MTE54 Fall 2024
Assignment 1

Please use computer-typed equations and text for the entire assignment, avoid hand-writing when possible.

1 Homogeneous Transformation Matrix & its inverse [5 pts]

Given $\mathbf{R} \in SO(2)$ and $\mathbf{q} \in \mathbb{R}^2$, and the following matrix:

$$\mathbf{G} = \begin{bmatrix} \mathbf{R} & \mathbf{q} \\ \mathbf{0}_{1\times 2} & 1 \end{bmatrix}$$

Show that

$$\mathbf{G}^{-1} = \begin{bmatrix} \mathbf{R}^\top & -\mathbf{R}^\top \mathbf{q} \\ \mathbf{0}_{1\times 2} & 1 \end{bmatrix}$$

2 Forward Kinematics [20 pts]

For this exercise, you may use MATLAB or Python. Given the planar serial robot in Figure 1, where each link is connected via a revolute (rotational) joint, except for the end-effector (the gripper), which is fixed to the final link:

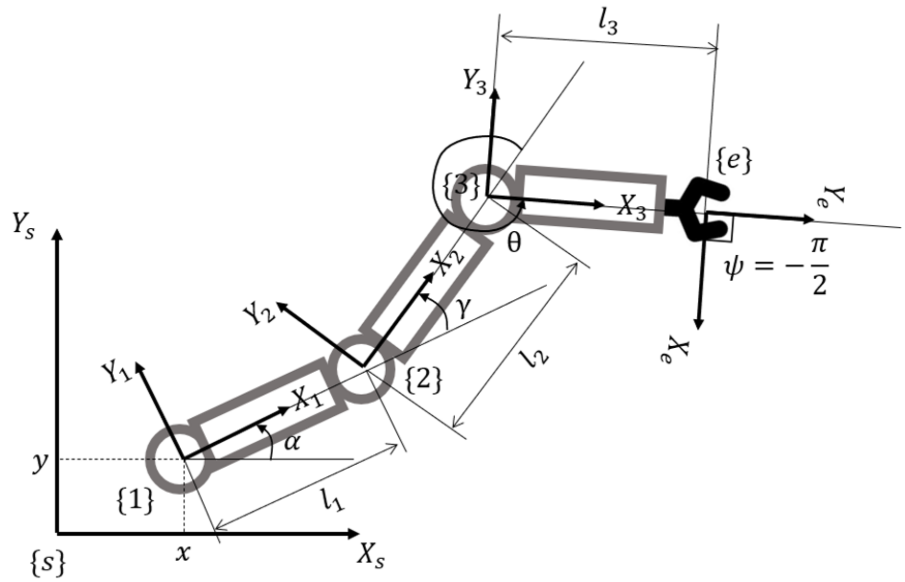

Figure 1: Planar serial manipulator

(a) Write the transformation matrices between subsequent reference frames, starting from the end-effector frame $\{e\}$ ($\mathbf{G}_e^3, \mathbf{G}_3^2, \ldots, \mathbf{G}_1^s$), and compute the transformation matrix from the end-effector frame $\{e\}$ to the global frame $\{s\}$. [10 pts]

(b) At the following configuration of the robot:

$$\langle \alpha, \gamma, \theta \rangle = \langle \pi/4, \pi/4, -\pi/3 \rangle, \quad l_1 = l_2 = l_3 = 1, \quad x = y = 1$$

What are the coordinates of these point in the spatial frame $\{s\}$? $\mathbf{p}_1^e = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{p}_2^e = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ [5 pts]

(c) Suppose now that the robot base is fixed at $x = y = 0$ (the origin of frame $\{1\}$ coincides with the origin of frame $\{s\}$), what is the expression of the velocity of a generic point $\mathbf{p}$ in the frame $\{2\}$ ($\mathbf{p}^2 = \begin{bmatrix} x' \\ y' \end{bmatrix}$) with respect to the base frame $\{s\}$? Assume that the point $\mathbf{p}$ is rigidly fixed to frame $\{2\}$. Report the procedure and the final coordinates, no need to report the full symbolic equations at each step. [5 pts.]

3 Differential-Drive Simulation [20 pts]

Create a simulation for the 2WD robot with Python which takes as inputs the linear and angular velocities $\begin{bmatrix} v, \omega \end{bmatrix}^\top$ (you can use the state space form). You should obtain the poses of the robot $\begin{bmatrix} x(t), y(t), \theta(t) \end{bmatrix}^\top$ as outputs.

(a) Simulate and plot the trajectories obtained for the following velocities (use at least 30 seconds with a timestep of 0.1s):

i. Constant velocities

1. $\begin{bmatrix} v, \omega \end{bmatrix}^\top = \begin{bmatrix} 1, 0 \end{bmatrix}^\top$
2. $\begin{bmatrix} v, \omega \end{bmatrix}^\top = \begin{bmatrix} 0, 0.3 \end{bmatrix}^\top$
3. $\begin{bmatrix} v, \omega \end{bmatrix}^\top = \begin{bmatrix} 1, 0.3 \end{bmatrix}^\top$

ii. Velocity profiles: $\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} 1 + 0.1 \cdot \sin(t) \\ 0.2 + 0.5 \cdot \cos(t) \end{bmatrix}$

Linear velocities are [m/s] and angular velocities are [rad/s]. For each case, plot the top view of the 2D trajectory (x vs y), and then each variable against time, including orientation θ. Discuss briefly your observations on the outcome (feel free to perform more tests). [15 pts]

(b) Assuming the robot has dimensions T (track) = 0.2 [m], and wheel radius $r = 0.1$ [m], what are the wheel speeds u_l,u_r needed to achieve the constant velocities in (a).i?

Briefly comment on how you would need to change your implemented simulation if instead of linear and angular velocities, the input control parameters were wheel speeds.

4 3-Omniwheel Simulation [55 pts]

(a) Derive the equation of motion (as we did for the 4W during the lectures) for the omnidirectional wheeled robot with three wheels shown in see Figure 2 [25 pts]

(b) Create a simulation for this robot with MATLAB or Python (you want to obtain x, y, θ for your simulation, you can obtain $\dot{x}, \dot{y}, \dot{\theta}$ as function of the wheels speeds by inverting the relationship obtained in (a)). Use $r = 10$ cm and $l = 25$ cm.

I. Apply rotation inputs (wheel rotation speeds) of $u_1 = -2$ rad/s, $u_2 = 1.0$ rad/s and $u_3 = 1.0$ rad/s to the wheels and plot the true robot motion (x, y, θ) for 30 seconds. Plot the variables against time, and also a top view of the trajectory (x vs y).

II. Design control signals (u_1, u_2, u_3) that will allow the robot to move in the following paths (simulation time is up to you, we recommend using at least 30 seconds):

1) a straight line with a slope of 60 degrees
2) turn in a 2-meter diameter circle.

Use your simulation to confirm the obtained control signals by plotting the two cases, plot the variables against time, and also a top view of the trajectory (x vs y).

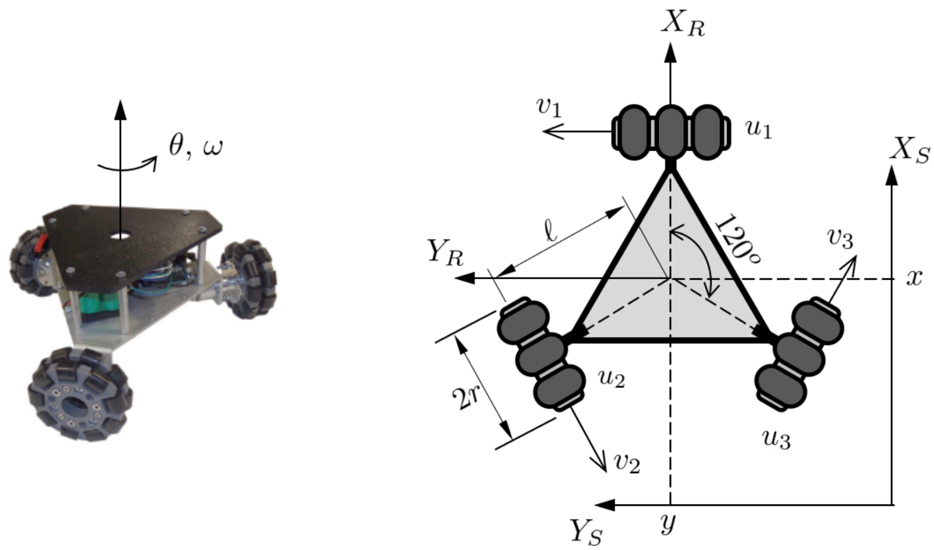

Figure 2: 3W omnidirectional robot

Ask TAs for help if you are facing difficulties with using MATLAB or Python for simulations.

Submission Guideline

Include details of all analytical computations and assumptions made (if any) and required plots in your assignment report. There is no page limit. Comment your code appropriately. Submit a single PDF for the report and a single zip file with all the codes on Dropbox by September 30th, 11:59PM. Late penalty applies as per syllabus if not properly justified. Overlaps with other assignments and/or course duties are not accepted as justifications.