Numerical Methods

Laboratory no. 1

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Documentation of laboratory work results

Task 1

Given a function

$$y = \frac{\cos(x)}{x^3} - x^2$$

we want to calculate it's T(x) coefficient. We will use the property

$$\sigma[\tilde{v}^k] = T^k \cdot \sigma[\tilde{x}^{k-1}] + \nu^k$$

In our function we use:

$$\begin{aligned}
v_1 &= \cos(x) \\
v_2 &= x^3 \\
v_4 &= \frac{v_1}{v_2} \\
v_4 &= x^2 \\
y &= v_4 - v_4
\end{aligned}$$

Therefore we obtain:

$$\begin{aligned}
\tilde{v_1} &= \cos(x(1+\epsilon_1)) \\
\tilde{v_2} &= (x(1+\epsilon))^3 = x^3(1+3\epsilon_2) \\
\tilde{v_3} &= \frac{\tilde{v_1}}{\tilde{v_2}} \\
\tilde{v_4} &= (x(1+\epsilon))^2 = x^2(1+2\epsilon_4) \\
\tilde{y} &= \tilde{v_3} - \tilde{v_4}
\end{aligned}$$

Let us focus on $\tilde{v_1}$:

$$\cos(x(1+\epsilon)) = \cos(x + x\epsilon) = \cos(x)\cos(x\epsilon) - \sin(x)\sin(x\epsilon)$$

Now we need to use property of trigonometric functions:

$$For\ \alpha \to 0, \sin(\alpha) \approx \alpha, \cos(\alpha) \approx 1 - \frac{\alpha^2}{2}$$

Using those properties:

$$\begin{aligned}
\cos(x(1+\epsilon)) &= \cos(x)\cos(x\epsilon) - \sin(x)\sin(x\epsilon) \\
&= \cos(x)(1 - \frac{(x\epsilon)^2}{2}) - \sin(x)x\epsilon \\
&= \cos(x) - \sin(x)x\epsilon \\
&= \cos(x) \cdot (1 - \tan(x)x\epsilon)
\end{aligned}$$

Now solve v_3:

$$\begin{aligned}
\tilde{v_3} &= \frac{\cos(x) \cdot (1 - x\tan(x)\epsilon_1)}{x^3(1+3\epsilon_2)} \\
&= \frac{\cos(x)}{x^3}(1 - x\tan(x)\epsilon_1) \cdot (1+3\epsilon_2)^{-1} \\
&= \frac{\cos(x)}{x^3}(1 - x\tan(x)\epsilon_1) \cdot (1-3\epsilon_2) \\
&= \frac{\cos(x)}{x^3}(1 - x\tan(x)\epsilon_1 - 3\epsilon_2)
\end{aligned}$$

Finally, we get:

$$\begin{aligned}
\tilde{y} &= \frac{\cos(x)}{x^3}(1 - x\tan(x)\epsilon_1 - 3\epsilon_2) - x^2(1+2\epsilon_4) \\
&= \frac{\cos(x)}{x^3} - \frac{\cos(x)}{x^3}(x\tan(x)\epsilon_1 + 3\epsilon_2) - x^2 - 2x^2\epsilon_4 \\
&= y - \frac{\cos(x)}{x^3}(x\tan(x)\epsilon_1 + 3\epsilon_2) - 2x^2\epsilon_4 \\
&= y(1 + (-\frac{\cos(x)}{x^3}(x\tan(x)\epsilon_1 + 3\epsilon_2) - 2x^2\epsilon_4)\frac{1}{y}) \\
&= y(1 + (-\frac{\sin(x)}{x^2}\epsilon_1 - 3\frac{\cos(x)}{x^3}\epsilon_2 - 2x^2\epsilon_4)\frac{1}{y})
\end{aligned}$$

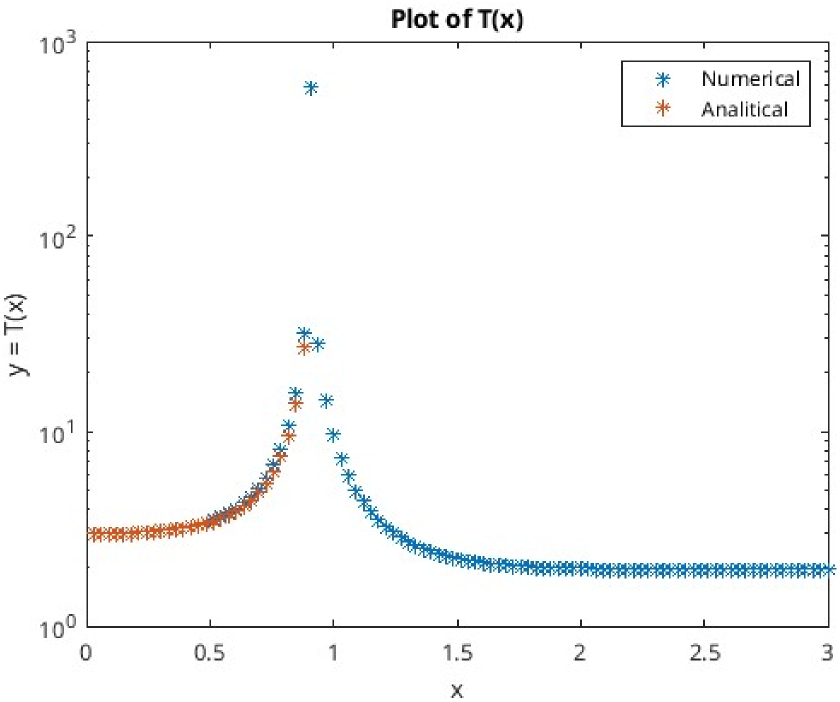
Therefore,

$$T(x) = \frac{\left|-\frac{\sin(x)}{x^2}\right| + \left|-3\frac{\cos(x)}{x^3}\right| + \left|-2x^2\right|}{\frac{\cos(x)}{x^3} + x^2}$$

This yields results simmilar to the ones we got via calculating the error numerically. Function

$$T(x) = \frac{1}{\epsilon_{sim}}\left|\frac{y(\tilde{x}) - y(x)}{y(x)}\right|$$

Does not differ significantly from the one we got from analitical solution.
Graph that visualises both:



Code I used to obtain such graph:

```
% clear previous experiment results
clc, clearvars, close all

% define domain and function
x = linspace(0,3,100);
y = @(x) cos(x)./x.^3 + x.^2;
% define erronous domain
esim = 1.0e-8;
x_eps = x.*(1+esim);
% calculate true y and epsilon y
ydot = y(x);
yeps = y(x_eps);
% calculate T numerically
numerator = yeps - ydot;
abs_error = abs(numerator ./ ydot);
tx = 1/esim * abs_error;

% calculate T analiticaly (formula calculated in the report)
T1 = -sin(x)./(x.^2);
T2 = -3*cos(x)./(x.^3);
T3 = x.^2;

Tn = (abs(T1) + abs(T2) + abs(T3));
tx_analitical = Tn./ydot;

% plot results to validate both methods result in simmilar results
semilogy(x,tx, '*');
hold on
semilogy(x,tx_analitical, '*');
title("Plot of T(x)"), xlabel("x"), ylabel("y = T(x)")
legend("Numerical", "Analitical")
```

1 Task 2