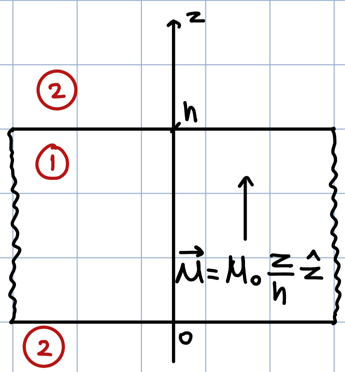
Παράδειγμα 1:



$\vec{B}, \vec{H}$ $\quad \frac{d}{dx} = 0 = \frac{d}{dy}$

i) μοντέλο ρευμάτων μαγνήτισης

$$\vec{J}_{M_1} = \nabla \times \left(M_0 \frac{z}{h}\hat{z} \right) = 0$$

$$\vec{J}_{M_2} = \nabla \times \vec{M}_2 = 0$$

$$\vec{K}_M (z=0) = -(-\hat{z}) \times M_0 \cdot \frac{0}{h}\hat{z} = 0$$

$$\vec{K}_M (z=h) = -(+\hat{z}) \times M_0 \hat{z} = 0$$

$$\underbrace{\begin{array}{ll} \nabla \times \vec{B} = \mu_0 (\cancelto{0}{\vec{J}} + \vec{J}_M) = 0 & \quad \hat{n} \times (\vec{B}_2 - \vec{B}_1) = \mu_0 (\cancelto{0}{K} + \cancelto{0}{K_M}) = 0 \\ \nabla \cdot \vec{B} = 0 & \quad \hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \end{array}}$$

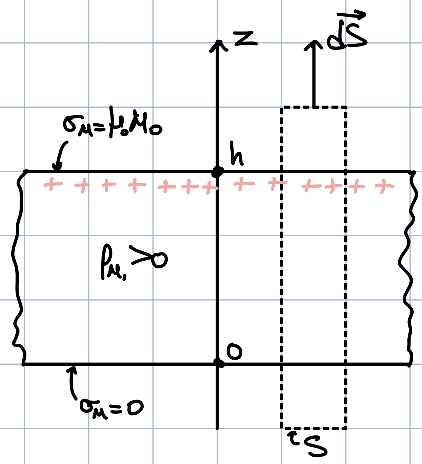
$$\vec{B}_1 = \vec{B}_2 = 0$$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_0} - \vec{M} = -M_0 \frac{z}{h}\hat{z}$$

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_0} - \cancelto{0}{\vec{M}_2} = 0$$

ii) Μοντέλο μαγνητικών φορτίων

$$\left.\begin{aligned} \rho_{M_1} &= \nabla(\mu_0 \vec{M}) = -\mu_0 \frac{M_0}{h} \\ \rho_{M_2} &= -\mu_0 \nabla \cdot \vec{M_2} = 0 \quad (\text{αέρας}) \\ \sigma_M(z=0) &= -\hat{z}\cdot\left(\mu_0 M_0 \tfrac{0}{h}\right)\hat{z} = 0 \\ \sigma_M(z=h) &= \hat{z}\cdot(\mu_0 M_0)\hat{z} = \mu_0 M_0 \end{aligned}\right\} \Rightarrow$$



$$Q_{M,\text{συνολικό}} = -\mu_0 \frac{M_0}{\cancel{h}} S\cancel{h} + \mu_0 M_0 S = 0$$

$$\oint \mu_0 \vec{H} d\vec{S} = \int_V \rho_M dV$$

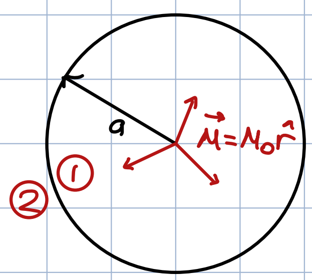
Όπως και στα ελεύθερα φορτία ξέρουμε ότι $\exists$ μόνο $H_z(z)$

$$2\mu_0 H_{z_2} S = Q_{M,\text{συνολικό}} = 0 \Rightarrow \vec{H}_{z_2} = 0 \Rightarrow \vec{H_2} = 0$$

περ① $\mu_0 H_{z_1}(z) \cdot S + 0 = -\mu_0 \frac{M_0}{h} S \cdot z \Rightarrow H_{z_1} = -\frac{M_0 z}{h} \Rightarrow \vec{H_1} = -\frac{M_0 z}{h}\hat{z}$

$$\vec{B_1} = \mu_0(\vec{H}+\vec{M}) = \mu_0(\cancel{-\vec{M}+\vec{M}}) = 0$$

Παράδειγμα 2 (εσωτερικός μαγνήτης)



$$\vec{H},\vec{B}=;\qquad \frac{\partial}{\partial\varphi}=0=\frac{\partial}{\partial\theta}$$

i) μοντέλο ρευμάτων μαγνήτισης

$$\vec{J}_{M_1}=\nabla\times\vec{M}_1=\nabla\times(M_0\hat{r})=0$$
$$\vec{J}_{M_2}=\nabla\times\vec{M}_2=0$$

$$\vec{K}_M(r=a)=-\hat{r}\times(M_0\hat{r})=0 \qquad \nabla\times\vec{B}=\mu_0(\vec{J}+\vec{J}_M)=0$$
$$\nabla\cdot\vec{B}=0$$

$$\left.\begin{array}{l}\hat{r}\times(\vec{B}_2-\vec{B}_1)=\mu_0(\cancelto{0}{\vec{K}}+\cancelto{0}{\vec{K}_M})=0\\ \hat{r}\cdot(\vec{B}_2-\vec{B}_1)=0\end{array}\right\}\vec{B}_{1,2}=0$$

$$\vec{H}_1=\frac{\vec{B}_1}{\mu_0}-\vec{M}_1=-M_0\hat{r} \qquad \text{και} \qquad \vec{H}_2=\frac{\vec{B}_2}{\mu_0}=0$$

ii) μοντέλο μαγνητικών φορτίων

$$\rho_{M_1}=-\nabla\cdot(\mu_0\vec{M})=-\frac{M_0}{r^2}\frac{d}{dr}(r^2M_0)=\frac{-2\mu_0M_0}{r}$$
$$\rho_{M_2}=0$$
$$\sigma_M(r=a)=\hat{r}\cdot(\mu_0M_0\hat{r})=\mu_0M_0$$

$$Q_{M,\text{συνολ}}=\int_V\rho_{M_1}dV+\sigma_M(r=a)\cdot4\pi a^2=\cdots=-8\pi\mu_0M_0\frac{a^2}{2}+\mu_0M_04\pi a^2=0$$

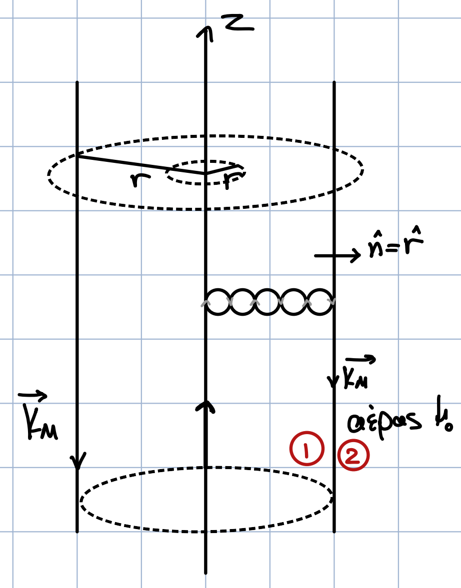
νόμος Gauss: $\oint\mu_0\vec{H}d\vec{S}=\int_V\rho_M dV$

περ. 1 $\quad \mu_0H_{r_1}\cdot4\pi r^2=-2\mu_0M_0\int_0^r\frac{4\pi(r')^2}{(r')}dr'=-8\pi\mu_0M_0\frac{r^2}{2}\Rightarrow\vec{H}_1=-M_0\hat{r}$

περ. 2 $\quad \mu_0H_{r_2}\cdot4\pi r^2=Q_{M,\text{συνολικό}}=0\Rightarrow H_{r_2}=0\Rightarrow\vec{H}_2=0$

$\vec{B}=\mu_0(\vec{H}+\vec{M})$, άρα $\vec{B}_1=\vec{B}_2=0$

Παράδειγμα 5 (απέραντος κύλινδρος)



$\vec{H}, \vec{B}, \vec{M}, \vec{J}_M, \vec{K}_M, \rho_M, \sigma_M = ;$ $\quad \frac{\partial}{\partial\varphi}=0=\frac{\partial}{\partial z}$

$\exists$ μόνο $\vec{H}=H_\varphi(r)\cdot\hat{\varphi}$

$\vec{H}_1=\frac{I}{2\pi r}\hat{\varphi}=\vec{H}_2$, $\vec{B}_1=\frac{\mu I}{2\pi r}\hat{\varphi}$ $\quad \vec{B}_2=\frac{\mu_0 I}{2\pi r}\hat{\varphi}$

$\vec{M}_1=\frac{\vec{B}_1}{\mu_0}-\vec{H}_1=\left(\frac{\mu}{\mu_0}-1\right)\vec{H}_1=\frac{\mu-\mu_0}{\mu_0}\cdot\frac{I}{2\pi r}\hat{\varphi}=\frac{\mu-\mu_0}{\mu_0}\frac{\vec{B}_1}{\mu}$

$\vec{M}_2=0$

$\vec{J}_M=\nabla\times\vec{M}_1=\frac{\mu-\mu_0}{\mu_0}\nabla\times\vec{H}_1=\frac{\mu-\mu_0}{\mu_0}\cdot\vec{J}_1 \Rightarrow \vec{J}_{M_1}=0$, $0<r<a$

$\vec{J}_1(r=0)=I\cdot\delta(\vec{r})\hat{z}=I\,\delta(x)\delta(y)\cdot\hat{z}$

$\vec{K}_M(r=a)=-\hat{r}\times\vec{M}_1(r=a)=-\frac{\mu-\mu_0}{\mu_0}\frac{I}{2\pi a}\hat{z}$

$\vec{J}_M=\nabla\times\vec{M}\Rightarrow\int_S\vec{J}_M d\vec{S}=\int_S(\nabla\times\vec{M})\,d\vec{S}=\oint_C\vec{M}d\vec{l}$

$I_M=M_\varphi(r)\cdot 2\pi r \longrightarrow$ (πιο σωστά: $I_M=\lim_{r\to 0}M_\varphi(r)\cdot 2\pi r$)

Εδώ $\mu=$ σταθ $\Rightarrow J_{M_1}=0$, $0<r<a$

$I_M=\frac{\mu-\mu_0}{\mu_0}I$, $\vec{K}_M\cdot 2\pi a=\frac{\mu-\mu_0}{\mu}I\cdot\hat{z}$

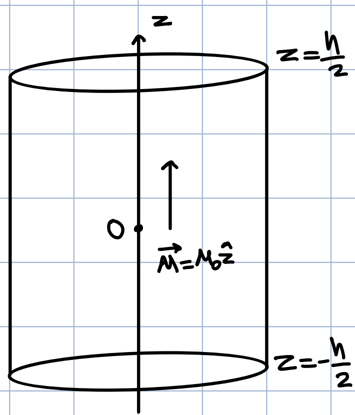
$\rho_{M_1}=-\mu_0\nabla\cdot\vec{M}_1=-\mu_0\frac{\mu-\mu_0}{\mu_0\mu}\nabla\cdot\cancelto{0}{\vec{B}}=0$

$\rho_{M_2}=0$ (αέρας)

$\sigma_M(r=a)=\hat{r}\left[\mu_0 M_1(r=a^-)\right]=0$

Άσκηση 12.1

a)



i) <u>μοντέλο ρευμάτων μαγνήτισης</u>

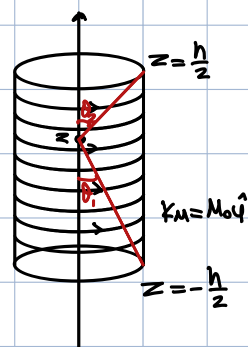
$$\vec J_M = \nabla\times\vec M = \nabla\times(M_0\hat z) = 0$$

$$\vec K_M(r=a) = -\hat r\times(M_0\hat z) = M_0\hat\varphi$$

$$\vec K_M(z=-\tfrac{h}{2}) = -(-\hat z)\times(M_0\hat z) = 0$$

$$\vec K_M(z=+\tfrac{h}{2}) = -\hat z\times M_0\hat z = 0$$

Από μοντελοποίηση:



$$\vec B = \frac{\mu_0 \underbrace{K_M}_{M_0}}{2}\left(\frac{\frac{h}{2}+z}{\sqrt{(\frac{h}{2}+z)^2+a^2}} + \frac{\frac{h}{2}-z}{\sqrt{(\frac{h}{2}-z)^2+a^2}}\right), \quad -\infty<z<\infty$$

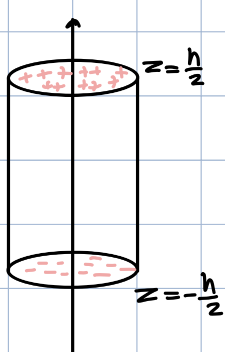
Για $|z|<\frac{h}{2} \Rightarrow \vec H = \frac{\vec B}{\mu_0} - \vec M = \frac{M_0}{2}(\dots)\hat z - M_0\hat z$ $\quad \left(-\frac{h}{2}<z<\frac{h}{2}\right)$

$$\vec H = \frac{\vec B}{\mu_0} = \frac{M_0}{2}(\dots)\hat z, \quad \left(z>\frac{h}{2},\ z<-\frac{h}{2}\right)$$

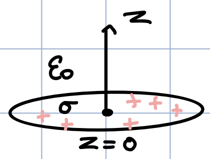
ii) Μοντέλο μαγνητικών φορτίων

$$\rho_M = -\nabla(\mu_0\vec M) = 0 \qquad \sigma_M(r=a) = \hat r\cdot(\mu_0 M_0\hat z) = 0$$

$$\sigma_M(z=-\tfrac{h}{2}) = -\hat z(\mu_0 M_0\hat z) = -\mu_0 M_0 \qquad \sigma_M(z=\tfrac{h}{2}) = \hat z\cdot(\mu_0 M_0\hat z) = \mu_0 M_0$$



Από προηγούμενο παράδειγμα:



$$\vec{E} = \begin{cases} \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right)\hat{z}, & z>0 \\ -\frac{\sigma}{2\varepsilon_0}\left(1 + \frac{z}{\sqrt{z^2+a^2}}\right)\hat{z}, & z<0 \end{cases}$$

Στο παράδειγμα μας, αντίστοιχα:

Για $z > \frac{h}{2}$: $H_z = \frac{\sigma_M}{2\mu_0}\left(\cancel{1} - \frac{z-\frac{h}{2}}{\sqrt{(z-\frac{h}{2})^2+a^2}} - \cancel{1} + \frac{z+\frac{h}{2}}{\sqrt{(z+\frac{h}{2})^2+a^2}}\right)$

$$= \frac{\sigma_M}{2\mu_0}\left(\frac{z+\frac{h}{2}}{\sqrt{(z+\frac{h}{2})^2+a^2}} - \frac{z-\frac{h}{2}}{\sqrt{(z-\frac{h}{2})^2+a^2}}\right)$$

αντίστοιχα βρίσκουμε το $\vec{H}$ και στις υπόλοιπες περιοχές

β)