EPRST - TEST

Instructions

- **You are obliged to copy and sign the following lines to the first page of the work you will be submitting:**

  I declare that this piece of work which is the basis for recognition of achieving learning outcomes in the EPRST course was completed on my own.
  First and last name:
  Student record book number (Student ID number):

- Be sure to show your work and explain your reasoning so that it is clear how you got your answers.

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1. Let $X \sim \mathcal{N}(-1, 4)$.
   (a) Find a, b such that $\mathbb{P}(X \leq a) = \mathbb{P}(a \leq X \leq b) = \mathbb{P}(X \geq b)$.
   (b) Compute $\mathbb{E}X(X-1)$.

2. Assume that Ω is a triangle with vertices $(0,0), (1,1), (0,1)$ and $\mathbb{P}$ stands for the geometrical probability on Ω. Let X be a random variable defined as
$$X(x,y) = y - x, \quad (x,y) \in \Omega.$$
   Find the distribution of X. Compute $\mathbb{E}X$.

3. There are two dice, one with four faces and one with six faces. We pick one dice at random and keep rolling it, summing the obtained numbers, until the sum exceeds 2. Let X be the number of rolls.
   (a) Find the distribution of X.
   (b) Suppose we have made two rolls. What is the probability that the four-sided dice was picked?

4. A random variable X has a continuous distribution with the density
$$f(x) = \begin{cases} x+2, & x \in [-2,-1], \\ 2-x, & x \in [1,2], \\ 0, & \text{otherwise.} \end{cases}$$
   (a) Find the density of $Y = |X+1|$.
   (b) Compute $\mathrm{Var}(\frac{1}{2}X - 1)$.

5. A random variable X has a discrete distribution with the PMF:
$$p_X(x) = \mathbb{P}(X = x) = \begin{cases} a, & x = -1, \\ bx, & x \in \{1,2\}, \\ 0, & \text{otherwise.} \end{cases}$$
   (a) Determine a and b given that $\mathbb{P}(X < 2) = \frac{3}{4}$.
   (b) Compute $\mathbb{E}\frac{X+1}{X}$.