_t, \omega_t) = \text{const}$$





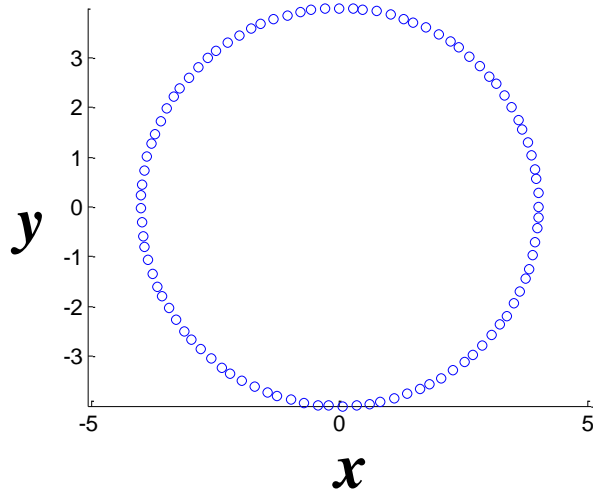
**42**

**$42+x-x$**

**$42+x/1000$**

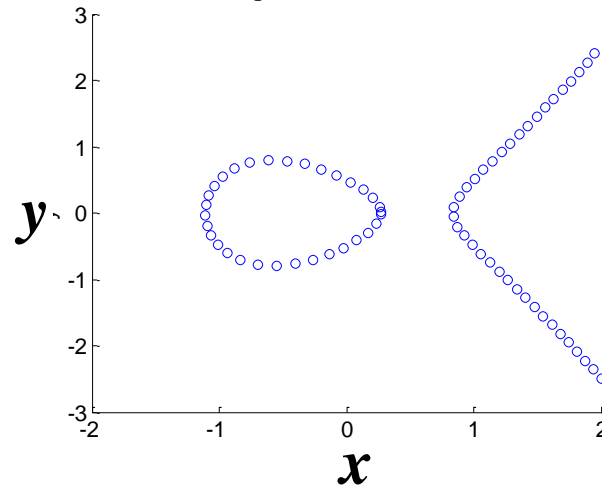
# Homework

**Circle**



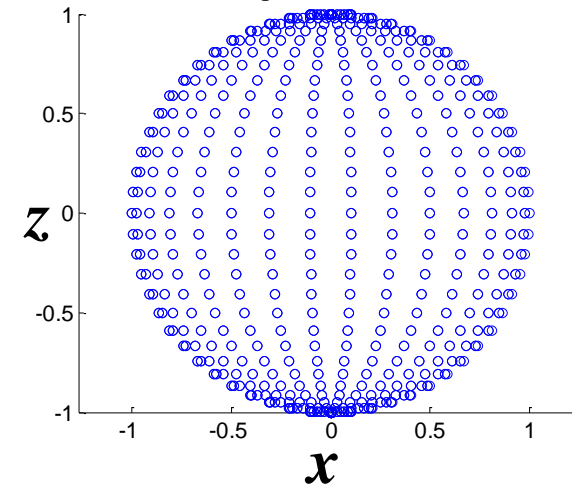
$$x^2 + y^2 - 16 = 0$$

**Elliptic Curve**



$$x^3 + x - y^2 - 1.5 = 0$$

**Sphere**



$$x^2 + y^2 + z^2 - 1 = 0$$

## From Data:

$x$	$y$	...
0.1	2.3	
0.2	4.5	
0.3	9.7	
0.4	5.1	
0.5	3.3	
0.6	1.0	
...	...	...

*Calculate partial derivatives Numerically:*



$$\frac{\delta x}{\delta y}, \quad \frac{\delta y}{\delta x}, \quad \dots$$

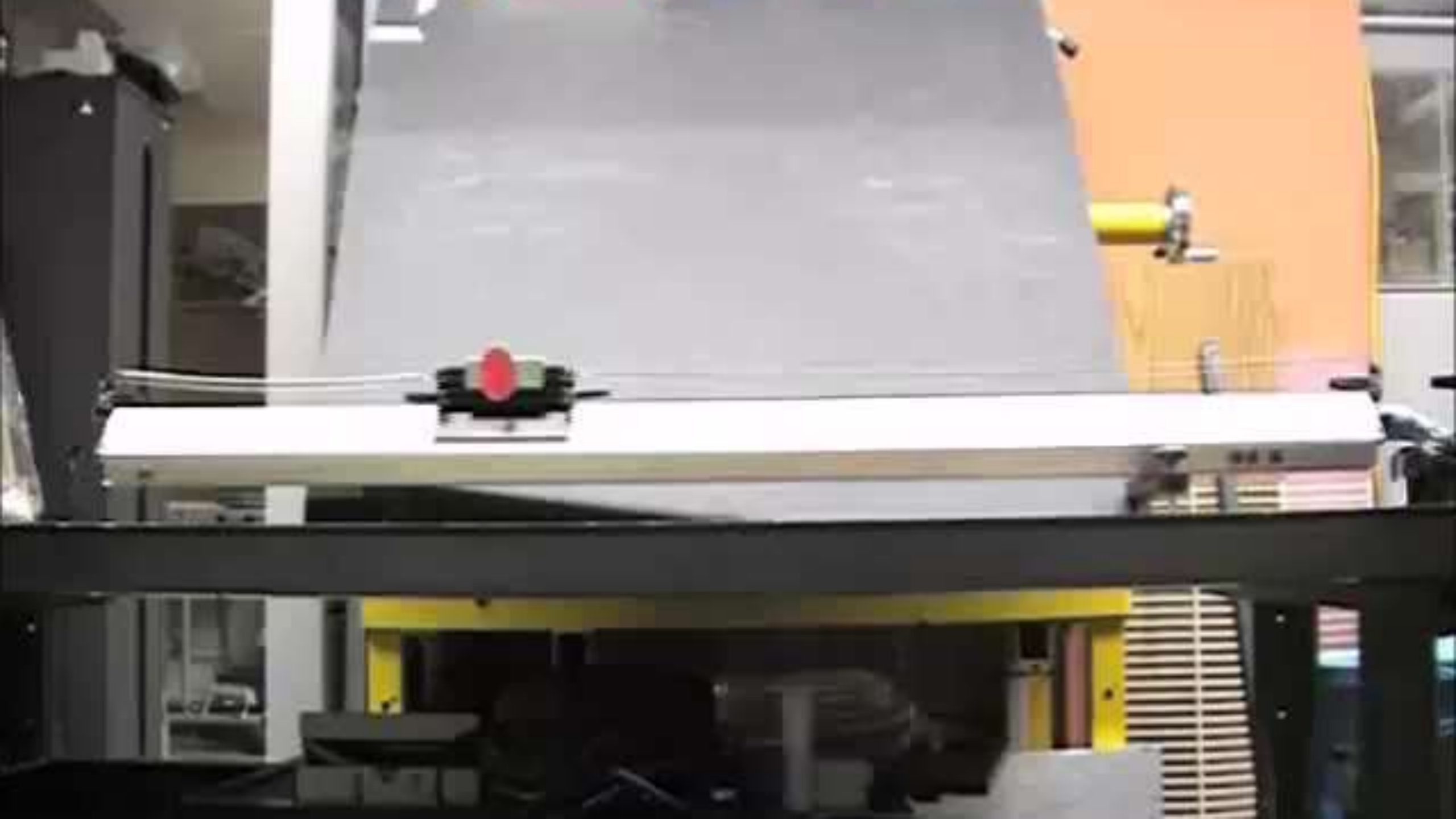
## From Equation:

*Calculate predicted partial derivatives Symbolically:*

$$\frac{\delta f}{\delta x} / \frac{\delta f}{\delta y}$$



$$\frac{\delta x'}{\delta y'}, \quad \frac{\delta y'}{\delta x'}, \quad \dots$$



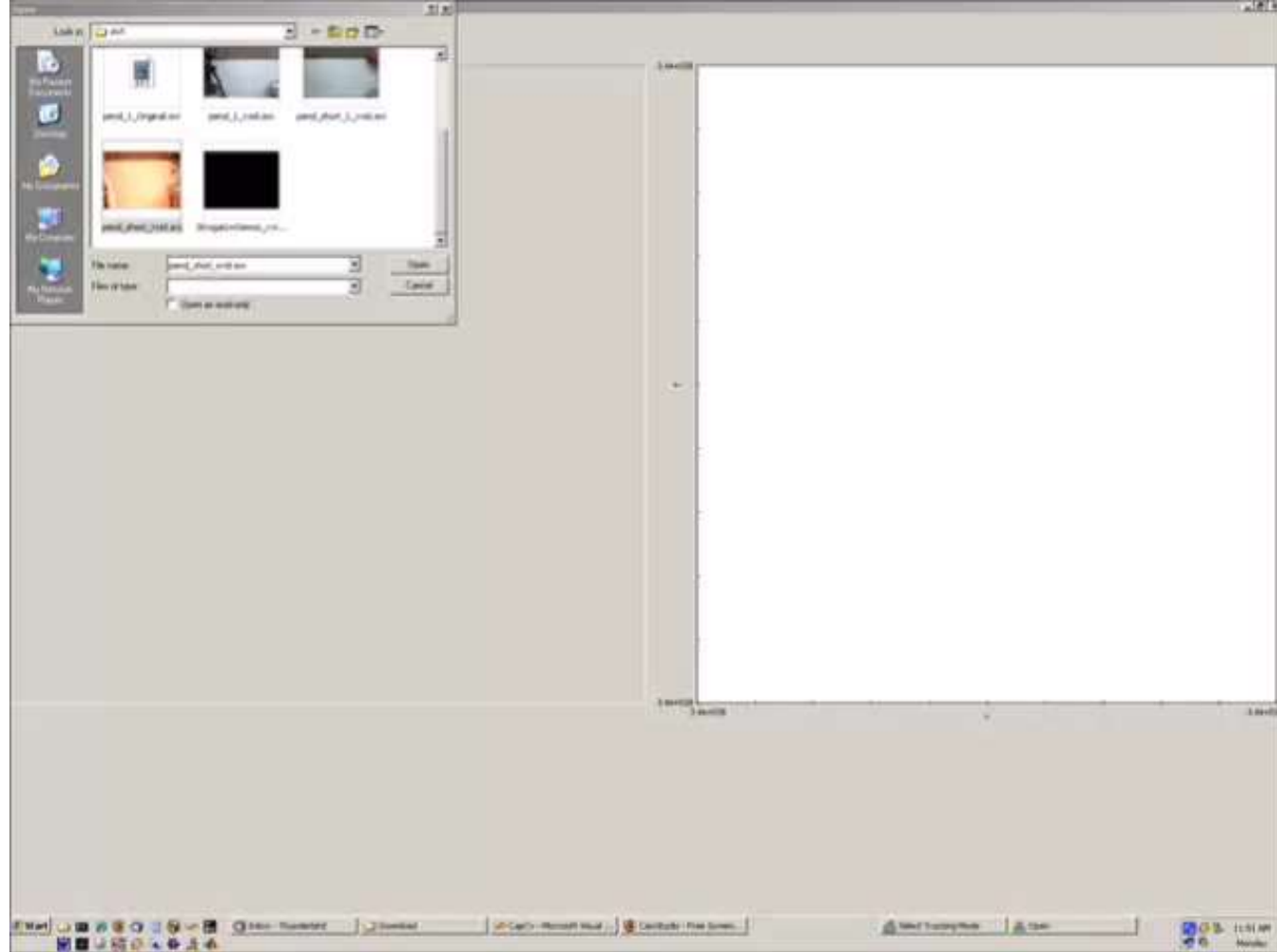


# Linear Oscillator



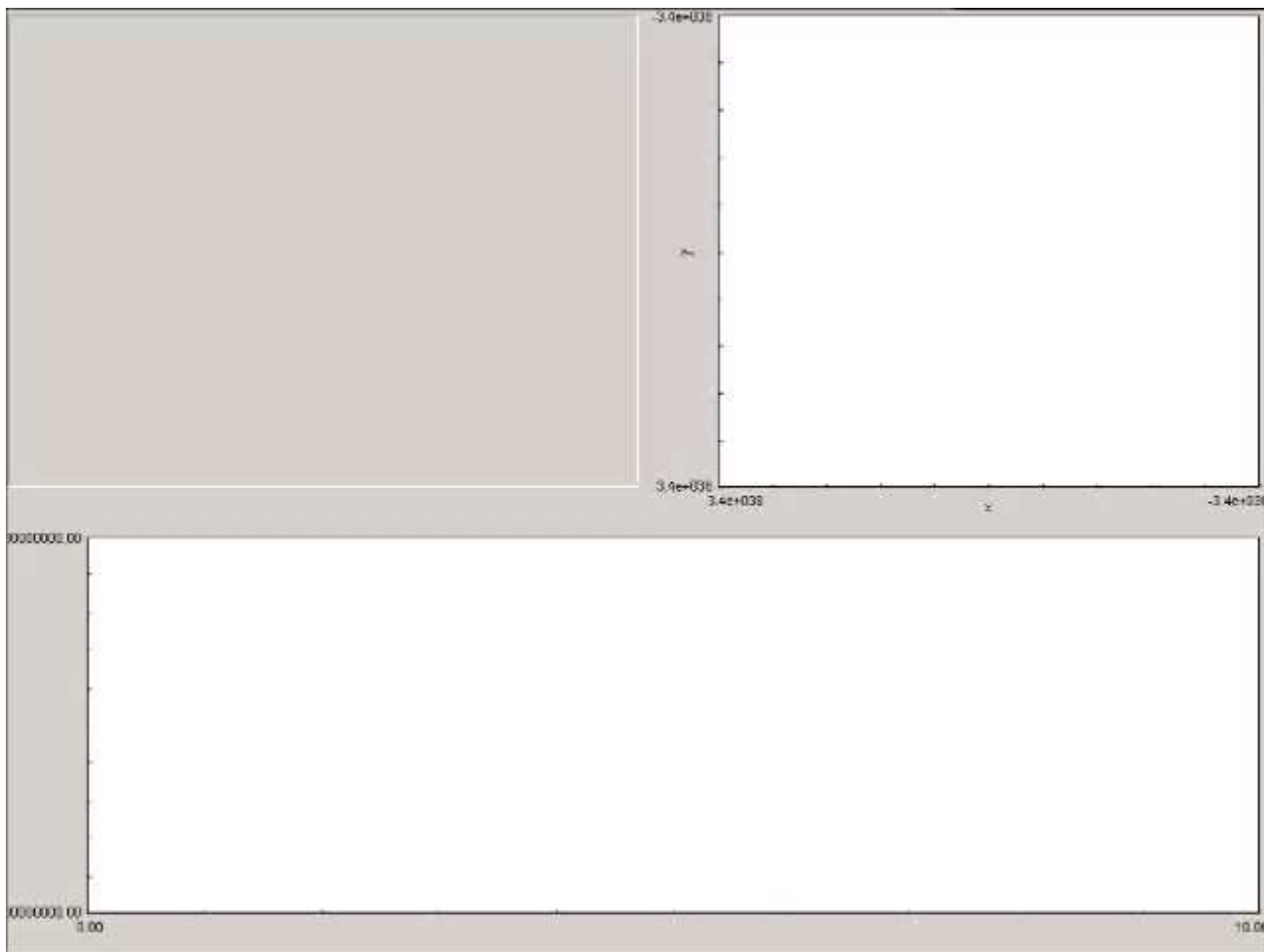
$$H = 114.28 * \left( \frac{dx}{dt} \right)^2 + 692.322 * x^2$$
$$L = 61.591 * \left( \frac{dx}{dt} \right)^2 - 369.495 * x^2$$

- Coefficients may have different scales and offsets each run



$$\mathbf{H} = \left( \frac{d\theta}{dt} \right)^2 + 2.42847 * \cos(\theta)$$
$$\mathbf{L} = 3.52768 * \left( \frac{d\theta}{dt} \right)^2 - 9.43429 * \cos(\theta)$$

# Double Linear Oscillator

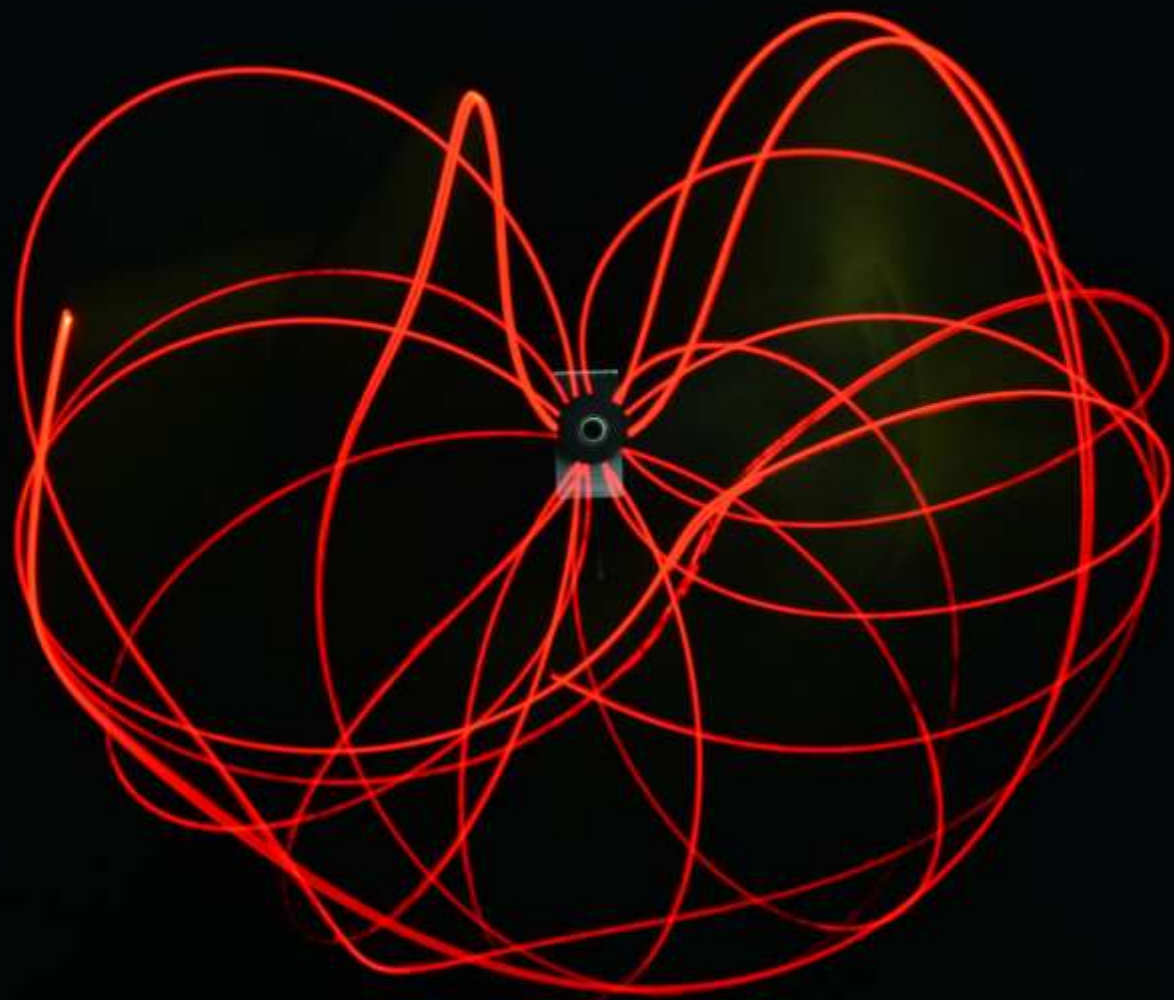


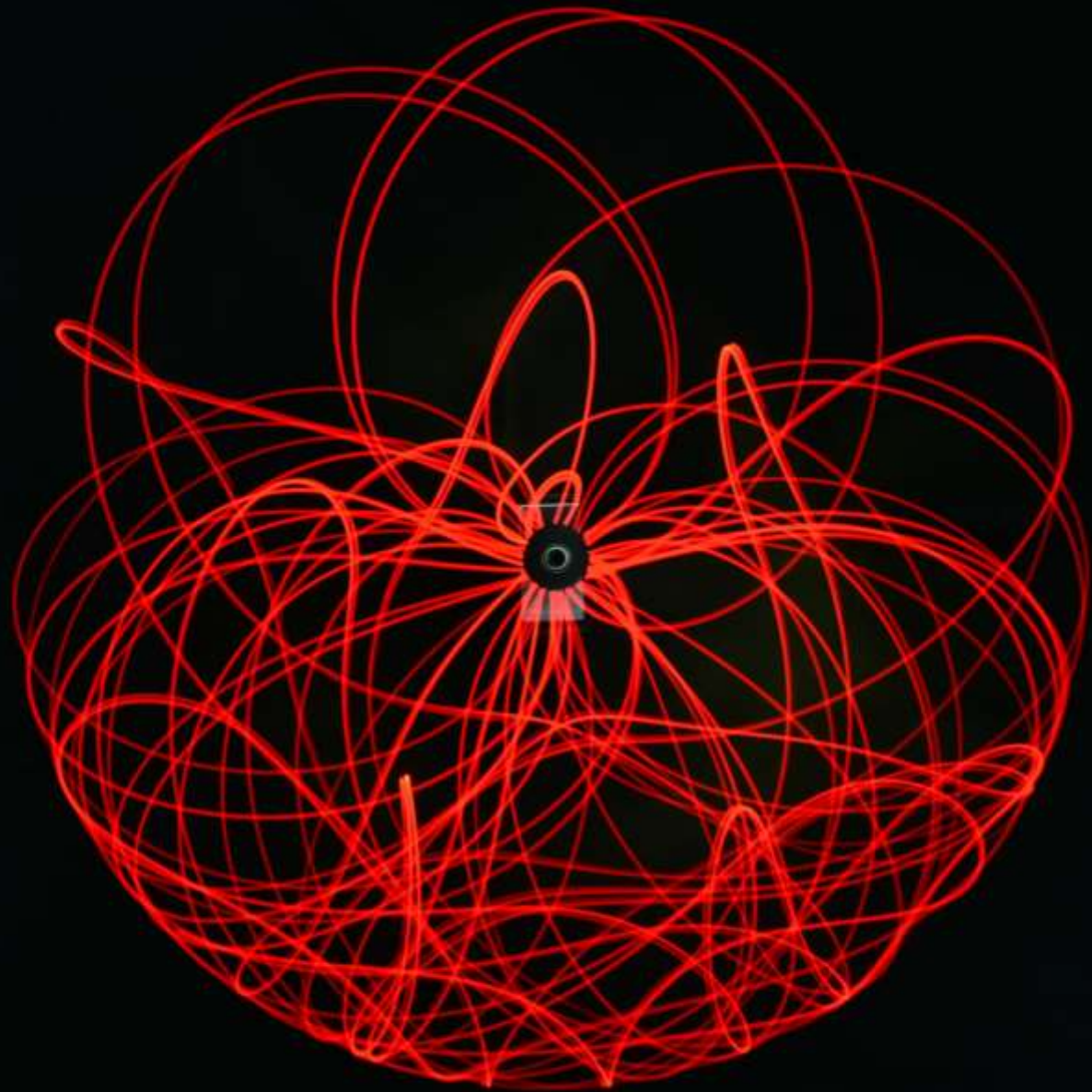
$$H = -14.691 * x_1^2 - 15.551 * x_2^2 - 21.676 * x_1 x_2 + 8.3808 * \left( \frac{dx_2}{dt} \right)^2 + 2.6046 * \left( \frac{dx_1}{dt} \right)^2$$

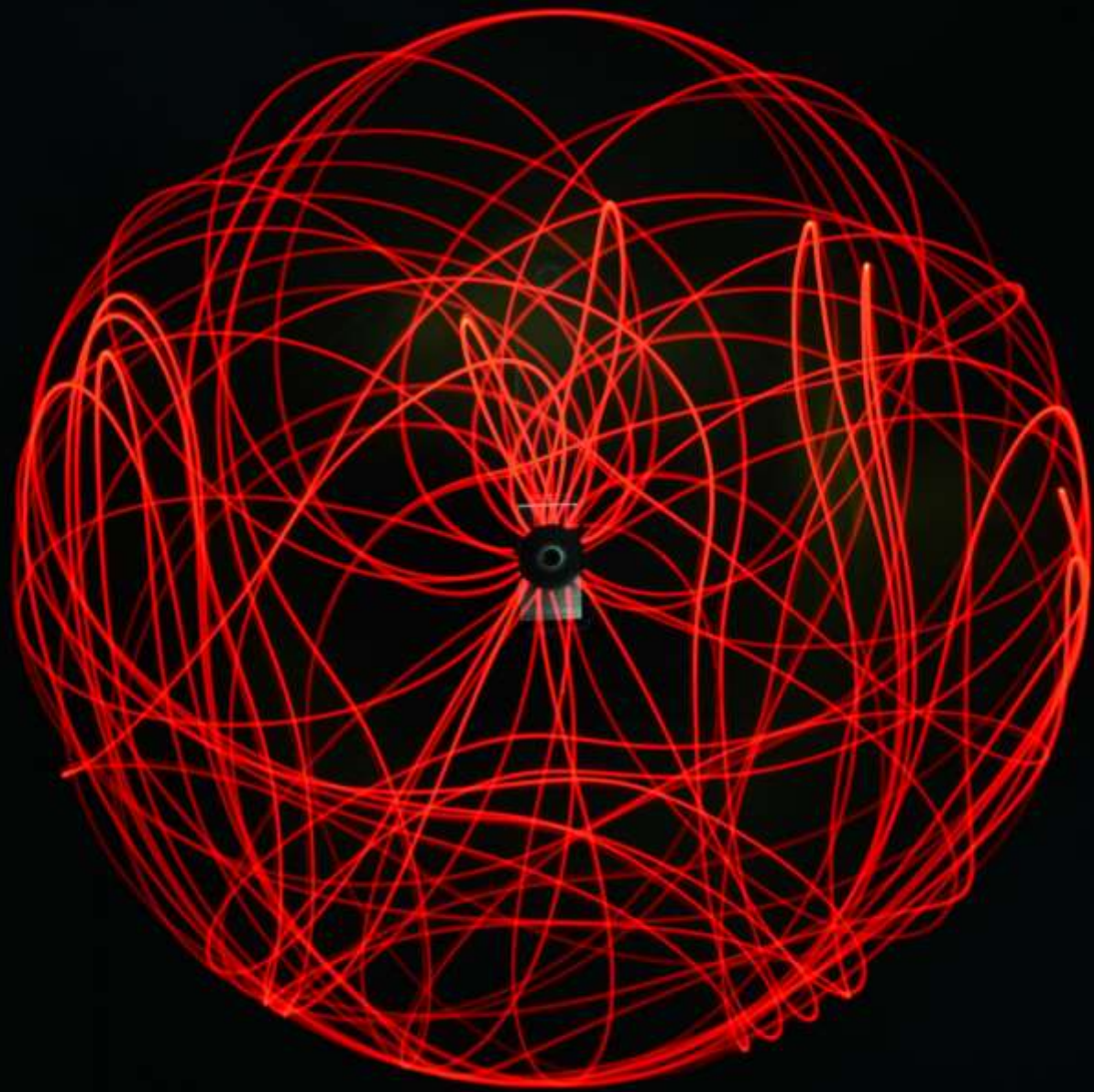
would be plus for Lagrangian

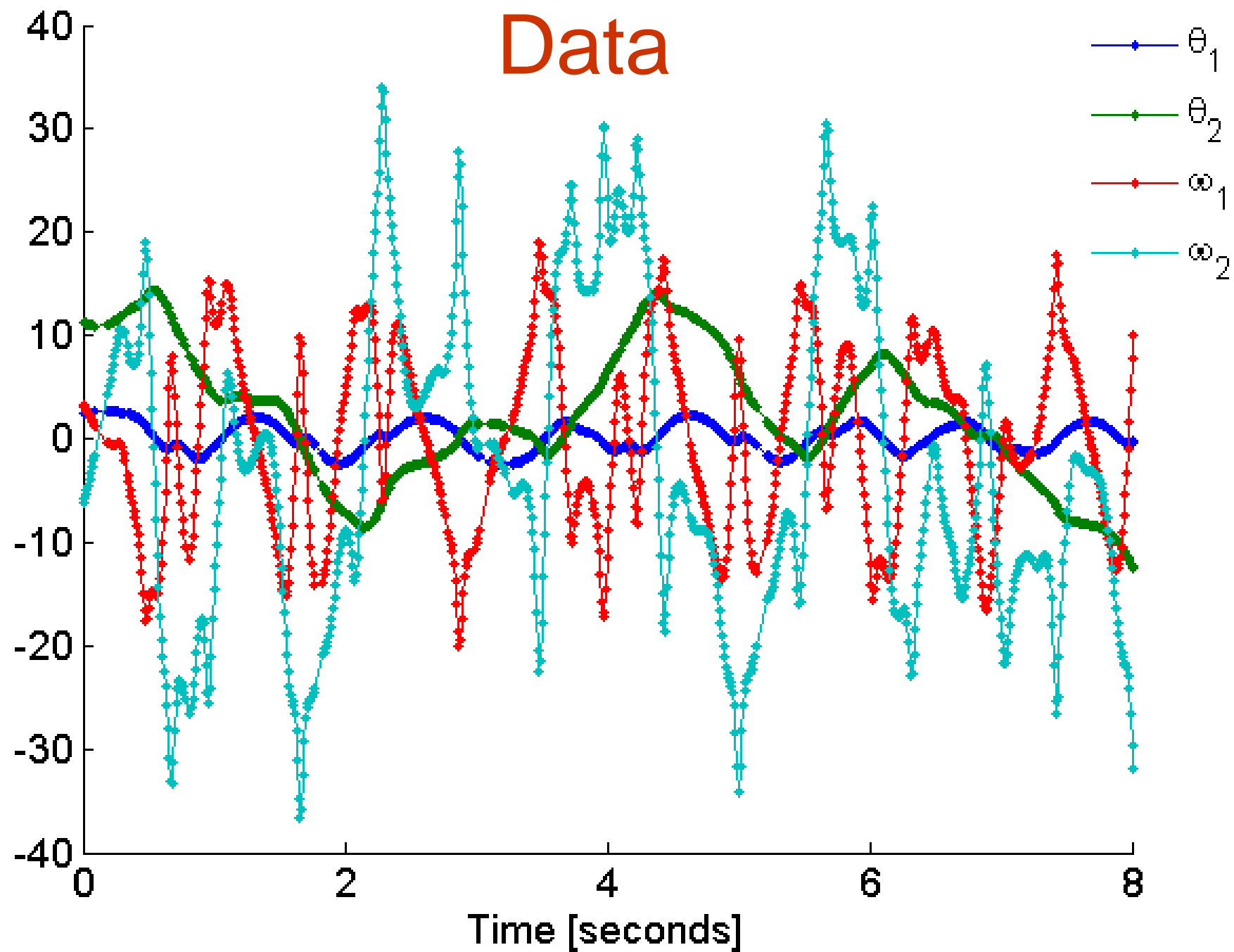




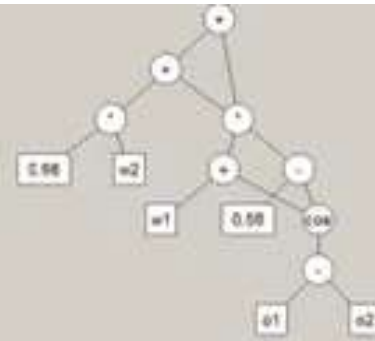




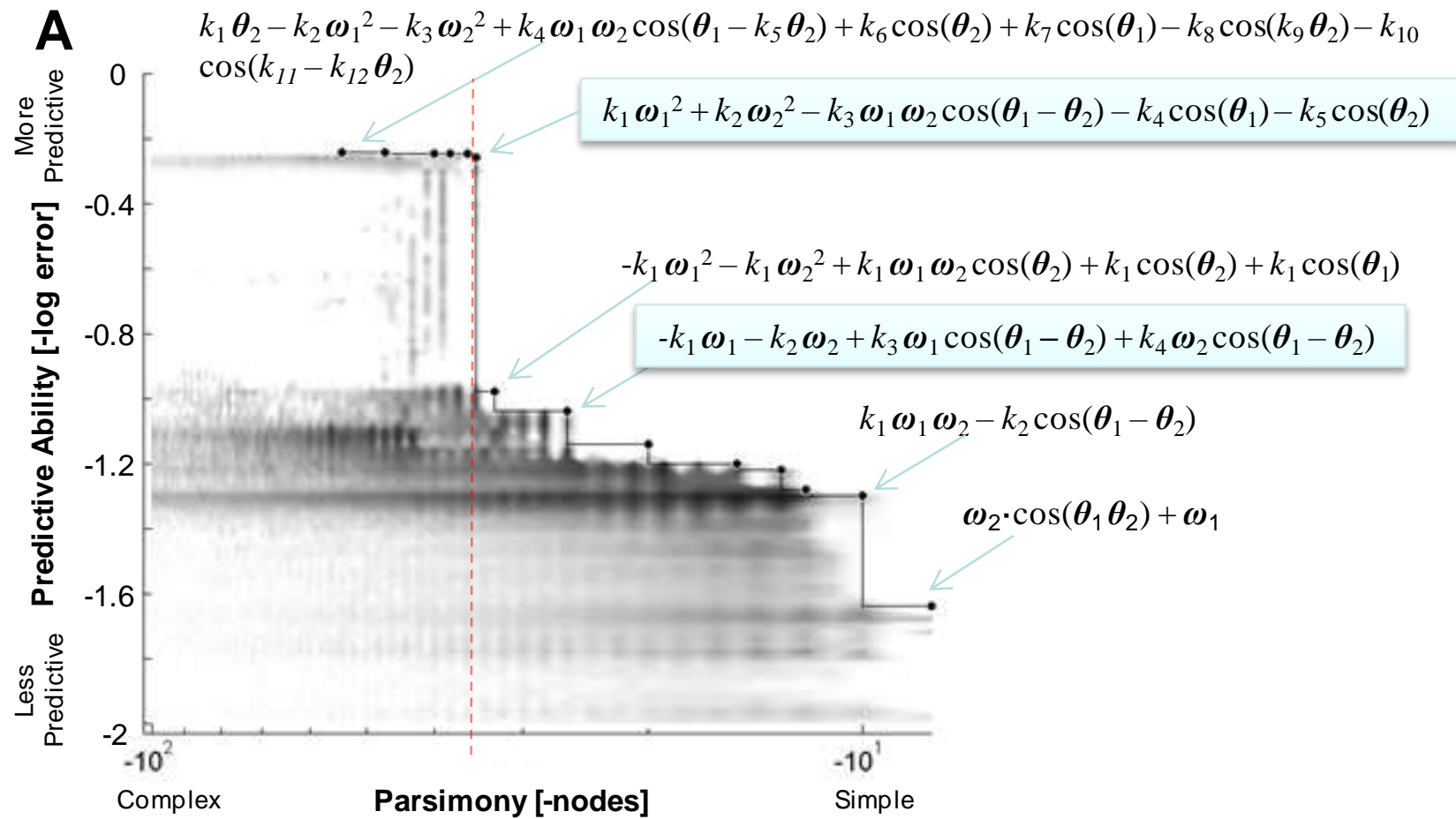


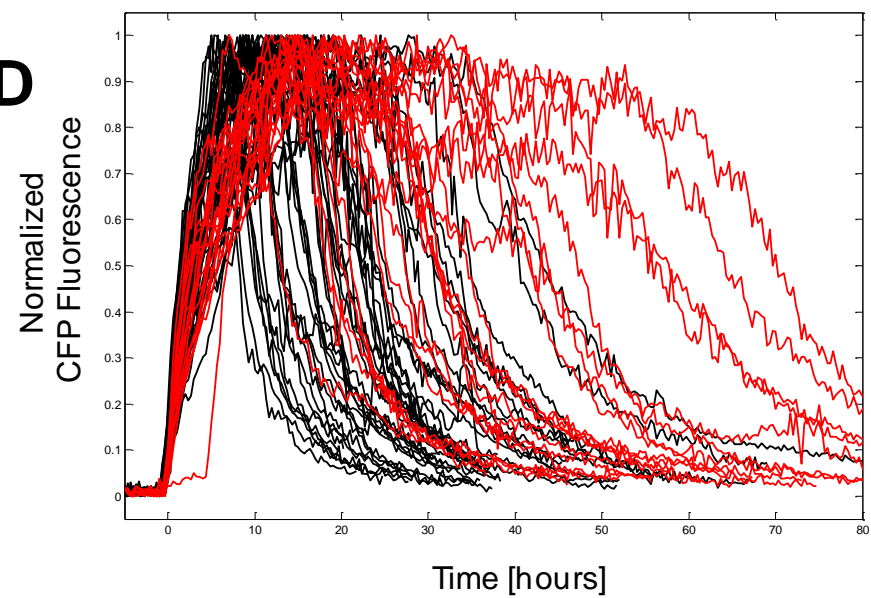
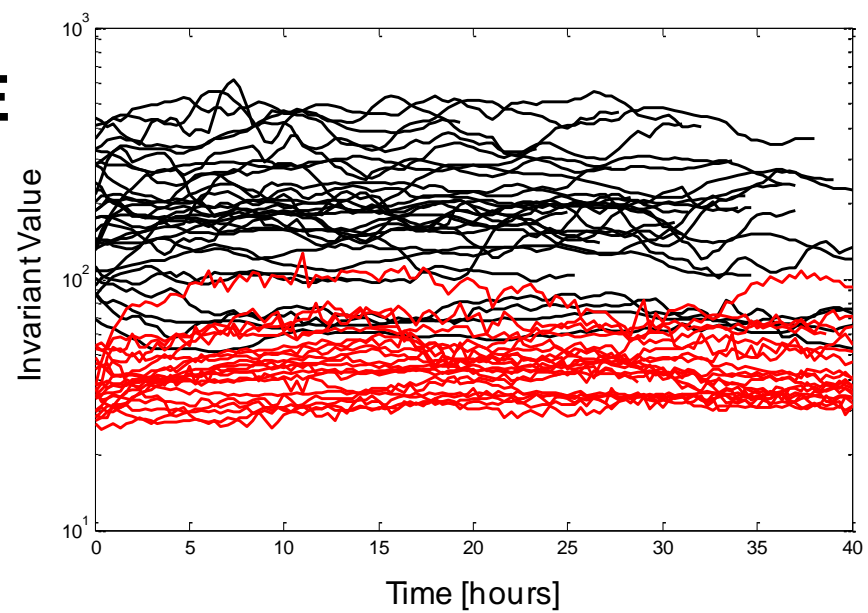






Search for Nontrivial  
Double Pendulum Law

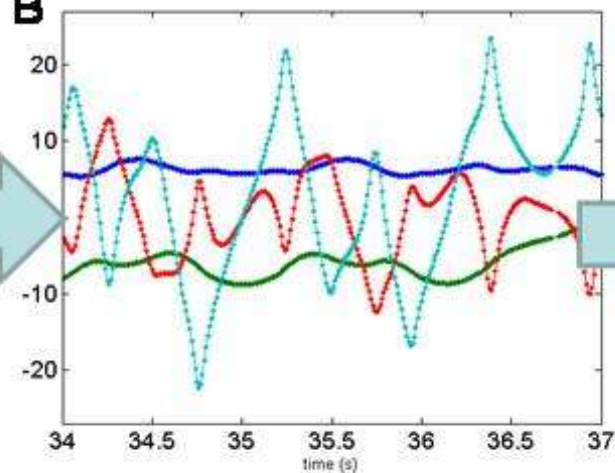


**D****E**



**A**

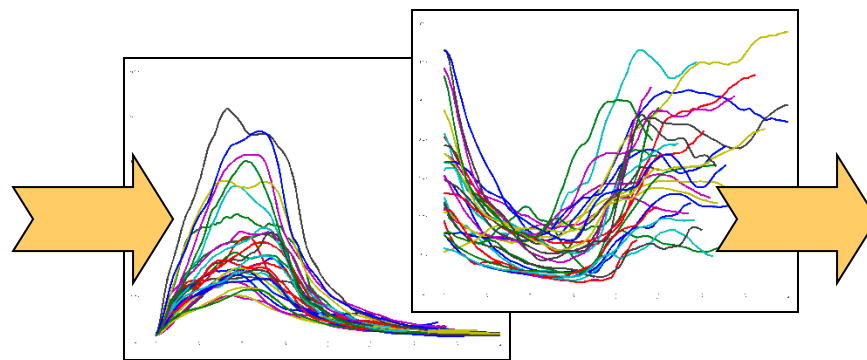
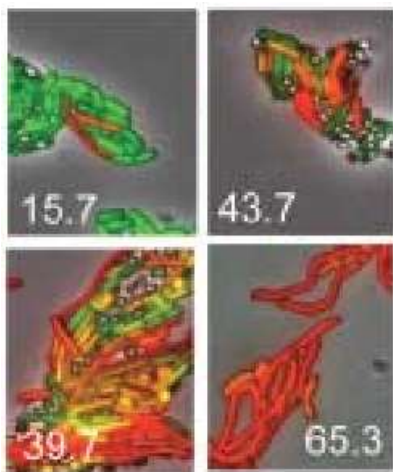
**B**



**C**

**Detected Invariance:**

$$L_1^2(m_1+m_2)\omega_1^2 + m_2L_2^2\omega_2^2 + m_2L_1L_2\omega_1\omega_2\cos(\theta_1-\theta_2) - 19.6L_1(m_1+m_2)\cos\theta_1 - 19.6m_2L_2\cos\theta_2$$

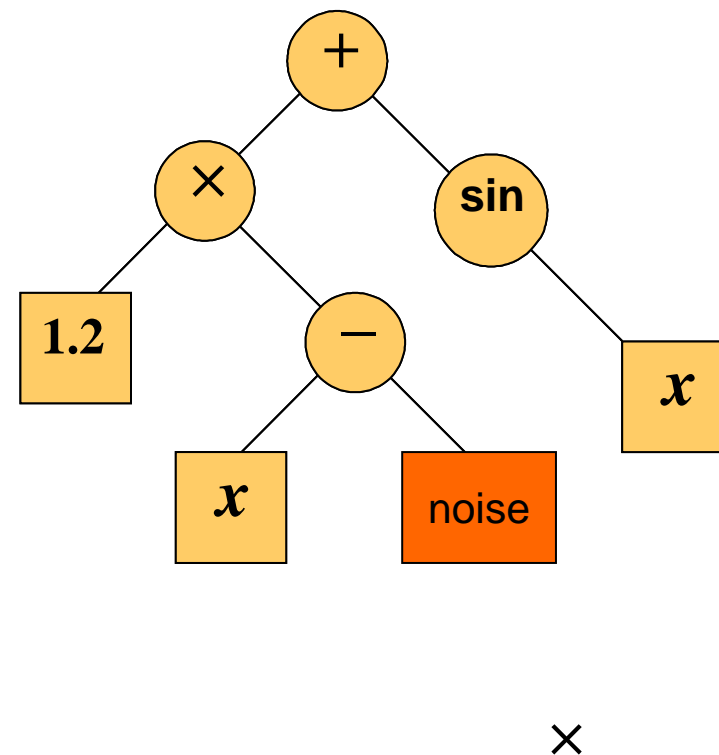
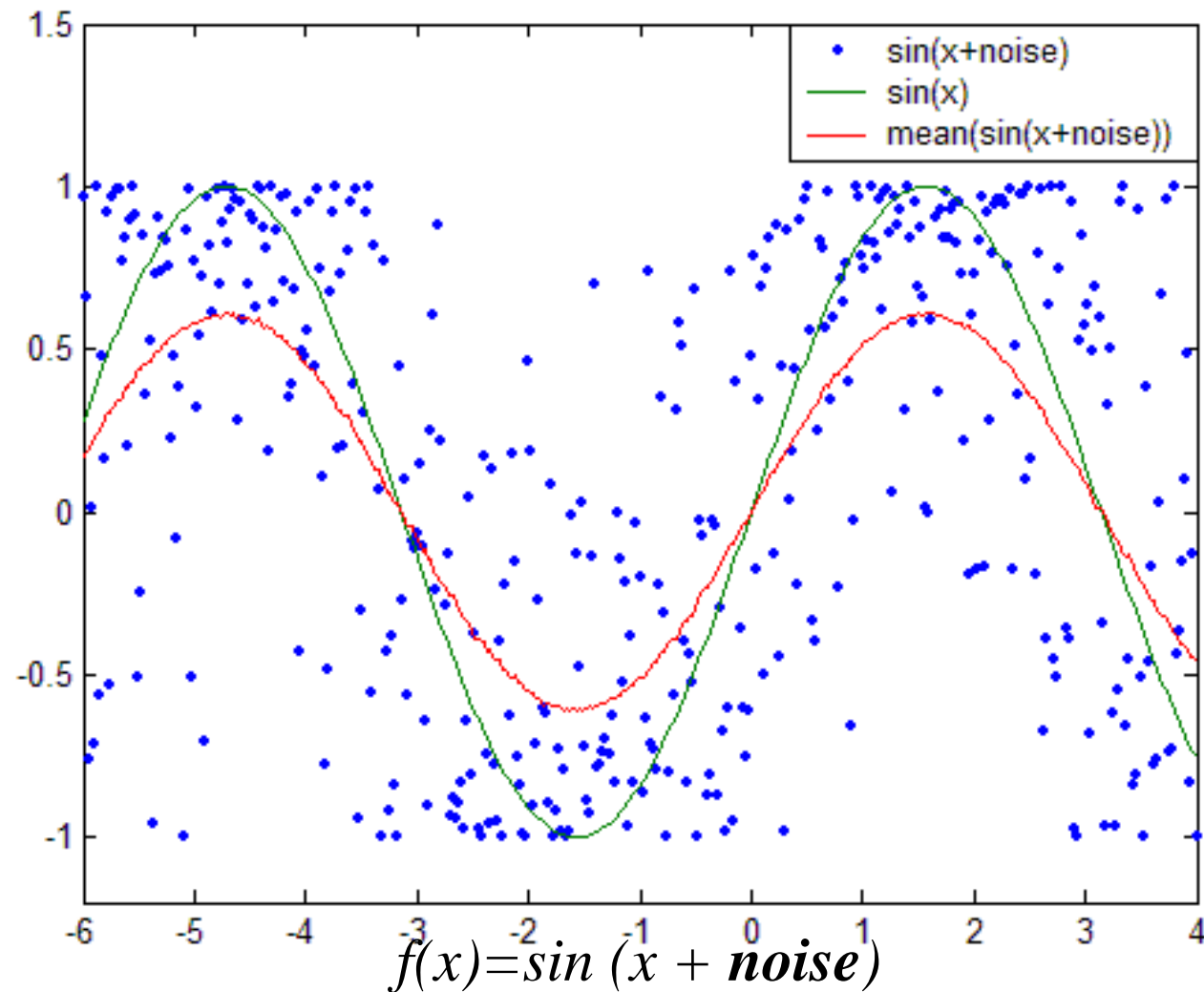


$$\frac{dK}{dt} = a_K + \frac{b_K + c_K S_{t-t_1}}{K_{t-t_2}}$$

$$\frac{dS}{dt} = a_S + \frac{b_S + c_S K_{t-t_3}}{S_{t-t_4}}$$

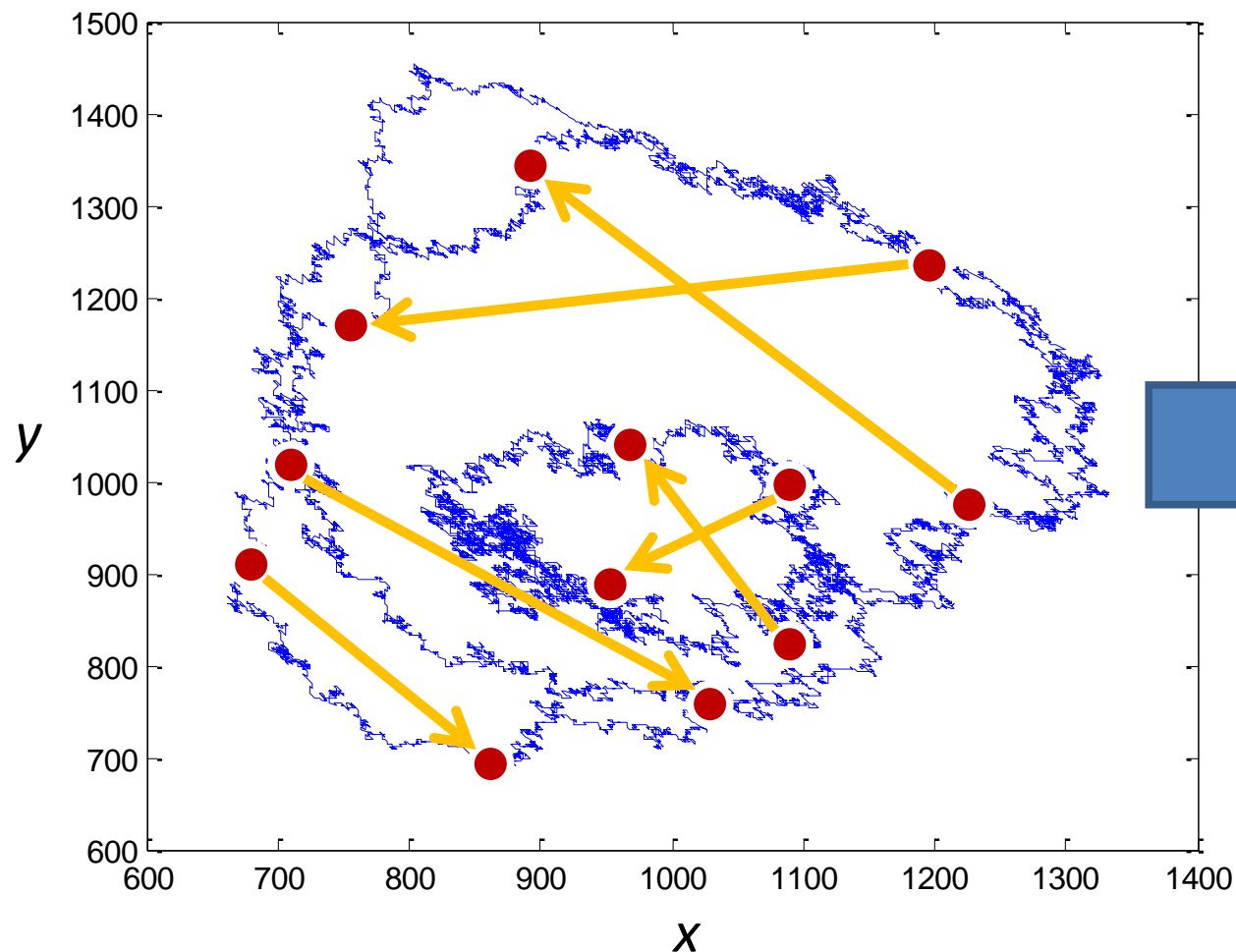


# Stochastic Models

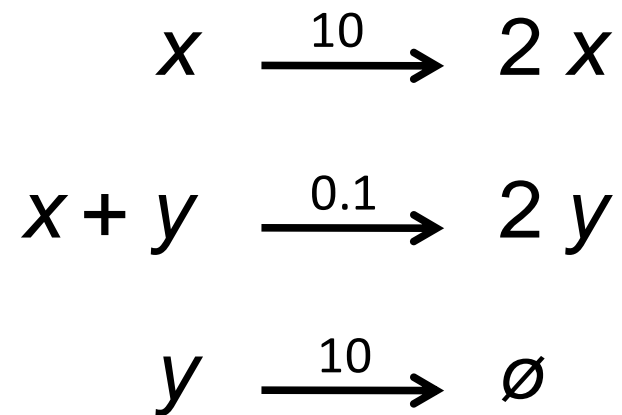


# Reaction Systems

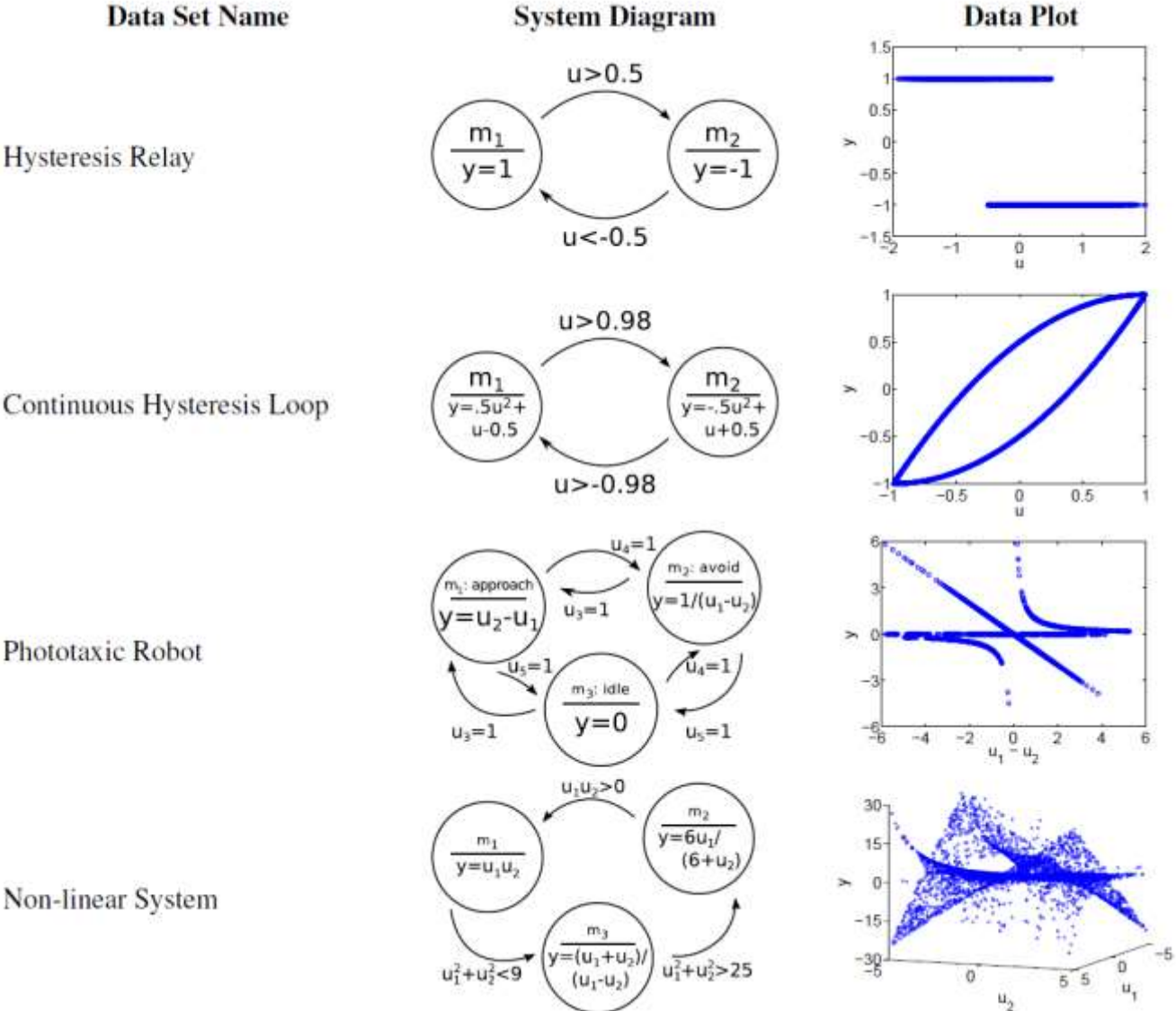
Periodic Samples of a  
Stochastic System



Maximum Likelihood  
Stochastic Model



# Hybrid Systems



$$\theta_t, \omega_t$$

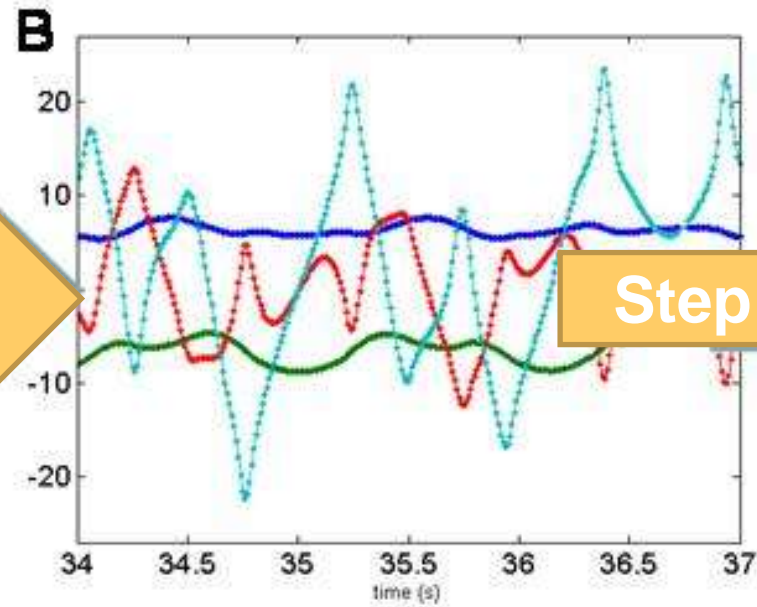


$$f(\theta_t, \omega_t) = \textit{const}$$

$$\theta_1, \omega_1, \theta_2, \omega_2$$



Step 1



Step 2

C

**Detected Invariance:**

$$L_1^2(m_1+m_2)\omega_1^2 + m_2L_2^2\omega_2^2 + m_2L_1L_2\omega_1\omega_2\cos(\theta_1 - \theta_2) - 19.6L_1(m_1+m_2)\cos\theta_1 - 19.6m_2L_2\cos\theta_2$$



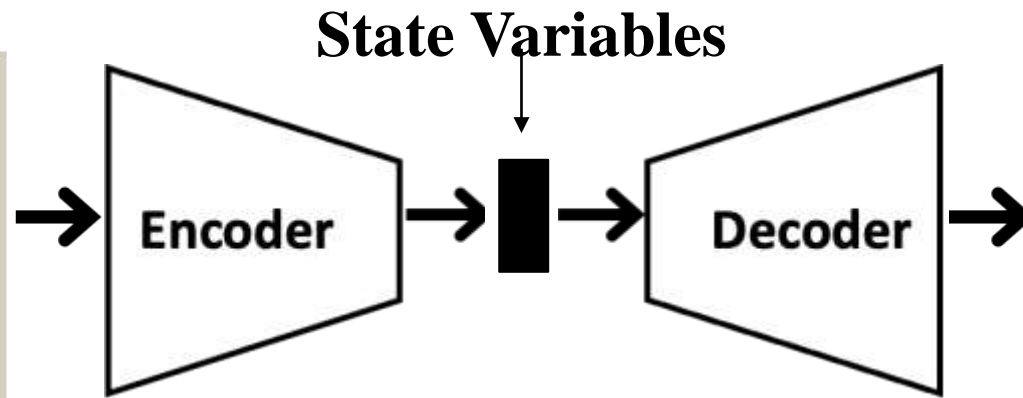
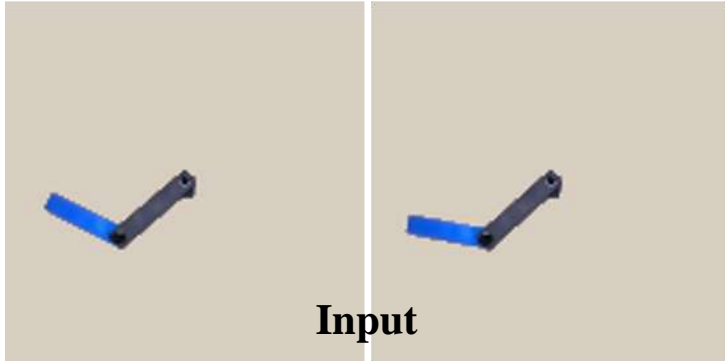


**Real**

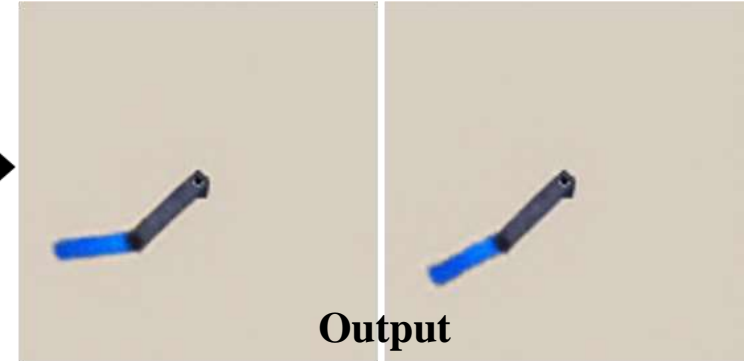


**Predicted**  
(~0.5 sec in advance)

**2<sup>nd</sup>-order system inputs two frames**



**2<sup>nd</sup>-order system predicts two frames**





4.7



**Real**

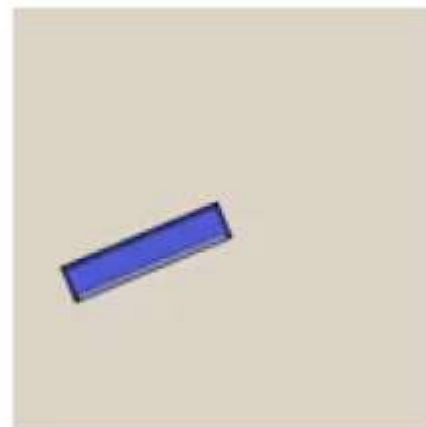
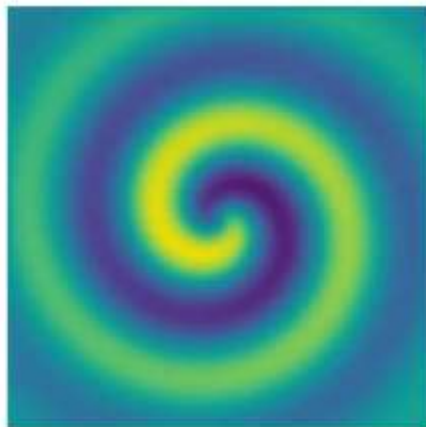
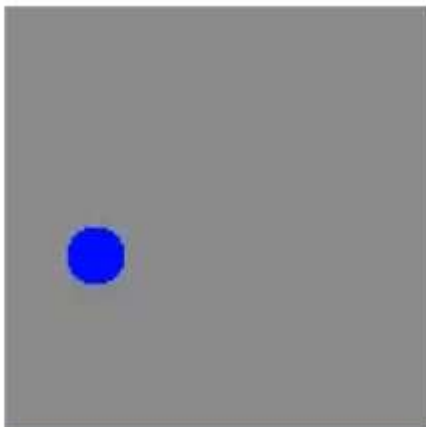
**Predicted**  
(~0.5 sec in advance)



**Real**

**24**

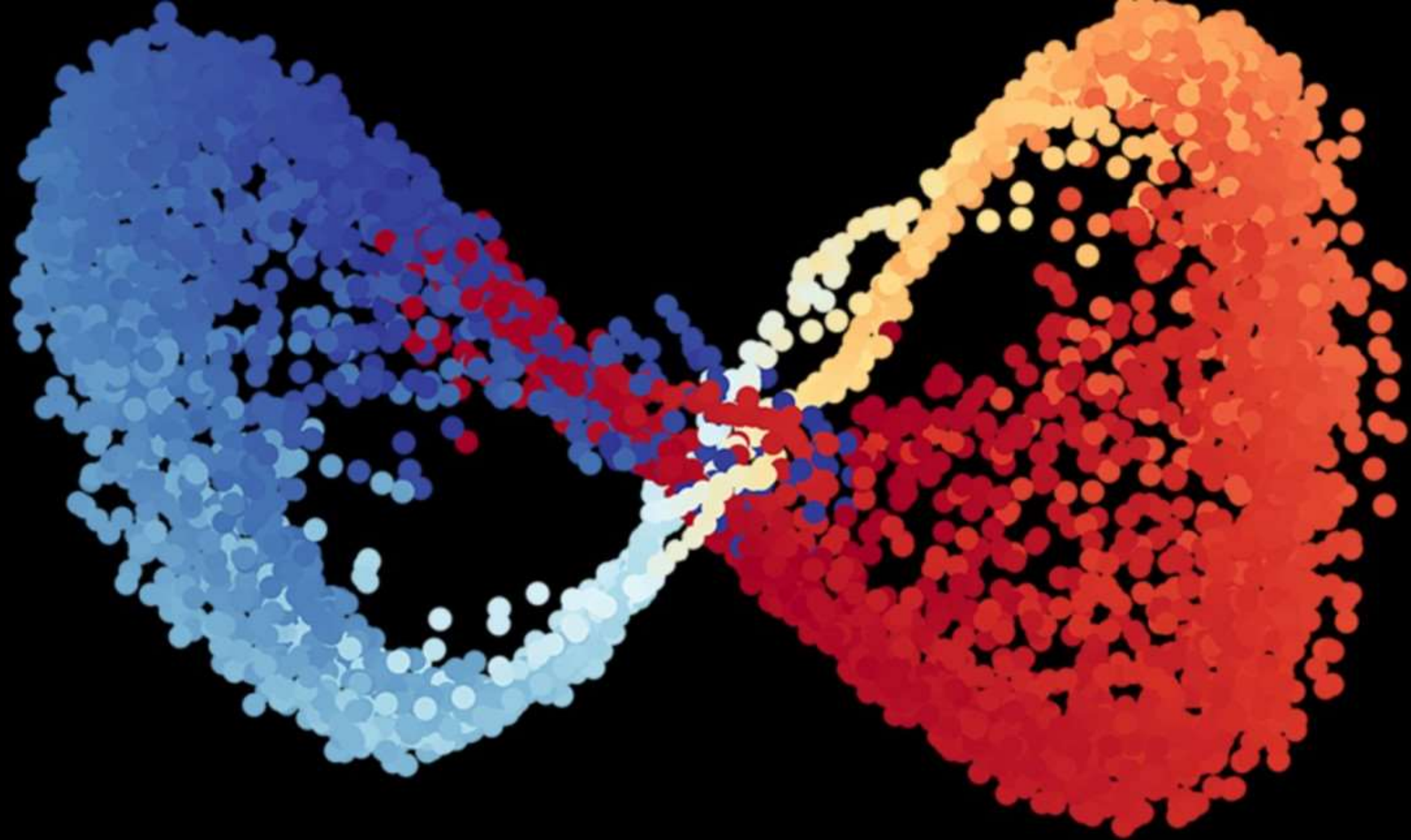
**Predicted**  
(~0.5 sec in advance)





System	ID from Latent Vectors	Ground Truth
Circular motion	2.19 ( $\pm$ 0.05)	2
Reaction diffusion	2.16 ( $\pm$ 0.14)	2
Single pendulum	2.05 ( $\pm$ 0.02)	2
Rigid double pendulum	4.71 ( $\pm$ 0.03)	4
Swing stick	4.89 ( $\pm$ 0.33)	4
Elastic double pendulum	5.34 ( $\pm$ 0.20)	6
Air dancer	7.57 ( $\pm$ 0.13)	Unknown
Lava lamp	7.89 ( $\pm$ 0.96)	Unknown
Fire	24.70 ( $\pm$ 2.02)	Unknown









$$f=ma$$

1687

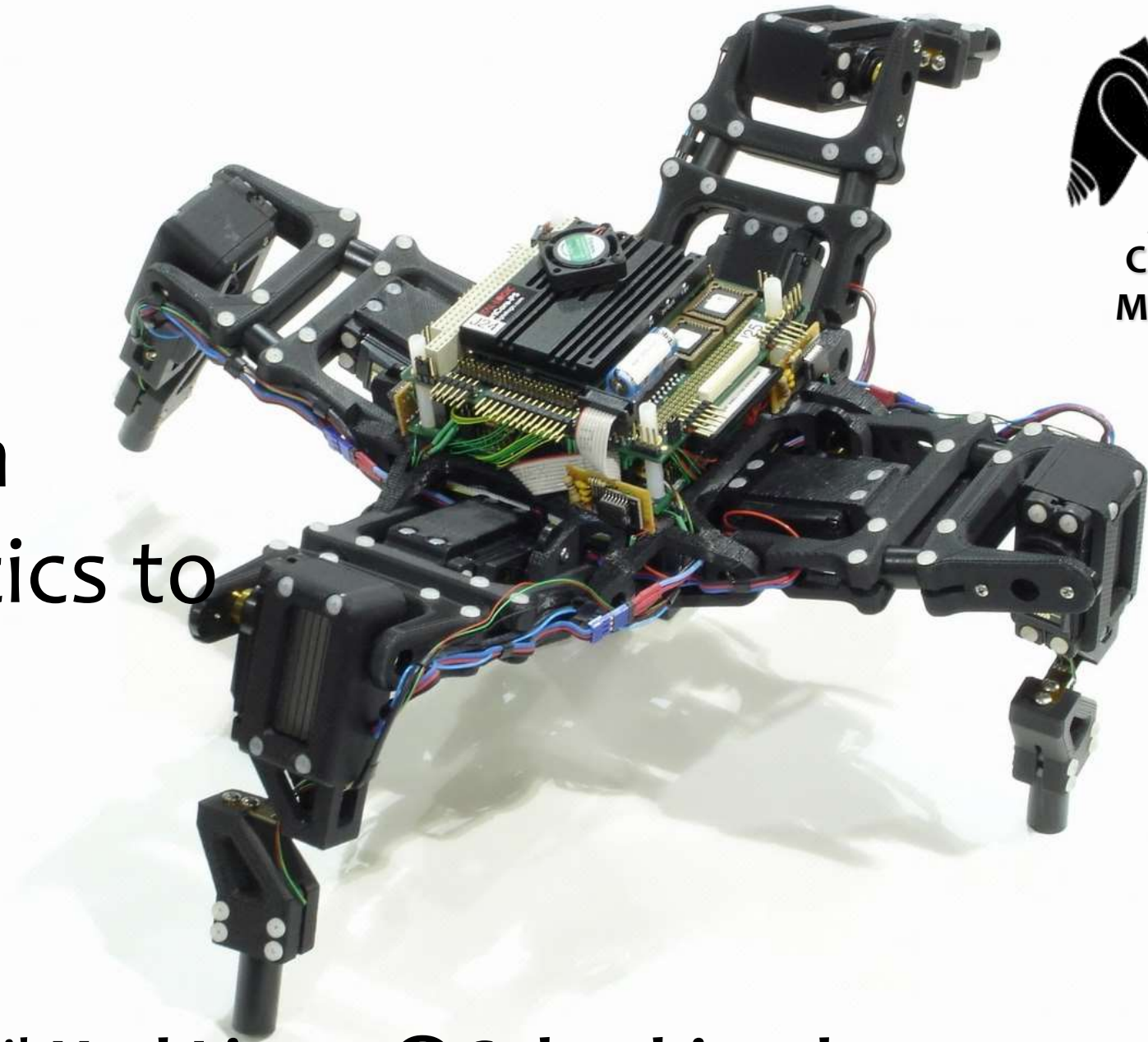
# The New York Times

**“... Theoretical physicists are not yet  
obsolete, but scientists have taken steps  
toward replacing themselves ...”**



# The AI Scientist

Automating  
discovery, from  
cognitive robotics to  
computational  
biology



Creative  
Machines  
Lab

For copy of slides email **Hod.Lipson@Columbia.edu**

This research was supported in part by

