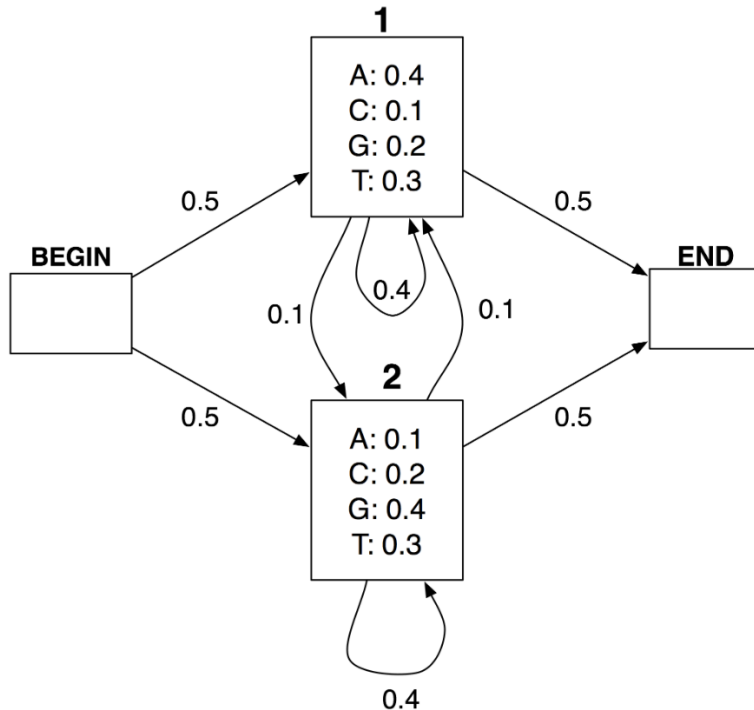


BMI/CS 576 – Day 18

- Today
 - Intro to hidden Markov models
- Next week
 - HMMs in depth with applications

Quiz

Given the HMM below and the sequence AC, which we assume has been generated by this HMM, what is the most likely path of hidden states?



- $^{1,1} P(x, \pi = (1,1)) = a_{BEGIN,1} e_1(A) a_{1,1} e_1(C) a_{1,END} = 0.5 \times 0.4 \times 0.4 \times 0.1 \times 0.5 = 0.004$
- $^{1,2} P(x, \pi = (1,2)) = a_{BEGIN,1} e_1(A) a_{1,2} e_2(C) a_{2,END} = 0.5 \times 0.4 \times 0.1 \times 0.2 \times 0.5 = 0.002$
- $^{2,2} P(x, \pi = (2,2)) = a_{BEGIN,2} e_2(A) a_{2,2} e_2(C) a_{2,END} = 0.5 \times 0.1 \times 0.4 \times 0.2 \times 0.5 = 0.002$
- $^{2,1} P(x, \pi = (2,1)) = a_{BEGIN,2} e_2(A) a_{2,1} e_1(C) a_{1,END} = 0.5 \times 0.1 \times 0.1 \times 0.1 \times 0.5 = 0.00025$

Most likely path?

$$\operatorname{argmax}_{\pi} P(\pi|x) = \operatorname{argmax}_{\pi} \frac{P(x,\pi)}{P(x)} = \operatorname{argmax}_{\pi} P(x,\pi)$$

constant with respect to π

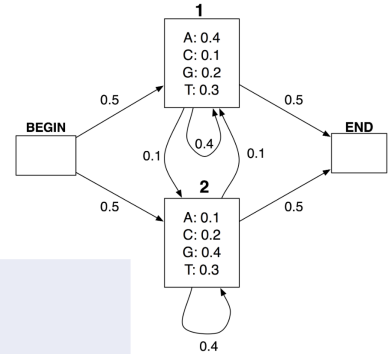
Using the quiz example:

$$\begin{aligned} P(x) &= P(x, \pi = (1,1)) + P(x, \pi = (1,2)) + P(x, \pi = (2,2)) + P(x, \pi = (2,1)) \\ &= 0.004 + 0.002 + 0.002 + 0.00025 \\ &= 0.00825 \end{aligned}$$

$$P(x | \pi = (1,1)) = \frac{P(x, \pi = (1,1))}{P(x)} = \frac{0.004}{0.00825} \approx 0.485$$

Viterbi example (from quiz)

		A		C
	0	1		2
BEGIN	1			
1	0	$e_1(A) \times \max \begin{cases} v_B(0)a_{B1} \\ v_1(0)a_{11} \\ v_2(0)a_{21} \end{cases}$ $= 0.4 \times 1 \times 0.5 = 0.2$		$e_1(C) \times \max \begin{cases} v_1(1)a_{11} \\ v_2(1)a_{21} \end{cases}$ $= 0.1 \times \max(0.2 \times 0.4, 0.05 \times 0.1)$ $= 0.1 \times 0.08 = 0.008$
2	0	$e_2(A) \times \max \begin{cases} v_B(0)a_{B2} \\ v_1(0)a_{12} \\ v_2(0)a_{22} \end{cases}$ $= 0.1 \times 1 \times 0.5 = 0.05$		$e_2(C) \times \max \begin{cases} v_1(1)a_{12} \\ v_2(1)a_{22} \end{cases}$ $= 0.2 \times \max(0.2 \times 0.1, 0.05 \times 0.4)$ $= 0.2 \times 0.02 = 0.004$
END				$\max \begin{cases} v_1(2)a_{1E} \\ v_2(2)a_{2E} \end{cases}$ $= \max(0.008 \times 0.5, 0.004 \times 0.5)$ $= 0.004$



Viterbi path from traceback: BEGIN, 1, 1, END