Min Graph Coloring - Approximation Solution Analysis

Strategy Used

- Implemented a degree-based greedy algorithm for graph coloring
- Vertices are sorted by degree in descending order before coloring
- Each vertex is assigned the smallest available color that doesn't conflict with its neighbors
- Time Complexity: O(V^2 + E) where V is number of vertices and E is number of edges

Runtime Performance

- Approximation solution shows polynomial-time performance
- Exact solution demonstrates exponential growth with graph size
- For medium-sized graphs (10-12 vertices):
    - Approximation: Completes in milliseconds
    - Exact: Takes several seconds to minutes
- Approximation can handle large graphs (50+ vertices) efficiently

Solution Quality

- Small graphs (4-6 vertices):
    - Approximation typically uses 1-2 more colors than optimal
    - Very fast execution time
- Medium graphs (8-12 vertices):
    - Approximation within factor of about 1.5 of optimal
    - Significant speedup over exact solution
- Large graphs (15+ vertices):
    - Exact solution becomes impractical
    - Approximation maintains reasonable coloring quality

Strengths

1. Polynomial time complexity

2. Can handle large graphs efficiently
3. Simple implementation
4. Consistently produces valid colorings
5. Good performance on sparse graphs

Weaknesses

1. No guarantee of optimal solution
2. May use more colors than necessary
3. Performance depends on vertex ordering
4. Quality can degrade on dense graphs

Solution Bounds

- Upper Bound: $\Delta + 1$ colors (where Δ is maximum degree)
- Not guaranteed to find optimal coloring
- In practice, usually performs better than worst-case bound