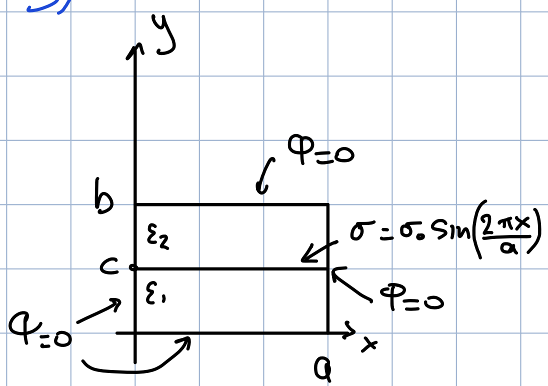
5)



για $y=c$: $\quad D_{y_2} - D_{y_1} = \sigma$

$$\Rightarrow -\varepsilon_2 \frac{\partial \Phi_2}{\partial y} + \varepsilon_1 \frac{\partial \Phi_1}{\partial y} = \sigma_0 \sin\left(\frac{2\pi x}{a}\right) \quad (1)$$

$$\Rightarrow \Phi_1(y=c^-) = \Phi_2(y=c^+) \quad (2)$$

Από (1) ξέρουμε ότι $\Phi_1, \Phi_2 \sim \sin\left(\frac{2\pi x}{a}\right)$

Από λύσεις Laplace:

$$X_1(x) = A_1 \sin\left(\frac{2\pi x}{a}\right) \Rightarrow K_x = -\left(\frac{2\pi}{a}\right)^2 = -k_x^2 < 0 \,, \quad K_y = -K_x = \left(\frac{2\pi}{a}\right)^2 = k_y^2 > 0$$

$$\Rightarrow Y_1(y) = B_1 \sinh\left(\frac{2\pi x}{a}\right)$$

$$\Phi_1 = \underbrace{A_1 B_1}_{C_1} \sin\left(\frac{2\pi x}{a}\right) \sinh\left(\frac{2\pi y}{a}\right) \quad (3)$$

Θέλουμε να μηδενίζεται στο $y=0$

και το Φ_2 ίδιας μορφής άρα

$$\Phi_2 = \underbrace{A_2 B_2}_{C_2} \sin\left(\frac{2\pi x}{a}\right) \sinh\left(\frac{2\pi (y-b)}{a}\right) \quad (4)$$

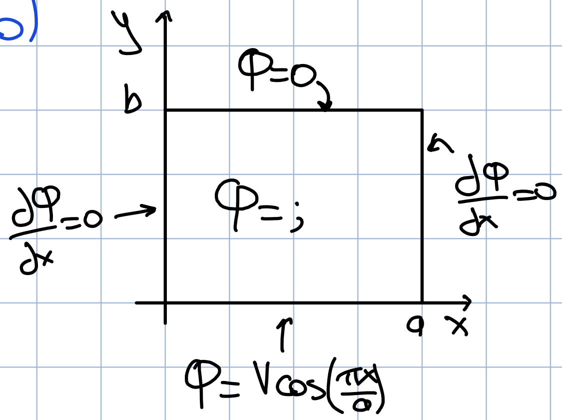
Θέλουμε να μηδενίζεται στο $y=b$

$$(2)-(4) \Rightarrow C_1 \cancel{\sin\left(\frac{2\pi x}{a}\right)} \sinh\left(\frac{2\pi c}{a}\right) = C_2 \cancel{\sin\left(\frac{2\pi x}{a}\right)} \sinh\left[\frac{2\pi}{a}(c-b)\right] \quad (5)$$

$$(1),(3),(4) \Rightarrow -\varepsilon_2 C_2 \cancel{\sin\left(\frac{2\pi x}{a}\right)} \frac{2\pi}{a} \cdot \cosh\left[\frac{2\pi(c-b)}{a}\right] + \varepsilon_1 C_1 \cancel{\sin\left(\frac{2\pi x}{a}\right)} \frac{2\pi}{a} \cosh\frac{2\pi c}{a} = \sigma_0 \cancel{\sin\left(\frac{2\pi x}{a}\right)} \quad (6)$$

βρίσκουμε τα C_1, C_2 άρα και τα Φ_1, Φ_2

6)



$$\Rightarrow X(x) = A\cos\left(\frac{\pi x}{a}\right) \Rightarrow K_x = -\left(\frac{\pi}{a}\right)^2 = -k_x^2 < 0$$

$$\Rightarrow K_y = -K_x = \left(\frac{\pi}{a}\right)^2 = k_y^2 > 0$$

$$Y(y) = B\sinh\left[\frac{\pi}{a}\cdot(y-b)\right]$$ αφού θέλουμε $\Phi=0$ για $y=b$

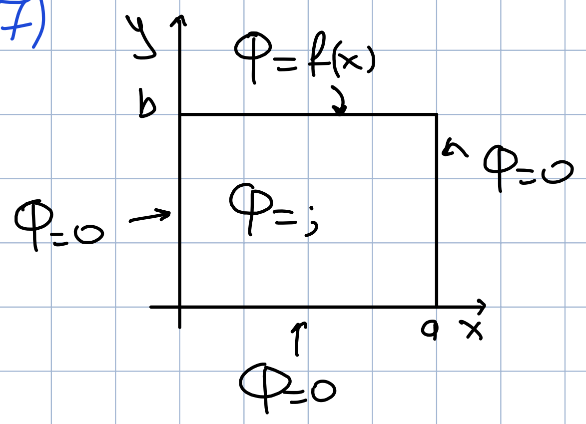
$$\Phi = C\cos\left(\frac{\pi x}{a}\right)\sinh\left[\frac{\pi}{a}(y-b)\right]$$

για $y=0$: $\Phi = C\cdot\cancel{\cos\left(\frac{\pi x}{a}\right)}\left[-\sinh\left(\frac{\pi b}{a}\right)\right] = V\cancel{\cos\left(\frac{\pi x}{a}\right)}$

$$C = -\frac{V}{\sinh\left(\frac{\pi b}{a}\right)}$$

$$\Phi = -\frac{V\cos\left(\frac{\pi x}{a}\right)\sinh\left[\frac{\pi}{a}(y-b)\right]}{\sinh\left[\frac{\pi b}{a}\right]}$$

7)



$$X(x) = A\sin(k_x \cdot x)$$

$$\sin(k_x \cdot a) = 0 \Rightarrow k_x a = m\pi, \quad m = 1, 2, \dots$$

$$k_x = \frac{m\pi}{a}$$

άρα $X(x) = A_m \sin\left(\frac{m\pi x}{a}\right)$

$$K_x = -\left(\frac{m\pi}{a}\right)^2 = -k_x^2 < 0 \Rightarrow k_y^2 = -K_x^2 = \left(\frac{m\pi}{a}\right)^2 = k_y^2 > 0$$

$$\varphi_m = \underbrace{A_m B_m}_{C_m} \sin\left(\frac{m\pi x}{a}\right)\sinh\left(\frac{m\pi y}{a}\right)$$ έτσι ικανοποιούνται οι 3 πλευρές για την 4η:

$$\varphi = \sum_{m=1}^{\infty} C_m \sin\left(\frac{m\pi x}{a}\right)\sinh\left(\frac{m\pi y}{a}\right) \Rightarrow$$

για $y = b$: $\varphi = \sum_{m=1}^{\infty} C_m \sin\left(\frac{m\pi x}{a}\right)\sinh\left(\frac{m\pi b}{a}\right) = f(x)$ (1)

Ορθογωνική σχέση για τα ημίτονα:

$$\int_{x=0}^{a} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{a}\right)dx = \begin{cases} \frac{a}{2}, & m = n \\ 0, & m \neq n \end{cases}$$

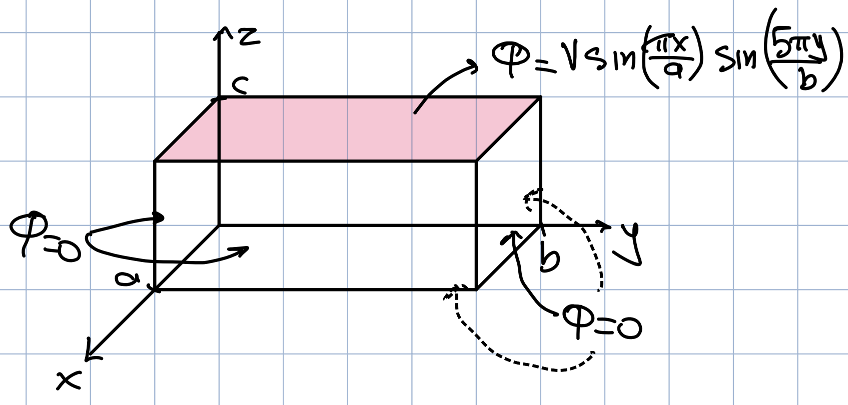
Ορθογωνική σχέση για τα συνημίτονα:

$$\int_{x=0}^{a} \cos\left(\frac{m\pi x}{a}\right)\cdot\cos\left(\frac{n\pi x}{a}\right)dx = \begin{cases} a, & m = n = 0 \\ \frac{a}{2}, & m = n \neq 0 \\ 0, & m \neq n \end{cases}$$

$$\varphi = \sum_{m=1}^{\infty} C_m \int_{x=0}^{a} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{a}\right)dx \cdot \sinh\left(\frac{m\pi b}{a}\right) = \int_{x=0}^{a} f(x)\sin\left(\frac{n\pi x}{a}\right)dx$$

$$\Rightarrow C_n \cdot \frac{a}{2} \cdot \sinh\left(\frac{n\pi b}{a}\right) = \int_{x=0}^{a} f(x)\sin\left(\frac{n\pi x}{a}\right)dx \Rightarrow C_n = \frac{2}{a}\,\frac{\int_{x=0}^{a} f(x)\sin\left(\frac{n\pi x}{a}\right)dx}{\sinh\left(\frac{n\pi b}{a}\right)}$$

8) 3-διάστατο πρόβλημα



$$K_x + K_y + K_z = 0$$

$$K_x = -\left(\frac{\pi}{a}\right)^2 = -k_x^2 < 0$$

$$K_y = -\left(\frac{5\pi}{b}\right)^2 = -k_y^2 < 0$$

$$K_z = -K_x - K_y = \left(\frac{\pi}{a}\right)^2 + \left(\frac{5\pi}{b}\right)^2 = k_z^2 > 0 \Rightarrow k_z = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{5\pi}{b}\right)^2}$$

$$\Phi = C\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{5\pi y}{b}\right)\cdot\sinh(k_z \cdot z)$$

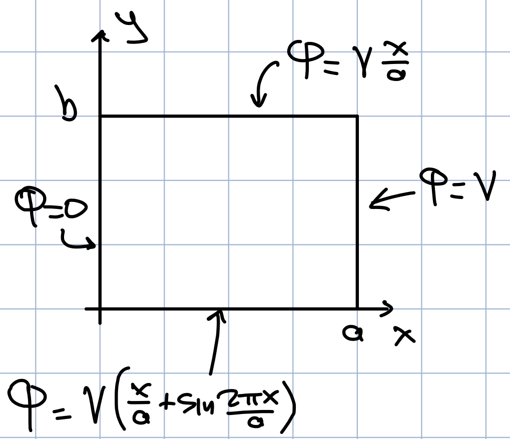
για $z=c$: $\Phi = C\cancel{\sin\left(\frac{\pi x}{a}\right)}\cancel{\sin\left(\frac{5\pi y}{b}\right)}\cdot\sinh(k_z \cdot c) = V\cancel{\sin\left(\frac{\pi x}{a}\right)}\cancel{\cos\left(\frac{5\pi y}{b}\right)}$

$$C = \frac{V}{\sinh(k_z \cdot c)}$$

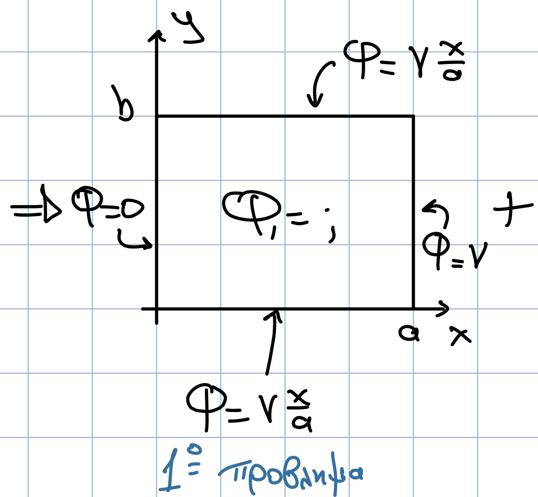
Άρα $\Phi = V\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{5\pi y}{b}\right)\cdot\frac{\sinh(k_z \cdot z)}{\sinh(k_z \cdot C)}$

Άσκηση 14.1

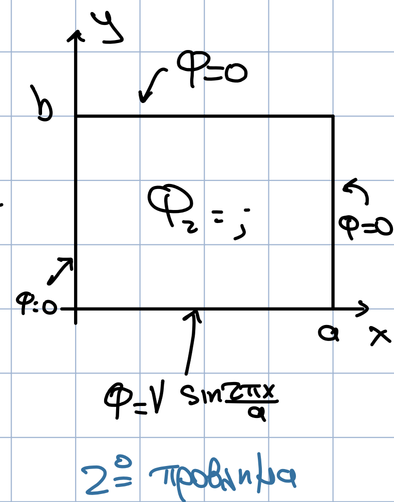
$$\frac{\partial}{\partial z}=0$$



$\Rightarrow$



$+$



1) $K_x = 0$, $X_1(x) = A_1 x + A_1'$

$K_y = -K_x = 0$ $\quad Y_1(y) = B_1 y + B_1' \Longrightarrow \varphi_1 = (A_1 x + A_1')(\cancel{B_1 y}^{\,0} + B_1') = C_1 x + C_1'$

Για $x=0$: $\varphi_1 = C_1' = 0$

$x=a$: $\varphi_1 = C_1 a = V \Rightarrow C_1 = \frac{V}{a} \Rightarrow \varphi_1 = V\cdot\frac{x}{a}$

2) $K_x = -\left(\frac{2\pi}{a}\right)^2 = -k_x^2 < 0$

$K_y = -K_x = \left(\frac{2\pi}{a}\right)^2 = k_y^2 > 0$

$$\left.\right\}\ \varphi_2 = C_2 \sin\left(\frac{2\pi x}{a}\right)\sinh\left[\frac{2\pi}{a}(y-b)\right]$$

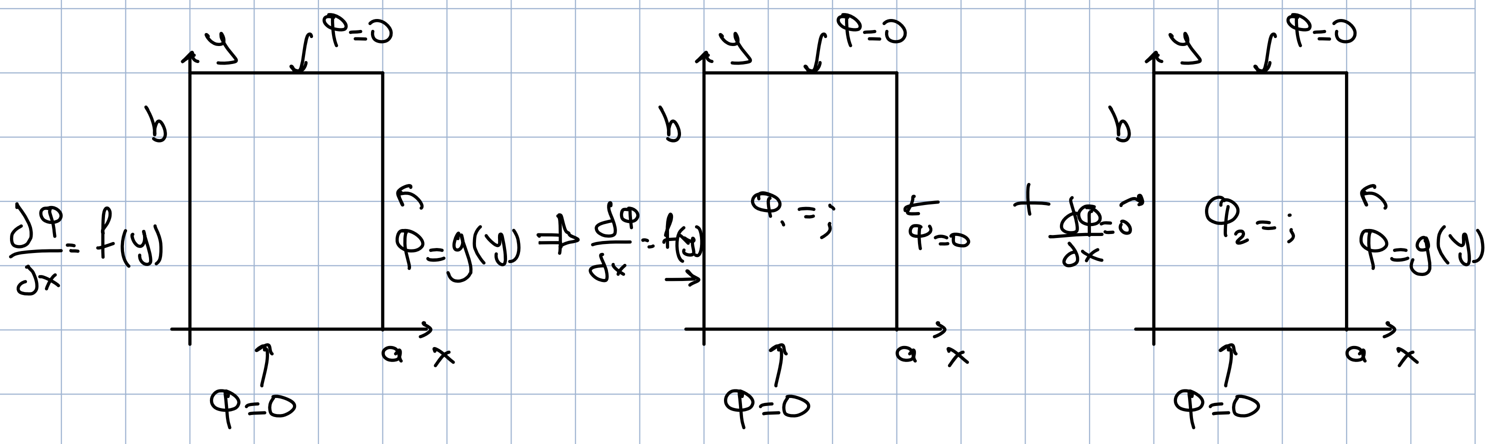
Για $y=0$: $\varphi_2 = -C_2 \sin\cancel{\frac{2\pi x}{a}}\cdot\sinh\frac{2\pi b}{a} = V\sin\cancel{\frac{2\pi x}{a}}$

$$C_2 = -\frac{V}{\sinh\left(\frac{2\pi b}{a}\right)}$$

$$\varphi = \varphi_1 + \varphi_2 = V\frac{x}{a} - V\sin\left(\frac{2\pi x}{a}\right)\frac{\sinh\left[\frac{2\pi}{a}(y-b)\right]}{\sinh\left(\frac{2\pi b}{a}\right)}$$

$$\vec{E} = -\nabla\varphi = -\frac{\partial\varphi}{\partial x}\hat{x} - \frac{\partial\varphi}{\partial y}\hat{y} = \dots$$

Άσκηση 14.2



1ο πρόβλημα 2ο πρόβλημα

1) $Y_m(y) = B_m \sin\left(\frac{m\pi y}{b}\right)$, $m = 1, 2, 3, \dots$

$$k_y = -\left(\frac{m\pi}{b}\right)^2 = -k_y^2 < 0$$

$$k_x = -k_y = \left(\frac{m\pi}{b}\right)^2 = k_x^2 > 0$$

$$X_{1,m} = A_m \sinh\left[\frac{m\pi}{b}(x-a)\right]$$

$$\varphi_1 = \sum_{m=1}^{\infty} C_m \sinh\left[\frac{m\pi}{b}(x-a)\right] \sin\left(\frac{m\pi y}{b}\right)$$

$$\frac{d\varphi_1}{dx} = \sum_{m=1}^{\infty} C_m \left(\frac{m\pi}{b}\right) \cosh\left[\frac{m\pi}{b}(x-a)\right] \sin\left(\frac{m\pi y}{b}\right)$$

για $x=0$: $\displaystyle \frac{d\varphi_1}{dx} = \sum_{m=1}^{\infty} C_{m,1} \frac{m\pi}{b} \cosh\left(\frac{m\pi a}{b}\right) \sin\left(\frac{m\pi y}{b}\right) = f(y)$

$$\Rightarrow \sum_{m=1}^{\infty} C_{m_1} \frac{m\pi}{b} \cdot \cosh\left(\frac{m\pi a}{b}\right) \int_{y=0}^{b} \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{n\pi y}{b}\right) dy = \int_{y=0}^{b} f(y) \sin\left(\frac{n\pi y}{b}\right) dy$$

$$\Rightarrow C_{n_1} \cdot \frac{n\pi}{b} \cosh\left(\frac{n\pi a}{b}\right) \cdot \frac{b}{2} = \int_{y=0}^{b} f(y) \sin\left(\frac{n\pi y}{b}\right) dy \Rightarrow C_{n_1} = \dots$$

(συνέχεια)

2)

$$\Phi_{m_2} = C_{m_2} \cosh\left(\frac{m\pi x}{b}\right) \sin\left(\frac{m\pi y}{b}\right)$$

$$\Phi_2 = \sum_{m=1}^{\infty} C_{m_2} \cosh\left(\frac{m\pi x}{b}\right) \cdot \sin\left(\frac{m\pi y}{b}\right)$$

Για $x=a$: $\Phi_2 = \sum_{m=1}^{\infty} C_{m_2} \cosh\left(\frac{m\pi a}{b}\right) \sin\left(\frac{m\pi y}{b}\right) = g(y)$

$$\Rightarrow \sum_{m=1}^{\infty} C_{m_2} \cosh\left(\frac{m\pi a}{b}\right) \int_{y=0}^{b} \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{n\pi y}{b}\right) dy = \int_{y=0}^{b} g(y) \sin\left(\frac{n\pi y}{b}\right) dy$$

$$\Rightarrow C_{n_2} \cosh\left(\frac{n\pi a}{b}\right) \cdot \frac{b}{2} = \int_{y=0}^{b} g(y) \sin\left(\frac{n\pi y}{b}\right) dy \Rightarrow C_{n_2} = \dots$$

$$\Phi = \Phi_1 + \Phi_2 = \dots$$