

Jump process

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Introduction

This implementation defines the model as a combination of two jump processes, infection and recovery, simulated using the [Doob-Gillespie algorithm](#).

Libraries

```
using DifferentialEquations
using SimpleDiffEq
using Random
using DataFrames
using StatsPlots
using BenchmarkTools
```

Transitions

For each process, we define the rate at which it occurs, and how the state variables change at each jump. Note that these are total rates, not *per capita*, and that the change in state variables occurs in-place.

```
function infection_rate(u,p,t)
    (S,I,R) = u
    ( $\beta$ ,c, $\gamma$ ) = p
    N = S+I+R
     $\beta$ *c*I/N*S
end
function infection!(integrator)
    integrator.u[1] -= 1
    integrator.u[2] += 1
end
infection_jump = ConstantRateJump(infection_rate,infection!);

function recovery_rate(u,p,t)
    (S,I,R) = u
    ( $\beta$ ,c, $\gamma$ ) = p
     $\gamma$ *I
end
function recovery!(integrator)
    integrator.u[2] -= 1
    integrator.u[3] += 1
end
recovery_jump = ConstantRateJump(recovery_rate,recovery!);
```

Time domain

```
tmax = 40.0
tspan = (0.0,tmax);
```

For plotting, we can also define a separate time series.

```
 $\delta t = 0.1$   
t = 0: $\delta t$ :tmax;
```

Initial conditions

```
u0 = [990,10,0]; #  $S, I, R$ 
```

Parameter values

```
p = [0.05,10.0,0.25]; #  $\beta, c, \gamma$ 
```

Random number seed

We set a random number seed for reproducibility.

```
Random.seed!(1234);
```

Running the model

Running this model involves:

- Setting up the problem as a `DiscreteProblem`;
- Adding the jumps and setting the algorithm using `JumpProblem`; and
- Running the model, specifying `SSAStepper()`

```
prob = DiscreteProblem(u0,tspan,p)
```

```
DiscreteProblem with uType Array{Int64,1} and tType Float64. In-place: true  
timespan: (0.0, 40.0)  
u0: [990, 10, 0]
```

```
prob_jump = JumpProblem(prob,Direct(),infection_jump,recovery_jump)
```

```
JumpProblem with problem DiscreteProblem and aggregator Direct  
Number of constant rate jumps: 2  
Number of variable rate jumps: 0
```

```
sol_jump = solve(prob_jump,SSAStepper());
```

Post-processing

In order to get output comparable across implementations, we output the model at a fixed set of times.

```
out_jump = sol_jump(t);
```

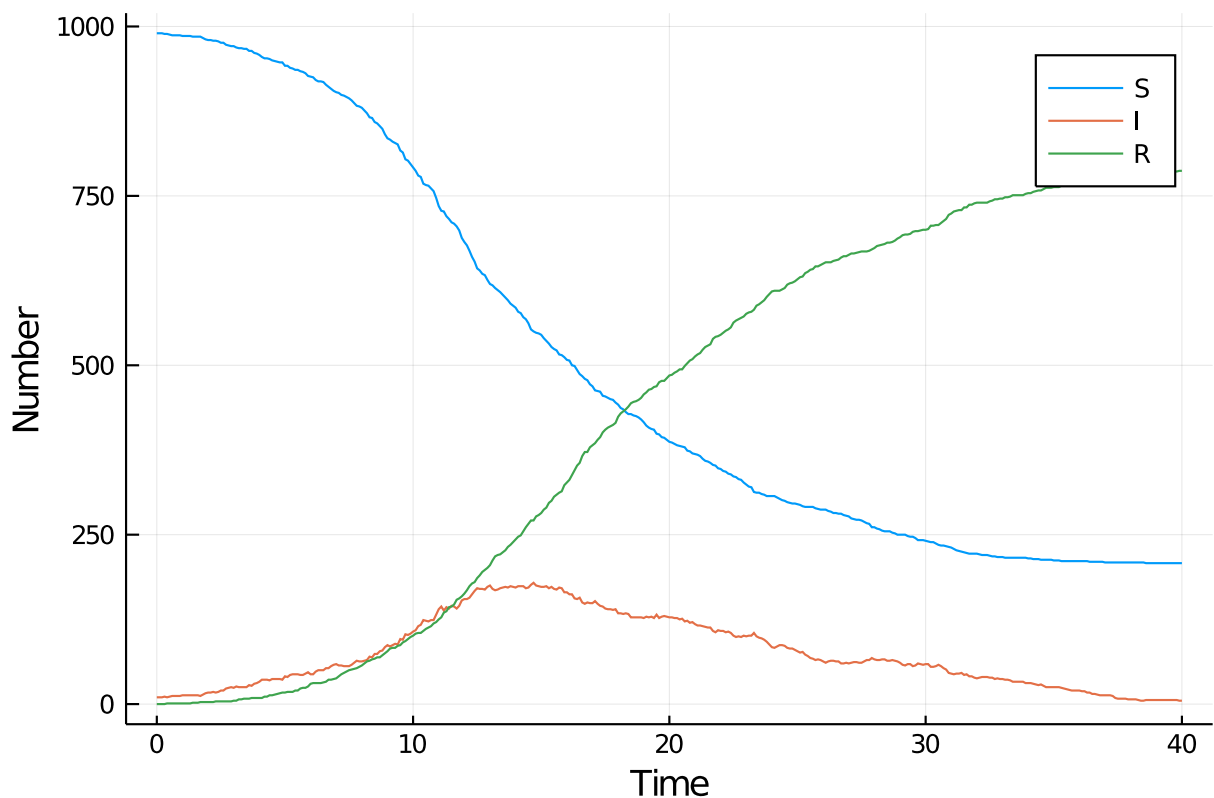
We can convert to a dataframe for convenience.

```
df_jump = DataFrame(out_jump')  
df_jump[!,:t] = out_jump.t;
```

Plotting

We can now plot the results.

```
@df df_jump plot(:t,  
  [:x1 :x2 :x3],  
  label=["S" "I" "R"],  
  xlabel="Time",  
  ylabel="Number")
```



Benchmarking

```
@benchmark solve(prob_jump,FunctionMap())
```

```
BenchmarkTools.Trial:  
  memory estimate: 18.70 KiB  
  allocs estimate: 149
```

```

-----
minimum time:      38.400  $\mu$ s (0.00% GC)
median time:       1.676 ms (0.00% GC)
mean time:         1.901 ms (5.03% GC)
maximum time:      30.268 ms (88.43% GC)
-----
samples:           2603
evals/sample:      1

```

Appendix

Computer Information

```

Julia Version 1.4.0
Commit b8e9a9ecc6 (2020-03-21 16:36 UTC)
Platform Info:
  OS: Windows (x86_64-w64-mingw32)
  CPU: Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_NUM_THREADS = 4

```

Package Information

```

Status `~\.julia\environments\v1.4\Project.toml`
[46ada45e-f475-11e8-01d0-f70cc89e6671] Agents 3.0.0
[b19378d9-d87a-599a-927f-45f220a2c452] ArrayFire 1.0.6
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.12.10
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[3a865a2d-5b23-5a0f-bc46-62713ec82fae] CuArrays 2.0.1
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[2445eb08-9709-466a-b3fc-47e12bd697a2] DataDrivenDiffEq 0.2.0
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[eb300fae-53e8-50a0-950c-e21f52c2b7e0] DiffEqBiological 4.3.0
[aae7a2af-3d4f-5e19-a356-7da93b79d9d0] DiffEqFlux 1.8.1
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[523d8e89-b243-5607-941c-87d699ea6713] Gillespie 0.1.0
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 [cf7118a7-6976-5b1a-9a39-7adc72f591a4] UUIDs