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great job!

ECE1396H Assignment 2

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A. Reference oscillator phase noise modal

As the question described, only $1/f^2$ noise dependence exists, with -140 dBc/Hz at 10 kHz offset, and a noise floor -160 dBc/Hz at large frequency offset. Its phase noise can be described as

$$L_{ref}(f) = \frac{en_{3,ref}}{f^3} + \frac{en_{2,ref}}{f^2} + \frac{en_{1,ref}}{f} + en_{0,ref} \quad (1)$$

And therefore $en_{3,ref} = en_{1,ref} = 0$, $en_{0,ref} = 10^{-16}$ rad^2/Hz. At 10 kHz frequency offset, it has

$$L_{ref}(f = 10kHz) = 10^{-14} = \frac{en_{2,ref}}{f^2} + en_{0,ref} \approx \frac{en_{2,ref}}{(10kHz)^2} \quad (2)$$

Therefore $en_{2,ref}$ is calculated as 10^{-6} red^2Hz.

$$L_{ref}(f) = \frac{10^{-6}}{f^2} + 10^{-16}\ rad^2/Hz \quad (3)$$

The plot of $L_{ref}(f)$ can be found in Fig. 1.

B. Voltage control oscillator phase noise modal

As the question described, only $1/f^3$ noise dependence exists, with -120 dBc/Hz at 1 MHz offset, and a noise floor -140 dBc/Hz at large frequency offset. Its phase noise can be described as

$$L_{vco}(f) = \frac{en_{3,vco}}{f^3} + \frac{en_{2,vco}}{f^2} + \frac{en_{1,vco}}{f} + en_{0,vco} \quad (4)$$

And therefore $en_{2,vco} = en_{1,vco} = 0$, $en_{0,vco} = 10^{-14}$ rad^2/Hz. At 1 MHz frequency offset, it has

$$L_{vco}(f = 1MHz) = 10^{-12} = \frac{en_{3,vco}}{f^3} + en_{0,vco} \approx \frac{en_{3,vco}}{(1MHz)^3} \quad (5)$$

Therefore $en_{3,vco}$ is calculated as 10^6 red^2Hz^2.

$$L_{vco}(f) = \frac{10^6}{f^3} + 10^{-14}\ rad^2/Hz \quad (6)$$

The plot of $L_{vco}(f)$ can be found in Fig. 1.

C. 3rd order PLL design

The design flow will basically follow the lecture note 13.

1) N = $f_{vco,max}/f_{ref}$ = 5.7 GHz/20 MHz = 285, which covers the worst phase noise scenario.
2) ω_{3dB} can be determined from Fig. 1, where the intersection of $L_{vco}(f)$ and $N^2L_{ref}(f)$ is. ω_{3dB} = $2\pi(5.2\times10^5)$ rad/s.
3) Select Q = 0.5, ω_{pll} = $0.4\omega_{3dB}$ = $0.8\pi(5.2\times10^5)$ rad/s.
4) $\frac{I_{ch}}{C_1} = \frac{w_{pll}^2 2\pi N}{K_{vco}}$, K_{vco} = 300 MHz/V, I_{ch} is the charge pump current. Select C_1 = 5 pf, then I_{ch} = 8.1 μA.
5) ω_z = Qω_{pll} = $0.4\pi(5.2\times10^5)$ rad/s
6) $R = \frac{1}{\omega_z C_1}$ = 306 kΩ
7) Select C_2 = $0.1C_1$ = 0.5 pF

The open-loop transfer function of loop filter is defined as

$$L_{lp}(s) = K_{lp}H_{lp}(s) = \frac{1}{RC_1C_2}\frac{\frac{s}{\omega_z}+1}{s^2 + s\omega_z(1+\frac{C_1}{C_2})} \quad (7)$$

Therefore:

$$K_{lp} = \frac{1}{RC_1C_2}, \qquad H_{lp}(s) = \frac{\frac{s}{\omega_z}+1}{s^2 + s\omega_z(1+\frac{C_1}{C_2})} \quad (8)$$

We know that the loop-gain of a PLL:

$$L_{pll}(s) = w_{pll}^2\frac{H_{lp}(s)}{s} \quad (9)$$

Plug $H_{lp}(s)$ from Eq. (8) to Eq. (9) leads to

$$L_{pll}(s) = \frac{w_{pll}^2}{s^2}\frac{\frac{s}{\omega_z}+1}{\frac{s}{\omega_p}+1} \quad (10)$$

Where

$$\omega_p = \frac{1}{(1+\frac{C_1}{C_1})\omega_z} \quad (11)$$

The Bode plot of the magnitude and phase of $L_{pll}(s)$ are shown in Fig. 2. The unity gain frequency $w_t = 2.73\times10^6$ rad/s, and phase margin is about 60 degrees. The output phase noise of the PLL L_{out} is plot in Fig. 1 (amber color, approximation). At 1 MHz frequency offset, the main noise contributor is VCO.

The rms phase error of the PLL can be broken down into four parts (amber color plot in Fig. 1), which can be seen also in Fig. 1. From 1 kHz to 100 kHz: L_{ref} dominates the noise source, specifically, $en_{2,ref}$ noise; from 100 kHz to f_{3dB}=518 kHz, L_{ref} dominates the noise source, specifically, $en_{0,ref}$ noise; from f_{3dB} = 518 kHz to 10 MHz, L_{vco} dominates the noise source, specifically, $en3, vco$ noise; from 10 MHz to 100 MHz, $L_{n,vco}$ dominates the noise source, specifically, $en_{0,vco}$ noise. Therefore, the rms phase error is calculated as

$$\begin{aligned}\theta_{rms,pll} &= \Bigg[2\Bigg(N^2\int_{10^3}^{10^5}\frac{en_{2,ref}}{f^2}df + N^2\int_{10^5}^{5.18\times10^5} en_{0,ref}df \\ &\qquad + \int_{5.18\times10^5}^{10^7}\frac{en_{3,vco}}{f^3}df + \int_{10^7}^{10^8} en_{0,vco}df\Bigg)\Bigg]^{1/2} \\ &= 0.013rad\end{aligned} \quad (12)$$

TABLE I: Some values asked by the assignment

$en_{2,ref}$	$en_{0,ref}$	$en_{3,vco}$	$en_{0,vco}$	N	I_{ch}
$10^{-6}\ rad^2Hz$	$10^{-16}\ rad^2/Hz$	$10^6\ rad^2Hz^2$	$10^{-14}\ rad^2/Hz$	285	8.1 μA

C_1	C_2	ω_t	phase margin	θ (rad)	σ (ps)
5 pf	0.5 pf	$2.73 \times 10^6\ rad/s$	60^o	0.013	0.36

The rms random jitter is calculated by (where f_{out} is 5.7 GHz)

$$\begin{aligned}\sigma_{rms,pll} &= \frac{1}{2\pi f_{out}}\theta_{rms,pll} \\ &= 0.36ps\end{aligned} \tag{13}$$

The close-loop response of PLL can be found by (with the fact that the feedback factor β is 1/N)

$$\begin{aligned}H_{pll}(s) &= \frac{NL_{pll}}{1+L_{pll}(s)} \\ &= \frac{N(\frac{s}{\omega_z}+1)}{\frac{s^3}{\omega_p\omega_{pll}^2}+\frac{s^2}{\omega_{pll}^2}+\frac{s}{\omega_z}+1}\end{aligned} \tag{14}$$

Its magnitude plot is in Fig. 3. An step input is fed into this transfer function and its response is plotted in Fig. 4. After about 6 μs, the system settles down to its steady state. So the lock time is smaller than 20 μs, satisfying the requirement.

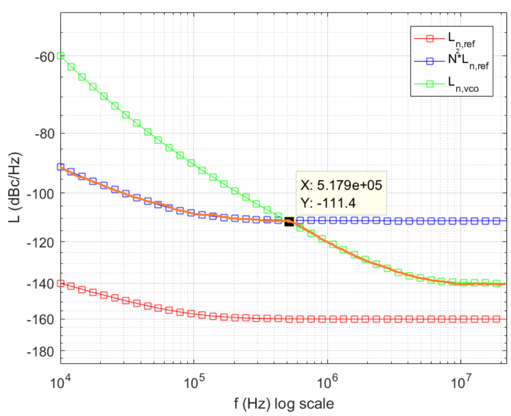

Fig. 1: Plot of L_{ref} in red, L_{vco} in green, N^2L_{ref} in blue, and L_{out} of PLL in amber (approximation). f_{3dB} is found at the intersection of N^2L_{ref} and L_{vco}, which is 518 kHz.

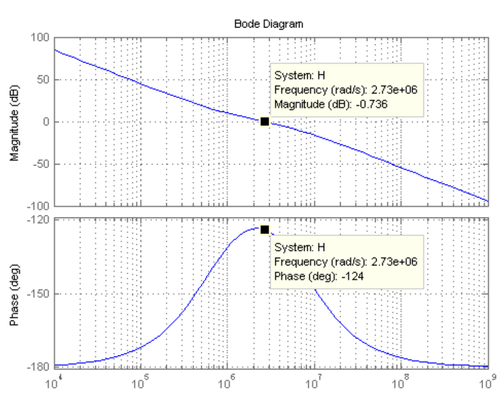

Fig. 2: Bode plot of PLL loop-gain (magnitude and phase). ω_t is found at about 2.73×10^6 rad/s, and phase margin is about 60 degrees.

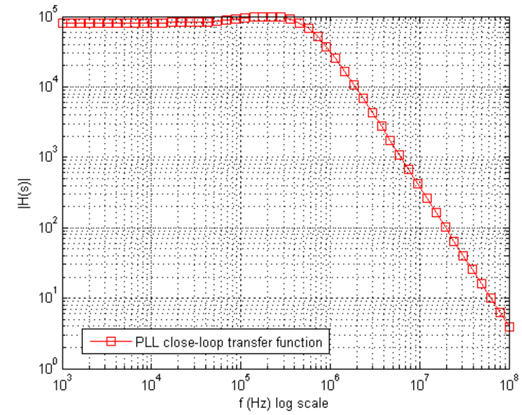

Fig. 3: Close-loop response of PLL

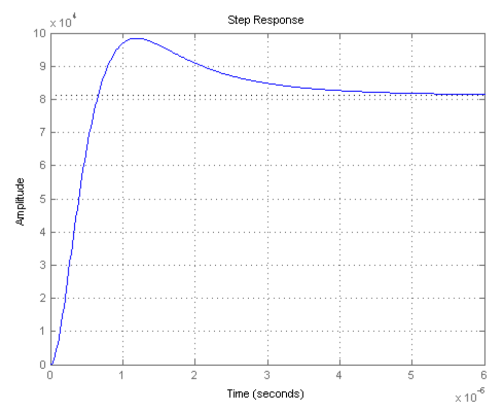

Fig. 4: Step-response of the PLL, which takes about 6 μs lock time to settles down to the steady state.

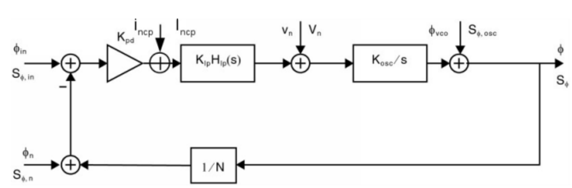

Fig. 5: Block diagram of PLL, where feedback factor is 1/N, and loop gain is $K_{pd}K_{lp}H_{lp}(s)K_{osc}/(Ns)$.