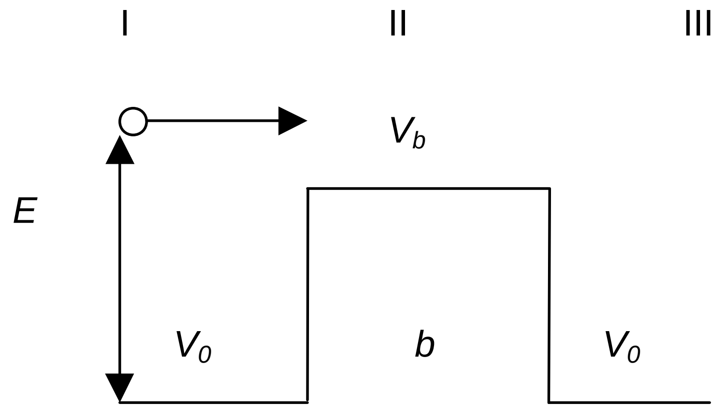
Problem 06: Overbarrier transition



Consider a potential barrier of a width ***b*** and a height $\boldsymbol{V_b}$**,** as in the figure. Assuming: $\boldsymbol{m_I=m_{II}=m_{III}=m^*}$, calculate the probability ***P*** of crossing the barrier by an electron of energy ***E***.

b*= 1 *nm, $\boldsymbol{V_b}$=**2 eV**, $\boldsymbol{V_0}$=**1 eV**, ***m*= 0.5 m₀*** , ***E***=**2.2 eV**

Within the classical physics, since the electron energy ***E*** is higher than the barrier energy $\boldsymbol{V_b}$, the electron would cross it without any obstacle. However, according to the Q-M approach, the electron either crossess the barrier with some probability ***P*** or is reflected from it with some probability ***R*** (note: ***P***+***R***=**1**), depending on both energies ***E***, $\boldsymbol{V_b}$, as well as on the barrier width ***b***.

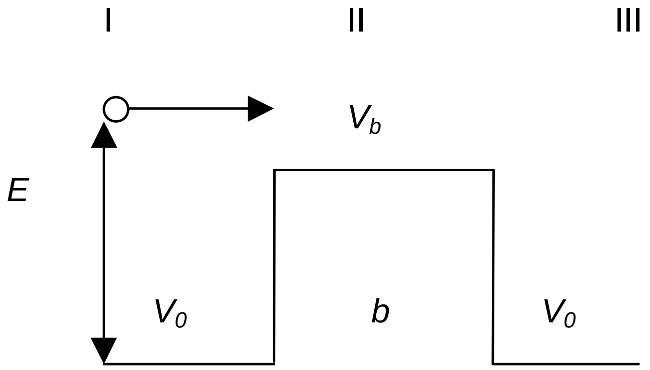
One can obtain (see Lecture) the following expression for the transition probability:

$$P = \frac{4k_1k_3}{(k_1+k_3)^2\cos^2(k_b b) + \left(\frac{k_1\cdot k_3}{k_b} + k_b\right)^2 \sin^2(k_b b)}$$

where $\boldsymbol{k_1, k_b, k_3}$ are wave vectors in regions I, II, and III correspondingly:

$$k = \frac{\sqrt{2m^*(E-V)}}{\hbar}$$

b= 1 nm, **V_b=2 eV**, **V_0=1 eV**, **m^*= 0.5 m_0**, **E=2.2 eV**



$$k_1 = k_3 = \frac{\sqrt{2m^*(E - V_0)}}{\hbar} =$$ **3.967E+09 [1/m]**

$$k_b = \frac{\sqrt{2m^*(E - V_b)}}{\hbar} =$$ **1.619E+09 [1/m]**

$$P = \frac{4k_1k_3}{(k_1 + k_3)^2 \cos^2(k_b b) + \left(\frac{k_1 \cdot k_3}{k_b} + k_b\right)^2 \sin^2(k_b b)} = 0.490$$

h = 6.626 x 10^{-34} [Js]

$$\hbar = \frac{h}{2\pi} = 1.055x10^{-34}\ [Js]$$

m_0 = 9.11 x 10^{-31} [kg]

q = 1.602 x 10^{-19} [C]

k_B = 1.381 x 10^{-23} [J/K]

ε_0 = 8.854 x 10^{-12} [F/m]

c = 3 x 10^8 [m/s]

Since its trigonometrical nature, the probability is fluctuating.
Lets find its maximum value:

$$P = \frac{4k_1k_3}{(k_1+k_3)^2\cos^2(k_bb) + \left(\frac{k_1\cdot k_3}{k_b}+k_b\right)^2 \sin^2(k_bb)}$$

$$P = P_{\max} \ \ if \ \ Den = (k_1+k_3)^2\cos^2(k_bb) + \left(\frac{k_1\cdot k_3}{k_b}+k_b\right)^2 \sin^2(k_bb) = \min \qquad if \ \ \frac{\partial Den}{\partial(k_bb)} = 0$$

$$\frac{\partial Den}{\partial k_bb} = C_1 2\cos(k_bb)(-\sin(k_bb)) + C_2 2\sin(k_bb)\cos(k_bb) = \sin(2k_bb)(C_2 - C_1) = 0$$

$$if \ \ 2k_bb = n\pi$$

Since the above condition corresponds to both minima and maxima, restricting it to $\boldsymbol{P_{max}}$ only yieds:

$$2k_bb = n2\pi$$

Find $\boldsymbol{P_{max}}$?

$$P_{\max} = \frac{4k_1k_3}{(k_1+k_3)^2}$$

If $\boldsymbol{k_1}$ = $\boldsymbol{k_3}$ then $\boldsymbol{P_{max}}$ = 1

Find the first three energies $\boldsymbol{E_n}$ for which the transition probability reaches $\boldsymbol{P_{max}}$?

b*= 1 nm**, ***V**$_b$*=2 eV**, ***V**$_0$*=1 eV**, ***m*= 0.5 m**$_0$**, ***E=2.2 eV**

Find the first three energies E_n for which the transition probability reaches P_{max} ?

$$k_b = \frac{\sqrt{2m^*(E - V_b)}}{\hbar} =$$

1.619E+09 [1/m]

$$2k_b b = n2\pi \qquad k_b = \frac{\sqrt{2m^*(E_n - V_b)}}{\hbar}$$

$$2 \cdot \frac{\sqrt{2m^*(E_n - V_b)}}{\hbar} \cdot b = n2\pi \longrightarrow E_n = V_b + \frac{\hbar^2 \pi^2}{2m^*} \frac{n^2}{b^2}$$

n	En [eV]
1	2.753
2	5.011
3	8.774

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