EPRST: Probability and Statistics
Problem set 0

1. Let A, B and C be events. Using the set operations: union, intersection and complementation, present the following events in terms of A, B and C:
   (a) all three events A, B and C occur,
   (b) at least one of the events A, B and C occurs,
   (c) none of the events A, B and C occurs,
   (d) at most one of the events A, B and C occurs,
   (e) exactly one of the events A, B, C occurs.
2. How many ways are there to permute the letters in the word
   - STATISTICS,
   - MISSISSIPPI.
3. How many ways are there to split 12 people into 3 teams
   (a) where one team has 2 people and the other two teams have 5 people each?
   (b) where each team has 4 people?
4. A student is obliged to take 7 out of a list of 20 courses, with the constraint that at least 1 of the 7 courses must be a probability course. Suppose that 5 of the 20 courses are probability courses. How many choices are there for 7 courses to take?
5. Three friends and seven other people are randomly seated in a row. How many possible ways are there for the friends to be seated next to each other?
6. Find the number of ways to put k indistinguishable balls into n distinguishable boxes.
7. There are k people in a room. What is the probability that two or more people in the group have the same birthday?
8. If we roll two fair dice, which is more likely: a sum of 11 or a sum of 12?
9. Which of the following events has the highest probability?
   A: at least one 6 appears when 6 fair dice are rolled?
   B: at least two 6's appear when 12 dice are rolled?
   C: at least three 6's appear when 18 fair dice are rolled?
10. Let $p \in [0, 1]$. On the real line, at points with natural coordinates ($n = 1, 2, 3, \ldots$), we put some weights. Let m_n be the mass of the weight placed at n. Calculate the total mass of all weights if
    (a) $m_n = 1/2^n$,
    (b) $m_n = p^n$,
    (c) $m_n = (1-p)^{n-1}p$,
    (d) $m_n = 1/n$.

    What is the total mass of the weights placed at even numbers if $m_n = (1-p)^{n-1}p$?
11. Let $n \in \mathbb{N}$.

(a) Prove **Newton's binomial formula** : for any $x, y \in \mathbb{R}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

(b) Let $p \in [0,1]$. Calculate in the closed form

$$\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}.$$

(c) Calculate in the closed form

$$\sum_{k=0}^{n} \binom{n}{k} 2^k.$$

(d) Calculate in the closed form

$$\sum_{k=0}^{n} \binom{n}{k}.$$