The prediction method for second order linear differential equations

Consider the differential equation

$$y'' + py' + qy = e^{ax}(P_1(x)\cos(bx) + P_2(x)\sin(bx)).$$

where $p, q, a, b \in \mathbb{R}$ and P_1, P_2 are given polynomials.

If a + bi is not a root of characteristic equation, then we predict PSNE in the form

$$y(x) = e^{ax}(Q_1(x)\cos(bx) + Q_2(x)\sin(bx)),$$

where Q_1 and Q_2 are some polynomials such that $\deg Q_1 = \deg Q_2 = \max \{\deg P_1, \deg P_2\}$.

If a + bi is a p-multiple root of characteristic equation, then we predict PSNE in the form

$$y(x) = x^p e^{ax} (Q_1(x)\cos(bx) + Q_2(x)\sin(bx)),$$

where Q_1 and Q_2 are some polynomials such that $\deg Q_1 = \deg Q_2 = \max \{\deg P_1, \deg P_2\}$.

The prediction method for recurrence equations

Consider the recurrence equation

$$ax_{n+2} + bx_{n+1} + cx_n = P_k(n) \cdot d^n$$

where $a, b, c, d \in \mathbb{R}$ and P_k is a polynomial of degree k.

If d is not a root of characteristic equation, then we predict PSNE in the form

$$x_n = Q_k(n) \cdot d^n$$
,

where Q_k is a polynomial such that $\deg Q_k = k$.

If d is a p-multiple root of characteristic equation, then we predict PSNE in the form

$$x_n = n^p \cdot Q_k(n) \cdot d^n$$

where Q_k is a polynomial such that deg $Q_k = k$.