

COSC264

Introduction to Computer Networks and the Internet

Introduction to Routing – Distance Vector Algorithm

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Outline – today

- Network layer overview
- Routing overview
- Link-state routing (Dijkstra's algorithm)
- **Distance-vector routing (Bellman-Ford)**
- **Summary**

Distance Vector Algorithm

- Dynamic
- Decentralised (Distributed)
- Load-sensitive/load-insensitive
- Asynchronous

Routing Algorithms and Routing Protocols

Intra-AS Routing

Routing Protocols

Routing Algorithms

RIP

Bellman-Ford (Distance-vector) Algorithm

OSFP

Dijkstra's Algorithm

BGP

Bellman-Ford (Distance-vector) Algorithm

Inter-AS Routing

Bellman-Ford Equation

Define

$d_x(y) :=$ cost of least-cost path from x to y

Then

$$d_x(y) = \min_v \{c(x,v) + d_v(y)\}$$

where \min is taken over all neighbours of x

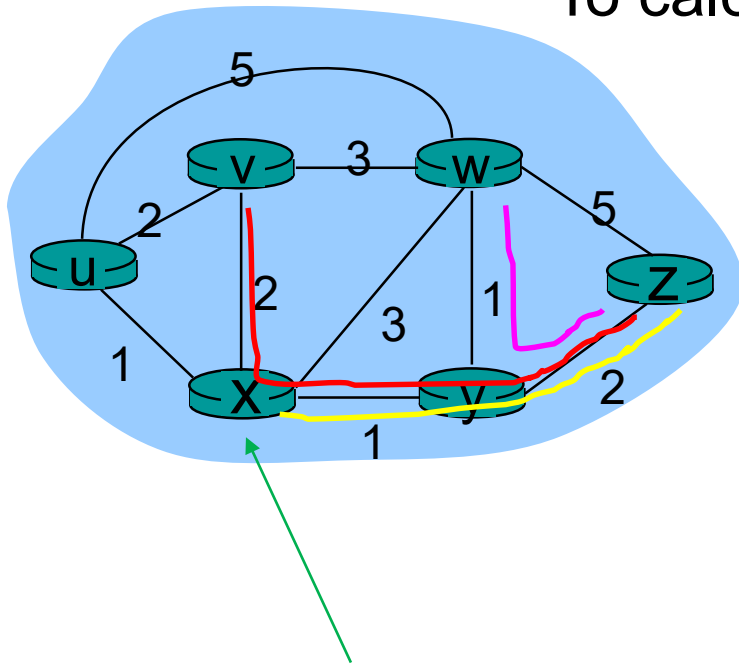
Bellman-Ford example

To calculate $d_u(z)$, according to B-F equation:

$$d_u(z) = \min \{ c(u,v) + d_v(z), \\ \mathbf{c(u,x) + d_x(z)}, \\ c(u,w) + d_w(z) \}$$

Clearly, $d_v(z) = 5$, $d_x(z) = 3$, $d_w(z) = 3$

$$= \min \{ 2 + 5, \\ \mathbf{1 + 3}, \\ 5 + 3 \} = 4$$

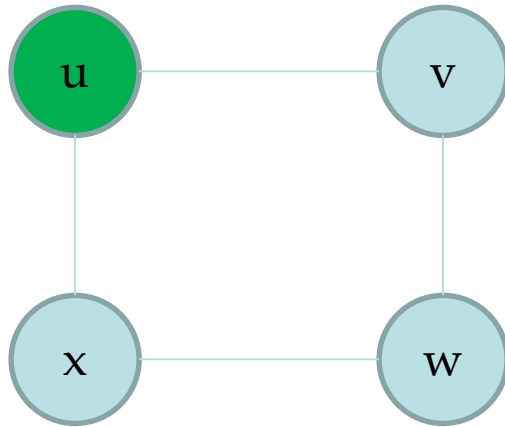


Node that achieves minimum is next hop in shortest path.

Distance Vector Algorithm

- Estimates:
 - $D_x(y)$ = estimate of least cost from x to y
 - Distance vector: $\mathbf{D}_x = [D_x(y): y \in N]$
- Each node x :
 - Node x knows cost to each neighbor v : $c(x,v)$
 - Node x maintains $\mathbf{D}_x = [D_x(y): y \in N]$
 - Node x also maintains its neighbors' distance vectors
 - For each neighbor v , x maintains $\mathbf{D}_v = [D_v(y): y \in N]$

An illustration



Distance vectors at node u

D_u

D_v

D_x

Distance vector algorithm

Basic idea:

- Each node periodically sends its own distance vector estimate to neighbours
- When a node x receives new DV estimate from neighbor, it updates its own DV using B-F equation:

$$D_x(y) \leftarrow \min_v \{c(x,v) + D_v(y)\} \quad \text{for each node } y \in N$$

- Amazingly, as long as all the nodes continue to exchange their distance vectors in an asynchronous fashion, the estimate $D_x(y)$ converges the actual least cost $d_x(y)$

Distance Vector Algorithm

Iterative, asynchronous:

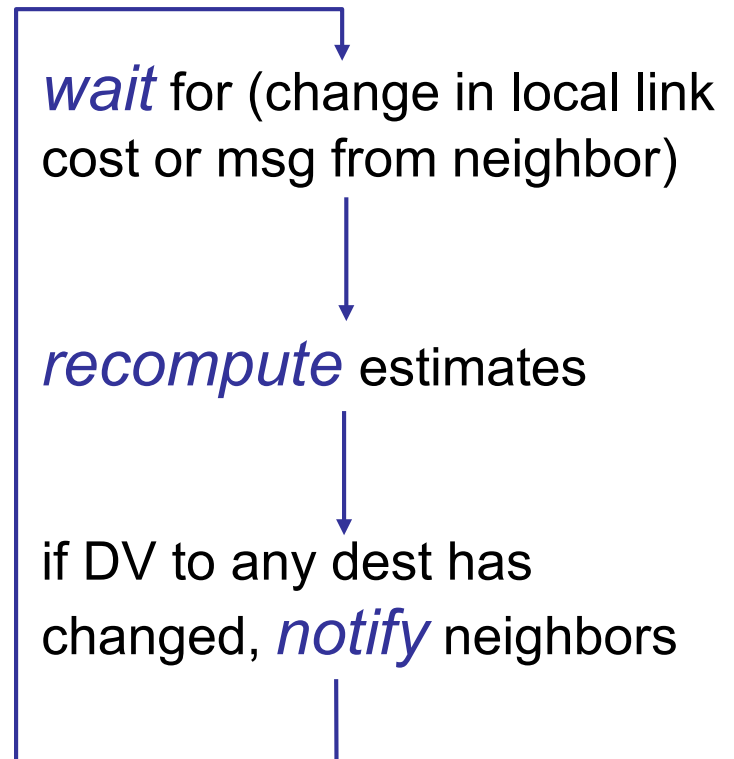
each local iteration caused by:

- local link cost change
- DV update message from neighbour

Distributed:

- each node notifies neighbours *only* when its DV changes
 - neighbours then notify their neighbours if necessary
 - The algorithm doesn't know the entire path – only knows the next hop

Each node:



Distance Vector Algorithm

At each node, x:

```
1  Initialization:
2      for all destinations y in N:
3           $D_x(y) = c(x,y)$  /*  $c(x,y) = \infty$  if y is not a neighbour */
4      for each neighbour w
5           $D_w(y) = \infty$  for all destinations y in N
6      for each neighbor w
7          send distance vector  $D_x = [D_x(y): y \text{ in } N]$  to w
8  loop
9      wait (until I see a link cost change to some neighbour w
10     or until I receive a distance vector from some neighbour w)
11     for each y in N:
12          $D_x(y) = \min_v \{c(x,v) + D_v(y)\}$  /* v is adjacent to x */
13     if  $D_x(y)$  changed for any destination y
14         send distance vector  $D_x = [D_x(y): y \text{ in } N]$  to all neighbours
15 forever
```

A hidden assumption – N (all destinations)

- *Q: if all nodes exchange distance vectors with their neighbours only, can each of them know all the destination nodes (N, in the pseudocode)?*



- Initially, x knows it has a path to y with cost 1; *but it does not know the existence of z*;
- y knows it has a path to x (and z) with cost 1;
- z knows it has a path to y with cost 1; *but it does not know the existence of x*;
- Then, y and x exchange distance vectors;
- x learned that there is **a new destination z** and it can reach z via y with cost 2;
- y and z exchange distance vectors;
- z learned that there is **a new destination x** and it can reach x via y with cost 2;
- Now both x and z know their destination nodes!

RIP allows at most 25 destination nodes.

$$D_x(y) = \min\{c(x,y) + D_y(y), c(x,z) + D_z(y)\}$$

$$= \min\{2+0, 7+1\} = 2$$

$$D_x(z) = \min\{c(x,y) + D_y(z), c(x,z) + D_z(z)\}$$

$$= \min\{2+1, 7+0\} = 3$$

node x table

from	x	y	z
x	0	2	7
y	∞	∞	∞
z	∞	∞	∞

from	x	y	z
x	0	2	3
y	2	0	1
z	7	1	0

from	x	y	z
x	0	2	3
y	2	0	1
z	3	1	0

node y table

from	x	y	z
x	∞	∞	∞
y	2	0	1
z	∞	∞	∞

from	x	y	z
x	0	2	7
y	2	0	1
z	7	1	0

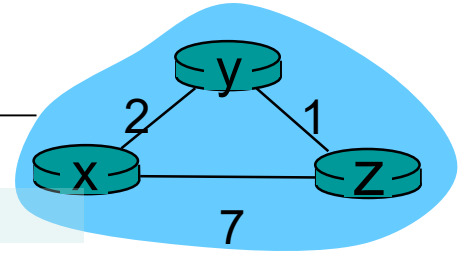
from	x	y	z
x	0	2	3
y	2	0	1
z	3	1	0

node z table

from	x	y	z
x	∞	∞	∞
y	∞	∞	∞
z	7	1	0

from	x	y	z
x	0	2	7
y	2	0	1
z	3	1	0

from	x	y	z
x	0	2	3
y	2	0	1
z	3	1	0



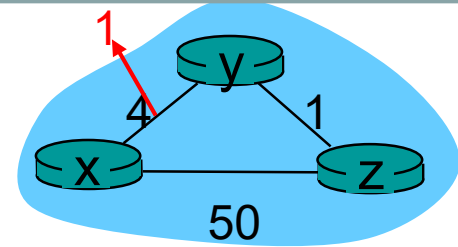
time

Distance Vector: link cost changes

Link cost changes:

- ❑ node detects local link cost change
- ❑ updates routing info, recalculates distance vector
- ❑ if DV changes, notify neighbors
- ❑ We consider y and z's DVs only here.

$$D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{1+0, 1+5\} = 1;$$



“good
news
travels
fast”

At time t_0 , y detects the link-cost change ($4 \rightarrow 1$), updates its DV, and informs its neighbors.

At time t_1 , z receives the update from y and updates its table. It computes a new least cost to x ($5 \rightarrow 2$) and sends its neighbors its DV.

At time t_2 , y receives z's update and updates its distance table. y's least costs do not change and hence y does *not* send any message to z.

Distance Vector: link cost changes

Link cost changes:

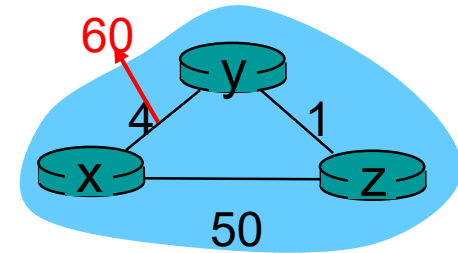
Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$ (only y, z's DV to dest. x)

At time t0, y detects the link-cost

change and re-compute its dv

- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+5\} = 6$;
- The assumption is that y stores its own DV and its neighbours' (and its link costs to its neighbours).



<i>y's DV table</i>	y	x	z
y	0	4 → 6 (via z)	1
x	4	0	5
z	1	5 (via y)	0

False

y's forwarding table

Dest.	Next-hop
x	z
z	z

z's forwarding table

Dest.	Next-hop
x	y
y	y

Distance Vector: link cost changes

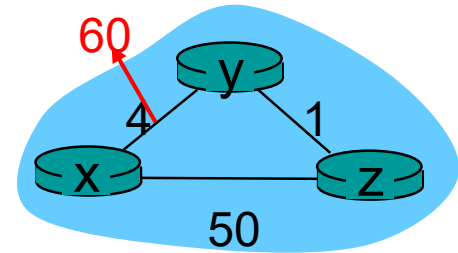
Link cost changes:

Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$ (again y, z's DV)

At time t0, y detects the link-cost change and re-compute its dv

- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+5\} = 6$;
- At time t1, y sends its new dv to z; after z receives y's new dv; z can update
 $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+6, 50+0\} = 7$;



<i>z's DV table</i>	y	x	z
y	0	6 (via z)	1
x	51	0	50
z	1	5 → 7 (via y)	0

y's forwarding table

Dest.	Next-hop
x	z
z	z

z's forwarding table

Dest.	Next-hop
x	y
y	y

Distance Vector: link cost changes

Link cost changes:

Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$ (again y, z's DV)

At time t0, y detects the link-cost change and re-compute its dv

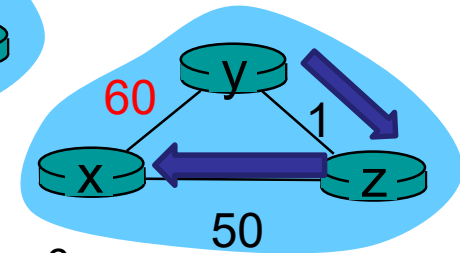
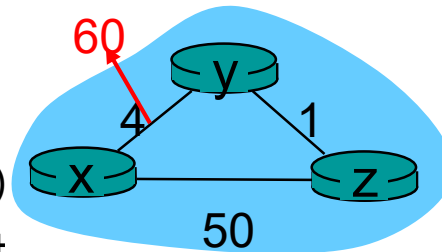
- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+5\} = 6$;
- At time t1, y sends its new dv to z; after z receives y's new dv; z can update
 $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+6, 50+0\} = 7$;
- At time t2, z sends its new dv to y; similarly y can update
 $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+7\} = 8$;
- Then $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+8, 50+0\} = 9$;
- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+9\} = 10$;
- ...
- $D_y(x) = \dots = 50$;
- $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+50, 50+0\} = 50$;
- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+50\} = 51$;
- $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+51, 50+0\} = 50$;

Next-hop changes!

Converged!

The bad news about the increase in link cost has travelled slowly!

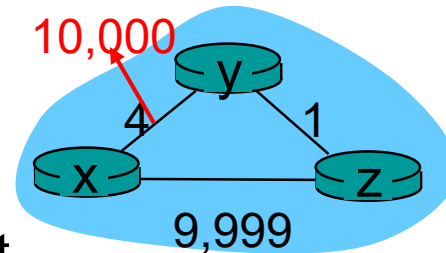
Count-to-infinity problem!



Distance Vector: link cost changes

Link cost changes:

- Before the link cost changes
 - $D_y(x) = 4$, $D_z(x) = 5$
- At time t_0 , y detects the link-cost



Change and re-compute its dv

- $D_y(x)$: $4 \rightarrow 6 \rightarrow 8 \rightarrow 10, \dots, \rightarrow 9998$;
- $D_z(x)$: $5 \rightarrow 7 \rightarrow 9 \rightarrow 11, \dots, \rightarrow 9999$; (causing a routing loop!)
- $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1 + 9998, 9999 + 0\} = 9999$;
- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{10000 + 0, 1 + 9999\} = 10000$;
- $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1 + 10000, 9999 + 0\} = 9999$;

Neighbours exchange distance vectors only!
Distance vectors provide limited information!
z tells y: "I have a path to x with a cost of 7."
It does NOT tell y that this path goes through y!

Distance Vector: link cost changes

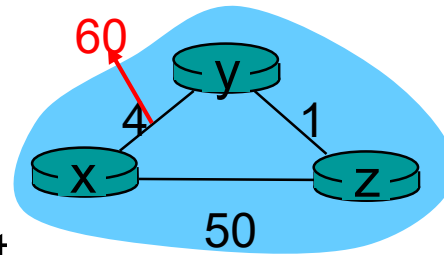
Link cost changes:

Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$ (only y, z's DV)

At time t_0 , y detects the link-cost change and re-compute its dv

- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+5\} = 6$;
- The assumption is that y stores its own DV and its neighbours' (and its link costs to its neighbours).



<i>y's DV table</i>	y	x	z
y	0	4 → 6 (via z)	1
x	4	0	5
z	1	5 (via y)	0

False

y's forwarding table

Dest.	Next-hop
x	z
z	z

z's forwarding table

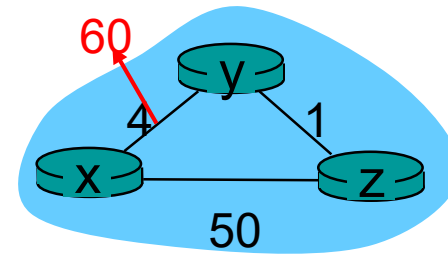
Dest.	Next-hop
x	y
y	y

Distance Vector: link cost changes

Poisoned reverse:

❑ If z routes through y to get to x :

- z tells y its (z's) distance to x is infinite (so y won't route to x via z)



❑ Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$, but z will lie to y saying “ **$D_z(x) = \infty$** ” (*poisoned reverse*)

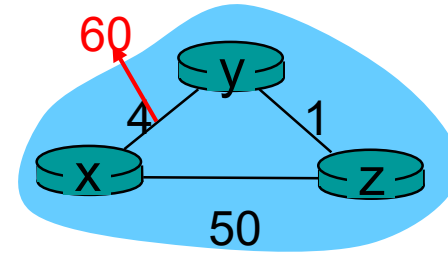
<i>y's DV table</i>	y	x	z
y	0	4	1
x	4	0	∞
z	1	∞	0

Distance Vector: link cost changes

Poisoned reverse:

□ If z routes through y to get to x :

- z tells y its (z's) distance to x is infinite (so y won't route to x via z)



□ Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$, but z will lie to y saying “ $D_z(x) = \infty$ ” (*poisoned reverse*)

□ At time t_0 , y detects the link-cost change and re-compute its dv

- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1 + \infty\} = 60$;

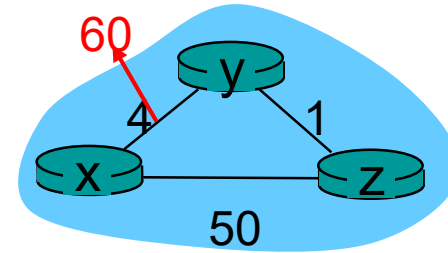
<i>y's DV table</i>	y	x	z
y	0	4 → 60 (via x)	1
x	4	0	∞
z	1	∞	0

Distance Vector: link cost changes

Poisoned reverse:

□ If z routes through y to get to x :

- z tells y its (z's) distance to x is infinite (so y won't route to x via z)



□ Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$, but z will lie to y saying “ $D_z(x) = \infty$ ” (*poisoned reverse*)

□ At time t_0 , y detects the link-cost change and re-compute its dv

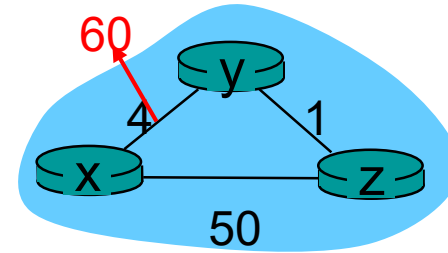
- $D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1 + \infty\} = 60$;
- Now y sends data directly to x;
- At time t_1 , y sends its new dv to z; after z receives y's new dv; z can update $D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+60, 50+0\} = 50$;

<i>z's DV table</i>	y	x	z
y	0	60	∞
x	∞	0	∞
z	1	50 (via x)	0

Distance Vector: link cost changes

Poisoned reverse:

- If z routes through y to get to x :
 - z tells y its (z's) distance to x is infinite (so y won't route to x via z)



- Before the link cost changes

○ *y's DV table*

	y	x	z	(se)
y	0	51 (via z)	1	
x	51	0	50	
z	1	50 (via x)	0	

- At chang

- At time t1, y sends its new dv to z; after z receives y's new dv; z can update

$$D_z(x) = \min\{c(z,y) + D_y(x), c(z,x) + D_x(x)\} = \min\{1+60, 50+0\} = 50;$$
- At time t2, z sends its new dv to y without lying since it will not route through y; similarly y can update

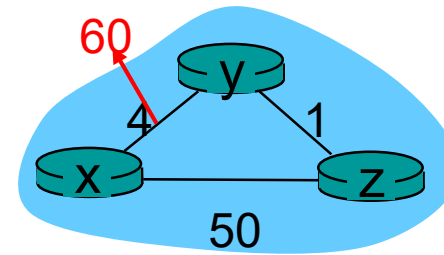
$$D_y(x) = \min\{c(y,x) + D_x(x), c(y,z) + D_z(x)\} = \min\{60+0, 1+50\} = 51;$$

Distance Vector: link cost changes

Poisoned reverse:

□ If z routes through y to get to x :

- z tells y its (z's) distance to x is infinite (so y won't route to x via z)



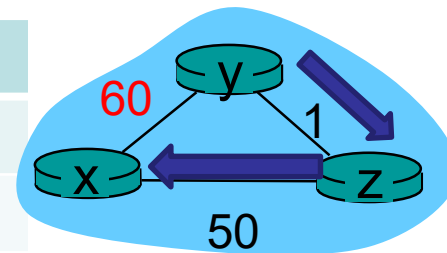
□ Before the link cost changes

- $D_y(x) = 4$, $D_z(x) = 5$, but z will lie to y saying “ **$D_z(x) = \infty$** ” (*poisoned reverse*)

□ At time t1, z sends its dv to y without lying since it will not route through y

-
-
-

z's DV table	y	x	z
y	0	∞	∞
x	∞	0	∞
z	1	50 (via x)	0



update

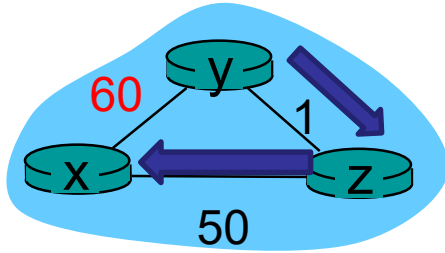
$$D_z(x) = \min\{c(z,y) + D_y(x), \text{c(z,x)+ D}_x(x)\} = \min \{1+60, 50+0\} = 50;$$

- At time t2, z sends its new dv to y without lying since it will not route through y; similarly y can update

$$D_y(x) = \min\{c(y,x) + D_x(x), \text{c(y,z)+ D}_z(x)\} = \min \{60+0, 1+50\} = 51;$$

- Then $D_z(x) = \min\{c(z,y) + D_y(x), \text{c(z,x)+ D}_x(x)\} = \min \{1+ \infty, 50+0\} = 50$; y lies to z this time because it routes through z;

A little lie helps!



y uses z as its next-hop and will lie to z by saying that “I have a path to x with a cost of *infinity*.”

(Do not count on me to route your traffic to x, --poisoned reverse.)

But a little lie helps only a little; it does not *solve* the problem!

Consider y,z,w distance table entries to

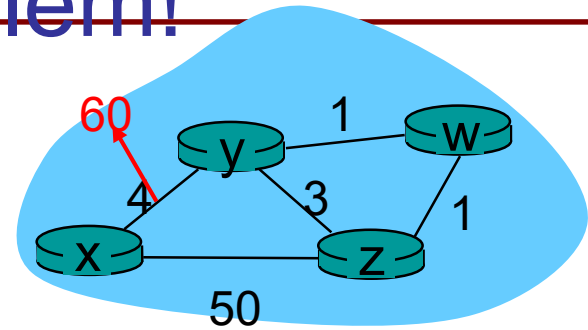
x only. *Using poisoned reverse,*

$z \rightarrow w$, $D_z(x) = \infty$; $z \rightarrow y$ $D_z(x) = 6$ (not lying);

$w \rightarrow y$, $D_w(x) = \infty$; $w \rightarrow z$ $D_w(x) = 5$ (not lying);

$y \rightarrow w$, $D_y(x) = 4$ (not lying); $y \rightarrow z$ $D_y(x) = 4$ (not lying);

Then there is link-cost change ($4 \rightarrow 60$);



But a little lie helps only a little; it does not *solve* the problem!

Co y re-computes:

$D_y(x) = \min\{c(y,z) + D_z(x), c(y,w) + D_w(x)\} =$
 $\min\{3+11, 1+\infty\} = 14$ (via z);
 y notifies z and w; $\rightarrow_z D_y(x) = \infty$ (*lying*);
 $\rightarrow_w D_y(x) = 14$ (telling truth);

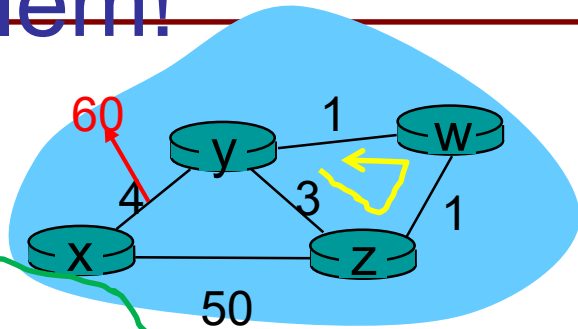
Th

At t1, y updates its $D_y(x) = 9$ (via z, z did not lie:);

Now y notifies w and z; $y \rightarrow w, D_y(x) = 9$; $y \rightarrow z D_y(x) = \infty$;

$D_y(x) = \min\{c(y,z) + D_z(x), c(y,w) + D_w(x), c(y,x) + D_x(x)\} = \min\{3+6, \infty, 60+0\} = 9$ (via z)

not lying);



w re-computes:

$D_w(x) = \min\{c(w,y) + D_y(x), c(w,z) + D_z(x)\}$
 $= \min\{1+9, 1+\infty\} = 10$ (via y);
 w notifies y and z; $\rightarrow_y D_w(x) = \infty$ (*lying*);
 $\rightarrow_z D_w(x) = 10$ (telling truth);

z re-computes:

$D_z(x) = \min\{c(z,w) + D_w(x), c(z,y) + D_y(x)\} =$
 $\min\{1+10, 3+\infty\} = 11$ (via w);
 z notifies w and y; $\rightarrow_w D_z(x) = \infty$ (*lying*);
 $\rightarrow_y D_z(x) = 11$ (telling truth);

But a little lie helps only a little; it does not *solve* the problem!

Consider y,z,w distance table entries to

x only. Using poisoned reverse,

$z \rightarrow w$, $D_z(x) = \infty$; $z \rightarrow y$ $D_z(x) = 6$ (not lying);

$w \rightarrow y$, $D_w(x) = \infty$; $w \rightarrow z$ $D_w(x) = 5$ (not lying);

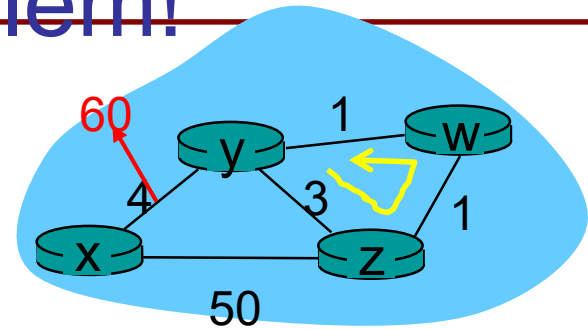
$y \rightarrow w$, $D_y(x) = 4$ (not lying); $y \rightarrow z$ $D_y(x) = 4$ (not lying);

Then there is link-cost change (4 \rightarrow 60);

At t1, y updates its $D_y(x) = 9$ (via z, z did not lie:);

Now y notifies w and z; $y \rightarrow w$, $D_y(x) = 9$; $y \rightarrow z$ $D_y(x) = \infty$;

$$D_y(x) = \min\{c(y,z)+D_z(x), c(y,w) + D_w(x), c(y,x) + D_x(x)\} = \min\{3+6, 1+ \infty, 60+0\} = 9 \text{ (via z)}$$



	t0	t1	t2	t3	t4
z	$\rightarrow w$, $D_z(x) = \infty$; $\rightarrow y$, $D_z(x) = 6$;		No change	$\rightarrow w$, $D_z(x) = \infty$; $\rightarrow y$, $D_z(x) = 11$;	
w	$\rightarrow y$, $D_w(x) = \infty$; $\rightarrow z$ $D_w(x) = 5$;		$\rightarrow y$, $D_w(x) = \infty$; $\rightarrow z$, $D_w(x) = 10$;		No change
y	$\rightarrow w$, $D_y(x) = 4$; $\rightarrow z$, $D_y(x) = 4$;	$\rightarrow w$, $D_y(x) = 9$; $\rightarrow z$, $D_y(x) = \infty$;		No change	$\rightarrow w$, $D_y(x) = 14$; $\rightarrow z$ $D_y(x) = \infty$;

This continues y-w-z-y-w-z-y-w-z; there is a routing loop (y-z, z-w, w-y). [ZL]

Distance Vector Algorithm

Iterative, asynchronous:

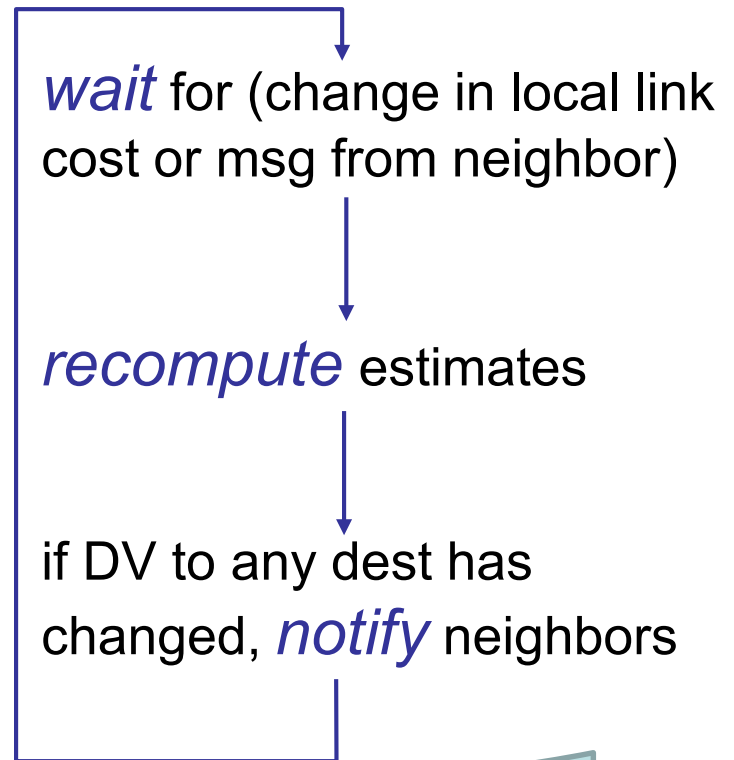
each local iteration caused by:

- local link cost change
- DV update message from neighbour

Distributed:

- each node notifies neighbours *only* when its DV changes
 - neighbours then notify their neighbours if necessary
 - The algorithm doesn't know the entire path – only knows the next hop

Each node:



DV has the count-to-infinity problem and poisoned reverse does not solve it. *In RIP the maximum cost of a path is limited to 15.*

Comparison of LS and DV algorithms

Message complexity

- LS: with n nodes, E links, $O(nE)$ msgs sent
- DV: exchange between neighbors only
 - convergence time varies

Speed of Convergence

- LS: $O(n^2)$ algorithm requires $O(nE)$ msgs
 - may have oscillations
- DV: convergence time varies
 - may be routing loops
 - count-to-infinity problem

Robustness: what happens if router malfunctions?

LS:

- node can advertise incorrect *link* cost
- each node computes only its *own* table

DV:

- DV node can advertise incorrect *path* cost
- each node's table used by others
 - error propagates thru network

Neither is an obvious winner over the other; both are used in deed!

Summary

- Network layer overview
- Routing overview
- Link-state routing (Dijkstra's algorithm)
- **Distance-vector routing (Bellman-Ford)**
 - B-F algorithm
 - Count-to-infinity problem and poisoned reverse
 - LS vs DV
- **Summary**

References

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https://users.cs.northwestern.edu/~akuzma/classes/CS340-w05/lecture_notes.htm