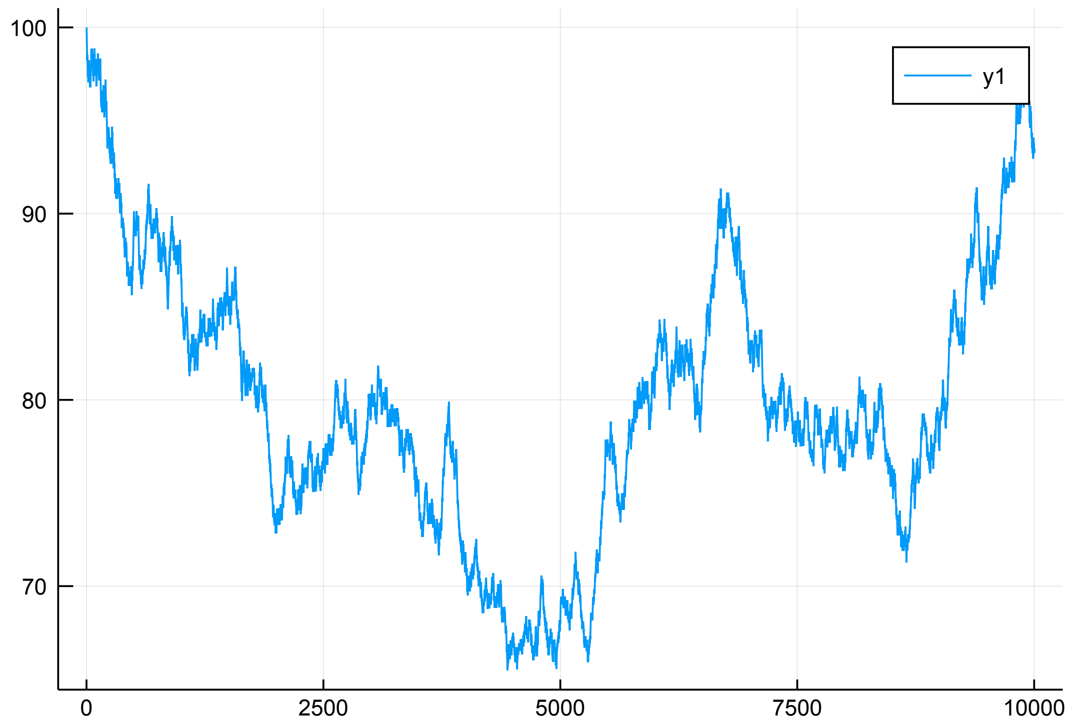
StockProblemAnswers

September 1, 2018

0.1 Problem 1 a)

In [2]:
```
using Plots

S = 100
r = 0.08
T = 1
n = 10000
sigma = 0.3
h = T/n

u = exp(r*h + sigma * sqrt(h))
d = exp(r*h - sigma * sqrt(h))

p_star = (exp(r*h) - d) / (u - d)

path = Array{Float64}(undef, n + 2)

#add in the starting price
path[1] = S

for k in 2:n+2
    if rand() < p_star
        #then we go up
        path[k] = path[k-1] * u
    else
        path[k] = path[k-1] * d
    end
end

plot(path)
```

Out[2]:

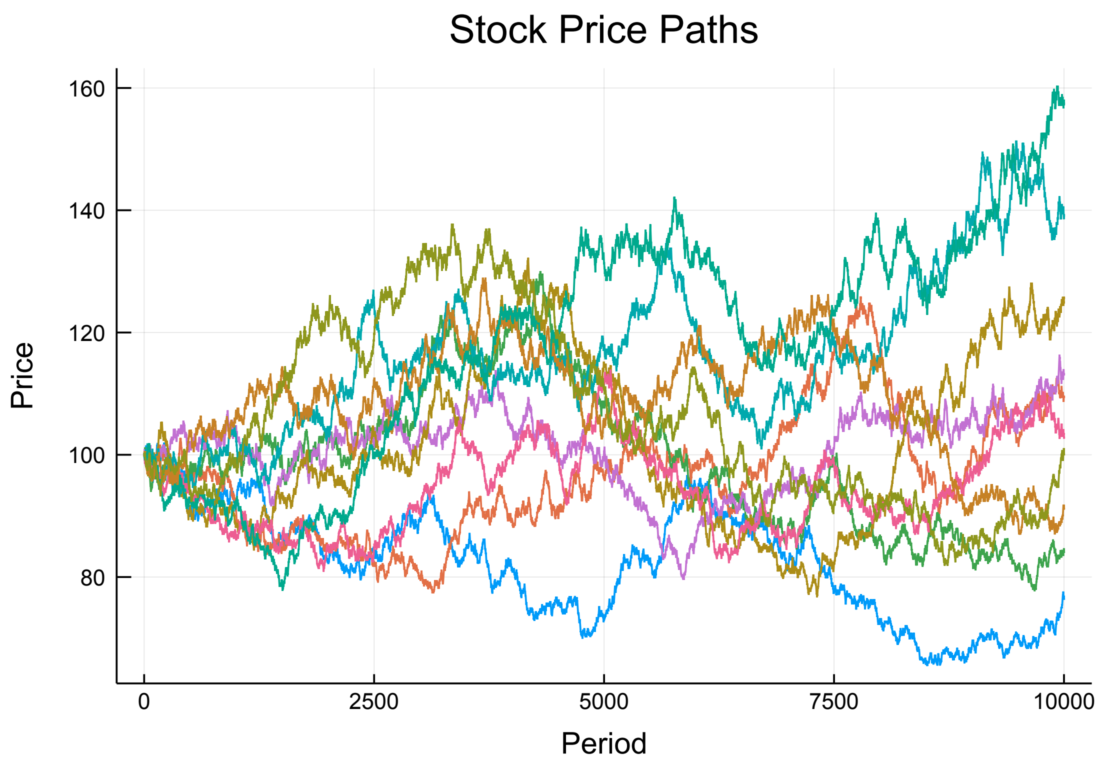


0.2 Problem 1 b)

```
In [9]: using Plots

        function createPath(S, r, sigma, T, n)
            h = T / n
            u = exp(r*h + sigma * sqrt(h))
            d = exp(r*h - sigma * sqrt(h))

            p_star = (exp(r*h) - d) / (u - d)

            path = Array{Float64}(undef,n + 2)

            #add in the starting price
            path[1] = S

            for k in 2:n+2
                if rand() < p_star
                    #then we go up
                    path[k] = path[k-1] * u
                else
                    path[k] = path[k-1] * d
                end
```

```
    end
    path
end

p = plot(createPath(100.0, 0.08, 0.3, 1.0, 10000));
plot!(p, title="Stock Price Paths", xlabel="Period", ylabel="Price", legend=false)

for k in 1:9
    plot!(p, createPath(100.0, 0.08, 0.3, 1.0, 10000))
end
p
```

Out[9]:



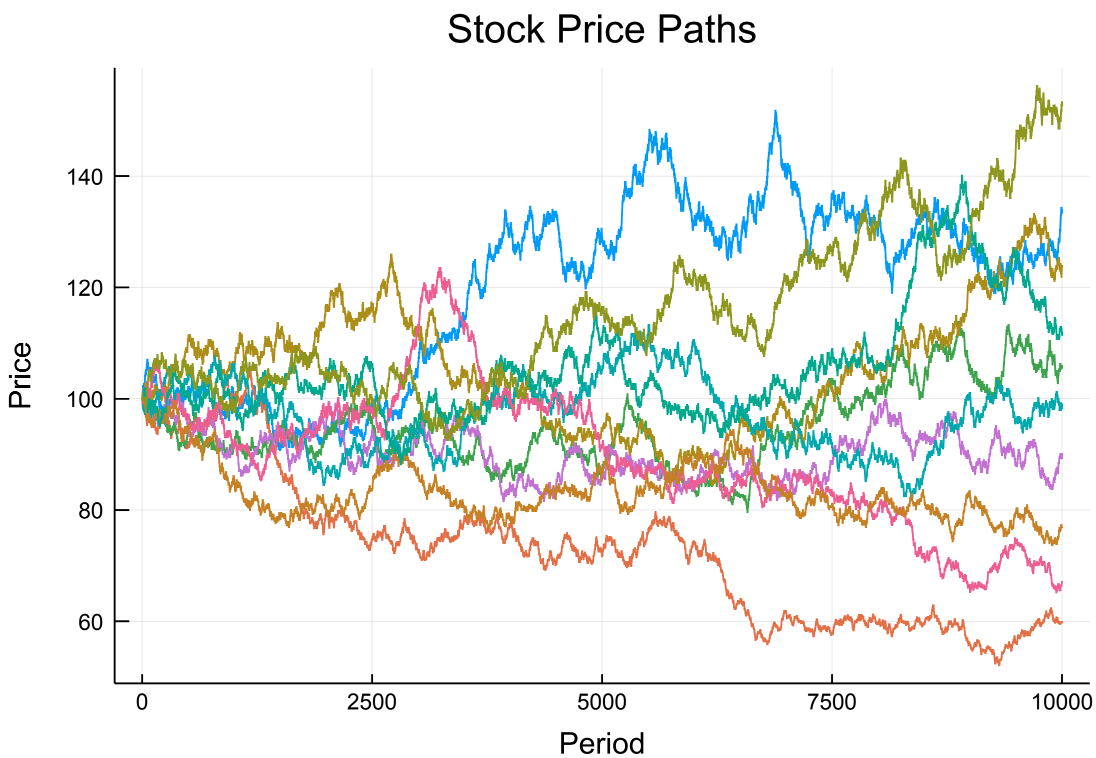
0.3 1 c) (Optional)

In [10]:

```
# Threads
# This will work on shared memory (multicore) machines

paths = Array{Array{Float64}}(undef,10)

Threads.@threads for i in 1:10
    paths[i] = createPath(100.0, 0.08, 0.3, 1.0, 10000)
end
```

```julia
p = plot(paths[1]);
plot!(p, title="Stock Price Paths", xlabel="Period", ylabel="Price", legend=false)

for k in 2:10
    plot!(p, paths[k])
end
p
```

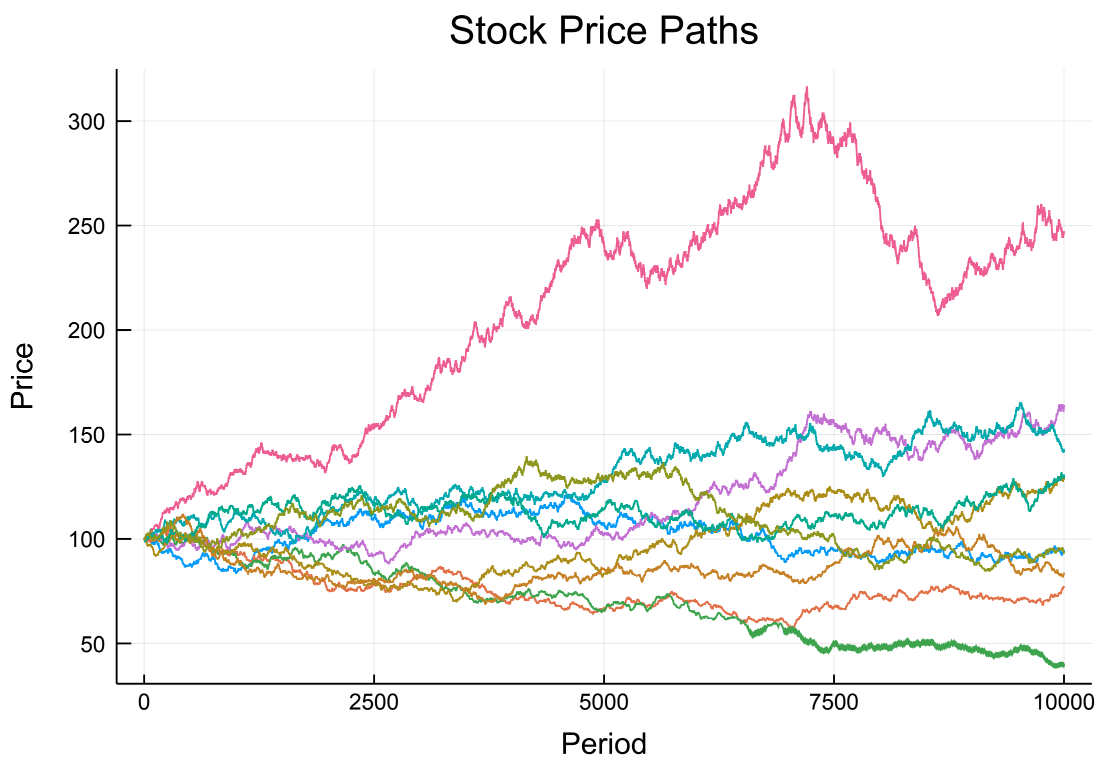
Out[10]:



In [14]:
```julia
# Distributed parallelism
# This will work across multiple computers and multiple nodes of a cluster

using Distributed

@everywhere function createPath(S, r, sigma, T, n)
    h = T / n
    u = exp(r*h + sigma * sqrt(h))
    d = exp(r*h - sigma * sqrt(h))

    p_star = (exp(r*h) - d) / (u - d)

    path = Array{Float64}(undef, n + 2)
```

```
    #add in the starting price
    path[1] = S

    for k in 2:n+2
        if rand() < p_star
            #then we go up
            path[k] = path[k-1] * u
        else
            path[k] = path[k-1] * d
        end
    end
    path
end

f = (i) -> createPath(100.0, 0.08, 0.3, 1.0, 10000)
addprocs()
paths = pmap(f,1:10)

p = plot(paths[1]);
plot!(p, title="Stock Price Paths", xlabel="Period", ylabel="Price", legend=false)

for k in 2:10
    plot!(p, paths[k])
end
p
```

Out[14]:



0.4 2 a)

The probabilities follow the binomial formula. Let S_T denote the final price.

$$\mathbb{P}[S_T = u^{n-k} d^k S] = \binom{n}{k} p^{*(n-k)} (1 - p^*)^k$$

0.5 2 b)

for $n = 100$

```
In [16]: function computeDistribution(S, r, sigma, T, n)

             h = T / n
             u = exp(r*h + sigma * sqrt(h))
             d = exp(r*h - sigma * sqrt(h))

             p_star = (exp(r*h) - d) / (u - d)

             prices = Array{Float64}(undef, n + 1)
             probs = Array{Float64}(undef, n + 1)
             for k in 1:n+1
                 prices[k] = S * (u ^ (n-k)) * (d ^ k)
                 probs[k] = binomial(BigInt(n), BigInt(k)) * p_star^(n-k) * (1 - p_star)^k
```

```
    end

    scatter(prices, probs)
    title = "Distrbution for Binomial Model with $n periods \n S = $S, r = $r, sigma =

    scatter!(title = title, xlabel= "Stock Price", ylabel = "Probability", legend = fa
end

computeDistribution(100.0, 0.08, 0.3, 1.0, 100)
```

Out[16]: