EPRST: Probability and Statistics
Problem set 4

1. The probability density function f of a continuous random variable X is given by

$$f(x) = \begin{cases} cx + 3, & -3 \le x \le -2, \\ 3 - cx, & 2 \le x \le 3, \\ 0, & \text{otherwise}. \end{cases}$$

   (a) Compute c.
   (b) Determine the cumulative distribution function of X.
   (c) Compute $\mathbb{P}(-1 < X < 1)$.

2. Let Z be a continuous random variable with probability density function

$$f(z) = \begin{cases} a(1 + z^2), & -2 < z < 1, \\ 0, & \text{otherwise}. \end{cases}$$

   (a) Determine a.
   (b) Find the cumulative distribution function of Z.
   (c) Compute $\mathbb{P}(|X| < 1/2)$.

3. We are flipping a coin until "6" comes up. Given that we've already been throwing 10 times find the probability that in the next 20 throws there will be no "6" as well.

4. Let X be chosen uniformly at random from the interval $[0, 3]$. What is the probability that $X^2 - 5X + 6 > 0$?

5. The distribution of the random variable X is **symmetric** with respect to 0, if $\forall x \in \mathbb{R}$: $\mathbb{P}(X \ge x) = \mathbb{P}(X \le -x)$. Show that for the random variable X having symmetric distribution, for all $t > 0$, the following equations hold:

   (a) $\mathbb{P}(|X| \le t) = 2F_X(t) - 1$,
   (b) $\mathbb{P}(|X| > t) = 2\,(1 - F_X(t))$,
   (c) $\mathbb{P}(X = t) = F_X(t) + F_X(-t) - 1$.

6. Andy passes by 5 traffic lights on his way to the EiTI building. If he doesn't need to stop at any lights, the trip takes him 18 minutes. Assume that each trafic light is red with probability $\frac{1}{3}$ independently of the other lights. If he needs to stop on the red light his trip is one minute longer.

   (a) Determine the distribution of the time T that Andy needs to get to the EiTI building. Compute $\mathbb{P}(T > 19)$.
   (b) Given that the trip took him at most 19 minutes, find the probability that he hadn't stopped at any lights.

7. Let n be a positive integer and

$$F_X(t) = \begin{cases} 0, & t < 0, \\ \frac{\lfloor t \rfloor}{n}, & 0 \le t < n, \\ 1, & t \ge n, \end{cases}$$

   where $\lfloor t \rfloor$ is the greatest integer less than or equal to t. Find the probability mass function (PMF) of X.

8. Andy is playing a game that has 7 levels. He starts at level 1 and has probability p_1 of reaching level 2. In general, given that he reaches level j, he has probability p_j of reaching level $j + 1$, for $1 \le j \le 6$. Let X be the heighest level that he reaches . Find the PMF and CDF of X (in terms of $p_1, \ldots, p_6$).