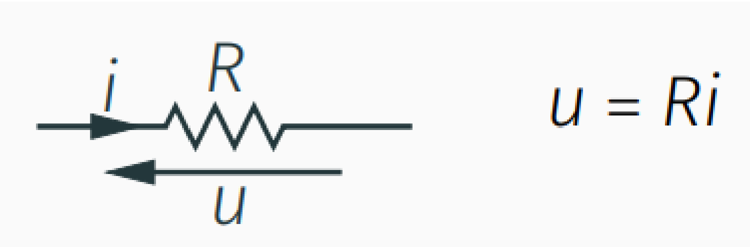
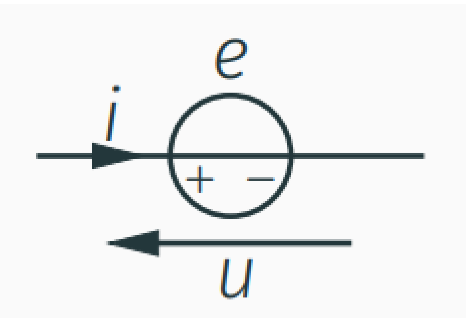
Summary of what we learned

Resistor



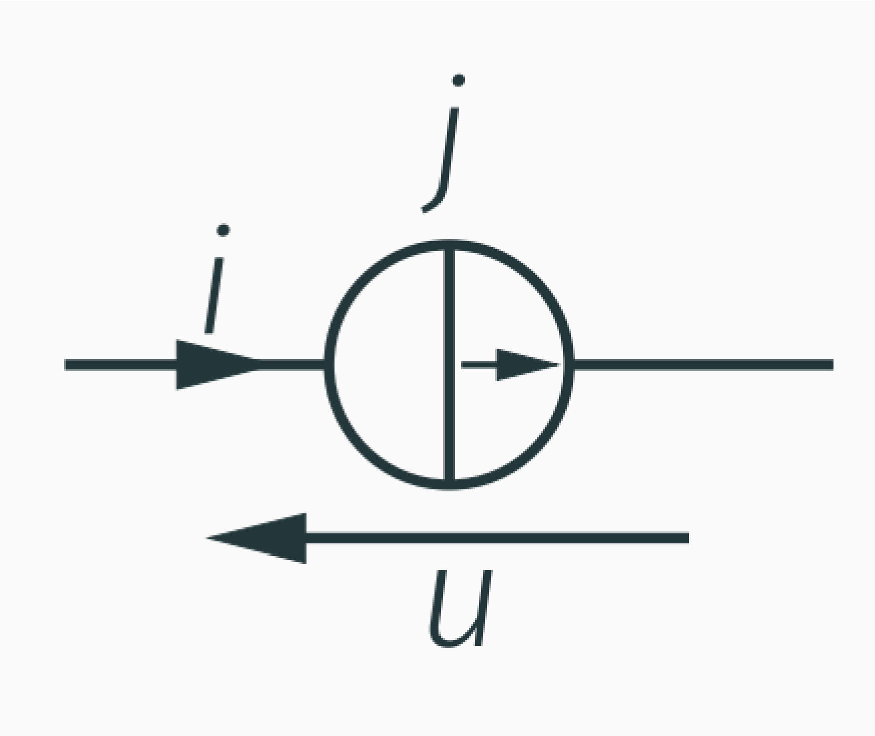
Arrow phrases matter! The usual $u = Ri$ is meaningless because when one of the arrows has the opposite direction, the formula is $u = -Ri$.

Voltage Source



And here $u = e$
Here $e[V]$ is called electromotive force.

Current Source

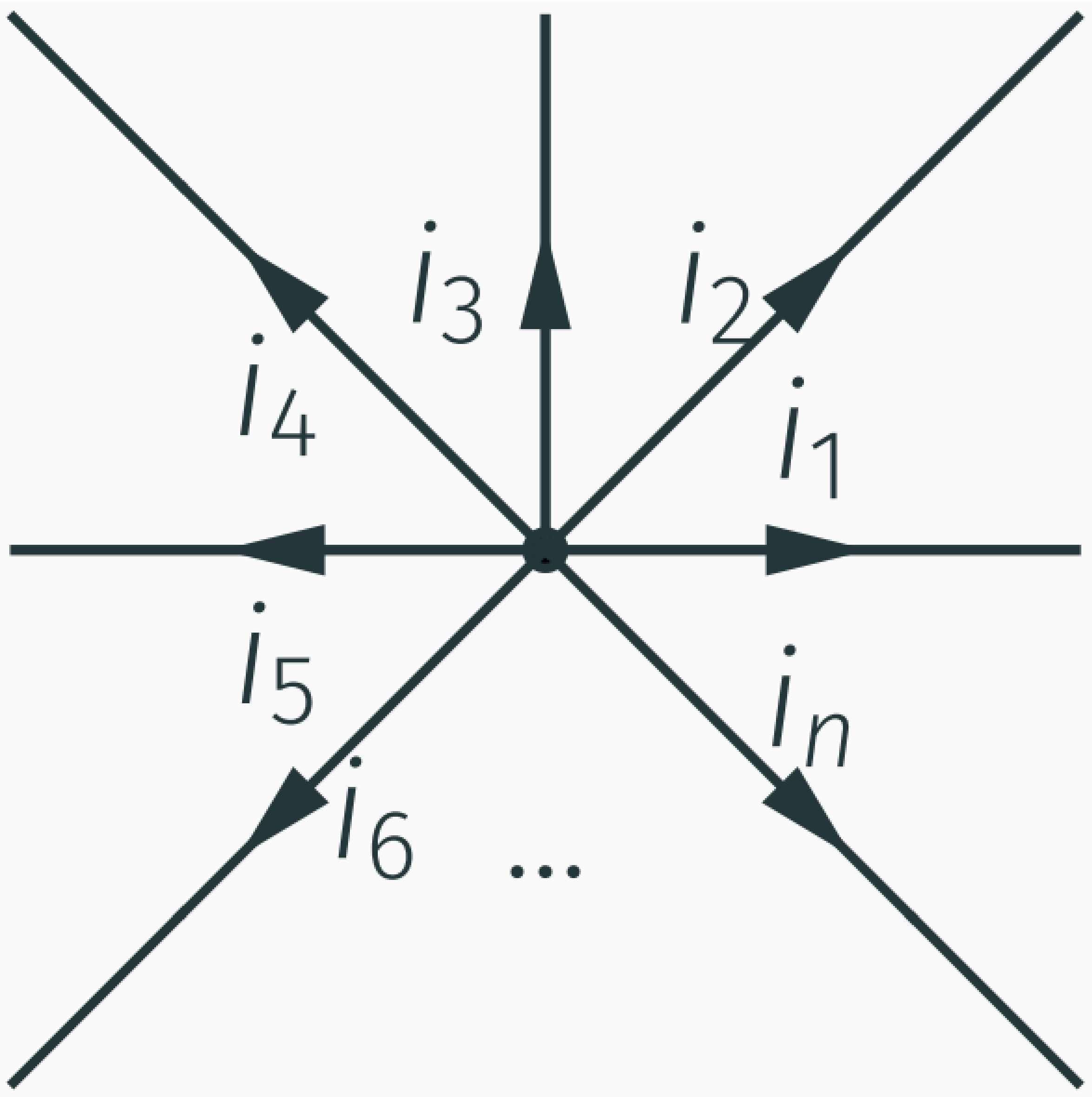


And here $i = j$.

Here $j[A]$ is the generated current

Kirchhoff's Current Law (KCL)

Given node like this:

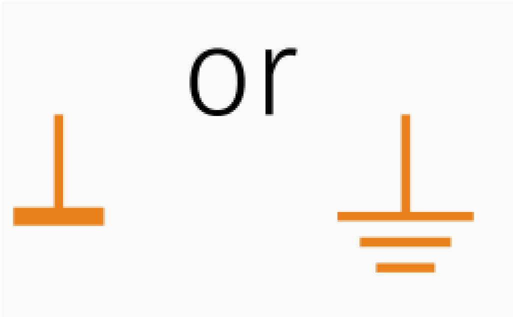


We have to note that: $i_1, i_2, i_3, \ldots, i_n = 0$.

Kirchhoff's Voltage Law (KVL)

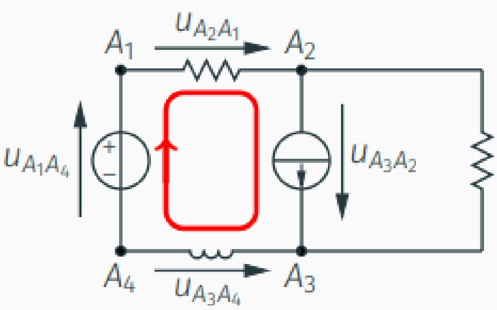
Electric potentials (with respect to any point) of the circuit's nodes are well defined.
The node with respect to which we measure

electric potentials is called ground or earth.



Kirchhoff's Voltage Law (KVL) — loop version

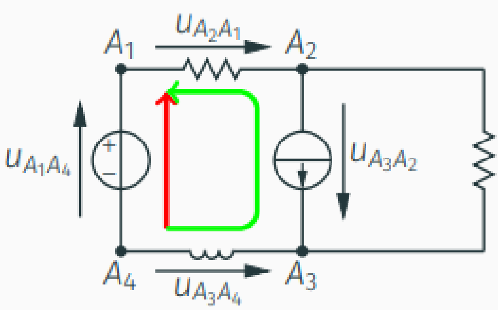
The algebraic sum of the voltage drops along any directed loop equals zero.



$$u_{A_1A_4} + u_{A_2A_1} + u_{A_3A_2} - u_{A_3A_4} = 0.$$

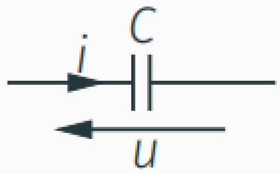
Kirchhoff's Voltage Law (KVL) — path version

The voltage between any two nodes does not depend on the (oriented) path along which it is computed.



$$u_{A_1A_4} = u_{A_3A_4} - u_{A_3A_2} - u_{A_2A_1}.$$

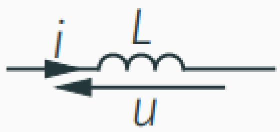
Capacitor



$$q = Cu, \quad \boxed{i = Cu'}.$$

C [F] is called capacitance.

Inductor



$$\psi = Li, \quad \boxed{u = Li'}.$$

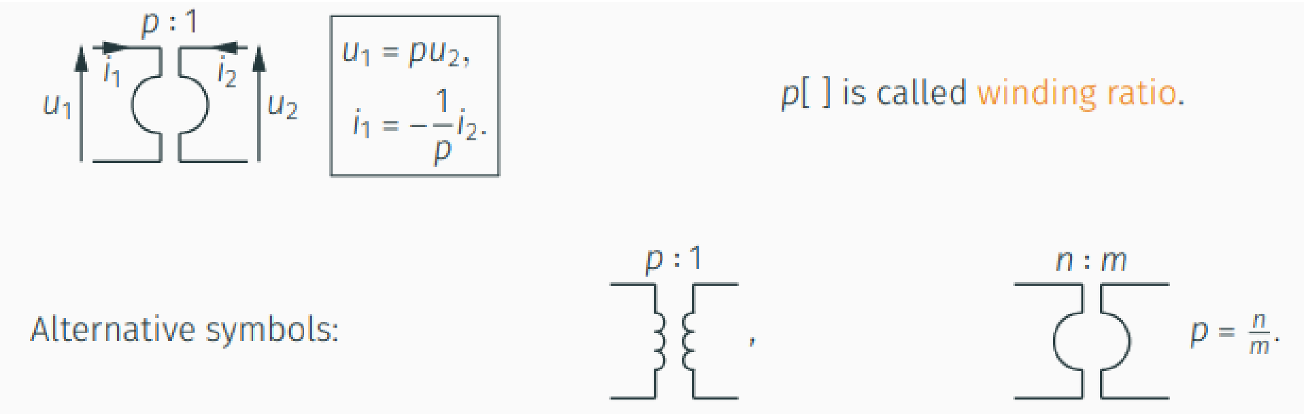
L [H] is called inductance.

Device port

A port of a device is a pair of device terminals such that any current flowing in through one terminal of a port must flow out through the oter terminal of the port.

Every 2-terimnal device is automatically a one-port!

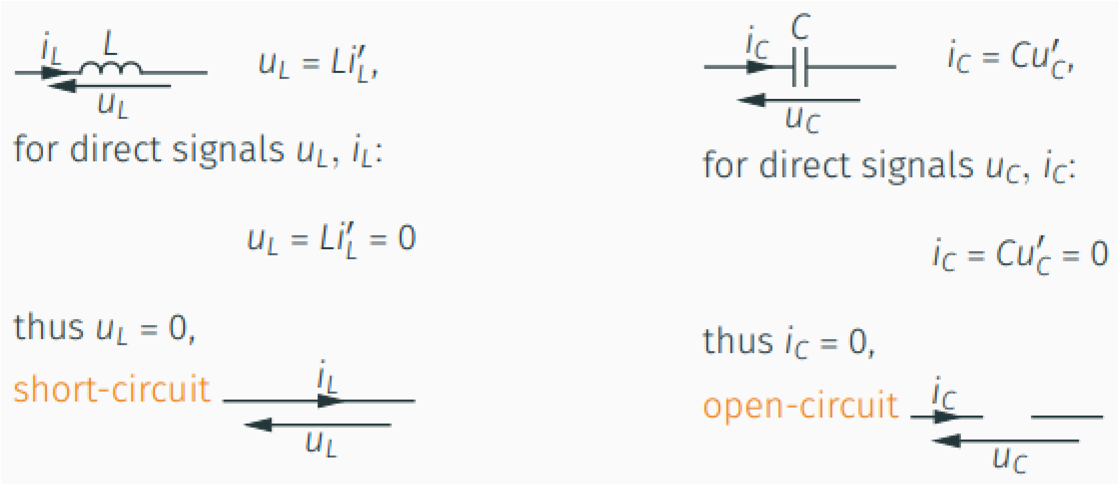
Transformers and DC-DC converters



DC analysis

A DC solution to a circuit is a solution consisting entirely of constant signals (all voltages and currents)

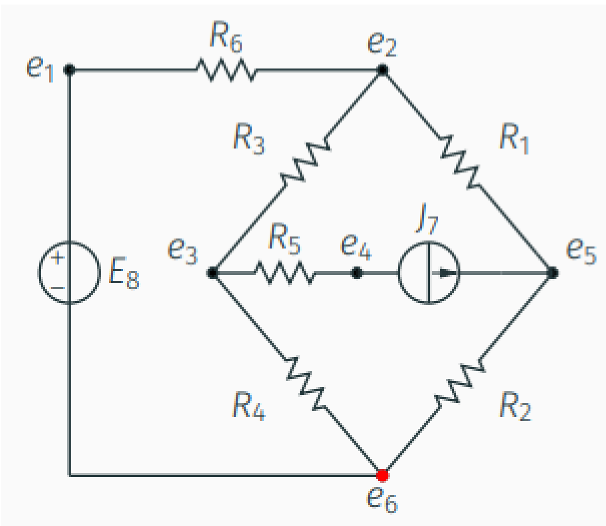
For DC all complicated ODEs normally ruling the circuits reduce to algebraic equations.



At our desposal we allways have Kirchhoff's laws (KCL, KVL) and device equations!

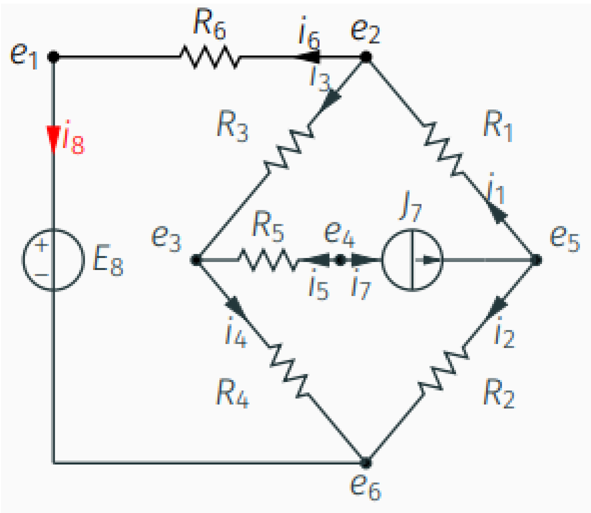
Nodal method

1. Label all the nodes with variables $e_1, e_2, \ldots, e_n$ denoting electric potentials



2. Labels all the currents $i_1, i_2, \ldots, i_n$ flowing into all the elements. For each n-terminal

element we introduce $n-1$ new variables in tis way.
The current flowing out of the last terminal of an n-terminal element equals the sum of the currents flowing into all the other terminals.



3. Write down the KVL equations (for each node)

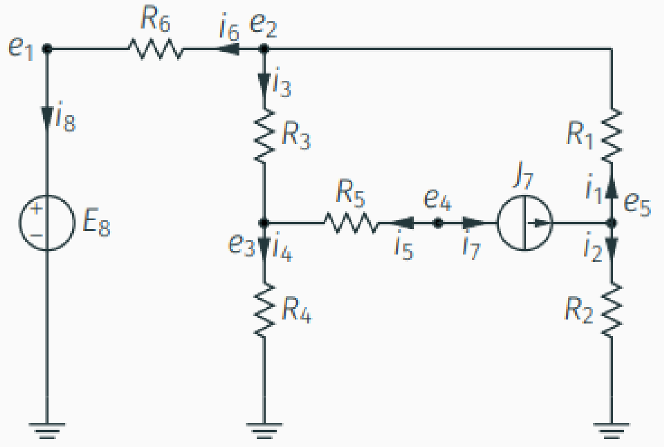
$$
\begin{aligned}
i_6 &= i_8, && (e_1)\\
i_1 &= i_3 + i_6, && (e_2)\\
i_3 + i_5 &= i_4, && (e_3)\\
0 &= i_5 + i_7, && (e_4)\\
i_7 &= i_1 + i_2, && (e_5)\\
i_2 + i_4 + i_8 &= 0. && (e_6)
\end{aligned}
$$

4. Write down element equations expressed in terms of the electric potentials $e_1, e_2, \ldots, e_n$ and currents $i_1, i_2, \ldots, i_n$.

$$
\begin{aligned}
e_5 - e_2 &= i_1 R_1, && (1)\\
e_5 - e_6 &= i_2 R_2, && (2)\\
e_2 - e_3 &= i_3 R_3, && (3)\\
e_3 - e_6 &= i_4 R_4, && (4)\\
e_4 - e_3 &= i_5 R_5, && (5)\\
e_2 - e_1 &= i_6 R_6, && (6)\\
i_7 &= J_7, && (7)\\
e_1 - e_6 &= E_8. && (8)
\end{aligned}
$$

5. Solve the equations

$i_6 = i_8,$	(e_1)	$e_5 - e_2 = i_1 R_1,$	(1)
$i_1 = i_3 + i_6,$	(e_2)	$e_5 - 0 = i_2 R_2,$	(2)
$i_3 + i_5 = i_4,$	(e_3)	$e_2 - e_3 = i_3 R_3,$	(3)
$0 = i_5 + i_7,$	(e_4)	$e_3 - 0 = i_4 R_4,$	(4)
$i_7 = i_1 + i_2,$	(e_5)	$e_4 - e_3 = i_5 R_5,$	(5)
$\cancel{i_2 + i_4 + i_8 = 0.}$	(e_6)	$e_2 - e_1 = i_6 R_6,$	(6)
		$i_7 = J_7,$	(7)
		$e_1 - 0 = E_8.$	(8)
		$e_6 = 0$	(9)



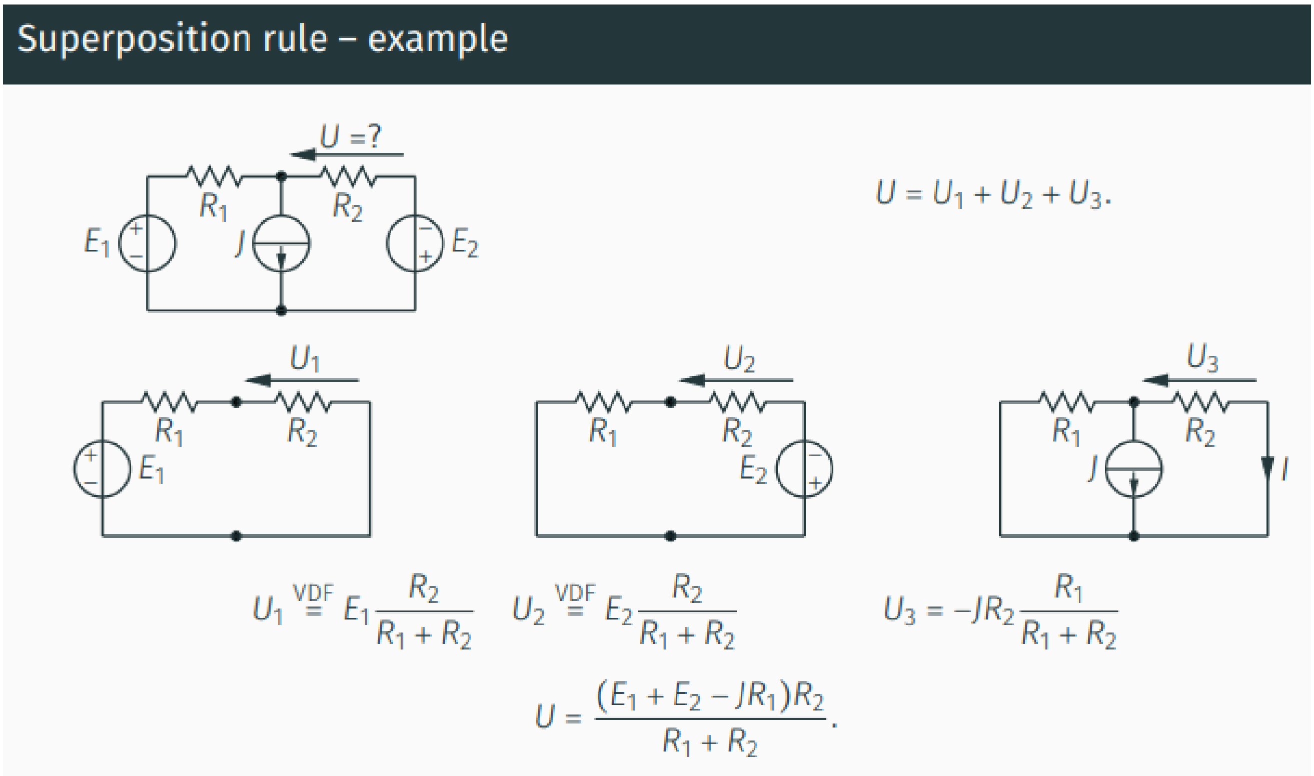
Electric potentials are measured with respect to ground node and KCL for that node is dropped.

Superposition rule

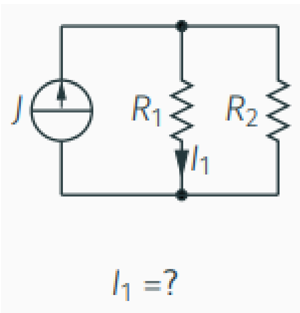
A solution to linear circuit with N independent sources is a sum of solutions to N circuits that result from the original circuit by reduction to zero all but one independent source (each time we let just a single independent source to act alone).

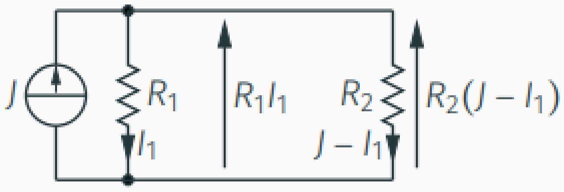
In this way one may find only the whole solutions but also the individual voltages and currents.

1. Reduction to zero



Current Divider Formula (CDF)





$$\begin{aligned} \text{Ohm's law:} \quad & U_1 = R_1 I_1 \\ \text{KCL:} \quad & I_2 = J - I_1 \\ \text{Ohm's law:} \quad & U_2 = R_2 I_2 \\ \text{KVL:} \quad & \underbrace{R_1 I_1}_{U_1} = \underbrace{R_2(J - I_1)}_{U_2} \end{aligned}$$

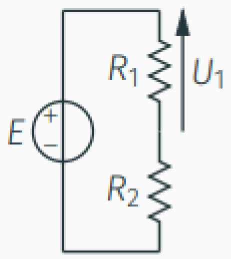
$$I_1(R_1 + R_2) = JR_2$$

$$I_1 = J\frac{R_2}{R_1 + R_2}$$

Similarly:

$$I_2 = J\frac{R_1}{R_1 + R_2}.$$

Voltage Divider Formula

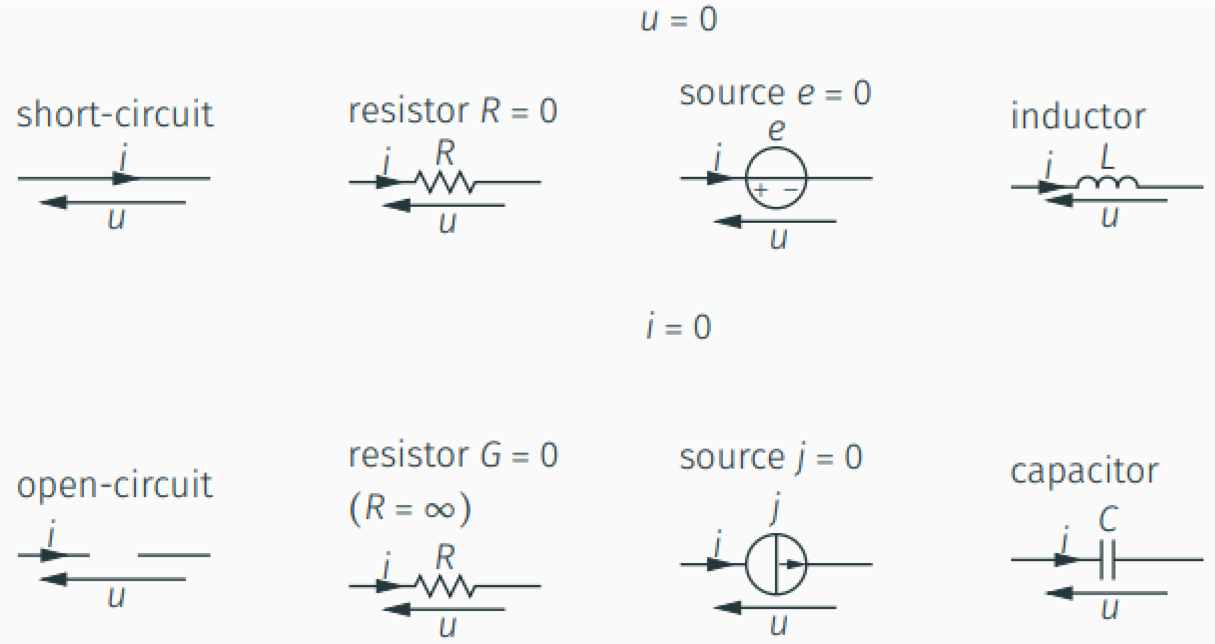


$$U_1 = E\frac{R_1}{R_1 + R_2}, \qquad U_2 = E\frac{R_2}{R_1 + R_2}.$$

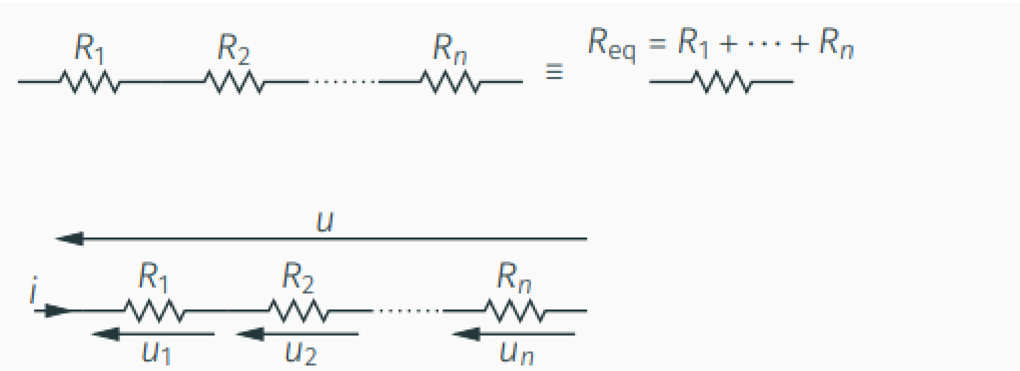
Equivalent devices

In some cases devices can act like other devices, therefor we can simplify the schematic to what we need. Ussually we will reduce elements to either a wire or open circuit (cuz they are the easiest).

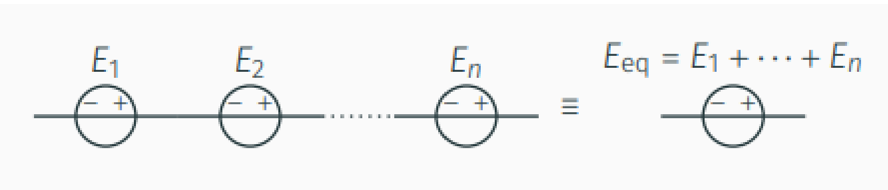
In DC case:

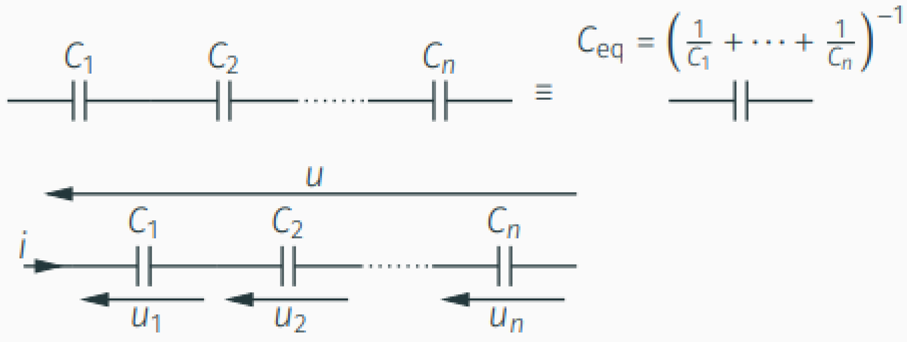


Series of connections



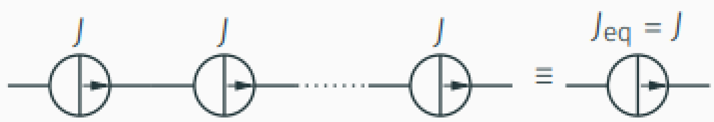
$$u \overset{\text{KVL}}{=} u_1 + \cdots + u_n \overset{u_k = R_k i}{=} i\,\underbrace{(R_1 + R_2 + \cdots + R_n)}_{R_{eq}}$$



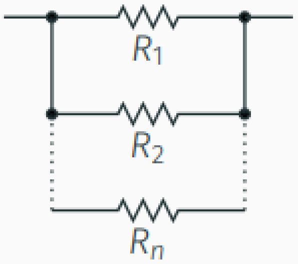


$$u' \overset{\text{KVL}}{=} u_1' + \cdots + u_n' \overset{i=C_k u_k'}{=} i\left(\frac{1}{C_1} + \cdots + \frac{1}{C_n}\right)$$

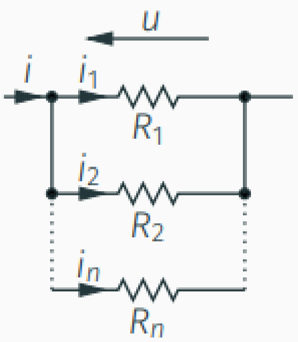
$$i = \underbrace{\left(\frac{1}{C_1} + \cdots + \frac{1}{C_n}\right)^{-1}}_{C_{eq}} u'.$$



Parallel connections of one-ports

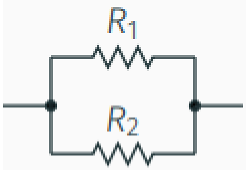


$$\equiv \quad R_{eq} = \left(\frac{1}{R_1}+\cdots+\frac{1}{R_n}\right)^{-1}$$

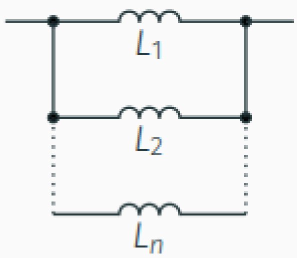


$$i \overset{\text{KCL}}{=} i_1+\cdots+i_n \overset{u=R_k i_k}{=} u\left(\frac{1}{R_1}+\cdots+\frac{1}{R_n}\right)$$

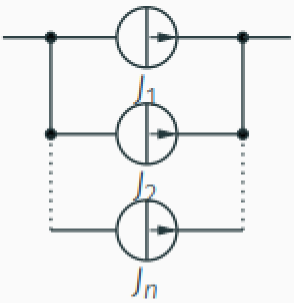
$$u = \underbrace{\left(\frac{1}{R_1}+\cdots+\frac{1}{R_n}\right)^{-1}}_{R_{eq}} i.$$



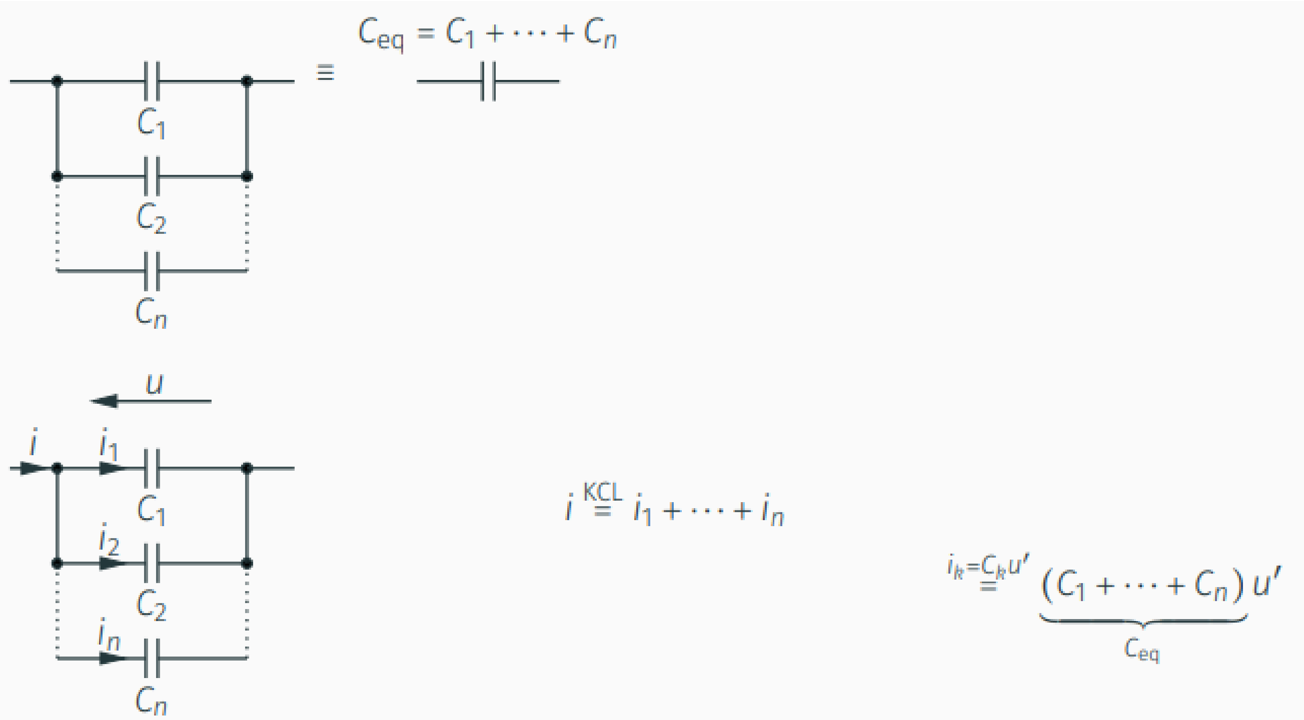
$$\equiv \quad R_{eq} = \frac{R_1 R_2}{R_1+R_2}$$

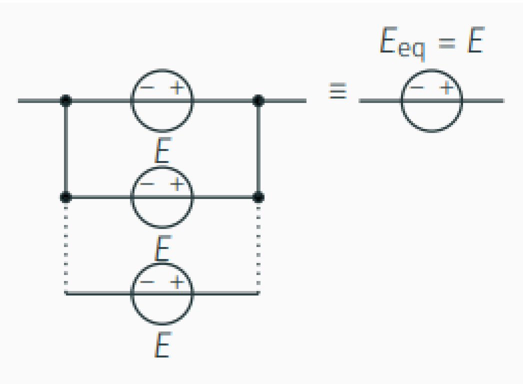


$$\equiv \quad L_{eq} = \left(\frac{1}{L_1}+\cdots+\frac{1}{L_n}\right)^{-1}$$

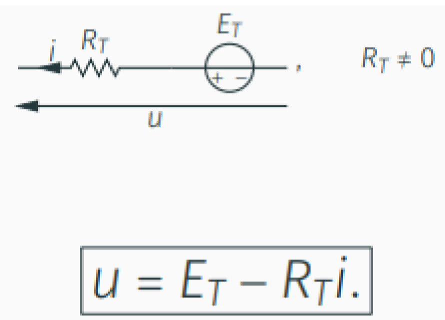


$$\equiv \quad J_{eq} = J_1+\cdots+J_n$$

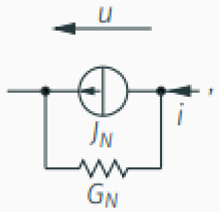




Thevenin's equivalent



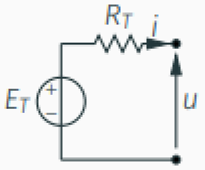
Norton's equivalent

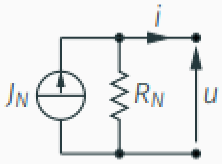


$$G_N \neq 0 \quad \left(R_N = \frac{1}{G_N}\right)$$

$$\boxed{i = J_N - uG_N = J_N - \frac{u}{R_N}.}$$

Thevenin's and Norton's equivalents equivalence



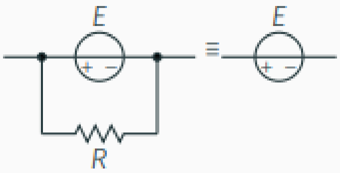


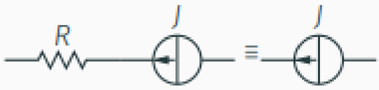
Thévenin's and Norton's equivalents are equivalent if and only if

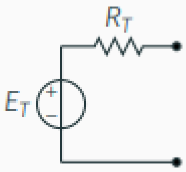
$$R_T = R_N \quad \text{and} \quad E_T = J_N R_N.$$

$$u = E_T - R_T i \qquad\qquad i = J_N - \frac{u}{R_N} \quad \equiv \quad u = J_N R_N - R_N i.$$

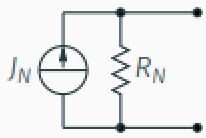
Attention:

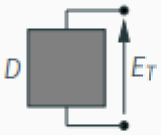




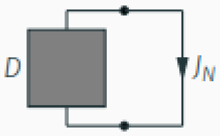


$$\overset{E_T = J_N R_N,\ R_T = R_N}{\equiv}$$





E_T is the open-circuit voltage,



J_N is the short-circuit current,

$$R_T = R_N = \frac{E_T}{J_N}.$$

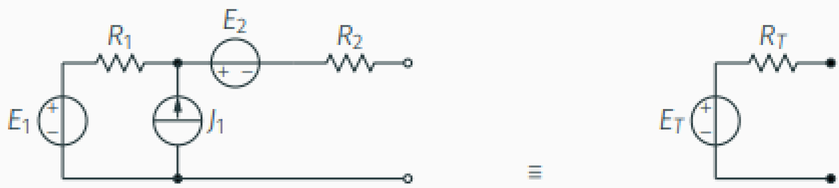
If short-circuit current is non-zero:

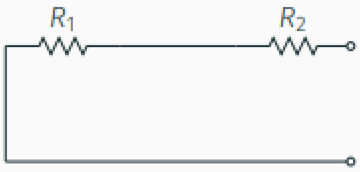
$$R_T = R_N = \frac{\text{open-circuit voltage}}{\text{short-circuit current}}.$$

In general:

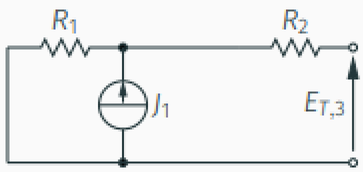
The internal (output) resistance $R_T = R_N$ of a one-port equals the equivalent resistance of the reduced one-port that results from the original one-port by reduction of all the independent sources to zeros.

Thévenin's equivalent parameters — an example



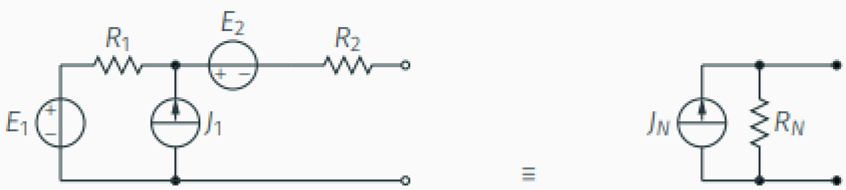


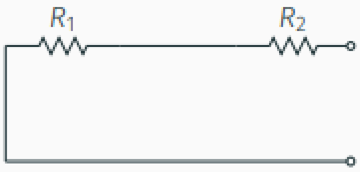
$$R_T = R_1 + R_2.$$



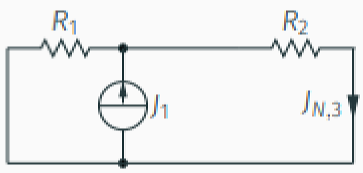
$$E_T = \underbrace{E_1} + \underbrace{(-E_2)} + \underbrace{J_1 R_1}.$$

Norton's equivalent parameters — an example





$$R_T = R_1 + R_2.$$



$$J_N = \underbrace{\frac{E_1}{R_1+R_2}}_{J_{N,1}} + \underbrace{\frac{-E_2}{R_1+R_2}}_{J_{N,2}} + \underbrace{J_1\frac{R_1}{R_1+R_2}}_{J_{N,3}}.$$

3 terminal device transformations

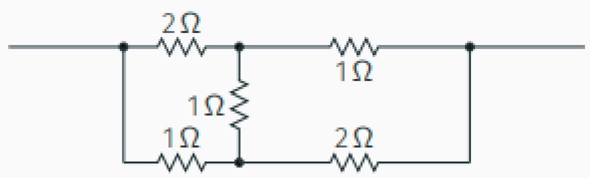
Equivalent if:

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$
$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

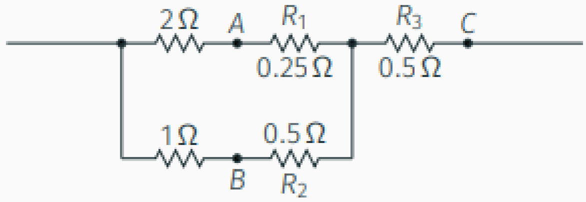
In other words:

$$R_1 = \frac{R_BR_C}{R_A + R_B + R_C},$$
$$R_2 = \frac{R_CR_A}{R_A + R_B + R_C},$$
$$R_3 = \frac{R_AR_B}{R_A + R_B + R_C}.$$

Example:

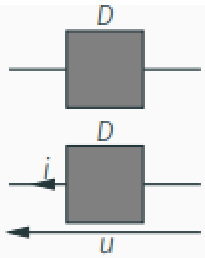


Becomes:



$$R_{eq} = \left(\frac{9}{4}\Big\|\frac{3}{2} + \frac{1}{2}\right)\Omega = \left(\frac{\frac{9}{4} \times \frac{3}{2}}{\frac{9}{4} + \frac{3}{2}} + \frac{1}{2}\right)\Omega = \left(\frac{27}{30} + \frac{1}{2}\right)\Omega = \frac{7}{5}\Omega.$$

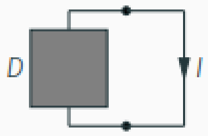
Table of linear DC one-port equivalence

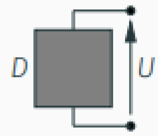


The following cases are possible:

$i = J_N$,	u arbitrary,	current source,
i arbitrary,	$u = E_T$,	voltage source,
i arbitrary,	$u = E_T - iR_T$, $R_T \neq 0$,	Thévenin's equivalent,
$i = J_N$,	$u = E_T$,	fixator,
i arbitrary,	u arbitrary,	norator.

Consider two connections (not necessarily circuits!):





I	U	one-port's type
$*$	$*$	norator
$\varnothing$	$\varnothing$	fixator
$\varnothing$	E	voltage source or fixator generating zero current
J	$\varnothing$	current source or fixator generating zero voltage
J	E	Thévenin's equivalent: $E_T = E$, $R_T = E/J$ (equivalently, Norton's equivalent: $J_N = J$, $R_N = E/J$)
0	0	nullator or resistor.

Current divider formula

Note: we only consider parrarel connection because current flowing through series elements is ALLWAYS the same (in ideal case).

For a pair resistors

R_1, R_2 and currents I_1, I_2 flowing though them then
$I_1 = I_{in}\frac{R_2}{R_1+R_2}$
$I_2 = I_{in}\frac{R_1}{R_1+R_2}$

Intution

We take the value from "the other resistor" because when resistance of that resistor is gonna get bigger->current though it is gonna get lower so current though "our" resistor is gonna get proportionally higher.

For more then two resistors

It's better to use conductance instead of resistance ($G = \frac{1}{R}$) and follow the formula:

$I_1 = I_{in}\frac{G_1}{G_1+G_2+G_3}$

Note: we HAVE TO change it into conductance, there is no easier/better way to do this!

Voltage divider formula

Note: we only consider series connection because voltage flowing through a parrarel elements is ALLWAYS the same (in ideal case).

Note: knowing the voltage and total resistance we can deduce the current and then calculate the voltage across the resistors in a two step fasion. Below is given the alternative to this method.

For a pair resistors

R_1, R_2 and currents I_1, I_2 flowing though them then
$V_1 = V_{in}\frac{R_1}{R_1+R_2}$, $V_2 = V_{in}\frac{R_2}{R_1+R_2}$.

Intution

We take the value from "the other resistor" because when resistance of that resistor is gonna get bigger->current though it is gonna get lower so current though "our" resistor is gonna get proportionally higher.

For more then two resistors

It's better to use conductance instead of resistance ($G = \frac{1}{R}$) and follow the formula:

$$I_1 = I_{in} \frac{G_1}{G_1 + G_2 + G_3}$$

Note: we HAVE TO change it into conductance, there is no easier/better way to do this!