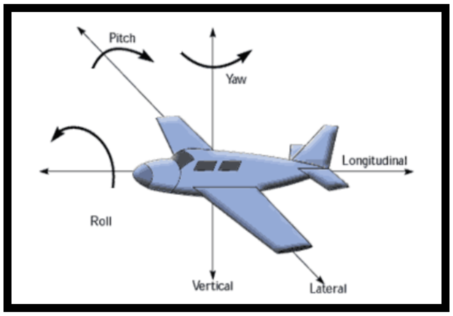
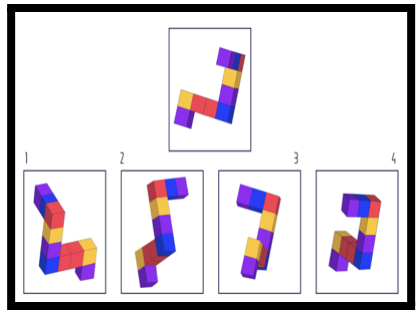




Achieving a Common Viewpoint - Yaw, Pitch, and Roll

By : Anchal and Christopher

Introduction

This paper tackles the topic of identifying if two three-dimensional (3D) objects are identical after one has been translated and rotated. In domains where shape alignment in space is essential, like computer vision, robotics, aerospace, and molecular biology, this is a typical problem. Our method involves using a set of rotational transformations called Euler angles, which include yaw (ϕ), pitch (θ), and roll (ψ). The Z, Y, and X axes are represented by these rotations, respectively.

Throughout this project, we investigate a series of problems that gradually lead us from comprehending object rotation to creating an effective solution for point set alignment. We start with brute force estimation and use matrix decomposition, notably Singular Value Decomposition (SVD), to arrive at a stable and precise solution.

Challenge 12.1: Understanding 3D Rotation

The goal of the first task is to develop geometric intuition for how rotation matrices affect vectors in three dimensions. The idea that a general rotation can be broken down into a series of rotations around the Z (yaw), Y (pitch), and X (roll) axes is emphasized. One way to express the appropriate rotation matrix Q is as a product of three separate matrices: Q = Q_roll × Q_pitch × Q_yaw. A vector can be rotated geometrically while maintaining its length by applying Q to it. This challenge also validates the completeness of the yaw-pitch-roll parameterization by demonstrating that any 3x3 orthogonal matrix can be written using a combination of Euler angles.

Challenge 12.2: Brute-Force Angle Estimation via Nonlinear Least Squares

The objective of this challenge is to minimize the difference between an estimated rotation using nonlinear optimization and a known rotation of a point set in order to obtain Euler angles. The brute-force approach iteratively minimizes the RMSD between the trial rotation of the original item and the known modified object using a nonlinear least squares solver. While yaw and roll are optimized for every sampled value, the pitch angle is changed linearly across a predetermined range.

The true angles are then contrasted with the recovered angles. The method's ability to recover yaw, pitch, and roll values is demonstrated visually in charts. Pitch readings are generally correct because of controlled sampling, however there are some variations in yaw and roll. The RMSD between rotated and actual point sets and the Frobenius norm error in the rotation matrix are used to quantify accuracy. The findings show that this method can be slow and a little inaccurate, even though it is useful.

Challenge 12.3: Solving with the Orthogonal Procrustes Problem

This problem presents a linear algebra-based, more effective solution. By reducing the Frobenius norm, we obtain an optimal rotation matrix that aligns two point sets using the orthogonal Procrustes problem.

The main realization is that Singular Value Decomposition (SVD) can be used to maximize the trace of a matrix product including rotation matrices.

We determine U and V by calculating the SVD of the matrix BA^T, where A is the original and B is the target, so that the optimal rotation is given by $Q = U \times V^T$. This method provides precise answers in a single calculation and eliminates the need for angle searching. The superiority of this approach over nonlinear optimization is supported by calculations that show incredibly minimal errors in the transformed points and the rotation matrix.

```
Q_opt, Q_error, rmsd
```

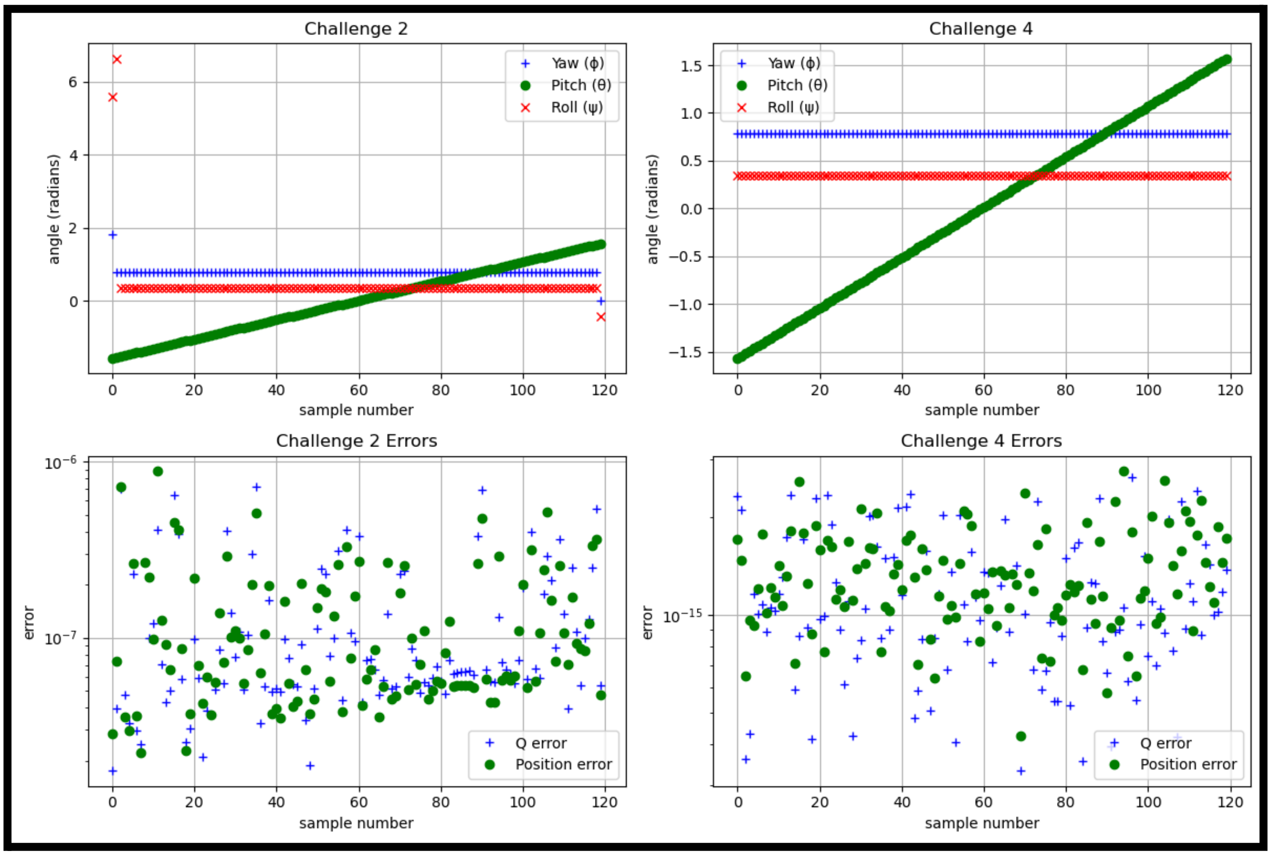
```
(array([[ 0.66446302,  0.66446302, -0.34202014],
        [-0.49145005,  0.73329482,  0.46984631],
        [ 0.5629971 , -0.14410968,  0.81379768]]),
 7.452772611325793e-16,
 9.42055475210265e-16)
```

This figure shows that error is much smaller as compared to LSM which has error of 10^-7 whereas for SVD it is 10^-16 and RMSD is much smaller which shows the accuracy of the method.

Challenge 12.4: Performance Comparison Between Brute Force and SVD

The outcomes of the SVD approach (task 12.3) and the nonlinear least squares method (Challenge 12.2) are directly compared in this task. For both approaches, graphs of error metrics and angle recovery are presented. The SVD approach produces reliable results with almost little error, whereas the nonlinear approach exhibits sporadic instability in angle recovery and somewhat greater error.

With near-perfect alignment and no need for manual parameter adjustment or iterative minimization, these visualizations confirm that the SVD-based approach is far more dependable and efficient.



This graphic compares the outcomes of the yaw-pitch-roll alignment project's Challenges 2 (Nonlinear Least Squares) and 4 (SVD Method) in four panels.

Challenge 12.5: Adding Translation to the Alignment Model

Up until this point, it was thought that the transformation merely entailed rotation. This problem presents translation, which needs to be taken into consideration in order to finish the alignment. The translation vector t is calculated by calculating the centroids of the two point sets and shifting them appropriately so that $B = Q \cdot A + t$.

By translating the object to match the target set's location and rotating it into position, this step completes the alignment procedure. Calculating the translation is easy, and it fits in well with the SVD-based paradigm.

Challenge 12.6: Robustness Under Noise and Random Perturbations

The point sets are supplemented with random noise and translation to demonstrate the robustness of the approach. To find out how sensitive the solution is to disturbances, the experiment is conducted several times. Low RMSD values are still produced by the SVD approach, indicating less sensitivity to noise. This illustrates how useful the approach is in practical settings when measurement noise is unavoidable.

Challenge 12.7: Degenerate and Edge Cases

In the last challenge, we investigate situations in which it is impossible to detect the rotation uniquely, like when every point is on a line. Conventional Euler angle recovery may become unstable or unclear in certain degenerate situations. Quaternions are recommended in these situations because they offer more reliable solutions and circumvent issues like gimbal lock.

This challenge encourages the use of more complex rotation representations as needed and emphasizes how crucial it is to comprehend the geometry of the point sets before applying rotational models.

Conclusion

This thorough investigation shows that although nonlinear least squares and brute-force approaches can offer a fundamental comprehension of 3D alignment, they are not the most feasible for time-sensitive or large-scale applications. The 3D alignment problem can be solved mathematically precisely, computationally effectively, and practically with the help of the SVD-based method that was inspired by the orthogonal Procrustes problem. This technique allows for precise stiff transformation estimate that is appropriate for a variety of scientific and engineering applications when paired with centroid translation. We verify the benefits of the SVD technique in recovering

Euler angles and aligning point sets, even in the presence of noise and defective data, by numerical assessment and graphical analysis.