A3 Part 2

By Christopher Jung, Tingyu Liang

Q1

a)

i) L -> MO

L^+ = LNOMQRSP   Satisfies BCNF

ii) M -> P

M^+ = MP   Violates BCNF

iii) N -> MQR

N^+ = NMQRP Violates BCNF

iv) O -> S

O^+ = OS   Violates BCNF

So, M -> P, N -> MQR, O ->S are FDs that violate BCNF

b)

We will decompose R using FD *N -> MQR*. N^+ = *NMQRP*, so this gives two relations:
R1 = LOS and *R2 = NMQRP*

Project the FDs onto *R1 = LOS*.

L	N	O	S	closure	FDs
✓				L^+ = LNOMQRSP	L -> MNOPQRS; L is a superkey of R1
	✓			N^+ = NMQRP	nothing
		✓		O^+ = OS	O -> S; Violates BCNF; abort the projection

We must decompose *R1* further.

We will decompose *R1* using FD *O ->S*. O^+ = *OS*, so this gives two relations:
R3 = LNO and *R4 = OS.*

Project the FDs on *R3 = LNO*

L	N	O	closure	FDs
✓			L^+ = LNOMQRSP	L -> NO; L is a superkey of R3
	✓		N^+ = NMQRP	nothing
		✓	O^+ = OS	nothing
✓	✓		irrelevant	Can only generate weaker FDs
✓		✓	irrelevant	Can only generate weaker FDs
	✓	✓	NO^+ = NOMQRS	nothing
✓	✓	✓	irrelevant	Can only generate weaker FDs

This relation, R3, satisfies BCNF.

Project the FDs on *R4 = OS*

O	S	closure	FDs
✓		O^+ = OS	L -> NO; L is a superkey of R3

	✓	$S^+ = S$	nothing
✓	✓	irrelevant	Can only generate weaker FDs

This relation, R4, satisfies BCNF.

We now return to *R2 = NMQRP*. Then, we project the FDs on *R2*.

N	M	Q	R	S	closure	FDs
✓					$N^+ = NMQRP$	N -> MQRP; N is a superkey of R2
	✓				$M^+ = MP$	M -> P; Violates BCNF; abort the projection

We must decompose R2 further.

We will decompose *R2* using FD *M ->P. M^+ = MP*, so this gives two relations:
R5 = NMQR and *R6 = MP.*

Project the FDs onto *R5 = NMQR*.

N	M	Q	R	closure	FDs
✓				$N^+ = NMQRP$	N -> MQR; N is a superkey of R5
	✓			$M^+ = MP$	nothing
		✓		$Q^+ = Q$	nothing
			✓	$R^+ = R$	nothing
✓	✓			irrelevant	Can only generate weaker FDs
✓		✓		irrelevant	Can only generate weaker FDs
✓			✓	irrelevant	Can only generate weaker FDs
	✓	✓		$MQ^+ = MQP$	nothing
	✓		✓	$MR^+ = MRP$	nothing

		✓	✓	$QR^+ = QR$	nothing
✓	✓	✓		irrelevant	Can only generate weaker FDs
✓	✓		✓	irrelevant	Can only generate weaker FDs
✓		✓	✓	irrelevant	Can only generate weaker FDs
	✓	✓	✓	$MQR^+ = MQRP$	nothing
✓	✓	✓	✓	irrelevant	Can only generate weaker FDs

This relation, *R5*, satisfies BCNF.

Project the FDs onto *R6 = MP*.

M	P	closure	FDs
✓		$M^+ = MP$	M -> P; M is a superkey of R6
	✓	$P^+ = P$	nothing
✓	✓	irrelevant	Can only generate weaker FDs

This relation, *R6*, satisfies BCNF.

Final Decomposition:

a) *R3 = LNO* with FD *L -> NO*,
b) *R4 = OS* with FD *O -> S*,
c) *R5 = NMQR* with FD *N -> MQR*,
d) *R6 = MP* with FD *M -> P*.

c) Since all 4 FDs from the original Relation *R* exist in the FDs for the decomposed relations where each relation satisfies the BCNF, there are no losses in the FDs. So, the schema does preserve dependencies.

d)

We will do a Chase Test to prove that this decomposition is a lossless-join decomposition. We have a relation *R* = *LMNOPQRS* with FDS
L -> NO, M -> P, N -> MQR, O -> S

We decomposed into relations:

R3 = *LNO* with FD *L -> NO*,
R4 = *OS* with FD *O -> S*,
R5 = *NMQR* with FD *N -> MQR*,
R6 = *MP* with FD *M -> P*.

We will show if a tuple <l,m,n,o,p,q,r,s> is in *R3* ⋈ *R4* ⋈ *R5* ⋈ *R6*, it is in *R*.
We will assume <l,m,n,o,p,q,r,s> is in *R3* ⋈ *R4* ⋈ *R5* ⋈ *R6*.

We will begin the Chase Test with:

	L	M	N	O	P	Q	R	S
R3	l	l	n	o	2	3	4	5
R4	6	7	8	o	9	10	11	s
R5	12	m	n	13	14	q	r	15
R6	16	m	17	18	p	19	20	21

L -> NO does not have any effect on the table.

Because *M -> P*, we make these changes:

	L	M	N	O	P	Q	R	S
R3	l	l	n	o	2	3	4	5
R4	6	7	8	o	9	10	11	s
R5	12	m	n	13	p	q	r	15
R6	16	m	17	18	p	19	20	21

Because *N -> MQR*, we make these changes:

	L	M	N	O	P	Q	R	S
R3	1	m	n	o	2	q	r	5
R4	6	7	8	o	9	10	11	s
R5	12	m	n	13	p	q	r	15
R6	16	m	17	18	p	19	20	21

Because *O -> S*, we make these changes:

	L	M	N	O	P	Q	R	S
R3	1	m	n	o	2	q	r	s
R4	6	7	8	o	9	10	11	s
R5	12	m	n	13	p	q	r	15
R6	16	m	17	18	p	19	20	21

We will do one more iteration to see if changes on the first iteration allowed us to move further on the Chase Test.

L -> NO has no effect once again.

Because M -> P, we make these changes:

	L	M	N	O	P	Q	R	S
R3	1	m	n	o	p	q	r	s
R4	6	7	8	o	9	10	11	s

R5	12	m	n	13	p	q	r	15
R6	16	m	17	18	p	19	20	21

We can stop here. We observe that the tuple <l,m,n,o,p,q,r,s> occurs in the first row. Therefore, the Chase Test succeeded.

Q2

a)
We first simplify the FDs to singleton RHS's. We have a set S1:

1. ACD -> E
2. B -> C
3. B -> D
4. BE -> A
5. BE -> C
6. BE -> F
7. D -> A
8. D -> B
9. E -> A
10. E -> C

We reduce the LHS of S1:

1. D^+ = DABCEF so we can reduce the LHS to D
4. B^+ = BCDAEF so we can reduce the LHS to B
   E^+ = EAC so we can reduce to E
5. B^+ = BCDAEF so we can reduce the LHS to B
   E^+ = EAC so we can reduce to E
6. B^+ = BCDAEF so we can reduce the LHS to B

We have the set S2 after reducing:

1. D -> E
2. ~~B -> C~~
3. B -> D
4. ~~B -> A~~
5. ~~E -> A~~
6. ~~B -> C~~
7. ~~E -> C~~
8. B -> F
9. ~~D -> A~~
10. D -> B
11. E -> A
12. E -> C

We look for the redundant FDs to eliminate:

FD	Exclude to compute clousure	Clousure	Decision
1	1	No way to get to E	keep
2	2	Duplicate to FD 6	discard

3	2, 3	No way to get to D	keep
4	2, 4	B^+= BDCEA	discard
5	2, 4, 5	Duplicate to FD 11	discard
6	2, 4, 5, 6	B^+ = BDEAC	discard
7	2, 4, 5, 6, 7	Duplicate to FD 12	discard
8	2, 4, 5, 6, 7	No way to get to F	keep
9	2, 4, 5, 6, 7, 9	D^+ = DEAC	discard
10	2, 4, 5, 6, 7, 9, 10	No way to get to B	keep
11	2, 4, 5, 6, 7, 9, 10, 11	No way to get to A	keep
12	2, 4, 5, 6, 7, 9, 10, 11, 12	No way to get to C	keep

The following set S3 is a minimal basis:

3. B -> D
8. B -> F
10. D -> B
1. D -> E
11. E -> A
12. E -> C

b)

	On LHS	On RHS	
GH	╳	╳	In every key
ACF	╳	✓	In no key
BDE	✓	✓	Must check

B	D	E	Closure	Attributes Included
✓			BGH^+ = BGHDFEAC	All
	✓		DGH^+ = DGHBEACF	All

		✓	$EGH^+ = EGHAC$	Missing B, D, F
✓	✓		irrelevant	Can only generate weaker FDs
✓		✓	irrelevant	Can only generate weaker FDs
✓	✓	✓	irrelevant	Can only generate weaker FDs

Keys for relation A are BGH and DGH

c)
Revise S3 into S4:

1. B -> DF
2. D -> BE
3. E -> AC

The set of relations that would have these attributes:
R1(B, D, F)   R2(B, D, E)   R3(A, C, E)

Add another relation that includes the key:
R4(B, G, H) or R4(D, G, H)

The resulting relation sets are:
R1(B, D, F)   R2(B, D, E)   R3(A, C, E)   R4(B, G, H)
Or
R1(B, D, F)   R2(B, D, E)   R3(A, C, E)   R4(D, G, H)

d)
No, the schema does not allow redundancy because none of the FDs in each relation violate BCNF.

R1: B^+ = BDF so B -> DF does not violate BCNF
R2: D^+ = DBE so D -> BE does not violate BCNF
R3: E^+ = EAC so E -> AC does not violate BCNF
R4: BGH or DGH is a key