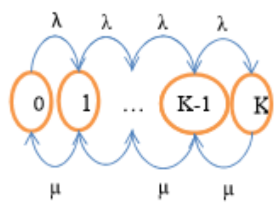
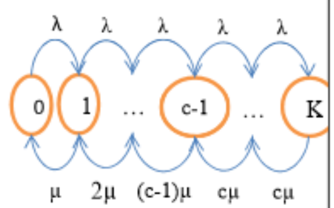
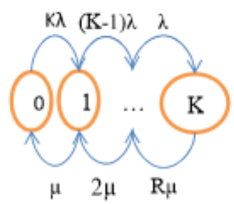
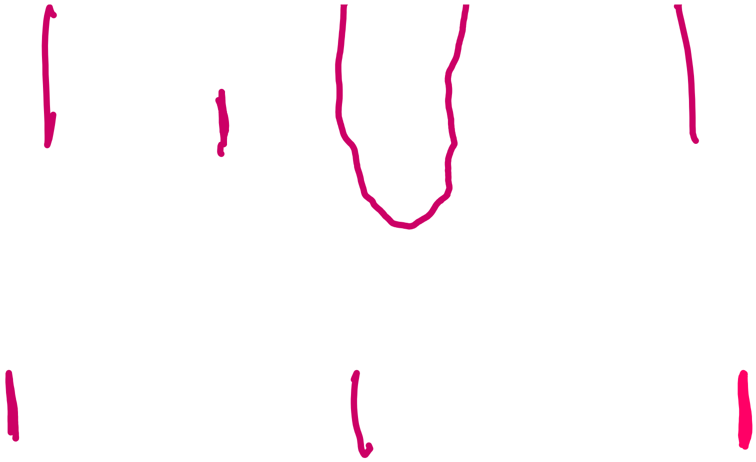
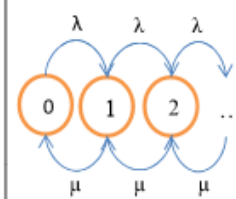
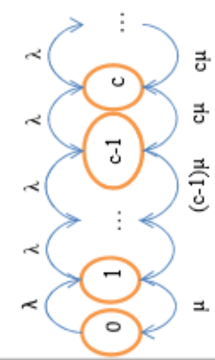
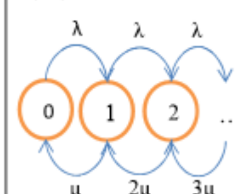
Queues Cheat Sheet

Thursday, 16 April 2020 12:13 PM

queue_c...

SSD	Performance Measures	Graph	SSD	Performance Measures
$\pi_0 = 1-\rho$ $\pi_j = \rho^j(1-\rho)$ $\rho = \frac{\lambda}{\mu}$ $\rho < 1\ SSD\exists$	$L = \frac{\rho}{1-\rho}$ $L_q = \frac{\rho^2}{1-\rho}$ $L_s = \rho$ $W_s = \frac{1}{\mu}$ $W = \frac{1}{\mu-\lambda} = \frac{L}{\lambda}$ $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$	M/M/1/K  $SSD\exists\ always$ $\lambda_{eff} = \lambda[1-\pi_K], \rho = \frac{\lambda}{\mu}$	$\lambda \neq \mu$ $\pi_0 = \frac{1-\rho}{1-\rho^{k+1}}$ $\pi_j = \rho^j \pi_0,$ $j = 1,2,\dots k$	$L = \frac{\rho[1-(k+1)\rho^k + k\rho^{k+1}]}{(1-\rho)(1-\rho^{k+1})}$ $L_s = \frac{\lambda_{eff}}{\mu} = 1-\pi_0$ $W = \frac{L}{\lambda_{eff}}$ $W_s = \frac{1}{\mu}$
			$\lambda = \mu$ $\pi_j = \frac{1}{K+1}$ $j = 0,1,\dots,K$	$L = \frac{K}{2}, W = \frac{L}{\lambda_{eff}}$ $L_s = 1-\pi_0, W_s = \frac{1}{\mu}$
$\pi_0 = \left[\sum_{j=0} \frac{\rho^j}{j!} + \frac{\rho^c}{c!}\frac{1}{\left(1-\frac{\rho}{c}\right)}\right]^{-1}$ $= \frac{\rho^j}{j!}\pi_0, j \le c$ $= \frac{\rho^{c+j}}{c!\,c^j}\pi_0, j \ge 0$ $\frac{\rho}{c} = \frac{\frac{\lambda}{\mu}}{c}$ $\frac{\rho}{c} < 1\ SSD\exists$	$L_q = \left(\frac{\rho^c}{c!}\pi_0\right)\frac{\frac{\rho}{c}}{\left(1-\frac{\rho}{c}\right)^2}$ $L_s = \rho$ $L = L_q + L_s$ $W_q = \frac{L_q}{\lambda}$ $W = \frac{L}{\lambda}$	M/M/c/K  $\rho = \frac{\lambda}{\mu}, \lambda_{eff} = \lambda(1-\pi_k)$ $SSD\exists, always$	$\frac{\rho}{c} \neq 1$ $\pi_0 = \left[\sum_{j=0}^{c-1}\frac{\rho^j}{j!} + \frac{\rho^c}{c!}\frac{1-\left(\frac{\rho}{c}\right)^{K-c+1}}{1-\frac{\rho}{c}}\right]^{-1}$ $\pi_j = \begin{cases} \frac{\rho^j}{j!}\pi_0, j \le c \\ \frac{\rho^j \pi_0}{c!\,c^{j-c}}, c < j \le K \end{cases}$	$\frac{\rho}{c} \neq 1$ $L_q = \pi_0 \frac{\rho^{c+1}}{c!\,c\left(1-\frac{\rho}{c}\right)^2}\left[1-\left(\frac{\rho}{c}\right)^{K-c+1} - (K-c+1)\left(\frac{\rho}{c}\right)^{K-c}\left(1-\frac{\rho}{c}\right)\right]$
			$\frac{\rho}{c} = 1$ $\pi_0 = \left[\sum_{n=0}^{c-1}\frac{\rho^n}{n!} + \frac{\rho^c}{c!}(K-c+1)\right]^{-1}$	$\frac{\rho}{c} = 1$ $L_q = \frac{\pi_0 \rho^c}{2c!}(K-c)(K-c+1)$
$\rho = \frac{\lambda}{\mu}$ $Poisson(\rho)$ $\pi_0 = e^{-\rho}$ $\frac{\rho^j}{j!}e^{-\rho}, j = 1,2\dots$ ote: $M/G/\infty$ so $Poisson(\rho)$	$L = L_s = \rho$ $L_q = 0$ $W = W_s = \frac{1}{\mu}$ $W_q = 0$	M/M/R/K/K  $\rho = \frac{\lambda}{\mu}$ $SSD\exists, always$	$\pi_0 = \left[\sum_{j=0}^{R}\binom{K}{j}\rho^j + \sum_{j=R+1}^{K}\binom{K}{j}\frac{j!\,\rho^j}{R!\,R^{j-R}}\right]^{-1}$ $\pi_j = \begin{cases} \binom{K}{j}\rho^j\pi_0, j \le R \\ \binom{K}{j}\frac{\rho^j j!}{R!\,R^{j-R}}\pi_0, R < j \le K \end{cases}$	$L_q = \sum_{j=R+1}^{K}(j-R)\pi_j$ $\lambda_{eff} = \sum_{j=0}^{K}\lambda_j\pi_j = \lambda(K-L)$ $L_s = \frac{\lambda_{eff}}{\mu}$



Graph	SSD	P
M/M/1 	$\pi_0 = 1-\rho$ $\pi_j = \rho^j(1-\rho)$ $\rho = \frac{\lambda}{\mu}$ $\rho < 1\ SSD\exists$	W W
M/M/c 	$\pi_0 = \left[\sum_{j=0}^{c-1}\frac{\rho^j}{j!} + \frac{\rho^c}{c!}\frac{1}{\left(1-\frac{\rho}{c}\right)}\right]^{-1}$ $\pi_j = \frac{\rho^j}{j!}\pi_0\ , j \le c$ $\pi_{c+j} = \frac{\rho^{c+j}}{c!\,c^j}\pi_0\ , j \ge 0$ $\frac{\rho}{c} = \frac{\frac{\lambda}{\mu}}{c}$ $\frac{\rho}{c} < 1\ SSD\exists$	L_q $= ($
M/M/∞ 	$\rho = \frac{\lambda}{\mu}$ $Poisson(\rho)$ $\pi_0 = e^{-\rho}$ $\pi_j = \frac{\rho^j}{j!}e^{-\rho}\ , j = 1,2\ldots$ $Note: M/G/\infty$ $also\ Poisson(\rho)$	