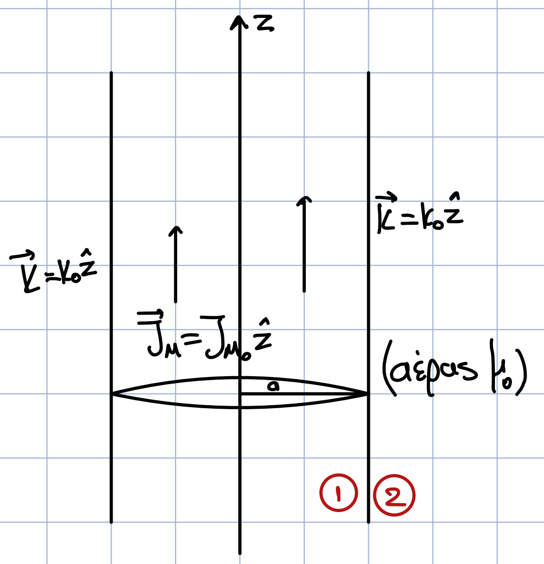
Άσκηση 12.2



$$\frac{\partial}{\partial\varphi} = 0 = \frac{\partial}{\partial z}$$

α) $\vec M = ;$ $\quad \vec K_M(r=a) = ;$

ξέρουμε ότι $\exists$ μόνο $B_\varphi, H_\varphi, M_\varphi$

$$\vec M = M_\varphi(r)\,\hat\varphi$$

$$\vec J_M = \nabla\times\vec M$$

$$I_M = \int_S \vec J_M\, d\vec S = \int_S (\nabla\times\vec M)\, d\vec S \Rightarrow \text{θεωρ. Stokes} = \oint_C \vec M\, d\vec l \Rightarrow M_\varphi(r)\cdot 2\pi r = J_{M_0}\pi r^2$$

$$\Rightarrow M_\varphi(r) = J_{M_0}\frac{r}{2}$$

$$K_M(r=a) = -\hat r\times\vec M(r=a) = -\hat z\cdot J_{M_0}\frac{a}{2}, \quad \text{πρέπει} \quad J_{M_0}\pi a^2 = J_{M_0}\frac{a}{\cancel{2}}\cdot\cancel{2}\pi a = J_{M_0}\pi a^2$$

β) $\vec H, \vec B, \vec A = ;$ $\qquad \oint_C \vec B\, d\vec l = \mu_0\int(\vec J_M + \cancel{\vec J}^{\,0})\, d\vec S$

Περιοχή ①:

$$B_{\varphi_1}\cdot 2\cancel{\pi}\cancel{r} = \mu_0 J_{M_0}\cancel{\pi} r^{\cancel{2}}$$

$$B_{\varphi_1} = \mu_0 J_{M_0}\frac{r}{2} \quad (1), \quad H_{\varphi_1} = \frac{B_{\varphi_1}}{\mu_0} - M_\varphi = J_{M_0}\frac{r}{2} - J_{M_0}\frac{r}{2} = 0$$

Περιοχή ②:

$$B_{\varphi_2}\cdot 2\pi r = \cancel{\mu_0 J_{M_0}\pi a^2} - \cancel{\mu_0 K_M(r=a)2\pi a} + \mu_0 K_0 2\pi a$$

$$B_{\varphi_2} = \mu_0 K_0\frac{a}{r} \quad (2), \quad H_{\varphi_2} = \frac{B_{\varphi_2}}{\mu_0} = K_0\frac{a}{r}$$

$\vec{A} = A_z(r)\hat{z}$, $\nabla \times \vec{A} = \vec{B} \Rightarrow -\frac{dA_z}{dr} = B_\varphi \Rightarrow A_z = -\int B_\varphi dr + C$

περ ①: (1) $\Rightarrow A_{z_1} = -\int \mu_0 J_{M_0} \frac{r}{2} dr + C_1 = -\mu_0 J_{M_0} \frac{r^2}{4} + C_1$

Για $r=0$: $A_{z_1} = 0$ (αφου δεν εχουμε ρευμα γραμμικό στον αξονα)

άρα $C_1 = 0$

$\Rightarrow A_{z_1} = -\mu_0 J_{M_0} \frac{r^2}{4}$ (3)

$A_{z_2} = -\mu_0 \frac{k_0}{2} \ln r + C_2$ (4)

Για $r=a$: $A_{z_2}(r=a) = A_{z_1}(r=a)$

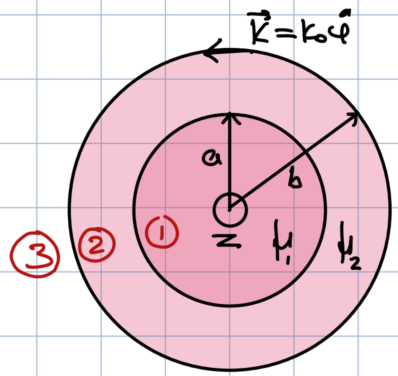
$\Rightarrow$ (3),(4) $\Rightarrow -\mu_0 \frac{k_0}{2} \ln a + C_2 = -\mu_0 J_{M_0} \frac{a^2}{4} \Rightarrow C_2 = -\mu_0 J_{M_0} \frac{a^2}{4}$

$A_{z_2} = \mu_0 k_0 a \ln\left(\frac{a}{r}\right) - \mu_0 J_{M_0} \frac{a^2}{4}$

γ) Μόνιμος μαγνήτης

Άσκηση 12.3

$$\frac{d}{d\varphi}=0=\frac{d}{dz} \qquad \alpha)\ \vec{B}, \vec{H}, \vec{A} = ;$$



$$\nabla\times\vec{H}=\vec{J} \Rightarrow \begin{cases} 0=0 \\ -\frac{dH_z}{dr}\hat{\varphi}=0 \Rightarrow H_z=\sigma\tau\alpha\vartheta \\ \frac{1}{r}\frac{d}{dr}(rH_\varphi)\hat{z}=0 \Rightarrow H_\varphi=\frac{C}{r} \end{cases}$$

$$\Rightarrow H_{\varphi_1}=\frac{C_1}{r}, \quad H_{\varphi_2}=\frac{C_2}{r}, \quad H_{\varphi_3}=\frac{C_3}{r}$$

$$\left.\begin{array}{l} \text{Για } r\to 0:\ H_{\varphi_1}\to 0 \Rightarrow C_1=0 \Rightarrow H_{\varphi_1}=0 \\ \text{Για } r=a:\ H_{\varphi_1}(r=a)=H_{\varphi_2}(r=a)=0 \Rightarrow C_2=0 \Rightarrow H_{\varphi_2}=0 \\ \text{Για } r=b:\ H_{\varphi_2}(r=b)=H_{\varphi_3}(r=b)=0 \Rightarrow C_3=0 \Rightarrow H_{\varphi_3}=0 \end{array}\right\} \begin{array}{l} H_\varphi=0 \\ \text{παντού} \end{array}$$

Έχουμε επίσης $H_{z_1}=C_1$, $H_{z_2}=C_2$, $H_{z_3}=C_3$

για $r\to\infty$: $H_{z_3}\to 0$, άρα $C_3=0$

για $r=b$: $\hat{r}\times(\vec{H}_3-\vec{H}_2)=K_0\hat{z} \Rightarrow H_{z_2}-\cancel{H_{z_3}}^{0}=K_0 \Rightarrow H_{z_2}=K_0$

για $r=a$: $H_{z_1}=H_{z_2} \Rightarrow H_{z_1}=K_0$

$$\vec{B}_1=\mu_1 K_0\hat{z}, \quad \vec{B}_2=\mu_2 K_0\hat{z}, \quad \vec{B}_3=0 \quad (1)$$

$$\vec{A}=A_\varphi(r)\cdot\hat{\varphi}, \quad \nabla\times\vec{A}=\vec{B} \Rightarrow \frac{1}{r}\frac{d}{dr}(rA_\varphi)=B_z \Rightarrow A_\varphi=\frac{1}{r}\int rB_z\,dr+\frac{C}{r}$$

Περ①: $A_{\varphi_1}=\mu_1 K_0\frac{r}{2}+\frac{C_1}{r}$ (2) Για $r\to 0 \Rightarrow C_1=0 \Rightarrow A_{\varphi_1}=\mu_1 K_0\frac{r}{2}$ (5)

Περ②: $A_{\varphi_2}=\mu_2 K_0\frac{r}{2}+\frac{C_2}{r}$ (3) Για $r=a$: $A_{\varphi_1}=A_{\varphi_2} \Rightarrow$ (3), (5) $\Rightarrow \mu_1 K_0\frac{a}{2}=\mu_2 K_0\frac{a}{2}+\frac{C_2}{a}$

$$\Rightarrow A_{\varphi_2}=\mu_2 K_0\frac{r}{2}+(\mu_1-\mu_2)K_0\frac{a^2}{2r} \quad (6)$$

Περ③: $A_{\varphi_3}=\frac{C_3}{r}$ (4) Για $r=b$: $A_{\varphi_2}=A_{\varphi_3} \Rightarrow$ (4), (6) $\Rightarrow \mu_2 K_0\frac{b}{2}+(\mu_1-\mu_2)K_0\frac{a^2}{2b}=\frac{C_3}{b}$

$$\Rightarrow A_{\varphi_3}=\mu_1 K_0\frac{a^2}{2r}+\mu_2 K_0\frac{b^2-a^2}{2r} \quad (7)$$

β) $\vec{M}_1 = \dfrac{\vec{B}_1}{\mu_0} - \vec{H}_1 = \dfrac{\mu_1 - \mu_0}{\mu_0} K_0 \hat{z}$ (8)

$\vec{M}_2 = \dfrac{\mu_2 - \mu_0}{\mu_0} K_0 \hat{z}$ (9)

$\vec{M}_3 = 0$

$\rho_{M_1} = -\mu_0 \nabla \cdot \vec{M}_1 = 0 \quad , \quad \rho_{M_2} = -\mu_0 \nabla \vec{M}_2 = 0 \quad , \quad \rho_{M_3} = 0$

$\sigma_M (r=a) = \hat{r}\left(\mu_0 \vec{M}_1 - \mu_0 \vec{M}_2\right)_{r=a} = 0$

$\vec{J}_{M_1} = \nabla \times \vec{M}_1 = 0 \quad , \quad \vec{J}_{M_2} = \vec{J}_{M_3} = 0$

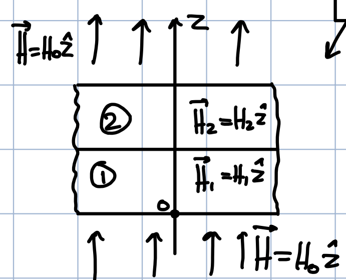
$\vec{K}_M (r=a) = \hat{r} \times \left(\mu_0 \vec{M}_2 - \mu_0 \vec{M}_1\right) \Rightarrow$ (8), (9) $\Rightarrow \dfrac{\mu_1 - \mu_2}{\mu_0} K_0 \hat{\varphi}$

γ) i) $\Psi_M = \int_S \vec{B} d\vec{S} = \mu_1 K_0 \pi a^2 + \mu_2 K_0 \pi (b^2 - a^2) = \mu_2 K_0 \pi b^2 + (\mu_1 - \mu_2) K_0 \pi a^2$

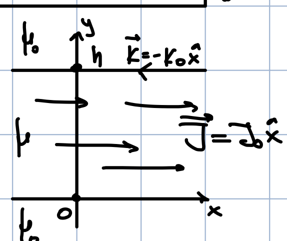
ίδια όπως το γράφουμε
διαφορετικό για ευκολία

ii) $\Psi_m = \oint_C \vec{A} d\vec{l} = A_{\varphi_2}(r=b) \cdot 2\pi b = \left[\mu_2 K_0 \dfrac{b}{2} + (\mu_1 - \mu_2) K_0 \dfrac{a^2}{2b}\right] \cdot 2\pi b = \mu_2 K_0 \pi b^2 + (\mu_1 - \mu_2) K_0 \pi a^2$

για παράδοση
12.4 & 12.7



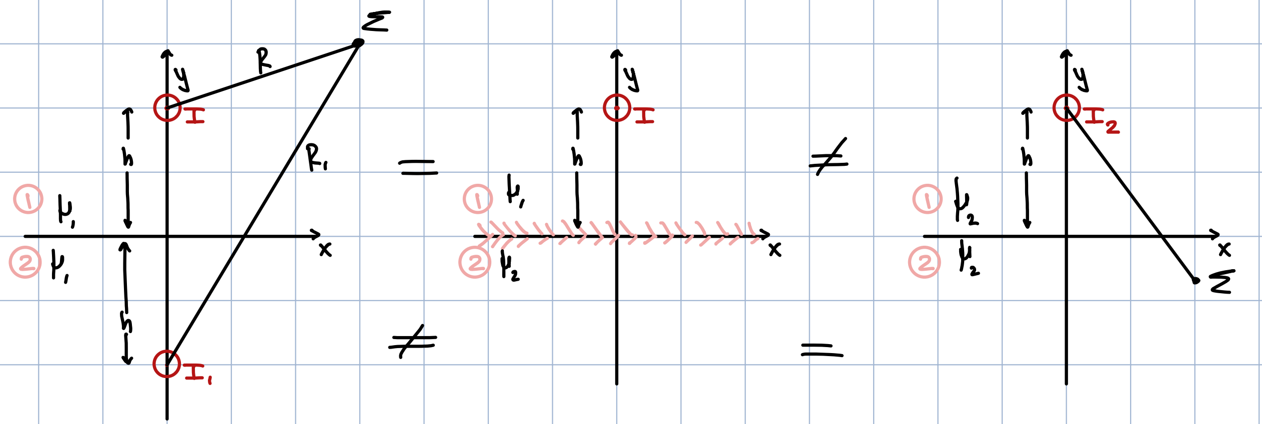
οριακές σχέσεις για $z=0$
$\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$ $z = d_1$
$z = d_1 + d_2$



Για ίδιο $\vec{H} = H_2 \hat{z}$

deadline 17/1/2023
ως 15:00

Μέθοδος των ειδώλων



$$A_{z_1} = -\frac{\mu_1 I}{2\pi}\ln R - \frac{\mu_1 I_1}{2\pi}\ln R_1 + C_1 \quad (1)$$

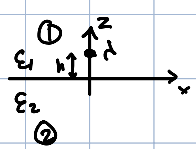
$$A_{z_2} = -\frac{\mu_2 I_2}{2\pi}\ln R_2 + C_2 \quad (2)$$

Ορ. συνθήκες: Για $y=0$: $A_{z_1} = A_{z_2}$

$y=0$: $\hat{y}\times(\vec{H}_2 - \vec{H}_1) = \vec{K} = 0$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} I_1 = \frac{\mu_2-\mu_1}{\mu_2+\mu_1} I, \quad I_2 = \frac{2\mu_1}{\mu_2+\mu_1} I$$

Από ηλεκτρικά υλικά:

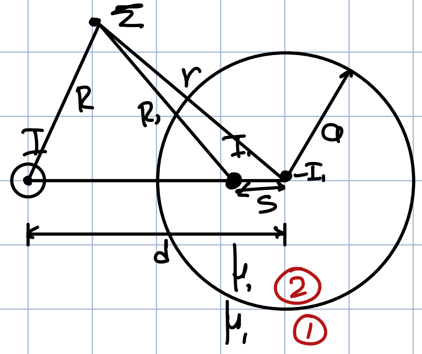


$$\lambda_1 = \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\lambda$$

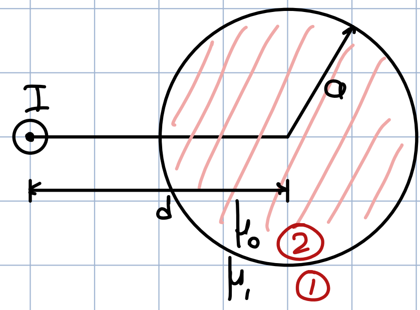
$$\lambda_2 = \frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}\lambda$$

$$\boxed{\begin{matrix} \lambda \leftrightarrow I \\ \varepsilon \leftrightarrow 1/\mu \end{matrix}}$$

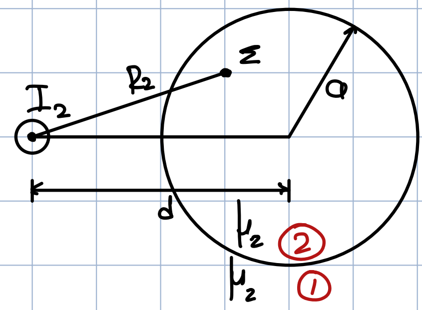
$$I_1 = \frac{\frac{1}{\mu_1}-\frac{1}{\mu_2}}{\frac{1}{\mu_1}+\frac{1}{\mu_2}} I = \frac{\mu_2-\mu_1}{\mu_2+\mu_1} I$$



Ισοδύναμο για περιοχή 1



Αρχικό πρόβλημα



Ισοδύναμο για περιοχή 2

$$A_{z_1} = -\frac{\mu_1 I}{2\pi}\ln R - \frac{\mu_1 I_1}{2\pi}\ln R_1 + \frac{\mu_1 I_1}{2\pi}\ln r + C_1$$

$$A_{z_2} = -\frac{\mu_2 I_2}{2\pi}\ln R_2 + C_2$$

Ορ. συνθ. για $r = a$ $\quad A_{z_1} = A_{z_2}$

$$\hat{r} \times (\vec{H}_1 - \vec{H}_2) = \vec{K} = 0$$