1 Truth Tables

Note 1 Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a)
$$P \wedge (Q \vee P) \equiv P \wedge Q$$

(b)
$$(P \lor Q) \land R \equiv (P \land R) \lor (Q \land R)$$

(c)
$$(P \land Q) \lor R \equiv (P \lor R) \land (Q \lor R)$$

2 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

- (a) There is a real number which is not rational.
- (b) All integers are natural numbers or are negative, but not both.
- (c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.
- (d) $(\forall x \in \mathbb{Z}) (x \in \mathbb{Q})$

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Note 1

- (e) $(\forall x \in \mathbb{Z}) (((2 \mid x) \lor (3 \mid x)) \Longrightarrow (6 \mid x))$
- (f) $(\forall x \in \mathbb{N}) ((x > 7) \implies ((\exists a, b \in \mathbb{N}) (a + b = x)))$

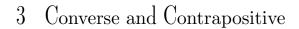
(a)
$$(\exists x \in \mathbb{R})(\exists a, b \in \mathbb{Z}) (x \neq \frac{a}{b})$$

<u>(b)</u> (∀x∈Z)((x∈N)∨(X<D)

$$(C)(AXEN)((PIX) \Rightarrow ((PIX) \land (PIX)))$$

(d) Any number which is integer is rational V

- (e) If a integer is divisible by 2 or by 3, it is divisible by 6x
- (f) Any natural number is greater than 7 can be divided into



Note 1 Note 2 Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

- (a) Write the statement in propositional logic. Prove that it is true or give a counterexample.
- (b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \Longrightarrow Q$ is $\neg P \Longrightarrow \neg Q$.)
- (c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.
- (d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample. Consider using part (a).

(a)
$$(\forall x \in \mathbb{N}) ((4|x) \Rightarrow (2|x))$$

- (b) If a natural number isn't divisible by 4, it isn't divisible by 2 $(\forall x \in \mathbb{N}) (\neg (4 \mid X) \Rightarrow \neg (2 \mid X))$ e. 6. $(\forall x \in \mathbb{N}) (\neg (2 \mid X) \Rightarrow \neg (4 \mid X))$ (C) If a natural number is n't divisible by 2, it isn't divisible by 4
- Assume xisn't divisible by 2 x is odd if x is divisible by 4
- 4 Logical Equivalence? 7=4k 7is even RATR Xis

Note 1

Decide whether each of the following logical equivalences is correct and justify your answer.

(a)
$$\forall x (P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)$$

(b)
$$\forall x (P(x) \lor Q(x)) \equiv \forall x P(x) \lor \forall x Q(x)$$

(c)
$$\exists x (P(x) \lor Q(x)) \equiv \exists x P(x) \lor \exists x Q(x)$$

(d)
$$\exists x (P(x) \land Q(x)) \equiv \exists x P(x) \land \exists x Q(x)$$