Circuits and Signals

Equivalent devices (continuation)

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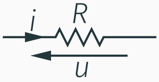
Equivalent devices

Devices are called equivalent if they are governed by equivalent equations.

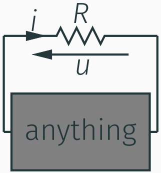
Device equations determines the behaviour of the device in any circuit!

Device equation — an example

Device equation:



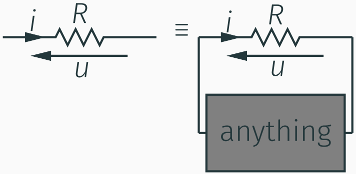
$u = Ri$



$u = Ri$

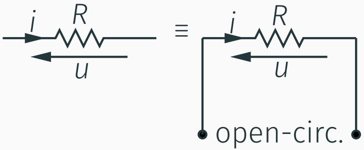
Device as a part of a bigger circuit vs device as independent circuit

Device equation:



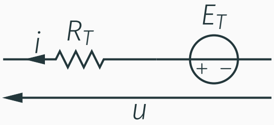
$u = Ri$

Device as independent circuit:



$u = Ri$, $i = 0$, $u = 0$.

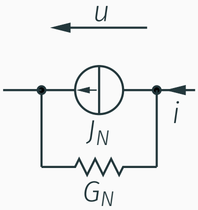
Thévenin's equivalent



, $R_T \neq 0$

$$\boxed{u = E_T - R_T i.}$$

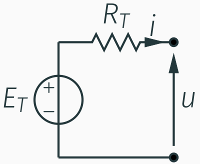
Norton's equivalent

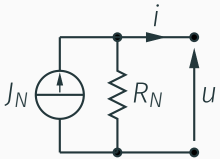


, $\quad G_N \neq 0 \quad \left(R_N = \frac{1}{G_N}\right)$

$$\boxed{i = J_N - uG_N = J_N - \frac{u}{R_N}.}$$

Thévenin's and Norton's equivalents equivalence



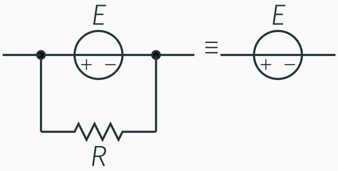


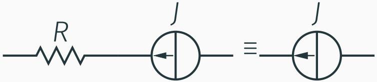
Thévenin's and Norton's equivalents are equivalent if and only if

$$R_T = R_N \quad \text{and} \quad E_T = J_N R_N.$$

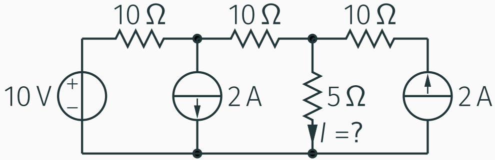
$$u = E_T - R_T i \qquad\qquad i = J_N - \frac{u}{R_N} \quad \equiv \quad u = J_N R_N - R_N i.$$

Attention:

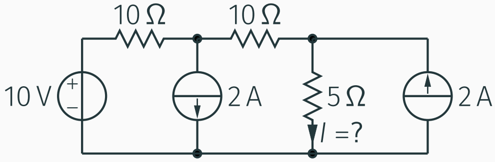




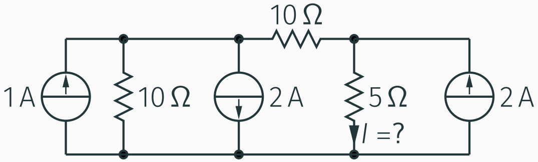
One-port equivalences — an example



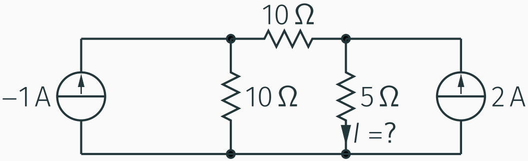
One-port equivalences — an example



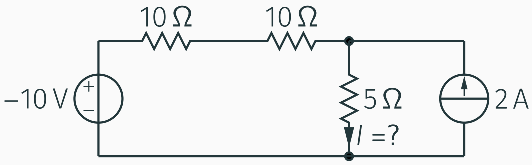
One-port equivalences — an example



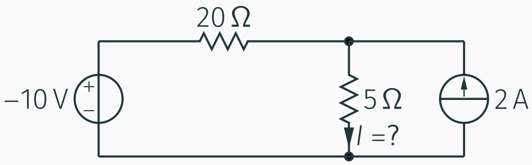
One-port equivalences — an example



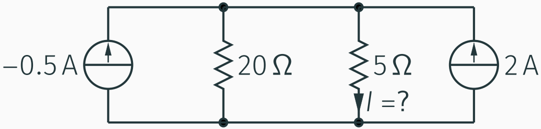
One-port equivalences — an example



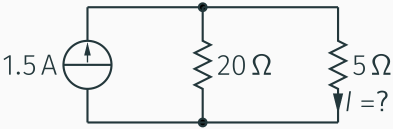
One-port equivalences — an example



One-port equivalences — an example



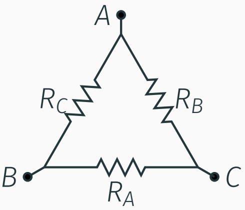
One-port equivalences — an example

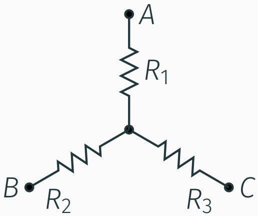


$$I \overset{\text{CDF}}{=} \frac{3}{2} \times \frac{20}{20+5} \text{A}$$

$$= \frac{6}{5} \text{A}$$

Δ – ★ and ★ – Δ transformations





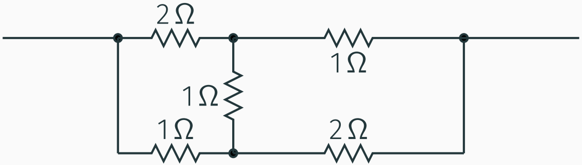
Equivalent if:

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1},$$
$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2},$$
$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}.$$

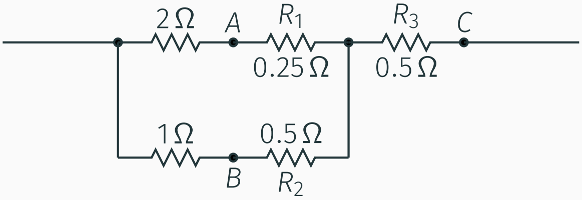
In other words:

$$R_1 = \frac{R_BR_C}{R_A + R_B + R_C},$$
$$R_2 = \frac{R_CR_A}{R_A + R_B + R_C},$$
$$R_3 = \frac{R_AR_B}{R_A + R_B + R_C}.$$

Every one-port comprising of resistors only is equivalent to a resistor

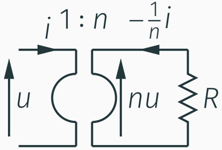


Every one-port comprising of resistors only is equivalent to a resistor



$$R_{eq} = \left(\frac{9}{4} \| \frac{3}{2} + \frac{1}{2}\right)\Omega = \left(\frac{\frac{9}{4} \times \frac{3}{2}}{\frac{9}{4} + \frac{3}{2}} + \frac{1}{2}\right)\Omega = \left(\frac{27}{30} + \frac{1}{2}\right)\Omega = \frac{7}{5}\Omega.$$

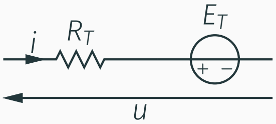
A one-port comprising of different devices can still be equivalent with a resistor



$$u \stackrel{?}{=} R_{\text{eq}} i$$

$$nu = -R(-\frac{1}{n}i), \qquad u = \underbrace{\frac{1}{n^2}R}_{R_{\text{eq}}} \, i.$$

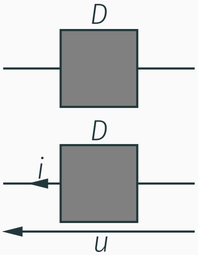
A one-port comprising of different devices does not have to be equivalent to a resistor



, $E_T \neq 0$

$$u = E_T + R_T i.$$

What is a linear DC one-port equivalent to?

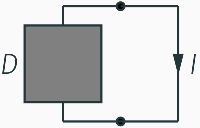


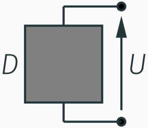
The following cases are possible:

$i = J_N$,	u arbitrary,		current source,
i arbitrary,	$u = E_T$,		voltage source,
i arbitrary,	$u = E_T - iR_T$,	$R_T \neq 0$,	Thévenin's equivalent,
$i = J_N$,	$u = E_T$,		fixator,
i arbitrary,	u arbitrary,		norator.

What is a linear DC one-port equivalent to?

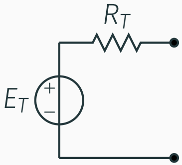
Consider two connections (not necessarily circuits!):



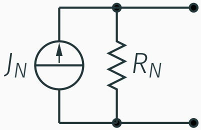


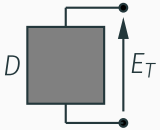
I	U	one-port's type
$*$	$*$	norator
$\varnothing$	$\varnothing$	fixator
$\varnothing$	E	voltage source or fixator generating zero current
J	$\varnothing$	current source or fixator generating zero voltage
J	E	Thévenin's equivalent: $E_T = E$, $R_T = E/J$ (equivalently, Norton's equivalent: $J_N = J$, $R_N = E/J$)
0	0	nullator or resistor.

Thévenin's and Norton's equivalents parameters

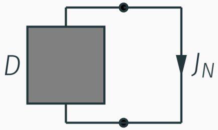


$$\overset{E_T=J_N R_N,\ R_T=R_N}{\equiv}$$





E_T is the open-circuit voltage,



J_N is the short-circuit current,

$$R_T = R_N = \frac{E_T}{J_N}.$$

An alternative way of determining internal (output) resistance

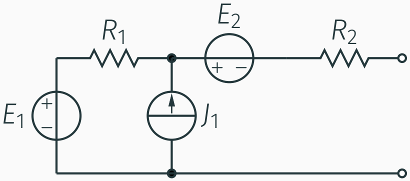
If short-circuit current is non-zero:

$$R_T = R_N = \frac{\text{open-circuit voltage}}{\text{short-circuit current}}.$$

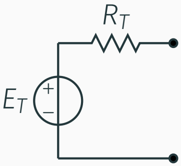
In general:

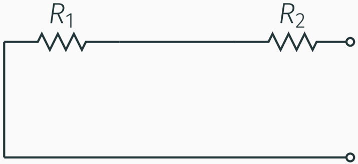
The internal (output) resistance $R_T = R_N$ of a one-port equals the equivalent resistance of the reduced one-port that results from the original one-port by reduction of all the independent sources to zeros.

Thévenin's equivalent parameters — an example

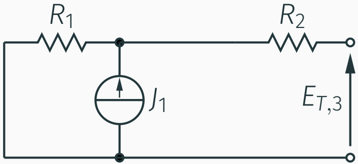


$\equiv$



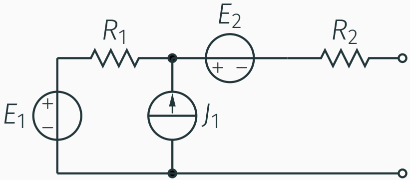


$$R_T = R_1 + R_2.$$

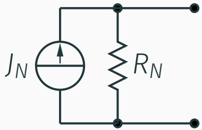


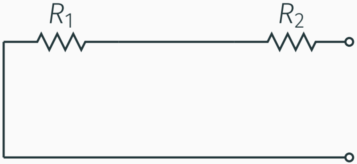
$$E_T = \underbrace{E_1}_{} + \underbrace{(-E_2)}_{} + \underbrace{J_1 R_1}_{}.$$

Norton's equivalent parameters — an example

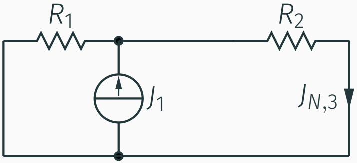


$\equiv$





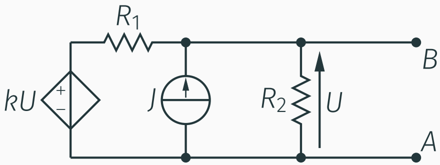
$$R_T = R_1 + R_2.$$



$$J_N = \underbrace{\frac{E_1}{R_1+R_2}}_{J_{N,1}} + \underbrace{\frac{-E_2}{R_1+R_2}}_{J_{N,2}} + \underbrace{J_1\frac{R_1}{R_1+R_2}}_{J_{N,3}}.$$

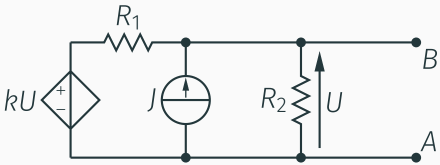
Thévenin equivalent – another example – 1st option (find E_T and R_T)

$J = 6\,\mathrm{mA},\ k = \frac{1}{4},\ R_1 = 1.5\,\mathrm{k\Omega},\ R_2 = 6\,\mathrm{k\Omega}.$



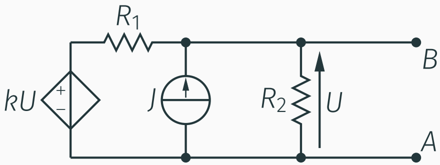
Thévenin equivalent – another example – 2nd option (find E_T and J_N; then $R_T = \frac{E_T}{J_N}$)

$J = 6\,\mathrm{mA},\ k = \frac{1}{4},\ R_1 = 1.5\,\mathrm{k\Omega},\ R_2 = 6\,\mathrm{k\Omega}.$



Thévenin equivalent – another example – 3rd option (find J_N and R_T; then $E_T = J_N R_T$)

$J = 6\,\mathrm{mA},\ k = \frac{1}{4},\ R_1 = 1.5\,\mathrm{k\Omega},\ R_2 = 6\,\mathrm{k\Omega}.$



Questions/Comments