Generalized Planck's postulate: *Any object with 1 degree of freedom, the parameter of which is a sinusoidal function of time (a harmonic oscillator), may only have such total energy that satisfies:*

$$E = nh\nu, \quad n = 0, 1, 2, 3, \ldots$$

(quantization of energy)

Problem 01a: Pendulum

h = 6.626 x 10^{-34} [Js]

$\hbar = \frac{h}{2\pi} = 1.055x10^{-34}$ [Js]

m_0 = 9.11 x 10^{-31} [kg]

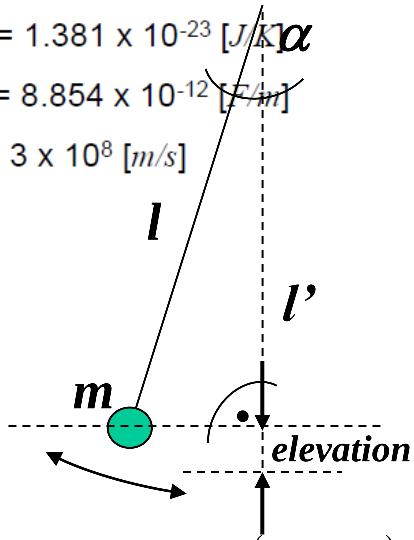
q = 1.602 x 10^{-19} [C]

k_B = 1.381 x 10^{-23} [J/K]

ε_0 = 8.854 x 10^{-12} [F/m]

c = 3 x 10^8 [m/s]

So, let's consider a pendulum consisting of a weight of a mass ***m*** **= 1 kg**, hanging on a strand of a length ***l*** = ***0.5 m***. Assume that the amplitude of the oscillations is such that at the highest position, the strand makes an angle ***α*** = ***0.5 rad*** with the vertical. The energy of the pendulum is decreasing, e.g., due to friction.



Is the energy decreasing continuously or discretely? Consider the ratio: $\frac{\Delta E}{E}$

$elev = l - l' = l \cdot (1 - \cos\alpha)$

= 0.061 m

Pendulum frequency:
gravity ***g*** = **9.8 m/s²**

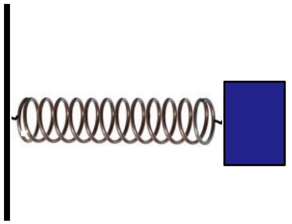
$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,[\frac{m}{s^2}]}{0.5\,[m]}} = 0.70\,[Hz]$$

$$\Delta E = h\nu = 6.626 \cdot 10^{-34}\,[Js] \cdot 0.70\,[Hz] = 4.67 \cdot 10^{-34}\,[J]$$

Total energy: $$E = m \cdot g \cdot elev = 1.0\,[kg] \cdot 9.8\,[\tfrac{m}{s^2}] \cdot 0.5\,[m] \cdot (1 - \cos\alpha) = 6.0 \cdot 10^{-1}\,[J]$$

The ratio: $$\frac{\Delta E}{E} = 7.78 \cdot 10^{-34}$$

Problem 01b: Micro spring

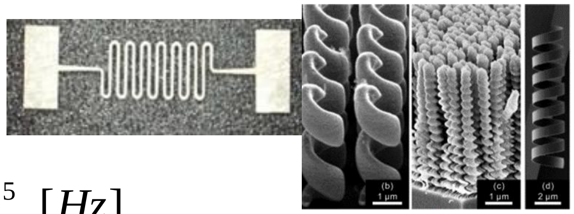


Consider a mass $\boldsymbol{m} = 10^{-7}$ **kg** oscillating on a micro spring of a spring constant $\boldsymbol{k} = 10^{6}\ N/m$. Assume the maximum deflection of the spring to be $\Delta\mathbf{x} = 10^{-7}$ **m.**

Is its energy decreasing continuously or discretely?

Consider the ratio: $\frac{\Delta E}{E}$

MEMS structures:



Pendulum frequency: $$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10^{6}\,[\frac{N}{m}]}{10^{-7}\,[kg]}} = 5.03\cdot 10^{5}\ [Hz]$$

$$\Delta E = h\nu = 6.626\cdot 10^{-34}\,[Js]\cdot 5.03\cdot 10^{5}\,[Hz] = 3.33\cdot 10^{-28}\,[J] = 2.08\cdot 10^{-9}\,[eV]$$

Max spring potential energy: $$E = \frac{1}{2}\cdot k\cdot \Delta x^{2} \qquad E = 0.5\cdot 10^{6}\,[\tfrac{N}{m}]\cdot\left(10^{-7}\,[m]\right)^{2} = 5.0\cdot 10^{-9}\,[J] = 3.12\cdot 10^{10}\,[eV]$$

The ratio: $$\frac{\Delta E}{E} = 6.67\cdot 10^{-20}$$