Name: ______________________

Passive Filters

Introduction

In the last couple of lessons, we studied the effect of complex impedances on performing AC circuit analysis. More specifically, we examined the phase difference between current and voltage associated with reactive components such as capacitors and inductors. In this lesson, we will learn how to use complex math and phasor addition to determine of simple passive filter circuits.

Discussion Overview

Processing audio signals, often times, involves separating the signal into low, mid and high frequencies. This is done by means of filtering. Although most modern audio processing equipment use complex and sophisticated active filters, there are simple passive filters that can be used in simple inexpensive hobby circuits and provide the basis for more complex filters. The circuit in Figure 1 is a simple "crossover" filter that separates an incoming audio signal into bass and treble components.

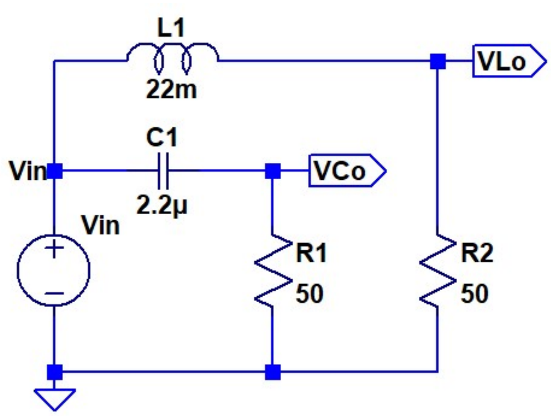

Figure 1 – Simple Crossover Circuit

Human ears can detect audio signals in the range of ~20Hz-20KHz. The circuit above places a crossover point at ~650Hz. For the purpose of our experiment in this lesson, we consider the signals lower than 650Hz as bass and the ones above as the treble components.

In the following sections, we will derive the expressions for the output voltages at V_{Lo} and V_{Co} separately.

Bass Output

We know that at low frequencies an inductor acts as a short while at high frequencies, it acts as an open. Therefore, we can expect that the voltage at V_{Lo} to be close to V_{in} at low frequencies and close to 0 at high frequencies. V_{Lo}, therefore, is our bass output.

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To analytically examine the signal at V_{Lo}, we use the voltage divider equation. However, we need to be cognizant of the complex nature of an inductor's impedance and perform all our math in complex domain.

$$V_{Lo} = V_{in}\frac{R_2}{R_2 + j\omega L} \ or \ \frac{V_{Lo}}{V_{in}} = \frac{R_2}{R_2 + j\omega L}$$

Factoring out R_2 in the denominator, we have

$$\frac{V_{Lo}}{V_{in}} = \frac{R_2}{R_2\left(1 + j\omega\frac{L}{R_2}\right)} = \frac{1}{1 + j\omega\frac{L}{R_2}} \quad \textbf{Eq. 1}$$

We are interested in the magnitude of the ratio above which can be found by dividing the magnitude of the numerator by the magnitude of the denominator.

$$\left|\frac{V_{Lo}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \left(\omega\frac{L}{R_2}\right)^2}} \quad \textbf{Eq. 2}$$

The ratio in Eq. 2 is called the gain of the circuit. To examine this gain, we first look at its value at $\omega_c = \frac{R_2}{L}$ called the corner frequency. At $\omega_c = \frac{R_2}{L}$,

$$\left|\frac{V_{Lo}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \left(\frac{R_2}{L}\frac{L}{R_2}\right)^2}} = \frac{1}{\sqrt{2}}$$

As seen, at the corner frequency, the gain is $\frac{1}{\sqrt{2}}$, or V_{Lo} is 3dB lower than V_{in} since

$$20\log\left(\frac{1}{\sqrt{2}}\right) = -3dB$$

At frequencies much lower than $\omega_c = \frac{R_2}{L}$, the denominator of the ratio in Eq. 2 is approximately 1; and therefore, the ratio is approximately 1.

$$\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega \ll \omega_c} \approx \frac{1}{\sqrt{1 + 0^2}} \approx 1$$

At frequencies much larger than the corner frequency, the denominator is dominated by the second term; and therefore,

$$\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega \gg \omega_c} \approx \frac{1}{\sqrt{\left(\omega\frac{L}{R_2}\right)^2}} \approx \frac{1}{\omega\frac{L}{R_2}}$$

In dB domain,

$$20\log\left(\left|\frac{V_{Lo}}{V_{in}}\right|_{\omega\ll\omega_c}\right) \approx 20\log\left(\frac{1}{\omega\frac{L}{R_2}}\right) = -20\log\left(\omega\frac{L}{R_2}\right) = -20\log\omega - 20\log\left(\frac{L}{R_2}\right)$$

which is a line with a slope of -20dB.

The behavior of the ratio in Eq. 2 is shown below.

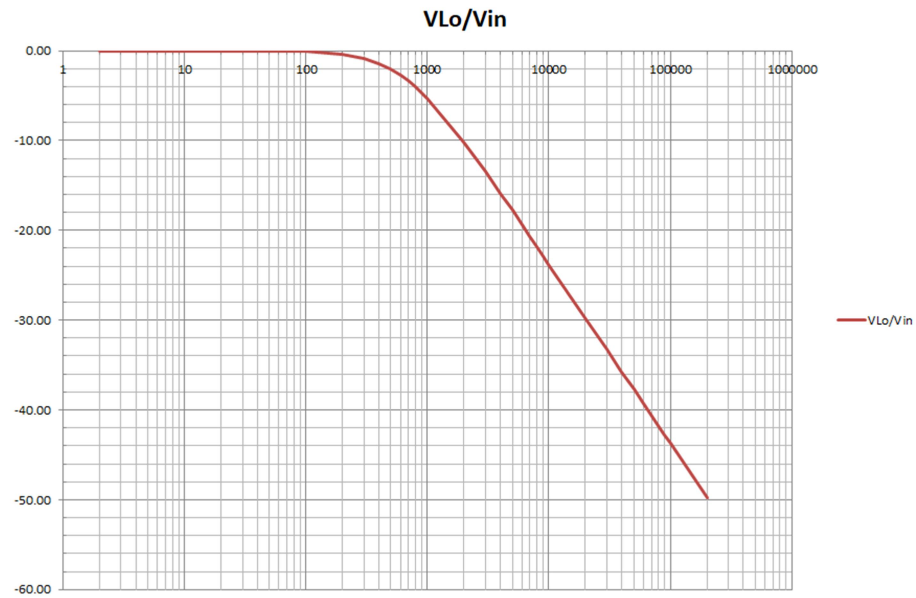

Figure 2 - Frequency Response of the Bass Output

Treble Output

A capacitor, at low frequencies, as an open while at high frequencies, it acts as a short. Therefore, we can expect that the voltage at V_{Co} to be close to V_{in} at high frequencies and close to 0 at low frequencies. V_{Co}, therefore, is our treble output.

The analytical examination of the signal at V_{Co}, follows that of V_{Lo} very closely.

$$V_{Co} = V_{in}\frac{R_1}{R_1 + \frac{1}{j\omega C}} \quad or \quad \frac{V_{Co}}{V_{in}} = \frac{j\omega R_1 C}{j\omega R_1 C + 1}$$

We are interested in the magnitude of the ratio above which can be found by dividing the magnitude of the numerator by the magnitude of the denominator.

$$\left|\frac{V_{Co}}{V_{in}}\right| = \frac{\omega R_1 C}{\sqrt{1 + (\omega R_1 C)^2}} \qquad \textbf{Eq. 3}$$

To examine this gain, we again first look at its value at its corner frequency $\omega_c = \frac{1}{R_1 C}$. At the corner frequency, Name: ______________________

$$\left|\frac{V_{Co}}{V_{in}}\right|_{\omega=\frac{1}{R_1C}} = \frac{1}{\sqrt{1+\left(\frac{1}{R_1C}R_1C\right)^2}} = \frac{1}{\sqrt{2}}$$

As seen, at the corner frequency, the gain is $\frac{1}{\sqrt{2}}$, or V_{Co} is 3dB lower than V_{in} since

$$20\log\left(\frac{1}{\sqrt{2}}\right) = -3dB$$

At frequencies much larger than the corner frequency, the denominator is dominated by the second term; and therefore,

$$\left|\frac{V_{Co}}{V_{in}}\right|_{\omega\gg\omega_c} \approx \frac{\omega R_1 C}{\sqrt{(\omega R_1 C)^2}} = 1$$

At frequencies much lower than $\omega_c = \frac{1}{R_1C}$, the denominator of the ratio in Eq. 3 is approximately 1; and therefore, the ratio is approximately

$$\left|\frac{V_{Co}}{V_{in}}\right|_{\omega\ll\omega_c} \approx \frac{\omega R_1 C}{\sqrt{1+0^2}} = \omega R_1 C$$

In dB domain,

$$20\log\left(\left|\frac{V_{Co}}{V_{in}}\right|_{\omega\ll\omega_c}\right) \approx 20\log(\omega R_1 C) = 20\log\omega + 20\log(R_1 C)$$

which is again a line with a slope of -20dB.

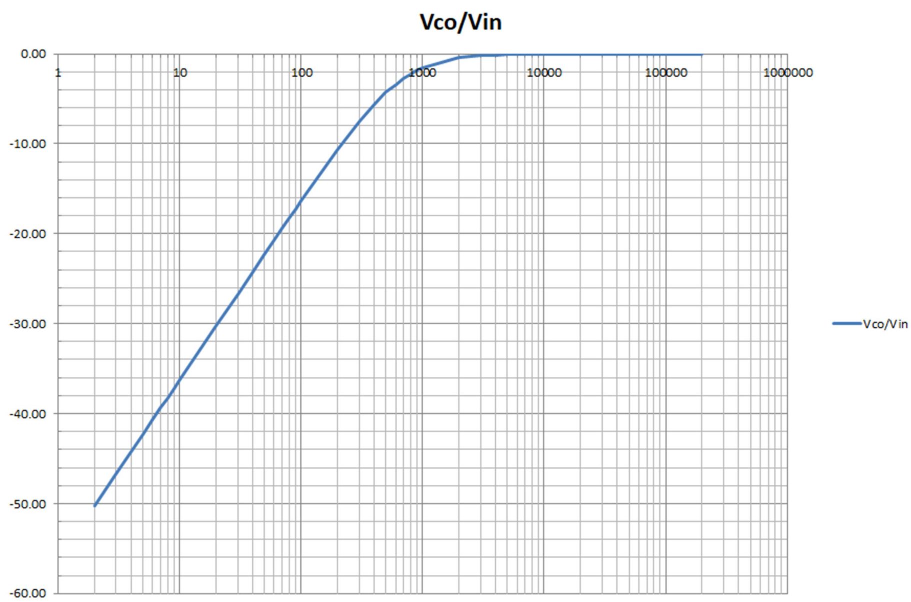

Figure 3 - Frequency Response of the Treble Output

The two frequency responses super imposed are shown in below.

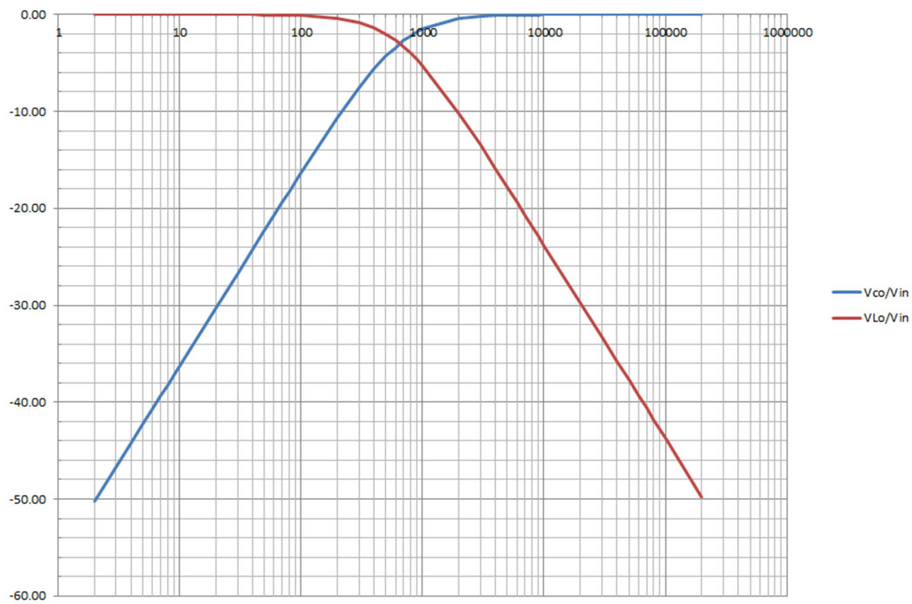

Figure 4 - Freqency Respone of the Crossover Filter

Procedure

SPICE Simulation

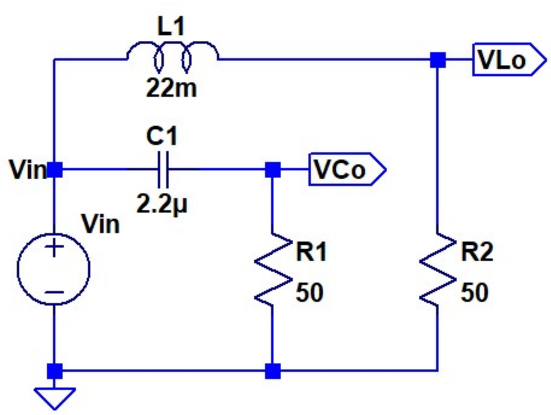

Figure 5 - Inductor and Current Source

A. Build the crossover filter circuit above in SPICE.
B. Configure the voltage source as a sine wave and set its AC amplitude to 3V.
C. Setup the model to run a “AC” simulation with the following parameters
    a. Steps set to decade,
    b. 100 points per decade,
    c. Starting frequency at 2Hz,

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d. Ending frequency at 200KHz

Below is the syntax for your reference:

.ac <oct, dec, lin> <Npoints> <StartFreq> <EndFreq]

D. Run the simulation and display the waveforms for the voltages.

E. What is the corner frequency for each output?

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