

Appendix VI: Backward Selection

```
In [1]: """Imports necessary packages"""

import itertools
import math
from typing import Dict, Iterable, List, Union

import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import pylab
import scipy
import scipy.stats as stats
import seaborn as sns
import statsmodels.api as sm
from sklearn.model_selection import train_test_split

sns.set_style("whitegrid")
```

```
In [2]: def backward_p_vals(model_str: str, data: Iterable) -> Dict[str, float]:
        """Obtains the p-values of variables in a model.

        Args:
            model_str (str): the model string as required by statsmodels.api.formulas.ols
            data (Iterable): the data used to fit the model.

        Returns:
            Dict[str, float]: the dictionary of variables of the model (keys) and their
            """
        splits = model_str.split(" ~ ")
        expl_vars = splits[-1].split(" + ")
        expl_vars = [var.replace("*", ":") for var in expl_vars]
        model = sm.formula.ols(formula=model_str, data=data)
        model_fitted = model.fit()
        p_vals = model_fitted.pvalues.to_dict()
        p_vals_to_return = {}
        for p_key, p_val in p_vals.items():
            if p_key in expl_vars:
                p_vals_to_return[p_key] = p_val
            for var in expl_vars:
                if var+"[" in p_key:
                    p_vals_to_return[var] = p_val
        return p_vals_to_return
```

```
In [3]: def print_p_vals_from_dict(model_dict: Dict[str, float]) -> None:
        """Prints p-values from a dictionary.

        Args:
            model_dict (Dict[str, float]): the dictionary of variables of the model (key
            """
        for var, value in model_dict.items():
            print("p-value of %s: %.8f" % (var, value))
```

```
In [4]: def get_new_backward_model(current_model: str, var_to_remove: str) -> str:
        """Obtains the new model by removing the `var_to_remove` from the `current_model`
```

Args:

current_model (str): the current model string as required by statsmodels.api
var_to_remove (str): the variable to remove.

Returns:

str: the new model string as required by statsmodels.api.formulas.ols
"""

```
var_to_remove = var_to_remove.replace(":", "")
splits = current_model.split(" ~ ")
expl_vars = splits[-1].split(" + ")
expl_vars = [var for var in expl_vars if var != var_to_remove]
return "%s ~ %s" % (splits[0], " + ".join(expl_vars))
```

In [5]:

```
def backward_selection(initial_model: str, data: Iterable) -> str:
    """Performs the backwards selection.

    Args:
        initial_model (str): The initial model string as required by statsmodels.api
        data (Iterable): the data used to fit the models.

    Returns:
        str: the best model string as required by statsmodels.api.formulas.ols.
    """
    i = 1
    current_model_str = initial_model
    while not current_model_str.endswith(" ~ "):
        print("--- STEP %i ---" % i)
        print("current model: %s" % current_model_str)
        p_vals = backward_p_vals(current_model_str, data)
        print_p_vals_from_dict(p_vals)
        max_key = max(p_vals, key=p_vals.get)
        max_val = p_vals[max_key]
        if max_val > 0.05:
            print("The maximal p-value is of %s" % max_key)
            i += 1
            current_model_str = get_new_backward_model(current_model_str, max_key.sp

        else:
            print("The maximal p-value is statistically significant.")
            return current_model_str

    return current_model_str
```

In [6]:

```
data = pd.read_csv("D:/School/frequentist-statistics/ITM-song-popularity/database/it
data = data.drop("Unnamed: 0", axis=1)
```

In [7]:

```
explanatory_vars = ["name_len", "track_number", "duration", "acousticness", "danceab
```

In [8]:

```
model_string = "popularity_abs ~ %s" % " + ".join(explanatory_vars)
backward_best_abs = backward_selection(model_string, data)
print("The best model for absolute popularity excluding correlations obtained via th
```

```
--- STEP 1 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode
p-value of mode: 0.35223609
p-value of name_len: 0.14958524
p-value of track_number: 0.00011500
p-value of duration: 0.12434777
```

```
p-value of acousticness: 0.82766002
p-value of danceability: 0.00535535
p-value of energy: 0.87226718
p-value of loudness: 0.21019295
p-value of speechiness: 0.26005450
p-value of valence: 0.33279373
p-value of tempo: 0.38971566
p-value of complexity: 0.83178724
p-value of age_days: 0.00000000
The maximal p-value is of energy
--- STEP 2 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + loudness + speechiness + valence + tempo + complexity + age_days + mo
de
p-value of mode: 0.33992165
p-value of name_len: 0.14901659
p-value of track_number: 0.00008886
p-value of duration: 0.12309395
p-value of acousticness: 0.87333415
p-value of danceability: 0.00404372
p-value of loudness: 0.07890515
p-value of speechiness: 0.15246822
p-value of valence: 0.32176101
p-value of tempo: 0.38286148
p-value of complexity: 0.82150439
p-value of age_days: 0.00000000
The maximal p-value is of acousticness
--- STEP 3 ---
current model: popularity_abs ~ name_len + track_number + duration + danceability +
loudness + speechiness + valence + tempo + complexity + age_days + mode
p-value of mode: 0.34310557
p-value of name_len: 0.14872869
p-value of track_number: 0.00006659
p-value of duration: 0.11810393
p-value of danceability: 0.00384267
p-value of loudness: 0.04537763
p-value of speechiness: 0.13265319
p-value of valence: 0.30237032
p-value of tempo: 0.38372737
p-value of complexity: 0.82866244
p-value of age_days: 0.00000000
The maximal p-value is of complexity
--- STEP 4 ---
current model: popularity_abs ~ name_len + track_number + duration + danceability +
loudness + speechiness + valence + tempo + age_days + mode
p-value of mode: 0.34470228
p-value of name_len: 0.14184872
p-value of track_number: 0.00006084
p-value of duration: 0.00074191
p-value of danceability: 0.00047049
p-value of loudness: 0.03591538
p-value of speechiness: 0.11102210
p-value of valence: 0.30993453
p-value of tempo: 0.36879421
p-value of age_days: 0.00000000
The maximal p-value is of tempo
--- STEP 5 ---
current model: popularity_abs ~ name_len + track_number + duration + danceability +
loudness + speechiness + valence + age_days + mode
p-value of mode: 0.35607847
p-value of name_len: 0.11969051
p-value of track_number: 0.00003659
p-value of duration: 0.00035380
p-value of danceability: 0.00060194
p-value of loudness: 0.01375100
p-value of speechiness: 0.05096657
p-value of valence: 0.38146249
p-value of age_days: 0.00000000
The maximal p-value is of valence
```

```

--- STEP 6 ---
current model: popularity_abs ~ name_len + track_number + duration + danceability +
loudness + speechiness + age_days + mode
p-value of mode: 0.24769969
p-value of name_len: 0.13156157
p-value of track_number: 0.00004335
p-value of duration: 0.00025288
p-value of danceability: 0.00054921
p-value of loudness: 0.02033360
p-value of speechiness: 0.04876473
p-value of age_days: 0.00000000
The maximal p-value is of mode
--- STEP 7 ---
current model: popularity_abs ~ name_len + track_number + duration + danceability +
loudness + speechiness + age_days
p-value of name_len: 0.19780413
p-value of track_number: 0.00003524
p-value of duration: 0.00016706
p-value of danceability: 0.00076148
p-value of loudness: 0.02780259
p-value of speechiness: 0.03859863
p-value of age_days: 0.00000000
The maximal p-value is of name_len
--- STEP 8 ---
current model: popularity_abs ~ track_number + duration + danceability + loudness +
speechiness + age_days
p-value of track_number: 0.00002241
p-value of duration: 0.00020915
p-value of danceability: 0.00065681
p-value of loudness: 0.04993284
p-value of speechiness: 0.06033662
p-value of age_days: 0.00000000
The maximal p-value is of speechiness
--- STEP 9 ---
current model: popularity_abs ~ track_number + duration + danceability + loudness +
age_days
p-value of track_number: 0.00008076
p-value of duration: 0.00116427
p-value of danceability: 0.00127984
p-value of loudness: 0.05466204
p-value of age_days: 0.00000000
The maximal p-value is of loudness
--- STEP 10 ---
current model: popularity_abs ~ track_number + duration + danceability + age_days
p-value of track_number: 0.00000849
p-value of duration: 0.00000126
p-value of danceability: 0.00054054
p-value of age_days: 0.00000000
The maximal p-value is statistically significant.
The best model for absolute popularity excluding correlations obtained via the backw
ard selection is `popularity_abs ~ track_number + duration + danceability + age_days
`.

```

In [9]:

```

model_string = "popularity_norm ~ %s" % " ".join(explanatory_vars)
backward_best_rel = backward_selection(model_string, data)
print("The best model for normalised popularity excluding correlations obtained via

```

```

--- STEP 1 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode
p-value of mode: 0.35223609
p-value of name_len: 0.14958524
p-value of track_number: 0.00011500
p-value of duration: 0.12434777
p-value of acousticness: 0.82766002
p-value of danceability: 0.00535535
p-value of energy: 0.87226718

```

```
p-value of loudness: 0.21019295
p-value of speechiness: 0.26005450
p-value of valence: 0.33279373
p-value of tempo: 0.38971566
p-value of complexity: 0.83178724
p-value of age_days: 0.00000000
The maximal p-value is of energy
--- STEP 2 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + loudness + speechiness + valence + tempo + complexity + age_days + mo
de
p-value of mode: 0.33992165
p-value of name_len: 0.14901659
p-value of track_number: 0.00008886
p-value of duration: 0.12309395
p-value of acousticness: 0.87333415
p-value of danceability: 0.00404372
p-value of loudness: 0.07890515
p-value of speechiness: 0.15246822
p-value of valence: 0.32176101
p-value of tempo: 0.38286148
p-value of complexity: 0.82150439
p-value of age_days: 0.00000000
The maximal p-value is of acousticness
--- STEP 3 ---
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loudness + speechiness + valence + tempo + complexity + age_days + mode
p-value of mode: 0.34310557
p-value of name_len: 0.14872869
p-value of track_number: 0.00006659
p-value of duration: 0.11810393
p-value of danceability: 0.00384267
p-value of loudness: 0.04537763
p-value of speechiness: 0.13265319
p-value of valence: 0.30237032
p-value of tempo: 0.38372737
p-value of complexity: 0.82866244
p-value of age_days: 0.00000000
The maximal p-value is of complexity
--- STEP 4 ---
current model: popularity_norm ~ name_len + track_number + duration + danceability +
loudness + speechiness + valence + tempo + age_days + mode
p-value of mode: 0.34470228
p-value of name_len: 0.14184872
p-value of track_number: 0.00006084
p-value of duration: 0.00074191
p-value of danceability: 0.00047049
p-value of loudness: 0.03591538
p-value of speechiness: 0.11102210
p-value of valence: 0.30993453
p-value of tempo: 0.36879421
p-value of age_days: 0.00000000
The maximal p-value is of tempo
--- STEP 5 ---
current model: popularity_norm ~ name_len + track_number + duration + danceability +
loudness + speechiness + valence + age_days + mode
p-value of mode: 0.35607847
p-value of name_len: 0.11969051
p-value of track_number: 0.00003659
p-value of duration: 0.00035380
p-value of danceability: 0.00060194
p-value of loudness: 0.01375100
p-value of speechiness: 0.05096657
p-value of valence: 0.38146249
p-value of age_days: 0.00000000
The maximal p-value is of valence
--- STEP 6 ---
current model: popularity_norm ~ name_len + track_number + duration + danceability +
loudness + speechiness + age_days + mode
```

```

p-value of mode: 0.24769969
p-value of name_len: 0.13156157
p-value of track_number: 0.00004335
p-value of duration: 0.00025288
p-value of danceability: 0.00054921
p-value of loudness: 0.02033360
p-value of speechiness: 0.04876473
p-value of age_days: 0.00000000
The maximal p-value is of mode
--- STEP 7 ---
current model: popularity_norm ~ name_len + track_number + duration + danceability +
loudness + speechiness + age_days
p-value of name_len: 0.19780413
p-value of track_number: 0.00003524
p-value of duration: 0.00016706
p-value of danceability: 0.00076148
p-value of loudness: 0.02780259
p-value of speechiness: 0.03859863
p-value of age_days: 0.00000000
The maximal p-value is of name_len
--- STEP 8 ---
current model: popularity_norm ~ track_number + duration + danceability + loudness +
speechiness + age_days
p-value of track_number: 0.00002241
p-value of duration: 0.00020915
p-value of danceability: 0.00065681
p-value of loudness: 0.04993284
p-value of speechiness: 0.06033662
p-value of age_days: 0.00000000
The maximal p-value is of speechiness
--- STEP 9 ---
current model: popularity_norm ~ track_number + duration + danceability + loudness +
age_days
p-value of track_number: 0.00008076
p-value of duration: 0.00116427
p-value of danceability: 0.00127984
p-value of loudness: 0.05466204
p-value of age_days: 0.00000000
The maximal p-value is of loudness
--- STEP 10 ---
current model: popularity_norm ~ track_number + duration + danceability + age_days
p-value of track_number: 0.00000849
p-value of duration: 0.00000126
p-value of danceability: 0.00054054
p-value of age_days: 0.00000000
The maximal p-value is statistically significant.
The best model for normalised popularity excluding correlations obtained via the bac
ward selection is `popularity_norm ~ track_number + duration + danceability + age_d
ays`.

```

```

In [10]: correlations = ["duration*complexity", "acousticness*energy", "energy*loudness", "tr
explanatory_vars.extend(correlations)

```

```

In [11]: model_string = "popularity_abs ~ %s" % " + ".join(explanatory_vars)
best_corr_bs_abs = backward_selection(model_string, data)
print("The best model for absolute popularity including correlations obtained via ba

```

```

--- STEP 1 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*complexity + track_number*duration + duration*loudness + duration*speechiness +
acousticness*loudness + danceability*valence + danceability*complexity + loudness*co
mplexity + valence*complexity
p-value of mode: 0.26547739
p-value of name_len: 0.60641621
p-value of track_number: 0.56951738

```

```

p-value of duration: 0.66954279
p-value of acousticness: 0.64949962
p-value of danceability: 0.23062817
p-value of energy: 0.64158889
p-value of loudness: 0.62797526
p-value of speechiness: 0.60388923
p-value of valence: 0.31553780
p-value of tempo: 0.47190290
p-value of complexity: 0.71701109
p-value of age_days: 0.00000001
p-value of duration:complexity: 0.25015697
p-value of acousticness:energy: 0.80797109
p-value of energy:loudness: 0.67935489
p-value of track_number:complexity: 0.93385058
p-value of track_number:duration: 0.92220040
p-value of duration:loudness: 0.85711795
p-value of duration:speechiness: 0.96147273
p-value of acousticness:loudness: 0.88251259
p-value of danceability:valence: 0.09558841
p-value of danceability:complexity: 0.01534522
p-value of loudness:complexity: 0.88802777
p-value of valence:complexity: 0.99871982
The maximal p-value is of valence:complexity
--- STEP 2 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*complexity + track_number*duration + duration*loudness + duration*speechiness +
acousticness*loudness + danceability*valence + danceability*complexity + loudness*co
mplexity
p-value of mode: 0.26024291
p-value of name_len: 0.59858751
p-value of track_number: 0.55921276
p-value of duration: 0.66592401
p-value of acousticness: 0.54098401
p-value of danceability: 0.15820714
p-value of energy: 0.56937739
p-value of loudness: 0.55407093
p-value of speechiness: 0.60085036
p-value of valence: 0.11596182
p-value of tempo: 0.46701812
p-value of complexity: 0.71471849
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.23531749
p-value of acousticness:energy: 0.80622163
p-value of energy:loudness: 0.60623585
p-value of track_number:complexity: 0.93319374
p-value of track_number:duration: 0.92153875
p-value of duration:loudness: 0.85549518
p-value of duration:speechiness: 0.96094150
p-value of acousticness:loudness: 0.85583677
p-value of danceability:valence: 0.08477034
p-value of danceability:complexity: 0.00744455
p-value of loudness:complexity: 0.88690138
The maximal p-value is of duration:speechiness
--- STEP 3 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*complexity + track_number*duration + duration*loudness + acousticness*loudness
+ danceability*valence + danceability*complexity + loudness*complexity
p-value of mode: 0.25657819
p-value of name_len: 0.59674396
p-value of track_number: 0.49809847
p-value of duration: 0.65778656
p-value of acousticness: 0.52113626
p-value of danceability: 0.15016908
p-value of energy: 0.53885458
p-value of loudness: 0.54828031

```

```
p-value of speechiness: 0.36427617
p-value of valence: 0.11156978
p-value of tempo: 0.46430586
p-value of complexity: 0.71401568
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.13947277
p-value of acousticness:energy: 0.80495779
p-value of energy:loudness: 0.58423331
p-value of track_number:complexity: 0.93843468
p-value of track_number:duration: 0.92301212
p-value of duration:loudness: 0.85414621
p-value of acousticness:loudness: 0.83209317
p-value of danceability:valence: 0.07733601
p-value of danceability:complexity: 0.00625925
p-value of loudness:complexity: 0.88739932
The maximal p-value is of track_number:complexity
--- STEP 4 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*duration + duration*loudness + acousticness*loudness + danceability*valence + d
anceability*complexity + loudness*complexity
p-value of mode: 0.25267729
p-value of name_len: 0.58922048
p-value of track_number: 0.47099829
p-value of duration: 0.53532889
p-value of acousticness: 0.50943084
p-value of danceability: 0.14492622
p-value of energy: 0.52560551
p-value of loudness: 0.51459721
p-value of speechiness: 0.33990331
p-value of valence: 0.10910615
p-value of tempo: 0.46247192
p-value of complexity: 0.61995980
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.13682948
p-value of acousticness:energy: 0.80656258
p-value of energy:loudness: 0.55867269
p-value of track_number:duration: 0.74155449
p-value of duration:loudness: 0.86588804
p-value of acousticness:loudness: 0.82138510
p-value of danceability:valence: 0.07524717
p-value of danceability:complexity: 0.00534540
p-value of loudness:complexity: 0.90429538
The maximal p-value is of loudness:complexity
--- STEP 5 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*duration + duration*loudness + acousticness*loudness + danceability*valence + d
anceability*complexity
p-value of mode: 0.21842488
p-value of name_len: 0.55349646
p-value of track_number: 0.47009619
p-value of duration: 0.02305363
p-value of acousticness: 0.51258279
p-value of danceability: 0.13292558
p-value of energy: 0.47559329
p-value of loudness: 0.48488511
p-value of speechiness: 0.33829925
p-value of valence: 0.10725964
p-value of tempo: 0.43170268
p-value of complexity: 0.04428301
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.12615075
p-value of acousticness:energy: 0.79367966
p-value of energy:loudness: 0.49758466
p-value of track_number:duration: 0.73524038
p-value of duration:loudness: 0.84938366
```


p-value of acousticness:loudness: 0.83136498
p-value of danceability:valence: 0.07339408
p-value of danceability:complexity: 0.00455784
The maximal p-value is of duration:loudness
--- STEP 6 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness + danceability + energy + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + acousticness*energy + energy*loudness + track_number*duration + acousticness*loudness + danceability*valence + danceability*complexity
p-value of mode: 0.21772903
p-value of name_len: 0.51763725
p-value of track_number: 0.47771375
p-value of duration: 0.00298869
p-value of acousticness: 0.51917716
p-value of danceability: 0.11831825
p-value of energy: 0.44889574
p-value of loudness: 0.41907086
p-value of speechiness: 0.34283168
p-value of valence: 0.10459647
p-value of tempo: 0.42022370
p-value of complexity: 0.02892376
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.12115890
p-value of acousticness:energy: 0.79488583
p-value of energy:loudness: 0.49048155
p-value of track_number:duration: 0.70824612
p-value of acousticness:loudness: 0.83844972
p-value of danceability:valence: 0.07084978
p-value of danceability:complexity: 0.00367089
The maximal p-value is of acousticness:loudness
--- STEP 7 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness + danceability + energy + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + acousticness*energy + energy*loudness + track_number*duration + danceability*valence + danceability*complexity
p-value of mode: 0.19265425
p-value of name_len: 0.50192257
p-value of track_number: 0.48962820
p-value of duration: 0.00282395
p-value of acousticness: 0.43125402
p-value of danceability: 0.11429392
p-value of energy: 0.45867060
p-value of loudness: 0.34405213
p-value of speechiness: 0.34755983
p-value of valence: 0.10168385
p-value of tempo: 0.38113621
p-value of complexity: 0.02594248
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.11557605
p-value of acousticness:energy: 0.73123129
p-value of energy:loudness: 0.49945177
p-value of track_number:duration: 0.63604583
p-value of danceability:valence: 0.06732664
p-value of danceability:complexity: 0.00348614
The maximal p-value is of acousticness:energy
--- STEP 8 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness + danceability + energy + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + energy*loudness + track_number*duration + danceability*valence + danceability*complexity
p-value of mode: 0.18520463
p-value of name_len: 0.46934625
p-value of track_number: 0.40616364
p-value of duration: 0.00275705
p-value of acousticness: 0.29910475
p-value of danceability: 0.09971190
p-value of energy: 0.46267130
p-value of loudness: 0.37214686

p-value of speechiness: 0.26509197
p-value of valence: 0.10681217
p-value of tempo: 0.29612133
p-value of complexity: 0.02336681
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.11166008
p-value of energy:loudness: 0.54364042
p-value of track_number:duration: 0.69672566
p-value of danceability:valence: 0.06988786
p-value of danceability:complexity: 0.00293740
The maximal p-value is of track_number:duration
--- STEP 9 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness + danceability + energy + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + energy*loudness + danceability*valence + danceability*complexity
p-value of mode: 0.17404335
p-value of name_len: 0.43996885
p-value of track_number: 0.00018253
p-value of duration: 0.00277951
p-value of acousticness: 0.27728952
p-value of danceability: 0.08660063
p-value of energy: 0.37167607
p-value of loudness: 0.32930283
p-value of speechiness: 0.22026282
p-value of valence: 0.09938519
p-value of tempo: 0.26117095
p-value of complexity: 0.02149086
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.07416482
p-value of energy:loudness: 0.48448817
p-value of danceability:valence: 0.06489464
p-value of danceability:complexity: 0.00192412
The maximal p-value is of energy:loudness
--- STEP 10 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness + danceability + energy + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.18601995
p-value of name_len: 0.50882424
p-value of track_number: 0.00020226
p-value of duration: 0.00322589
p-value of acousticness: 0.28459579
p-value of danceability: 0.10142439
p-value of energy: 0.57071532
p-value of loudness: 0.46222031
p-value of speechiness: 0.22232259
p-value of valence: 0.11227650
p-value of tempo: 0.31650120
p-value of complexity: 0.02640974
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.07982410
p-value of danceability:valence: 0.07104456
p-value of danceability:complexity: 0.00232285
The maximal p-value is of energy
--- STEP 11 ---
current model: popularity_abs ~ name_len + track_number + duration + acousticness + danceability + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.16805729
p-value of name_len: 0.53170931
p-value of track_number: 0.00014883
p-value of duration: 0.00356629
p-value of acousticness: 0.36035058
p-value of danceability: 0.10483980
p-value of loudness: 0.17269346
p-value of speechiness: 0.07746947
p-value of valence: 0.13217418
p-value of tempo: 0.28967260

```
p-value of complexity: 0.02906177
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.08084636
p-value of danceability:valence: 0.08311415
p-value of danceability:complexity: 0.00247099
The maximal p-value is of name_len
--- STEP 12 ---
current model: popularity_abs ~ track_number + duration + acousticness + danceabilit
y + loudness + speechiness + valence + tempo + complexity + age_days + mode + durati
on*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.19161807
p-value of track_number: 0.00012013
p-value of duration: 0.00295459
p-value of acousticness: 0.37635159
p-value of danceability: 0.09113097
p-value of loudness: 0.20810393
p-value of speechiness: 0.09153037
p-value of valence: 0.07370789
p-value of tempo: 0.27752951
p-value of complexity: 0.01950620
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.07360210
p-value of danceability:valence: 0.04231410
p-value of danceability:complexity: 0.00126091
The maximal p-value is of acousticness
--- STEP 13 ---
current model: popularity_abs ~ track_number + duration + danceability + loudness +
speechiness + valence + tempo + complexity + age_days + mode + duration*complexity +
danceability*valence + danceability*complexity
p-value of mode: 0.21696002
p-value of track_number: 0.00016342
p-value of duration: 0.00425423
p-value of danceability: 0.12494569
p-value of loudness: 0.30189830
p-value of speechiness: 0.14704264
p-value of valence: 0.09767807
p-value of tempo: 0.27627807
p-value of complexity: 0.02013614
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.11339202
p-value of danceability:valence: 0.05422690
p-value of danceability:complexity: 0.00178928
The maximal p-value is of loudness
--- STEP 14 ---
current model: popularity_abs ~ track_number + duration + danceability + speechiness
+ valence + tempo + complexity + age_days + mode + duration*complexity + danceabilit
y*valence + danceability*complexity
p-value of mode: 0.24582499
p-value of track_number: 0.00005171
p-value of duration: 0.00045069
p-value of danceability: 0.06612764
p-value of speechiness: 0.12800188
p-value of valence: 0.09239254
p-value of tempo: 0.23617846
p-value of complexity: 0.03113239
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00913997
p-value of danceability:valence: 0.05532644
p-value of danceability:complexity: 0.00073268
The maximal p-value is of mode
--- STEP 15 ---
current model: popularity_abs ~ track_number + duration + danceability + speechiness
+ valence + tempo + complexity + age_days + duration*complexity + danceability*valen
ce + danceability*complexity
p-value of track_number: 0.00003184
p-value of duration: 0.00040598
p-value of danceability: 0.09088147
p-value of speechiness: 0.08639794
p-value of valence: 0.12234764
```

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p-value of tempo: 0.28965134
p-value of complexity: 0.04762148
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00717445
p-value of danceability:valence: 0.06455031
p-value of danceability:complexity: 0.00118044
The maximal p-value is of tempo
--- STEP 16 ---
current model: popularity_abs ~ track_number + duration + danceability + speechiness
+ valence + complexity + age_days + duration*complexity + danceability*valence + dan
ceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of danceability: 0.10294844
p-value of speechiness: 0.04090825
p-value of valence: 0.05402147
p-value of complexity: 0.07444009
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is of danceability
--- STEP 17 ---
current model: popularity_abs ~ track_number + duration + speechiness + valence + co
mplexity + age_days + duration*complexity + danceability*valence + danceability*comp
lexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of speechiness: 0.04090825
p-value of valence: 0.05402147
p-value of complexity: 0.07444009
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is of complexity
--- STEP 18 ---
current model: popularity_abs ~ track_number + duration + speechiness + valence + ag
e_days + duration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of speechiness: 0.04090825
p-value of valence: 0.05402147
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is of valence
--- STEP 19 ---
current model: popularity_abs ~ track_number + duration + speechiness + age_days + d
uration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of speechiness: 0.04090825
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is statistically significant.
The best model for absolute popularity including correlations obtained via backward
selection is `popularity_abs ~ track_number + duration + speechiness + age_days + du
ration*complexity + danceability*valence + danceability*complexity`.

```

```

In [12]: model_string = "popularity_norm ~ %s" % " + ".join(explanatory_vars)
best_corr_bs_rel = backward_selection(model_string, data)
print("The best model for relative popularity including correlations obtained via ba

```

```

--- STEP 1 ---

```

```

current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*complexity + track_number*duration + duration*loudness + duration*speechiness +
acousticness*loudness + danceability*valence + danceability*complexity + loudness*co
mplexity + valence*complexity
p-value of mode: 0.26547739
p-value of name_len: 0.60641621
p-value of track_number: 0.56951738
p-value of duration: 0.66954279
p-value of acousticness: 0.64949962
p-value of danceability: 0.23062817
p-value of energy: 0.64158889
p-value of loudness: 0.62797526
p-value of speechiness: 0.60388923
p-value of valence: 0.31553780
p-value of tempo: 0.47190290
p-value of complexity: 0.71701109
p-value of age_days: 0.00000001
p-value of duration:complexity: 0.25015697
p-value of acousticness:energy: 0.80797109
p-value of energy:loudness: 0.67935489
p-value of track_number:complexity: 0.93385058
p-value of track_number:duration: 0.92220040
p-value of duration:loudness: 0.85711795
p-value of duration:speechiness: 0.96147273
p-value of acousticness:loudness: 0.88251259
p-value of danceability:valence: 0.09558841
p-value of danceability:complexity: 0.01534522
p-value of loudness:complexity: 0.88802777
p-value of valence:complexity: 0.99871982
The maximal p-value is of valence:complexity

```

--- STEP 2 ---

```

current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*complexity + track_number*duration + duration*loudness + duration*speechiness +
acousticness*loudness + danceability*valence + danceability*complexity + loudness*co
mplexity
p-value of mode: 0.26024291
p-value of name_len: 0.59858751
p-value of track_number: 0.55921276
p-value of duration: 0.66592401
p-value of acousticness: 0.54098401
p-value of danceability: 0.15820714
p-value of energy: 0.56937739
p-value of loudness: 0.55407093
p-value of speechiness: 0.60085036
p-value of valence: 0.11596182
p-value of tempo: 0.46701812
p-value of complexity: 0.71471849
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.23531749
p-value of acousticness:energy: 0.80622163
p-value of energy:loudness: 0.60623585
p-value of track_number:complexity: 0.93319374
p-value of track_number:duration: 0.92153875
p-value of duration:loudness: 0.85549518
p-value of duration:speechiness: 0.96094150
p-value of acousticness:loudness: 0.85583677
p-value of danceability:valence: 0.08477034
p-value of danceability:complexity: 0.00744455
p-value of loudness:complexity: 0.88690138
The maximal p-value is of duration:speechiness

```

--- STEP 3 ---

```

current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*complexity + track_number*duration + duration*loudness + acousticness*loudness

```

```
+ danceability*valence + danceability*complexity + loudness*complexity
p-value of mode: 0.25657819
p-value of name_len: 0.59674396
p-value of track_number: 0.49809847
p-value of duration: 0.65778656
p-value of acousticness: 0.52113626
p-value of danceability: 0.15016908
p-value of energy: 0.53885458
p-value of loudness: 0.54828031
p-value of speechiness: 0.36427617
p-value of valence: 0.11156978
p-value of tempo: 0.46430586
p-value of complexity: 0.71401568
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.13947277
p-value of acousticness:energy: 0.80495779
p-value of energy:loudness: 0.58423331
p-value of track_number:complexity: 0.93843468
p-value of track_number:duration: 0.92301212
p-value of duration:loudness: 0.85414621
p-value of acousticness:loudness: 0.83209317
p-value of danceability:valence: 0.07733601
p-value of danceability:complexity: 0.00625925
p-value of loudness:complexity: 0.88739932
The maximal p-value is of track_number:complexity
--- STEP 4 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*duration + duration*loudness + acousticness*loudness + danceability*valence + d
anceability*complexity + loudness*complexity
p-value of mode: 0.25267729
p-value of name_len: 0.58922048
p-value of track_number: 0.47099829
p-value of duration: 0.53532889
p-value of acousticness: 0.50943084
p-value of danceability: 0.14492622
p-value of energy: 0.52560551
p-value of loudness: 0.51459721
p-value of speechiness: 0.33990331
p-value of valence: 0.10910615
p-value of tempo: 0.46247192
p-value of complexity: 0.61995980
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.13682948
p-value of acousticness:energy: 0.80656258
p-value of energy:loudness: 0.55867269
p-value of track_number:duration: 0.74155449
p-value of duration:loudness: 0.86588804
p-value of acousticness:loudness: 0.82138510
p-value of danceability:valence: 0.07524717
p-value of danceability:complexity: 0.00534540
p-value of loudness:complexity: 0.90429538
The maximal p-value is of loudness:complexity
--- STEP 5 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*duration + duration*loudness + acousticness*loudness + danceability*valence + d
anceability*complexity
p-value of mode: 0.21842488
p-value of name_len: 0.55349646
p-value of track_number: 0.47009619
p-value of duration: 0.02305363
p-value of acousticness: 0.51258279
p-value of danceability: 0.13292558
p-value of energy: 0.47559329
p-value of loudness: 0.48488511
p-value of speechiness: 0.33829925
```

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p-value of valence: 0.10725964
p-value of tempo: 0.43170268
p-value of complexity: 0.04428301
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.12615075
p-value of acousticness:energy: 0.79367966
p-value of energy:loudness: 0.49758466
p-value of track_number:duration: 0.73524038
p-value of duration:loudness: 0.84938366
p-value of acousticness:loudness: 0.83136498
p-value of danceability:valence: 0.07339408
p-value of danceability:complexity: 0.00455784
The maximal p-value is of duration:loudness
--- STEP 6 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*duration + acousticness*loudness + danceability*valence + danceability*complexi
ty
p-value of mode: 0.21772903
p-value of name_len: 0.51763725
p-value of track_number: 0.47771375
p-value of duration: 0.00298869
p-value of acousticness: 0.51917716
p-value of danceability: 0.11831825
p-value of energy: 0.44889574
p-value of loudness: 0.41907086
p-value of speechiness: 0.34283168
p-value of valence: 0.10459647
p-value of tempo: 0.42022370
p-value of complexity: 0.02892376
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.12115890
p-value of acousticness:energy: 0.79488583
p-value of energy:loudness: 0.49048155
p-value of track_number:duration: 0.70824612
p-value of acousticness:loudness: 0.83844972
p-value of danceability:valence: 0.07084978
p-value of danceability:complexity: 0.00367089
The maximal p-value is of acousticness:loudness
--- STEP 7 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + acousticness*energy + energy*loudness + track_nu
mber*duration + danceability*valence + danceability*complexity
p-value of mode: 0.19265425
p-value of name_len: 0.50192257
p-value of track_number: 0.48962820
p-value of duration: 0.00282395
p-value of acousticness: 0.43125402
p-value of danceability: 0.11429392
p-value of energy: 0.45867060
p-value of loudness: 0.34405213
p-value of speechiness: 0.34755983
p-value of valence: 0.10168385
p-value of tempo: 0.38113621
p-value of complexity: 0.02594248
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.11557605
p-value of acousticness:energy: 0.73123129
p-value of energy:loudness: 0.49945177
p-value of track_number:duration: 0.63604583
p-value of danceability:valence: 0.06732664
p-value of danceability:complexity: 0.00348614
The maximal p-value is of acousticness:energy
--- STEP 8 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + energy*loudness + track_number*duration + dancea

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```
bility*valence + danceability*complexity
p-value of mode: 0.18520463
p-value of name_len: 0.46934625
p-value of track_number: 0.40616364
p-value of duration: 0.00275705
p-value of acousticness: 0.29910475
p-value of danceability: 0.09971190
p-value of energy: 0.46267130
p-value of loudness: 0.37214686
p-value of speechiness: 0.26509197
p-value of valence: 0.10681217
p-value of tempo: 0.29612133
p-value of complexity: 0.02336681
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.11166008
p-value of energy:loudness: 0.54364042
p-value of track_number:duration: 0.69672566
p-value of danceability:valence: 0.06988786
p-value of danceability:complexity: 0.00293740
The maximal p-value is of track_number:duration
--- STEP 9 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + energy*loudness + danceability*valence + danceab
ility*complexity
p-value of mode: 0.17404335
p-value of name_len: 0.43996885
p-value of track_number: 0.00018253
p-value of duration: 0.00277951
p-value of acousticness: 0.27728952
p-value of danceability: 0.08660063
p-value of energy: 0.37167607
p-value of loudness: 0.32930283
p-value of speechiness: 0.22026282
p-value of valence: 0.09938519
p-value of tempo: 0.26117095
p-value of complexity: 0.02149086
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.07416482
p-value of energy:loudness: 0.48448817
p-value of danceability:valence: 0.06489464
p-value of danceability:complexity: 0.00192412
The maximal p-value is of energy:loudness
--- STEP 10 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + energy + loudness + speechiness + valence + tempo + complexity + age_
days + mode + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.18601995
p-value of name_len: 0.50882424
p-value of track_number: 0.00020226
p-value of duration: 0.00322589
p-value of acousticness: 0.28459579
p-value of danceability: 0.10142439
p-value of energy: 0.57071532
p-value of loudness: 0.46222031
p-value of speechiness: 0.22232259
p-value of valence: 0.11227650
p-value of tempo: 0.31650120
p-value of complexity: 0.02640974
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.07982410
p-value of danceability:valence: 0.07104456
p-value of danceability:complexity: 0.00232285
The maximal p-value is of energy
--- STEP 11 ---
current model: popularity_norm ~ name_len + track_number + duration + acousticness +
danceability + loudness + speechiness + valence + tempo + complexity + age_days + mo
de + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.16805729
```



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p-value of name_len: 0.53170931
p-value of track_number: 0.00014883
p-value of duration: 0.00356629
p-value of acousticness: 0.36035058
p-value of danceability: 0.10483980
p-value of loudness: 0.17269346
p-value of speechiness: 0.07746947
p-value of valence: 0.13217418
p-value of tempo: 0.28967260
p-value of complexity: 0.02906177
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.08084636
p-value of danceability:valence: 0.08311415
p-value of danceability:complexity: 0.00247099
The maximal p-value is of name_len
--- STEP 12 ---
current model: popularity_norm ~ track_number + duration + acousticness + danceability + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.19161807
p-value of track_number: 0.00012013
p-value of duration: 0.00295459
p-value of acousticness: 0.37635159
p-value of danceability: 0.09113097
p-value of loudness: 0.20810393
p-value of speechiness: 0.09153037
p-value of valence: 0.07370789
p-value of tempo: 0.27752951
p-value of complexity: 0.01950620
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.07360210
p-value of danceability:valence: 0.04231410
p-value of danceability:complexity: 0.00126091
The maximal p-value is of acousticness
--- STEP 13 ---
current model: popularity_norm ~ track_number + duration + danceability + loudness + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.21696002
p-value of track_number: 0.00016342
p-value of duration: 0.00425423
p-value of danceability: 0.12494569
p-value of loudness: 0.30189830
p-value of speechiness: 0.14704264
p-value of valence: 0.09767807
p-value of tempo: 0.27627807
p-value of complexity: 0.02013614
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.11339202
p-value of danceability:valence: 0.05422690
p-value of danceability:complexity: 0.00178928
The maximal p-value is of loudness
--- STEP 14 ---
current model: popularity_norm ~ track_number + duration + danceability + speechiness + valence + tempo + complexity + age_days + mode + duration*complexity + danceability*valence + danceability*complexity
p-value of mode: 0.24582499
p-value of track_number: 0.00005171
p-value of duration: 0.00045069
p-value of danceability: 0.06612764
p-value of speechiness: 0.12800188
p-value of valence: 0.09239254
p-value of tempo: 0.23617846
p-value of complexity: 0.03113239
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00913997
p-value of danceability:valence: 0.05532644
p-value of danceability:complexity: 0.00073268
The maximal p-value is of mode

```

```

--- STEP 15 ---
current model: popularity_norm ~ track_number + duration + danceability + speechiness + valence + tempo + complexity + age_days + duration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00003184
p-value of duration: 0.00040598
p-value of danceability: 0.09088147
p-value of speechiness: 0.08639794
p-value of valence: 0.12234764
p-value of tempo: 0.28965134
p-value of complexity: 0.04762148
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00717445
p-value of danceability:valence: 0.06455031
p-value of danceability:complexity: 0.00118044
The maximal p-value is of tempo
--- STEP 16 ---
current model: popularity_norm ~ track_number + duration + danceability + speechiness + valence + complexity + age_days + duration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of danceability: 0.10294844
p-value of speechiness: 0.04090825
p-value of valence: 0.05402147
p-value of complexity: 0.07444009
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is of danceability
--- STEP 17 ---
current model: popularity_norm ~ track_number + duration + speechiness + valence + complexity + age_days + duration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of speechiness: 0.04090825
p-value of valence: 0.05402147
p-value of complexity: 0.07444009
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is of complexity
--- STEP 18 ---
current model: popularity_norm ~ track_number + duration + speechiness + valence + age_days + duration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of speechiness: 0.04090825
p-value of valence: 0.05402147
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is of valence
--- STEP 19 ---
current model: popularity_norm ~ track_number + duration + speechiness + age_days + duration*complexity + danceability*valence + danceability*complexity
p-value of track_number: 0.00000826
p-value of duration: 0.00029939
p-value of speechiness: 0.04090825
p-value of age_days: 0.00000000
p-value of duration:complexity: 0.00308498
p-value of danceability:valence: 0.02843870
p-value of danceability:complexity: 0.00146201
The maximal p-value is statistically significant.
The best model for relative popularity including correlations obtained via backward

```

```
selection is `popularity_norm ~ track_number + duration + speechiness + age_days + d
uration*complexity + danceability*valence + danceability*complexity`.
```