EPRST: Probability and Statistics
Problem set 8

1. Two-dimensional random variable (X, Y) has the following distribution:

X\Y	-1	0	1
0	0.1	0.1	0
1	0.2	0.2	0.1
2	0.1	0.1	0.1

   Are X and Y independent?

2. The distribution of random vector (X, Y) is shown in the table:

X\ Y	-1	0	1
0	1/11	3/11	2/11
1	2/11	1/11	2/11

   Are X and Y independent?

3. Verify that
$$f(x,y) = \begin{cases} -\frac{x}{8}, & -2 \le x \le 0, 0 \le y \le 2, \\ \frac{y}{8}, & 0 < x \le 2, 0 \le y \le 2, \\ 0, & \text{otherwise}, \end{cases}$$
   is the density of a random vector (X, Y). Find the marginal densities. Compute $\mathbb{P}(X + Y > 2)$.

4. The joint density of a random vector (X, Y) is given by
$$f(x,y) = \begin{cases} 1, & 0 < x < 1,\ 0 < y < 2x, \\ 0, & \text{otherwise}, \end{cases}$$
   Find the marginal distributions of X and Y. Are X and Y independent? Compute $\mathbb{P}(X < Y)$.

5. The joint density of a random vector (X, Y) is given by
$$f(x,y) = \begin{cases} \frac{1}{3}, & x \in [0,1] \text{ and } y \in [0,1], \\ -\frac{1}{3}y, & x \in [0,1] \text{ and } y \in [-2,0), \\ 0, & \text{otherwise}. \end{cases}$$
   Find the marginal densities. Are X and Y independent?

6. X and Y are independent random variables, $X \sim Exp(\lambda_1)$ and $Exp(\lambda_2)$. Compute $\mathbb{P}(X > Y)$.

7. Let X and Y have the joint probability density function
$$f(x,y) = \begin{cases} 1, & x \in [0,1], y \in [0,1], \\ 0, & \text{otherwise}. \end{cases}$$
   Calculate $\mathbb{P}(X + Y \le \frac{1}{2})$, $\mathbb{P}(X - Y \le \frac{1}{2})$, $\mathbb{P}(XY \le \frac{1}{4})$, $\mathbb{P}(X^2 + Y^2 \le 1)$.

8. Stores A and B, which belong to the same owner, are located in two different towns. If the probability density function of the weekly profit of each store, in thousands of zlotys, is given by
$$f(x) = \begin{cases} \frac{x}{4}, & 1 < x < 3, \\ 0, & \text{otherwise}, \end{cases}$$
   and the profit of one store is independent of the other, what is the probability that next week one store makes at least 500 zl more than the other store?

9. Let T denote the triangle with vertices at points $(0, 0)$, $(-1, 0)$ and $(0, 1)$. Consider random vector (X, Y) uniformly distributed on T. Compute $\mathbb{P}(Y > |X|)$.

10. The joint density of the vector (X, Y) is of the form
$$f(x,y) = \begin{cases} |x|, & -1 < x < y < 1; \\ 0, & \text{otherwise}. \end{cases}$$
   Are X and Y independent? Compute $P(Y > X + \frac{1}{2})$.