Circuits and Signals

Non-linear circuits

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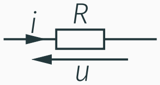
Non-linear circuit

Definition

Non-linear circuit is a circuit, in which there is at least one non-linear device.

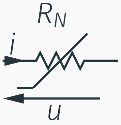
Non-linear resistor

resistor (linear):



$$u = Ri.$$

non-linear resistor:



$$u = f(i),$$

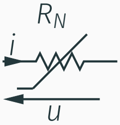
f is non-linear function, whose graph lies in 1 i 3 quadrants and crosses the origin of coordinates i-u. alternatively:

$$i = g(u),$$

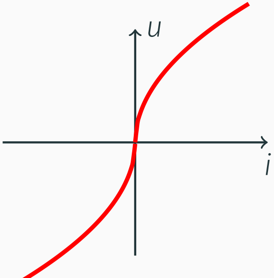
where g is non-linear function, whose graph lies in 1 i 3 quadrants and crosses the origin of coordinates u-i.

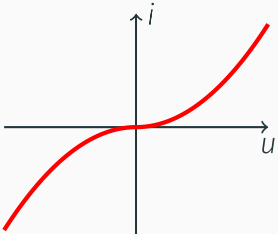
Non-linear resistor — cont.

example:

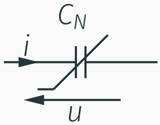


$$u = \underbrace{\alpha(\operatorname{sgn} i)\sqrt{|i|}}_{f(i)}, \qquad \alpha > 0 \qquad i = \underbrace{(\operatorname{sgn} u)(u/\alpha)^2}_{g(i)}.$$





Non-linear capacitor — (revisited)

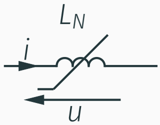


$$\boxed{q = q(u)}$$ lin. case: $q = Cu$

$$i(t) = q'(t) = \frac{\mathrm{d}q}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}t}(t) = \frac{\mathrm{d}q(u(t))}{\mathrm{d}u}u'(t).$$

The graph of q lies in 1 i 3 quadrants and crosses the origin of coordinates u-q.

Non-linear inductor — (revisited)

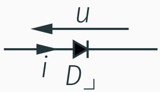


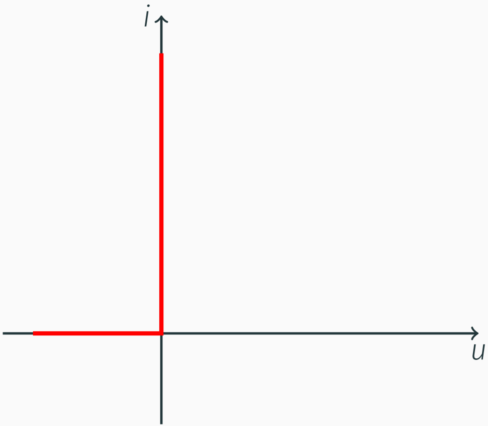
$$\boxed{\psi = \psi(i)} \quad \text{lin. case: } \psi = Li$$

$$u(t) = \psi'(t) = \frac{\mathrm{d}\psi}{\mathrm{d}i}\frac{\mathrm{d}i}{\mathrm{d}t}(t) = \frac{\mathrm{d}\psi(i(t))}{\mathrm{d}i}i'(t).$$

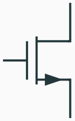
The graph of ψ lies in 1 i 3 quadrants and crosses the origin of coordinates i-ψ.

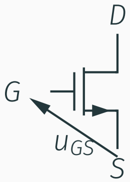
Diode — basic (short-/open- circuit) model



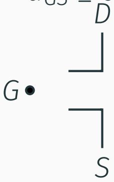


FET transistor as a switch

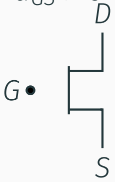




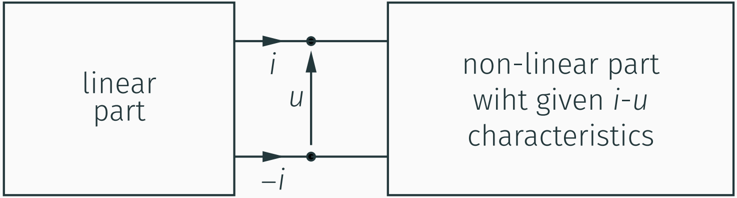
$u_{GS} \leq 0$



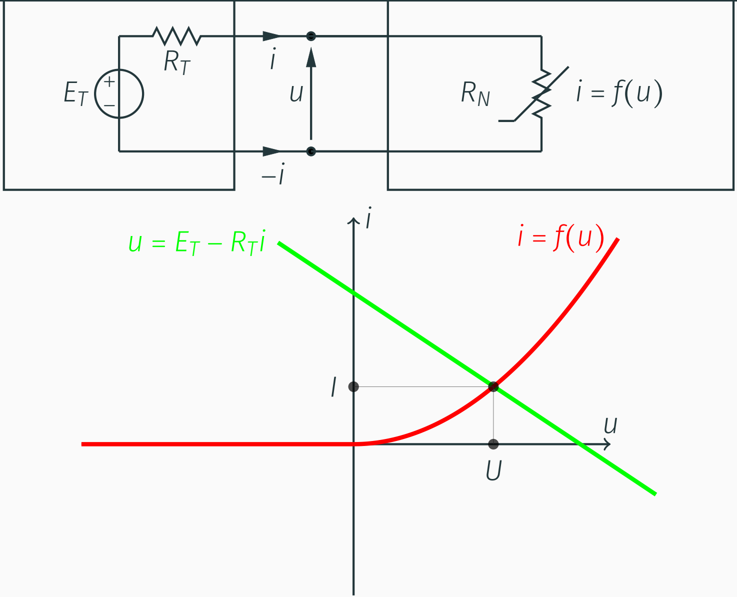
$u_{GS} > 0$



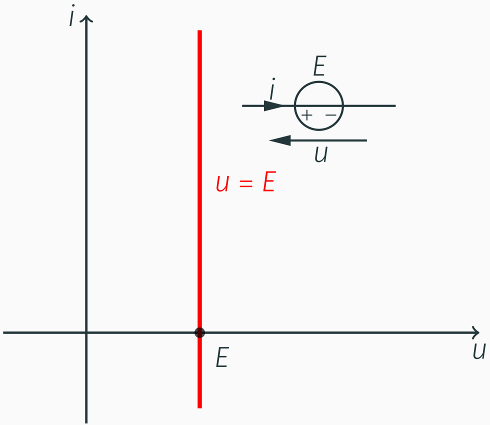
Load line method (DC case)



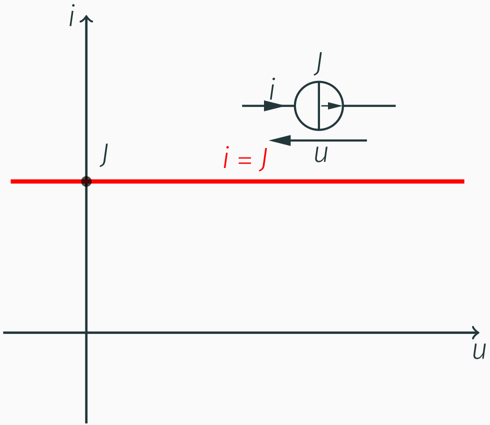
Load line method (DC case)



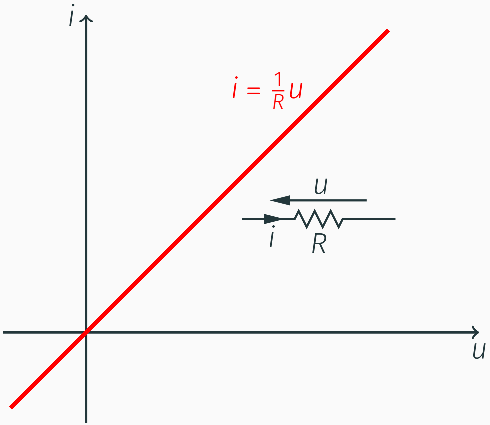
Composing the characteristics



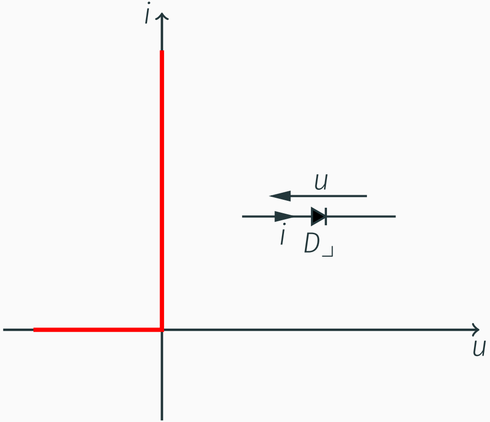
Composing the characteristics



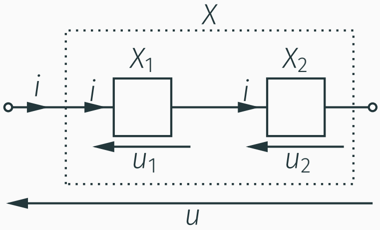
Composing the characteristics



Composing the characteristics



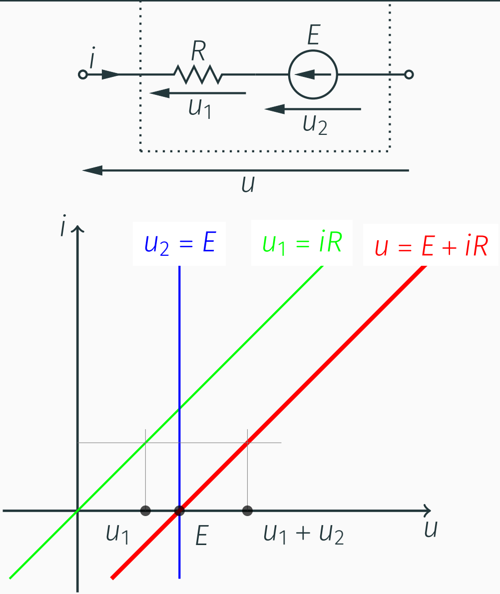
Composing the characteristics



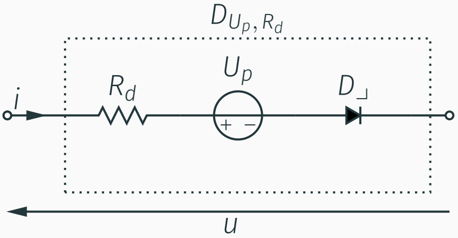
The current through individual one-ports is the current through combined one-port and

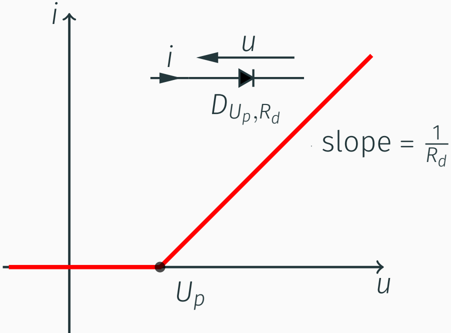
$$u = u_1 + u_2$$

Composing the characteristics

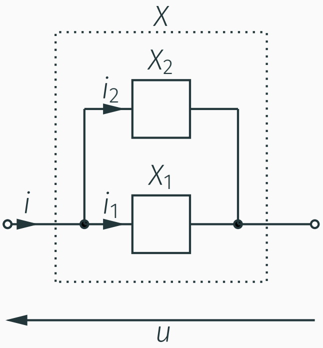


Composing the characteristics





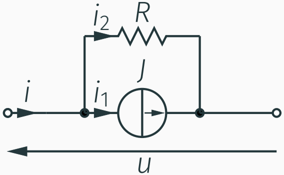
Composing the characteristics

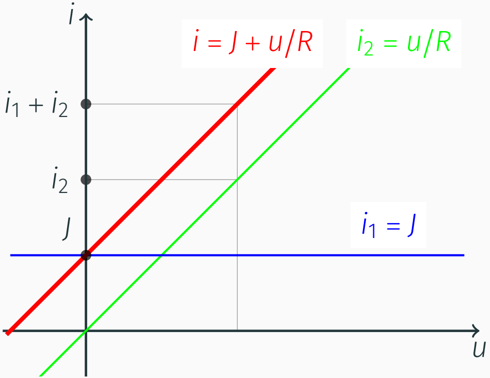


Voltage across the combined one-port equal voltages across individual one-ports and

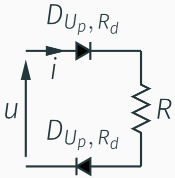
$$i = i_1 + i_2.$$

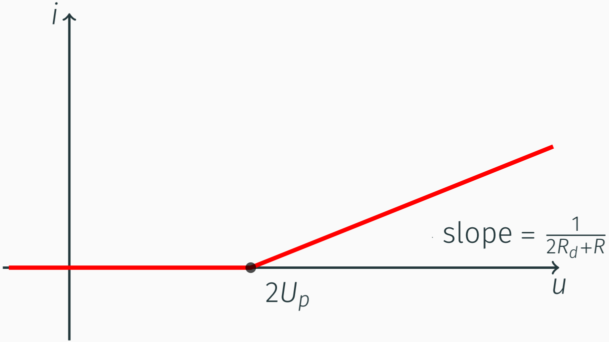
Composing the characteristics



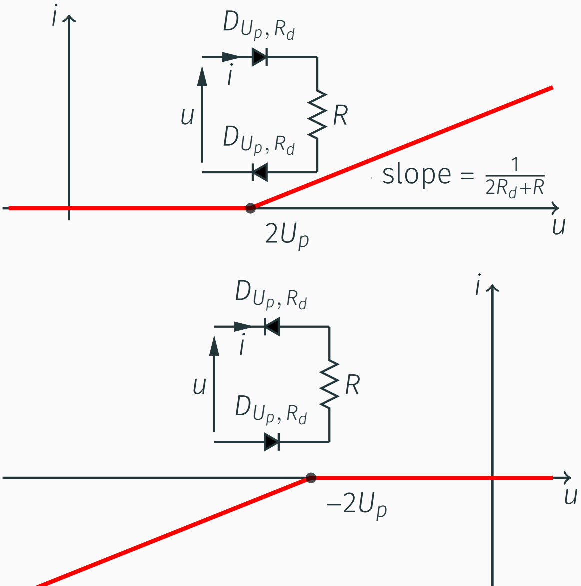


Full-wave rectifier — introduction

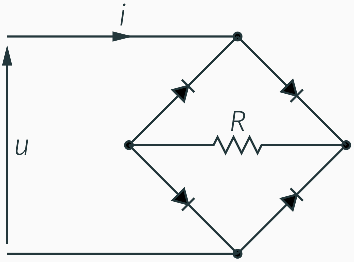


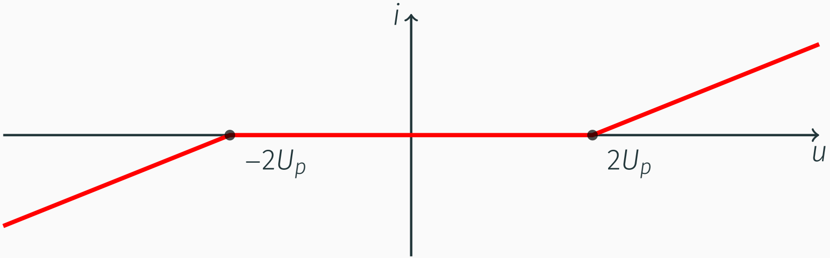


Full-wave rectifier — introduction

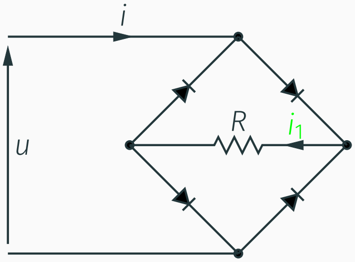


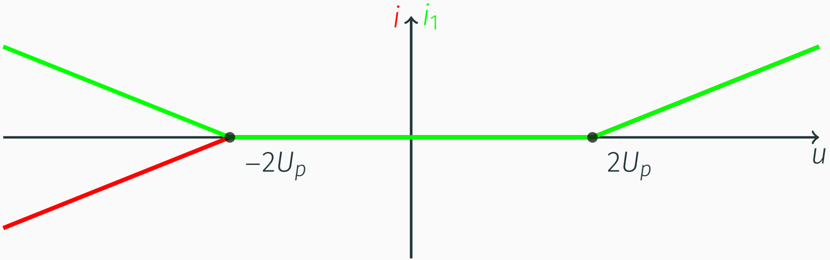
Full-wave rectifier



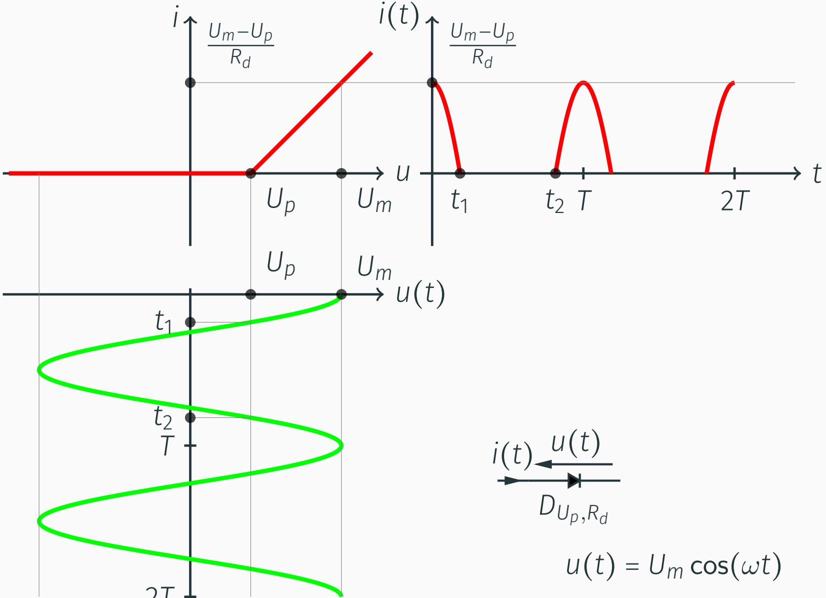


Full-wave rectifier



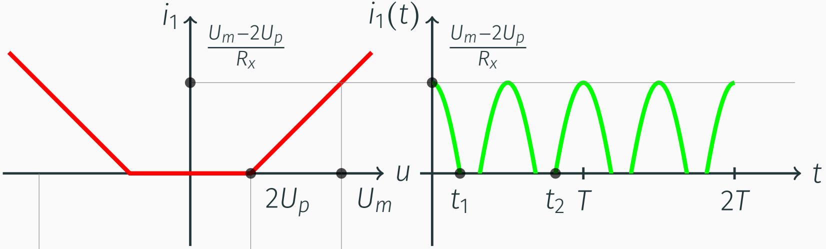


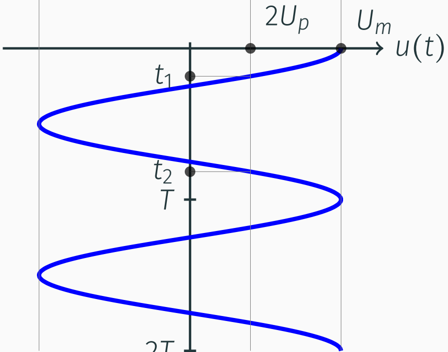
Projection through a characteristics

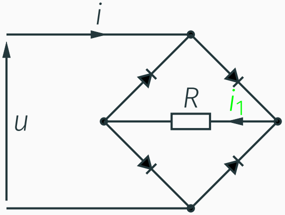


$$u(t) = U_m \cos(\omega t)$$

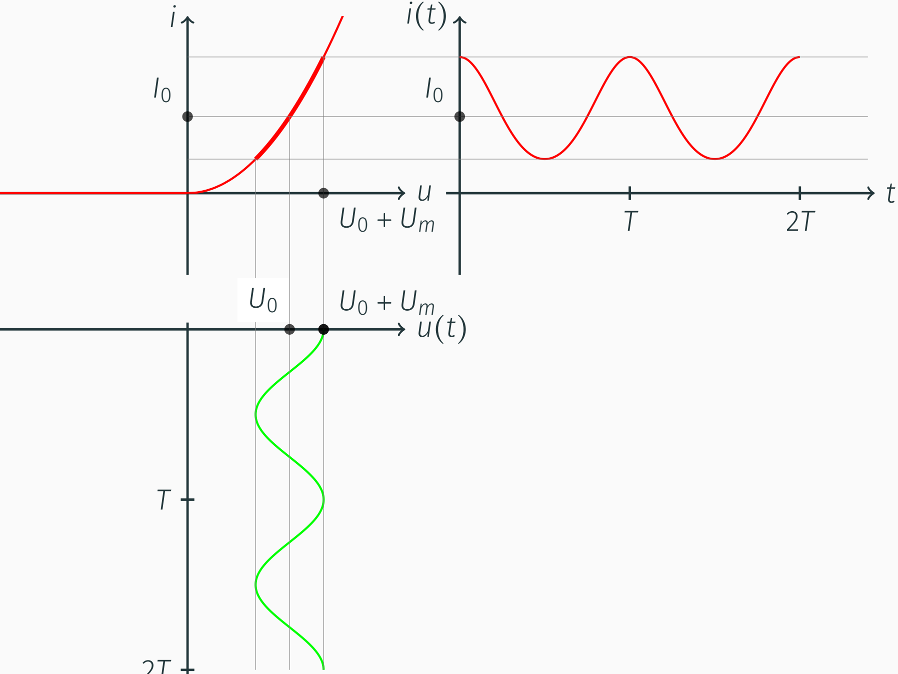
Projection — full-wave rectifier



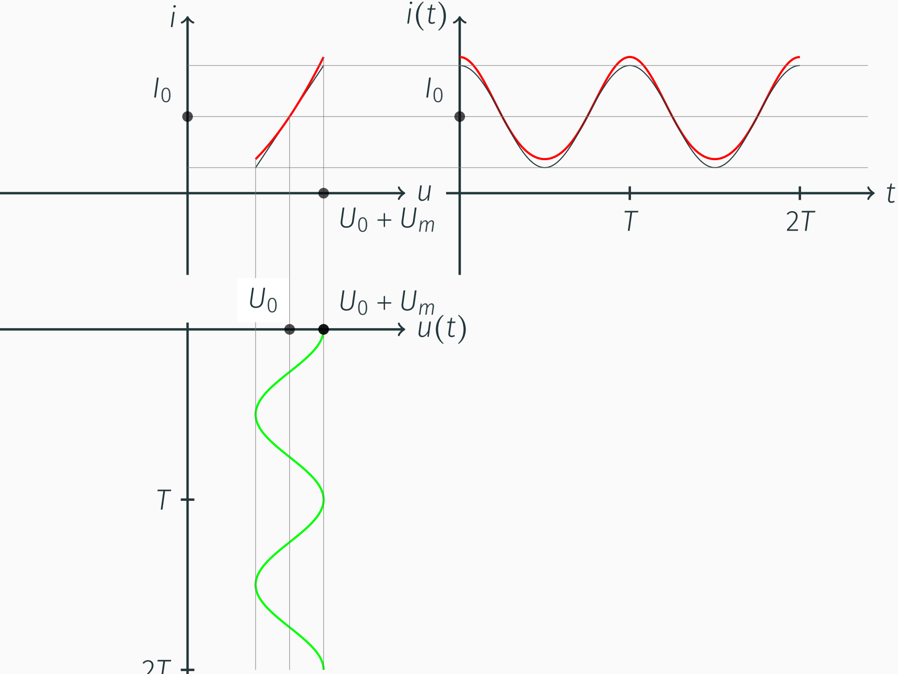




$$u(t) = U_m \cos(\omega t)$$



i
i(t)
I0
I0
u
t
U0 + Um
T
2T
U0
U0 + Um
u(t)
T
2T



i
$i(t)$
I_0
I_0
u
t
$U_0 + U_m$
T
$2T$
U_0
$U_0 + U_m$
$u(t)$
T
$2T$

Small-signal method

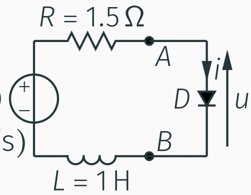
Small-signal method is a method of approximation of the solution to a non-linear circuit by the sum of:

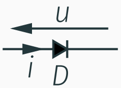
- DC solution of the original non-linear circuit (with all time-varying signals reduced to zero),
- periodic solution (can be harmonic in particular case) to a linear circuit that is obtained through linearization (around the DC solution) of the original circuit and reducing the DC components to zero.

The smaller are non-DC components of the signals, the better is the approximation obtained.

Small-signal analysis

$$e(t) = E_0 + E_m \cos(\omega t)$$
$$= 3\,\mathrm{V} + 1\,\mathrm{mV} \cos(t \cdot 1\,\mathrm{rad/s})$$





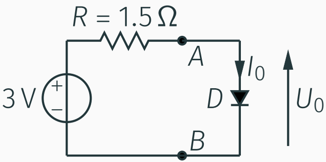
,

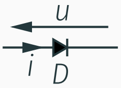
$$i = f(u) = \begin{cases} 0 & \text{if } u < 0, \\ \frac{u^2}{\alpha} & \text{if } u \geq 0, \end{cases} \qquad \alpha = \tfrac{9}{2}\,\mathrm{V}\,\Omega.$$

$$i = ?, \qquad u = ?$$

$$i(t) \approx I_0 + I_m \cos(\omega t + \psi), \qquad u(t) \approx U_0 + U_m \cos(\omega t + \phi).$$

Small-signal analysis





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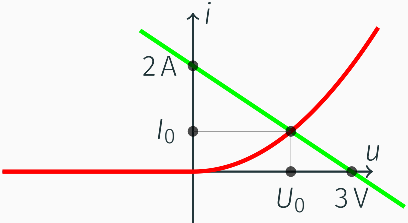
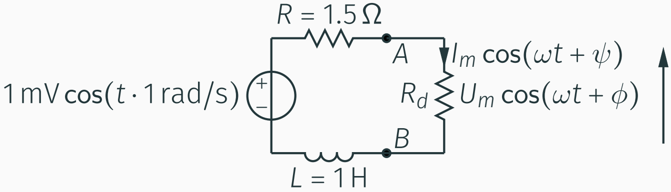
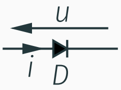
$i =?$, $u =?$



$$U_0 \approx 1.9\,\mathrm{V}, \quad I_0 \approx 0.8\,\mathrm{A}.$$

Small-signal analysis





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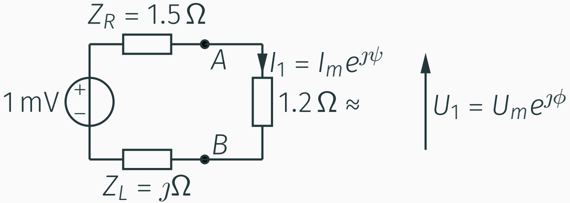
$$i = f(u) = \begin{cases} 0 & \text{if } u < 0, \\ \frac{u^2}{\alpha} & \text{if } u \geq 0, \end{cases} \qquad \alpha = \tfrac{9}{2}\,\mathrm{V}\,\Omega.$$

$$R_d = \frac{1}{f'(U_0)} = \frac{\alpha}{2U_0} \approx 1.2\,\Omega.$$

Small-signal analysis



$$I_1 = \frac{E_m}{R + R_d + \jmath\omega L} \approx (0.3 - \jmath 0.1)\text{mA} \approx 0.3e^{-\jmath 0.4}\text{mA}.$$

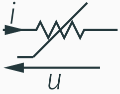
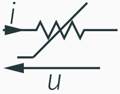
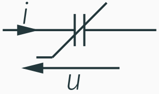
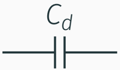
From Ohm's law:

$$U_1 = R_d I_1 \approx 0.4e^{-\jmath 0.4}\text{mV}.$$

$$i(t) \approx 0.8\,\text{A} + 0.3\cos(\omega t - 0.4)\text{mA},$$

$$u(t) \approx 1.9\,\text{V} + 0.4\cos(\omega t - 0.4)\text{mV}.$$

One-ports linearization

one-port	linearization	formula
R_N: $i = f(u)$ 		$R_d = \frac{1}{f'(U_0)}$
R_N: $u = g(i)$ 		$R_d = g'(I_0)$
C_N: $q = f(u)$ 		$C_d = f'(U_0)$
L_N: $\psi = f(i)$ 		$L_d = f'(I_0)$

Example

Find $i(t)$. $e(t) = E_0 + E_1 \cos \omega t$, $E_0 = 1\,\mathrm{V}$, $E_1 = 22\,\mathrm{mV}$, $R = 1\,\mathrm{k\Omega}$, $L = 4\,\mathrm{mH}$, L_N: $\psi(i) = l_0 i + \alpha i^2$, $l_0 = 4\,\mathrm{mH}$, $\alpha = -\frac{1}{2}\mathrm{Wb/A^2}$, $\omega = 1\,\mathrm{Mrad/s}$.