Circuits and Signals

DC analysis

Marek Rupniewski

2022 spring semester

Faculty of Electronics and Information Technology

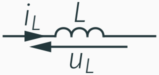
WARSAW UNIVERSITY OF TECHNOLOGY

DC analysis

A DC (direct current) solution to a circuit is a solution consisting entirely of constant signals (all voltages and currents).

DC analysis

For DC analysis all ODEs reduce to algebraic equations:

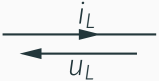


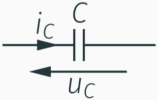
$u_L = Li'_L,$

for direct signals u_L, i_L:

$$u_L = Li'_L = 0$$

thus $u_L = 0$,

short-circuit 

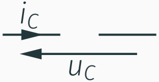


$i_C = Cu'_C,$

for direct signals u_C, i_C:

$$i_C = Cu'_C = 0$$

thus $i_C = 0$,

open-circuit 

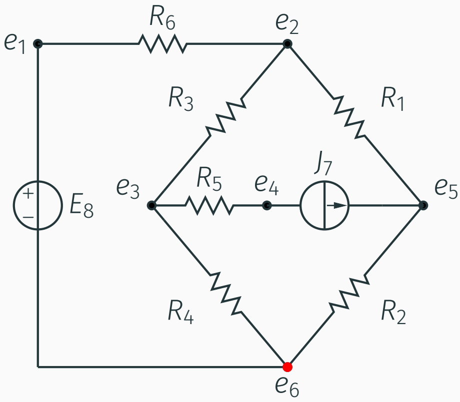
What do we have at our disposal?

- Kirchhoff's laws (KCL, KVL),
- device equations.

Nodal method

1. Label all the nodes with variables $e_1, e_2, \ldots, e_N$ denoting electric potentials,

Nodal method

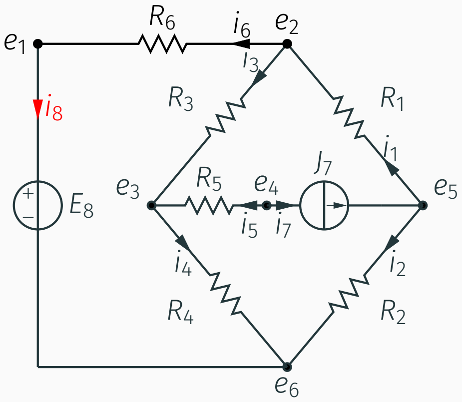


Nodal method

1. Label of all the nodes with variables $e_1, e_2, \ldots, e_N$ denoting electric potentials,
2. Label of the currents $(i_1, i_2, \ldots, i_M)$ flowing into all the elements. For each n-terminal element we introduce $n - 1$ new variables in this way,

The current flowing out of the last terminal of an n-terminal element equals the sum of the currents flowing into all the other terminals.

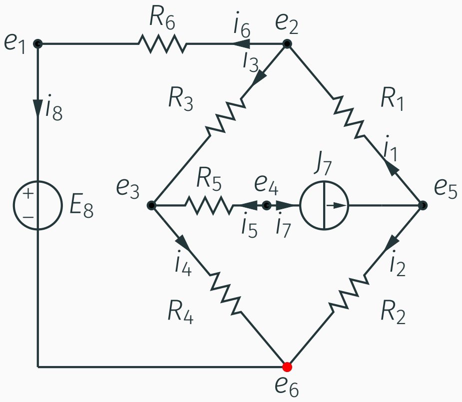
Nodal method



Nodal method

1. Label of all the nodes with variables $e_1, e_2, \dots, e_N$ denoting electric potentials,
2. Label of the currents $(i_1, i_2, \dots, i_M)$ flowing into all the elements. For each n-terminal element we introduce $n - 1$ new variables in this way,
3. Write down the KCL equations (for each node),

Nodal method

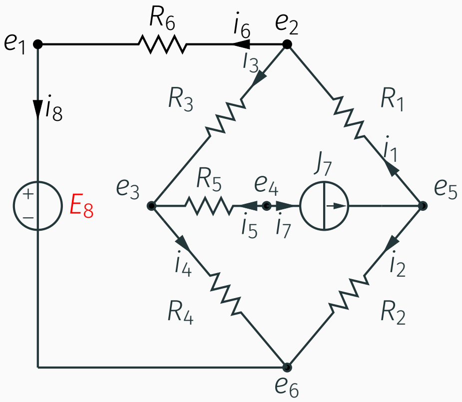


$$
\begin{aligned}
i_6 &= i_8, && (e_1)\\
i_1 &= i_3 + i_6, && (e_2)\\
i_3 + i_5 &= i_4, && (e_3)\\
0 &= i_5 + i_7, && (e_4)\\
i_7 &= i_1 + i_2, && (e_5)\\
i_2 + i_4 + i_8 &= 0. && (e_6)
\end{aligned}
$$

Nodal method

1. Label of all the nodes with variables $e_1, e_2, \ldots, e_N$ denoting electric potentials,
2. Label of the currents $(i_1, i_2, \ldots, i_M)$ flowing into all the elements. For each n-terminal element we introduce $n - 1$ new variables in this way,
3. Write down the KCL equations (for each node),
4. Write down element equations expressed in terms of electric potentials $e_1, \ldots, e_N$ and currents $i_1, \ldots, i_M$.

Nodal method



$$e_5 - e_2 = i_1 R_1, \tag{1}$$

$$e_5 - e_6 = i_2 R_2, \tag{2}$$

$$e_2 - e_3 = i_3 R_3, \tag{3}$$

$$e_3 - e_6 = i_4 R_4, \tag{4}$$

$$e_4 - e_3 = i_5 R_5, \tag{5}$$

$$e_2 - e_1 = i_6 R_6, \tag{6}$$

$$i_7 = J_7, \tag{7}$$

$$e_1 - e_6 = E_8. \tag{8}$$

Nodal method

1. Label of all the nodes with variables $e_1, e_2, \dots, e_N$ denoting electric potentials,
2. Label of the currents $(i_1, i_2, \dots, i_M)$ flowing into all the elements. For each n-terminal element we introduce $n - 1$ new variables in this way,
3. Write down the KCL equations (for each node),
4. Write down elements equations expressed in terms of electric potentials $e_1, \dots, e_N$ and currents $i_1, \dots, i_M$.
5. Solve the obtained set of equations.

Every voltage can be recovered as the difference of appropriate electric potentials.

Nodal method

$$i_6 = i_8, \qquad (e_1)$$
$$i_1 = i_3 + i_6, \qquad (e_2)$$
$$i_3 + i_5 = i_4, \qquad (e_3)$$
$$0 = i_5 + i_7, \qquad (e_4)$$
$$i_7 = i_1 + i_2, \qquad (e_5)$$
$$i_2 + i_4 + i_8 = 0. \qquad (e_6)$$

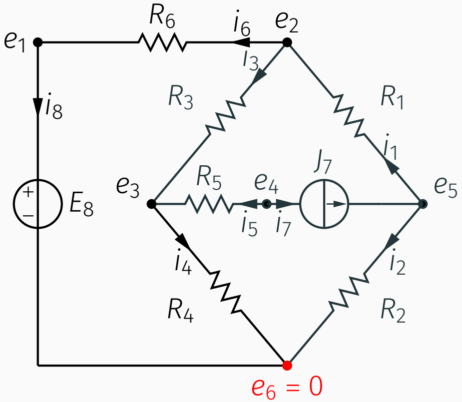
$$e_5 - e_2 = i_1 R_1, \qquad (1)$$
$$e_5 - e_6 = i_2 R_2, \qquad (2)$$
$$e_2 - e_3 = i_3 R_3, \qquad (3)$$
$$e_3 - e_6 = i_4 R_4, \qquad (4)$$
$$e_4 - e_3 = i_5 R_5, \qquad (5)$$
$$e_2 - e_1 = i_6 R_6, \qquad (6)$$
$$i_7 = J_7, \qquad (7)$$
$$e_1 - e_6 = E_8. \qquad (8)$$

Nodal method

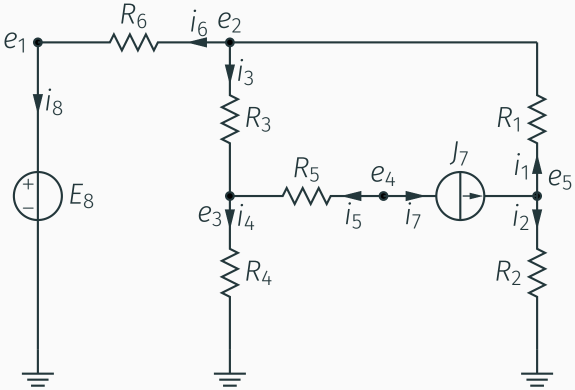
$$
\begin{aligned}
i_6 &= i_8, && (e_1)\\
i_1 &= i_3 + i_6, && (e_2)\\
i_3 + i_5 &= i_4, && (e_3)\\
0 &= i_5 + i_7, && (e_4)\\
i_7 &= i_1 + i_2, && (e_5)\\
\cancel{i_2 + i_4 + i_8} &\cancel{= 0.} && (e_6)
\end{aligned}
$$

$$
\begin{aligned}
e_5 - e_2 &= i_1 R_1, && (1)\\
e_5 - 0 &= i_2 R_2, && (2)\\
e_2 - e_3 &= i_3 R_3, && (3)\\
e_3 - 0 &= i_4 R_4, && (4)\\
e_4 - e_3 &= i_5 R_5, && (5)\\
e_2 - e_1 &= i_6 R_6, && (6)\\
i_7 &= J_7, && (7)\\
e_1 - 0 &= E_8. && (8)\\
e_6 &= 0 && (9)
\end{aligned}
$$

Nodal method



Nodal method



Electric potentials are measured with respect to ground node and KCL for that node is dropped.