1. The cumulative distribution function of a random variable X is given by

$$F_X(t) = \begin{cases} 0, & t < 0, \\ \frac{t^2}{4}, & 0 \leq t < 2, \\ 1, & t \geq 2. \end{cases}$$

   (a) Compute variance and standard deviation of X.

   (b) Determine the median of distribution of the random variable X.

2. The random variable X has a Rayleigh distribution with the density

$$f(x) = \begin{cases} xe^{-x^2/2}, & x > 0, \\ 0 & x \leq 0. \end{cases}$$

   Find the first quartile (q_1), median (q_2), and the third quartile (q_3) of the distribution of the random variable X, i.e. q_j such that $\mathbb{P}(X \leq q_j) = \frac{j}{4}$, $j = 1, 2, 3$.

3. Determine the median of the exponential distribution with parameter λ. Are the median and the expected value equal?

4. We roll a fair die six times. Let X be the number of ones and Y the number of twos obtained in six rolls. Determine the distribution of the random vector (X, Y).

5. On the probability space $(\Omega, \mathbb{P})$, where $\Omega = \{0, 1, \ldots, 9\}$, $\mathbb{P}(\{\omega\}) = 0.1 \ \forall \ \omega \in \Omega$, we define random variables $X(\omega)$ − the remainder from the division of ω by 2, $Y(\omega)$ − the remainder from the division of ω by 3. Find the distribution of the vector (X, Y). What is $\mathbb{P}(X = Y)$?

6. Two-dimensional random variable (X, Y) has the following distribution:

X\Y	-1	0	1
0	0.1	0.1	0
1	0.2	0.2	0.1
2	0.1	0.1	0.1

   . Compute the marginal distributions. Compute $\mathbb{P}(X \leq 1, Y \leq 2)$ and $\mathbb{P}(X \leq 4, Y \leq 0.5)$. What is $\mathbb{P}(X > 2Y)$?

7. Let a be a real number, and let the distribution of the random vector (X, Y) be given by the following table

X\ Y	−1	0	1
4	$a - 1/16$	$1/4 - a$	0
5	$1/8$	$3/16$	$1/8$
6	$a + 1/16$	$1/16$	$1/4 - a$

   (a) What values can a take?

   (b) Determine parameter a such that $\mathbb{E}Y^2 = \frac{5}{8}$.

8. We roll a die. On the probability space $(\Omega, \mathbb{P})$, where $\Omega = \{1, 2, \ldots, 6\}$ and $\mathbb{P}(\omega) = \frac{1}{6}$, $\forall \omega \in \Omega$, we define two random variables: $X(\omega) = \omega$ and $Y(\omega) = \omega(\text{mod } 4)$. Determine ditribution of the random vector (X, Y).

9. Roll a pair of fair dice. If X denotes the smaller and Y denotes the larger outcome on the dice, what is the joint distribution of the random vector (X, Y)?