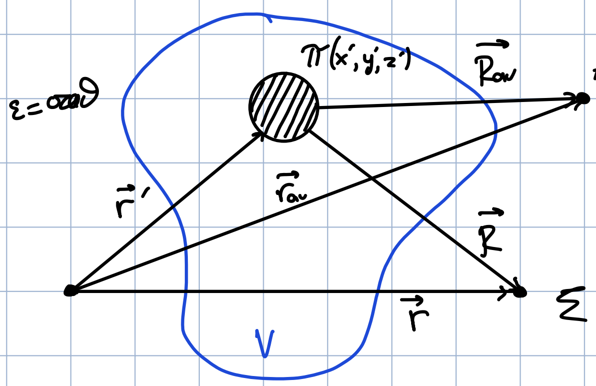
# Αρχή επαλληλίας (υπερθέσεως)




$$\vec{E}_\Sigma = \frac{1}{4\pi\varepsilon} \int_{\substack{\text{σε όλα} \\ \text{τα φορτία}}} \frac{dq}{R^2}\hat{R} \quad *$$

$$\vec{R} = \vec{r} - \vec{r}\,' = \hat{x}(x-x') + \hat{y}(y-y') + \hat{z}(z-z')$$

$$|\vec{R}| = R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}, \quad \hat{R} = \frac{\vec{R}}{R}$$

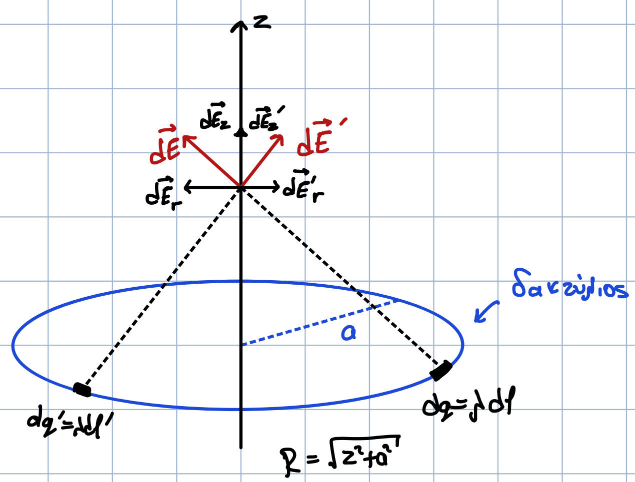
$$* \Rightarrow \vec{E}_\Sigma = \frac{1}{4\pi\varepsilon}\left[\int_V \frac{\rho(\vec{r}\,')}{R^2} dV' \hat{R} + \int \frac{\sigma(\vec{r}\,')}{R^2} ds' \hat{R} + \int \frac{\lambda(\vec{r}\,')}{R^2} d\ell' \hat{R} + \sum_{i=1}^{n} \frac{q_i}{R_i} \hat{R}\right]$$

$$\varphi_\Sigma = \frac{1}{4\pi\varepsilon}\int dq\left(\frac{1}{R} - \frac{1}{R_{αν}}\right) = \frac{1}{4\pi\varepsilon}\int_{\substack{\text{σε} \\ \text{όλα τα} \\ \text{φορτία}}} dq\left(\frac{1}{R} - \frac{1}{R_{αν}}\right)$$

$$\varphi_\Sigma = \frac{1}{4\pi\varepsilon}\left[\int_V \rho(\vec{r}\,')dV'\left(\frac{1}{R} - \frac{1}{R_{αν}}\right) + \int_S \sigma(\vec{r}\,')ds'\left(\frac{1}{R} - \frac{1}{R_{αν}}\right) + \int_\ell \lambda(\vec{r}\,')d\ell'\left(\frac{1}{R} - \frac{1}{R_{αν}}\right) + \sum_{i=1}^{n} q_i\left(\frac{1}{R_i} - \frac{1}{R_{iαν}}\right)\right]$$

$$\vec{E}_\Sigma = -\nabla\varphi_\Sigma$$

## Παράδειγμα 1 (Σελίδα 17)



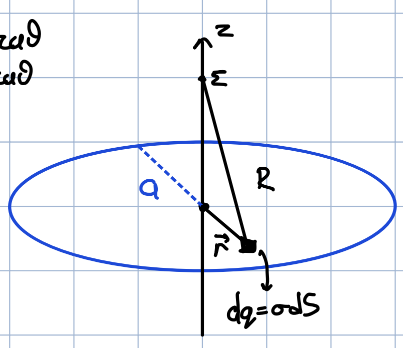

$$\Phi_\Sigma = \oint \frac{\lambda dl}{4\pi\varepsilon R} = \frac{\lambda}{4\pi\varepsilon R}\cdot 2\pi a$$

$$\Phi_\Sigma = \frac{\lambda a}{2\varepsilon\sqrt{z^2+a^2}} \quad (-\infty < z < \infty)$$

$$\vec{E}_\Sigma = -\nabla\Phi_\Sigma = -\frac{d\Phi}{dz}\hat{z} = \frac{\lambda a \frac{1}{2}\cdot 2z}{2\varepsilon(z^2+a^2)^{3/2}}\cdot\hat{z} = \frac{\lambda a z}{2\varepsilon(z^2+a^2)^{3/2}}\hat{z}$$

## Παράδειγμα 2 (δίσκος)

$\sigma = \sigma\tau\alpha\theta$
$\varepsilon = \sigma\tau\alpha\theta$



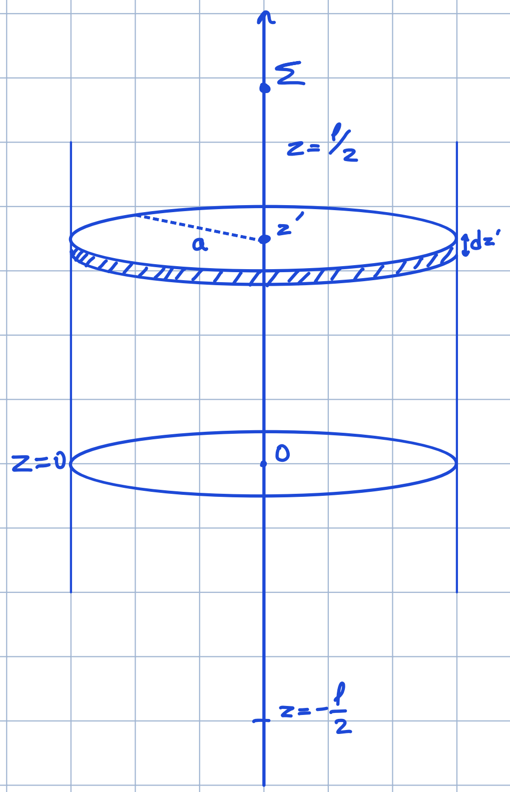

$$\Phi_\Sigma = \frac{\sigma}{4\pi\varepsilon}\int_{r'=0}^{a}\int_{0}^{2\pi}\frac{r'dr'd\varphi}{\sqrt{z^2+r'^2}}$$

$$= \frac{\sigma}{2\varepsilon}\sqrt{z^2+r'^2}\Big|_0^a$$

$$= \frac{\sigma}{2\varepsilon}\left[\sqrt{z^2+a^2}-|z|\right] \quad -\infty < z < \infty$$

$$E_\Sigma = -\nabla\Phi_\Sigma = -\frac{d\Phi_\Sigma}{dz}\hat{z} = -\frac{\sigma}{2\varepsilon}\left[\frac{z}{\sqrt{z^2+a^2}} - \mathrm{sgn}(z)\right]$$

# Παράδειγμα 4 (απέραντος κύλινδρος)



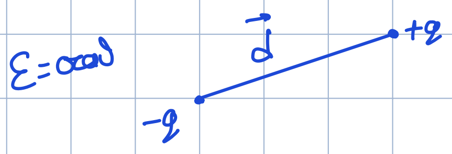

$$dq = \sigma dz'$$

$$\Phi_\Sigma = \int_{z'=-\ell/2}^{\ell/2} \frac{\sigma a\, dz'}{2\varepsilon\sqrt{(z-z')^2+a^2}}$$

$$\Phi_\Sigma = \frac{\sigma a}{2\varepsilon} \ln\left(\frac{z+\frac{\ell}{2}+\sqrt{(z+\frac{\ell}{2})^2+a^2}}{z-\frac{\ell}{2}+\sqrt{(z-\frac{\ell}{2})^2+a^2}}\right)$$

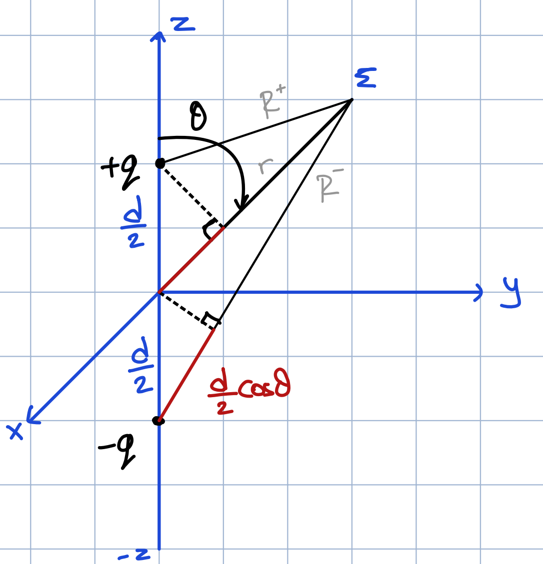
$$\vec{E}_\Sigma = -\nabla\Phi_\Sigma = \frac{\sigma a}{2\varepsilon}\left[\frac{1}{\sqrt{(z-\frac{\ell}{2})^2+a^2}} - \frac{1}{\sqrt{(z+\frac{\ell}{2})^2+a^2}}\right]\hat{z}\ , \quad -\infty < z < +\infty$$

# Ηλεκτρικό δίπολο




Ροπή ηλ. διπόλου: $\vec{p} = q\vec{d}$

Ακριβής ορισμός: $\vec{p} = \lim\limits_{\substack{d\to 0\\ q\to\infty}} q\vec{d}$ = πεπερασμένο



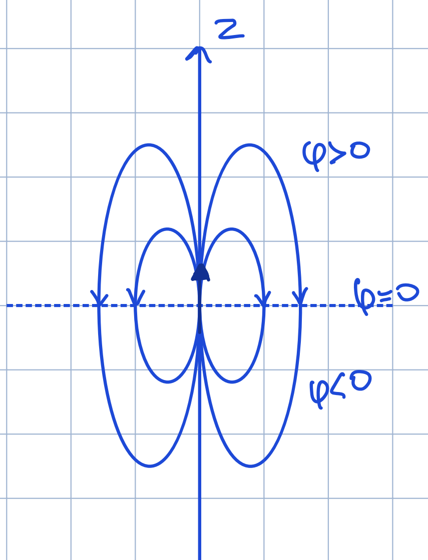

Για $r \gg d$ $\quad R^+ \simeq r - \frac{d}{2}\cos\theta$

$$R^- \simeq r + \frac{d}{2}\cos\theta$$

$$\Phi_\Sigma = \frac{q}{4\pi\varepsilon}\left(\frac{1}{R^+} - \frac{1}{R^-}\right) = \frac{q}{4\pi\varepsilon}\,\frac{R^- - R^+}{R^- R^+}$$

$$= \frac{q\,(d\cos\theta)}{4\pi\varepsilon\left(r^2 - \frac{d^2}{4}\cos^2\theta\right)} \simeq \frac{q d\cos\theta}{4\pi\varepsilon r^2} = \frac{p\cos\theta}{4\pi\varepsilon r^2}$$

$$\vec{E}_\Sigma = -\nabla\Phi_\Sigma = -\frac{\partial\Phi_\Sigma}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial\Phi_\Sigma}{\partial\theta}\hat{\theta} = \frac{p}{4\pi\varepsilon r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

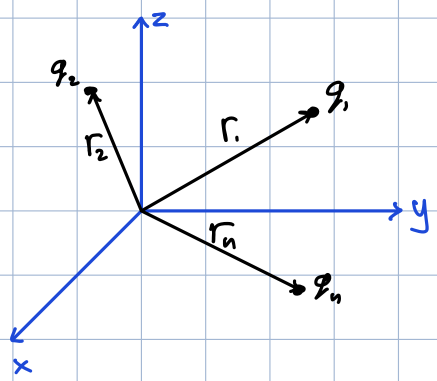



$$\Phi_\Sigma = \frac{\vec{p}\,\hat{r}}{4\pi\varepsilon r^2}$$

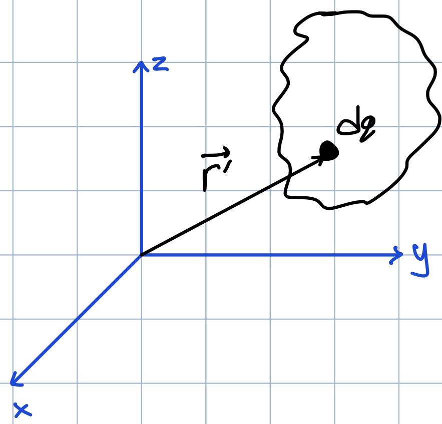
Για τυχαία θέση του διπόλου ισχύει:

$$\Phi_\Sigma = \frac{\vec{p}\,\hat{r}}{4\pi\varepsilon r^2} \Rightarrow \vec{E}_\Sigma = \frac{3(\vec{p}\,\hat{r})\,\hat{r} - \vec{p}}{4\pi\varepsilon r^3}$$

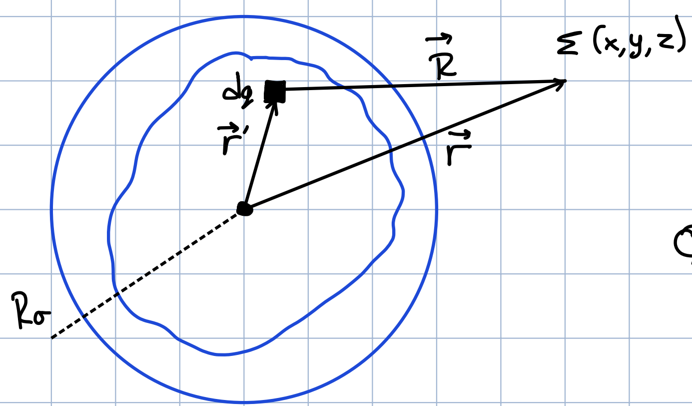
# Γενίκευση της διπολικής ροπής




$$\vec{p} = \sum_{i=0}^{n} q_i \vec{r_i}$$




$$\vec{p} = \int_{\substack{\text{σε όλα} \\ \text{τα φορτία}}} \vec{r}' dq$$




$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\Phi_\Sigma = \frac{1}{4\pi\varepsilon} \int_V \frac{dq}{R}$$

Για $r > R_\sigma >> r' \Rightarrow \Phi_\Sigma = \dots = \underbrace{\Phi_0 + \Phi_1 + \dots}_{\text{Ανάπτυγμα Maclaurin}}$

Ηλεκτρικό μονόπολο: $\Phi_0 = \frac{q}{4\pi\varepsilon r}$

Ηλεκτρικό δίπολο : $\Phi_1 = \frac{\vec{r}\vec{p}}{4\pi\varepsilon r^2}$ , $\vec{p} = \int \vec{r} dq$