For 1 and 11 see the picture after.

2. x_1, x_3 and x_4 are support vectors.

3.

$$\bar{\alpha} = [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4]^T$$

$$L(\bar{w}, b, \alpha_j) = \frac{1}{2}||\bar{w}||^2 - \Sigma_j \alpha_j[(\bar{w} \cdot x_j + b)y_j - 1]$$

4. Fix α_j

$$\left.\begin{aligned} \nabla_{\bar{w}} L(\bar{w}, b, \alpha_j) = \bar{w} - \Sigma_j \alpha_j y_j x_j = 0 \rightarrow \bar{w} = \Sigma_j \alpha_j y_j x_j \\ \nabla_b L(\bar{w}, b, \alpha_j) = -\Sigma_j \alpha_j b y_j = 0 \rightarrow \Sigma_j \alpha_j y_j = 0 \end{aligned}\right\} \text{KKT conditions}$$

5. Plug $\bar{w} = \Sigma_j \alpha_j y_j x_j$ back to $L(\bar{w}, b, \alpha_j)$

$$L(\bar{w}, b, \alpha_j) = \frac{1}{2}(\Sigma_j \alpha_j y_j x_j)^T(\Sigma_j \alpha_j y_j x_j) - \Sigma_j(\alpha_j y_j \bar{w} \cdot x_j + \alpha_j b y_j - \alpha_j)$$

$$= \frac{1}{2}(\Sigma_j \alpha_j y_j x_j)^T(\Sigma_j \alpha_j y_j x_j) - \Sigma_j \alpha_j y_j \bar{w} \cdot x_j \overset{\to \text{zero}}{\boxed{- b\Sigma_j \alpha_j y_j}} + \Sigma_j \alpha_j$$

$$= \frac{1}{2}(\Sigma_j \alpha_j y_j x_j)^T(\Sigma_j \alpha_j y_j x_j) - \Sigma_j \alpha_j y_j(\Sigma_j \alpha_j y_j x_j) \cdot x_j + \Sigma_j \alpha_j$$

$$= \frac{1}{2}(\Sigma_j \alpha_j y_j x_j)^T(\Sigma_j \alpha_j y_j x_j) - \Sigma_j \alpha_j y_j(\Sigma_j \alpha_j y_j x_j)^T x_j + \Sigma_j \alpha_j$$

$$= (\Sigma_j \alpha_j y_j x_j)^T[\frac{1}{2}(\Sigma_j \alpha_j y_j x_j) - \Sigma_j \alpha_j y_j x_j] + \Sigma_j \alpha_j$$

$$= \Sigma_j \alpha_j - \frac{1}{2}(\Sigma_j \alpha_j y_j x_j)^T(\Sigma_j \alpha_j y_j x_j)$$

$$maximize_{\alpha_j}\ \Sigma_j \alpha_j - \frac{1}{2}(\Sigma_j \alpha_j y_j x_j)^T(\Sigma_j \alpha_j y_j x_j)$$

6. We know that the first point x_2 is not the support vector and should be removed from the constraint

$$L(\bar{w}, b, \alpha_j) = \frac{1}{2}||\bar{w}||^2 - \Sigma_{j \neq j^*} \alpha_j[(\bar{w} \cdot x_j + b)y_j - 1]$$

7.

$$\Sigma_{j \neq j^*} \alpha_j y_j = 0 \qquad \text{complementary slackness}$$

8.

$$Y = [y_1\ y_2\ y_3\ y_4]^T$$

$$X = [x_1\ x_2\ x_3\ x_4]^T$$

$$\Sigma_j \alpha_j y_j x_j = \alpha_1 y_1 x_1 + \alpha_2 y_2 x_2 + \alpha_3 y_3 x_3 + \alpha_4 y_4 x_4$$

$$= \alpha_1 x_1 - \alpha_2 x_2 - \alpha_3 x_3 - \alpha_4 x_4$$

$$= [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4][x_1\ -x_2\ -x_3\ -x_4]^T$$

Therefore

$$\left(\Sigma_j \alpha_j y_j x_j\right)^T \left(\Sigma_j \alpha_j y_j x_j\right) = [x_1 \, -x_2 \, -x_3 \, -x_4][\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4]^T [\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4][x_1 \, -x_2 \, -x_3 \, -x_4]^T$$

$$= [\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4][x_1 \, -x_2 \, -x_3 \, -x_4]^T [x_1 \, -x_2 \, -x_3 \, -x_4][\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4]^T$$

$$[\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4] \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}^T \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} [\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4]^T$$

$$= \mathrm{A}^T H A$$

Where

$$A = [\alpha_1 \, \alpha_2 \, \alpha_3 \, \alpha_4]^T$$

$$H = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}^T \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -1 & -1 \\ 2 & 2 & -1 & -1 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix}$$

Rewrite the original optimization problem into quadratic programming (Q.P.) form:

$$minimize_A \;\; \frac{1}{2}\mathrm{A}^T HA - 1^T A$$

$$subject\ to \;\; A^T Y = 0$$

$$A \geq 0$$

By solving the above optimization problem, it gives

$$A = [4\ 0\ 2\ 2]^T$$

Which means

$$\alpha_1 = 4, \alpha_2 = 0, \alpha_3 = \alpha_4 = 2$$

9.

$$\overline{w} = \Sigma_j \alpha_j y_j x_j = 4[1\ 1]^T + 0 - 2[1\ 0]^T - 2[0\ 1]^T = [2\ \ 2]^T$$

10.

$$\left(\overline{w} \cdot x_j + b\right) y_j - 1 = 0$$

Select a support vector, say x_1, plug it into the above equation:

$$([2\ 2][1\ 1]^T + b)(1) = 1 \rightarrow b = -3$$

Therefore:

$$\overline{w} \cdot x + b = 0$$

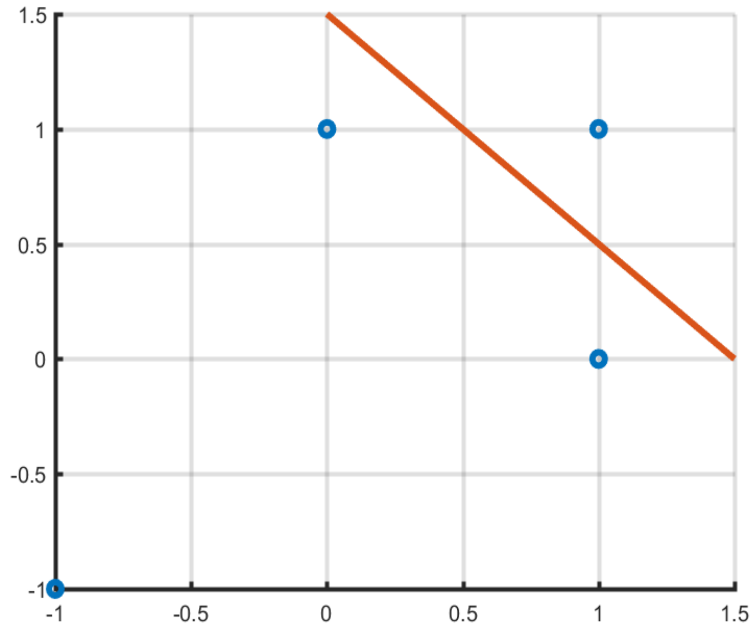
$$[2\ 2]x - 3 = 0 \rightarrow \ -[1\ 1]x + 1.5 = 0$$

Therefore:

$$\overline{w}^* = [-1\ \ -1]^T$$

$$b^* = 1.5$$

Which is the same as the initial guess in q1.




```
close all
clear all

x1 = [1;1];
x2 = [-1;-1];
x3 = [1;0];
x4 = [0;1];
x = [x1 x2 x3 x4];
j = 4;

Y(:,:,4) = [1;-1;-1;-1];
m = 0:0.1:1.5;
k = - m + 1.5;

figure(1)
p1 = scatter(x(1,:), x(2,:));
set(p1, 'linewidth', 3)
hold on
p2 = plot(m,k);
set(p2, 'linewidth', 3)
grid on
set(gca,'linewidth',2)

% support point: x1, x3 and x4

X = transpose(x);

% Formulating Q.P.
H = [2 2 -1 -1; 2 2 -1 -1; -1 -1 1 0; -1 -1 0 1];
f = [-1; -1; -1; -1];
lb = zeros(4,1);
Aeq = [1 -1 -1 -1];
beq = 0;

[x,fval,exitflag,output,lambda] = ...
   quadprog(H,f,[],[],Aeq, beq, lb, []);
```