

Homework 5

5.1

a. State Transition Matrix

	0	1	2	3	4
0	0.520	0.413	0.044	0.019	0.004
1	0.284	0	0.716	0	0
2	0.284	0	0	0.716	0
3	0.284	0	0	0	0.716
4	1	0	0	0	0

b.

i) $q(t+1 | q(t)=0) = \begin{cases} 0 & \text{if } r \leq 0.520 \\ 1 & \text{if } 0.520 < r \leq 0.933 \\ 2 & \text{if } 0.933 < r \leq 0.977 \\ 3 & \text{if } 0.977 < r \leq 0.996 \\ 4 & \text{if } 0.996 < r \leq 1 \end{cases}$

ii) $q(t+1 | q(t) \in \{1, 2, 3\}) = \begin{cases} 0 & \text{if } r \leq 0.284 \\ q(t)+1 & \text{if } 0.284 < r \leq 1 \end{cases}$

iii) $q(t+1 | q(t)=4) = 0$

c.

c)

t	r	q(t)	q(t+1)
0	0.765	0	1
1	0.557	1	2
2	0.347	2	3
3	0.098	3	0
4	0.039	0	0
5	0.132	0	0

d.

*See Jupyter Notebook File

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estimated stationary distribution (solved using simulation):
P(q=0) = 0.466 (out)
P(q=1) = 0.195 (first)
P(q=2) = 0.153 (second)
P(q=3) = 0.106 (third)
P(q=4) = 0.080 (home)
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5.2

a.

$P_a = 0.75$, $\bar{W} = 6$

$P_b = 0.6$, $\bar{W} = 3$

b.

c.

i	$t(i)$	$g(i)$	$t_{arrival}(i)$	$t_{service}(i)$	$\Delta t(i)$	$g(i+1)$	$t(i+1)$	$b(i)$	$c(i)$	$w(i)$
0	0	0	2.97	1.19	2.97	1	2.97	0	1	0
1	2.97	1	2.52	2.67	1.52	2	5.49	2.52	1	2.52
2	5.49	2	3.07	0.13	0.13	1	5.62	0.13	0	0.26
3	5.62	1	2.42	2.03	2.03	0	7.65	2.03	0	2.03
4	7.65	0	2.56	1.25	2.56	1	10.21	0	1	0

d.

*See Excel

e.

The utilization ratio and average waiting time decreased from part a to part d from both candidates. You could get a more accurate estimation of the steady-state utilization ratio and average waiting time by increasing the number of events to a larger value of N .

f. For small values of N , outliers have more influence over the final results because there are fewer normal values to balance them out.