

Decibels

Comparison of power and reference power: $P_{dB} = 10 \log_{10} \frac{P}{P_r}$

Comparison of 2 amplitudes of rms values: $P_{dB} = 10 \log_{10} (\frac{A}{A_r})^2 = 20 \log_{10} \frac{A}{A_r}$

Signal energy and power

Instantaneous power: $P(t) = x^2(t)$

Energy: $E = \int_0^T x^2(t) dt$

Average power: $P = \frac{1}{T} \int_0^T x^2(t) dt$

Periodic signals

$\forall x(t+T) = x(t)$ $T > 0$ is a period

Fourier Transform

$$X(f) = F[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

Inverse Fourier Transform

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

Shift theorem

$$F[x(t-t_0)] = e^{-j2\pi ft_0} X(f)$$

Inv Fourier Transform Calculation

$$X(f) = \text{rect}_{2B}(f)$$

$$x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df = 2B \frac{\sin(2\pi Bt)}{2\pi Bt}$$

Z transform

A series of samples $\{x_n\}$, ideal sampling: $x_s(t) = \sum_n x_n \delta(t - nT)$

$$X_s(f) = \sum x_n F[\delta(t - nT)] = \sum x_n e^{-j2\pi fnT}$$

substitute $z = e^{j2\pi fT}$

f	z
0	1
$\frac{1}{4T}$	j
$\frac{1}{2T}$	-1
$\frac{3}{4T}$	-j
$\frac{1}{T}$	1

Z Transform: $X(z) = Z[\{x_n\}] = \sum_n x_n z^{-n}$ defined for any complex variable z.

Properties

$$Z[\{ax_n + by_n\}] = aX(z) + bY(z)$$

$$Z[\{x_{n+k}\}] = z^k X(z)$$

$$F[x(t) * y(t)] = F[x(t)]F[y(t)] = X(f)Y(f) \quad F^{-1}[x(t)y(t)] = X(f) * Y(f)$$