Complex Numbers

Read Appendix B of your textbook before attending this session

ELEN 50L

Week 7 Workshop

Complex Numbers

A complex number **z** is written as **z = x + *j y***, with $\boldsymbol{j = \sqrt{-1}}$

with **real part = *x*** and **imaginary part = *y***

Rectangular form: **z = x + *j y***

Polar form: $\mathbf{z} = \boldsymbol{re^{j\phi} = r\angle\phi}$

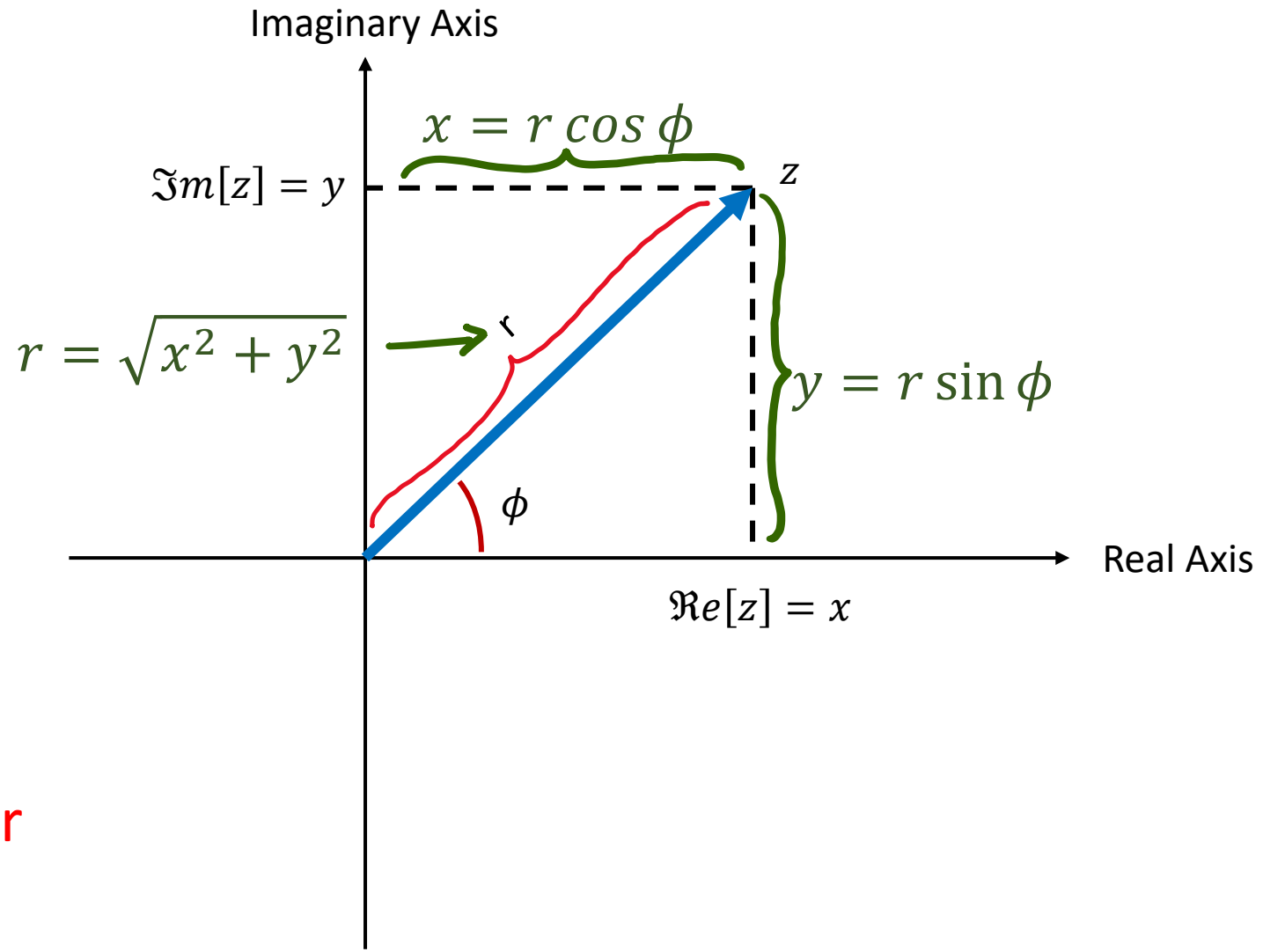
Complex Numbers: Polar/Rectangular Forms

A complex number can be represented as a point on a plane, called the Complex Plane. The line from the origin of the plane to that point is similar to a 2D vector, with x and y components. Therefore, as in 2D vectors, a complex number can be expressed in Cartesian (rectangular) or polar form.

Rectangular: $z = x + jy$

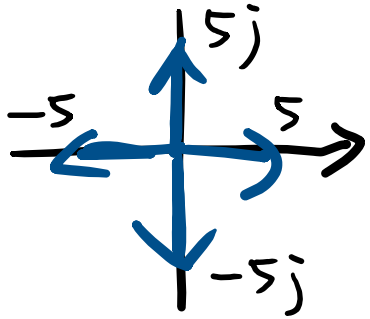
Polar: $z = r\angle\phi$

Prelab assignment #1: verify the equivalency of rectangular and polar forms



Complex Number Arithmetic

RECTANGULAR ⟷ POLAR



$5 = 5 + j0 = 5\angle 0°$

$-5 = -5 + j0 = 5\angle -180°$

$5j = 0 + j5 = 5\angle 90°$

$-5j = 0 - j5 = 5\angle -90°$

Remember: $j = \sqrt{-1} \Rightarrow j^2 = -1 \Rightarrow \frac{1}{j} = -j$

Complex Number Arithmetic

$$Z_1 = x_1 + jy_1 = r_1 \angle \phi_1$$

$$Z_2 = x_2 + jy_2 = r_2 \angle \phi_2$$

ADDITION

$$Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

SUBTRACTION

$$Z_1 - Z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

} Rectangular form is convenient

MULTIPLICATION

$$Z_1 \cdot Z_2 = r_1 r_2 \angle (\phi_1 + \phi_2)$$

DIVISION

$$Z_1 / Z_2 = \frac{r_1}{r_2} \angle (\phi_1 - \phi_2)$$

} Polar form is convenient

Complex Number Arithmetic

$$z = r\angle\phi = x + jy$$

Reciprocal

$$\frac{1}{z} = \frac{1}{r}\angle -\phi$$

Square Root

$$\sqrt{z} = \sqrt{r}\angle \frac{1}{2}\phi$$

Complex Conjugate

$$z^* = x - jy = r\angle -\phi$$

Prelab assignment #2: show $zz^* = x^2 + y^2 = r^2$

Use a complex conjugate to realize the denominator of the ratio of 2 complex numbers

$$z = (3 + j4)/(1 - j2)$$

Prelab assignment #3: compute z as a single complex number in rectangular and polar forms. Indicate in which quadrant the number resides.

In-lab Exercise 1: Complex number arithmetic

$$z = \frac{5\angle 30^\circ + j2}{8 - j6} - 2\angle 15^\circ$$

Exercise 1 (continued)

$$z = \frac{5\angle 30^\circ + j2}{8 - j6} - 2\angle 15^\circ$$

Exercise 1 results

$$z = \frac{5\angle 30^\circ + j2}{8 - j6} - 2\angle 15^\circ$$

In-lab Exercise 2: Solve an equation with one unknown complex number and express it in rectangular and polar forms

$$10\angle 0° = \frac{\tilde{\mathrm{V}}}{5} + \frac{\tilde{\mathrm{V}}}{-j20/9} + \frac{\tilde{\mathrm{V}}}{j5} + \frac{\tilde{\mathrm{V}} + \mathrm{j}100}{20}$$

Assignment

(to be completed during lab, using calculator if needed)

- Worksheet has 5 problems.
- Solve all problems by pencil-and-paper, showing each step explicitly.
- Optional: if time permits, you may use any tool at your disposal, such as programmable calculator, MATLAB, HP Prime, etc. to check your answers.
- Hand in your worksheet before leaving the lab.