#38: Krzysztof Watras (318339)

Numerical Methods (ENUME) – Project
Assignment C: Ordinary differential equations

1. Use the MATLAB function ***ode45*** to solve the Lotka-Volterra equations:

$$\begin{cases} \dfrac{dx(t)}{dt} = p_1 x(t) - p_2 x(t) y(t) \\ \dfrac{dy(t)}{dt} = p_3 x(t) y(t) - p_4 y(t) \end{cases} \tag{1}$$

for $t \in [0, 1]$, $p_1 = 14$, $p_2 = 0.11$, $p_3 = 0.04$, $p_4 = 10$, $x(0) = 530$, $y(0) = 30$.

2. Solve the set of equations (1) for $t \in [0, 1]$ by means of:

a) Euler explicit method,

b) Euler implicit method,

c) Adams-Moulton method of order 2,

d) Adams-Bashforth method of order 3.

Use the step of integration $h = 0.005$.

3. Compute the relative aggregated errors Δ_y of the solutions obtained in Task #2:

$$\Delta_y \equiv \frac{\sum_{n=1}^{N} (\hat{y}_n - \dot{y}_n)^2}{\sum_{n=1}^{N} \dot{y}_n^2} \tag{2}$$

where:

- $\hat{y}_1, ..., \hat{y}_N$ are the estimates of y, obtained by means of one of the methods specified in Task #2;
- $\dot{y}_1, ..., \dot{y}_N$ are the reference values of y, obtained by means of the function ***ode45*** with the parameters $RelTol = 10^{-8}$ and $AbsTol = 10^{-12}$.

4. Determine the dependence of the relative aggregated error Δ_y on the step of integration $h \in \left[10^{-4}, 10^{-2}\right]$ for all four methods specified in Task #2.

5. Use the MATLAB functions ***fminsearch*** and ***ode45*** to determine the parameters p_1, p_2, p_3, p_4 minimising the following criterion:

$$J(\mathbf{p}) = \sum_{n=1}^{N} (\hat{x}_n(\mathbf{p}) - \tilde{x}_n)^2 + \sum_{n=1}^{N} (\hat{y}_n(\mathbf{p}) - \tilde{y}_n)^2 \tag{3}$$

where:

- $\mathbf{p} \equiv [p_1, p_2, p_3, p_4]^{\mathrm{T}}$;
- $\tilde{x}_1, ..., \tilde{x}_N$ and $\tilde{y}_1, ..., \tilde{y}_N$ are the data stored in the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, in the file *data38.csv*;
- $\hat{x}_1(\mathbf{p}), ..., \hat{x}_N(\mathbf{p})$ and $\hat{y}_1(\mathbf{p}), ..., \hat{y}_N(\mathbf{p})$ are the estimates of x and y, respectively, corresponding to the time instants $t_1, ..., t_N$ specified in the vector $\boldsymbol{t}$ in the file *data38.csv*, obtained by solving the system of equations (1) by means of the function ***ode45***.

Use $\tilde{x}_1$ and $\tilde{y}_1$ as the initial values when solving the system of equations (1) using ***ode45***. Use the parameters p_1, p_2, p_3, p_4 specified in Task #1 as the initial values for optimisation using ***fminsearch***.