#38: Krzysztof Watras (318339)

Numerical Methods (ENUME) – Project
Assignment B: Nonlinear algebraic equations

1. Use MATLAB's function ***fzero*** to solve the equation:

$$f(x) = 0 \tag{1}$$

with:

$$f(x) \equiv 2.5\,[\cos(-x/7 - 1.5)]^3 - 0.01\,(x/3)^3 + 2, \quad x \in [1, 10]$$

Plot the function $f(x)$ for $x \in [1, 10]$. Mark the horizontal line corresponding to $y = 0$ and the point corresponding to the solution of Eq. (1).

2. Write a MATLAB function for solving nonlinear algebraic equations using the bisection method. The function should have the following input arguments:

- a *function_handle*, representing the function $f(x)$ defining the equation to be solved;
- a two-element vector $[x_{\min}, x_{\max}]$, representing the boundaries of the interval of x in which the solution is to be sought;
- a scalar, representing the maximum acceptable magnitude Δ of the absolute error of the solution.

The function should return a vector containing the approximations of the solution obtained in consecutive iterations. Test the function by using it for solving Eq. (1) with $\Delta = 10^{-12}$.

3. Write three more MATLAB functions for solving nonlinear algebraic equations using:

- the secant method,
- the Newton's method,
- version II of Muller's method.

The syntax of these functions should be analogous to that of the function from Task #2. In the case of secant method, the values $x_0 = x_{\min}$ and $x_1 = x_{\max}$ should be the starting points. In the case of the Newton's method, the value $x_0 = x_{\max}$ should be the starting point. In the case of the Muller's method, the values $x_0 = x_{\min}$, $x_1 = (x_{\min} + x_{\max})/2$ and $x_2 = x_{\max}$ should be the starting points. Test the functions by using them for solving Eq. (1) with $\Delta = 10^{-12}$.

4. For all four methods implemented in Tasks #2 and #3, plot the absolute errors of the approximations of the solution, obtained in consecutive iterations for $\Delta = 10^{-12}$. Determine those errors by taking the solution obtained using ***fzero*** as reference.

5. For all four methods implemented in Tasks #2 and #3, plot the number of iterations performed before reaching the acceptable accuracy of the solution for $\Delta \in [10^{-15}, 10^{-1}]$.

6. Determine the number of iterations necessary to reach the acceptable accuracy of the solution using the bisection method with $\Delta \in [10^{-15}, 10^{-1}]$ according to the formula presented on the lecture slide #4-13. Compare the results with those obtained in Task #5.

7. Plot and compare the number of iterations necessary to reach the acceptable accuracy of the solution using the Newton's method with $\Delta \in [10^{-15}, 10^{-1}]$ for two different starting points: $x_0 = x_{\min}$ and $x_0 = x_{\max}$.