Error Analysis of an Algorithm for Magnetic Compensation of Aircraft

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Abstract

The Tolles and Lawson equations were programmed on the IBM 370-65 computer. These equations predict the magnetic signals which are generated by the permanent moments, the induced moments and the eddy-current terms of an aircraft as it maneuvers in the Earth's magnetic field. The least mean squared (LMS) solution of these equations, which was developed for the microprocessor compensation program, was also simulated on the IBM 370 computer. By measuring the simulated figure of merit (FOM) before and after applying compensation, several types of error were studied. It was found that the error introduced by truncation effects on a 16-bit microprocessor limits the FOM to 50 milligammas. Since this figure is an order of magnitude smaller than the design goal, it is concluded that the algorithm is not limited by the microprocessor capacity. Other studies show that although the baseline model is insensitive to bearing separation, the bearing separation should be 90° ± 7.5° if 3 iterations are used in the post processor and 90° ± 15° if 5 iterations are used.

Although the algorithm is robust and relatively insensitive to various types of noise, experimental evidence suggests that hysteresis can cause temporal variations in the Tolles and Lawson model. It is suggested that whitening and/or adaptive techniques could be used to minimize such variations.

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I. Introduction

In 1950 W.E. Tolles and J.D. Lawson[1] first reported on the problem of maneuver noise associated with the magnetic field of magnetic airborne detection (MAD) equipped aircraft. Tolles and Lawson identified three permanent, five induced, and eight eddy-current fields as three separate sources of magnetic interference associated with air frame maneuvers. An exact solution to those equations has been found when the aircraft maneuvers are small [1]. Digital effects and additive random noise produce only small errors. However, the Tolles and Lawson equations may not completely describe noise due to aircraft maneuvers. Effects such as hysteresis which cause a time-varying field produce time variations in Tolles and Lawson coefficients. This can cause the coefficients to vary with maneuver frequency. These effects can be minimized by employing whitening filters [1].

Another source of error is caused by motion in the Earth's gradient field. The gradient field effects can be compensated in part by altitude compensation. During a turn maneuver, horizontal gradients can cause significant errors. The figure of merit (FOM) tests summarized in Table I were conducted under close to ideal conditions. The aircraft autopilot performed the training maneuvers and the FOM flight was made immediately after the training flight. This minimized magnetic hysteresis effects. To study the stability of the compensation and possible hysteresis effects, training data from each day was tested against FOM runs of different days during the week of October 6, 1975.

Table I shows that a single takeoff and landing of the aircraft can have a significant effect on its magnetic properties. The FOM of about 0.5 gamma is observed under ideal conditions, but due to hysteresis this FOM increases from 0.5 to a neighborhood of 1.2 to 2.0 gammas after one takeoff and landing. This data suggests that when FOMs of less than 1.5 or so are required, the compensation of coefficients will have to be updated on each flight. This could either be done by a dedicated compensation trim flight or by an adaptive algorithm which continuously updates coefficients for random flight paths.

The errors to be considered in this study are due to the finite word length of the computer and additive random noise. These errors are an order of magnitude less than those shown in Table I. That means that FOMs of less than 0.05 gamma are obtained. In order to test the software a digital simulation of the Tolles and Lawson equations was made on the IBM 370 computer. This simulation considered the effects of permanent, induced, and eddy-current terms and gave the aircraft either pure or mixed maneuvers as specified by data input cards. Fig. 1 shows the results of this simulation for the microprocessor program as implemented in 16-bit words. It is shown in the curve that if the sample rate drops below 6 Hz or increases

[1] W.E. Tolles and J.D. Lawson, "Magnetic compensation of MAD equipped aircraft," Airborne Instruments Lab., Inc., Mineola, N.Y., Rep. 201-1, June 1950. No longer available.

TABLE I
Hysteresis Effects in Gammas on Compensated FOM

Date of Training	Date of FOM Flight Oct. 6	Oct. 7	Oct. 8	Oct. 10
Oct. 6	0.45	1.04	1.16	1.38
Oct. 7	0.73	0.50	1.21	1.23
Oct. 8	1.32	1.24	0.53	2.03
Oct. 10	1.26	1.23	1.95	0.56

above 12 Hz, the microprocessor program fails to compensate the data. This is not due to any inherent limitation of the algorithm or the mathematical solution but rather in the fixed word length of the processor. A future generation of this software could dynamically scale the input data so that the compensation would be effective over all sample rates.

The curve labeled 370 generation software was a simulation of these equations on the IBM 370 computer using floating-point 32-bit arithmetic. The solution is mathematically ideal and the FOM should tend to zero, independent of sample rate. However, we see that the curve tends to flatten out at a sample rate around 10 Hz with an FOM less than 8 milligammas. Then it begins to increase slowly as the sample rate is increased beyond 10 Hz. This dependency of the FOM on sample rate is due to the method of estimating the first derivative data. We find that we are approximating the derivative by the first difference, so that at low sample frequencies the time between samples increases causing larger errors. On the other hand, at the high frequency range the sample interval becomes finer and finer. From one data point to the next the change approaches the least significant bit of the computer and again we begin to lose accuracy. The increased error at 20 Hz is due to the finite word length of the 370 computer.

The microprocessor software on the other hand has an FOM of around 50 milligammas in the 6- to 12-Hz range. This is because truncation effects are limiting the FOM. However, this FOM is an order of magnitude below those FOMs that can be obtained in the field and would not be observed in actual operating systems. This report is largely dedicated to design curves which show that the implementation of the mathematical solution is close to the mathematical model. It is not expected that measurements made with real world data would be as small as the errors that are shown here. Curves such as the one shown in Fig. 1 result from noise-free parameter studies. This means that the error is due only to digital noise induced because of finite word length implementation. The second set of curves generated is performance versus random noise and here small amounts of random noise were added to the various sensors. Again, it is felt that these random noises that were added in the study were much smaller than noise that would be measured in the real world. This is largely because in addition to the random noise, the real world has hysteresis or time variation in the Tolles and Lawson coefficients. In addition, real world data would have gradient effects due to imperfect altitude compensation and/or

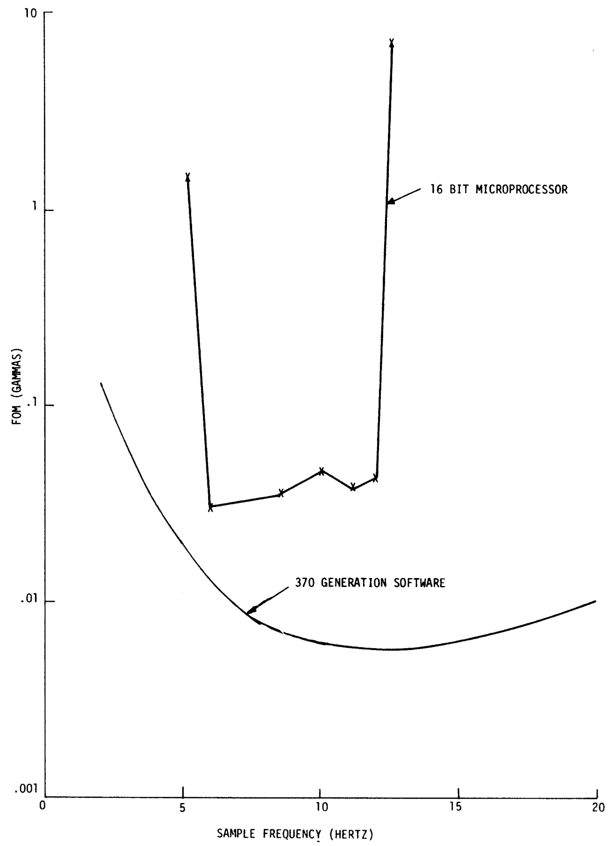

Fig. 1. FOM as a function of sample rate mixed maneuver and 3 iterations.

horizontal gradients.

The lower curve in Fig. 1 can be used as a baseline to determine the sample rate. At 8 Hz an FOM of 8 milligammas is obtained; correspondingly, at 5 Hz and 2 Hz FOMs of 20 milligammas and 120 milligammas are observed. The 16-bit design shows a sample rate of 8 Hz and the final implementation has an FOM of 50 milligammas. The 370 generation of the software could in principle obtain an FOM of 20 milligammas by paying more detailed attention to the scaling and the word lengths of the imput data and subsequent calculations.

II. Noise-Free Parameter Study

Errors in the noise-free case can be segmented into three parts. The first part is due to an error in the analytical solution of the Tolles and Lawson equation. The second source of error is errors due to the finite word length of the floating-point arithmetic in the 370 computer which was used to simulate the mathematical solution. The third source of error is truncation or scaling errors in the 16-bit microprocessor implementation used in the airborne system. Since the mathematical solution is exact we will neglect the first error source.

The subsequent simulations verify that this can be done, since the errors can always be reduced by improving the computational accuracy. Fig. 2(A) gives an example of the errors due to the implementation on the 370 computer.

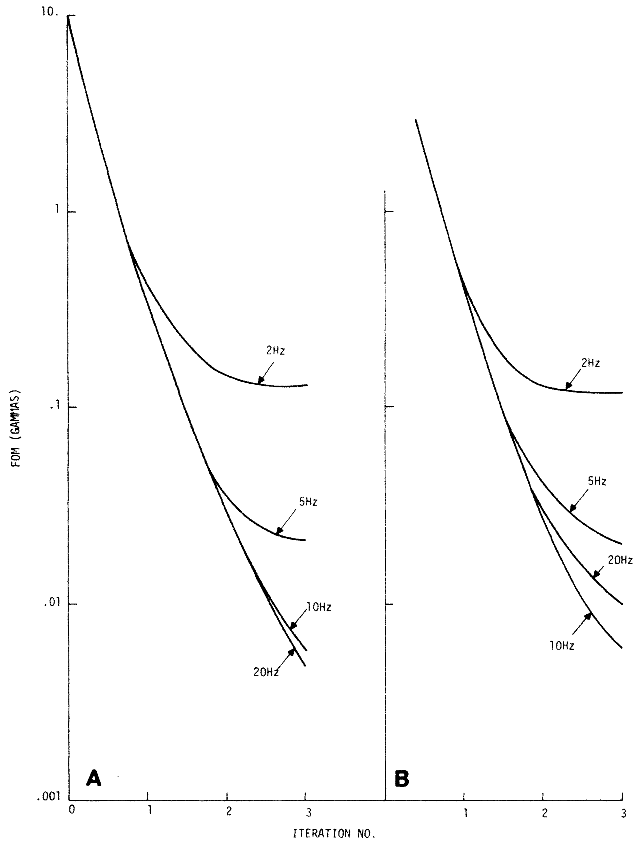

Fig. 2. FOM versus number of iterations for various sample rates (noise-free simulation of 32-bit machine). (A) Pure maneuver. (B) Mixed maneuver.

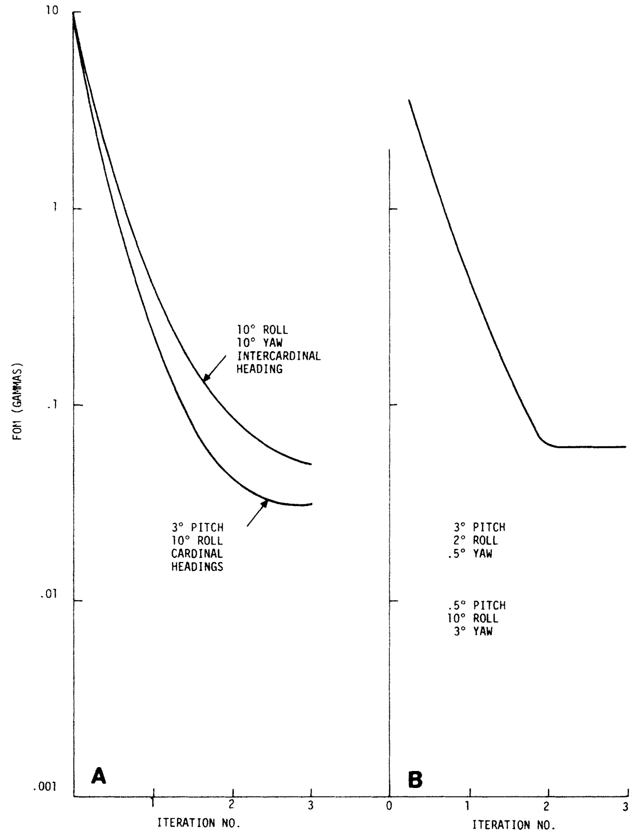

Fig. 3. FOM versus number of iterations at 10-Hz sample frequency (noise-free simulation of 16-bit implementation). (A) Pure maneuver. (B) Mixed maneuver.

The parameters here are the number of iterations for 4 different sample rates versus the FOM which is used for the goodness of criteria. In Fig. 2(A) the maneuvers were assumed to be pure. The airframe maneuvers were 3° in pitch and 10° of roll along cardinal headings. As we see in the figure the FOM gets lower as the sampling rate is increased and the lowest FOMs were obtained at 20 Hz. This is due to the error introduced by the finite difference in estimating the derivative of the vector cosine data which is used in the estimation of the eddy-current terms.

Fig. 2(B) shows the same effect but this time the maneuvers were changed from pure maneuvers to mixed maneuvers. The first maneuvers were 3° pitch, 2° roll, and 0.5° yaw. The second maneuver was 0.5° pitch, 10° roll, and 3° yaw. Instead of flying cardinal headings the data in Fig. 2(B) were taken with the box offset 25° from the cardinal direction. Using the 32-bit machine we again find that usually the higher the sampling rate the lower the FOM. However, in this case the 10-Hz sampling rate produces a lower FOM than the 20-Hz sampling rate. This is probably due to the fact that the samples at 20 Hz were finely spaced and the differences between samples exceeded the finite word lengths of the 370 computer. Therefore we are seeing the effect of truncation error rather than the error caused by too large a spacing between the samples.

The comparison of Fig. 2(A) and (B) shows that the mixed maneuvers and flying on cardinal headings have very little effect on the FOM. The governing factors are the number of iterations and the sample rate. It was found that the FOM decreases until the number of iterations reaches about 3 or 4 beyond which no decrease was found by increasing the number of iterations. In a case of 2 Hz no improvement was seen after two iterations. Again we are being limited not by iterations but by the finite time interval between samples and the associated error in estimating the derivative. For noise-free data the 2-Hz sample rate could be used if one were to use a polynomial or least mean square (LMS) technique to estimate the derivative from the sampled data. This was not done but the sample rate was increased for the 16-bit hardware. The sample rate of 8 Hz was chosen.

Fig. 3 shows the similar errors in the microprocessor implementation software. In comparison with Fig. 2 we see than the microprocessor implementation performed better than 32-bit floating-point implementation with a 2-Hz sample rate but not as well as an implementation using floating point and a 5-Hz sample rate. Fig. 3(A) was constructed from the case of pure maneuvers. Here two cases were considered. The first case was 3° pitch and 10° yaw along cardinal headings. In the second case, intercardinal headings were considered and the pitch maneuver was eliminated. In this case 10° of roll and 10° of yaw were the two maneuvers used for the training data. We see that FOM of less than 100 milligammas after three iterations is possible in both cases. And it may be concluded that training can be done with either cardinal or intercardinal headings. If one uses intercardinal headings the pitch maneuver could be replaced by a yaw maneuver. In Fig. 3(B) the 16-bit software was tested using a mixed maneuver. As before, the first maneuver consisted of 3° pitch, 2° roll, and 0.5° yaw. The second maneuver consisted of 0.5° pitch, 10° roll, and 3° yaw. It is observed that no improvement was obtained after two iterations. Again the mixed maneuver provided an FOM less than 0.1 gamma. In all cases these FOMs are lower than the 0.5 gamma that would be observed in the actual practice.

In Figs. 2 and 3 a separation of 90° was assumed between each bearing for the maneuver training flight. In all cases the FOM of 10 gammas was compensated to below 0.1 gamma after three iterations. This means the FOM was improved by a factor of 100 to 1.

Fig. 4 shows the FOM as a function of the bearing separation for the microprocessor software. As a point of reference in this figure, the 370 simulation is also shown. A bearing separation study was conducted by 370 simulation with separations of 30°, 60°, and 90°. The 90° separation means, for example, that the aircraft was flown at 0°, 90°, 180°, and 270°. The 30° separation shows that the aircraft bearings were taken at 0°, 30°, 60°, and 90°. The figure shows that in the range from 30° to 90° bearing separation had little or no effect on the final FOM. This is in agreement with the theory which says that the maneuver flight and bearing separation does not determine the accuracy of the solution, since the solution is exact in the noise-free case.

However, when we look at the 16-bit software we find that after 2 iterations the FOM stabilized to 70 or 80 milligammas for 90° of separation. However, for 60° separation, the FOM was only 1 gamma after 2 iterations. It took 16 iterations for the FOM to reach the value of 0.2 gammas. For 75° separation the FOM stabilized to that of the 90° separation after 5 iterations. The slow conversion of the FOM as a function of separation is due to the method of solution of the matrix of equations. The Gauss-Seidel technique was used and it converges most rapidly for orthogonal separations. As the separation is decreased the iteration time increases. For the second generation software, the 370 computer relied upon a utility matrix inversion program and one does not have a problem with convergence and therefore, whether one uses 1. 2, or 3 iterations, the convergence is the same, independent of separation. The 16-bit software was structured with the assumption that the separation between bearings will be 90° ± 7.5°. Three iterations are hardwired into the post processor of the 16-bit software. If the bearing separation is to be decreased to say 75° between bearings then the number of iterations will have to be increased by an order of 5 or 6 in order to maintain a low FOM. If the bearing separation is further decreased to 60°, then the number of iterations would have to be increased further, say to 25 or 30, to obtain the same performance. If the separation is further decreased, say to 20° or 30°, then the

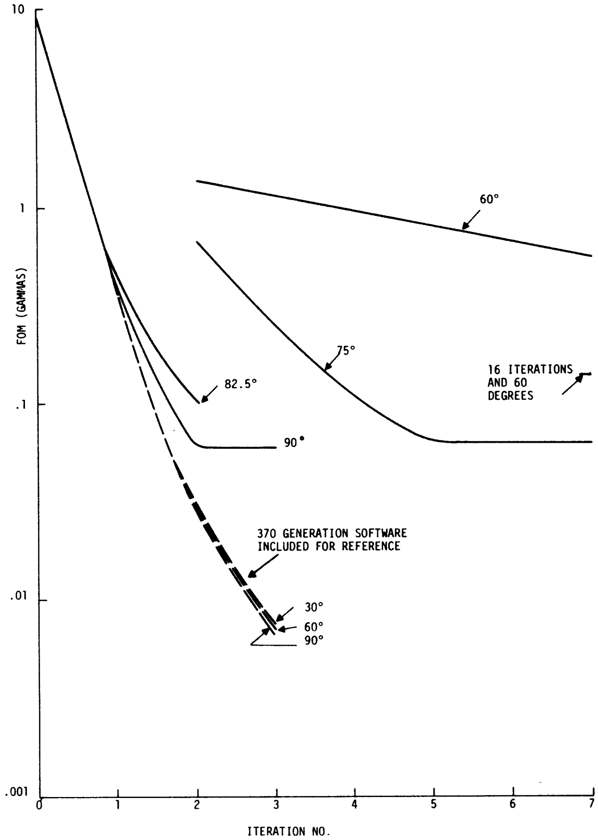

Fig. 4. FOM as a function of iteration number for various bearing separations (mixed maneuver) for the 16-bit microprocessor.

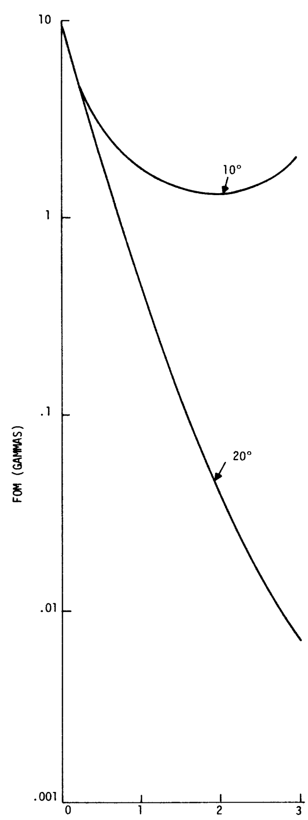

Fig. 5. FOM versus number of iterations for various bearings separation angles (noise-free).

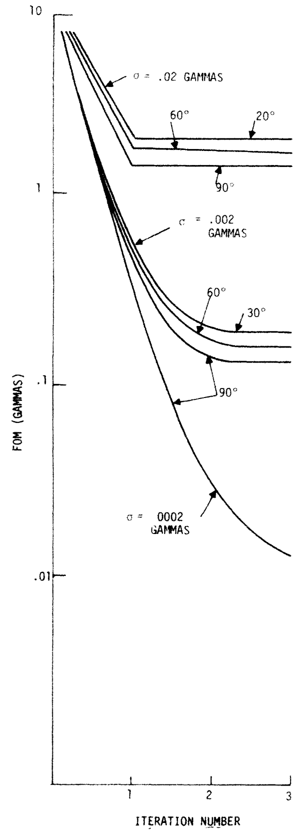

Fig. 6. FOM with noise for various bearing separation angles. Noise added to total field only.

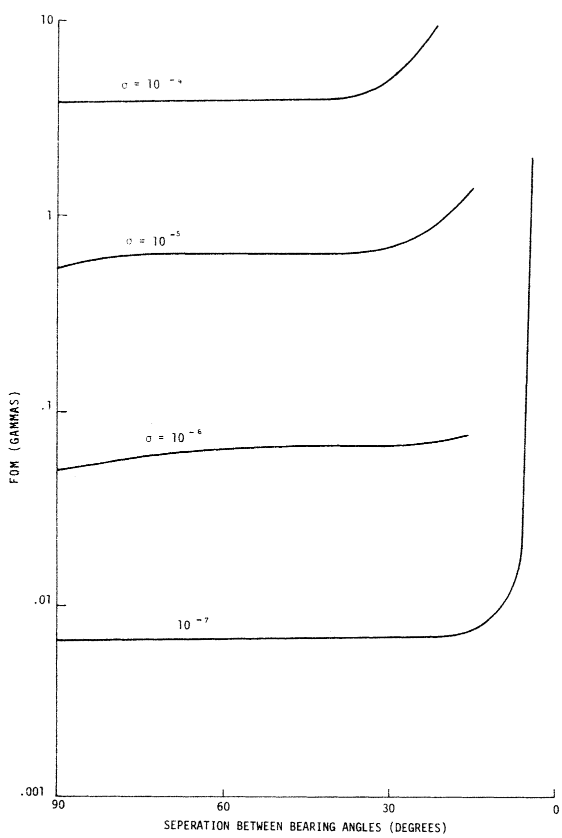

Fig. 7. FOM versus bearing separation with noise added to direction cosines.

post processor will have to be rewritten and a different algorithm employed for the matrix inversion.

As we see from looking at the 370 generation curve, the problem is not in the algorithm or inherent accuracy of storing the word, but rather in the technique used to solve the matrix of equations. This is shown in Fig. 5 where the bearing separation is decreased to 20° and an FOM of 10 milligammas was obtained in the 370 simulation. Also shown in Fig. 5 is a simulation where a bearing separation was decreased to 10°. In this case the algorithm finally broke down and FOM of only 2 gammas was obtained. This is expected because in the case where there is no separation between bearing angles the system equation is singular and no solution is possible [1]. In the case of a 10° separation the system equation is near singular and an FOM of only 2 gammas was obtained. A lower FOM can only be obtained by increasing the word length on the 370 computer. This means going to double precision arithmetic in order to solve the system equation which is now almost singular. As you recall if the bearing separation goes exactly to 0, the Tolles and Lawson equations go to a singular equation and no solution is possible. For practical purposes we say that the separation has to be 20° or better from Fig. 5. In the next section we see that the addition of noise does not alter these conclusions.

III. Performance Versus Noise

In this section we discuss the effects of noise added to the 370 simulation of the Tolles and Lawson equations. Performance is given in terms of the FOM and the LMS error as a function of the noise added for various bearing angles. Noise is added to both the total field and the direction cosines. The cosine noise is a result of noise in the vector magnetometers as well as the noise in the total field, since the total field measurement is used to normalize the vector data to obtain direction cosines.

Fig. 6 shows a continuation of the bearing-separation study. In this figure Gaussian noise has been added to the total field measurement. We find that for an rms noise level of 0.2 milligamma the FOM is essentially that of the noise-free case. However, when 2 milligammas of noise are added we find that the FOM is of the order of 10 to 30 milligammas depending upon the bearing separation. If the rms noise is increased further yet to 20 milligammas, we find that FOMs between 2 and 4 gammas are obtained. In a practical system, FOMs of the order of 50 milligammas have been measured. It is concluded from Fig. 6 that the rms noise level is of the order of 20 or 30 milligammas. Fig. 6 also shows that the FOM is relatively insensitive to the bearing separation as long as the separation is above 20° or so.

Fig. 7 shows the effects of noise added to the direction cosines. The noise was varied from 10^{-4} to 10^{-7}. It is seen that a noise level of 10^{-7} produces an FOM better than 10 milligammas; at 10^{-6} the FOM is below 0.01 gamma and at 10^{-5} the FOM is less than 1 gamma. These curves are very flat and do not depend upon the separation between bearing angles from 90° down to 30°. It can be concluded from these curves that the algorithm is not sensitive to bearing angle separation. The microprocessor implementation requires that the separation be 90° ± 7.5° in order to converge with only three iterations. As you recall the tolerance can be relaxed to 90° ± 15° if the number of iterations is increased to 5.

Finally, a study was made to determine if the integration time could be increased to offset the effects of noise, in a low signal-to-noise ratio environment. The result of this study is shown in Fig. 8. When noise is added to the direction cosines, increasing integration time does not improve the FOM. Here large amounts of noise ($10^{-4}\ \sigma$) were added to the direction cosines. In this case the observed rms error was 9 gammas as integration was varied from 10s to 300s. In the second experiment a σ of 10 milligammas was added to the total field measurement. In this case we see that the rms error decreased from 0.2 gammas to 0.02 gammas as integration time was increased from 10 s to 1000 s. This varies inversely as the square root of time as shown by the curve. However, there is a large amount of variance in the curve as shown in the scattering of the data points. It can be concluded that although increasing the integration time will improve the FOM somewhat, the improvement is not great and that increasing the integration factor time by a factor of 10 or 100 will not produce a corresponding reduction in the FOM.

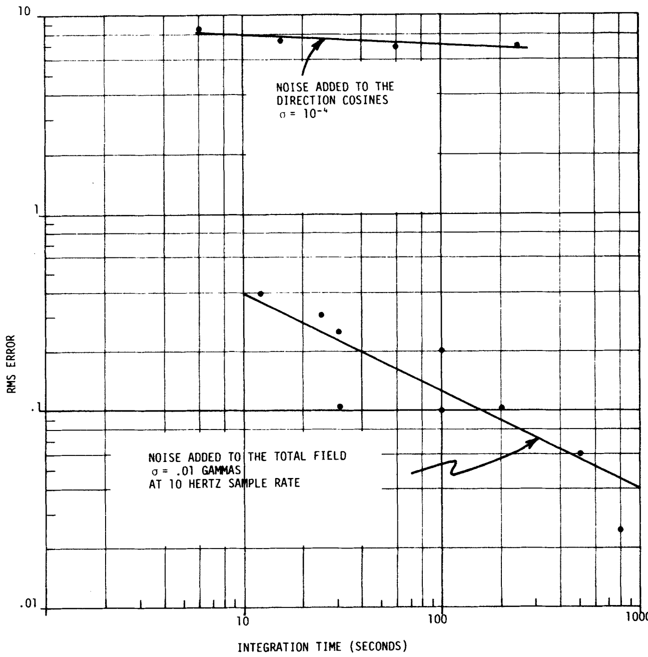

Fig. 8. Error in compensation coefficients.

IV. Conclusions

The major conclusion of this study is that hysteresis limits the performance of the compensation algorithm. It is found that an FOM of ½ gamma can be obtained when the FOM flight is made immediately after the training flight. After a single takeoff and landing, however, the FOM increases to 1.5 gammas. This implies that the Tolles and Lawson coefficients vary as a function of time. The mathematical model used in the microprocessor algorithm assumes that the Tolles and Lawson coefficients are constant and do not vary with time. This is a basic limitation. One way this could be overcome would be to use an adaptive technique to solve for the coefficients so that when they vary with time they could be continuously updated during flight. This requires a reformulation of equations and possibly the application of Kalman filtering techniques.

The errors analyzed in this study relate to a mathematical solution of the Tolles and Lawson equations when they do not vary with time. Therefore these errors are for the most part very small compared to the errors that one would actually encounter during an actual mission. This report does show, however, that the mathematical solution is not very sensitive to the bearing angle or the mixture of maneuvers. It is also insensitive to the separation between bearing angles. However, the 16-bit algorithm, because of the implementaion of matrix inversion, requires a large number of iterations if the separation btween bearing angles deviates much from 90°. This problem could be alleviated by either increasing the number of iterations or by rewriting the matrix inversion program. However, this would not produce any improved performance in the presence of hysteresis. To obtain an improved performance against hysteresis, one would have to use some kind of adaptive technique or possibly whitening filters in the design algorithm. Whitening filters have been shown to be effective in reducing the effects of frequency variations. The maneuver frequency with manual pilot maneuvers is roughly a factor of 2 lower than that of the P3-C autopilot when performing FOM tests. Variations in the solution of the compensation coefficients have been observed, when training with manual versus autopilot data. These variations are thought to be partially due to frequency variations in the coefficients themselves. It was found that whitening filters could reduce these effects by a factor of 2 or more [1]. However, experiments need to be conducted to learn how effective the whitening filter is in compensating for the frequency variations which are brought about by hysteresis effects due to takeoffs and landings.

Finally, experiments were conducted to learn the effect of integration time on performance. It was found that increasing the integration time on each path increased the performance somewhat. However, after one has acquired say 30 s or so of data per maneuver, increasing the lengths of the sample to 300 s or 3000 s does not improve the performance significantly.

It is concluded that there are two areas where future work could be profitable in improving system performance. First, one could incorporate whitening filters in the microprocessor algorithm in its present form. By rewriting the code and incorporating the whitening filter one could increase the performance and at the same time reduce the number of multiples per sample point by 90 percent. If this is done the sample rate requirement shown in Fig. 1 could be relaxed so that the computer program could operate from 5 Hz to 20 or 30 Hz without degradation. One would be able to perform the compensation fast enough so that one could operate in real time at the increased sample rates. Whitening filters would permit us to do this. Dynamic scaling would have to be incorporated in the program so that at different sample rates the program does not underflow or overflow the storage allocated.

The second area where improvements might be made is in the design of the post processor. The goal would be to update the compensation coefficients at the end of each bearing. In the present algorithm the coefficients are updated after four bearings are taken and an LMS solution is obtained over all the four bearings. However, if one were to update at the end of each bearing the solution could then be cast into a form to permit adaptive updating at the end of any specified length of time. This adaptive updating would then be used to offset the temporal variations brought about by the hysteresis effects. Since the errors observed in the present algorithm are orders of magnitude below the error that is produced as a result of hysteresis, it is felt that further improvements of the algorithm without incorporating adaptive and/or prewhitening features would be unproductive.

References

[1] S.H. Bickel, "Small signal compensation of the magnetic fields resulting from aircraft maneuver," to be published.

Samuel H. Bickel was born in York, Pa., on April 15, 1936. He received the B.S.E.E. degree from the Drexel Institute of Technology, Philadelphia, Pa., the M.S. degree from the University of Southern California, Los Angeles, and the Ph.D. degree from the University of California at Berkeley in 1958, 1960, and 1964, respectively.

From 1963 to 1965 he was a member of the technical staff of the MITRE Corporation, where his work included antenna noise temperature, simulation studies, electromagnetic scattering theory, and polarization studies. Following this he joined Texas Instruments Incorporated in Dallas, where he developed spectral techniques for analyzing radiometric infrared signatures. In 1974 he developed a technique for extracting antisubmarine warfare (ASW) tactical parameters from the magnetic gradient coherence tensor. During 1977 he analyzed synthetic-aperature radar target signature data and developed a feedback procedure for real-time correction of quadratic phase errors. Other work experience in the ASW group included a least mean square regression procedure for estimating target position from multiple sensor measurements and analysis of magnetic airborne detection range in the presence of geological noise. In 1979 he joined the Atlantic Richfield Company as a research associate where his current interests are devoted to the deconvolution of VIBROSEIS^{T} seismic data for the purpose of geophysical exploration.

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