Summary of EDDE

Methods to solve ODEs:

1. Separation of Variables
2. Homogenous equations
3. Linear differential equations
4. Bernoulli equations
5. Exact method with optional integrating factor

Separation of Variables

Step 1: Change the equation to the form

$$F(y)dy = G(x)dx$$

Step 2: Integrate both sides

$$\int F(y)dy = \int G(x)dx$$

Step 3: Solve this new equation like any other

Homogenous equations

Step 1: Let $y = vx$
Step 2: Differentiate both sides using product rule

$$dy = x \cdot dv + v \cdot dx$$

Step 3: Substitute
Step 4: Simplify
Step 5: Separation of Variables
Step 6: Substitute back $v = \frac{x}{y}$

Linear differential equations

Given a function:

$$y' + P(x)y = Q(x)$$

Step 1: Calculate an integrating factor

$$I(x) = e^{\int P(x)dx}$$

Step 2: Calculate general solution

$$y = \frac{1}{I(x)}\left[\int Q(x)I(x)dx\right]$$

Bernoulli equations

For a function in a form

$$y + P(x)y' = Q(x) \cdot y^n$$

Step 1: Find an integrating factor following this formula

$$I(x) = e^{\int (1-n)P(x)dx}$$

Step 2: Solve equation

$$y^{1-n} = \frac{1}{I(x)}\left[\int (1-n)Q(x)I(x)dx\right]$$

Exact method

Step 1: Turn equation into a form: $f(x, y) := P(x, y)dx + Q(x, y)dy = 0$

Step 2: Calculate P_y, Q_x
Step 3: Calculate $Q_x - P_y$.
if equal to 0: follow to **Step 4**, else follow the [Integrating factor for exact method]

Step 4: Integrate P over x or Q over y.

$$f(x, y) = \int Pdx = \int Qdy$$

Step 5: Use one of $f_x = P, f_y = Q$ properies.
Just differentiate the f over dy (or dx if we integrated Q)

Integrating factor for exact equations

For a original function

$$f(x, y) := P(x, y)dx + Q(x, y)dy = 0$$

we transform it into

$$f(x, y) := P(x, y)\mu dx + Q(x, y)\mu dy = 0$$

Calculating μ

1. $$\frac{\mu'}{\mu} = \frac{P_y - Q_x}{Q} \text{ and dx in the integral}$$

   or, if that is dependant on both x and y:
2. $$\frac{\mu'}{\mu} = \frac{Q_x - P_y}{P} \text{ and dy in the integral}$$

   to get μ just integrate both side using dx or dy depending how u got $\frac{\mu'}{\mu}$.

Table of derivatives

Function	Derivative
scalars	0
$x^n,\ n \in \mathbb{N} - \{0\}$	$n \cdot x^{n-1}$
$ln(x)$	$\frac{1}{x}$
e^x	e^x
$e^{c \cdot x}$	$c \cdot e^{cx}$
$\mathbb{c}^x$	$\mathbb{c}^x = ln(\mathbb{c}) \cdot \mathbb{c}^x$
sin(x)	cos(x)
cos(x)	-sin(x)
tan(x)	$\frac{1}{cos^2(x)}$
$(fg)'$	$f'g + fg'$
$\frac{f}{g}$	$\frac{f'g - fg'}{g^2}$
$arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$arccos(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$arctan(x)$	$\frac{1}{1-x^2}$