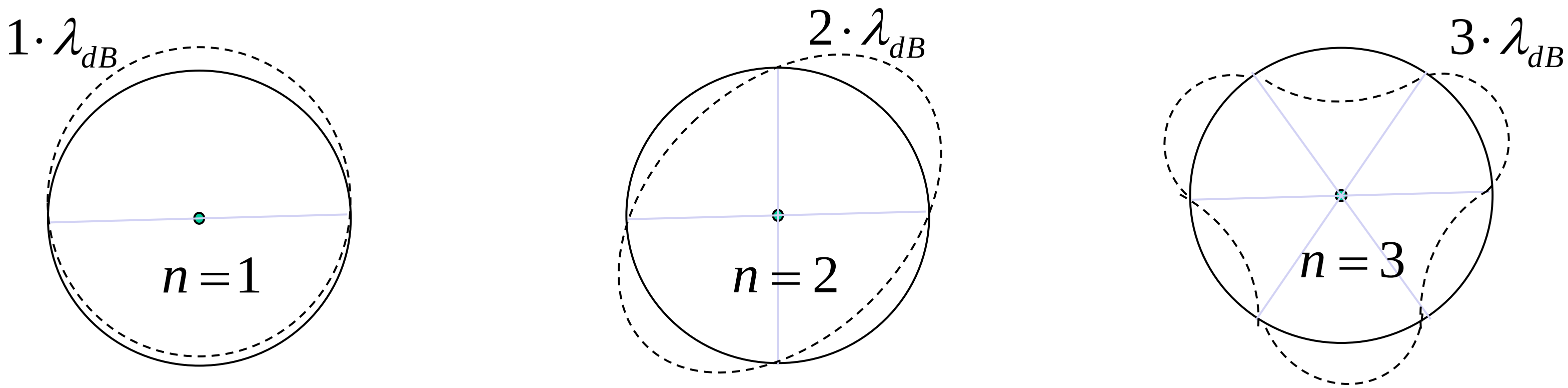
De Broglie's hypothesis: wavelike property of a material particle

de Broglie interpretation of the **H** atom orbits:
the circumference of the orbit must be an integer multiple of the wavelength; the stationary orbit corresponds to a standing wave around the nucleus

$$mv_n r_n = n\frac{h}{2\pi} \qquad p\cdot r_n = n\frac{h}{2\pi} \qquad 2\pi r_n = n\lambda_{dB} \quad \Rightarrow \quad \boxed{\lambda_{dB} = \frac{h}{p}}$$



Problem 04a: de Broglie wavelength

Consider the following systems and determine which approach is more relevant: the classical physics approach or the quantum mechanical approach.

a) an electron in the beam of a cathode-ray tube (CRT) accelerated by a voltage of **4 KV,** **Length ~ 20 cm**

The laws of classical physics apply to the motion of a particle if its de Broglie wavelength is much smaller than the dimensions of the system under consideration. Otherwise the quantum mechanical approach must be applied. So, one must find the de Broglie wavelength for the particle and compare it with the dimensions of a system in which it is considered.

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{m_0 v}$$

The electron gains its kinetic energy from the potential energy of the field:

$$\frac{m_0 v^2}{2} = qV \longrightarrow v = \sqrt{\frac{2qV}{m_0}}$$

$$v = \sqrt{\frac{2 \cdot 1.601 \cdot 10^{-19}[C] \cdot 4000[V]}{9.11 \cdot 10^{-31}[kg]}} = 3.75 \cdot 10^{7}[m/s]$$

$\lambda_{dB} << L$, ***classical***

$$\lambda_{dB} = \frac{h}{m_0 v} = \frac{6.626 \cdot 10^{-34}[J \cdot s]}{9.11 \cdot 10^{-31}[kg] \cdot 3.75 \cdot 10^{7}[m/s]} = 1.94 \cdot 10^{-11}[m]$$

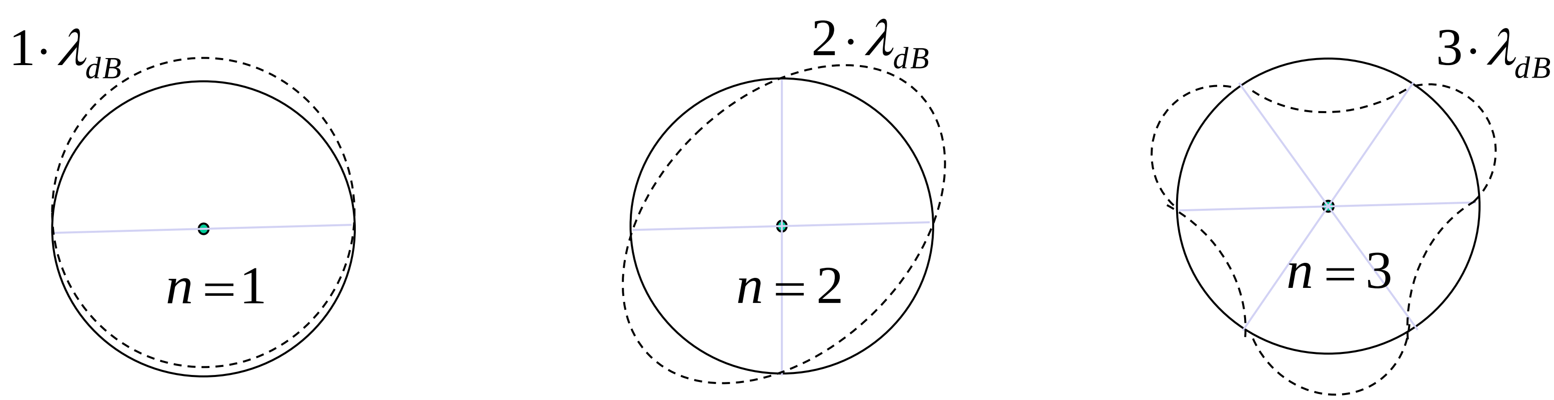
homework

Calculate λ_{dB} for an electron on *your (EELE1 list)* orbit in the **H** atom.

Hard way:

$$\lambda_{dB} = \frac{h}{p} \qquad v_n = \frac{q^2}{4\pi\varepsilon_0\hbar}\frac{Z}{n}$$

Smart way: utilize de Broglie interpretation of the **H** atom orbits: $2\pi r_n = n\lambda_{dB}$



$$\lambda_{dB} = \frac{2\pi r_n}{n} \qquad r_n = \frac{4\pi\varepsilon_0\hbar^2}{m_0 q^2}\frac{n^2}{Z} \overset{Z=1}{=} a_0 n^2 \qquad a_0 = 0.53\,[\text{Å(angstrom)}]$$

Problem 04b: de Broglie's wavelength and dimensions of semiconductor structures

Electrons in semiconductors have different kinetic energy $\boldsymbol{E_{kin} = m_e v^2/2}$ *and their average energy is equal to* $\boldsymbol{3/2 k_B T}$*, where* k_B *is Boltzmann's constant, T is the temperature and* $\boldsymbol{m_e}$ *is the electron effective mass. Dynamics of motion of electrons in solid state is described by the effective mass* $\boldsymbol{m_e}$ *instead of the free electron mass* $\boldsymbol{m_0 = 9.1 \times 10^{-31}}$ ***kg.***

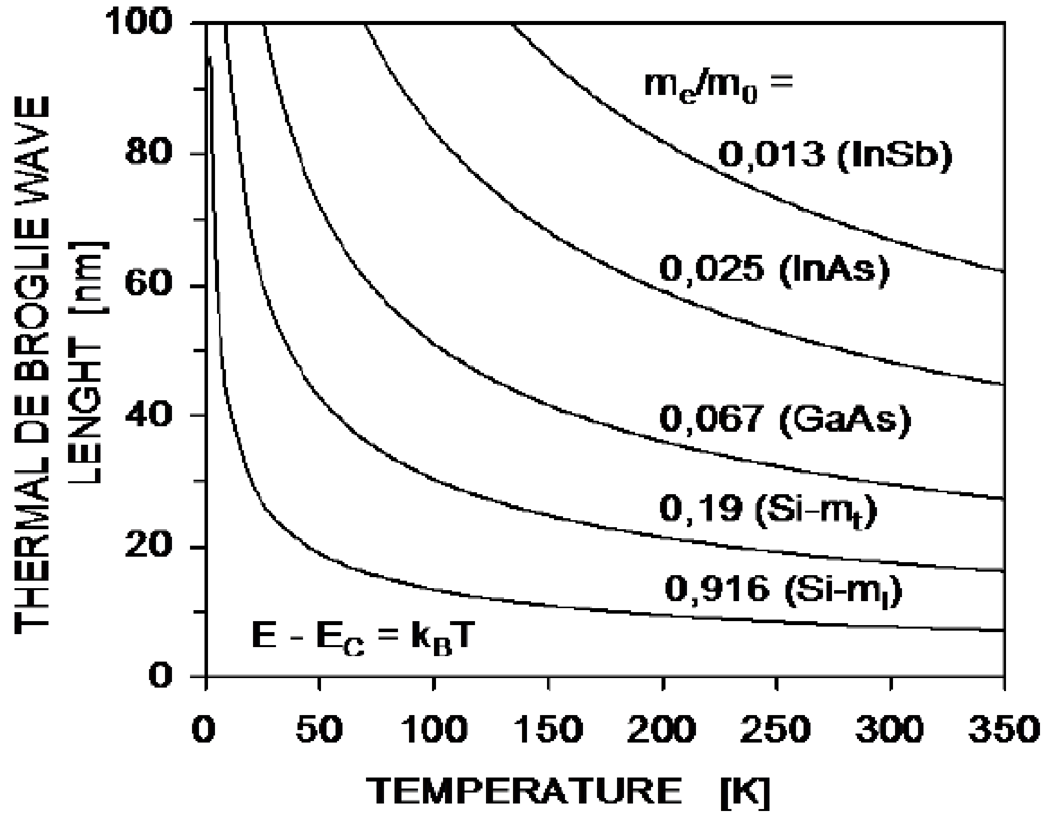
How small should be the length of the active region of a device made of a given semiconductor (Si, GaAs, InAs, InSb) in order to be comparable with the de Broglie's wavelength of electrons having a kinetic energy of the order of $\boldsymbol{3/2 k_B T}$ at **T = 300K**?

At T=300K

Solution:

$$\lambda_{dB} = \frac{h}{p} \qquad p = \sqrt{2 m_e E} \qquad E_k = \frac{3}{2} k_B T \qquad E_k = 6.21\ x10^{-21}\,[J]$$

Semiconductor	$\boldsymbol{m_e}$ (effective mass)	***p*** [kg x m/s]	***v*** [m/s]	$\boldsymbol{\lambda_{dB}}$ [nm]
Si	0.916 m_0	1.02E-25	1.22E+05	**6.5**
GaAs	0.067 m_0	2.75E-26	4.51E+05	**24.1**

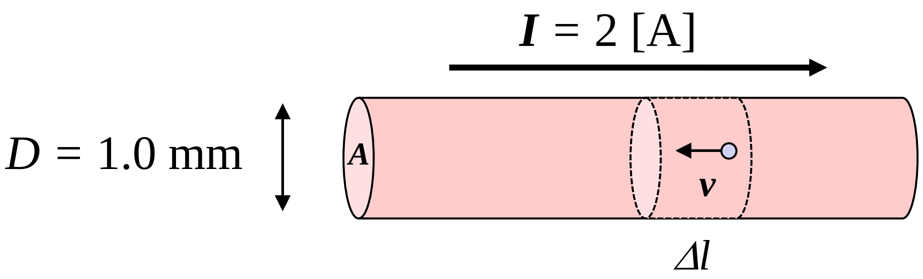


Conclusions:

- Electrons in silicon structures of dimensions less than 10 nm should exhibit wave properties even at room temperature.
- This should be easier in nanostructures made of other semiconductors of less effective mass.
- A decrease in temperature allows observation of electron wave phenomena in larger devices.

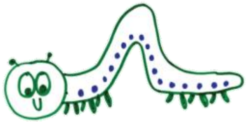
Problem 04c: de Broglie wavelength

Find the de Broglie wavelength λ_{dB} of electrons flowing in a copper wire of a diameter D in a stream constituting current I. Assume the density of electrons in the copper material to be ρ **= 10^{21}** [cm^{-3}] and free electron mass.

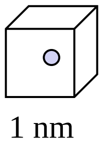


$$I = \frac{\Delta Q}{\Delta t} = \frac{q\rho \Delta V}{\Delta t} = \frac{q\rho A \Delta l}{\Delta t} = q\rho A v$$

$$v = \frac{I}{q\rho A} = \frac{2\,[A]}{1.602\cdot 10^{-19}[C]\cdot 10^{27}[m^{-3}]\cdot \pi \cdot \left(\frac{0.001}{2}\right)^2 [m^2]} = 0.0159\ [\text{m/s}] = 1.59\ [\text{cm/s}]$$



Average distance between electrons: $l_{aver} = \sqrt[3]{\frac{1}{\rho}} = 1\,nm$



$$\lambda_{dB} = \frac{h}{p}$$

λ_{dB} = 4.58 [cm]