Circuits and Signals

Signals, Devices, Circuits, Kirchhoff's Laws

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Lumped Device Model

Real-life circuits are governed by Maxwell (PD) equations. Making some simplifying assumptions (given below) one may reduce Maxwell equations to algebraic or ODE equations.

- electric devices are defined completely in terms of their terminal currents and inter-terminal voltages only,
- the devices are joined by a network of perfectly conducting wires,
- electric signals propagation times may be neglected.

Electric Current

Electric current is the rate of flow of electric charge through a conductor.

e.g. $\quad i_1 = 1\,\text{A} \qquad i_2 = -1\,\text{A}$

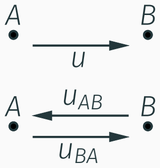
or $\quad i_1 = -2\,\text{mA} \qquad i_2 = 2\,\text{mA}.$

SI prefixes

Prefix	Symbol	factor	Prefix	Symbol	factor
yocto	y	10^{-24}	deca	da	10^{1}
zepto	z	10^{-21}	hecto	h	10^{2}
atto	a	10^{-18}	kilo	k	10^{3}
femto	f	10^{-15}	mega	M	10^{6}
pico	p	10^{-12}	giga	G	10^{9}
nano	n	10^{-9}	tera	T	10^{12}
micro	μ	10^{-6}	peta	P	10^{15}
milli	m	10^{-3}	exa	E	10^{18}
centi	c	10^{-2}	zetta	Z	10^{21}
deci	d	10^{-1}	yotta	Y	10^{24}

Electric Voltage

Voltage across two terminals is the work (energy) required to move a unit positive charge from one terminal to the other. It can be computed as the difference between electric potentials of the terminals.



$$u_{BA} = e_B - e_A,$$

$$u_{AB} = e_A - e_B = -u_{BA},$$

$$u_{AA} = e_A - e_A = 0,$$

$$u_{BA} = \underbrace{e_B - e_C}_{u_{BC}} + \underbrace{e_C - e_A}_{u_{CA}} = u_{BC} + u_{CA}.$$

Voltage u_{BA} between terminals A and B is also called a (electric) potential of B with respect to A.

Electric Signals

Voltage and current are fundamental (electric) signals that we will deal with.

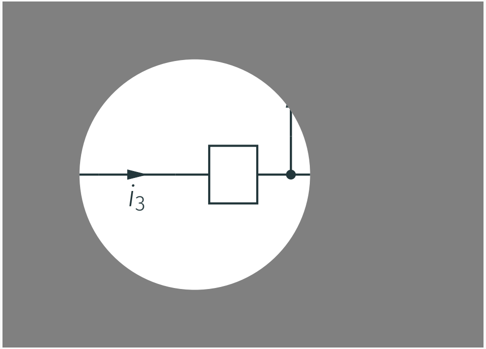
These are usually continuous signals, i.e., they are continuous functions of time: $u = u(t)$, $i = i(t)$.

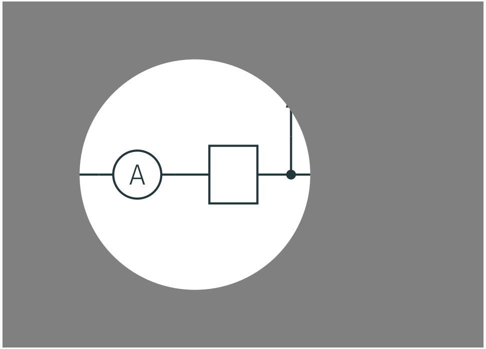
Some subclasses of continuous signals have simple algebraic representations.

constant signals (e.g. $u(t) = 1\,\mathrm{V}$) — numbers ($U = 1\,\mathrm{V}$),
alternating signals (e.g. $i(t) = I_m \cos(\omega t + \phi)$) — triples (ω, I_m, ϕ) or phasors $I = I_m e^{\jmath\phi}$,
periodic signals — Fourier series coefficients.

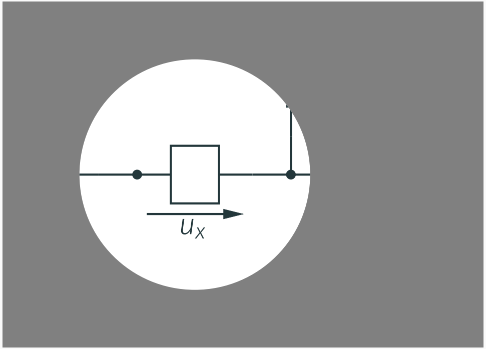
We will learn how to make an effective use of these representations.

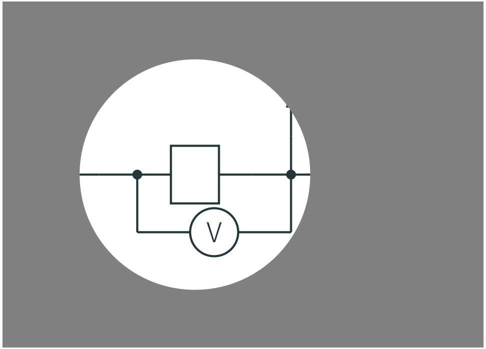
How to measure currents



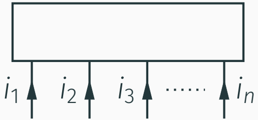


How to measure voltages





Lumped Devices do not accumulate/produce charge



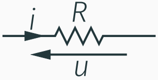
We assume that $i_1 + i_2 + \cdots + i_n = 0$ (the total charge stored in the device is preserved).

The device is described by a set of relations (equations) that relates terminal currents and inter-terminal voltages.

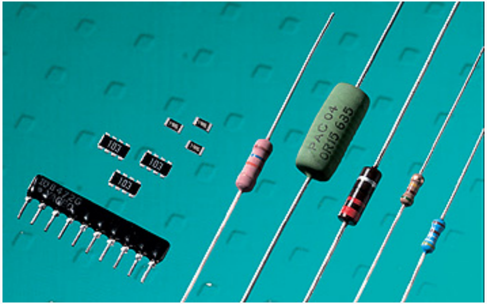
Linear devices — examples

real-life ...

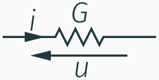
... and its symbol



$$\boxed{u = Ri}$$



R [Ω] is called resistance.

Alternative description: 

, $\boxed{i = Gu}$, $G = \frac{1}{R}$,

G [S] = [℧] is called conductance.

Alternative symbols: 

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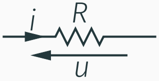
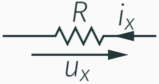
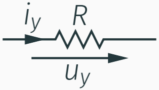
Voltage and current directions



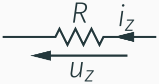
$$u = Ri$$



$$u_x = Ri_x \qquad (u_x = -u, \quad i_x = -i)$$

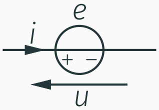


$$u_y = -Ri_y \qquad (u_y = -u, \quad i_y = i)$$



$$u_z = -Ri_z \qquad (u_z = u, \quad i_z = -i)$$

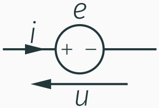
(Independent) Voltage Source

$$\boxed{u = e},$$

e [V] is called electromotive force.

Alternative symbols:



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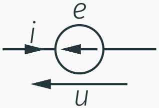

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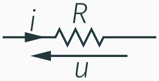
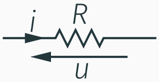
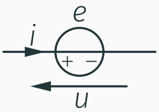
Symmetry of device's terminals



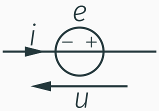
$u = Ri$

rotated by 180°: 

$u = Ri$

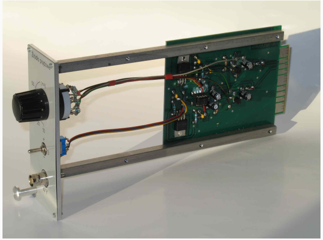


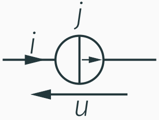
$u = e$

rotated by 180°: 

$u = -e.$

(Independent) Current Source

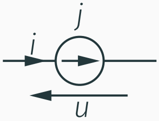




, $\boxed{i = j}$,

j [A] is the generated current.

Alternative symbols:



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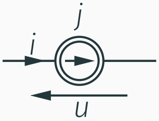

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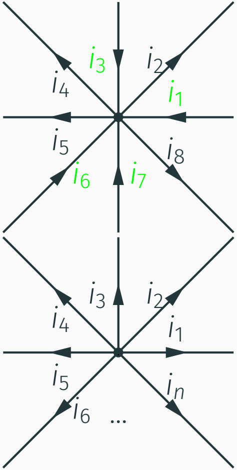
Kirchhoff laws

Kirchhoff's Current Law (KCL)

The total sum of the currents converging to a node equals the sum of all the currents diverging from the node.

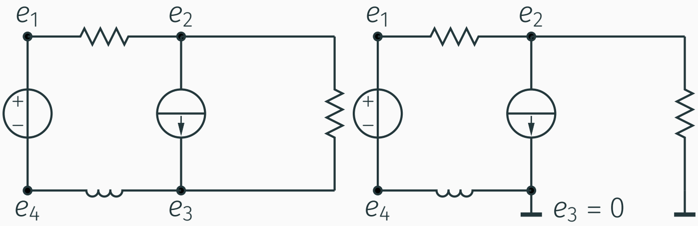


$$i_1 + i_3 + i_6 + i_7 = i_2 + i_4 + i_5 + i_8.$$

$$i_1 + i_2 + \cdots + i_n = 0.$$

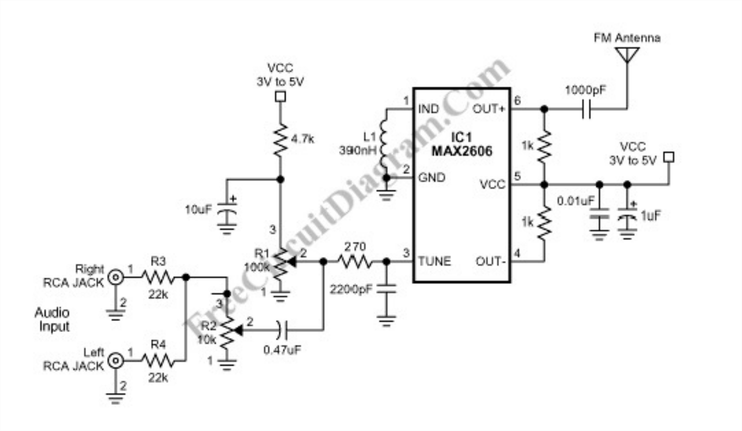
Kirchhoff's Voltage Law (KVL)

Electric potentials (with respect to any point) of the circuit's nodes are well defined.



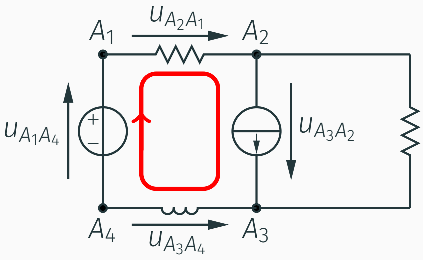
The node with respect to which we measure electric potentials is called ground or earth ($\perp$ or ⏚).

Ground convention — an example



Kirchhoff's Voltage Law (KVL) — loop version

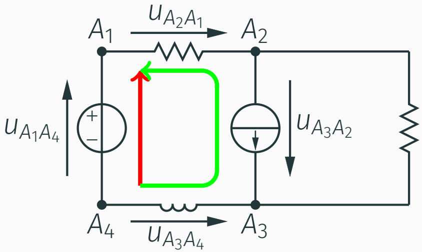
The algebraic sum of the voltage drops along any directed loop equals zero.



$$u_{A_1A_4} + u_{A_2A_1} + u_{A_3A_2} - u_{A_3A_4} = 0.$$

Kirchhoff's Voltage Law (KVL) — path version

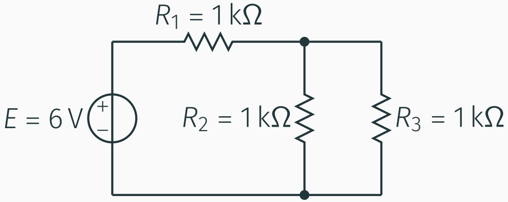
The voltage between any two nodes does not depend on the (oriented) path along which it is computed.



$$u_{A_1A_4} = u_{A_3A_4} - u_{A_3A_2} - u_{A_2A_1}.$$

Circuit solving — example

How to find all the currents and voltages in a circuit



Questions/Comments