## SAMPLE TEST - PROBLEMS

1. There are four dice in a drawer: one tetrahedron (4 sides with numbers from 1 to 4), one hexahedron (6 sides with numbers form 1 to 6) and two octahedra (8 sides with numbers from 1 to 8). You pick one dice at random and you roll it.
   (a) What is the probability that 3 comes up on the chosen dice?
   (b) Given that the result of the roll is 3, find the probability that the chosen dice has 8 sides.
2. There is an urn with two black and three white balls, and a dice. We roll the dice once. We repeat the following steps that many times as the number on the dice indicates: we draw a ball from the urn, note its color aside, and put the ball back into the urn. Compute the probability that we had got "five" on the dice, if there were exactly three occurences of black balls noted on the piece of paper.
3. There are two urns: the one with 1 white and 4 black balls and the other one with 1 black and 4 white balls. We pick one urn at random and draw three balls out of it (after we draw ba ball, we return it back to the urn). It turnes out that three drawn balls are black. What is the probability that the balls were taken from the first urn?
4. There are 10 coins and 2 of them have tails on both sides. We pick one coin at random and we toss it three times. Let $X$ be a number of tails obtained in three tosses. Determine the distribution of the random variable $X$.
5. We filp a biased coin until a head comes up. At each flip $H$ is twice as likely as $T$. Compute the probability that
   (a) we will make at least 5 tosses,
   (b) we will make an even number of tosses.
6. Assume that $\Omega = [-1,1] \times [-1,1]$ and $P$ stands for geometrical probability on $\Omega$. Find the distribution function of the random variable $X(x,y) = \max(x,y)$.
7. Assume that $\Omega = [-1,1] \times [0,1]$ and $\mathbb{P}$ stands for geometrical probability on $\Omega$. Find the distribution of the random variable $X(x,y) = \min(|x|, y)$.
8. Random variable $X$ has a continuous distribution with the density:
$$f(x) = \begin{cases} -\frac{1}{3}x, & x \in (-2,0), \\ ax^2, & x \in (0,1), \\ 0, & \text{otherwise.} \end{cases}$$
   (a) Determine $a$.
   (b) Determine the cumulative distribution function of $X$.
9. Random variable $X$ has a normal distribution $\mathcal{N}(-2,1)$.
   (a) Find the value of $\mathbb{P}(X < -2)$.
   (b) Compute $\mathbb{P}(X > -5) - \mathbb{P}(X < 1)$.
10. The cumulative distribution function of a random variable $X$ is given by
$$F_X(t) = \begin{cases} 0, & t < -2, \\ 1/8, & -2 \le t < -1, \\ 1/2, & -1 \le t < 1, \\ 2/3, & 1 \le t < 3, \\ 1, & t \ge 3. \end{cases}$$

(a) Determine probability mass function (PMF) of $X$.

(b) Compute $\mathbb{P}(X(X+1) > 0)$.

11. A box contains 4 red and 5 green balls. Two balls are selected at random and discarded without their colors being seen. If a third ball is drawn randomly and observed to be red, what is the probability that both discarded balls were green?

12. The weight of any person in a group of people is described (in kgs) by the normal distribution $\mathcal{N}(\mu, 25)$.

(a) Determine $\mu$ given that there are equal chances of choosing a person that weighs less than 70 kgs and a person that weighs more than 70 kgs.

(b) Which probability is larger: a randomly picked person weighs at least 80 kgs or a randomly chosen person weighs less then 60?