Circuits and Signals

Transient states in RLC circuits

Marek Rupniewski

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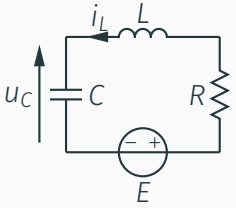
Faculty of Electronics and Information Technology

WARSAW UNIVERSITY OF TECHNOLOGY

RLC circuit



$$u_C(t_0) = U_0, \quad i_L(t_0) = I_0.$$

$$u_C + Li_L' + Ri_L = E.$$

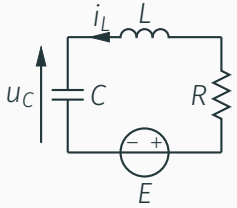
$$i_L = Cu_C', \qquad i_L' = Cu_C''.$$

$$LCu_C'' + RCu_C' + u_C = E,$$

$$\underbrace{\frac{L}{CR^2}}_{Q^2}\underbrace{(RC)^2}_{\tau^2}u_C'' + \underbrace{RC}_{\tau}u_C' + u_C = E,$$

$$(Q\tau)^2 u_C'' + \tau u_C' + u_C = E.$$

RLC circuit



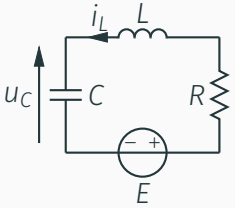
$$(Q\tau)^2 u_C'' + \tau u_C' + u_C = E, \qquad \tau = RC, \quad Q = \frac{\sqrt{L/C}}{R}.$$

$$\Delta = \tau^2 - 4(Q\tau)^2 = \tau^2(1-2Q)(1+2Q).$$

There are 3 qualitatively different cases:

- $Q < \frac{1}{2}$ — the overdamped case,
- $Q > \frac{1}{2}$ — the underdamped case.
- $Q = \frac{1}{2}$ — the critically damped case,

RLC circuit — overdamped case



$$(Q\tau)^2 u_C'' + \tau u_C' + u_C = E, \qquad \tau = RC, \quad Q = \frac{\sqrt{L/C}}{R} < \frac{1}{2}.$$

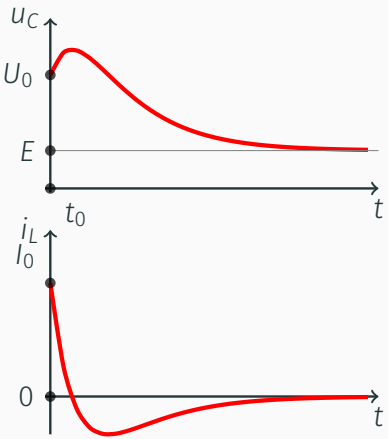
$$u_C(t) = E + Ae^{\omega_1(t-t_0)} + Be^{\omega_2(t-t_0)},$$

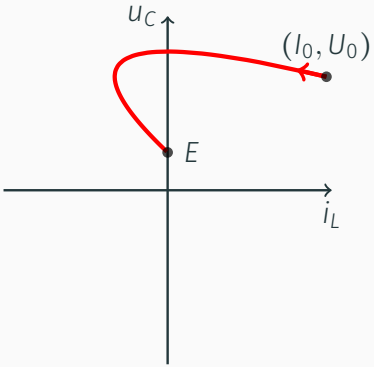
where ω_1 and ω_2 are real roots of the characteristic equation:

$$(Q\tau)^2 s^2 + \tau s + 1 = 0,$$

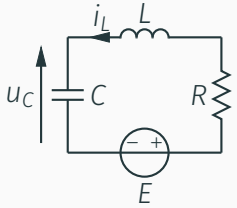
and A, B are constants to be determined (using IC).

RLC circuit — overdamped case





RLC circuit — underdamped case



$$(Q\tau)^2 u_C'' + \tau u_C' + u_C = E, \qquad \tau = RC, \quad Q = \frac{\sqrt{L/C}}{R} > \frac{1}{2}.$$

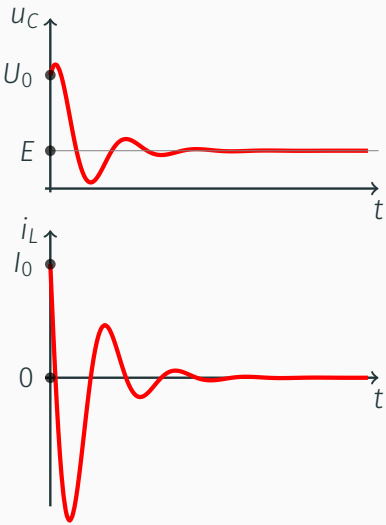
$$u_C(t) = E + Ae^{-\omega_r(t-t_0)} \cos\big(\omega_s(t-t_0) + \varphi\big),$$

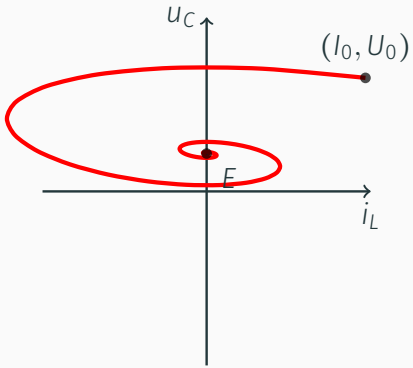
where $\omega_r \pm \jmath\omega_s$ are (conjugated) complex roots of the characteristic equation:

$$(Q\tau)^2 s^2 + \tau s + 1 = 0,$$

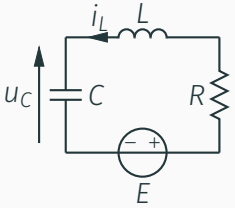
and A, φ are constants to be determined (using IC).

RLC circuit — underdamped case





RLC circuit — critically damped case



$$(Q\tau)^2 u_C'' + \tau u_C' + u_C = E, \qquad \tau = RC, \quad Q = \frac{\sqrt{L/C}}{R} = \frac{1}{2}.$$

$$u_C(t) = E + Ae^{-\frac{t-t_0}{\tau/2}} + B(t-t_0)e^{-\frac{t-t_0}{\tau/2}},$$

where $-\tau/2$ is the double (real) root of the characteristic equation:

$$(Q\tau)^2 s^2 + \tau s + 1 = 0,$$

and A, B are constants to be determined (using IC).

RLC circuit — critically damped case