

BasicProblems

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0.1 Starter Problems

Note: The 4 * problems are from https://lectures.quantecon.org/jl/julia_by_example.html

Strang Matrix Problem Use Julia's array and control flow syntax in order to define the $N \times N$ Strang matrix:

$$\begin{bmatrix} -2 & 1 & & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & 1 \\ & & & 1 & -2 \end{bmatrix}$$

i.e. a matrix with -2 on the diagonal, 1 on the off-diagonals, and 0 elsewhere.

Factorial Problem* Using a `for` loop, write a function `my_factorial(n)` that computes the n th factorial. Try your function on integers like `15`

Bonus: Use `BigInt` inputs like `big(100)`. Make your function's output type match the input type for `n`. You'll know that you'll successfully matched the input type if your output does not "overflow" to negative, and you can check `typeof(x)`. Hint, you may want to initialize a value using `one(x)`, which is the value `1` in the type that matches `x`.

Binomial Problem* A random variable $X \sim \text{Bin}(n, p)$ is defined the number of successes in n trials where each trial has a success probability p . For example, if `Bin(10, 0.5)`, then X is the number of coin flips that turn up heads in 10 flips.

Using only `rand()` (uniform random numbers), write a function `binomial_rv(n, p)` that produces one draw of $\text{Bin}(n, p)$.

Monte Carlo π Problem* Use random number generation to estimate π . To do so, mentally draw the unit circle. It is encompassed in the square $[-1, 1] \times [-1, 1]$. The area of the circle is $\pi r^2 = \pi$. The area of the square is 4. Thus if points are randomly taken evenly from $[-1, 1] \times [-1, 1]$, then the probability they land in the circle ($x^2 + y^2 \leq 1$) is $\frac{\pi}{4}$. Use this to estimate π .

0.2 Integration Problems

These problems integrate basic workflow tools to solve some standard data science and scientific computing problems.

Timeseries Generation Problem* An AR1 timeseries is defined by

$$x_{t+1} = \alpha x_t + \epsilon_{t+1}$$

where $x_0 = 0$ and $t = 0, \dots, T$. The shocks ϵ_t are i.i.d. standard normal ($N(0, 1)$), given by `randn()`. Using $T = 200$

1. $\alpha = 0$
2. $\alpha = 0.5$
3. $\alpha = 0.9$

use `Plots.jl` to plot a timecourse for each of the parameters. Label the lines for the values of α that generate them using the `label` argument in `plot`.

Logistic Map Problem The logistic difference equation is defined by the recursion

$$b_{n+1} = r * b_n(1 - b_n)$$

where b_n is the number of bunnies at time n . Starting with $b_0 = .25$, by around 400 iterations this will reach a steady state. This steady state (or steady periodic state) is dependent on r . Write a function which plots the steady state attractor. This is done as follows:

- 1) Solve for the steady state(s) for each given r (i.e. iterate the relation 400 times).
- 2) Calculate "every state" in the steady state attractor. This means, at steady state (after the first 400 iterations), save the next 150 values. Call this set of values $y_s(r)$.
- 3) Do steps (1) and (2) with $r \in (2.9, 4)$, $dr = .001$. Plot r x-axis vs $y_s(r)$ =value seen in the attractor) using `Plots.jl`. Your result should be the [Logistic equation bifurcation diagram](#).