EPRST: Probability and Statistics
Problem set 1

1. Given that $\mathbb{P}(A') = \frac{1}{3}$, $\mathbb{P}(A \cap B) = \frac{1}{4}$ and $\mathbb{P}(A \cup B) = \frac{2}{3}$, compute $\mathbb{P}(B')$, $\mathbb{P}(A \cap B')$ and $\mathbb{P}(B \setminus A)$.
2. Prove that if $C \supset A \cap B$ then $\mathbb{P}(C) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$.
3. Let A and B be events. Prove that if $A \subseteq B$, then
$$\mathbb{P}(B \setminus A) = \mathbb{P}(B) - \mathbb{P}(A).$$
4. Out of a group of children: 50% like strawberry ice cream, 70% like chocolate ice cream and 30% like both. Find the probability that a randomly chosen child in the group does not like strawberry nor chocolate ice cream.
5. Flip a fair coin 3 times. Compute the probability of the following events:
   (a) three tails: TTT,
   (b) any sequence of 2 heads and 1 tail,
   (c) any sequence where the number of heads is less then or equal to the number of tails.
6. We roll a symmetric dice until we get one or five. Compute the probability that
   (a) we will make k rolls ($k \geq 1$),
   (b) we will make an odd number of rolls,
   (c) we will make at least 3 rolls.
7. * Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the randomness, Alice decides to register for 7 randomly selected classes out of 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?
8. * A fair die is rolled 10 times. What is the probability that at least one of the 6 values never appears?
9. * Matching problem: n men pick a hat at random from the hat rack when they leave a party. Find the probability that at least one of them picks his own hat.
10. Alice and Barbara arrange a meeting at a specified place. They arrive at the place at random times between 16:00 and 17:00. The girl who arrives first is going to wait 15 minutes and then leave the place if the other girl does not arrive.
    (a) What is the probability that the girls will meet each other?
    (b) What is the probability that they will arrive at exactly the same moment?
    (c) What is the probability that Alice will arrive at least 5 minutes after Barbara?
11. We draw two points p and q at random from the interval $[-1, 1]$. Let x_1 and x_2 denote the roots of the equation $x^2 + px + q = 0$. Find the probability that
    (a) $x_1, x_2 \in \mathbb{R}$,
    (b) $x_1^2 + x_2^2 < 1$.

* For solving problems: 7, 8, 9 you may find useful the following:

Theorem 1 (inclusion-exclusion formula)**.** *For any finite sequence of events $A_1, A_2, \ldots, A_n$, $n \geq 2$,*

$$\mathbb{P}\left(\bigcup_{k=1}^{n} A_k\right) = \sum_{k=1}^{n} \mathbb{P}(A_k) - \sum_{1 \leq i_1 < i_2 \leq n} \mathbb{P}(A_{i_1} \cap A_{i_2}) + \sum_{1 \leq i_1 < i_2 < i_3 \leq n} \mathbb{P}(A_{i_1} \cap A_{i_2} \cap A_{i_3}) - \ldots + (-1)^{n-1}\mathbb{P}(A_1 \cap \ldots \cap A_n).$$