

Stochastic differential equation model using StochasticDiffEq.jl

Simon Frost (@sdwfrost), 2020-04-27

Introduction

A stochastic differential equation version of the SIR model is:

- Stochastic
- Continuous in time
- Continuous in state

This implementation uses `StochasticDiffEq.jl`, which has a variety of SDE solvers.

Libraries

```
using DifferentialEquations
using StochasticDiffEq
using Random
using SparseArrays
using DataFrames
using StatsPlots
```

Transitions

We begin by specifying the ODE kernel.

```
function sir_ode!(du,u,p,t)
    (S,I,R) = u
    (β,c,γ) = p
    N = S+I+R
    @inbounds begin
        du[1] = -β*c*I/N*S
        du[2] = β*c*I/N*S - γ*I
        du[3] = γ*I
    end
    nothing
end;

# Define a sparse matrix by making a dense matrix and setting some values as not zero
A = zeros(3,2)
A[1,1] = 1
A[2,1] = 1
A[2,2] = 1
A[3,2] = 1
A = SparseArrays.sparse(A);
```

```

# Make `g` write the sparse matrix values
function sir_noise!(du,u,p,t)
    (S,I,R) = u
    ( $\beta$ ,c, $\gamma$ ) = p
    N = S+I+R
    ifrac =  $\beta$ *I/N*S
    rfrac =  $\gamma$ *I
    du[1,1] = -sqrt(ifrac)
    du[2,1] = sqrt(ifrac)
    du[2,2] = -sqrt(rfrac)
    du[3,2] = sqrt(rfrac)
end;

```

Time domain

Note that even though I'm using fixed time steps, `DifferentialEquations.jl` complains if I pass integer timespans, so I set the timespan to be `Float64`.

```

 $\delta t$  = 0.1
tmax = 40.0
tspan = (0.0,tmax)
t = 0.0: $\delta t$ :tmax;

```

Initial conditions

```

u0 = [990.0,10.0,0.0]; #  $S, I, R$ 

```

Parameter values

```

p = [0.05,10.0,0.25]; #  $\beta, c, \gamma$ 

```

Random number seed

```

Random.seed!(1234);

```

Running the model

```

prob_sde = SDEProblem(sir_ode!,sir_noise!,u0,tspan,p,noise_rate_prototype=A)

```

```

SDEProblem with uType Array{Float64,1} and tType Float64. In-place: true
timespan: (0.0, 40.0)
u0: [990.0, 10.0, 0.0]

```

```

sol_sde = solve(prob_sde,SRA1());

```

Post-processing

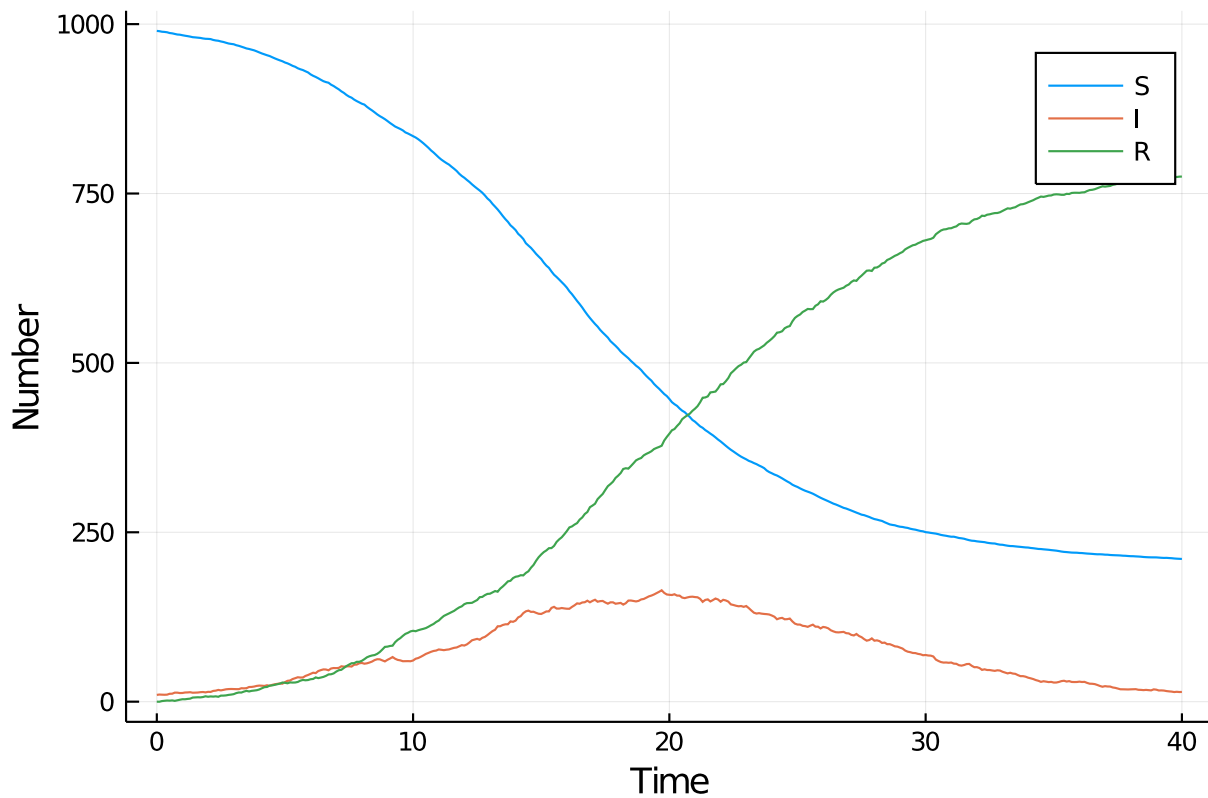
We can convert the output to a dataframe for convenience.

```
df_sde = DataFrame(sol_sde(t)')  
df_sde[!,:t] = t;
```

Plotting

We can now plot the results.

```
@df df_sde plot(:t,  
  [:x1 :x2 :x3],  
  label=["S" "I" "R"],  
  xlabel="Time",  
  ylabel="Number")
```



Appendix

Computer Information

```
Julia Version 1.4.0  
Commit b8e9a9ecc6 (2020-03-21 16:36 UTC)  
Platform Info:  
  OS: Windows (x86_64-w64-mingw32)  
  CPU: Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz  
  WORD_SIZE: 64
```

```
LIBM: libopenlibm
LLVM: libLLVM-8.0.1 (ORCJIT, skylake)
Environment:
  JULIA_NUM_THREADS = 4
```

Package Information

```
Status `~\.julia\environments\v1.4\Project.toml`
[46ada45e-f475-11e8-01d0-f70cc89e6671] Agents 3.0.0
[b19378d9-d87a-599a-927f-45f220a2c452] ArrayFire 1.0.6
[c52e3926-4ff0-5f6e-af25-54175e0327b1] Atom 0.12.10
[6e4b80f9-dd63-53aa-95a3-0cdb28fa8baf] BenchmarkTools 0.5.0
[be33ccc6-a3ff-5ff2-a52e-74243cff1e17] CUDAnative 3.0.4
[3a865a2d-5b23-5a0f-bc46-62713ec82fae] CuArrays 2.0.1
[717857b8-e6f2-59f4-9121-6e50c889abd2] DSP 0.6.6
[2445eb08-9709-466a-b3fc-47e12bd697a2] DataDrivenDiffEq 0.2.0
[a93c6f00-e57d-5684-b7b6-d8193f3e46c0] DataFrames 0.20.2
[ebbdde9d-f333-5424-9be2-dbf1e9acfb5e] DiffEqBayes 2.12.1
[eb300fae-53e8-50a0-950c-e21f52c2b7e0] DiffEqBiological 4.3.0
[aae7a2af-3d4f-5e19-a356-7da93b79d9d0] DiffEqFlux 1.8.1
[c894b116-72e5-5b58-be3c-e6d8d4ac2b12] DiffEqJump 6.6.3
[41bf760c-e81c-5289-8e54-58b1f1f8abe2] DiffEqSensitivity 6.13.0
[6d1b261a-3be8-11e9-3f2f-0b112a9a8436] DiffEqTutorials 0.1.0
[0c46a032-eb83-5123-abaf-570d42b7fbba] DifferentialEquations 6.13.0
[31c24e10-a181-5473-b8eb-7969acd0382f] Distributions 0.23.2
[634d3b9d-ee7a-5ddf-bec9-22491ea816e1] DrWatson 1.10.2
[587475ba-b771-5e3f-ad9e-33799f191a9c] Flux 0.10.4
[0c68f7d7-f131-5f86-a1c3-88cf8149b2d7] GPUArrays 3.1.0
[28b8d3ca-fb5f-59d9-8090-bfdbd6d07a71] GR 0.48.0
[523d8e89-b243-5607-941c-87d699ea6713] Gillespie 0.1.0
[7073ff75-c697-5162-941a-fcdaad2a7d2a] IJulia 1.21.2
[e5e0dc1b-0480-54bc-9374-aad01c23163d] Juno 0.8.1
[23fbe1c1-3f47-55db-b15f-69d7ec21a316] Latexify 0.12.0
[d8e11817-5142-5d16-987a-aa16d5891078] MLStyle 0.4.0
[961ee093-0014-501f-94e3-6117800e7a78] ModelingToolkit 3.0.2
[429524aa-4258-5aef-a3af-852621145aeb] Optim 0.20.6
[1dea7af3-3e70-54e6-95c3-0bf5283fa5ed] OrdinaryDiffEq 5.34.1
[91a5bcd7-55d7-5caf-9e0b-520d859cae80] Plots 1.0.12
[e6cf234a-135c-5ec9-84dd-332b85af5143] RandomNumbers 1.4.0
[c5292f4c-5179-55e1-98c5-05642aab7184] ResumableFunctions 0.5.1
[428bdadb-6287-5aa5-874b-9969638295fd] SimJulia 0.8.0
[05bca326-078c-5bf0-a5bf-ce7c7982d7fd] SimpleDiffEq 1.1.0
[2913bbd2-ae8a-5f71-8c99-4fb6c76f3a91] StatsBase 0.33.0
[f3b207a7-027a-5e70-b257-86293d7955fd] StatsPlots 0.14.5
[789caeaf-c7a9-5a7d-9973-96adeb23e2a0] StochasticDiffEq 6.19.2
[a759f4b9-e2f1-59dc-863e-4aeb61b1ea8f] TimerOutputs 0.5.5
[fce5fe82-541a-59a6-adf8-730c64b5f9a0] Turing 0.11.0
[44d3d7a6-8a23-5bf8-98c5-b353f8df5ec9] Weave 0.9.4
```

[37e2e46d-f89d-539d-b4ee-838fcccc9c8e] LinearAlgebra
[cf7118a7-6976-5b1a-9a39-7adc72f591a4] UUIDs