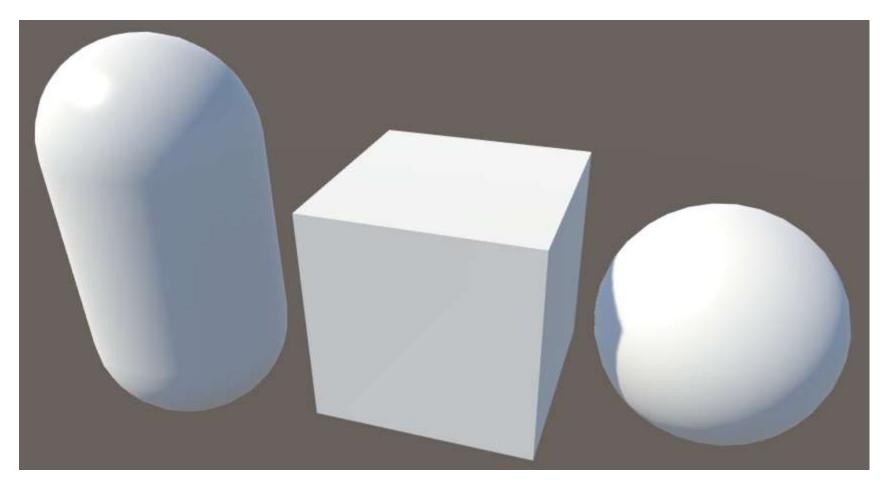
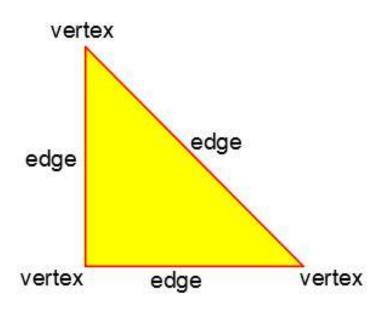
Significant content "borrowed" from Dan Chang @ Nintendo NTD "with permission"

Chris Ryan

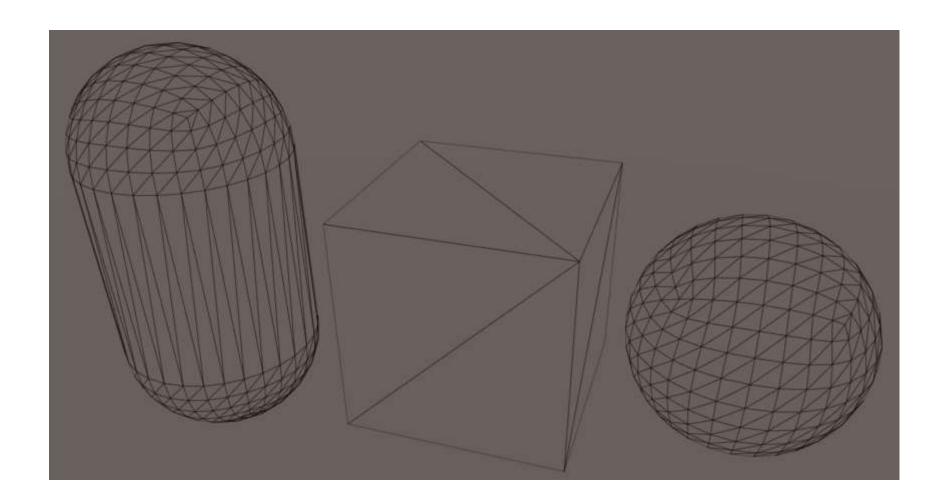
Northwest C++ User Group Nov 18th 2020 github.com/ChrisRyan98008/NwCpp-Nov2020

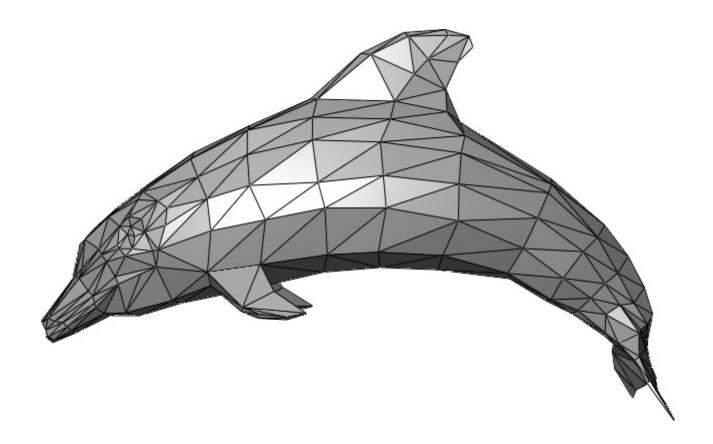


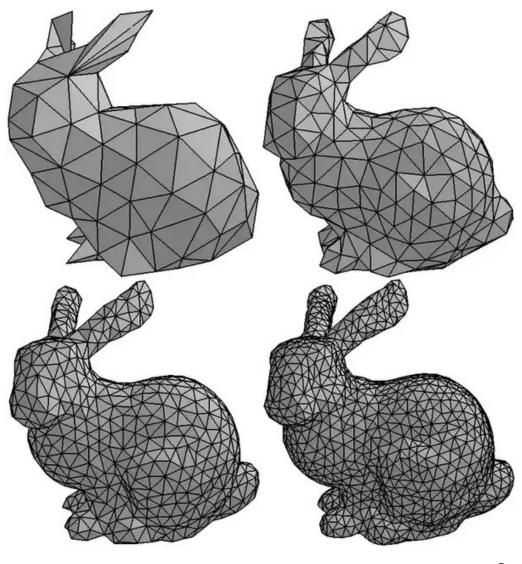
Draw Lots of Triangles



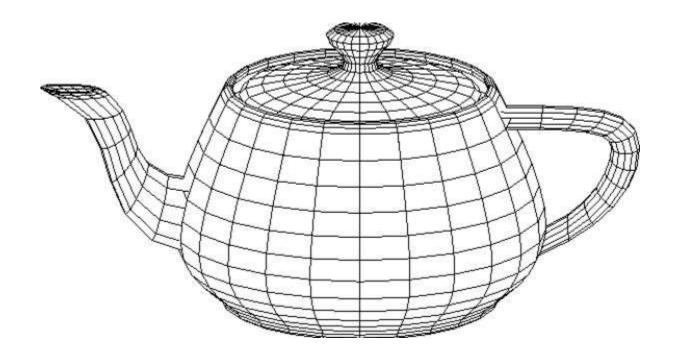
sometimes also called polygons or polys

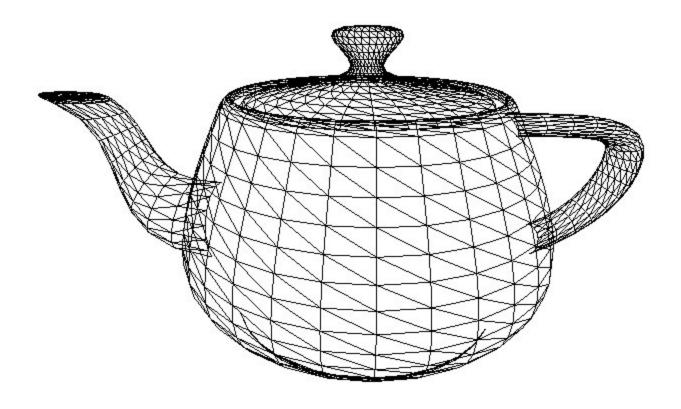


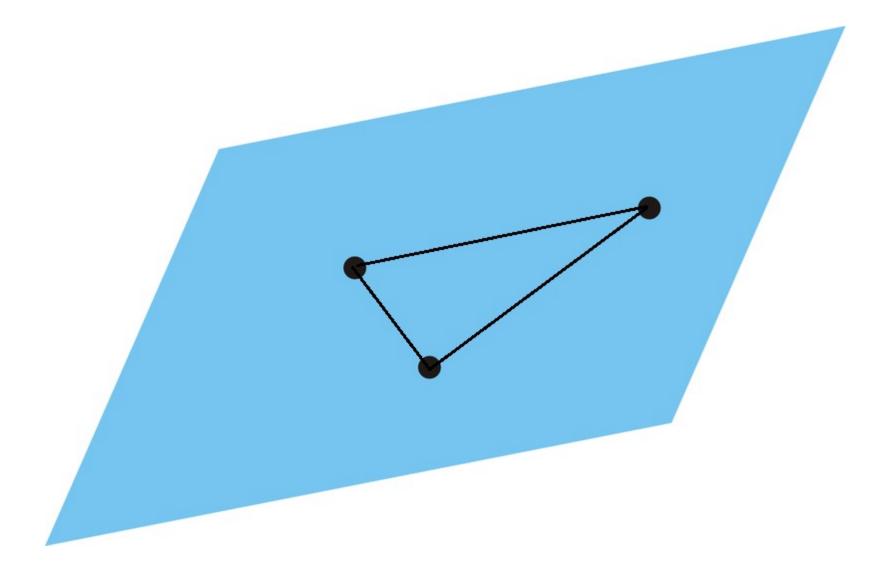


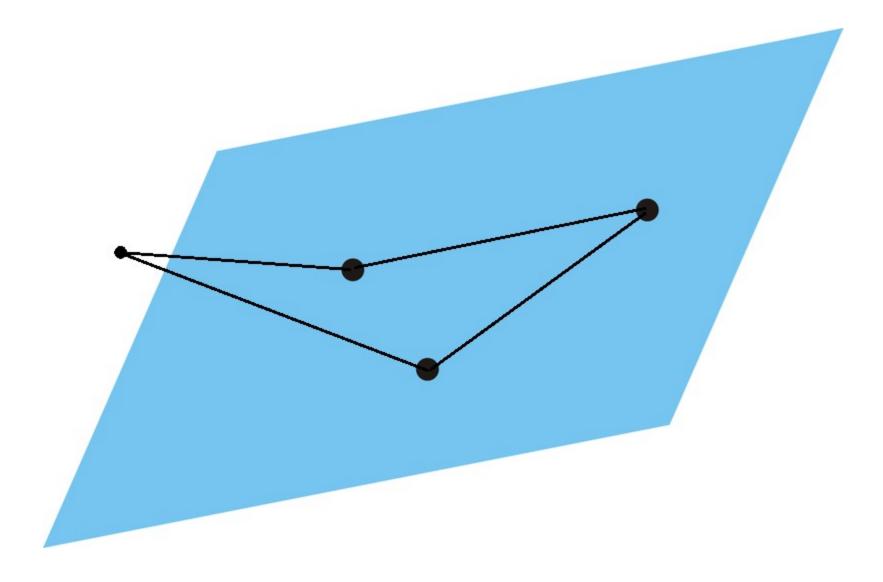


6









A Model

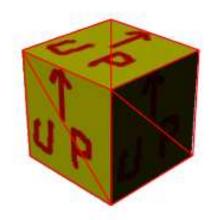
Only care about visible surfaces (no data about interior)



Sometimes also called a 3D object or an object

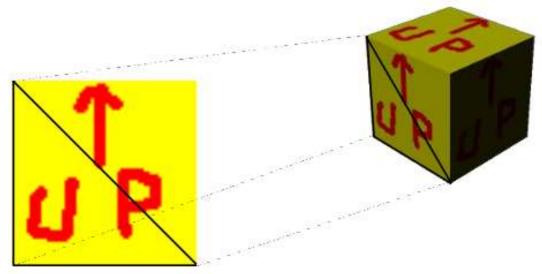
A Model

- Surfaces represented with a collection of triangles
- Mesh a collection of triangles
- Model a collection of meshes

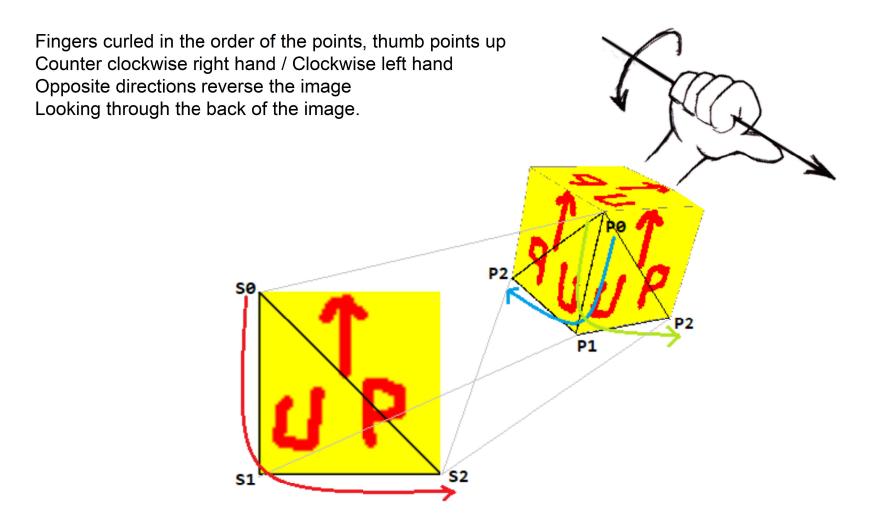


Textures

- Are 2D images which are applied to triangles
- Rough analogy: stickers or decals
 - Except textures can be stretched
 - Each triangle specifies which part of texture gets applied to it



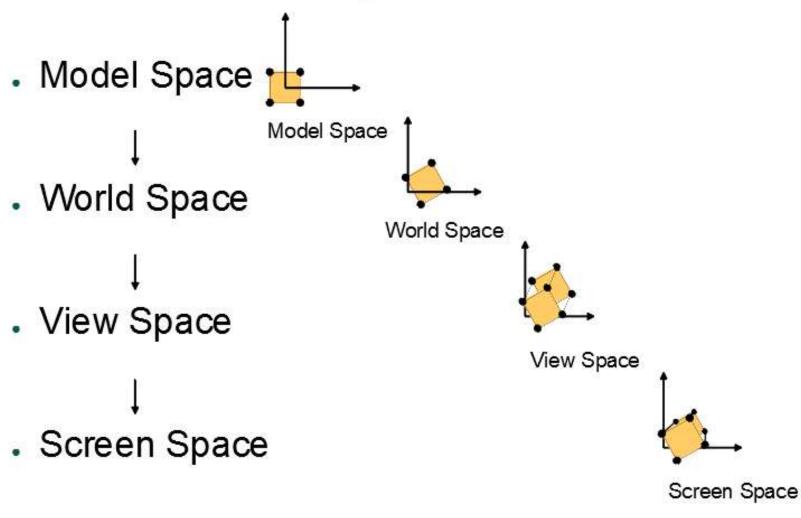
Right Hand / Left Hand Rule / Winding



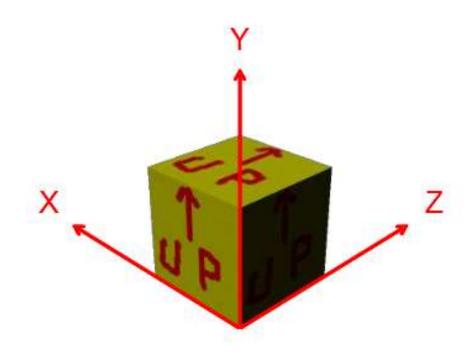
Model has Matrix

- Every model has a Matrix
 - Specifies Position in the world
 - Specifies Orientation in the world
 - Specifies Scale (size) in the world
- A Matrix used in this way is also called an "model to world space matrix"
 - or "model to world matrix"
 - or sometimes just "world matrix"

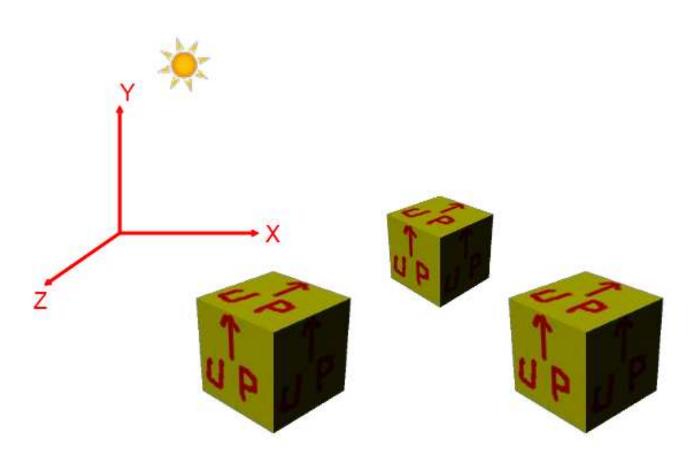
Coordinate Spaces Overview



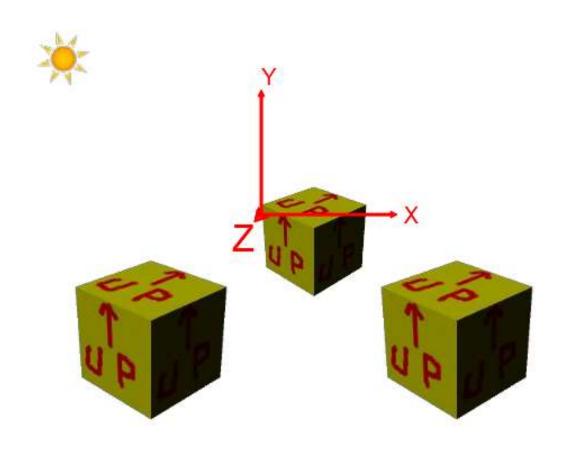
Model Space



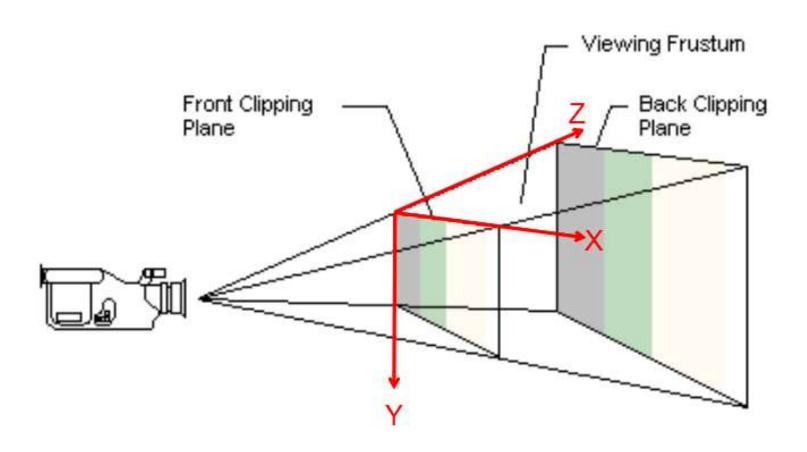
World Space



View Space

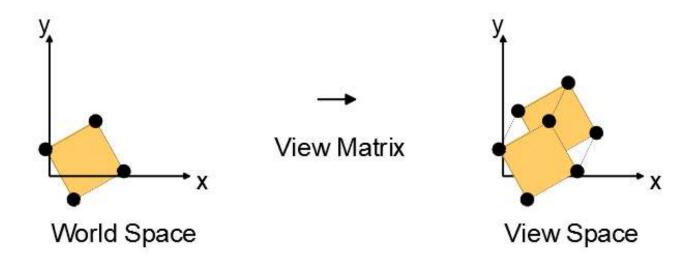


Screen Space



World Space to View Space

- View Matrix ("oriented from camera view")
- World Space × View Matrix = View Space
- View Space sometimes called Camera Space



Creating View Matrices

```
PointOfView(Point cameraPosition,
Point cameraTarget,
Vector cameraUpVector);
```

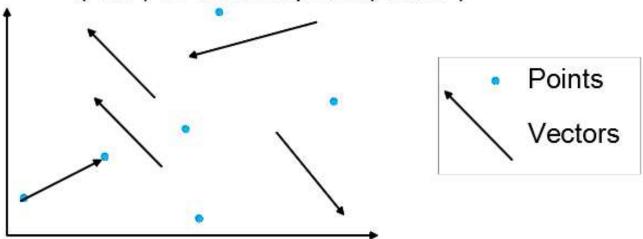
- cameraPosition position of the camera in the world
- cameraTarget the position in the world we're looking at
- cameraUpVector the camera's up vector

Creating Projection Matrices

- fieldOfView Field of view in the y direction
- aspectRatio Width divided by height
- nearPlaneDistance Distance to the near plane
- farPlaneDistance Distance to the far plane

Points and Vectors

- Compare points and vectors:
 - Point has only one property: <u>location</u>
 - Vector has two properties: <u>length</u> & <u>direction</u>
 - One interpretation: vectors get you from one point ("tail") to another point ("head")

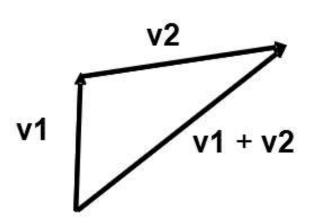


Points and Vectors

- Usually write either this way: (3, 4, 5)
 - For point in 3D, write as (3, 4, 5, <u>1</u>)
 - For vector in 3D, write as (3, 4, 5, <u>0</u>)
- When you subtract two points, you get a vector!
 - Adding two points is undefined
- When you add or subtract two vectors, you get a vector

Vector addition

- Takes two vectors as input (v1 and v2)
 - \circ (v1x, v1y, v1z)
 - (v2x, v2y, v2z)
- Output is a vector
- Place vectors "head to tail"
- Output = (v1x + v2x, v1y + v2y, v1z + v2z)



Skipping:

- Vector * scalar
- Vector Length
- Normalize Vector
- Vector Dot Product
- Vector Cross Product
- Reflection Vector
- Triangle and Plane Normals
- Plane Equation
- "Square up" vectors

Row Vectors

- MonoGame uses row vectors
 - Written (3, 4, 5, 0), or [3 4 5 0]
- Unity uses column vectors
 - Written (3, 4, 5, 0)^T, or

Matrix Math

```
v * M =
[vx vy vz vw] * m11 m12 m13 m14
m21 m22 m23 m24
m31 m32 m33 m34
m41 m42 m43 m44
```

```
= [(vx*m11 + vy*m21 + vz*m31 + vw*m41)
(vx*m12 + vy*m22 + vz*m32 + vw*m42)
(vx*m13 + vy*m23 + vz*m33 + vw*m43)
(vx*m14 + vy*m24 + vz*m34 + vw*m44)]
```

```
Supported operations
type correct and type coherent: vector, point, matrix, mesh
types:
    matrix (4x4)
   vector (1x4) (xyz,w=0)
   point (1x4) (xyz,w=1)
           (poly collection)
    mesh
matrix operators
   matrix *= matrix;
   matrix = matrix * matrix;
vector operators
   vector = vector * matrix;
   vector *= matrix;
   vector = vector * vector;
                                 // vector cross product
   vector *= vector;
                                  // vector cross product
   vector = Normalize(vector);
point operators
    point = point * matrix;
    point *= matrix;
    point = point + vector;
    point += vector;
   vector = point - point;
mesh operators (poly collection) used as a model, world, or screen
   mesh = mesh + mesh;
   mesh += mesh;
   mesh = mesh * matrix;
   mesh *= matrix;
   mesh.PerspectiveDivide();
```

```
Manipulator Matrices:

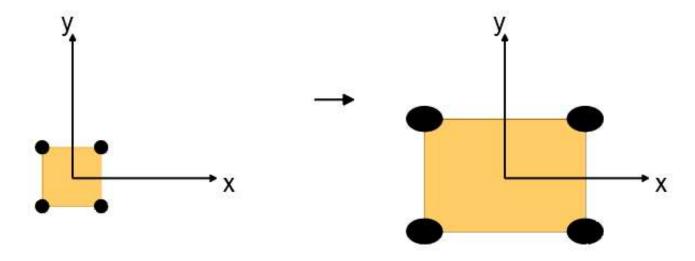
Matrix Identity();
Matrix RotateX(float degrees);
Matrix RotateY(float degrees);
Matrix RotateZ(float degrees);
Matrix Scale(float x, float y, float z);
Matrix Translate(float x, float y, float z);

Transforms Matrices:

Matrix PointOfView(const Point& eye, const Point& target, const Vector& up);
Matrix FieldOfView(float fovAngle, float aspectRatio, float nearPlaneDistance, float farPlaneDistance);
Matrix Viewport(Rect& view, float minZ, float maxZ);
```

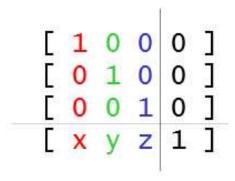
Scaling Matrix

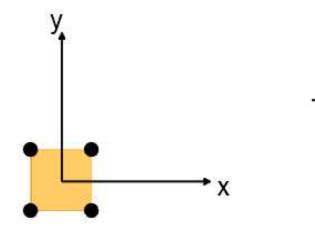
```
float x = 3, y = 2, z = 1;
Matrix mat = Scale(x, y, z);
```

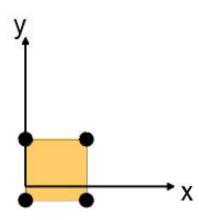


Translation Matrix

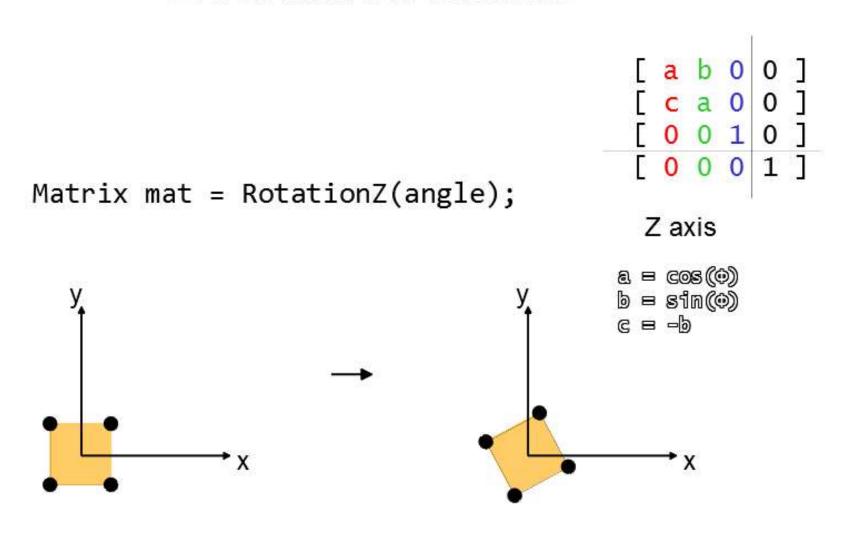
float x = 5, y = 2, z = 0;
Matrix mat = Translate(x, y, z);







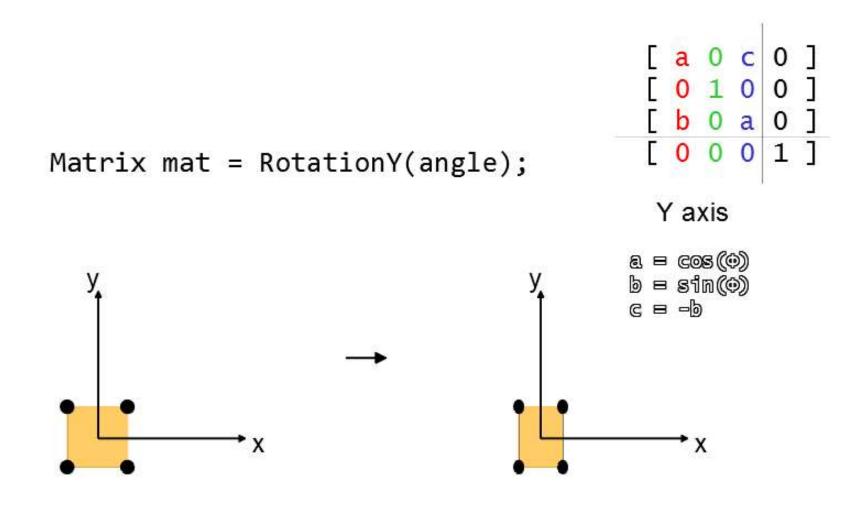
Z Rotation Matrix



X Rotation Matrix

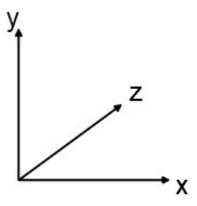
Matrix mat = RotationX(angle); X axis $a = cos(\phi)$ b = sin(Φ)

Y Rotation Matrix



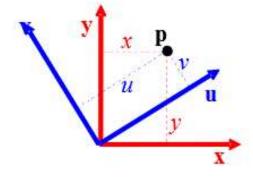
Orthonormal Basis

- Basis: a space is totally defined by a set of vectors – any point is a linear combination of the basis
- Orthogonal: dot product of any two vector is zero
- Normal: magnitude is one
- Orthonormal: orthogonal + normal
- Most common example:

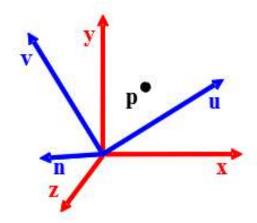


Change of Orthonormal Basis

Given:
 coordinate frames
 xyz and uvn
 point p = (px, py, pz)



• Find: $p = (p_u, p_v, p_n)$

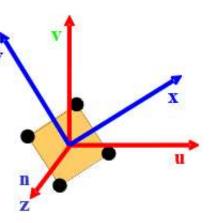


Change of Orthonormal Basis

Simply fill in columns with vectors
 u, v, n (coordinates of new basis
 vector relative to old)

Ε	u _x	٧,	n _x	0]
Γ	u	V	, n	0]
Ε	u_{7}	V,	n_{z}	0]
	0	0	0	1]

- Dotting with vectors u, v, n
- Can think of X, Y or Z rotations as changes in orthonormal basis
- Basis deals only w/ upper-left 3x3!



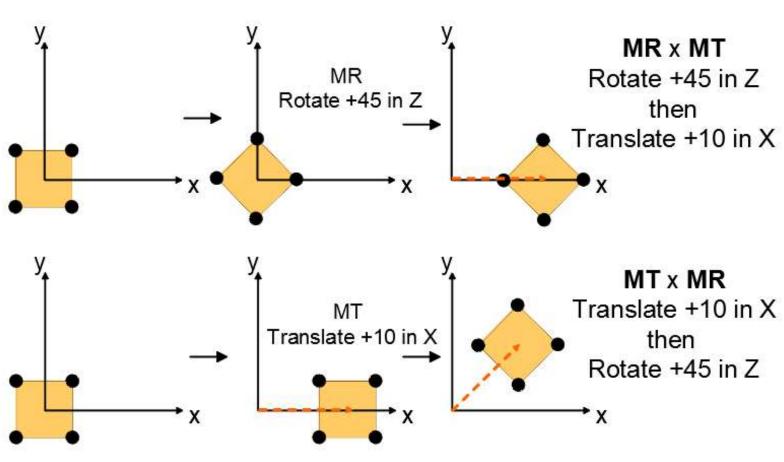
Matrix Math Multiply Order

v * M1 * M2 * M3 = [(v * M1) * M2] * M3]= v * (M1 * M2 * M3)M = M1 * M2 * M3= v * M

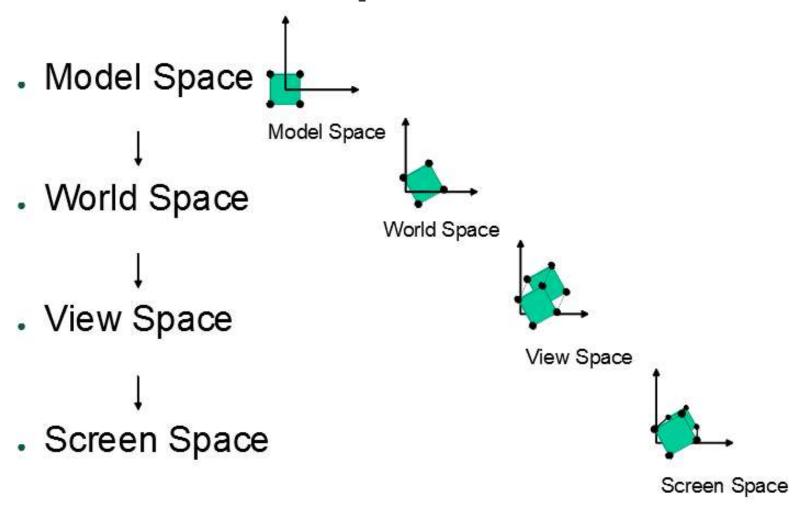
Combining Matrices

```
Matrix matRot = RotationZ(angle);
float x = 5, y = 2, z = 0;
Matrix matTrans = Translation(x, y, z);
Matrix mat = matRot * matTrans;
                                                        Z axis
                                                      a = cos(\phi)
                                                      b = sin()
                                                      c = -b
```

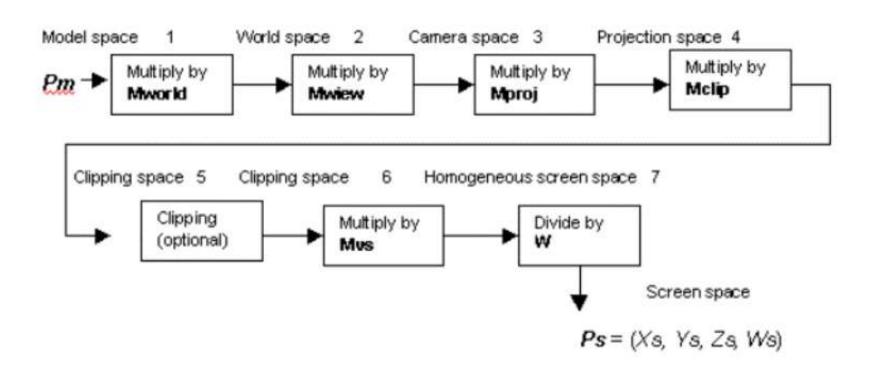
Geometric Interpretation: Matrix Multiplication is Not Commutative



Coordinate Spaces Overview



Transformation Pipeline



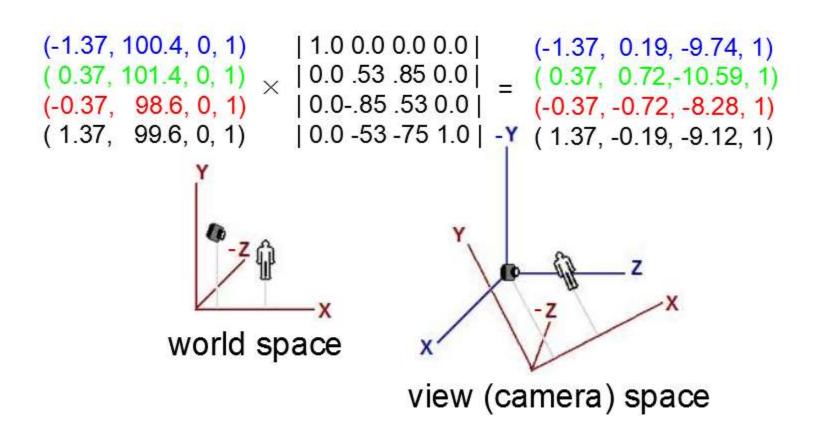
World Matrix

Model Space to World Space

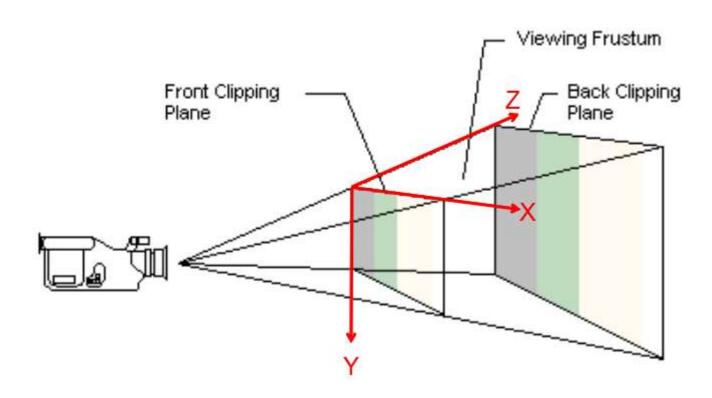
Model Space to World Space

View Matrix

World Space to View Space



Add Perspective: View Frustum



Projection Matrix

```
float fieldOfView = 45;
float aspectRatio = 1.333;
float nearPlaneDistance = 1;
float farPlaneDistance = 100;
Matrix projection = FieldOfView(fieldOfView,
                                    aspectRatio,
                                    nearPlaneDistance,
                                    farPlaneDistance);
                 D * tan(fov/2)
                                  1.811 0.000 0.000
                                                       0.000
                 ZFRONT
                         ZBACK
Camera
                                  0.000 2.414 0.000
                                                        0.000
Position
                                  0.000 0.000 -1.010 -1.000
                                  0.000 0.000 -1.010
                                                       0.000
```

View Space to Projection Space

Clipping

Clipping volume for all points (px, py, pz, pw)

```
-pw < px <= pw
-pw < py <= pw
0 < pz <= pw
```

Viewport Scale Matrix

Derived from Viewport settings

```
\begin{bmatrix} dwWidth/2 & 0 & 0 & 0 \\ 0 & -dwHeight/2 & 0 & 0 \\ 0 & 0 & dvMaxZ-dvMinZ & 0 \\ dwX+dwWidth/2 & dwHeight/2 + dwY & dvMinz & 1 \end{bmatrix}
```

```
| 320.0 0.000 0.000 0.000 |
| 0.000 -240.0 0.000 0.000 |
| 0.000 0.000 1.000 0.000 |
| 320.0 240.0 0.000 1.000
```

Projection Space to Homogeneous Screen Space

```
(-2.47, 0.47, 8.83, 9.74) | 320 0.0 0.0 0.0 | (2327, 2226, 8.83, 9.74)

(0.66, 1.75, 9.69, 10.59) | 0.0 -240 0.0 0.0 | (3602, 2123, 9.69, 10.59)

(-0.66, -1.75, 7.35, 8.28) | 0.0 0.0 1.0 0.0 | (2436, 2406, 7.35, 8.28)

(2.47, -0.47, 8.21, 9.12) | 320 240 0.0 1.0 | (3711, 2302, 8.21, 9.12)
```

Divide by w

Ready to show on screen

$$Xs = \frac{X}{W}, Ys = \frac{Y}{W}, Zs = \frac{Z}{W}, Ws = \frac{1}{W}$$

Divide by w also called perspective divide

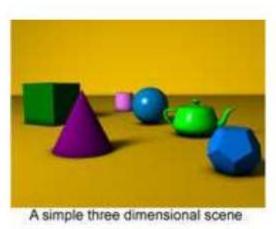
Homogeneous Screen Space to Screen Space

```
Homogeneous Screen / W = Screen Space
(2327, 2226, 8.83, 9.74) / 9.74 = (238.8, 228.5, 0.906, 1) (blue)
(3602, 2123, 9.69, 10.59) / 10.59 = (340.0, 200.4, 0.915, 1) (green)
(2436, 2406, 7.35, 8.28) / 8.28 = (294.4, 290.7, 0.888, 1) (red)
(3711, 2302, 8.21, 9.12) / 9.12 = (406.8, 252.3, 0.899, 1) (black)
                                                   Are
                                                   screen space
                                                   Z values
                                                   used?
      Red is
   closest to
     camera
                                                 (640, 480)
```

Image Buffers and Z-Buffer

Two Image Buffers:

- One holds previously rendered frame which is being displayed on-screen
- One holds the frame currently being rendered
- Additional Z-Buffer (or Depth Buffer) stores depth information

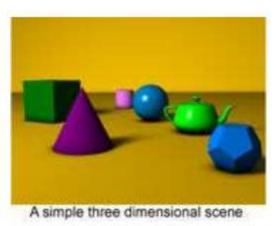




Z-buffer representation

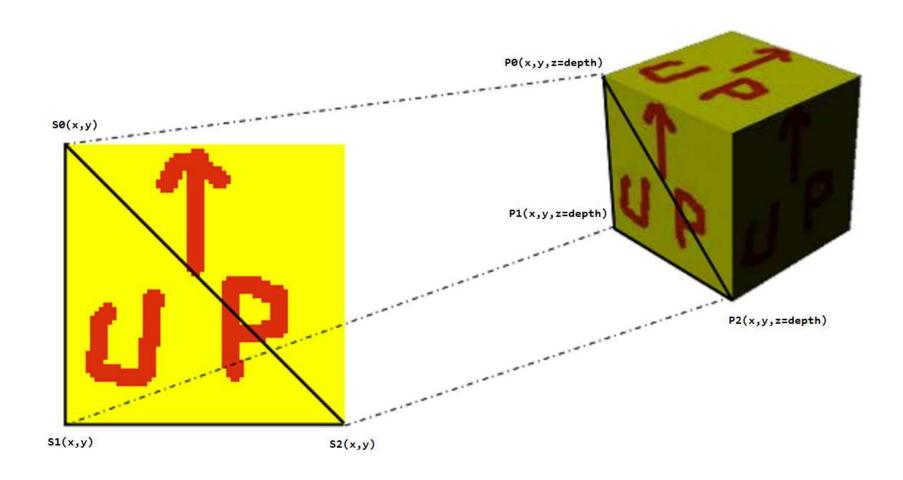
Z-Buffer Operation

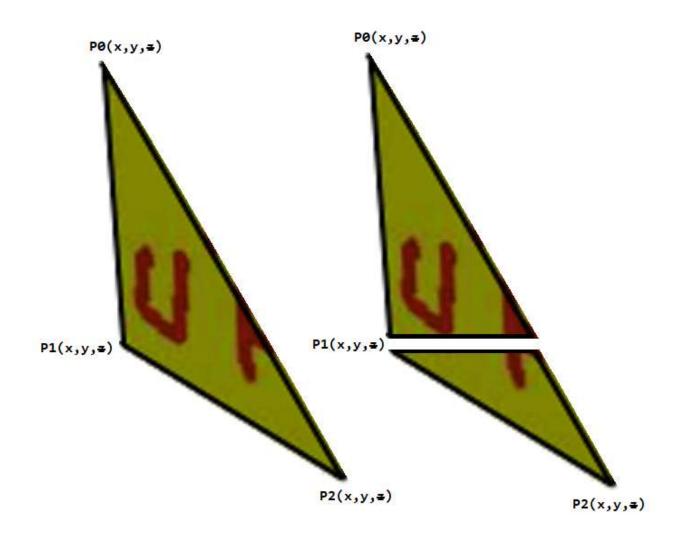
- Before drawing each pixel on screen:
- Z-Test: compare Z value ([0 1] in view frustum) against value in Z-buffer
- If (Z value < value in Z-buffer), update image buffer & Zbuffer

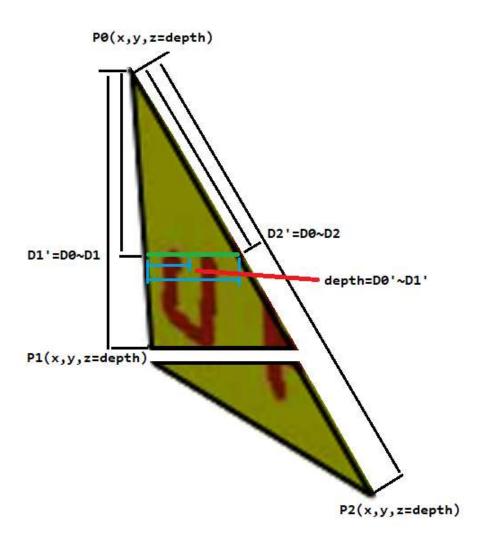


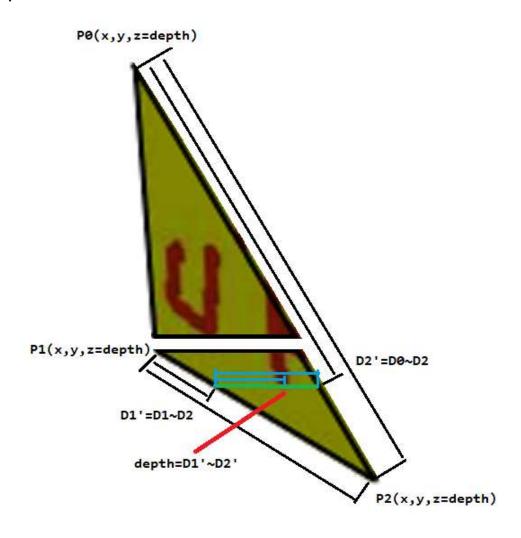


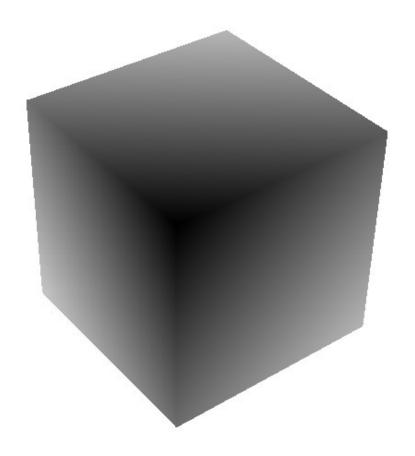
Depth / Texturing / Applying Surfaces



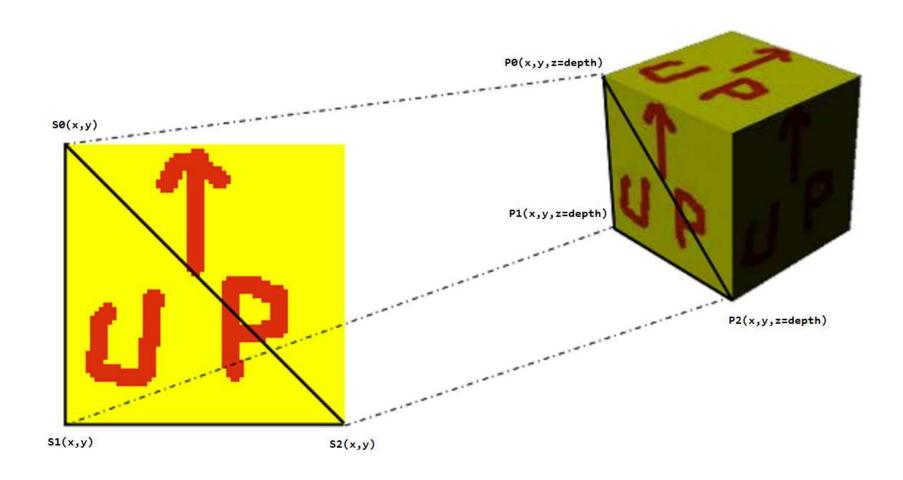




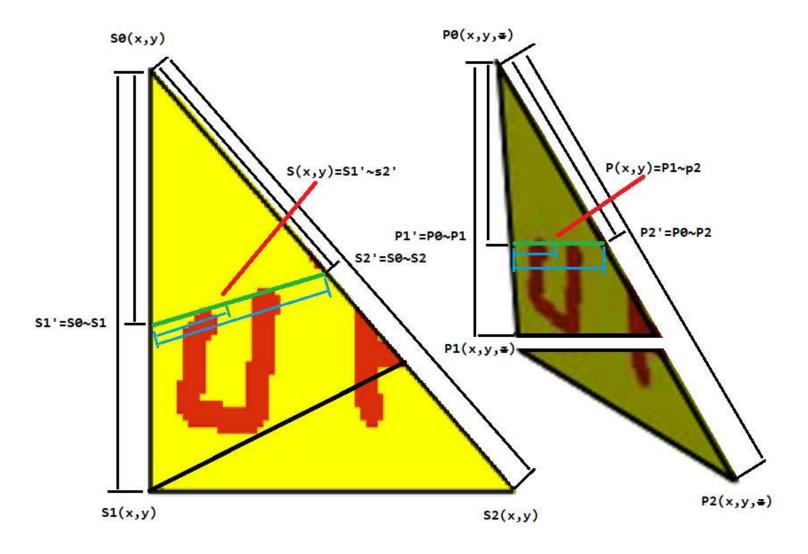




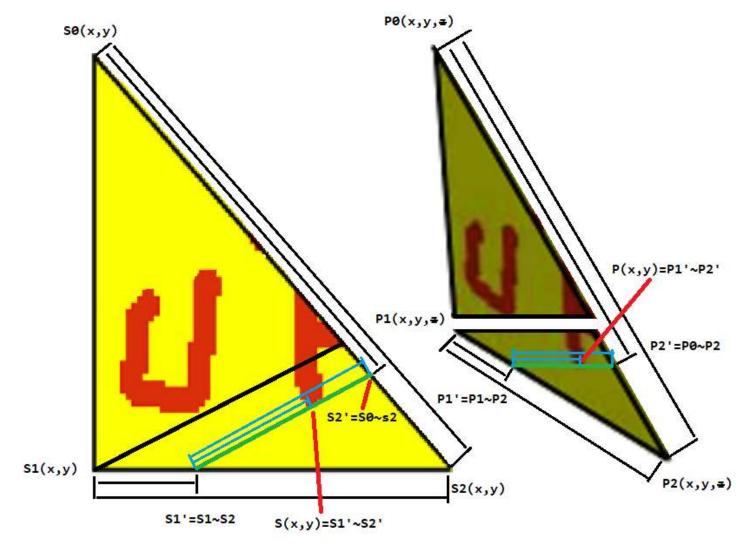
Depth / Texturing / Applying Surfaces



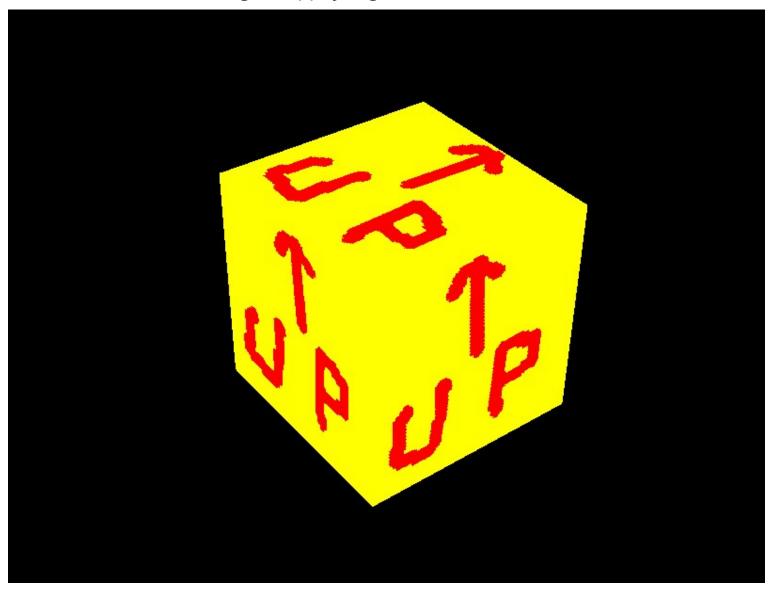
Rasterization: Texturing / Applying Surfaces



Rasterization: Texturing / Applying Surfaces

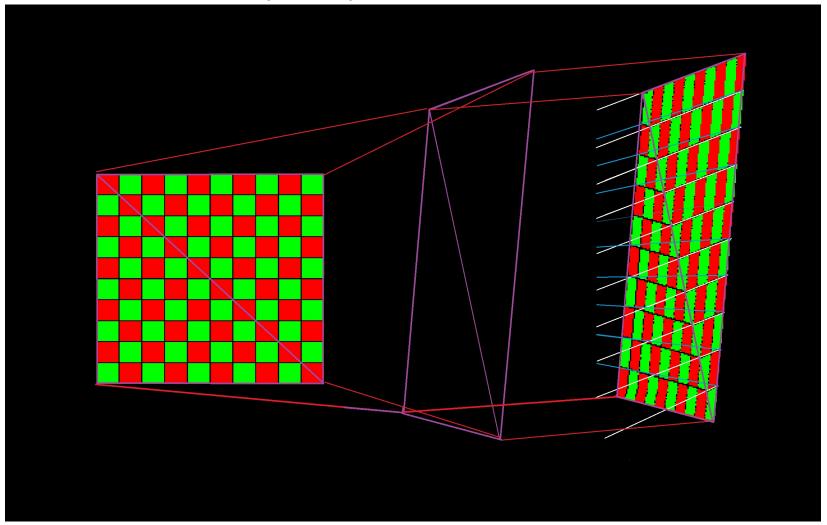


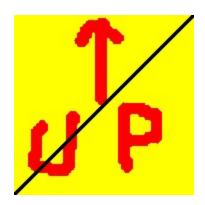
Rasterization: Texturing / Applying Surfaces

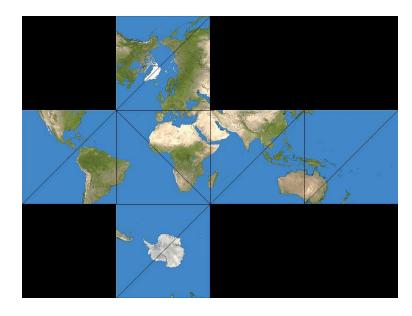


Rasterization: Affine Perspective Error with Triangle/Poly Interpolation

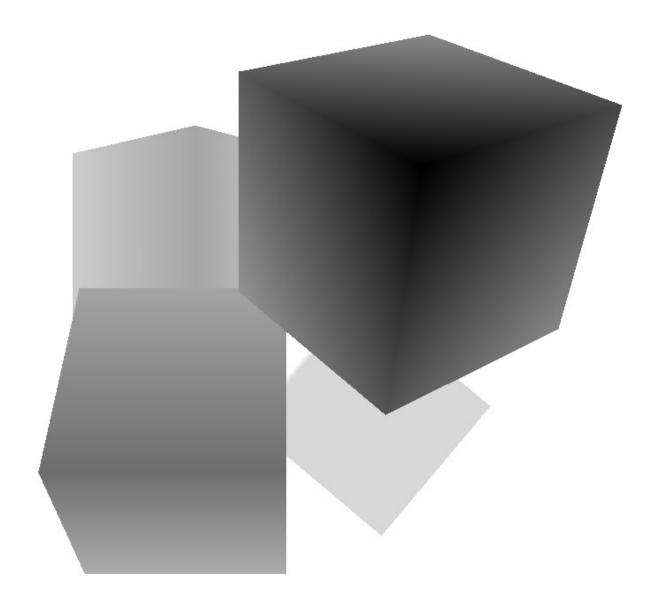
a.k.a.: Derivative Discontinuity in Interpolation

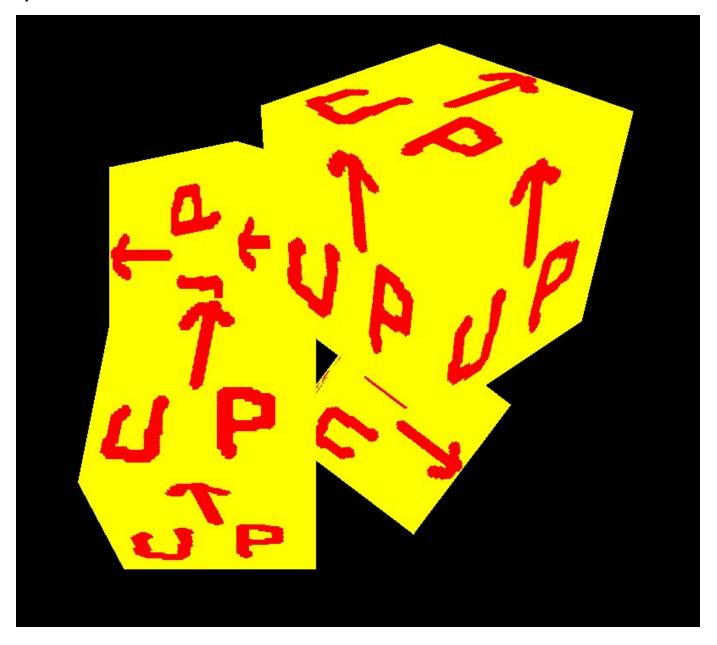


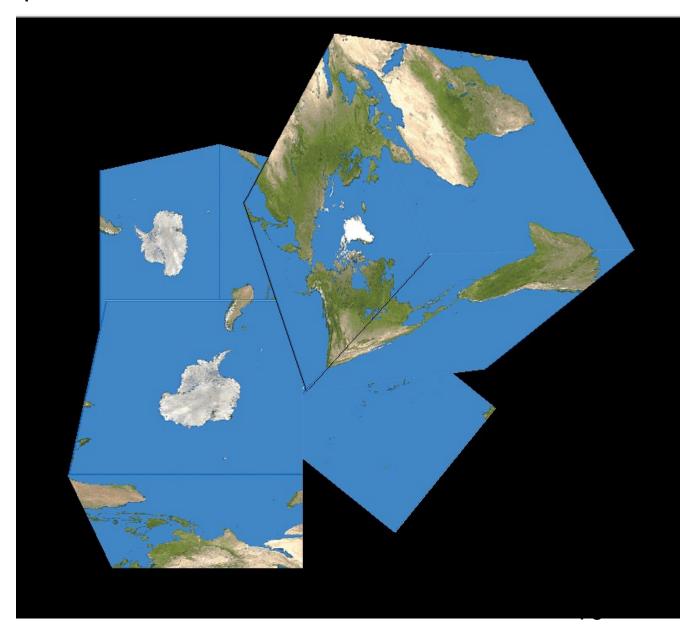




```
Screen CreateWorld(Rect& rect, Model& model, float angle, float fScale, float fOffset)
   Model modelX = model * Scale(fScale, fScale, fScale);
   Model modelY = modelX * RotateZ(90);
   Model modelZ = modelX * RotateY(90);
   World world;
   world += modelX * (RotateX(angle) *
                                                                          Translate(-f0ffset, -f0ffset, -20));
   world += modelY * (RotateY(angle) *
                                                                          Translate(-f0ffset, f0ffset, -40));
   world += modelZ * (RotateZ(angle) *
                                                                          Translate( f0ffset, -f0ffset, -60));
   world += modelX * (RotateX(angle) * RotateY(angle) * RotateZ(angle) * Translate( fOffset, fOffset, 0));
   Matrix pov = PointOfView({ 0, 0, 100 }, { 0, 0, 0 }, { 0, 1, 0 });
   Matrix fov = FieldOfView(45, rect.AspectRatio(), 1, 100);
   Matrix view = Viewport(rect, 0, 100);
   Screen screen = world * (pov * fov * view);
    screen.PerspectiveDivide();
    return screen;
```







DEMO

Questions?