1 Truth Tables

Note 1

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

(a)

P	Q	$P\wedge(Q\vee P)$	$P\wedge Q$
T	F	T	F
F	T	F	F
T	T	T	T
F	F	F	F

(b) (c)

P	Q	R	$(P\vee Q)\wedge R$	$(P\wedge R)\vee(Q\wedge R)$	$(P\wedge Q)\vee R$	$(P\vee R)\wedge(Q\vee R)$
T	T	T	T	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	F	F	F	F
F	F	T	F	F	T	T
F	F	F	F	F	F	F

2 Propositional Practice

Note 1

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{Z})\ (x \in \mathbb{Q})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

(a) $(\exists x\in\mathbb{R})(\exists a,b\in\mathbb{Z})\ (x\neq\frac{a}{b})$ ✓

(b) $(\forall x\in\mathbb{Z})((x\in\mathbb{N})\vee(x<0))$ ✓

(c) $(\forall x\in\mathbb{N})((6\mid x)\Rightarrow((2\mid x)\vee(3\mid x)))$ ✓

(d) Any number which is integer is rational ✓

(e) If a integer is divisible by 2 or by 3, it is divisible by 6 ✗

(f) Any natural number is greater than 7 can be divided into the sum of two natural numbers. ✓

3 Converse and Contrapositive

Note 1
Note 2

Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample. Consider using part (a).

(a) $(\forall x \in N)((4|x) \Rightarrow (2|x))$

(b) If a natural number isn't divisible by 4, it isn't divisible by 2.
$(\forall x \in N)(\neg(4|x) \Rightarrow \neg(2|x))$ e. 6 . $(\forall x \in N)(\neg(2|x) \Rightarrow \neg(4|x))$

(c) If a natural number isn't divisible by 2, it isn't divisible by 4
Assume x isn't divisible by 2 x is odd if x is divisible by 4

4 Logical Equivalence?

$x = 4k$ x is even $R \wedge \neg R$ x is

Note 1

Decide whether each of the following logical equivalences is correct and justify your answer.

(a) $\forall x\,\big(P(x) \wedge Q(x)\big) \equiv \forall x\,P(x) \wedge \forall x\,Q(x)$

(b) $\forall x\,\big(P(x) \vee Q(x)\big) \equiv \forall x\,P(x) \vee \forall x\,Q(x)$

(c) $\exists x\,\big(P(x) \vee Q(x)\big) \equiv \exists x\,P(x) \vee \exists x\,Q(x)$

(d) $\exists x\,\big(P(x) \wedge Q(x)\big) \equiv \exists x\,P(x) \wedge \exists x\,Q(x)$