**Answers - sample problems**

1. (a) $\frac{1}{4}\left(1 + \left(\frac{5}{6}\right)^2 + 2\left(\frac{5}{8}\right)^2\right)$

   (b) $\frac{2\left(\frac{5}{8}\right)^2}{1+\left(\frac{5}{6}\right)^2+2\left(\frac{5}{8}\right)^2}$

2. $\frac{\binom{5}{3}\left(\frac{3}{5}\right)^2}{\sum_{k=0}^{3}\binom{3+k}{3}\left(\frac{3}{5}\right)^k}$

3. $\frac{64}{65}$

4. $S_X = \{0,1,2,3\}$, $\mathbb{P}(X=k) = \binom{3}{k}\left(\frac{1}{2}\right)^3 \frac{8}{10}$ $\qquad k = 0,1,2,$

   $\mathbb{P}(X=3) = \frac{2}{10} + \frac{8}{10}\left(\frac{1}{2}\right)^3$

5. (a) $\mathbb{P}(X \geq 5) = \left(\frac{1}{3}\right)^6$

   (b) $\frac{1}{4}$

6.
$$F_X(t) = \begin{cases} 0, & t < -1, \\ \frac{(t+1)^2}{4}, & -1 \leq t < 1, \\ 1, & t \geq 1. \end{cases}$$

7.
$$F_X(t) = \begin{cases} 0, & t < 0, \\ 1 - (1-t)^2, & 0 \leq t < 1, \\ 1, & t \geq 1. \end{cases}$$

8. (a) $a = 1$

   (b) $F_X(t) = \begin{cases} 0, & t < -2, \\ \int_{-2}^{t}\left(-\frac{1}{3}x\right), & -2 \leq t < 0, \\ \int_{-2}^{0}\left(-\frac{1}{3}x\right) + \int_0^t x^2 dx, & 0 \leq t < 1, \\ 1 & t \geq 1. \end{cases}$

9. (a) $\mathbb{P}(X < -2) = \frac{1}{2}$,

   (b) $\mathbb{P}(X > -5) - \mathbb{P}(X < 1) = 0$,

10. (a) $S_X = \{-2,-1,1,3\}$, $p_X(-2) = \frac{1}{8}$, $p_X(-1) = \frac{3}{8}$, $p_X(1) = \frac{1}{6}$, $p_X(3) = \frac{1}{3}$,

    (b) $\mathbb{P}(X(X+1) > 0) = \frac{5}{8}$.

11. $\frac{4\binom{5}{2}}{3\binom{4}{1}\binom{5}{1}+2\binom{4}{2}+4\binom{5}{2}}$

12. (a) $\mu = 70$,

    (b) These two probabilities are equal.