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Numerical Methods (ENUME) – Project
Assignment A: Accuracy of computation

1. Determine the coefficient $T(x)$ characterising the propagation of the relative error corrupting the datum x to the result of computing the following expression:

$$y(x) = \frac{\cos(x)}{x^3} - x^2 \quad \text{for } x \in [0, 3]$$

Compare the values of the coefficient $T(x)$ obtained using: (a) the method of *epsilon calculus* and (b) the method of simulation, implemented in the following way:

(i) For a given value of x, compute $y(x)$ and $y(\tilde{x})$ with $\tilde{x} = x(1 + \varepsilon_{\text{sim}})$ and $\varepsilon_{\text{sim}} = 10^{-8}$.

(ii) Determine $T(x)$ according to the formula:

$$T(x) = \frac{1}{\varepsilon_{\text{sim}}} \left| \frac{y(\tilde{x}) - y(x)}{y(x)} \right|$$

(iii) Repeat the above steps for 100 values of $x \in [0, 3]$.

2. Determine the coefficient $K_{\text{A1}}(x)$ characterising the propagation of the relative errors caused by rounding the intermediate results of computing $y(x)$ according to the following algorithm:

$$\text{A1}: \quad [x] \rightarrow \begin{bmatrix} v_1 = \cos(x) \\ v_2 = x^3 \end{bmatrix} \rightarrow \begin{bmatrix} v_3 = v_1/v_2 \\ v_4 = x^2 \end{bmatrix} \rightarrow [y = v_3 - v_4]$$

Compare the values of the coefficient $K_{\text{A1}}(x)$ obtained using (a) the method of *epsilon calculus*, (b) the method of simulation, implemented in the following way:

(i) For a given value of x, compute $y(x)$ and $\tilde{y}(x)$ – the result of the algorithm presented above, obtained under the assumption that the result of each elementary operation is corrupted with a relative error equal either ε_{sim} or $-\varepsilon_{\text{sim}}$. Pick the sign of each error randomly.

(ii) Determine $K_{\text{A1}}(x)$ according to the formula:

$$K_{\text{A1}}(x) = \frac{1}{\varepsilon_{\text{sim}}} \left| \frac{\tilde{y}(x) - y(x)}{y(x)} \right|$$

(iii) Repeat the above steps 300 times, each time randomly picking the signs of the errors, and select the largest among the obtained values of $K_{\text{A1}}(x)$.

(iv) Repeat the above steps for 100 values of $x \in [0, 3]$.

3. Determine the coefficient $K_{\text{A2}}(x)$ characterising the propagation of the relative errors caused by rounding the intermediate results of computing $y(x)$ according to the following algorithm:

$$\text{A2}: \quad [x] \rightarrow \begin{bmatrix} v_1 = \cos(x) \\ v_2 = x^5 \end{bmatrix} \rightarrow \begin{bmatrix} v_3 = v_1 - v_2 \\ v_4 = x^3 \end{bmatrix} \rightarrow [y = v_3/v_4]$$

Compare the values of the coefficient $K_{\text{A2}}(x)$ obtained using (a) the method of *epsilon calculus*, (b) the method of simulation, implemented in the same way as in Task 2.

4. Present graphically and compare the dependencies of $T(x)$, $K_{\text{A1}}(x)$ and $K_{\text{A2}}(x)$ on x for $x \in [0, 3]$. Discuss the observations made on that basis.