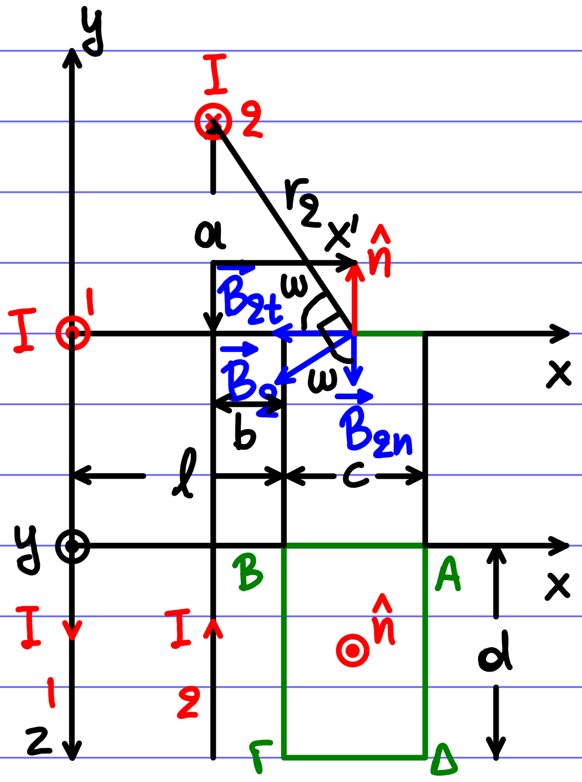
02/12/2022

Άσκηση 11.1



α) $\vec{B}_1 = \frac{\mu_0 I}{2\pi x}\hat{y}$, $B_2 = \frac{\mu_0 I}{2\pi r_2}$

$$\Psi_1 = \iint_S \vec{B}_1 \cdot d\vec{S} = \frac{\mu_0 I d}{2\pi}\int_{\ell}^{\ell+c}\frac{dx}{x} = \frac{\mu_0 I d}{2\pi}\ln\left(\frac{\ell+c}{\ell}\right)$$

$$\Psi_2 = -\iint_S B_{2n}\, dS = -\frac{\mu_0 I d}{2\pi}\int_{\ell}^{\ell+c}\frac{\cos\omega}{r_2}dx =$$

$$= -\frac{\mu_0 I d}{2\pi}\int_b^{b+c}\frac{x'}{r_2^2}dx' = -\frac{\mu_0 I d}{2\pi}\int_b^{b+c}\frac{x'}{x'^2+a^2}dx' =$$

$$= -\frac{\mu_0 I d}{4\pi}\int_b^{b+c}\frac{d(x'^2+a^2)}{x'^2+a^2} = -\frac{\mu_0 I d}{4\pi}\Big[\ln(x'^2+a^2)\Big]_b^{b+c} =$$

$$= -\frac{\mu_0 I d}{4\pi}\ln\left[\frac{(b+c)^2+a^2}{b^2+a^2}\right]$$

$$\Psi_m = \Psi_1 + \Psi_2 = \frac{\mu_0 I d}{4\pi}\ln\left[\frac{(\ell+c)^2\,(b^2+a^2)}{\ell^2\left[(b+c)^2+a^2\right]}\right]$$

β) $\vec{A} = A_z\hat{z}$

$$A_z = -\frac{\mu_0 I}{2\pi}\ln(r_1) + \frac{\mu_0 I}{2\pi}\ln(r_2) + k = \frac{\mu_0 I}{2\pi}\ln\left(\frac{r_2}{r_1}\right) + k$$

$$\Psi_m = \oint_C \vec{A}\cdot d\vec{\ell} = \int_{AB}\vec{A}\cdot\hat{x}\,dx + \int_{B\Gamma}\vec{A}\cdot\hat{z}\,dz + \int_{\Gamma\Delta}\vec{A}\cdot\hat{x}\,dx + \int_{\Delta A}\vec{A}\cdot\hat{z}\,dz =$$

$$= 0 + \int_0^d \frac{\mu_0 I}{2\pi}\ln\left(\frac{r_2}{r_1}\right)dz + kd + 0 + \int_d^0\frac{\mu_0 I}{2\pi}\ln\left(\frac{r_2}{r_1}\right)dz - kd =$$

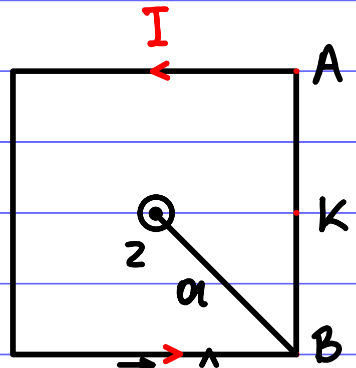
$$= \int_0^d\frac{\mu_0 I}{2\pi}\ln\left(\frac{\sqrt{b^2+a^2}}{\ell}\right)dz + \int_d^0\frac{\mu_0 I}{2\pi}\ln\left(\frac{\sqrt{(b+c)^2+a^2}}{\ell+c}\right)dz$$

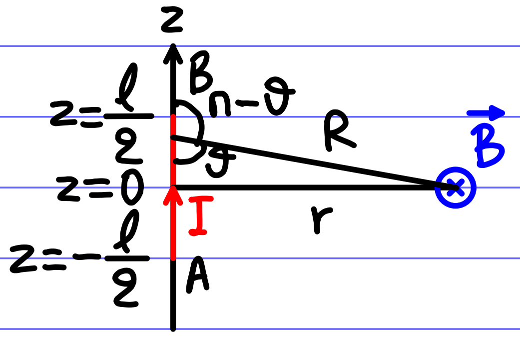
$$= \frac{\mu_0 I d}{4\pi}\ln\left(\frac{b^2+a^2}{\ell^2}\right) - \frac{\mu_0 I d}{4\pi}\ln\left[\frac{(b+c)^2+a^2}{(\ell+c)^2}\right] \Rightarrow$$

$$\Rightarrow \Psi_m = \frac{\mu_0 I d}{4\pi}\ln\left[\frac{(\ell+c)^2\,(b^2+a^2)}{\ell^2\left[(b+c)^2+a^2\right]}\right]$$

γ) $\Psi_m = 0 \Rightarrow \frac{(\ell+c)\sqrt{b^2+a^2}}{\ell\sqrt{(b+c)^2+a^2}} = 1 \Rightarrow \ell = \frac{c\sqrt{b^2+a^2}}{\sqrt{(b+c)^2+a^2}-\sqrt{b^2+a^2}}$

Άσκηση 11.2



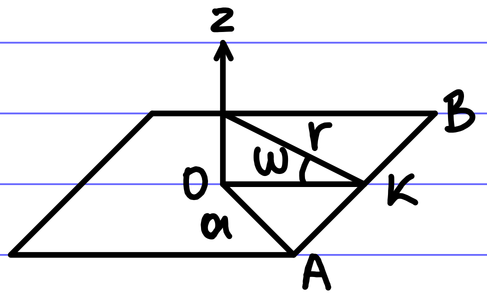


$\sin\theta = \frac{r}{R}$

$$d\vec{H} = \frac{I\,d\vec{l}'\times\hat{R}}{4\pi R^2} = \frac{I\,dz'\sin(\pi-\theta)}{4\pi R^2}\hat{\varphi} = \frac{I\,dz'\sin\theta}{4\pi R^2}\hat{\varphi} = \frac{I r\,dz'}{4\pi R^3}\hat{\varphi} =$$

$$= \frac{I r\,dz'}{4\pi(z'^2+r^2)^{3/2}}\hat{\varphi} \Rightarrow \vec{H} = \hat{\varphi}\int_{-l/2}^{l/2}\frac{Ir}{4\pi(z'^2+r^2)^{3/2}}dz' =$$

$$= \hat{\varphi}\frac{Ir}{4\pi}\left[\frac{z'}{r^2\sqrt{z'^2+r^2}}\right]_{-l/2}^{l/2} = \hat{\varphi}\frac{I}{4\pi r}\cdot\frac{l}{\sqrt{\frac{l^2}{4}+r^2}} = \hat{\varphi}\frac{Il}{4\pi r(\Sigma A)}$$



$$H_{z,AB} = \frac{Il}{4\pi r(\Sigma A)}\cos\omega = \frac{Il}{4\pi r(\Sigma A)}\cdot\frac{(OK)}{r} =$$

$$= \frac{Il}{4\pi r^2}\,\frac{\sqrt{a^2-\frac{l^2}{4}}}{\sqrt{z^2+a^2}} =$$

$$= \frac{Il}{4\pi}\cdot\frac{\sqrt{a^2-\frac{l^2}{4}}}{(z^2+(OK)^2)\sqrt{z^2+a^2}} \Rightarrow$$

$$\Rightarrow H_z = N H_{z,AB} = \frac{NIl\sqrt{a^2-\frac{l^2}{4}}}{4\pi\left(z^2+a^2-\frac{l^2}{4}\right)\cdot\sqrt{z^2+a^2}}$$

Για $N\to+\infty$: $l\to 0$ και $Nl\to 2\pi a$

$$H_z = \frac{2\pi a I\sqrt{a^2-\frac{0^2}{4}}}{4\pi\left(z^2+a^2-\frac{0^2}{4}\right)\cdot\sqrt{z^2+a^2}} \Rightarrow$$

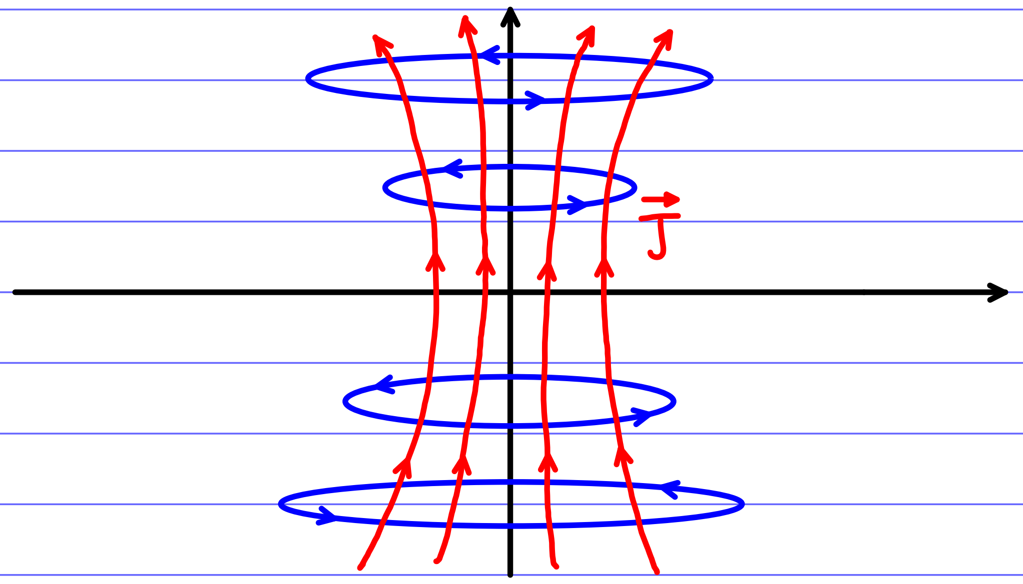
$$\Rightarrow H_z = \frac{Ia^2}{2(z^2+a^2)^{3/2}}$$

Άσκηση 11.3

$$\frac{\partial}{\partial\varphi}=0$$

α) $J_\varphi=0 \ (K_\varphi=0) \Rightarrow H_r=H_z=0$

$$\frac{\partial}{\partial\varphi}=0\,,\ J_\varphi=0\,,\ d\vec{A}=\frac{\mu\vec{J}dV'}{4\pi}\Rightarrow\vec{A}=A_r(r,z)\hat{r}+A_z(r,z)\hat{z}$$



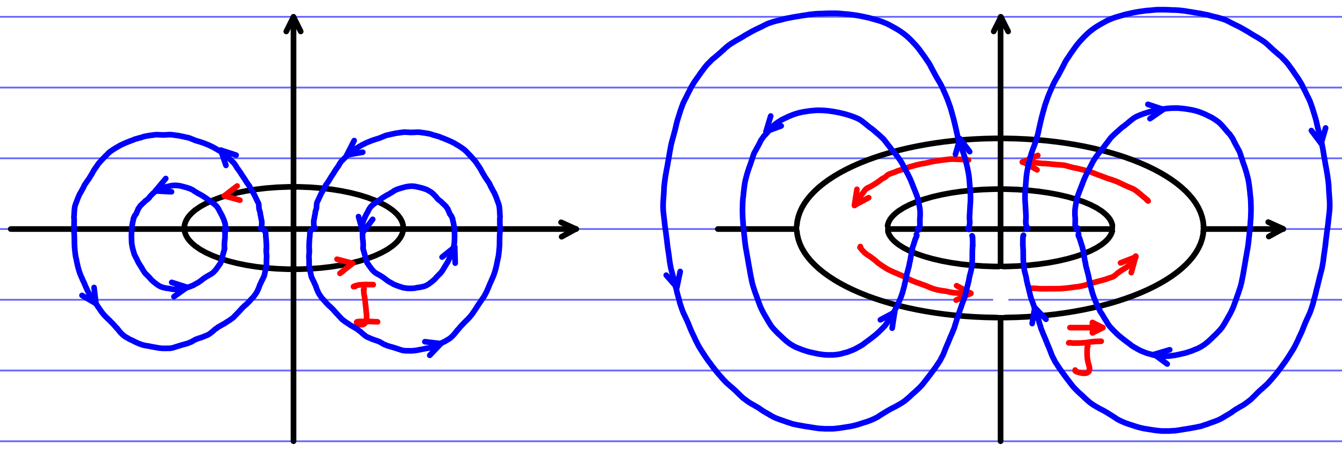
$$\vec{H}=\frac{1}{\mu}\vec{\nabla}\times\vec{A}=$$

$$=\hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\varphi}-\frac{\partial A_\varphi}{\partial z}\right)-\hat{\varphi}\left(\frac{\partial A_z}{\partial r}-\frac{\partial A_r}{\partial z}\right)+\hat{z}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)-\frac{1}{r}\frac{\partial A_r}{\partial\varphi}\right)\Rightarrow$$

$$\Rightarrow\vec{H}=-\hat{\varphi}\left(\frac{\partial A_z}{\partial r}-\frac{\partial A_r}{\partial z}\right)\Rightarrow\vec{H}=H_\varphi(r,z)\hat{\varphi}$$

β) $J_r=J_z=0 \ (K_r=K_z=0)\Rightarrow H_\varphi=0$

$$\frac{\partial}{\partial\varphi}=0\,,\ J_r=J_z=0\,,\ d\vec{A}=\frac{\mu\vec{J}dV'}{4\pi}\Rightarrow\vec{A}=A_\varphi(r,z)\hat{\varphi}$$



$$\vec{H}=\frac{1}{\mu}\vec{\nabla}\times\vec{A}=$$

$$=\hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\varphi}-\frac{\partial A_\varphi}{\partial z}\right)-\hat{\varphi}\left(\frac{\partial A_z}{\partial r}-\frac{\partial A_r}{\partial z}\right)+\hat{z}\left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)-\frac{1}{r}\frac{\partial A_r}{\partial\varphi}\right)\Rightarrow$$

$$\Rightarrow\vec{H}=-\frac{\partial A_\varphi}{\partial z}\hat{r}+\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\hat{z}\Rightarrow\vec{H}=H_r(r,z)\hat{r}+H_z(r,z)\hat{z}$$