1 Truth Tables

Note 1

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \land (Q \lor P) \equiv P \land Q$

(b) $(P \lor Q) \land R \equiv (P \land R) \lor (Q \land R)$

(c) $(P \land Q) \lor R \equiv (P \lor R) \land (Q \lor R)$

(a)

P	Q	$P \land (Q \lor P)$	$P \land Q$
T	F	T	F
F	T	F	F
T	T	T	T
F	F	F	F

(b) (c)

P	Q	R	$(P \lor Q) \land R$	$(P \land R) \lor (Q \land R)$	$(P \land Q) \lor R$	$(P \lor R) \land (Q \lor R)$
T	T	T	T	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	F	F	F	F
F	F	T	F	F	T	T
F	F	F	F	F	F	F

2 Propositional Practice

Note 1

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{Z})\ (x \in \mathbb{Q})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \lor (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

3 Converse and Contrapositive

Note 1
Note 2

Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample. Consider using part (a).

4 Logical Equivalence?

Note 1

Decide whether each of the following logical equivalences is correct and justify your answer.

(a) $\forall x \, \big(P(x) \wedge Q(x)\big) \; \equiv \; \forall x \, P(x) \wedge \forall x \, Q(x)$

(b) $\forall x \, \big(P(x) \vee Q(x)\big) \; \equiv \; \forall x \, P(x) \vee \forall x \, Q(x)$

(c) $\exists x \, \big(P(x) \vee Q(x)\big) \; \equiv \; \exists x \, P(x) \vee \exists x \, Q(x)$

(d) $\exists x \, \big(P(x) \wedge Q(x)\big) \; \equiv \; \exists x \, P(x) \wedge \exists x \, Q(x)$