

Problem 05a: Heisenberg's uncertainty principle



Both, a **bullet of a mass 50 g** and a **free electron** are traveling with the same velocities of **300 m/s** known with a precision of **0.01%**.

What is the maximum precision Δx with which one can find the positions of these objects when measured along with the velocities in the same experiment?



Let's follow Heisenberg's uncertainty principle in the following form:

$$\Delta x \cdot \Delta p \geq h$$

$$\Delta p = m \Delta v \quad \Delta v = \quad \Delta x \geq$$

$$h = 6.626 \times 10^{-34} \text{ [Js]}$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ [Js]}$$

$$m_0 = 9.11 \times 10^{-31} \text{ [kg]}$$

$$q = 1.602 \times 10^{-19} \text{ [C]}$$

$$k_B = 1.381 \times 10^{-23} \text{ [J/K]}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ [F/m]}$$

$$c = 3 \times 10^8 \text{ [m/s]}$$

	$\Delta v \text{ [m/s]}$	$\Delta p \text{ [kg*m/s]}$	$\Delta x \text{ [m]}$
bullet:	0.03	0.0015	4.42E-31
electron:	0.03	2.733E-32	0.024

$$E = \frac{p^2}{2m} \longrightarrow \frac{\Delta E}{\Delta p} = \frac{2p}{2m} = \frac{p}{m} \longrightarrow \Delta E = \frac{p}{m} \Delta p$$

$$\Delta E = v \cdot \Delta p = \frac{\Delta x}{\Delta t} \cdot \Delta p \longrightarrow \Delta E \cdot \Delta t = \Delta x \cdot \Delta p \longrightarrow \Delta E \cdot \Delta t \geq h$$

Problem 05b: Heisenberg's uncertainty principle

How long one needs to observe/measure these objects to improve the precision of finding their velocities v up to **0.0001%**?

$$prec = \frac{\Delta v}{v}$$

$$\Delta E = v \cdot \Delta p = v \cdot m \Delta v$$

$$\Delta E = prec \cdot v^2 \cdot m$$

$$\Delta t \geq \frac{h}{prec \cdot m \cdot v^2}$$

$$\Delta E \cdot \Delta t = \Delta x \cdot \Delta p \geq h$$

$$h = 6.626 \times 10^{-34} \text{ [Js]}$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ [Js]}$$

$$m_0 = 9.11 \times 10^{-31} \text{ [kg]}$$

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$$c = 3 \times 10^8 \text{ [m/s]}$$

	m [kg]	v [m/s]	Δt [s]
bullet:	0.05	300	1.47E-31
electron:	9.11E-31	300	8.08E-03