

EPRST: Probability and Statistics
Problem set 7

1. We toss a non-symmetric coin (the probability that the coin will come up a tail is p , $p \in (0, 1)$) until we get a tail or two heads in a row. Compute the expected value of the number of tosses.
2. Let X be a random variable with probability mass function:

$$p_X(x) = \mathbb{P}(X = x) = \begin{cases} \frac{x}{8} & \text{for } x = 1, 2, 5, \\ 0 & \text{otherwise.} \end{cases}$$

Find

- (a) $\mathbb{E}(X)$, $\text{Var}(X)$,
 - (b) $\mathbb{E}(2X + 3)$, $\text{Var}(2X + 3)$,
 - (c) $\mathbb{E}(3X - 5X^2 + 1)$.
3. Compute the first two moments (that is $\mathbb{E}Y$ and $\mathbb{E}Y^2$) of $Y = 2^{-X}$ if $X \sim b(n, p)$.
 4. Four buses carry 40, 33, 25, and 50 students (total 148), respectively. One student is randomly selected. Let X denote the number of students that were on the bus that carries this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus. Which of $\mathbb{E}(X)$ and $\mathbb{E}(Y)$ is larger?
 5. Twelve people, consisting of 6 married couples, are to be seated at three different tables, with four people at each table. If the seating is done at random, what is the expected number of married couples that are seated at the same table?
 6. A distracted man has n keys, one of which opens the door to his office. He tries the keys at random, one by one, and independently. Compute the mean of the number of trials required to open the door if the wrong keys
 - (a) are not eliminated,
 - (b) are eliminated.
 7. A group of N people throw their hats into the center of a room. The hats are mixed and each person randomly selects one. Find the expected number of people that select their own hat.
 8. The radius of a circle, R , has a probability density function:

$$f(x) = \begin{cases} 6r(1-r), & 0 < r < 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the expected value of the radius.
 - (b) Find the expected circumference.
 - (c) Find the expected area.
9. The density of random variable X is $f(x) = c x^4 \mathbb{1}_{[-1,1]}(x)$. Find the value of constant c and compute the expected value $\mathbb{E}(3X + 5)$ and the variance $\text{Var}(2X - 1)$.
 10. Let

$$f(x) = \begin{cases} 1 - |x - 1|, & x \in [0, 2], \\ 0, & x \notin [0, 2], \end{cases}$$

be the density of random variable X . Compute $\text{Var}(3X + 5)$.

11. Let $X \sim \mathcal{N}(-3, 4)$. Calculate $\mathbb{E}(X^2 - 3X + 2)$.
12. The random variable X has the following cumulative distribution function F :

$$F(t) = \begin{cases} 0, & t < 0, \\ t(2-t), & 0 \leq t < 1, \\ 1, & t \geq 1. \end{cases}$$

Compute $\mathbb{E}X$, $\text{Var}(X)$, $\mathbb{E}e^X$.