Technion – Israel Institute of Technology
Viterbi Faculty of Electrical and Computer Engineering

Advanced Circuits and Architectures with Memristors

Homework 1

Memristor Modeling

Submission until 4/12/2022 at 11:59pm

If you have questions, please either post on the course forum (on Moodle) or send an email to course046265@gmail.com

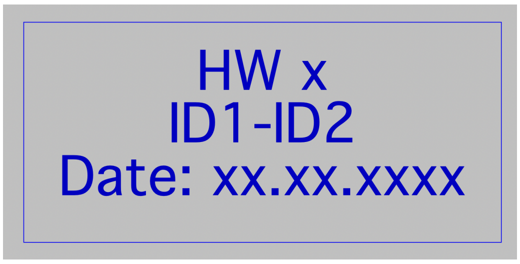
Technion Israel Institute of Technology

General Instructions

- Items marked with  are "Dry" and only require calculations, explanations and/or simple graph plotting.
- Items marked with  are "Wet" and require simulation using MATLAB/LTspice.
- The answer sheet should be submitted **in PDF format**, with answers to all questions, and with all requested plots pasted as pictures inside.
- **All written code** should be submitted in separate files.
- **The entire submission should be zipped into one file named ID1-ID2.zip for ID1 and ID2 the ID numbers of the students**. The submission is to be uploaded to Moodle.
- **Late Submission Policy:** Submission of the assignment past the deadline without permission from the course staff reduces 5 points per day, for a maximum of 3 days.

Wet Instructions

- Circuit schematics, graphs, and waveforms requested in the question prompt must be **explicitly shown** (pasted into the PDF).
- **Refer to the Appendices for guides on LTspice usage in this homework.**
- **All schematics must include the following header (shown with the schematic):**



Question 1 – Ideal Memristor

Consider a memristor with the following memristance,

$$M(q) = \frac{R_0}{\sqrt{1 + \left(\frac{q}{Q_0}\right)^2}}, \tag{1}$$

where Q_0 is the memristor's charge at $t = 0$.

Guidance: Parts b. and c. can be solved either analytically or empirically. For an empirical solution, you may use MATLAB to numerically solve the instantaneous memristance at each point in time and the change in the memristance due to the value of the current at this time. For simplicity, you may draw curves in this question with all parameters equal to 1.

a. What is the flux-charge curve? Find an expression and plot it.

b. The memristor is connected to a sinusoidal **current source** $I(t) = \cos(t)$. Plot:
   (a) Voltage $V(t)$ and current $I(t)$ (on the same plot)
   (b) Memristance $M(t)$
   (c) I-V curve

c. The memristor is connected to a sinusoidal **voltage source** $V(t) = \cos(t)$. Plot:
   (a) Voltage $V(t)$ and current $I(t)$ (on the same plot)
   (b) Memristance $M(t)$
   (c) I-V curve

d. Plot $M(t)$, the memristance over time, for a square wave *current*. Show changes in both directions (i.e., for negative and positive currents).

Question 2 – Memristor Modeling in MATLAB

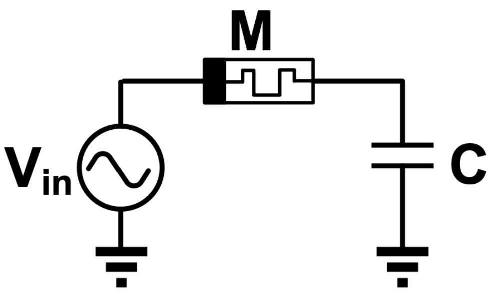
This question investigates the linear ion-drift model with Prodromakis' window function in MATLAB. For this model,

a. Write the equations that define the model.

Using the attached multimodel script for MATLAB (*memristor.m*), simulate the following conditions. For each of the configurations, choose parameters that will allow dynamic memristor behavior (switching). You will need to modify the MATLAB script in both questions.

b. Single memristor connected to a sine wave **voltage** input – notice that the provided MATLAB model is for a sine wave **current** input. Attach **transient** graphs for the state variable $w(t)/D$, for the memristance, and for the I-V curve. Explain.

c. Change the script to simulate the MC circuit drawn below, with a symmetric square wave voltage input. Choose the magnitude and frequency of the source such that the memristor switches fully to each direction, and choose the capacitance to be sufficiently small such that the effect of the capacitor is noticeable. Attach **transient** graphs for the input voltage, the state variable $w(t)/D$, the memristance, and the memristor voltage. Why is the memristance at the start of the second period different from the memristance at the start of the first period?



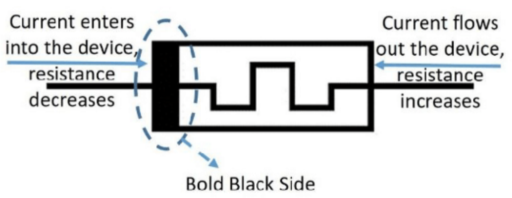
Question 3 – Memristor Simulation in LTspice (Linear Drift with Joglekar Window)

a. Write the equations that define this model (linear-drift with Joglekar window function).

b. Show the below waveforms[1] for this model (see the Appendix for loading the model into LTspice). Include the schematic used to create the waveforms.

   (a) Voltage $V(t)$
   (b) Current $I(t)$
   (c) I-V curve
   (d) State variable $x(t)$
   (e) Charge $q(t)$

   Hint: Use the following memristor symbol convention to connect the memristor properly:



c. Show the memristor voltage and current waveforms using the following parameters,

Parameter	Value
R_{on} (Ron)	130Ω
R_{off} (Roff)	$16k\Omega$
R_{init} (Rinit)	$10k\Omega$
D (D)	$11N$
μ (uv)	$9F$
p (p)	1.2

   Explain the behavior of these waveforms and explain why the current waveform does not have the same shape as the voltage waveform (i.e., why do the voltage and current waveforms have maximums at different times?).

d. Show the memristor voltage, current and normalized state variable (w/D) waveforms using the following parameters,

Parameter	Value
R_{on}	130Ω
R_{off}	$16k\Omega$
R_{init}	$10k\Omega$
D	$8N$
μ	$9F$
p	1.2

   Explain the current behaviour. When are there current spikes and why do they occur?

[1] You may need to change the memristor model parameters and the source parameters to get meaningful waveforms.

e. Show the normalized state variable (w/D) waveform using the following parameters,

Parameter	Value
R_{on}	130Ω
R_{off}	$16k\Omega$
R_{init}	$10k\Omega$
D	$10N$
μ	$9F$
p	$sweep$

where p is swept from 0.1 to 10. Explain the effect of parameter p on the state variable behaviour. Why does this occur?

f. Show the current and normalized state variable (w/D) waveforms using the following parameters,

Parameter	Value
R_{on}	130Ω
R_{off}	$16k\Omega$
R_{init}	$10k\Omega$
D	$5N$
μ	$sweep$
p	1

where μ is swept from $1F$ to $10F$. Explain the effect of parameter μ on the memristor current and the state variable behaviour.

g. For the following parameters,

Parameter	Value
R_{on}	100Ω
R_{off}	$16k\Omega$
R_{init}	$5k\Omega$
D	$5N$
μ	$29F$
p	0.5

find the cut-off frequency of the memristor (the frequency at which the memristor behaves the same as a normal resistor). Explain your methodology.

Question 4 – Memristor Simulation in LTSpice (Binarized Model)

This question introduces a simplified memristor model which will be used in later exercises. The model is defined according to,

$$\frac{dx}{dt} = \begin{cases} e^{a_1 - x} & v > 1.2 \\ 0 & -1.2 \leq v \leq 1.2 \\ -e^{a_2 + x} & v < -1.2 \end{cases} \tag{2}$$

$$R(x) = \begin{cases} R_{on} & x \geq 0 \\ R_{off} & x < 0 \end{cases} \tag{3}$$

$$I(t) = \frac{1}{R(x)} v(t) \tag{4}$$

a. Explain the behavior of this model. Why there is no need for a window function?

b. Simulate the memristor of this model (see the Appendix for importing the model into LTspice) using the following parameters:

Parameter	Value
x_0	-2
R_0	300
R_1	100
a_1	5
a_2	5
V_f	0.8
V_r	-1.0

Simulate the memristor connected to a sinusoidal voltage source. Include the schematic in the report. Show the following waveforms:

- Voltage $V(t)$
- Current $I(t)$
- I-V curve
- Memristor resistance $R(t)$

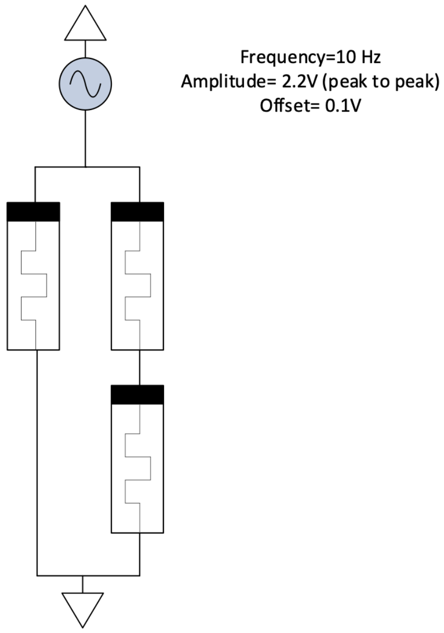
Explain the waveform behaviour.

c. How do the parameters a_1 and a_2 influence the dynamics of the memristor model? Design and simulate a schematic that shows it (include the schematic in the report), and present the waveforms that justify your claim.

d. We will now start using memristors to build more complex and interesting circuits using the binarized memristor model with the following parameters,

Parameter	Value
x_0	-2
R_0	1000
R_1	100
a_1	20
a_2	20
V_f	1.0
V_r	-1.0

Build a schematic for the following circuit (and include it in the report):



Simulate this circuit and draw the equivalent resistance of this circuit as function of time. Explain its behavior (why and when does it change).

Bonus Question – Memristor Simulation in LTspice (Yakopcic Model)

This question is a *bonus* question that adds up to 5 points to the grade of this homework assignment. The grade of the assignment is capped at 100.

The Yakopcic memristor model is a model similar to VTEAM that enables fitting the model to different memristor devices. This question will deal with understanding the dynamics of this model.

The model I-V relation is given by:

$$I(t) = \begin{cases} a_1 x(t)\sinh(bV(t)) & V(t) \geq 0 \\ a_2 x(t)\sinh(bV(t)) & V(t) < 0 \end{cases} \tag{5}$$

The state variable ($x(t)$), which is normalized between 0 and 1, changes according to:

$$\frac{dx}{dt} = g(V(t))f(x(t)) \tag{6}$$

where $g(V(t))$ is

$$g(V(t)) = \begin{cases} A_p(e^{V(t)} - e^{V_p}) & V(t) > V_p \\ -A_n(e^{-V(t)} - e^{V_n}) & V(t) < -V_n \\ 0 & -V_n \leq V(t) \leq V_p \end{cases} \tag{7}$$

and $f(x(t))$ limits the state variable dynamics according to

$$f(x) = \begin{cases} \begin{cases} e^{-\alpha_p(x-x_p)} w_p(x,x_p) & x \geq x_p \\ 1 & x < x_p \end{cases} & V(t) \geq 0 \\ \begin{cases} e^{\alpha_n(x+x_n-1)} w_n(x,x_n) & x \leq 1-x_n \\ 1 & x > 1-x_n \end{cases} & V(t) < 0 \end{cases} . \tag{8}$$

The window functions $w_{p,n}$ used in $f(x(t))$ keep the state variable between 0 and 1 and are defined by

$$w_p(x,x_p) = \frac{x_p - x}{1-x_p} + 1 \qquad w_n(x,x_n) = \frac{x}{1-x_n} \tag{9}$$

a. Explain how the window functions $w_{p,n}$ keep in the interval $[0,1]$.

b. Simulate the memristor of this model (see the Appendix for instructions on importing the model into LTspice) using the following parameters:

Parameter	Value
a_1	0.5
a_2	0.5
b	0.05
V_p	0.8
V_n	1
A_p	400000
A_n	400000
X_p	0.3
X_n	0.5
α_p	5
α_n	5
X_0	0.05
eta	1

Connect the memristor to a voltage source[2] in order to be able to simulate the I-V curve of the device and other waveforms.

Hint: You can connect a wire and net label to the state pin of the device to be able to draw the state variable of the device ($x(t)$).

Which parameters control the threshold voltage of the device (the voltage at which it changes its state from low to high resistance and vice versa)? How is this switching behavior different from the previous model (from Question 3)? Explain your answer and show the relevant waveforms and equations to justify your claims.

Hint: Modify the parameters A_p and A_n to study their influence on the device behavior.

c. For the same parameters from the previous step, what are the R_{on} (low resistance state) and R_{off} (high resistance state) values of the device? Show and explain your reasoning.

d. What are the limitations of this memristor model for modeling the device resistance values? **Hint:** Look at how the device resistance is modeled in a simpler model (e.g., as in Question 3).

[2]You may need to change the amplitude of the voltage source in order to get meaningful results.

Appendix 1 – Importing Memristor Models into LTspice

Step 1

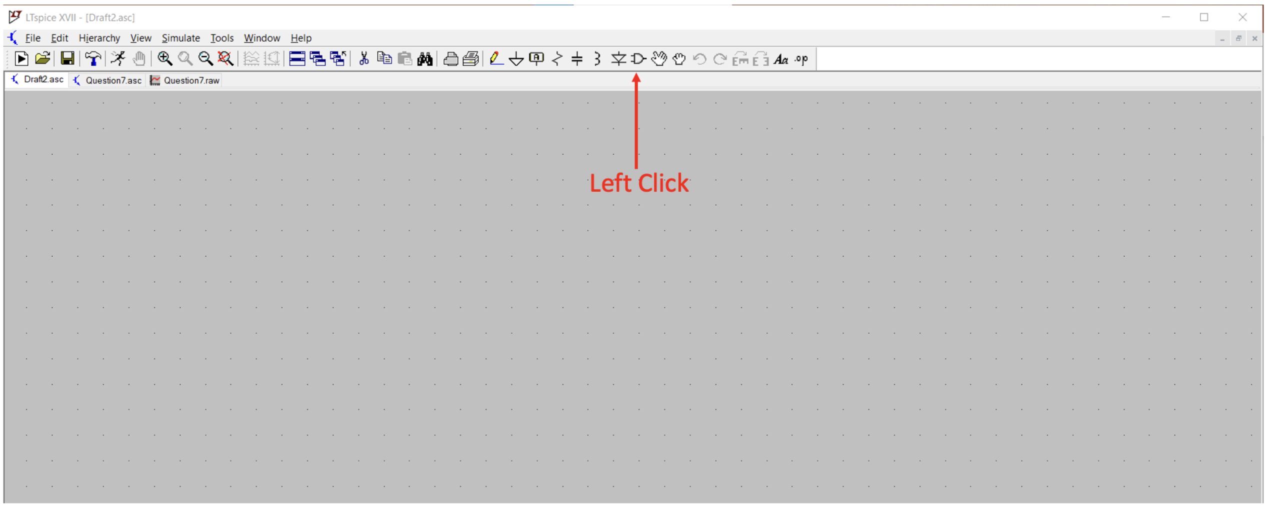
In the **same folder** as your schematic design, place the two files for each model that you import into LTspice:

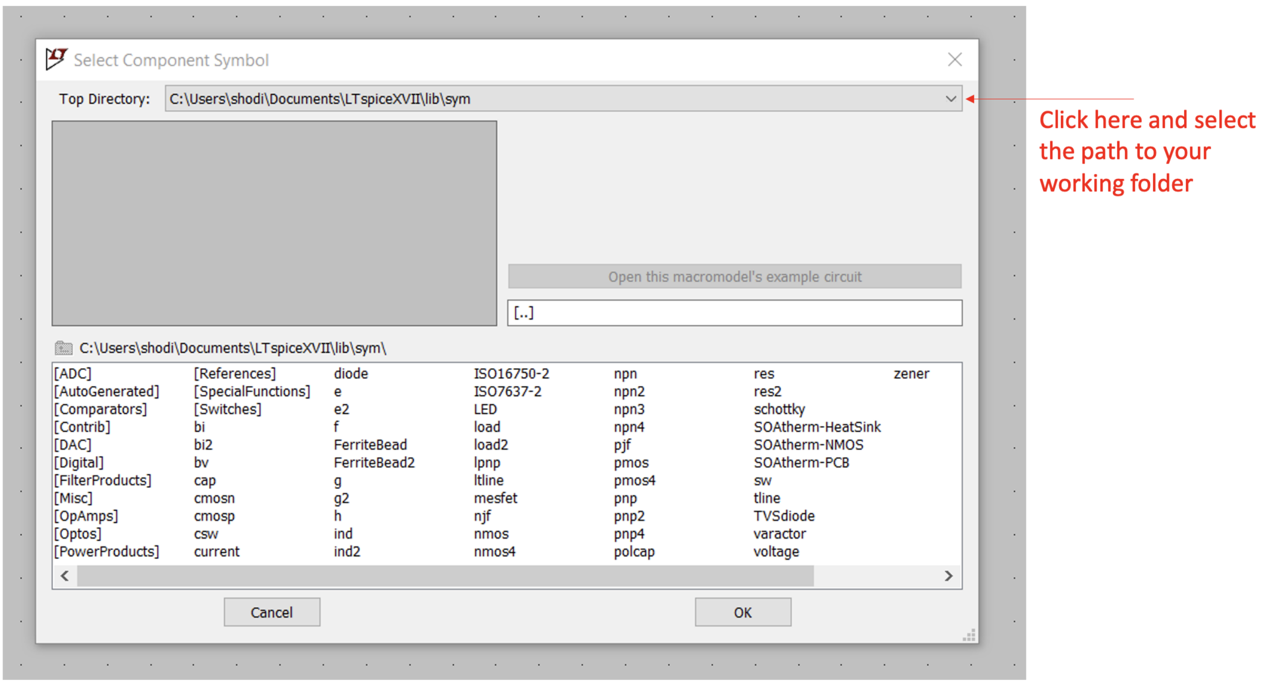
- .LIB file (the model description in SPICE format)
- .asy file (the model symbol-graphical design)

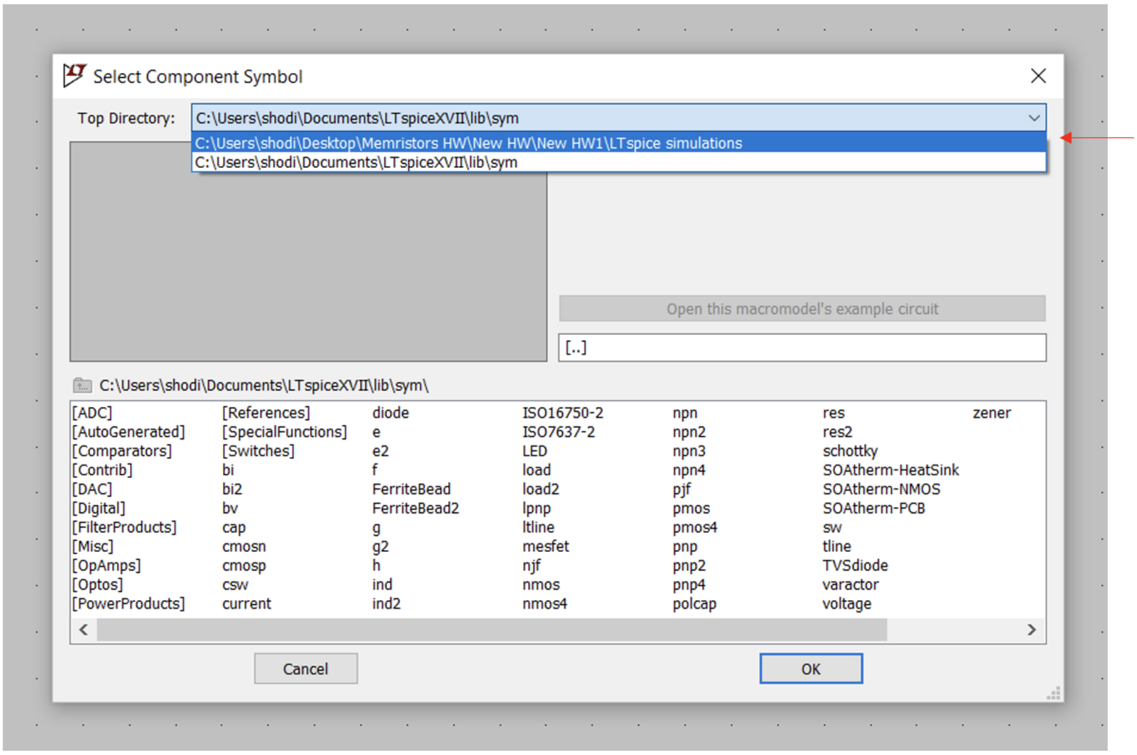
The files are provided on the course Moodle site.

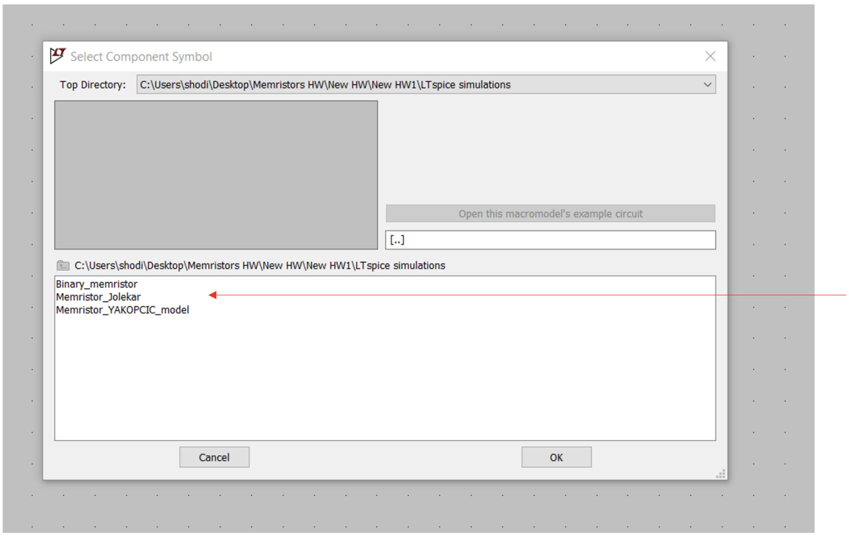
Step 2

Import a memristor model into your schematic as follows:







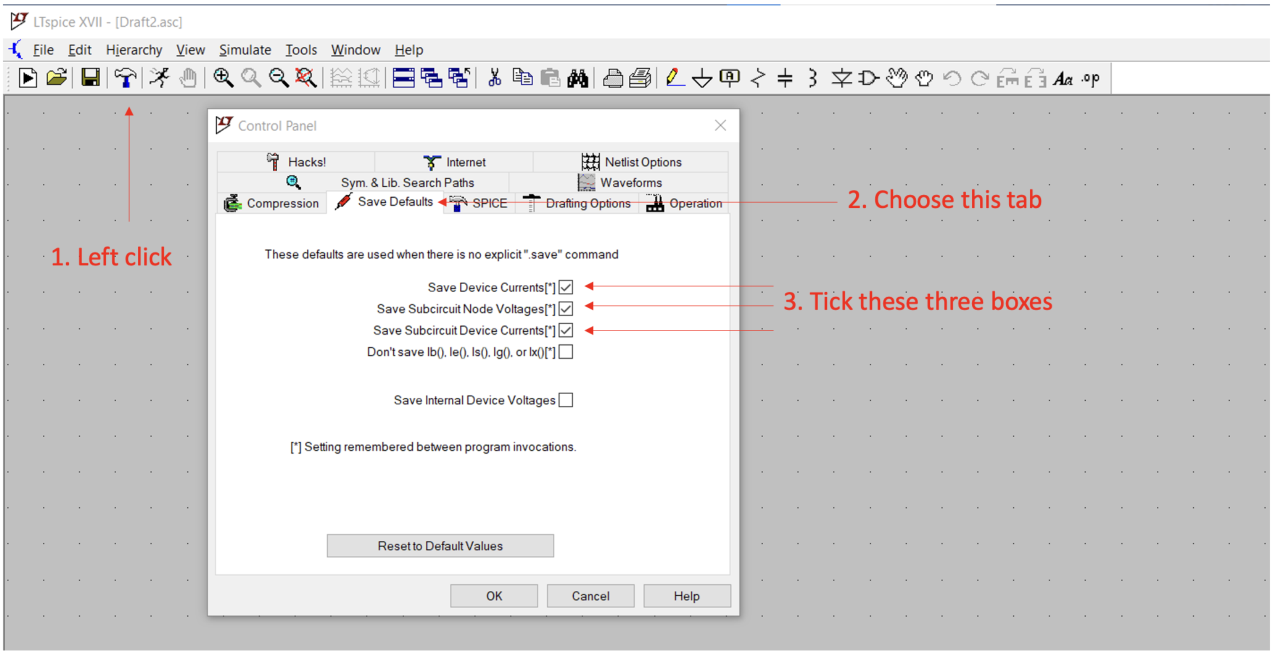


Double left click on the model to be imported into the design

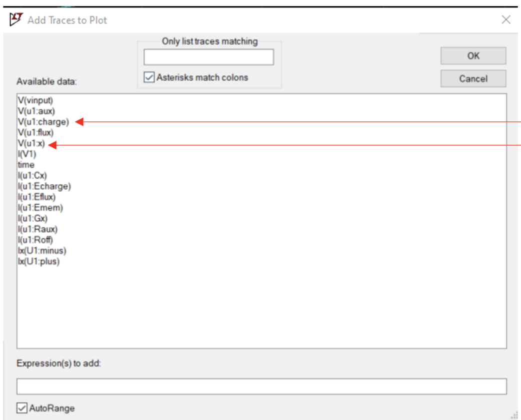
Now the model can be imported into the design just like any other component.

Appendix 2 – Saving Internal Voltages and Currents of Model

The internal voltages and currents of the model can be saved as follows,



Specifically for the Joglekar model, use the following parameters:



Choose to draw the charge (Q) through the memristor

Choose to draw the normalized state variable of the memristor [w/D]