Assignment 2

Tuesday, January 28, 2020 4:27 PM

Problem 1

1. $v = x - (w \cdot x + b) w$

$$w \cdot v + b = w\left[x - (w \cdot x + b) w\right] + b$$

$$= w \cdot x - w \cdot (w \cdot x + b) w + b$$

$$= w \cdot x - (\|w\|_2 x + w \cdot b) w + b$$

$$= w \cdot x - w \cdot x - b + b = 0$$

2. Say: since $\min \|x - u\|_2 = 0$ when $u = x$

$\therefore \quad v = u - (w \cdot u + b) w$

since: $u \in R^d$, s.t. $w \cdot u + b = 0$

$\therefore \quad v = u$

$\therefore$ when: $u = v = x - (w \cdot x + b) w = -bw = x$

$\|x - u\|_2$ has the minimum

$$\|x - u\|_2^2 = (x + bw)^2 = x^2 + 2wxb + b^2 w^2$$

$$= x^2 + 2wxb + b^2.$$

$$|w \cdot x + b|^2 = w^2 x^2 + 2wxb + b^2$$

$$= x^2 + 2wxb + b^2.$$

$$\Rightarrow \min_{u \in R^d \text{ s.t. } wu + b = 0} \|x - u\|_2 = |wx + b| \le \|w \cdot x + b\|_2 \|w\|_2$$

let: $u = v$

$= x - (w \cdot x + b) w$:

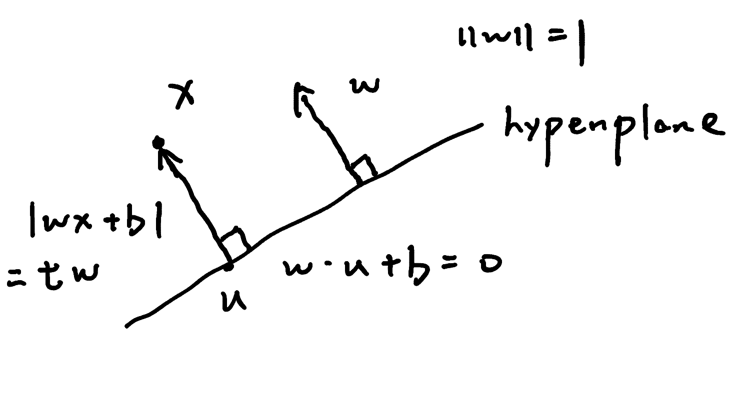
$LHS = \|x - x + (w \cdot x + b) w\|_2$

$= \|(w \cdot x + b) w\|_2$

since: $\|w\|_2 = 1$

$\therefore LHS \le \|w \cdot x + b\|_2 = RHS$

$|wx+b|$ can be shown (in 4) later that is the shortest distance from point x to the hyperplane defined by normal vector w and offset b, $\|x-u\|_2$ is the Euclidean distance from point x to the point u that are on the hyperplane. Therefore, there is no solution to find point u that satisfies $\|x-u\|_2 < |w.x+b|$, only exists $\|x-u\|_2 = |w.x+b|$, where $x-u = tw$, where "t" is a scalar that is a magnitude of the distance from x to the hyperplane. When $t=0$, $x=u$, and the "=" is hold (proven above); when $t \neq 0$, $\|x-u\|_2 = |t|$, $|wx+b| = |t|$, "=" is hold, too.



$$\|x-u\|_2^2 = \|(x-v)+(v-u)\|_2^2 = [(x-v)^T+(v-u)^T][(x-v)+(v-u)]$$

$$= \|x-v\|_2^2 + 2(x-v)^T(v-u) + \|v-u\|_2^2 \geq \|x-v\|_2^2$$

$$\therefore \|x-u\|_2^2 \geq \|x-v\|_2^2$$

3. $v = -bw$, when $u = v = -bw$, $\|x-u\|_2 = \|x-v\|_2$

$$\|x-v\|_2^2 = \|x\|^2 - 2x\cdot v + \|v\|^2$$

$$\|x-u\|_2^2 = \|x\|^2 - 2x\cdot u + \|u\|^2$$

proof: $\|x-u\|_2^2 - \|x-v\|_2^2 = 2x\cdot v - 2x\cdot u + \|u\|^2 - \|v\|^2 \geq 0$

$$= 2x\cdot(v-u) + \|u\|^2 - \|v\|^2$$

not sufficient conditions to deterministically saying that $\|x-u\| > \|x-v\|$, u & v do share the same properties and are on the same hyperplane.

It only possible $\|x-u\|_2 > \|x-w\|$ when x is selected that is closer to the point v that v in terms of Euclidean distance.

4. pick a point $u \xrightarrow{} u \in R^d$ from the hyperplane, its distance d to this hyperplane:

$$d = \|x-u\| = \frac{|(x-u)\cdot w|}{\|w\|}$$

$$\overset{\|w\|=1}{=} |w\cdot x - w\cdot u|$$

$$\overset{w\cdot u+b=0}{=} |w\cdot x + b|$$

$$= |w \cdot x + b|$$

Problem 2

1. $z = w^T x$

$$w' = w + \alpha t x$$

$$w'^T x = (w + \alpha t x)^T x = (w^T + x^T t^T \alpha^T) x$$

$$= w^T x + \alpha t \|x\|_2^2 = z' \quad (\alpha > 0)$$

if: $t = -1$, z' will be smaller than z

if: $t = 1$, z' will be larger than z

$\therefore$ it does not change the prediction

2 and 3:

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"""
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Assignment 2 Q2
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"""

from sklearn.datasets import load_breast_cancer
import numpy as np

# import data
breast_cancer = load_breast_cancer()
X = breast_cancer.data
Y = breast_cancer.target
train_X = X[:450]
test_X = X[450:]
# Convert "0" to "-1"
train_Y = 2 * Y[:450] - np.ones(len(Y[:450]))
```

```
test_Y = 2 * Y[450:] - np.ones(len(Y[450:]))

# perceptron
epoch = 100
alpha = 0.01
w = np.zeros(np.shape(train_X)[1])
z = np.matmul(train_X, w)
for j in range(epoch):
   for i in range(450):
      if z[i] * train_Y[i] <= 0:
         #print z[i]
         w = w + train_X[i] * train_Y[i] * alpha
         #print w
         z[i] = np.dot(train_X[i], w)
         #print z[i]

# normalize the data
for k in range(450):
   if z[k] > 0:
      z[k] = 1
   else:
      z[k] = -1
# calculate training accuracy
p_train = np.shape(np.nonzero(z - train_Y))[1]
precision_train = 1 - float(p_train)/450
print precision_train # 100%

z_prime = np.matmul(test_X, w)

for n in range(119):
   if z_prime[n] > 0:
      z_prime[n] = 1
   else:
      z_prime[n] = -1
#calculate testing accuracy
p = np.shape(np.nonzero(z_prime - test_Y))[1]
precision = 1 - float(p)/119
print precision # 77.3%
```