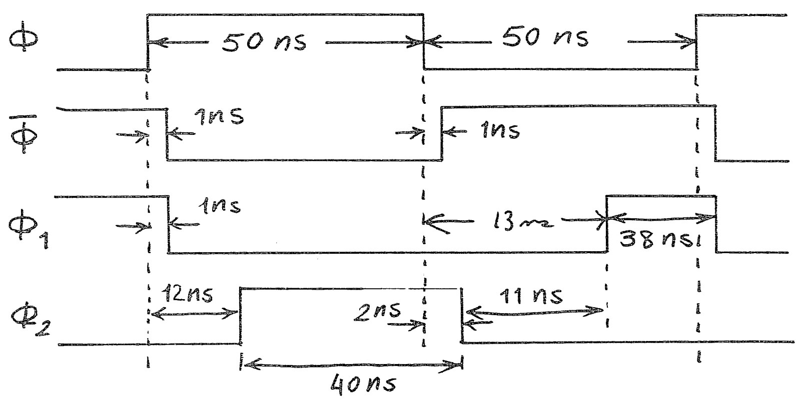
Chapter 10 – Problems

10.1) The clock period is $\frac{1}{10\ \text{MHz}} = 100\ ns$. Assuming 50 % duty cycle, we have the following timing diagram:

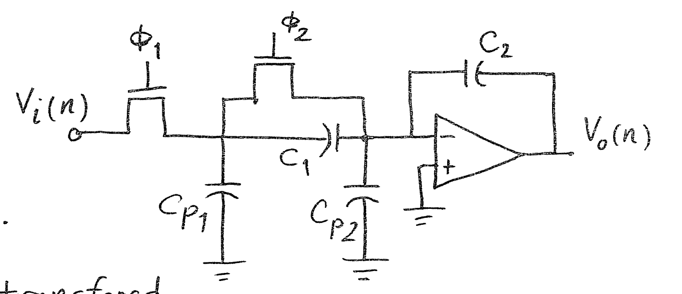


(not to scale)

10.2) $C_2 V_o(n) = C_2 V_o(n-1) - C_1 V_i(n)$

$$\Rightarrow C_2 V_o(z) = C_2 z^{-1} V_o(z) - C_1 V_i(z) \Rightarrow \frac{V_o(z)}{V_i(z)} = \frac{-C_1/C_2}{1-z^{-1}}$$

10.3) C_{P2} is always discharged since its voltage is virtually ground.



During Φ_1, C_{P1} is charged to $V_i(n)\,C_{P1}$.
This charge will be transfered to C_2 during Φ_2. Therefore: $C_2 V_o(n) = C_2 V_o(n-1) - C_{P1} V_i(n-1) - C_1 V_i(n)$

$$\Rightarrow \frac{V_o(z)}{V_i(z)} = -\frac{\frac{C_1}{C_2} + \frac{C_{P1}}{C_2} z^{-1}}{1-z^{-1}}$$

10.4) A finite gain of A implies the voltage of the inverting terminal is $\frac{-1}{A}V_o(n)$ if the output voltage is $V_o(n)$.

$$C_2 V_o(n)\left[1+\frac{1}{A}\right] = C_2 V_o(n-1)\left[1+\frac{1}{A}\right] - C_1\left[V_i(n-1)+\frac{V_o(n)}{A}\right]$$

Note that at the end of ϕ_2, the voltage across C_1, which is $V_i(n-1)$, reduces to $\frac{-V_o(n)}{A}$, not ground.

Also, assuming $\frac{1}{A} \ll 1$, we have

$$C_2 V_o(n) = C_2 V_o(n-1) - C_1\left(V_i(n-1)+\frac{V_o(n)}{A}\right)$$

$$\Rightarrow \frac{V_o(z)}{V_i(z)} = \frac{-C_1/C_2}{z\left(1+\frac{C_1}{C_2A}\right)-1}$$

At dc $(z=1)$, this gain is equal to $\frac{-C_1/C_2}{\cancel{1}+\frac{C_1}{C_2A}-\cancel{1}} = -A$

$$Z_p = \frac{1}{1+\frac{C_1}{C_2A}} \simeq 1-\frac{C_1}{C_2A}$$ assuming $C_1 \ll C_2A$.

10.5) $$H(z) = \frac{kz}{z-0.53327}$$

Setting $H(1)=1 \Rightarrow H(z) = \frac{0.46673z}{z-0.53327}$

Equating $H(z)$ with that of Eq. (10.33) and assuming

$C_A = 10\,pF \Rightarrow C_1 = 0 \quad C_2 = -8.752\ pF,\ C_3 = 8.752\,pF$

The new gain at 50 KHz $= H(-1) = -0.304 = -10.3\ dB$

10.6) $\omega_{3dB} = \dfrac{1\,\text{kHz}}{50\,\text{kHz}} \times 2\pi = 0.04\pi$ RAD/SAMPLE

Since the zero is at 0 rather than −1, we cannot use a bilinear transform. From example 9.5 in chapter 9, we have

$$\omega_{3dB} = \cos^{-1}\left(2 - \frac{a}{2} - \frac{1}{2a}\right)$$

This example assumed a zero at ∞ which has same magnitude response as a zero at 0.

$$\cos(0.4\pi) = 2 - \frac{a}{2} - \frac{1}{2a} \Rightarrow a = 0.8821$$

$H(z) = \dfrac{kz}{z - 0.8821}$ Forcing $H(1) = 1 \Rightarrow k = 0.1179$

Equating coeff with (10.33) results in

$\underline{C_1 = 0}$, $\underline{-C_2 = C_3 = 6.683\,pF}$

25 kHz corresponds to $z = -1$ ∴ $H(-1) = 0.0626 = \underline{-24.1\,dB}$

10.7) The transfer function is given by (10.33). Substituting the given capacitances, we have:

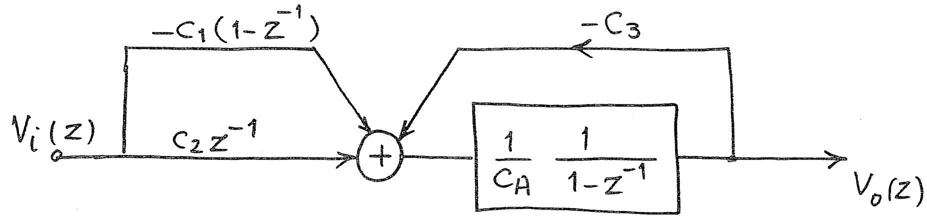
$$\underline{H(z) = \frac{0.1z}{1.1z - 1}}$$

At dc: $z = 1$, $\underline{H(1) = 1, \ \angle H(1) = 0}$

At $f_s/4$: $z = j$, $H(j) = \underline{0.673\angle -42.27^\circ}$

At $f_s/2$: $z = -1$, $\underline{H(-1) = 0.0476\angle 0^\circ}$

10.8) The equivalent signal flow graph is :



$$\left\{ V_i(z)\left[C_2 z^{-1} - C_1 + C_1 z^{-1}\right] - C_3 V_o(z) \right\} \frac{1}{C_A} \frac{1}{1-z^{-1}} = V_o(z)$$

$$\Rightarrow \quad H(z) = \frac{V_o(z)}{V_i(z)} = -\frac{C_1 - (C_1 + C_2) z^{-1}}{(C_A + C_3) - C_A z^{-1}}$$

10.9) The transfer function poles are the zeros of the denominator.

Using (10.49) with $K_6 = 0$, we have

$$z^2 + (K_4 K_5 - 2) z + 1 = 0$$

Since all the coefficients are real numbers, this equation has, in general, two complex conjugate roots, z_1 & z_1^*, with their product equal to the constant term of the equation (i.e. 1).

$$\therefore \quad z_1 z_1^* = 1 \Rightarrow |z_1|^2 = 1 \Rightarrow |z_1| = 1$$

$\therefore$ both z_1 & z_1^* lie on the unit circle.

$\boxed{10.10)}$ $\omega_0 = \frac{2\pi}{100} = 0.062832$ RAD/SAMPLE & $Q = 20$

Equivalent $\Omega_0 = \tan\left(\frac{\omega_0}{2}\right) = 0.0314263$ RAD/S

$$\therefore \quad H_a(s) = \frac{\frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} = \frac{0.0015713\, s}{s^2 + 0.0015713\, s + 0.00098761}$$

Transform using the bilinear transform $s = \frac{z-1}{z+1}$

$$H(z) = \frac{0.001572\, z^2 - 0.001572}{1.003145\, z^2 - 1.9991875\, z + 1}$$

FOR LOW-Q biquad from (10.51) -(10.56)

$k_3 = -0.001572$ $\quad k_2 = 0.003144$

$k_6 = 0.003145$ $\quad k_4 = k_5 = 0.06291$

$k_1 = 0$

FOR HIGH-Q biquad from (10.69)-(10.74)

$$H(z) = \frac{0.001567\, z^2 - 0.001567}{z^2 - 1.99292\, z + 0.99687}$$

$k_3 = 0.001567$ $\quad k_1 = 0$ $\quad k_2 = 0.0499$

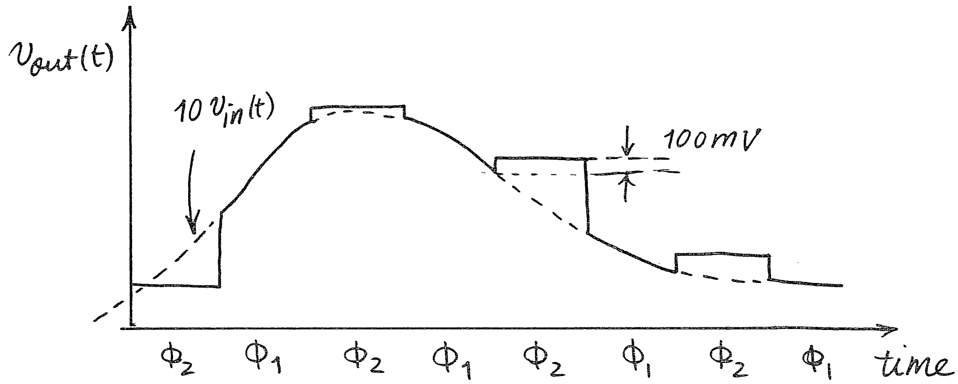
$k_4 = k_5 = 0.06285$ $\quad k_6 = 0.0498$

Ignoring k_3 (since it only sets zero at -1)

CAP ratio for low-Q is 318 while it is 20 for high-Q biquad.

14.18 Using the bilinear transform, find the transfer function in the z-domain of a discrete-time bandpass filter with a $Q = 20$ (in the continuous-time domain) and a peak gain of one near $f_s/100$. Find the largest to smallest capacitor ratio if this transfer function is realized using the high-Q biquad circuit, and compare it to that which would be obtained if the low-Q circuit were used instead. Let $C_1 = C_2 = 1$ in both cases.

10.11)



10.12) Assuming $V_{out} = -V_{SS}$,

$$\Delta V_X = -K_2 V_{SS} = -K_{in} V_{in}$$

the positive jump of V_X is still $K_1(V_{SS}+V_{DD})$ as before.

$$\Rightarrow T_{osc} = 2\,\frac{K_1(V_{SS}+V_{DD})}{K_2 V_{SS} + K_{in}V_{in}}\,T$$

Or, equivalently (assuming $V_{SS} = V_{DD}$)

$$f_{osc} = \frac{1}{4}\left(\frac{K_2}{K_1} + \frac{K_{in}V_{in}}{K_1 V_{DD}}\right) f$$