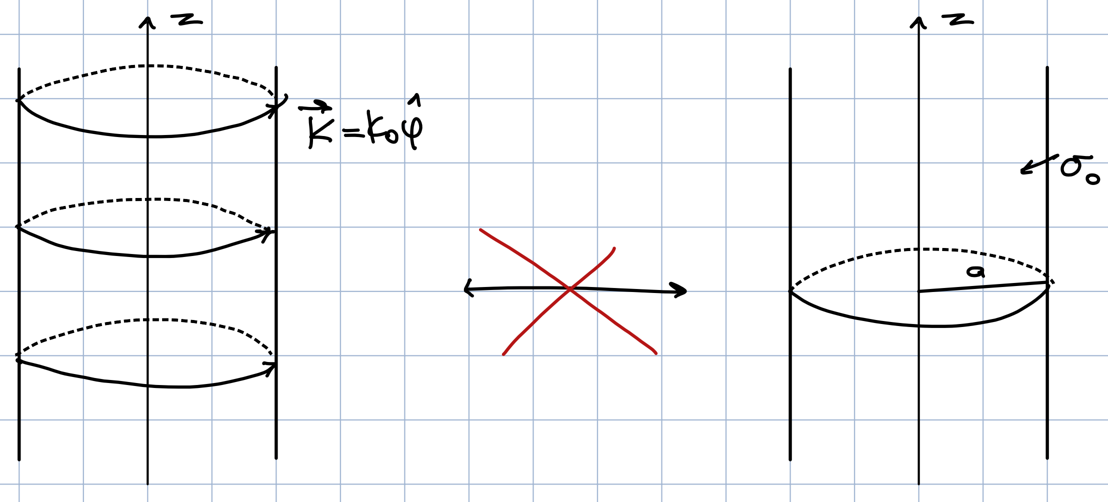
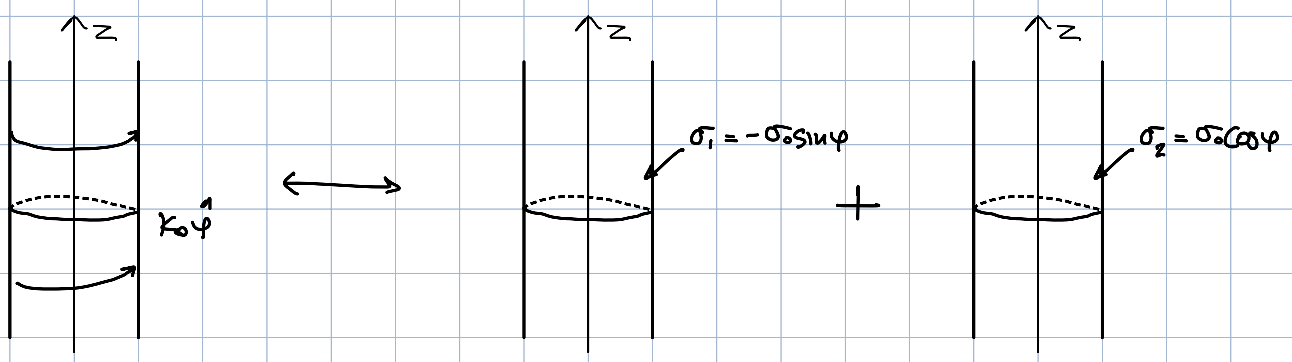
Παραδείγματα:

1)

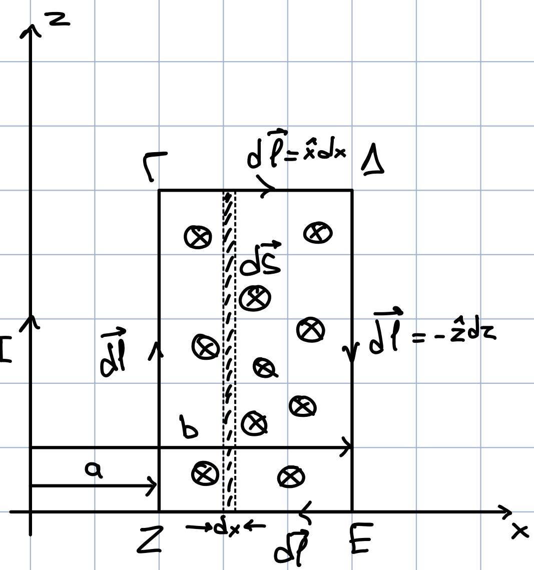


δεν ισχυει αυτό, αφού η αντιστοιχηση είναι σε καρτεσιανες συνιστωσες αρα:

$$\vec{K} = K_0\hat{\varphi} = \underbrace{-K_0 \sin\varphi}_{K_x}\,\hat{x} + \underbrace{K_0 \cos\varphi}_{K_y}\,\hat{y} \;:$$



2)



$$\vec{B}=\frac{\mu_0 I}{2\pi r}\hat{\varphi}$$

$$\Psi_m=\int_S \vec{B}d\vec{S}=\frac{\mu_0 I h}{2\pi}\int_a^b \frac{dx}{x}=\frac{\mu_0 I h}{2\pi}\ln\frac{b}{a}$$

$$\Psi_m=\oint_c \vec{A}d\vec{\ell}$$

$$\vec{A}=\left(-\frac{\mu_0 I}{2\pi}\ln x+\sigma\tau\alpha\theta\right)\hat{z}$$

$$\Psi_m=\underbrace{-\frac{\mu_0 I h}{2\pi}\ln a+\cancel{(\sigma\tau\alpha\theta)}h}_{Z\Gamma}+\underbrace{0}_{\Gamma\Delta}+\underbrace{\frac{\mu_0 I h}{2\pi}\ln b-\cancel{(\sigma\tau\alpha\theta)}h}_{\Delta E}+\underbrace{0}_{EZ}=\frac{\mu_0 I h}{2\pi}\ln\left(\frac{b}{a}\right)$$

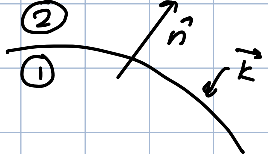
Οριακές συνθήκες για το A

$$\vec{B}=\nabla\times\vec{A}\Rightarrow\int_S\vec{B}d\vec{S}=\int_S\vec{B}d\vec{S}=\int_S(\nabla\times\vec{A})d\vec{S}=\oint_c\vec{A}d\vec{\ell}=\Psi_m\quad(1)\Rightarrow$$

$$\hat{n}\times(\vec{A}_2-\vec{A}_1)=0$$
$$A_{z_2}=A_{z_1}$$

$$\nabla\cdot\vec{A}=0\ (\text{συνθήκη Coulomb})\Rightarrow\int_V(\nabla\cdot\vec{A})dV\ \underset{=}{\overset{\text{Θ. Gauss}}{}}\ \oint_S\vec{A}d\vec{S}=0\quad(2)\Rightarrow$$

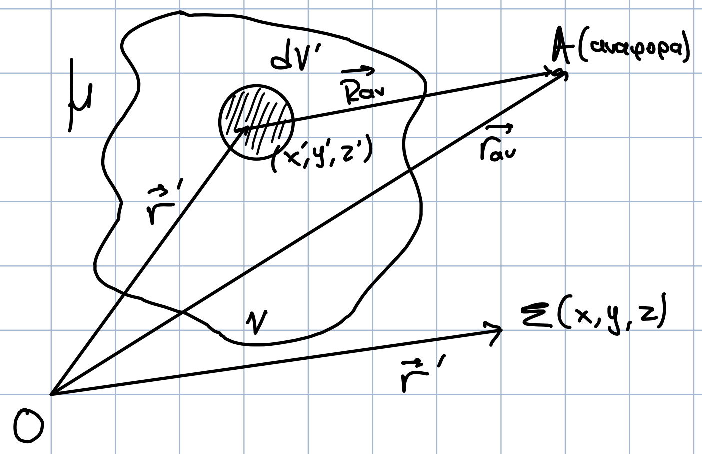
$$\hat{n}\cdot(A_2-A_1)=0$$
$$A_{n_2}=A_{n_1}$$



$$\oint_c\vec{E}d\vec{\ell}=-\frac{d}{dt}\underbrace{\int_S\vec{B}d\vec{S}}_{\Psi_m}=-\frac{d\Psi_m}{dt}\quad(3)\Rightarrow\hat{n}\times(\vec{E}_2-\vec{E}_1)=0$$
$$E_{z_2}=E_{z_1}$$

$$\oint_S\vec{B}d\vec{S}=0\quad(4)\Rightarrow\hat{n}(\vec{B}_2-\vec{B}_1)=0$$
$$B_{n_2}=B_{n_1}$$

Νόμος Biot-Savart (σελ 329)



$$R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$R_{\alpha\nu}=\sqrt{(x'-x_{\alpha\nu})^2+(y'-y_{\alpha\nu})^2+(z'-z_{\alpha\nu})^2}$$

$$\vec{A}_\Sigma(\vec{r})=\frac{\mu}{4\pi}\int_V \vec{J}(\vec{r}\,')dV'\left(\frac{1}{R}-\frac{1}{R_{\alpha\nu}}\right) \quad (1)$$

$$\vec{B}_\Sigma=\nabla\times\vec{A}_\Sigma=\frac{\mu}{4\pi}\int_V \nabla\times\left[\frac{\vec{J}(\vec{r}\,')}{R}\right]dV', \quad \nabla=\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}$$

Θέτω $f=\frac{1}{R}$, $\nabla\times(f\vec{J})=f\,\cancelto{0}{\nabla\times\vec{J}(x',y',z')}+\nabla f\times\vec{J}=\nabla\left(\frac{1}{R}\right)\times\vec{J}=-\frac{\hat{R}}{R^2}\times\vec{J}$

$$=\frac{\vec{J}(\vec{r}\,')\times\hat{R}}{R^2} \quad (2)$$

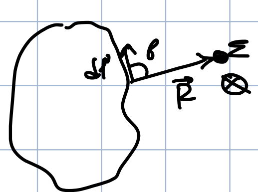
$$\vec{B}_\Sigma(\vec{r})=\frac{\mu}{4\pi}\int_V\frac{\vec{J}(\vec{r}\,')\times\hat{R}}{R^2}dV' \quad (3) \qquad \text{(Ν. Biot Savart)}$$

Για επιφανειακά ρεύματα $\vec{K}$:

$$\vec{B}_\Sigma(\vec{r})=\frac{\mu}{4\pi}\int_S\frac{\vec{K}(\vec{r}\,')\times\hat{R}}{R^2}dS'$$

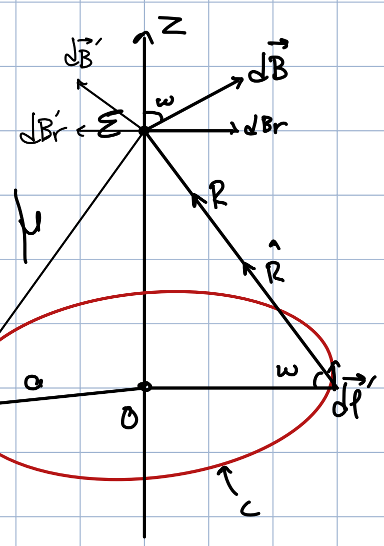
Για γραμμικά ρεύματα I:

$$\vec{B}_\Sigma(\vec{r})=\frac{\mu I}{4\pi}\oint_C\frac{d\vec{l}\,'\times\hat{R}}{R^2}$$



$$d\vec{B}_\Sigma=\frac{\mu I}{4\pi}\frac{dl\sin\theta}{R^2}\hat{n}$$

Παράδειγμα 3 (σελίδα 334)



$$d\vec{B} = \frac{\mu I}{4\pi}\oint_C \frac{d\vec{l}' \times \hat{R}}{R^2}$$

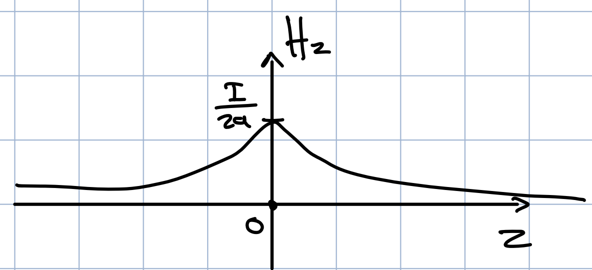
όλα τα dB_r θα φύγουν αφού για το καθένα υπάρχει το ίδιο με ένα μείον λόγω συμμετρίας.

$$dB_z = \frac{\mu I}{4\pi}\,\frac{a\,d\varphi'}{R^2}\cos\omega = \frac{\mu I a\,d\varphi'}{4\pi R^2}\,\frac{a}{R}$$

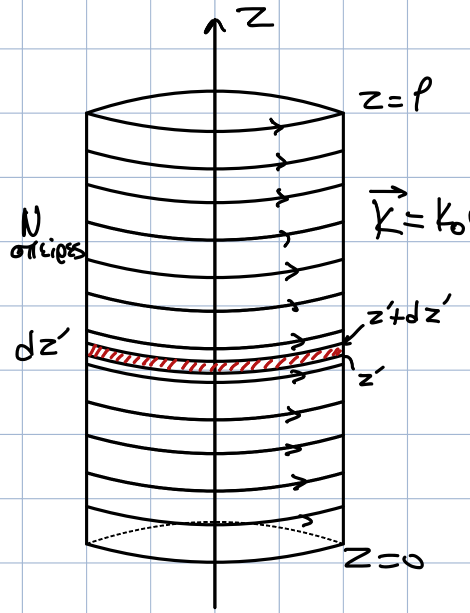
άρα $dB_z = \frac{\mu I a^2}{4\pi R^3}\,d\varphi'$, $B_z = \frac{\mu I a^2}{4\pi(z^2+a^2)^{3/2}}\int_0^{2\pi} d\varphi' = \frac{\mu I a^2}{2\,(z^2+a^2)^{3/2}}$

$$H_z = \frac{B_z}{\mu} = \frac{I a^2}{2(z^2+a^2)^{3/2}}, \quad -\infty < z < \infty$$

$$H_{z,max} = \frac{I a^2}{2a^3} = \frac{I}{2a}$$



Παράδειγμα 5 (σελίδα 337)

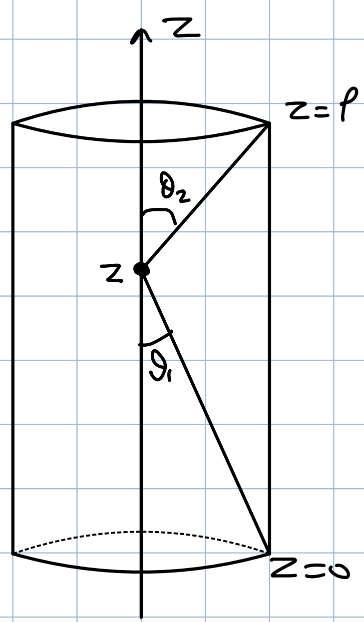


αμπερελίγματα

$$K_0 = \frac{NI}{\ell} \quad \left(\frac{A}{m}\right)$$

$$B_z(z) = \int_{z'=0}^{\ell} \frac{\mu a^2 K_0 \, dz'}{2\left[(z-z')^2+a^2\right]^{3/2}}$$

$$B_z(z) = \frac{\mu a^2 K_0}{2}\int_0^{\ell} \frac{dz'}{\left[(z-z')^2+a^2\right]^{3/2}} = \frac{\mu \cancel{a^2} K_0}{2\cancel{a^2}}\left[\frac{z}{\sqrt{z^2+a^2}} + \frac{\ell-z}{\sqrt{(\ell-z)^2+a^2}}\right], \quad -\infty < z < \infty$$



$$\left.\begin{aligned} \cos\theta_1 &= \frac{z}{\sqrt{z^2+a^2}} \\ \cos\theta_2 &= \frac{\ell-z}{\sqrt{(\ell-z)^2+a^2}} \end{aligned}\right\} \quad \begin{aligned} B_z &= \frac{\mu K_0}{2}(\cos\theta_1+\cos\theta_2) \\ H_z &= \frac{K_0}{2}(\cos\theta_1+\cos\theta_2) \end{aligned}$$

$\Rightarrow$ Για $\ell \ll a$: $H_z = \dfrac{NI}{2a}$

$\ell \gg a$: $H_z(z=\ell/2) = K_0$

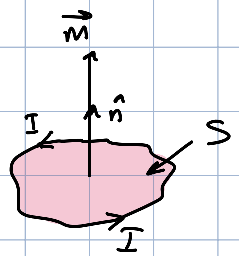
$\theta_1, \theta_2 \approx 0$

$$\underset{\substack{\cos\theta_1=0 \\ \cos\theta_2=1}}{H_z(z=0)} = \frac{K_0}{2} = \underset{\substack{\cos\theta_2=0 \\ \cos\theta_1=1}}{H_z(z=\ell)}$$

Μαγνητικό δίπολο (σελίδα 344)

Ροπή του μαγνητικού διπόλου:

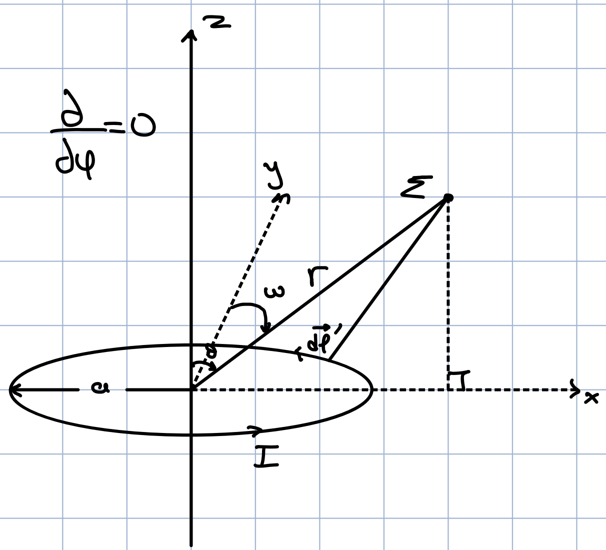
$$\vec m = I\vec S$$



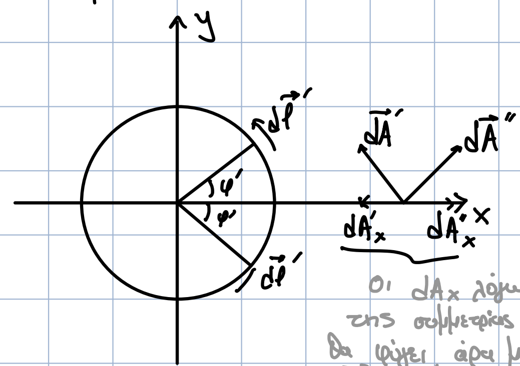
(ακανόνιστο το σχήμα)

Ακριβής ορισμός: $\vec m = \lim_{\substack{S\to 0\\ I\to\infty}} I\vec S$ = πεπερασμένο

$\frac{\partial}{\partial\varphi} = 0$



$$d\vec A = \frac{\mu I}{4\pi}\frac{d\vec\ell'}{R}$$



Οι dA_x λόγω της συμμετρίας θα φύγει, άρα μένει το $d\vec A_y$, η οποία στο (r,φ,z) είναι το $\hat\varphi$

$$\Rightarrow dA_y = \frac{\mu I}{4\pi}\cdot\frac{a\,d\varphi'}{R}\cos\varphi'$$

μόνο φ αφού μόνο y έχουμε

$$A_\varphi = \frac{\mu I a}{4\pi}\int_{\varphi'=0}^{2\pi}\frac{\cos\varphi'\,d\varphi'}{\sqrt{r^2+a^2-2ra\cos\omega}} = \frac{\mu I a}{4\pi}\int_0^{2\pi}\frac{\cos\varphi'\,d\varphi'}{\sqrt{r^2+a^2-2ra\sin\theta\cos\varphi'}}$$

$$= \frac{\mu I a}{4\pi r}\int_0^{2\pi}\frac{\cos\varphi'\,d\varphi'}{\sqrt{1+\frac{a^2}{r^2}-2\frac{a}{r}\sin\theta\cos\varphi'}} \overset{r\gg a}{=}$$

και $\frac{1}{\sqrt{1+x}} \simeq 1+\frac{x}{2}$, $|x| \ll 1$

$$= \frac{\mu I a}{4\pi r}\int_0^{2\pi}\cos\varphi'\,d\varphi'\left(1+\frac{a}{r}\sin\theta\cos\varphi'\right) = \frac{\mu I\pi a^2}{4\pi r^2}\sin\theta$$

$$\Rightarrow \cos\omega = \hat r\cdot\hat r'$$
$$= (\hat x\sin\theta + \hat z\cos\theta)(\hat x\cos\varphi' + \hat y\sin\varphi')$$
$$= \sin\theta\cos\varphi'$$

Άρα $\Rightarrow$ $A_\varphi = \frac{\mu m}{4\pi r^2}\sin\theta$