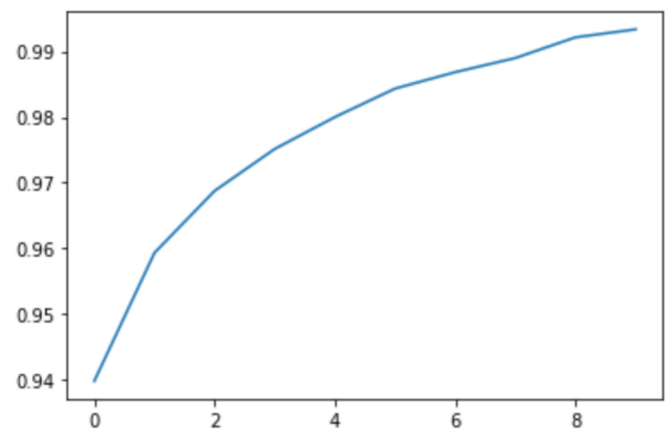
Problem 1.

1-3: see the code in the back

4. (Training accuracy plot on the right)

learning rate = 0.1

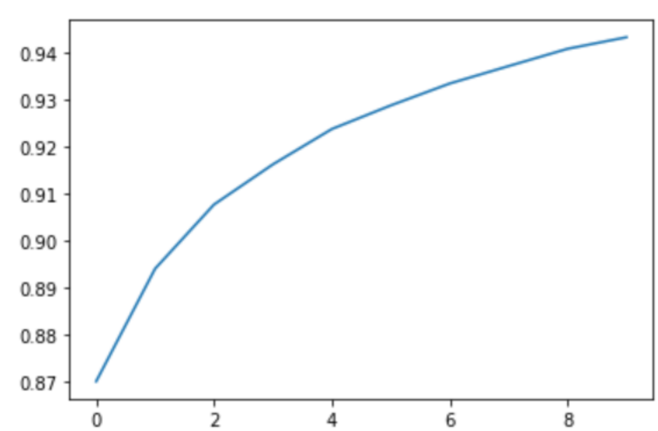
```
Epoch 0 in 3.29 sec
Training set accuracy 0.9398333430290222
Test set accuracy 0.9395000338554382
Epoch 1 in 0.50 sec
Training set accuracy 0.9593333601951599
Test set accuracy 0.9532000422477722
Epoch 2 in 0.50 sec
Training set accuracy 0.9687666893005371
Test set accuracy 0.9602000713348389
Epoch 3 in 0.50 sec
Training set accuracy 0.9751499891281128
Test set accuracy 0.9661000370979309
Epoch 4 in 0.49 sec
Training set accuracy 0.9800000190734863
Test set accuracy 0.9682000279426575
Epoch 5 in 0.49 sec
Training set accuracy 0.984333336353302
Test set accuracy 0.969200074672699
Epoch 6 in 0.50 sec
Training set accuracy 0.9868333339691162
Test set accuracy 0.9705000519752502
Epoch 7 in 0.51 sec
Training set accuracy 0.9889833331108093
Test set accuracy 0.971000075340271
Epoch 8 in 0.49 sec
Training set accuracy 0.9921333193778992
Test set accuracy 0.9735000729560852
Epoch 9 in 0.50 sec
Training set accuracy 0.9933500289916992
Test set accuracy 0.9730000495910645
```



5. (training accuracy plot on the right)

Slow convergence: learning rate = 0.01

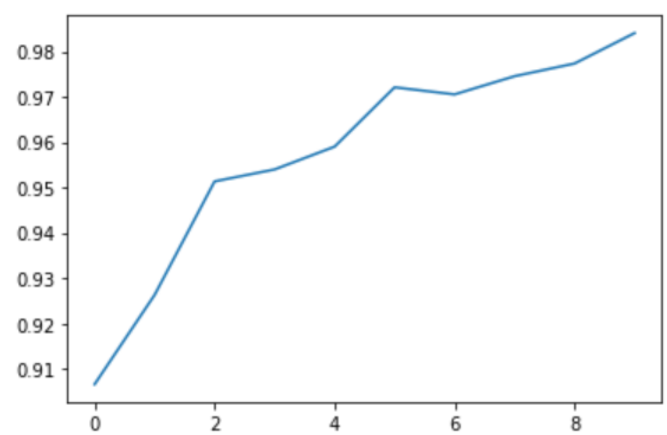
```
Epoch 0 in 0.90 sec
Training set accuracy 0.8627166748046875
Test set accuracy 0.8671000599861145
Epoch 1 in 0.45 sec
Training set accuracy 0.8939833641052246
Test set accuracy 0.8987000584602356
Epoch 2 in 0.48 sec
Training set accuracy 0.9076666831970215
Test set accuracy 0.9103000164031982
Epoch 3 in 0.44 sec
Training set accuracy 0.9162333607673645
Test set accuracy 0.9165000319480896
Epoch 4 in 0.45 sec
Training set accuracy 0.9237666726112366
Test set accuracy 0.9234000444412231
Epoch 5 in 0.44 sec
```



```
Training set accuracy 0.9287999868392944
Test set accuracy 0.9283000230789185
Epoch 6 in 0.46 sec
Training set accuracy 0.9334666728973389
Test set accuracy 0.9324000477790833
Epoch 7 in 0.43 sec
Training set accuracy 0.9371833205223083
Test set accuracy 0.9353000521659851
Epoch 8 in 0.47 sec
Training set accuracy 0.9408666491508484
Test set accuracy 0.9381000399589539
Epoch 9 in 0.48 sec
Training set accuracy 0.9433333277702332
Test set accuracy 0.9393000602722168
```

Oscillation but convergence: learning rate = 1

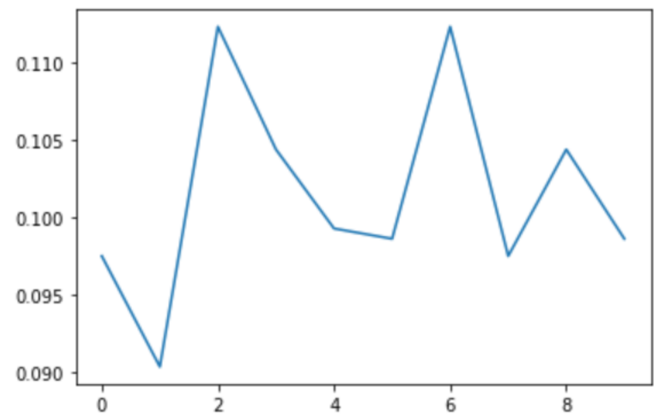
```
Epoch 0 in 0.47 sec
Training set accuracy 0.8757833242416382
Test set accuracy 0.8737000226974487
Epoch 1 in 0.50 sec
Training set accuracy 0.9419833421707153
Test set accuracy 0.9349000453948975
Epoch 2 in 0.52 sec
Training set accuracy 0.9506666660308838
Test set accuracy 0.9444000720977783
Epoch 3 in 0.56 sec
Training set accuracy 0.9575666785240173
Test set accuracy 0.9467000365257263
Epoch 4 in 0.58 sec
Training set accuracy 0.9659833312034607
Test set accuracy 0.9533000588417053
Epoch 5 in 0.58 sec
Training set accuracy 0.9727333188056946
Test set accuracy 0.9635000228881836
Epoch 6 in 0.50 sec
Training set accuracy 0.9703166484832764
Test set accuracy 0.9602000713348389
Epoch 7 in 0.52 sec
Training set accuracy 0.9745500087738037
Test set accuracy 0.961400032043457
Epoch 8 in 0.50 sec
Training set accuracy 0.9807167053222656
Test set accuracy 0.9663000702857971
Epoch 9 in 0.52 sec
Training set accuracy 0.9824333190917969
Test set accuracy 0.969700038433075
```



Oscillation but non-convergence: learning rate = 2

```
Epoch 0 in 1.02 sec
Training set accuracy 0.09751667082309723
Test set accuracy 0.09740000218153
Epoch 1 in 0.50 sec
Training set accuracy 0.09035000205039978
Test set accuracy 0.08920000493526459
Epoch 2 in 0.47 sec
```

```
Training set accuracy 0.11236666887998581
Test set accuracy 0.11350000649690628
Epoch 3 in 0.46 sec
Training set accuracy 0.10441666841506958
Test set accuracy 0.10280000418424606
Epoch 4 in 0.45 sec
Training set accuracy 0.09930000454187393
Test set accuracy 0.10320000350475311
Epoch 5 in 0.46 sec
Training set accuracy 0.09863333404064178
Test set accuracy 0.0958000048995018
Epoch 6 in 0.46 sec
Training set accuracy 0.11236666887998581
Test set accuracy 0.11350000649690628
Epoch 7 in 0.47 sec
Training set accuracy 0.09751667082309723
Test set accuracy 0.09740000218153
Epoch 8 in 0.45 sec
Training set accuracy 0.10441666841506958
Test set accuracy 0.10280000418424606
Epoch 9 in 0.47 sec
Training set accuracy 0.09863333404064178
Test set accuracy 0.0958000048995018
```



6. Underfitting: `layer_sizes = [784, 1, 1, 10]`

```
Epoch 0 in 1.22 sec
Training set accuracy 0.21115000545978546
Test set accuracy 0.2086000144481659
Epoch 1 in 0.44 sec
Training set accuracy 0.21220000088214874
Test set accuracy 0.21090000867843628
Epoch 2 in 0.40 sec
Training set accuracy 0.21213333308696747
Test set accuracy 0.21000000834465027
Epoch 3 in 0.40 sec
Training set accuracy 0.21303333342075348
Test set accuracy 0.2111000120639801
Epoch 4 in 0.40 sec
Training set accuracy 0.21076667308807373
Test set accuracy 0.2079000025987625
Epoch 5 in 0.40 sec
Training set accuracy 0.2117166668176651
Test set accuracy 0.20940001308918
Epoch 6 in 0.44 sec
Training set accuracy 0.21164999902248383
Test set accuracy 0.20900000631809235
Epoch 7 in 0.40 sec
Training set accuracy 0.21295000612735748
Test set accuracy 0.21040001511573792
Epoch 8 in 0.43 sec
Training set accuracy 0.21310000121593475
Test set accuracy 0.20940001308918
Epoch 9 in 0.39 sec
Training set accuracy 0.21338333189487457
Test set accuracy 0.21040001511573792
```

7. Overfitting: `layer_sizes = [784, 1024, 1024, 10]; num_epochs = 50; create_outliers=True`

```
Epoch 0 in 1.00 sec
Training set accuracy 0.4868333339691162
Test set accuracy 0.8271000385284424
Epoch 1 in 0.49 sec
Training set accuracy 0.5063333511352539
Test set accuracy 0.8406000137329102
Epoch 2 in 0.46 sec
Training set accuracy 0.5160666704177856
Test set accuracy 0.8385000228881836
Epoch 3 in 0.50 sec
Training set accuracy 0.5252666473388672
Test set accuracy 0.8416000604629517
Epoch 4 in 0.45 sec
Training set accuracy 0.5402666926383972
Test set accuracy 0.8326000571250916
Epoch 5 in 0.47 sec
Training set accuracy 0.5518666505813599
Test set accuracy 0.7945000529289246
Epoch 6 in 0.44 sec
Training set accuracy 0.5556833148002625
Test set accuracy 0.7870000600814819
Epoch 7 in 0.47 sec
Training set accuracy 0.5856500267982483
Test set accuracy 0.8047000169754028
Epoch 8 in 0.46 sec
Training set accuracy 0.5877333283424377
Test set accuracy 0.7749000191688538
Epoch 9 in 0.48 sec
Training set accuracy 0.6187000274658203
Test set accuracy 0.7550000548362732
Epoch 10 in 0.47 sec
Training set accuracy 0.6121833324432373
Test set accuracy 0.7252000570297241
Epoch 11 in 0.45 sec
Training set accuracy 0.6636000275611877
Test set accuracy 0.7686000466346741
Epoch 12 in 0.46 sec
Training set accuracy 0.6808333396911621
Test set accuracy 0.7438000440597534
Epoch 13 in 0.44 sec
Training set accuracy 0.6863666772842407
Test set accuracy 0.706000030040741
Epoch 14 in 0.46 sec
Training set accuracy 0.7089499831199646
Test set accuracy 0.6793000102043152
Epoch 15 in 0.42 sec
Training set accuracy 0.7196999788284302
Test set accuracy 0.6629000306129456
Epoch 16 in 0.46 sec
Training set accuracy 0.7662667036056519
Test set accuracy 0.6637000441551208
Epoch 17 in 0.42 sec
Training set accuracy 0.785966694355011
Test set accuracy 0.6829000115394592
Epoch 18 in 0.45 sec
Training set accuracy 0.8090500235557556
Test set accuracy 0.6338000297546387
Epoch 19 in 0.43 sec
```

```
Training set accuracy 0.824916660785675
Test set accuracy 0.659000039100647
Epoch 20 in 0.52 sec
Training set accuracy 0.8390333652496338
Test set accuracy 0.6429000496864319
Epoch 21 in 0.45 sec
Training set accuracy 0.8325166702270508
Test set accuracy 0.6018000245094299
Epoch 22 in 0.44 sec
Training set accuracy 0.8649333715438843
Test set accuracy 0.6093000173568726
Epoch 23 in 0.44 sec
Training set accuracy 0.8950833678245544
Test set accuracy 0.6377000212669373
Epoch 24 in 0.43 sec
Training set accuracy 0.9064666628837585
Test set accuracy 0.5840000510215759
Epoch 25 in 0.43 sec
Training set accuracy 0.9083666801452637
Test set accuracy 0.6249000430107117
Epoch 26 in 0.46 sec
Training set accuracy 0.9273000359535217
Test set accuracy 0.6517000198364258
Epoch 27 in 0.44 sec
Training set accuracy 0.9330166578292847
Test set accuracy 0.6289000511169434
Epoch 28 in 0.43 sec
Training set accuracy 0.9493499994277954
Test set accuracy 0.6568000316619873
Epoch 29 in 0.44 sec
Training set accuracy 0.956516683101654
Test set accuracy 0.6318000555038452
Epoch 30 in 0.42 sec
Training set accuracy 0.9604499936103821
Test set accuracy 0.6577000021934509
Epoch 31 in 0.42 sec
Training set accuracy 0.9635666608810425
Test set accuracy 0.659500002861023
Epoch 32 in 0.43 sec
Training set accuracy 0.9565500020980835
Test set accuracy 0.6285000443458557
Epoch 33 in 0.44 sec
Training set accuracy 0.9696000218391418
Test set accuracy 0.629800021648407
Epoch 34 in 0.44 sec
Training set accuracy 0.9702000021934509
Test set accuracy 0.6482000350952148
Epoch 35 in 0.45 sec
Training set accuracy 0.9759666919708252
Test set accuracy 0.6378000378608704
Epoch 36 in 0.43 sec
Training set accuracy 0.972516655921936
Test set accuracy 0.6288000345230103
Epoch 37 in 0.42 sec
Training set accuracy 0.9734833240509033
Test set accuracy 0.6488000154495239
Epoch 38 in 0.42 sec
Training set accuracy 0.9805166721343994
Test set accuracy 0.6380000114440918
Epoch 39 in 0.42 sec
```

```
Training set accuracy 0.9778500199317932
Test set accuracy 0.6522000432014465
Epoch 40 in 0.42 sec
Training set accuracy 0.978783369064331
Test set accuracy 0.6229000091552734
Epoch 41 in 0.42 sec
Training set accuracy 0.9770500063896179
Test set accuracy 0.6455000042915344
Epoch 42 in 0.54 sec
Training set accuracy 0.9789833426475525
Test set accuracy 0.6625000238418579
Epoch 43 in 0.43 sec
Training set accuracy 0.9837333559989929
Test set accuracy 0.6265000104904175
Epoch 44 in 0.44 sec
Training set accuracy 0.9850000143051147
Test set accuracy 0.6150000095367432
Epoch 45 in 0.45 sec
Training set accuracy 0.9859499931335449
Test set accuracy 0.6467000246047974
Epoch 46 in 0.45 sec
Training set accuracy 0.985883355140686
Test set accuracy 0.6465000510215759
Epoch 47 in 0.44 sec
Training set accuracy 0.9857833385467529
Test set accuracy 0.6454000473022461
Epoch 48 in 0.45 sec
Training set accuracy 0.9869833588600159
Test set accuracy 0.6300000548362732
Epoch 49 in 0.45 sec
Training set accuracy 0.9899166822433472
Test set accuracy 0.6450000405311584
```

Problem 2.

The following hyperparameters are used: learning rate = 0.01, batch size = 32, epoch = 10, momentum = 0.9 architecture of the network:

```
init_random_params, predict = stax.serial(

    stax.Conv(32, (3, 3), strides=(1, 1)),
    stax.Relu,
    stax.MaxPool((2, 2), strides=(2, 2)),

    stax.Conv(64, (3, 3), strides=(1, 1)),
    stax.Relu,
    stax.Conv(64, (3, 3), strides=(1, 1)),
    stax.Relu,
    stax.MaxPool((2, 2), strides=(2, 2)),

    stax.Flatten,
    stax.Dense(100),
    stax.Relu,

    stax.Dense(10),
)
```

The highest accuracy with this setup is 98.26%.

```
Test set loss, accuracy (%): (0.84, 96.11)
Test set loss, accuracy (%): (0.75, 97.28)
Test set loss, accuracy (%): (0.70, 97.47)
Test set loss, accuracy (%): (0.71, 97.99)
Test set loss, accuracy (%): (0.66, 98.04)
Test set loss, accuracy (%): (0.65, 98.12)
Test set loss, accuracy (%): (0.71, 98.26)
Test set loss, accuracy (%): (0.71, 97.95)
Test set loss, accuracy (%): (0.67, 98.16)
Test set loss, accuracy (%): (0.72, 98.07)
```

It is just taking too long to blindly searching for a better hyperparameter set that reaches 99% accuracy rate. However, with the same architecture, in TensorFlow it manages to reach above 99% (99.05%) accuracy rate…

The code in TF is below (https://machinelearningmastery.com/how-to-develop-a-convolutional-neural-network-from-scratch-for-mnist-handwritten-digit-classification/):

```
# deeper cnn model for mnist
from numpy import mean
from numpy import std
from matplotlib import pyplot
from sklearn.model_selection import KFold
from keras.datasets import mnist
from keras.utils import to_categorical
from keras.models import Sequential
from keras.layers import Conv2D
from keras.layers import MaxPooling2D
from keras.layers import Dense
```

```
from keras.layers import Flatten
from keras.optimizers import SGD

# load train and test dataset
def load_dataset():
  # load dataset
  (trainX, trainY), (testX, testY) = mnist.load_data()
  # reshape dataset to have a single channel
  trainX = trainX.reshape((trainX.shape[0], 28, 28, 1))
  testX = testX.reshape((testX.shape[0], 28, 28, 1))
  # one hot encode target values
  trainY = to_categorical(trainY)
  testY = to_categorical(testY)
  return trainX, trainY, testX, testY

# scale pixels
def prep_pixels(train, test):
  # convert from integers to floats
  train_norm = train.astype('float32')
  test_norm = test.astype('float32')
  # normalize to range 0-1
  train_norm = train_norm / 255.0
  test_norm = test_norm / 255.0
  # return normalized images
  return train_norm, test_norm

# define cnn model
def define_model():
  model = Sequential()
  model.add(Conv2D(32, (3, 3), activation='relu', kernel_initializer='he_uniform', in
put_shape=(28, 28, 1)))
  model.add(MaxPooling2D((2, 2)))
  model.add(Conv2D(64, (3, 3), activation='relu', kernel_initializer='he_uniform'))
  model.add(Conv2D(64, (3, 3), activation='relu', kernel_initializer='he_uniform'))
  model.add(MaxPooling2D((2, 2)))
  model.add(Flatten())
  model.add(Dense(100, activation='relu', kernel_initializer='he_uniform'))
  model.add(Dense(10, activation='softmax'))
  # compile model
  opt = SGD(lr=0.01, momentum=0.9)
  model.compile(optimizer=opt, loss='categorical_crossentropy', metrics=['accuracy'])
  return model

# evaluate a model using k-fold cross-validation
def evaluate_model(dataX, dataY, n_folds=5):
  scores, histories = list(), list()
  # prepare cross validation
  kfold = KFold(n_folds, shuffle=True, random_state=1)
  # enumerate splits
  for train_ix, test_ix in kfold.split(dataX):
```

```
    # define model
    model = define_model()
    # select rows for train and test
    trainX, trainY, testX, testY = dataX[train_ix], dataY[train_ix], dataX[test_ix],
dataY[test_ix]
    # fit model
    history = model.fit(trainX, trainY, epochs=10, batch_size=32, validation_data=(te
stX, testY), verbose=0)
    # evaluate model
    _, acc = model.evaluate(testX, testY, verbose=0)
    print('> %.3f' % (acc * 100.0))
    # stores scores
    scores.append(acc)
    histories.append(history)
  return scores, histories

# plot diagnostic learning curves
def summarize_diagnostics(histories):
  for i in range(len(histories)):
    # plot loss
    pyplot.subplot(2, 1, 1)
    pyplot.title('Cross Entropy Loss')
    pyplot.plot(histories[i].history['loss'], color='blue', label='train')
    pyplot.plot(histories[i].history['val_loss'], color='orange', label='test')
    # plot accuracy
    pyplot.subplot(2, 1, 2)
    pyplot.title('Classification Accuracy')
    pyplot.plot(histories[i].history['accuracy'], color='blue', label='train')
    pyplot.plot(histories[i].history['val_accuracy'], color='orange', label='test')
  pyplot.show()

# summarize model performance
def summarize_performance(scores):
  # print summary
  print('Accuracy: mean=%.3f std=%.3f, n=%d' % (mean(scores)*100, std(scores)*100, le
n(scores)))
  # box and whisker plots of results
  pyplot.boxplot(scores)
  pyplot.show()

# run the test harness for evaluating a model
def run_test_harness():
  # load dataset
  trainX, trainY, testX, testY = load_dataset()
  # prepare pixel data
  trainX, testX = prep_pixels(trainX, testX)
  # evaluate model
  scores, histories = evaluate_model(trainX, trainY)
  # learning curves
  summarize_diagnostics(histories)
```

```
  # summarize estimated performance
  summarize_performance(scores)

# entry point, run the test harness
run_test_harness()
```

Results:

```
> 99.033
> 99.050
> 98.742
> 99.183
> 98.700
```

Python Code:

```
# -*- coding: utf-8 -*-
"""Copy of ECE1513H - Assignment 4 boilerplate

Automatically generated by Colaboratory.

Original file is located at
    https://colab.research.google.com/drive/1lmsYRDtum0ot25W11w8UI6g0SmkGJ0ei

Let's first get the imports out of the way.
"""

import array
import gzip
import itertools
import numpy
import numpy.random as npr
import os
import struct
import time
from os import path
import urllib.request
import matplotlib.pyplot as plt

import jax.numpy as np
from jax.api import jit, grad
from jax.config import config
from jax.scipy.special import logsumexp
from jax import random

"""The following cell contains boilerplate code to download and load MNIST data."""

_DATA = "/tmp/"

def _download(url, filename):
  """Download a url to a file in the JAX data temp directory."""
  if not path.exists(_DATA):
    os.makedirs(_DATA)
  out_file = path.join(_DATA, filename)
  if not path.isfile(out_file):
    urllib.request.urlretrieve(url, out_file)
    print("downloaded {} to {}".format(url, _DATA))


def _partial_flatten(x):
  """Flatten all but the first dimension of an ndarray."""
  return numpy.reshape(x, (x.shape[0], -1))


def _one_hot(x, k, dtype=numpy.float32):
  """Create a one-hot encoding of x of size k."""
  return numpy.array(x[:, None] == numpy.arange(k), dtype)


def mnist_raw():
```

```
  """Download and parse the raw MNIST dataset."""
  # CVDF mirror of http://yann.lecun.com/exdb/mnist/
  base_url = "https://storage.googleapis.com/cvdf-datasets/mnist/"

  def parse_labels(filename):
    with gzip.open(filename, "rb") as fh:
      _ = struct.unpack(">II", fh.read(8))
      return numpy.array(array.array("B", fh.read()), dtype=numpy.uint8)

  def parse_images(filename):
    with gzip.open(filename, "rb") as fh:
      _, num_data, rows, cols = struct.unpack(">IIII", fh.read(16))
      return numpy.array(array.array("B", fh.read()),
                         dtype=numpy.uint8).reshape(num_data, rows, cols)

  for filename in ["train-images-idx3-ubyte.gz", "train-labels-idx1-ubyte.gz",
                   "t10k-images-idx3-ubyte.gz", "t10k-labels-idx1-ubyte.gz"]:
    _download(base_url + filename, filename)

  train_images = parse_images(path.join(_DATA, "train-images-idx3-ubyte.gz"))
  train_labels = parse_labels(path.join(_DATA, "train-labels-idx1-ubyte.gz"))
  test_images = parse_images(path.join(_DATA, "t10k-images-idx3-ubyte.gz"))
  test_labels = parse_labels(path.join(_DATA, "t10k-labels-idx1-ubyte.gz"))

  return train_images, train_labels, test_images, test_labels


#def mnist(create_outliers=False):
def mnist(create_outliers=True):
  """Download, parse and process MNIST data to unit scale and one-hot labels."""
  train_images, train_labels, test_images, test_labels = mnist_raw()

  train_images = _partial_flatten(train_images) / numpy.float32(255.)
  test_images = _partial_flatten(test_images) / numpy.float32(255.)
  train_labels = _one_hot(train_labels, 10)
  test_labels = _one_hot(test_labels, 10)

  if create_outliers:
    mum_outliers = 30000
    perm = numpy.random.RandomState(0).permutation(mum_outliers)
    train_images[:mum_outliers] = train_images[:mum_outliers][perm]

  return train_images, train_labels, test_images, test_labels

def shape_as_image(images, labels, dummy_dim=False):
  target_shape = (-1, 1, 28, 28, 1) if dummy_dim else (-1, 28, 28, 1)
  return np.reshape(images, target_shape), labels

#train_images, train_labels, test_images, test_labels = mnist(create_outliers=False)
train_images, train_labels, test_images, test_labels = mnist(create_outliers=True)
num_train = train_images.shape[0]

"""# **Problem 1**

This function computes the output of a fully-connected neural network (i.e.,
multilayer perceptron) by iterating over all of its layers and:
```

```
1. taking the `activations` of the previous layer (or the input itself for the first
hidden layer) to compute the `outputs` of a linear classifier. Recall the lectures:
`outputs` is what we wrote $z=w\cdot x + b$ where $x$ is the input to the linear
classifier.
2. applying a non-linear activation. Here we will use $tanh$.

Complete the following cell to compute `outputs` and `activations`.
"""

def predict(params, inputs):
  activations = inputs
  for w, b in params[:-1]:
    outputs = np.dot(activations, w) + b
    activations = np.tanh(outputs)

  final_w, final_b = params[-1]
  logits = np.dot(activations, final_w) + final_b
  return logits - logsumexp(logits, axis=1, keepdims=True)

"""The following cell computes the loss of our model. Here we are using cross-entropy
combined with a softmax but the implementation uses the `LogSumExp` trick for
numerical stability. This is why our previous function `predict` returns the logits
to which we substract the `logsumexp` of logits. We discussed this in class but you
can read more about it [here](https://blog.feedly.com/tricks-of-the-trade-
logsumexp/).

Complete the return line. Recall that the loss is defined as :
$$ l(X, Y) = -\frac{1}{n} \sum_{i\in 1..n}  \sum_{j\in 1.. K}y_j^{(i)}
\log(f_j(x^{(i)})) = -\frac{1}{n} \sum_{i\in 1..n}  \sum_{j\in 1.. K}y_j^{(i)}
\log\left(\frac{z_j^{(i)}}{\sum_{k\in 1..K}z_k^{(i)}}\right) $$
where $X$ is a matrix containing a batch of $n$ training inputs, and $Y$ a matrix
containing a batch of one-hot encoded labels defined over $K$ labels. Here
$z_j^{(i)}$ is the logits (i.e., input to the softmax) of the model on the example
$i$ of our batch of training examples $X$.
"""

def loss(params, batch):
  inputs, targets = batch
  preds = predict(params, inputs)
  ce = -np.mean(np.sum(targets*preds, axis=1))
  print(ce)
  return ce

"""The following cell defines the accuracy of our model and how to initialize its
parameters."""

def accuracy(params, batch):
  inputs, targets = batch
  target_class = np.argmax(targets, axis=1)
  predicted_class = np.argmax(predict(params, inputs), axis=1)
  return np.mean(predicted_class == target_class)

def init_random_params(layer_sizes, rng=npr.RandomState(0)):
  scale = 0.1
  return [(scale * rng.randn(m, n), scale * rng.randn(n))
```

```
        for m, n, in zip(layer_sizes[:-1], layer_sizes[1:])]

"""The following line defines our architecture with the number of neurons contained
in each fully-connected layer (the first layer has 784 neurons because MNIST images
are 28*28=784 pixels and the last layer has 10 neurons because MNIST has 10
classes)"""

layer_sizes = [784, 1024, 1024, 10]
# [784, 1024, 128, 10]

"""The following cell creates a Python generator for our dataset. It outputs one
batch of $n$ training examples at a time."""

batch_size = 32
num_complete_batches, leftover = divmod(num_train, batch_size)
num_batches = num_complete_batches + bool(leftover)
def data_stream():
  rng = npr.RandomState(0)
  while True:
    perm = rng.permutation(num_train)
    for i in range(num_batches):
      batch_idx = perm[i * batch_size:(i + 1) * batch_size]
      yield train_images[batch_idx], train_labels[batch_idx]
batches = data_stream()

"""We are now ready to define our optimizer. Here we use mini-batch stochastic
gradient descent. Complete `<w UPDATE RULE>` and `<b UPDATE RULE>` using the update
rule we saw in class. Recall that `dw` is the partial derivative of the `loss` with
respect to `w` and `learning_rate` is the learning rate of gradient descent."""

learning_rate = 0.1
# 0.01: slow
# 1: oscillate but converge
# 2: oscillate but non-converge

@jit
def update(params, batch):
  grads = grad(loss)(params, batch)
  return [(w - learning_rate * dw, b - learning_rate * db)
          for (w, b), (dw, db) in zip(params, grads)]

"""This is now the proper training loop for our fully-connected neural network."""

num_epochs = 50
#num_epochs = 10
params = init_random_params(layer_sizes)
for epoch in range(num_epochs):
  start_time = time.time()
  for _ in range(num_batches):
    params = update(params, next(batches))
  epoch_time = time.time() - start_time

  train_acc = accuracy(params, (train_images, train_labels))
  test_acc = accuracy(params, (test_images, test_labels))

  print("Epoch {} in {:0.2f} sec".format(epoch, epoch_time))
```

```
  print("Training set accuracy {}".format(train_acc))
  print("Test set accuracy {}".format(test_acc))


"""# **Problem 2**

Before we get started, we need to import two small libraries that contain boilerplate
code for common neural network layer types and for optimizers like mini-batch SGD.
"""

from jax.experimental import optimizers
from jax.experimental import stax

"""Here is a fully-connected neural network architecture, like the one of Problem 1,
but this time defined with `stax`"""

init_random_params, predict = stax.serial(

    stax.Conv(32, (3, 3), strides=(1, 1)),
    stax.Relu,
    stax.MaxPool((2, 2), strides=(2, 2)),

    stax.Conv(64, (3, 3), strides=(1, 1)),
    stax.Relu,
    stax.Conv(64, (3, 3), strides=(1, 1)),
    stax.Relu,
    stax.MaxPool((2, 2), strides=(2, 2)),

    stax.Flatten,
    stax.Dense(100),
    stax.Relu,

    stax.Dense(10),
)

"""We redefine the cross-entropy loss for this model. As done in Problem 1, complete
the return line below (it's identical)."""

def loss(params, batch):
  inputs, targets = batch
  logits = predict(params, inputs)
  preds  = stax.logsoftmax(logits)
  return -np.mean(np.sum(targets*preds, axis=1))

"""Next, we define the mini-batch SGD optimizer, this time with the optimizers
library in JAX."""

learning_rate = 0.01
opt_init, opt_update, get_params = optimizers.momentum(learning_rate, 0.9)

@jit
def update(_, i, opt_state, batch):
  params = get_params(opt_state)
  return opt_update(i, grad(loss)(params, batch), opt_state)

"""The next cell contains our training loop, very similar to Problem 1."""
```

```
num_epochs = 10

key = random.PRNGKey(123)
_, init_params = init_random_params(key, (-1, 28, 28, 1))
opt_state = opt_init(init_params)
itercount = itertools.count()

for epoch in range(1, num_epochs + 1):
  for _ in range(num_batches):
    opt_state = update(key, next(itercount), opt_state,
shape_as_image(*next(batches)))

  params = get_params(opt_state)
  test_acc = accuracy(params, shape_as_image(test_images, test_labels))
  test_loss = loss(params, shape_as_image(test_images, test_labels))
  print('Test set loss, accuracy (%): ({:.2f}, {:.2f})'.format(test_loss, 100 *
test_acc))
```