EPRST: Probability and Statistics
Problem set 11

1. Let $(X_n)_n$ be a sequence of i.i.d. random variables distributed with the distribution function

$$F(t) = \begin{cases} 0, & t < 0, \\ t/4, & 0 \leq t < 4, \\ 1, & t \geq 4. \end{cases}$$

   Then (answer *yes* or *no*):

   (a) $\mathbb{P}\left(\sum_{n=1}^{100} X_n < 200\right) = 0.5$,

   (b) $\mathbb{P}\left(\sum_{n=1}^{300} X_n < 560\right) + \mathbb{P}\left(\sum_{n=1}^{300} X_n < 640\right) = 1$,

   (c) $\mathbb{P}\left(580 < \sum_{n=1}^{300} X_n < 620\right) > 0.5$,

   (d) $\mathbb{P}\left(\left|\sum_{n=1}^{100} X_n\right| \leq 20\right) > 0.5$.

2. We toss an ordinary coin 1000 times. Use the Central Limit Theorem to answer: what is the probability that the number of tails will be between 475 and 525?

3. There are 100 rooms in a hotel. Since the owner knows that 10% of the early reservations are canceled before the arrival, he ordered to accept reservations for more than 100 rooms. What is the probability that after accepting 104 reservations the hotel will run out of vacant rooms? Use the Central Limit Theorem.

4. We roll a dice 144 times. What is the probability that the number of appearances of 6 will be between 20 and 26? Use the Central Limit Theorem to justify your answer.

5. We toss a biased coin ($\mathbb{P}(\mathrm{T}) = 1/4$) one hundred times. What is the probability that the number of tails will exceed 30? Use the Central Limit Theorem to justify your answer.

6. A plane is loaded with 100 packages whose weights (in kgs) are independent random variables with uniform distribution $\mathcal{U}(5, 55)$. What is the probability that the total weight will exceed 3000?

7. Let $(X_n)_n$ be a sequence of i.i.d. random variables with $\mathbb{E}(X_1) = 5$ and $\mathrm{Var}(X_1) = 9$. Using the Central Limit Theorem find the largest value of n such that:

$$\mathbb{P}\left(X_1 + \ldots + X_n \geq 200 + 5n\right) \leq 0.05.$$