

1. Find the solution of the differential equation

$$y''(x) - 4y'(x) + 3y(x) = 2e^x - 4e^{2x}.$$

satisfying the initial conditions $y(0) = 1$ and $y'(0) = 4$.

ANSWER: $y(x) = 4e^{2x} - (x + 3)e^x$

2. Find the general solution of the differential equation

$$y''(x) - 4y'(x) + 4y(x) = \frac{2e^{2x}}{x^2 - 1}.$$

ANSWER: $y(x) = (C_1 + C_2 x)e^{2x} + ((x-1)\log(x-1) - (x+1)\log(x+1))e^{2x}$

3. Consider the differential equation

$$y''(x) - 6y'(x) + 8y(x) = 4xe^{2x}.$$

Assume that we introduce a new unknown function by substituting $y(x) = z(x) \cdot e^{2x}$ and divide the equation by e^{2x} . Write down the new equation and find its solution (at least one).

ANSWER: $z''(x) - 2z'(x) = 4x$ with a solution $z(x) = -x^2 - x$.

4. Find the general form of sequences (a_n) satisfying the equation

$$a_{n+2} - 6a_{n+1} + 8a_n = 3^n - 12.$$

ANSWER: $a_n = A \cdot 4^n + B \cdot 2^n - 3^n - 4$