Stochastic differential equation

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Introduction

A stochastic differential equation version of the SIR model is:

- Stochastic
- Continuous in time
- Continuous in state

Libraries

```
using DifferentialEquations
using SimpleDiffEq
using Distributions
using Random
using DataFrames
using StatsPlots
using BenchmarkTools
```

Transitions

```
function sir_sde!(du,u,p,t)
    (S,I,R) = u
    (β,c,γ,δt) = p
    N = S+I+R
    ifrac = β*c*I/N*S*δt
    rfrac = γ*I*δt
    ifrac_noise = sqrt(ifrac)*rand(Normal(0,1))
    rfrac_noise = sqrt(rfrac)*rand(Normal(0,1))
    @inbounds begin
        du[1] = S-(ifrac+ifrac_noise)
        du[2] = I+(ifrac+ifrac_noise) - (rfrac + rfrac_noise)
        du[3] = R+(rfrac+rfrac_noise)
    end
    for i in 1:3
        if du[i] < 0 du[i]=0 end
    end
    nothing
end
```

```
sir_sde! (generic function with 1 method)
```

Time domain

Note that even though we're using fixed time steps, `DifferentialEquations.jl` complains if I pass integer timespans, so I set the timespan to be `Float64`.

```
δt = 0.1
nsteps = 400
tmax = nsteps*δt
tspan = (0.0,nsteps)
t = 0.0:δt:tmax;
```

Initial conditions

```
u0 = [990.0,10.0,0.0]
```

```
3-element Array{Float64,1}:
 990.0
  10.0
   0.0
```

Parameter values

```
p = [0.05,10.0,0.25,δt]
```

```
4-element Array{Float64,1}:
  0.05
 10.0
  0.25
  0.1
```

Random number seed

```
Random.seed!(1234);
```

Running the model

```
prob_sde = DiscreteProblem(sir_sde!,u0,tspan,p);
```

```
sol_sde = solve(prob_sde,solver=FunctionMap)
```

```
retcode: Success
Interpolation: left-endpoint piecewise constant
t: 401-element Array{Float64,1}:
   0.0
   1.0
   2.0
   3.0
   4.0
```

```
   5.0
   6.0
   7.0
   8.0
   9.0
   ⋮
 392.0
 393.0
 394.0
 395.0
 396.0
 397.0
 398.0
 399.0
 400.0
u: 401-element Array{Array{Float64,1},1}:
 [990.0, 10.0, 0.0]
 [988.8947671522094, 11.30610475571904, 0.0]
 [988.7055274430037, 11.69272699154772, 0.0]
 [987.4706638900711, 11.439388577532924, 1.4882019669473676]
 [986.5057382330858, 12.263646921629789, 1.6288692798357647]
 [985.5104541149187, 13.238597190929257, 1.6492031287034035]
 [985.3109877737326, 13.118197077827924, 1.9690695829908509]
 [984.5620377717174, 12.478152468995, 3.3580641938389024]
 [984.5966431703534, 12.070101461308104, 3.731509802889783]
 [984.1962508229602, 11.965649933497922, 4.23635367809324]
 ⋮
 [203.93798918731858, 20.11670266219165, 776.3435625850418]
 [203.93331045829538, 17.890067678388853, 778.5748762978678]
 [203.96220308765527, 17.43879995813703, 778.9972513887598]
 [203.76541827427786, 17.296012920495063, 779.3368232397792]
 [203.69082753114262, 15.647193547495467, 781.060233355914]
 [203.4023893577508, 16.33385318992283, 780.6620118868784]
 [203.51099967961449, 16.85237753507438, 780.0348772198632]
 [203.65506395608364, 15.662816464260057, 781.0803740142084]
 [203.17463327529836, 14.383186887926252, 782.8404342713275]
```

Post-processing

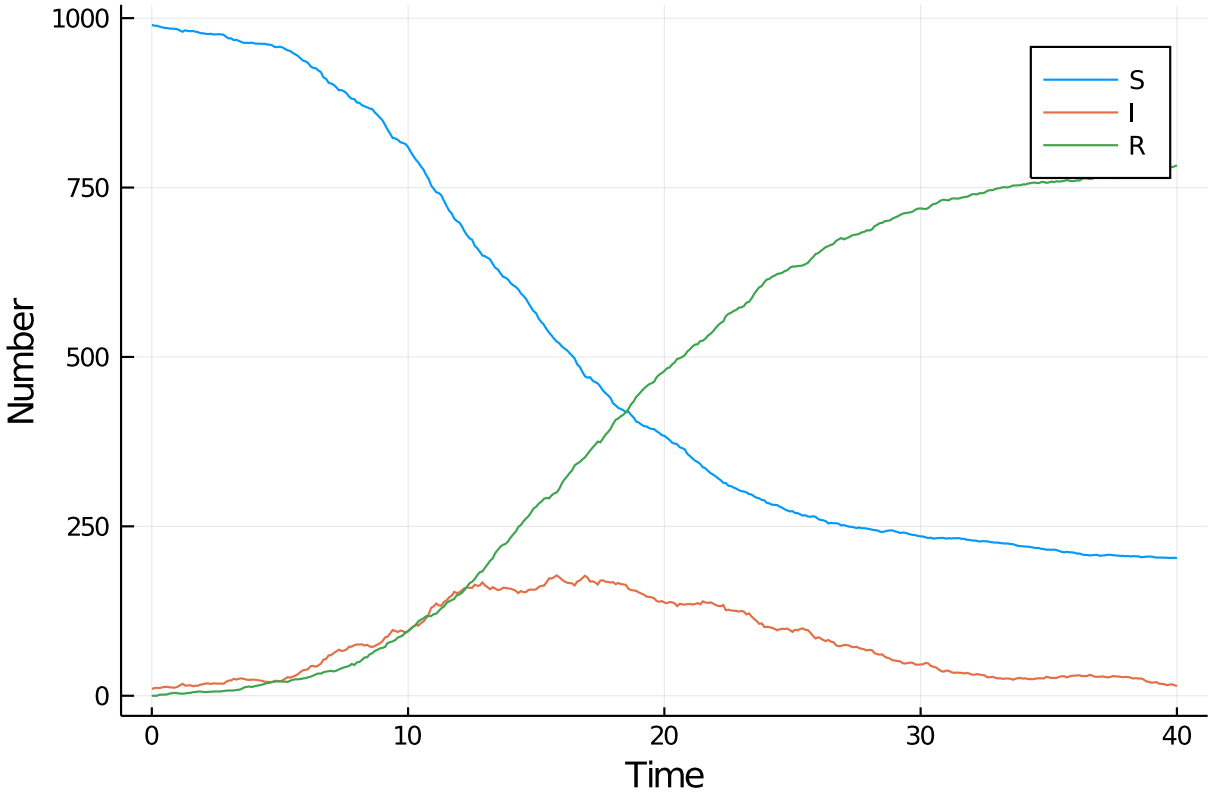
We can convert the output to a dataframe for convenience.

```
df_sde = DataFrame(sol_sde')
df_sde[!,:t] = t;
```

Plotting

We can now plot the results.

```
@df df_sde plot(:t,
    [:x1 :x2 :x3],
    label=["S" "I" "R"],
    xlabel="Time",
    ylabel="Number")
```



Benchmarking

```
@benchmark solve(prob_sde,solver=FunctionMap)
```

```
BenchmarkTools.Trial:
  memory estimate:  59.11 KiB
  allocs estimate:  479
  --------------
  minimum time:     69.700 μs (0.00% GC)
  median time:      74.299 μs (0.00% GC)
  mean time:        89.563 μs (6.49% GC)
  maximum time:     12.675 ms (98.99% GC)
  --------------
  samples:          10000
  evals/sample:     1
```

Appendix

Computer Information

```
Julia Version 1.4.0
Commit b8e9a9ecc6 (2020-03-21 16:36 UTC)
Platform Info:
  OS: Windows (x86_64-w64-mingw32)
  CPU: Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
```

```
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_NUM_THREADS = 4
```

Package Information

```
Status `~\.julia\environments\v1.4\Project.toml`
[46ada45e-f475-11e8-01d0-f70cc89e6671] Agents 3.0.0
[b19378d9-d87a-599a-927f-45f220a2c452] ArrayFire 1.0.6
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.12.10
[6e4b80f9-dd63-53aa-95a3-0cdb28fa8baf] BenchmarkTools 0.5.0
[be33ccc6-a3ff-5ff2-a52e-74243cff1e17] CUDAnative 3.0.4
[3a865a2d-5b23-5a0f-bc46-62713ec82fae] CuArrays 2.0.1
[717857b8-e6f2-59f4-9121-6e50c889abd2] DSP 0.6.6
[2445eb08-9709-466a-b3fc-47e12bd697a2] DataDrivenDiffEq 0.2.0
[a93c6f00-e57d-5684-b7b6-d8193f3e46c0] DataFrames 0.20.2
[aae7a2af-3d4f-5e19-a356-7da93b79d9d0] DiffEqFlux 1.8.1
[41bf760c-e81c-5289-8e54-58b1f1f8abe2] DiffEqSensitivity 6.13.0
[6d1b261a-3be8-11e9-3f2f-0b112a9a8436] DiffEqTutorials 0.1.0
[0c46a032-eb83-5123-abaf-570d42b7fbaa] DifferentialEquations 6.13.0
[31c24e10-a181-5473-b8eb-7969acd0382f] Distributions 0.23.2
[634d3b9d-ee7a-5ddf-bec9-22491ea816e1] DrWatson 1.10.2
[587475ba-b771-5e3f-ad9e-33799f191a9c] Flux 0.10.4
[0c68f7d7-f131-5f86-a1c3-88cf8149b2d7] GPUArrays 3.1.0
[28b8d3ca-fb5f-59d9-8090-bfdbd6d07a71] GR 0.48.0
[523d8e89-b243-5607-941c-87d699ea6713] Gillespie 0.1.0
[7073ff75-c697-5162-941a-fcdaad2a7d2a] IJulia 1.21.2
[e5e0dc1b-0480-54bc-9374-aad01c23163d] Juno 0.8.1
[961ee093-0014-501f-94e3-6117800e7a78] ModelingToolkit 3.0.2
[429524aa-4258-5aef-a3af-852621145aeb] Optim 0.20.6
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.34.1
[91a5bcdd-55d7-5caf-9e0b-520d859cae80] Plots 1.0.12
[e6cf234a-135c-5ec9-84dd-332b85af5143] RandomNumbers 1.4.0
[c5292f4c-5179-55e1-98c5-05642aab7184] ResumableFunctions 0.5.1
[428bdadb-6287-5aa5-874b-9969638295fd] SimJulia 0.8.0
[05bca326-078c-5bf0-a5bf-ce7c7982d7fd] SimpleDiffEq 1.1.0
[f3b207a7-027a-5e70-b257-86293d7955fd] StatsPlots 0.14.5
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.19.2
[44d3d7a6-8a23-5bf8-98c5-b353f8df5ec9] Weave 0.9.4
[37e2e46d-f89d-539d-b4ee-838fcccc9c8e] LinearAlgebra
[cf7118a7-6976-5b1a-9a39-7adc72f591a4] UUIDs
```