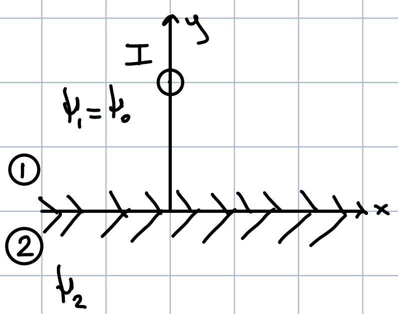
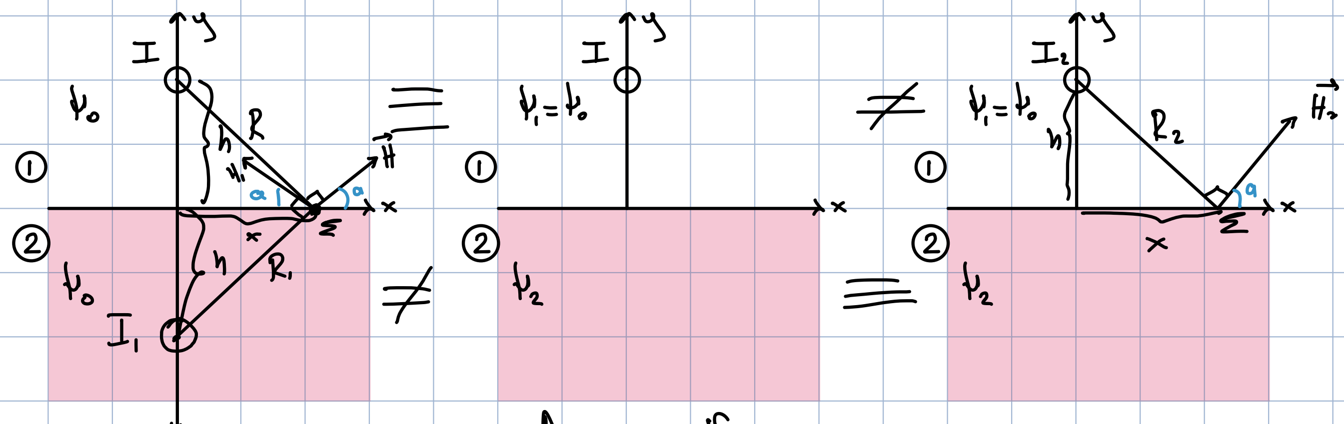
Άσκηση 12.5



β) Με την μέθοδο των ειδώλων:



Ισοδύναμο για περιοχή 1 — Αρχικό σχέδιο — Ισοδύναμο για περιοχή 2

(το H_1 είναι κάθετο στο R_1 και το H κάθετο στο R)

$$I_1=\frac{\mu_2-\mu_1}{\mu_2+\mu_1}I \quad (1)$$

$$I_2=\frac{2\mu_1}{\mu_2+\mu_1}I \quad (2)$$

Για $y=0_+$ (περιοχή 1):

$$\vec{H}_{1,ολ}=\left(\frac{I}{2\pi R}\cdot\underbrace{\cos a}_{=\left(\frac{h}{R}\right)}-\frac{I_1}{2\pi R}\cdot\frac{h}{R}\right)\hat{x}+\left(\frac{I}{2\pi R}\cdot\frac{x}{R}+\frac{I_1}{2\pi R}\cdot\frac{x}{R}\right)\hat{y} \quad (3)$$

$(R=R_1)$

Για $y=0_-$ (περιοχή 2):

$$\vec{H}_2=\left(\frac{I_2}{2\pi R}\cdot\frac{h}{R}\right)\hat{x}+\left(\frac{I_2}{2\pi R}\cdot\frac{x}{R}\right)\hat{y} \quad (4),$$

Οριακή συνθήκη: Για $y=0$: $\hat{y}\times(\vec{B}_{1,ολ}-\vec{B}_2)=\mu_0(\cancel{\vec{K}}+\vec{K}_M)$ $(\hat{y}\times\hat{y}=0,\ \hat{y}\times\hat{x}=-\hat{z})$

άρα $$\vec{K}_M(y=0)=\frac{1}{\mu_0}\left(B_{2x}-B_{1,ολ,x}\right)\hat{z}=\frac{1}{\mu_0}\left[\frac{\mu_2 I_2 h}{2\pi R^2}-\frac{\mu_1(I-I_1)h}{2\pi R^2}\right]\hat{z}$$

Αντικαθιστούμε (1), (2) και $R^2=h^2+x^2$

$$\vec{K}_M(y=0)=\hat{z}\cdot\frac{\cancel{\mu_0}Ih(\mu_2-\mu_0)}{\pi\cancel{\mu_0}(\mu_0+\mu_2)(h^2+x^2)} \quad (5)$$

$$\sigma_M(y=0) = \hat{y}\cdot\left(\mu_0 \vec{H}_{1,ow} - \mu_0 \vec{H}_2\right) = \frac{\mu_0 (I+I_1)x}{2\pi R^2} - \frac{\mu_0 I_2 \cdot x}{2\pi R^2} = (1),(2) = \frac{\mu_0(\mu_2-\mu_0) I \cdot x}{\pi(\mu_0+\mu_2)(x^2+h^2)} \quad (6)$$

a) $\mu_2 \to \infty$: (1) $I_1 \to I$

(2) $I_2 \to 0$

(5) $\vec{K}_M(y=0) \to \hat{z}\frac{Ih}{\pi(x^2+h^2)}$

(6) $\sigma_M(y=0) \to \frac{\mu_0 I x}{\pi(x^2+h^2)}$

Κεφάλαιο 13

Λύση της εξ. Laplace με χωρισμό μεταβλητών

Καρτεσιανές συντεταγμένες:

$$\nabla^2 \varphi = 0 \quad (\text{για } \rho = 0)$$

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad , \quad \varphi = \varphi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2}$$

$$\frac{\nabla^2 \varphi}{\varphi(=XYZ)} = \underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2}}_{k_x} + \underbrace{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{k_y} + \underbrace{\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}}_{k_z} = 0$$

k_x, k_y, k_z : σταθερές διαχωρισμού

Άρα $k_x + k_y + k_z = 0$

$$\frac{1}{X}\frac{d^2 X}{dx^2} = k_x \quad , \quad \frac{1}{Y}\frac{d^2 Y}{dy^2} = k_y \quad , \quad \frac{1}{Z}\frac{d^2 Z}{dz^2} = k_z$$

Λύνουμε $\frac{1}{X}\frac{d^2X}{dx^2}=K_x$ και τα άλλα δύο έχουν ίδια λύση

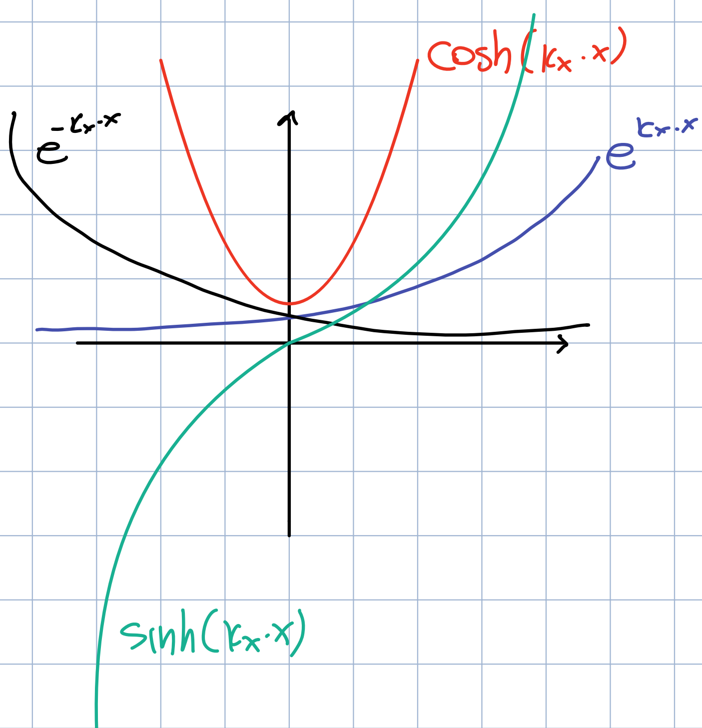
i) $K_x=0 \Rightarrow X=A_1x+A_2$

ii) $K_x=-k_x^2<0$, $\cos(k_x\cdot x)$, $\sin(k_x\cdot x)$, $e^{jk_x\cdot x}$, $e^{-jk_x x}$

Πχ
$$\left\{\begin{array}{l} X=A_1'\cos(k_x\cdot x)+A_2'\sin(k_x\cdot x) \\ X=A_1''e^{jk_x\cdot x}+A_2''e^{-jk_x\cdot x}\end{array}\right.$$

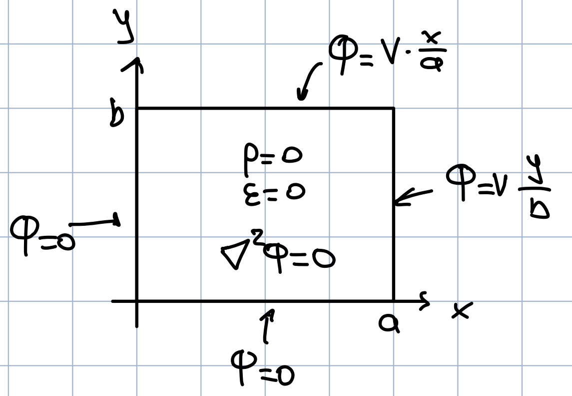
iii) $K_x=k_x^2>0$, $\cosh(k_x\cdot x)$, $\sinh(k_x\cdot x)$, $e^{k_x\cdot x}$, $e^{-k_x\cdot x}$

Πχ.
$$\left\{\begin{array}{l} X=A_1'''\cosh(k_x\cdot x) \\ \quad +A_2'''\sinh(k_x\cdot x)\end{array}\right.$$



Παραδείγματα (Δισδιάστατα προβλήματα)

1)



$$\frac{d^2\varphi}{dz^2}=0 \Rightarrow k_z=0$$

$$\boxed{k_x+k_y=0}$$

$$\varphi(x,y)=X(x)\,Y(y)$$

$$\varphi(x,y)=(A_1x+A_2)(B_1y+B_2)$$

Οριακές σχέσεις:

$x=0 \qquad \varphi=0 \quad \forall y,\ 0<y<b \Rightarrow A_2=0$

$y=0 \qquad \varphi=0 \quad \forall x,\ 0<x<a \Rightarrow B_2=0$

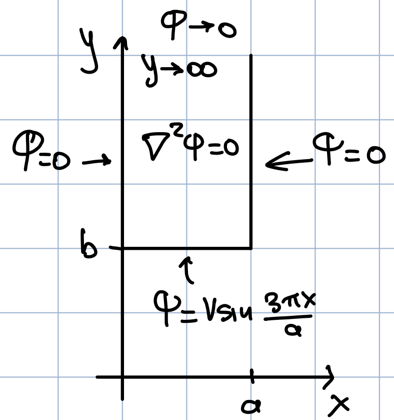
Άρα $\quad \varphi=\underbrace{A_1B_1}_{C_1}xy = C_1xy$

για $\quad x=a \qquad \varphi = C_1ay = V\frac{y}{b}$

$$C_1=\frac{V}{ab}$$

$$\varphi = V\frac{x\cdot y}{ab}$$

2)



$$K_x = -(k_x)^2 = -\left(\frac{3\pi}{a}\right)^2 < 0$$

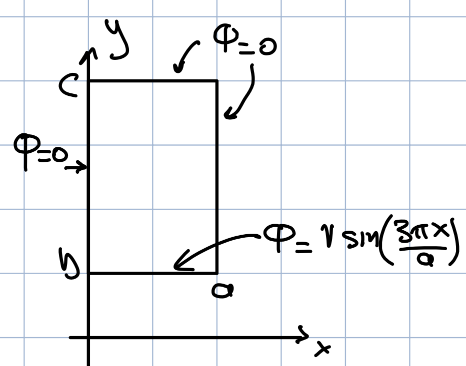
$$K_y = -K_x = \left(\frac{3\pi}{a}\right)^2 = k_y^2 > 0$$

$$\left.\begin{aligned} X(x) &= A \sin\left(\frac{3\pi x}{a}\right) \\ Y(y) &= B e^{-\frac{3\pi}{a} y} \end{aligned}\right\} \Phi(x,y) = C \sin\left(\frac{3\pi x}{a}\right) e^{-\frac{3\pi}{a} y}$$

$y = b$ $\quad \Phi(x,b) = C \sin\left(\frac{3\pi x}{a}\right) \cdot e^{-\frac{3\pi b}{a}} = V \sin\left(\frac{3\pi x}{a}\right)$

$$C = V \cdot e^{\frac{3\pi b}{a}} \Rightarrow \Phi(x,y) = V \sin\left(\frac{3\pi x}{a}\right) e^{-\frac{3\pi}{a}(y-b)}$$

3)



$$X(x) = A \sin\left(\frac{3\pi x}{a}\right)$$

$$K_x = -k_x^2 = -\left(\frac{3\pi}{a}\right)^2 < 0$$

$$K_y = -K_x = \left(\frac{3\pi}{a}\right)^2 = k_y^2 > 0$$

αφού μηδενίζεται στο c

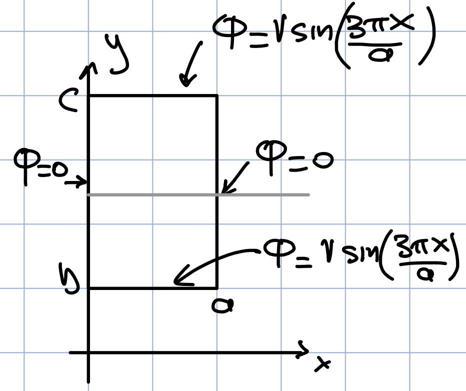
$$\Phi = A \sin\left(\frac{3\pi x}{a}\right) B \sinh\left[\frac{3\pi}{a}(y-c)\right]$$

για $y = b$ $\quad \Phi(x,b) = C \cancel{\sin\left(\frac{3\pi x}{a}\right)} \sinh\left(\frac{3\pi}{a}(b-c)\right) = V \cancel{\sin \frac{3\pi x}{a}}$

$$C = \frac{V}{\sinh\left(\frac{3\pi}{a}(b-c)\right)}$$

$$\Phi = V \sin\left(\frac{3\pi x}{a}\right) \frac{\sinh\left[\frac{3\pi}{a}(y-c)\right]}{\sinh\left[\frac{3\pi}{a}(b-c)\right]}$$

4)



Αφου εχουμε συμμετρια

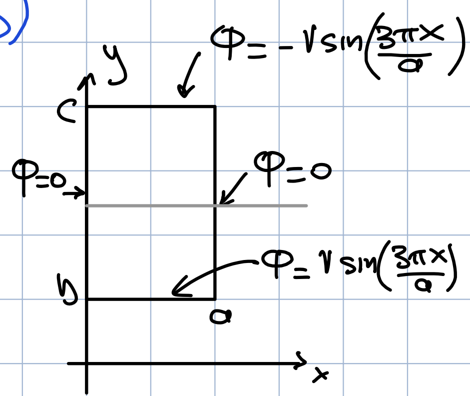
$$\Phi = C\sin\left(\frac{3\pi x}{a}\right)\cdot\cosh\left[\frac{3\pi}{a}\left(y-\frac{b+c}{2}\right)\right]$$

για $y=b$: $\Phi = C\cdot\cancel{\sin\left(\frac{3\pi x}{a}\right)}\cdot\cosh\left[\frac{3\pi}{a}\left(b-\frac{b+c}{2}\right)\right] = V\cancel{\sin\left(\frac{3\pi x}{a}\right)}$

$$C = \frac{V}{\cosh\left[\frac{3\pi}{a}\left(b-\frac{b+c}{2}\right)\right]}$$

$$\Phi = V\sin\left(\frac{3\pi x}{a}\right)\frac{\cosh\left[\frac{3\pi}{a}\left(y-\frac{b+c}{2}\right)\right]}{\cosh\left[\frac{3\pi}{a}\,\frac{b-c}{2}\right]}$$

5)



Αφου εχουμε αντισυμμετρια

$$\Phi = C\sin\left(\frac{3\pi x}{a}\right)\cdot\sinh\left[\frac{3\pi}{a}\left(y-\frac{b+c}{2}\right)\right]$$