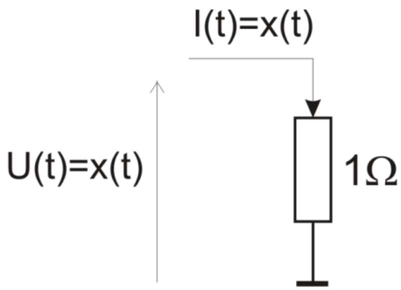
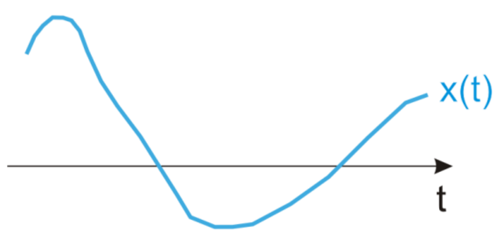
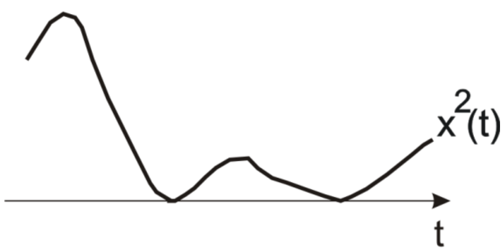
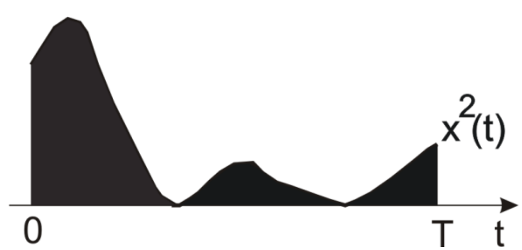
Signal energy and power



Instantaneous power
$P(t)=x^2(t)$





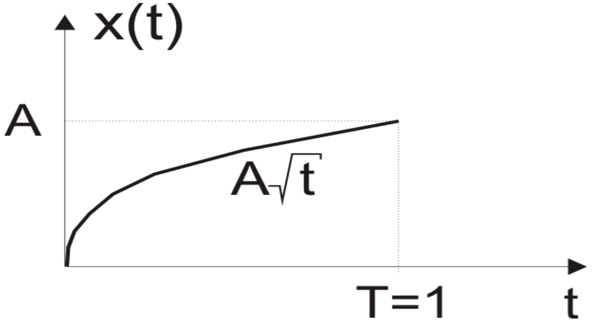


Energy $$E=\int_0^T x^2(t)\,dt$$

Average power $$P=\frac{1}{T}\int_0^T x^2(t)\,dt$$

Signal energy and power

Task 1
Calculate the **energy** of the signal x(t) drawn below



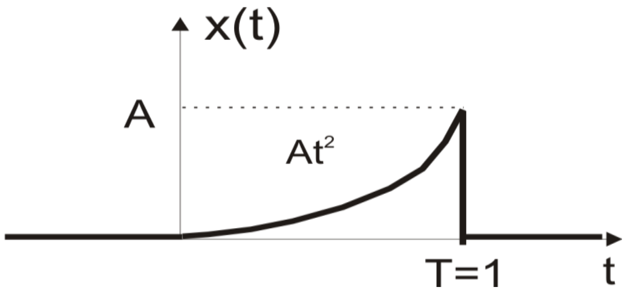
Solution:

$$E = \int_0^1 x^2(t)dt = \int_0^1 A^2 t\, dt = \frac{A^2}{2}$$

Signal energy and power

Task 2

Calculate the **energy** of the signal x(t) drawn below



Solution:

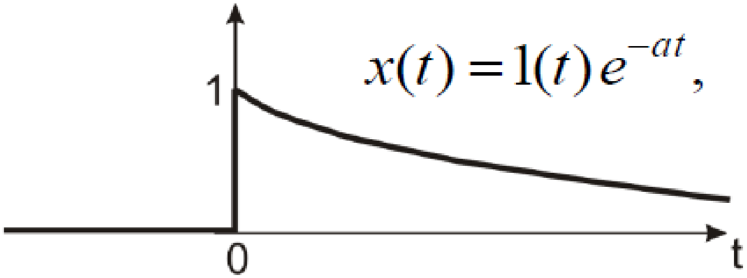
$$E = \int_0^1 x^2(t)dt = \int_0^1 A^2 t^4 dt = \frac{A^2}{5}$$

Energy of signals of infinite duration

Signals of infinite duration may have finite or infinite energy:

e.g. $x(t) = \exp(-at), \qquad t \in (0, \infty), \quad a > 0$

has energy equal to $\dfrac{1}{2a}$



$$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^{\infty} (e^{-at})^2\,dt = \int_0^{\infty} e^{-2at}\,dt = \tfrac{1}{-2a} e^{-2at}\Big|_0^{\infty} = \tfrac{1}{-2a}[0-1] = \tfrac{1}{2a}$$

Periodic signals

Periodic signal: $\forall_t \; x(t+T) = x(t)$

T > 0 – period
Minimum value of *T* - fundamental period or just period

e.g. For x(t) = cos(2πft), fundamental period *T*=1/f

$x(t) = 1 + \sin(2\pi t)$ Periodic signal, period T=1

$x(t) = \cos^2(2\pi t)$ Periodic, period T=0.5, because $\cos^2(2\pi t) = \frac{1}{2} + \frac{1}{2}\cos(4\pi t)$

$x(t) = \cos(2\pi t) + \cos(\pi t)$ periodic, T_1=1 , T_2=2, common period T=2

$x(t) = \cos(6\pi t) + \cos(4\pi t)$ periodic, $T_1 = \frac{1}{3}$, $T_2 = \frac{1}{2}$ common period T=1

$x(t) = \cos(2\pi t) + \cos(\frac{2\pi t}{\sqrt{2}})$ first period T_1=1 , second $T_2 = \sqrt{2}$

There is no common period, because T_1/T_2 is not a rational number. The signal x(t) is not periodic.

Power of periodic signals

Periodic signals have infinite duration and infinite energy

We may calculate energy per period $E_T = \int_0^T x^2(t)dt$ where T - period

Average power $P = E_T/T$

For example $x(t) = A\cos(2\pi f t)$ has period $T=1/f$

$$E_T = \int_0^T x^2(t)dt = A^2 \int_0^T cos^2(2\pi f t)dt = A^2 \int_0^T [0.5 + 0.5\, cos(4\pi f t)]dt = 0.5\, A^2 T$$

$$P = \frac{A^2}{2}$$

Calculation of signal power using probability density function (pdf) of signal samples

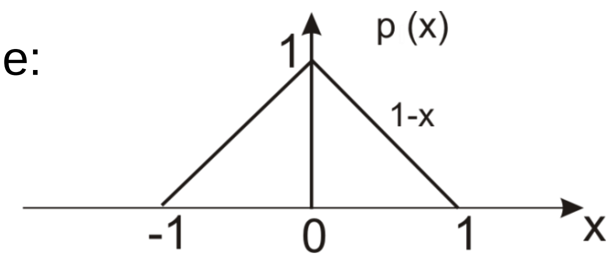
Given p(x) - pdf of samples x – we may calculate:

Mean value of signal samples $m_x = E[x] = \int x\, p(x)\, dx$ (here E – statistical average)

Instatnateous power: x^2

Mean power $P = E[x^2] = \int x^2 p(x)\, dx$

Example:



$$P = \int_{-1}^{1} x^2 p(x) dx = 2\int_0^1 x^2 (1-x) dx = 2\int_0^1 x^2 dx - 2\int_0^1 x^3 dx = \frac{2}{3} - \frac{2}{4} = \frac{1}{6}$$

Correlation coefficient, correlation function

Similarity of real signals x(t) and y(t) may be characterized by the scalar product:

$$< x, y > = \int x(t)\, y(t)\, dt$$

Scalar product $< x, y >$ is also called a **correlation** of x(t) and y(t),
but **correlation coefficient** of x and y is usually calculated using the normalized signals

$$\rho(x, y) = < \frac{x}{||x||}, \frac{y}{||y||} > = \frac{< x, y >}{\| x \| \| y \|}$$

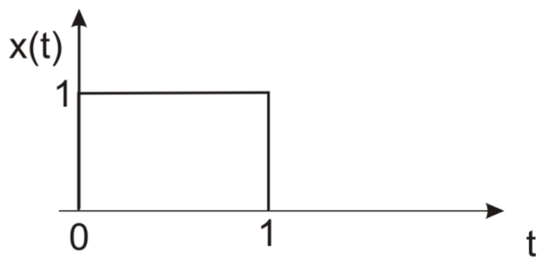
If x(t)=y(t), then $\rho(x, y) = 1$. If $< x, y > = 0$, signals x and y are orthogonal.

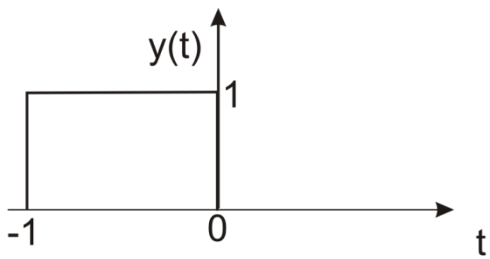
Correlation function of signals x(t) and y(t) is a function of t_0. t_0 is a shift of signal y(t).

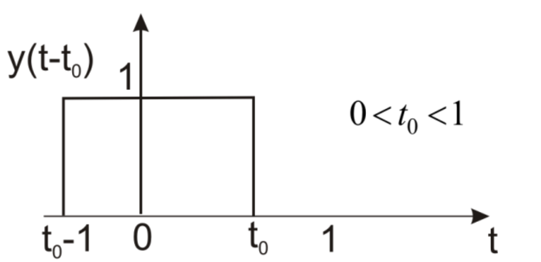
$$R_{xy}(t_0) = \int x(t)\, y(t - t_0)\, dt$$

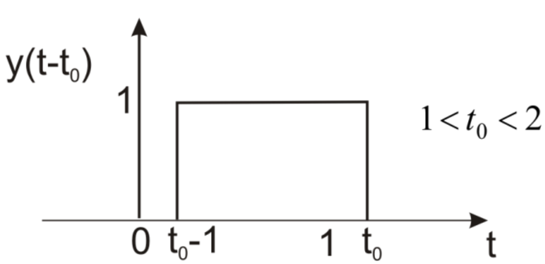
If y(t)=x(t) we call it **autocorrelation function** $R_x(t_0) = \int x(t)\, x(t - t_0)\, dt$

Calculation of correlation function – an example









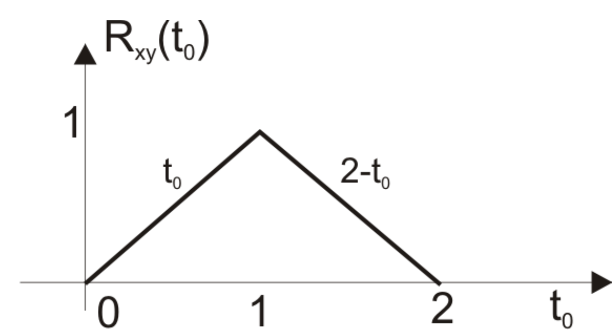
$$R_{xy}(t_0) = \int x(t)\, y(t - t_0)\, dt$$

For $t_0 > 2$ and for $t_0 < 0$ pulses x(t) and $y(t\text{-}t_0)$ do not overlap, $x(t)\, y(t\text{-}t_0) = 0$ and $R_{xy}(t_0)=0$

For $0 < t_0 < 1$ pulses x(t) and $y(t\text{-}t_0)$ overlap from 0 to t_0 and $y(t)=t_0$

For $1 < t_0 < 2$ pulses x(t) and $y(t\text{-}t_0)$ overlap from $t_0\text{-}1$ to 1 and $y(t)=2\text{-}t_0$

Finally the correlation function equals:



Convolution

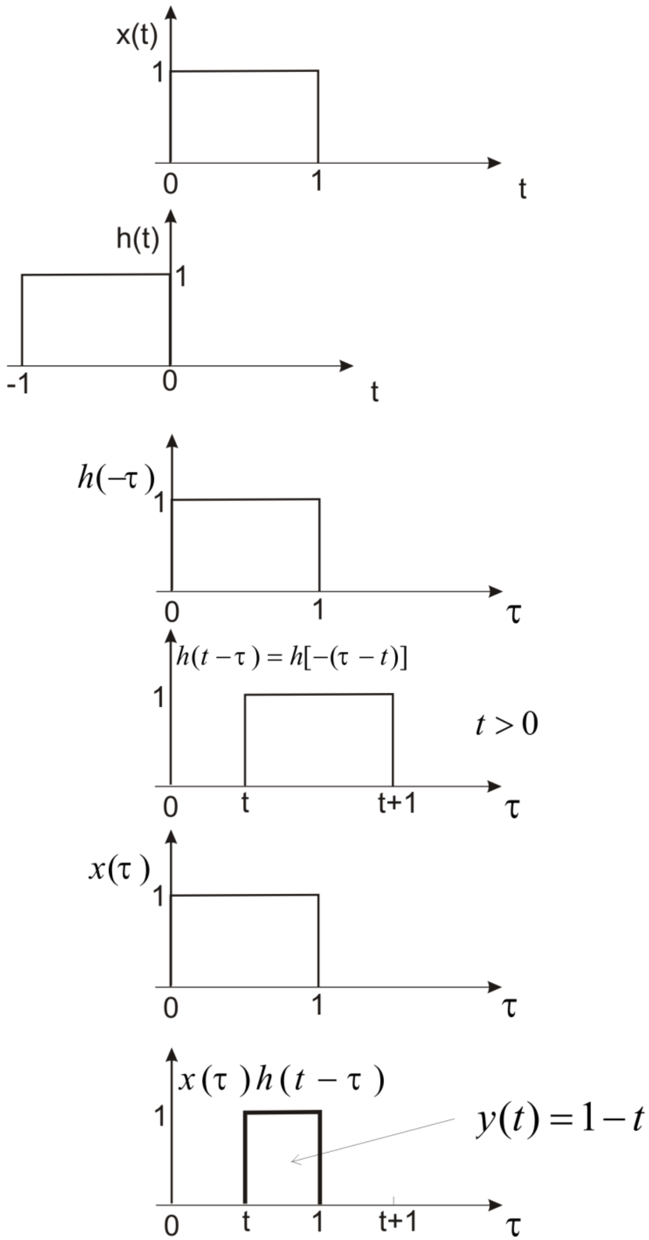
The function $$y(t) = x(t) * h(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

is called a **convolution** of x(t) and h(t).
It is a function of time shift t.

$$h(t-\tau) = h[-(\tau - t)]$$

mirror image shift

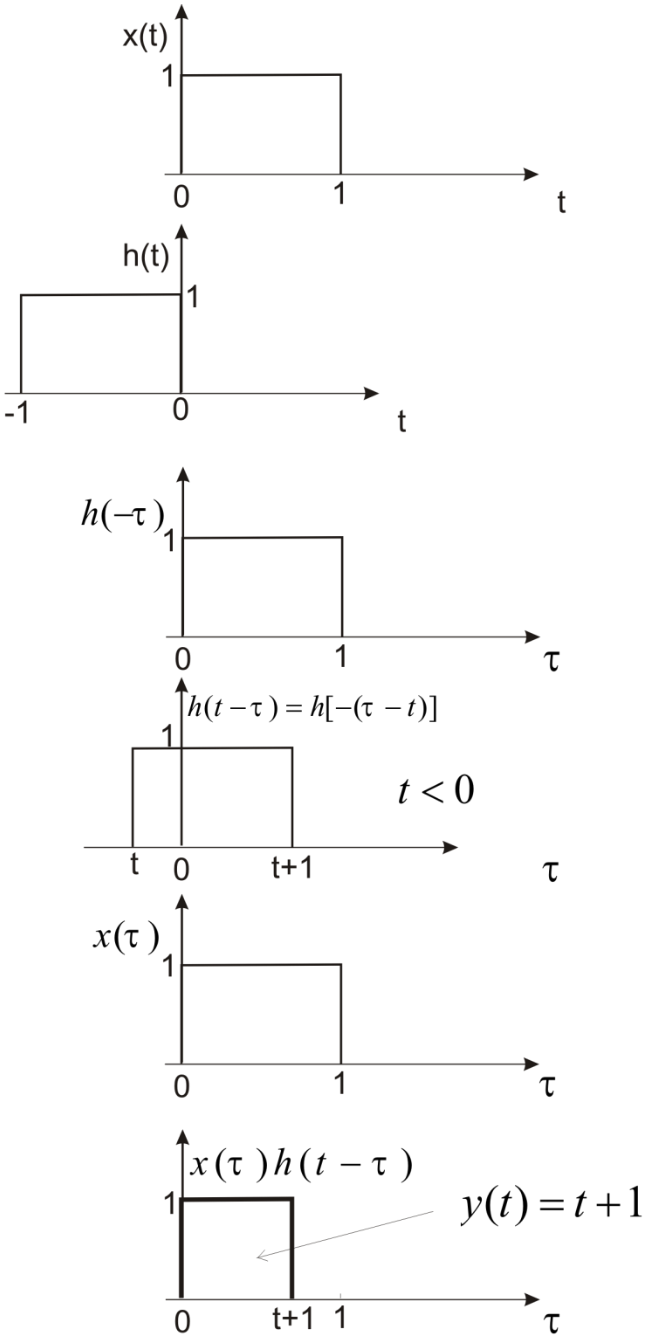
Calculation of convolution – an example



$$y(t) = x(t) * h(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

For t > 1 and for t < -1 pulses $x(\tau)$ and $h(t-\tau)$ do not overlap , $x(\tau)h(t-\tau) = 0$ and y(t)=0

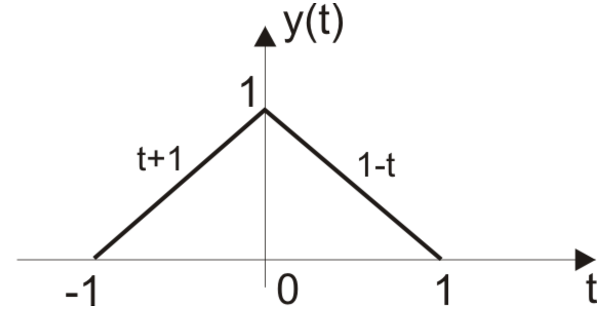
For 0 < t < 1 pulses overlap from t to 1 and y(t)=1-t



$$y(t)=x(t)*h(t)=\int x(\tau)h(t-\tau)\,d\tau$$

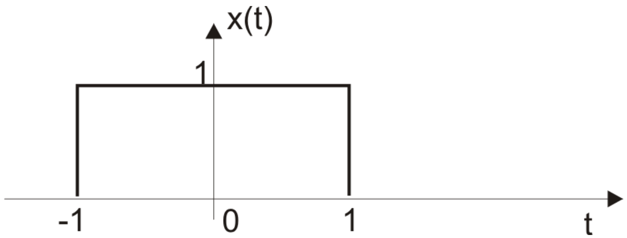
For $-1 < t < 0$ pulses overlap from 0 to $t+1$ and $y(t)=t+1$

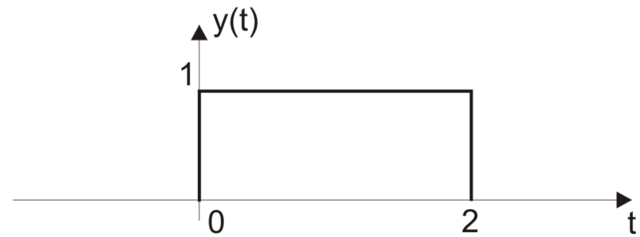
Finally
the convolution
equals:

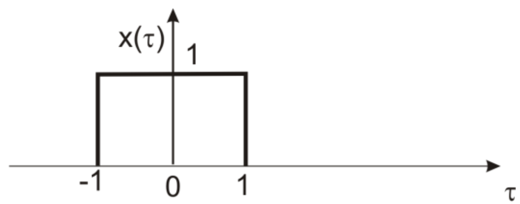


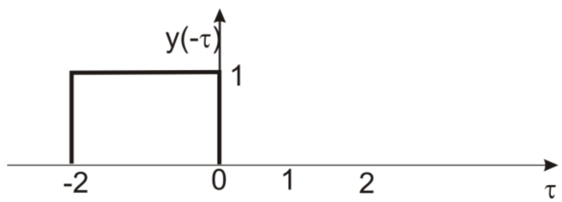
Calculation of convolution – another example

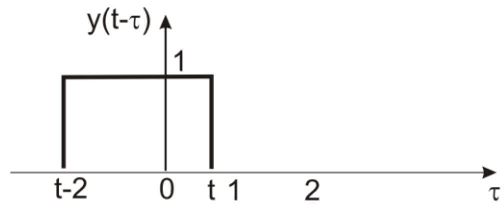
Calculate **and draw** the **convolution** of signals x(t) and y(t)

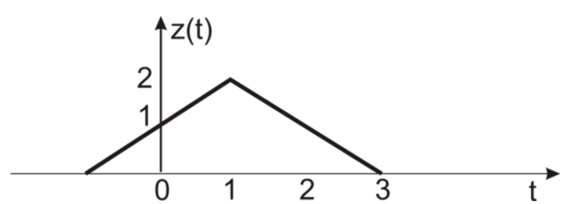










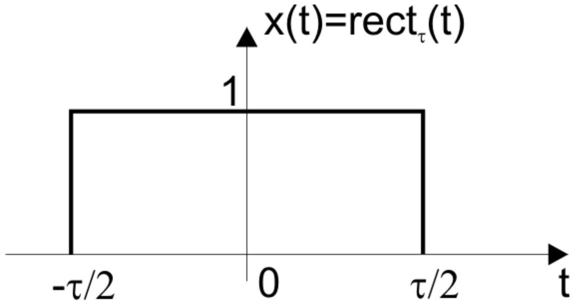


Calculation of Fourier Transform

Fourier Transform $$X(f) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

Inverse Fourier Transform $$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

An example:



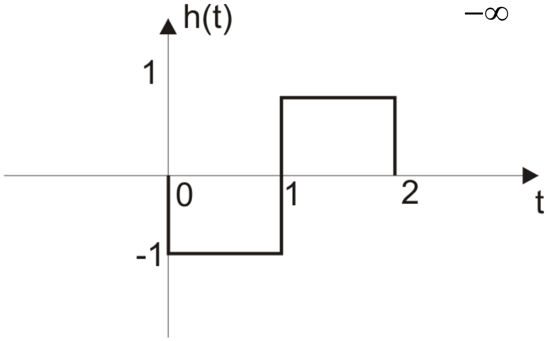
$$X(f) = F[x(t)] = \int_{-\tau/2}^{\tau/2} e^{-j2\pi f t}\, dt = \frac{1}{-j2\pi f}\, e^{-j2\pi f t}\Big|_{-\tau/2}^{\tau/2} = \frac{1}{-j2\pi f}\left[e^{-j\pi f \tau} - e^{j\pi f \tau}\right] =$$

$$= \frac{1}{\pi f}\left[\frac{e^{+j\pi f \tau} - e^{-j\pi f \tau}}{2j}\right] = \frac{\sin(\pi f \tau)}{\pi f} = \tau\, \frac{\sin(\pi f \tau)}{\pi f \tau}$$

Calculation of Fourier Transform

Fourier Transform $$X(f) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

Another example:



$$H(f) = -\int_0^1 e^{-j2\pi f t} dt + \int_1^2 e^{-j2\pi f t} dt = \frac{1}{-j2\pi f}[-e^{-j2\pi f} + 1 + e^{-j4\pi f} - e^{-j2\pi f}] =$$

$$= \frac{1}{-j2\pi f}[1 - 2e^{-j2\pi f} + e^{-j4\pi f}] = \frac{-1}{j2\pi f}[1 - e^{-j2\pi f}]^2$$

Calculation of Fourier Transform using its properties

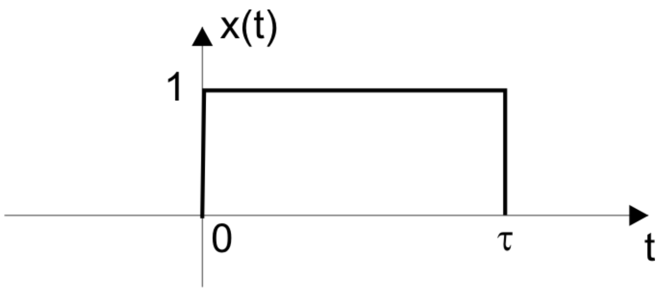
Fourier Transform $$X(f) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

Shift theorem:

$$F[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0) e^{-j2\pi f t} dt = e^{-j2\pi f t_0} \int_{-\infty}^{\infty} x(t-t_0) e^{-j2\pi f (t-t_0)} d(t-t_0) =$$

$$= e^{-j2\pi f t_0} F[x(t)] = e^{-j2\pi f t_0} X(f)$$

An example



$$x(t) = rect_\tau(t - \tfrac{\tau}{2})$$

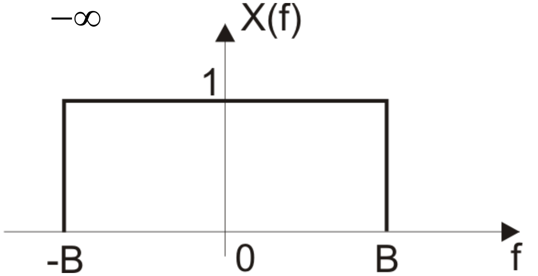
$$F[rect_\tau(t)] = \tau \frac{\sin(\pi f \tau)}{\pi f \tau}$$

$$F[rect_\tau(t - \tfrac{\tau}{2})] = \tau \frac{\sin(\pi f \tau)}{\pi f \tau} e^{-j\pi f \tau}$$

Calculation of Inverse Fourier Transform

Inverse Fourier Transform $$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

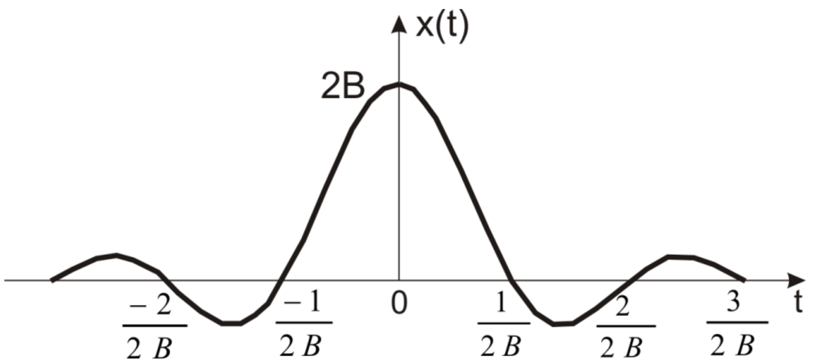
An example: $$X(f) = rect_{2B}(f)$$



$$x(t) = F^{-1}\left[X(f)\right] = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df = \int_{-B}^{B} e^{j2\pi ft} df =$$

$$= \tfrac{1}{j2\pi t}[e^{j2\pi Bt} - e^{-j2\pi Bt}] = \tfrac{1}{\pi t} \frac{e^{j2\pi Bt} - e^{-j2\pi Bt}}{2j} = \tfrac{1}{\pi t}\sin(2\pi Bt) =$$

$$= 2B \frac{\sin(2\pi Bt)}{2\pi Bt}$$

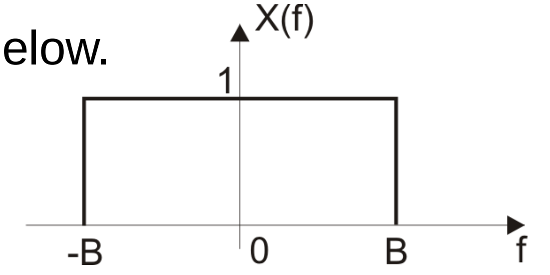


Fourier transform of a product of two signals

Fourier Transform of a product of x(t) y(t) is a convolution of their Fourier Transforms

$$F\left[x(t)\,y(t)\right] = X(f) * Y(f)$$

Example: signal x(t) has the Fourier Transform drawn below.
Draw Fourier transform of the signal $v(t) = x(t)\cos 2\pi f_0 t$

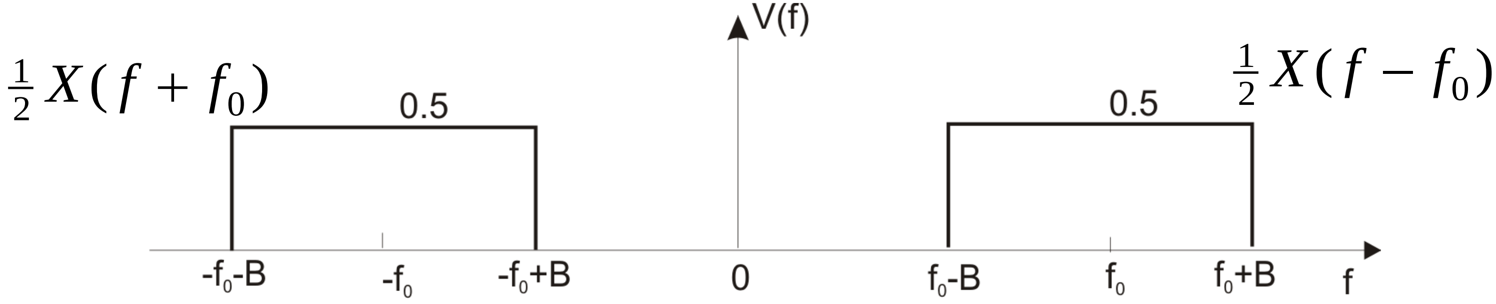


Fourier transform of $\cos 2\pi f_0 t$ equals

$$\tfrac{1}{2}\delta(f - f_0) + \tfrac{1}{2}\delta(f + f_0)$$

Fourier transform of v(t) equals

$$F[v(t)] = X(f) * [\tfrac{1}{2}\delta(f - f_0) + \tfrac{1}{2}\delta(f + f_0)] = \tfrac{1}{2}X(f - f_0) + \tfrac{1}{2}X(f + f_0)$$



Some examples of Fourier Transforms

time: $x(t)=F^{-1}[X(f)]$	**frequency:** $X(f)=F[x(t)]$
$x(t) = rect_\tau(t)$	$X(f) = F\left[rect_\tau(t)\right] = \tau \dfrac{\sin(\pi\tau f)}{\pi\tau f}$
$x(t) = F^{-1}\left[X(f)\right] = 2B\dfrac{\sin(2\pi Bt)}{2\pi Bt}$	$X(f) = rect_{2B}(f)$
$x(t)=\delta(t)$	$X(f)=1$
$x(t)=1$	$X(f)=\delta(f)$
$x(t)=\delta(t-t_0)$	$X(f)= \exp(-j2\pi ft_0) = \cos(2\pi ft_0) - j\sin(2\pi ft_0)$
$x(t)=\exp(j2\pi f_0 t)$	$X(f) = \delta(f-f_0)$
$x(t) = \cos(2\pi f_0 t)$	$X(f) = \frac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$
$x(t) = \sin(2\pi f_0 t)$	$X(f) = \frac{1}{2j}\left[\delta(f - f_0) - \delta(f + f_0)\right]$

LTI Filter

$$x(t) \longrightarrow \boxed{\text{imp. resp. } h(t)} \longrightarrow y(t)=x(t)*h(t)$$

The output signal is a convolution of the input signal and impulse response of the LTI filter

For stable filters: **Y(f) = X(f) H(f)**

x(t) - input signal, X(f) =F[x(t)] - its spectrum (Fourier transform)

h(t) – impulse response of LTI filter, H(f) =F[h(t)] – frequency response of LTI filter

|X(f)| - the amplitude spectrum of the input signal: $|X(f)| = \sqrt{\{\mathrm{Re}[X(f)]\}^2 + \{\mathrm{Im}[X(f)]\}^2} = \sqrt{X(f)X^*(f)}$

$|X(f)|^2$ – energy spectral density of the input signal (for signals of finite energy)

$G_x(f)$ - power spectral density (psd) of the input signal (for signals of finite power)

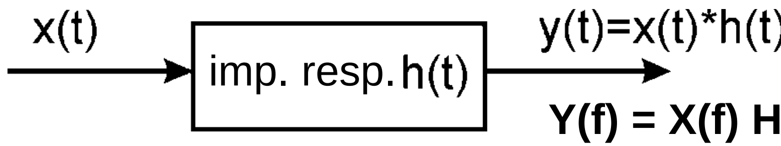
y(t) – the output signal, Y(f) =F[y(t)] – its Fourier transform , |Y(f)| - its amplitude spectrum, $|Y(f)|^2$ - its energy spectral density, $G_y(f)$ - its power spectral density (psd)

Y(f)=X(f) H(f)	|Y(f)|=|X(f)| |H(f)|	$\|Y(f)\|^2=\|X(f)\|^2 \|H(f)\|^2$	$G_y(f) = G_x(f)\,\|H(f)\|^2$

Energy of the input signal: $E_x = \int_{-\infty}^{\infty} |X(f)|^2\, df$, energy of the output signal $E_y = \int_{-\infty}^{\infty} |Y(f)|^2\, df$

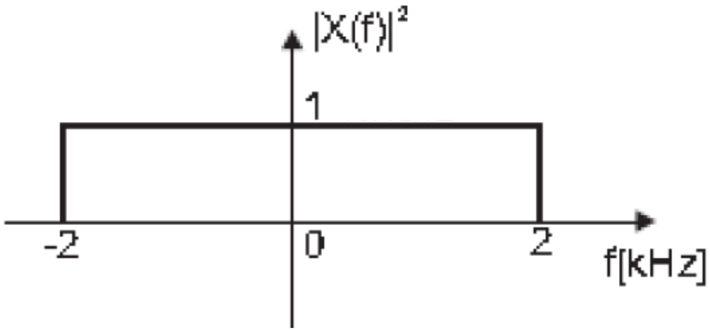
Power of the input signal: $P_x = \int_{-\infty}^{\infty} G_x(f)\, df$, power of the output signal $P_y = \int_{-\infty}^{\infty} G_y(f)\, df$

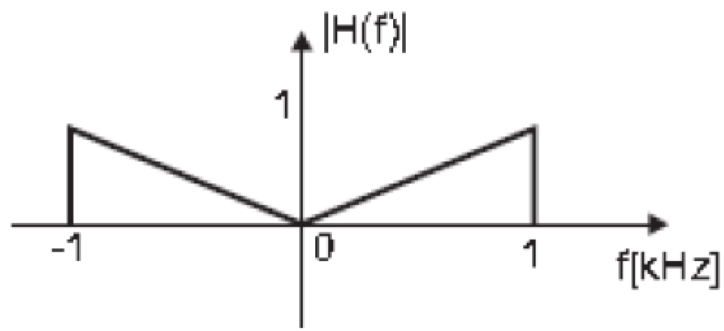
Filtering



Y(f) = X(f) H(f)

Given: Energy spectrum of the input signal $|X(f)|^2$ and absolute value of frequency response $|H(f)|$
Calculate energy spectrum of the output signal $|Y(f)|^2$ and energies of the input and output signals



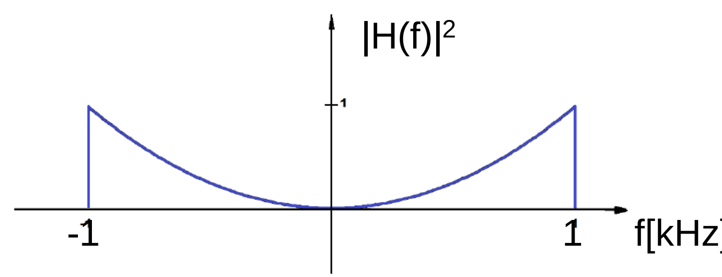


Energy spectral density (energy spectrum) of the output signal is calculated according to the formula $|Y(f)|^2=|X(f)|^2\,|H(f)|^2$

$|H(f)|^2 = f^2$ (see figure)

$|Y(f)|^2=|X(f)|^2\,|H(f)|^2 = |H(f)|^2 = f^2$

because $|X(f)|^2 = 1$



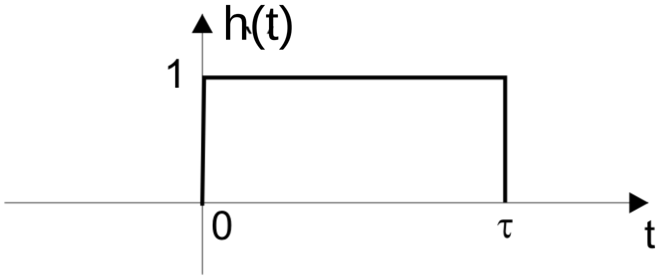
Energies are obtained by interration of energy spectrums: $E_x = \int_{-\infty}^{\infty} |X(f)|^2\,df = 4$

$E_y = \int_{-\infty}^{\infty} |Y(f)|^2\,df = 2/3$

Response of stable LTI filter to sine/cosine (1)

The input signal $x(t) = A\cos(2\pi f_0 t + \varphi)$

Impulse response of the LTI filter: h(t) – see figure



Frequency response of LTI filter H(f) = F [h(t)]
(see slide Properties of Fourier Transform):

$$H(f) = |H(f)|\, e^{j\arg[H(f)]} = \tau \frac{\sin \pi f \tau}{\pi f \tau} e^{-j\pi f \tau}$$

where $|H(f)| = |\tau \frac{\sin \pi f \tau}{\pi f \tau}|$, $\arg\big(H(f)\big) = -\pi f \tau$ if $\frac{\sin \pi f \tau}{\pi f \tau} > 0$

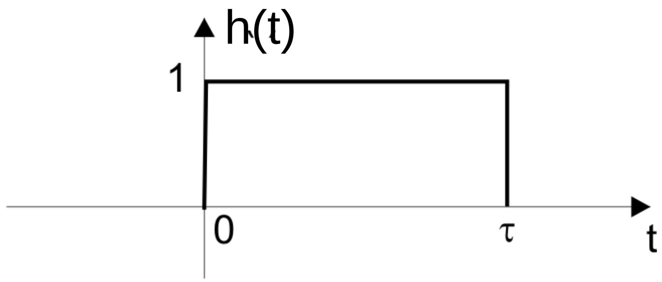
, $\arg\big(H(f)\big) = -\pi f \tau + \pi$ if $\frac{\sin \pi f \tau}{\pi f \tau} < 0$

The output signal $y(t) = A|H(f_0)| \cos[2\pi f_0 t + \varphi + \arg(H(f_0))]$

Amplitude of y(t): $A|H(f_0)| = A|\tau \frac{\sin \pi f_0 \tau}{\pi f_0 \tau}|$

Initial phase of y(t): $\varphi + \arg(H(f_0)) = \varphi - \pi f_0 \tau$ or $\varphi - \pi f_0 \tau + \pi$

Response of stable LTI filter to sine/cosine (2)



The same filter (see impulse response h(t))

Frequency response: ($\tau = 0.001\ s$)

$$H(f) = |H(f)|\, e^{j\arg[H(f)]} = \tau \frac{\sin \pi f \tau}{\pi f \tau} e^{-j\pi f \tau}$$

Input signal $x(t) = A\cos(2\pi f_0 t + \varphi)$ A=2, $\varphi = 0,\ f_0 = 1000\ Hz$ $f_0\tau = 1$

$$H(f_0) = 0.001 \frac{\sin \pi}{\pi} e^{-j\pi} = 0$$ because $\sin \pi = 0$

The output signal $y(t) = A|\,H(f_0)\,|\cos[2\pi f_0 t + \varphi + \arg(H(f_0))]$= 0
This filter attenuates completely (sets to zero) signals of frequency 1000 Hz.

Input signal as above, but frequency f_1 = 10 Hz. $f_1\tau = 0.01$

$$H(f_1) = 0.001 \frac{\sin 0.01\pi}{0.01\pi} e^{-j0.01\pi} \approx 0.001 e^{-j0.01\pi}$$

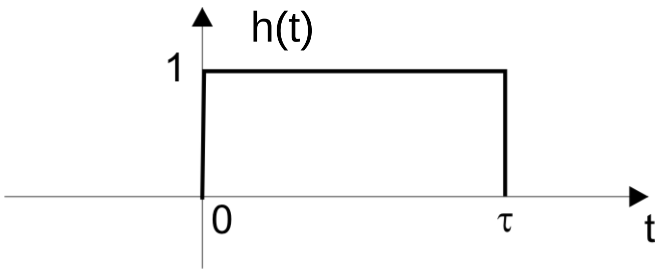
$|H(f_1)| \approx 0.001$
$\arg\big(H(f_1)\big) = -0.01\pi$

$$y(t) = A\,|H(f_1)|\cos[2\pi f_1 t + \varphi + \arg\big(H(f_1)\big)] = 0.002\ \cos[2\pi f_1 t - 0.01\pi]$$

Calculation of the frequency response of LTI filter given the impulse response

H(f)=F[h(t)] – frequency response of the LTI filter is the Fourier transform of its impulse response

Example:



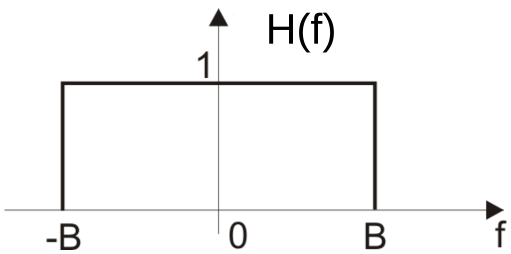
$$H(f) = F[h(t)] = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft} dt = \int_{0}^{\tau} e^{-j2\pi ft} dt =$$

$$= \frac{1}{-j2\pi f} e^{-j2\pi ft} \Big|_{0}^{\tau} = \frac{1}{-j2\pi f} [e^{-j2\pi f\tau} - 1]$$

Calculation of the impulse response of LTI filter given the frequency response

$h(t)=F^{-1}[H(f)]$ – impulse response of the LTI filter is the Inverse Fourier transform of its frequency response

Example:

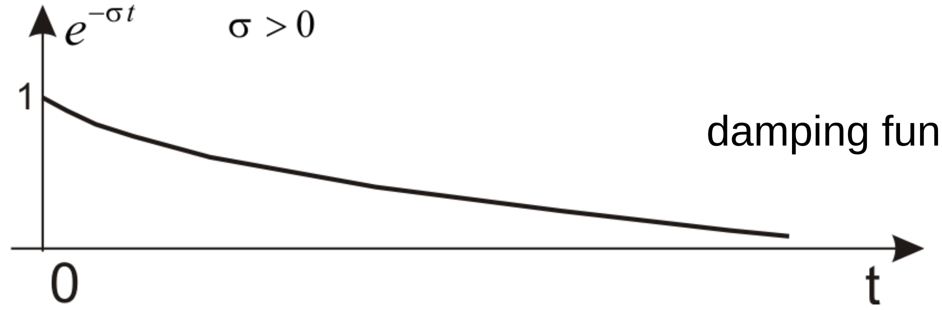


$$h(t) = F^{-1}\left[H(f)\right] = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df = \int_{-B}^{B} e^{j2\pi f t} df =$$

$$= \frac{1}{j2\pi t}[e^{j2\pi Bt} - e^{-j2\pi Bt}] = \frac{1}{\pi t} \frac{e^{j2\pi Bt} - e^{-j2\pi Bt}}{2j} = \frac{1}{\pi t}\sin(2\pi Bt)$$

Laplace transform

If Fourier transform does not converge, then Laplace transform may be used. Laplace transform is some generalization of Fourier transform, obtained by damping the signal before transformation.



Let's consider signal $x(t)$, $x(t) = 0 \text{ for } t < 0$ of nonconvergent Fourier Transform.

We may calculate Fourier transform for damped signal $\bar{x}(t) = e^{-\sigma t} x(t)$

$$\bar{X}(f) = F[x(t)e^{-\sigma t}] = \int_0^\infty x(t)e^{-\sigma t}e^{-j2\pi f t}dt = \int_0^\infty x(t)e^{-st}dt$$

where $s = \sigma + j2\pi f = \sigma + j\omega \qquad \omega = 2\pi f$

Laplace transform $$\boxed{L[x(t)] = X(s) = \int_0^\infty x(t)e^{-st}dt}$$

Laplace transform - properties

Properties of Laplace transform are similar to properties of Fourier transform

1. Linearity: $L[ax(t)+by(t)] = aL[x(t)] + bL[y(t)] = aX(s) + bY(s)$
2. Differentiation: $L[\frac{d}{dt}x(t)] = sL[x(t)] = sX(s)$
3. Convolution: $L[x(t)*y(t)] = L[x(t)]\ L[y(t)] = X(s)Y(s)$

Example: Convolution of unit step function with itself is a linearly increasing function

$$1(t)*1(t) = t1(t)$$

Its Laplace transform equals

$$L[1(t)*1(t)] = L[1(t)]\ L[1(t)] = \frac{1}{s}\frac{1}{s} = \frac{1}{s^2}$$

Using Laplace transform to test stability

LTI system has the impulse response $h(t) = 1(t)e^{-\alpha t}$.

For $\alpha > 0$ the system is stable, for $\alpha < 0$ it is unstable.

Let's calculate Laplace transform of h(t), i.e. transfer function od our system:

$$H(s) = L[1(t)e^{-\alpha t}] = \int_0^\infty e^{-\alpha t} e^{-st} dt = \int_0^\infty e^{(-\alpha - s)t} dt = \frac{1}{-\alpha - s} e^{(-\alpha-s)t}|_0^\infty =$$

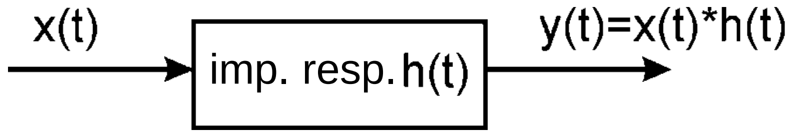
$$= \frac{1}{-\alpha - s}(0-1) = \frac{1}{s+\alpha}$$

Pole of H(s) is at $s_1 = -\alpha$, because $H(s_1) = \infty$.

For $\alpha > 0$ the transfer function H(s) has **a pole** is on the **left side** of the complex plane and the system is **stable**.

For $\alpha < 0$ the transfer function H(s) has **a pole** is on the **right side** of the complex plane and the system is **unstable**.

Stability of LTI systems and poles of transfer function - an example



LTI system is described by system of differential equations:

$$1.25y(t)+\frac{d}{dt}y(t)+\frac{d^2}{dt^2}y(t)=x(t)-\frac{d}{dt}x(t)$$

Differentiation is represented by $j\omega$, which becomes $s=j\omega$ in Laplace transform

$$1.25Y(s)+sY(s)+s^2Y(s)=X(s)-sX(s)$$

$$Y(s)[s^2+s+1.25]=X(s)[1-s]$$

Transfer function (output over input) $\quad H(s)=\dfrac{Y(s)}{X(s)}=\dfrac{1-s}{s^2+s+1.25}$

$H(s)$ has two poles (zeros of denominator) $\quad s_1=-0.5+j \quad s_2=-0.5-j$

Real part of both poles is negative $\quad \mathrm{Re}(s_i)=-0.5, \quad i=1,2$ so the system is stable.

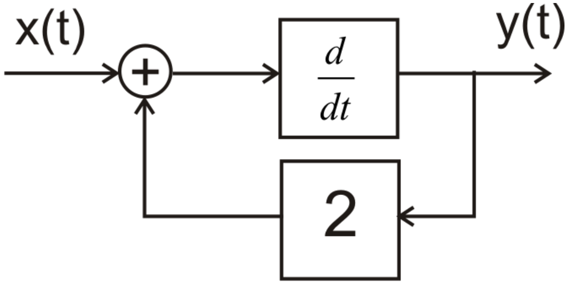
$H(s)$ has also a zero (of a numerator) at s=1 but zeros do not influence the stability

Example: block scheme

Description in time domain:

$$y(t) = \tfrac{d}{dt}[x(t) + 2y(t)]$$

$$y(t) - 2\tfrac{d}{dt}\, y(t) = \tfrac{d}{dt}\, x(t)$$



Description in frequency domain (only for stable systems):

$$Y(f) - 2j2\pi f\,Y(f) = j2\pi f\,X(f)$$

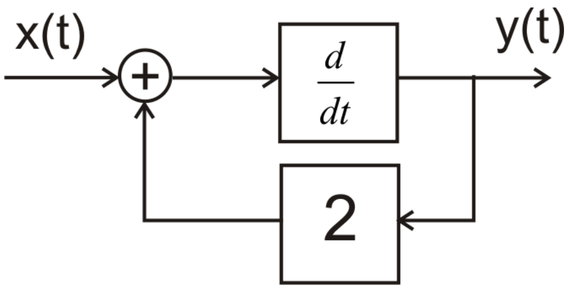
$$H(f) = \frac{Y(f)}{X(f)} = \frac{j2\pi f}{1 - 4j\pi f}$$ If system is stable this is frequency response

Is this system stable? We use Laplace transform to check it:

$$Y(s) - 2sY(s) = s\,X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{1-2s}$$ For $s = \tfrac{1}{2}$ $H(s) \to \infty$. The transfer function H(s) has **a pole** at s > 0 (right side of complex plane), so **the system is unstable.**

Example: block scheme



Description in time domain:

$$y(t) = \tfrac{d}{dt}[x(t) + 2y(t)]$$

$$y(t) - 2\tfrac{d}{dt}y(t) = \tfrac{d}{dt}x(t)$$

Description in frequency domain (only for stable systems):

$$\cancel{Y(f) - 2j2\pi f\,Y(f) = j2\pi f\,X(f)}$$

$$\cancel{H(f) = \frac{Y(f)}{X(f)} = \frac{j2\pi f}{1-4j\pi f}}$$

If system is stable this is frequency response

Is this system stable? We use Laplace transform to check it:

$$Y(s) - 2sY(s) = s\,X(s)$$

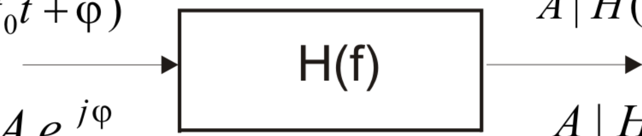
$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{1-2s}$$

For $s = \frac{1}{2}$ $H(s) \to \infty$. The transfer function H(s) has **a pole** at s > 0 (right side of complex plane), so **the system is unstable.**
There is no frequency response, steady state analysis makes no sense.

Time domain analysis (transient analysis) using differential equations or Laplace transform is still possible.

Steady state analysis of LTI systems

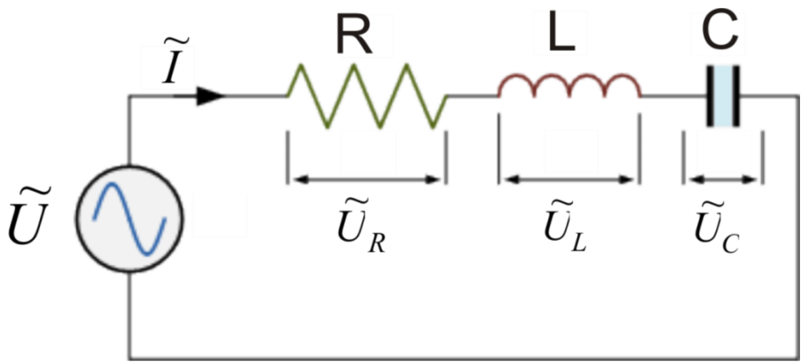
$A\cos(2\pi f_0 t + \varphi)$ $\quad$ $A\,|H(f_0)|\cos(2\pi f_0 t + \varphi + \psi(f_0))$



$A\,e^{j\varphi}$ $\quad$ $A\,|H(f_0)|\,e^{j[\varphi+\psi(f_0)]}$

$$H(f_0) = |H(f_0)|\,e^{j\psi(f_0)}$$

Example: amplitude of input voltage $|\tilde{U}| = A$, frequency $f_{0,}$ find amplitude of current $|\tilde{I}|$



Inductor – impedance $j2\pi f\,L = j\omega L$

Capacitor - impedance $\frac{1}{j2\pi f\,C} = \frac{1}{j\omega C}$

$\omega = 2\pi f \qquad \omega_0 = 2\pi f_0$

$$\tilde{U} = \tilde{U}_R + \tilde{U}_L + \tilde{U}_C = \tilde{I}R + \tilde{I}\,j\omega L + \frac{\tilde{I}}{j\omega C} \qquad \tilde{I} = \frac{\tilde{U}}{R + j\omega L + \frac{1}{j\omega C}} = |\tilde{I}|\,e^{j\psi}$$

Amplitude of current $|\tilde{I}| = \dfrac{|\tilde{U}|}{|R + j\omega_0 L + \frac{1}{j\omega_0 C}|} = \dfrac{A}{|R + j[\omega_0 L - \frac{1}{\omega_0 C}]|} = \dfrac{A}{\sqrt{R^2 + [\omega_0 L - \frac{1}{\omega_0 C}]^2}}$