ECE 1505 Convex Optimization
Winter 2020

Homework Set #2

1. Problem 4.11(a,c,e) of Boyd and Vandenberghe
2. Problem 4.16 of Boyd and Vandenberghe
3. Problem 4.21(a) of Boyd and Vandenberghe. (Consider the convex case only.)
4. Problem 4.25 of Boyd and Vandenberghe
5. Problem 4.30 of Boyd and Vandenberghe
6. Problem 4.43(a-b) of Boyd and Vandenberghe
7. Consider the following optimization problem:

$$\underset{x\in\mathbf{R}^n,\ z\in\mathbf{R}}{\text{minimize}} \sum_{m=1}^{M} \max(a_m^T x, z) + \tau\|x\|_2^2.$$

where $a_m \in \mathbf{R}^n$ and $\tau \in \mathbf{R}$. Formulate the above problem as an LP, QP, SOCP or SDP. Clearly outline each step in the derivation of your formulation.

8. **Portfolio Design**.
After some research, you discovered that annual mean return and the fluctuation of the following stocks are as follows:

IBM	10%
Google	35%
Apple	25%
Intel	5%

	IBM	Google	Apple	Intel
IBM	0.2	-0.2	-0.12	0.02
Google	-0.2	1.4	0.02	0
Apple	-0.12	0.02	1	-0.4
Intel	0.02	0	-0.4	0.2

In other words, suppose that you invest \$1 in each stock. Let x be the value of the stocks after one year. Then, $\mathbb{E}[x] = \overline{x} = [1.1 \quad 1.35 \quad 1.25 \quad 1.05]^T$, and $\mathbb{E}[(x-\overline{x})(x-\overline{x})^T] = \Sigma$ as shown in the table on the right. We wish to design a portfolio (i.e., the proportion of money invested in each company) to minimize the variance of the investment subject to some fixed minimum expected return.

(a) Formulate the optimization problem as a quadratic programming problem. Plot the tradeoff curve between the variance and the expected return;
(`MATLAB` hint: You may find the `MATLAB` routine `quadprog` useful.)

(b) Plot the composition of the portfolio as you move from one extreme of the risk-return tradeoff curve to the other extreme. Comment on the benefit of diversification.

9. **Optimal Control of a Unit Mass**.
Consider a unit mass with position $x(t)$ and velocity $\dot{x}(t)$ subject to force $f(t)$, where $f(t) = p_i$ for $i-1 < t \leq i$, $i = 1, \cdots, 10$. Ignore friction.

   (a) Assume the mass has zero initial position and velocity, i.e., $x(0) = \dot{x}(0) = 0$. Find p_i that minimizes
$$\sum_{i=1}^{10} p_i^2$$
subject to the following specifications: $x(10) = 1$, $\dot{x}(10) = 0$. Plot the optimal f, the resulting x and $\dot{x}$. Give a short intuitive explanation of what you see.

   (b) Suppose that we add one more specification $x(5) = 0$, i.e., we require the mass to be at position 0 at time 5. Plot the optimal f, the resulting x and $\dot{x}$. Give a short intuitive explanation of what you see.

10. **Least-Square Deconvolution.**
A communications channel is modelled by a finite-impulse response (FIR) filter:
$$y(t) = \sum_{\tau=0}^{n-1} u(t-\tau)h(\tau), \tag{1}$$
where u is the channel input sequence, y is the channel output, and $h = [h(0), \cdots, h(n-1)]$ is the impulse response of the channel. Equivalently, we can write this as $y = h * u$.

You will design a *deconvolution filter* or *equalizer* $g = [g(0), \cdots, g(m-1)]$ which cancels the channel impulse response. The output of the equalizer is $z = g * y$. In other words, the goal is to choose g so that the filter output is approximately the channel input delayed by D samples, i.e., $z(t) \approx u(t-D)$. Since $z = g * h * u$, this means we want the *least-square* equalizer g that minimizes the sum of squared errors
$$\sum_{t \neq D} \left((g * h)(t)\right)^2, \tag{2}$$
subject to the constraint
$$(g * h)(D) = 1. \tag{3}$$

In this question, we let $n = 10$ and $m = 20$. The channel impulse response is given as the vector h in `hw2data.mat`.

   (a) Note that if the delay D is fixed, this problem is a least-square problem. Please find the best D such that the sum of squared errors is minimized.

(b) The vector y (defined in `hw2data.mat`) contains the channel output corresponding to a binary signal $[u(0), \cdots, u(9999)]$ (i.e., $u(t) \in \{-1, 1\}$) passed through the channel. Pass y through the best least-square equalizer found in part (a), to form the signal z. Give a histogram plot of the amplitude distribution of both y and z. (You should only use 10000 samples from z to make the histogram plot. Choose them appropriately.) Comment on what you find.

(`MATLAB` hints: `conv` convolves two vectors; `hist` plots a histogram. Also note that `MATLAB` vector indices start with 1, not 0.)