Introduction

In this investigation, we explore the reliability of measuring the depth of a 4-kilometer vertical mine shaft located at Earth's equator by analyzing the motion of a 1-kilogram test mass dropped from the surface. The idea is to relate the time it takes for the mass to reach the bottom with the depth of the shaft, but the physics involved goes far beyond simple free fall learned in high school. Our analysis incorporates increasingly realistic physical effects, including the variation of gravitational acceleration with depth, air drag forces that depend on velocity, and the Coriolis effect due to Earth's rotation arround its own axis.

Using a combination of analytical methods and numerical techniques (such as solving systems of differential equations with `“solve_ivp”`), we simulate the motion of the falling mass under these complex conditions. We also examine the influence of Earth's internal density distribution on fall times through idealized trans-planetary tunnels.

Section 2:

First we calculated the time for a simple falling scenario(constant gravity and no air resistance). Using the equation “time=sqrt((2*height)/gravity)” defined as the function freefall_time in the code yields a result of 28.55 seconds, and the solve_ivp for this case yields 28.56(Confirming theoretical time). Next we calculate the result if we consider a variable gravity, and no air resistance. The equation in this case we will be using a differential equation.

$$\frac{d^2y}{dt^2} = -g + \alpha \left| \frac{dy}{dt} \right|^{\gamma} \quad (1) \qquad g(r) = g_o \left(\frac{r}{R_{\oplus}} \right) \quad (2)$$

Here the first term in (1) is acceleration, the second term in (1) is replaced by the right hand side of (2), and the third term is “turned off” by setting alpha to 0: eliminating drag. The second result that it yields is 28.56s, which makes sense as the distance traveled is arround 0.07% the radius of earth, and it makes not much of a difference to gravity. The third time is given by the same things done in our result 2, but this time with a value of alpha nonzero. If we assume that the terminal speed of the test mass is 50m/s that means that when the body reaches that speed gravity is no longer creating acceleration, as velocity is not changing. So we equate Drag force to gravity. Solving for alpha in this equation, and calculating using the function called “alpha_test_mass” in the code, we get 0.003924. Using this value in (1), and solving the differential equationthe result for the time yielded is 83.55s

We can see clearly that just considering linear gravity variation, is not enough to approximate what would be a real scenario, this is due to the fact that the height of the shaft is not big enough to create a big difference in gravity, but big enough for the mass to find its terminal velocity due to drag force. Turning drag on, we can see how as soon as the test mass reaches 50m/s it stops accelerating, making the free fall way much slower. This is really important, as in the conclusion I give more insight on future considerations of drag.

Section 3

Consider coriolis force, which as mentioned is the force due to earth's rotation arround its own axis, we will have the object moving to the east, the positive side of our Y axis.

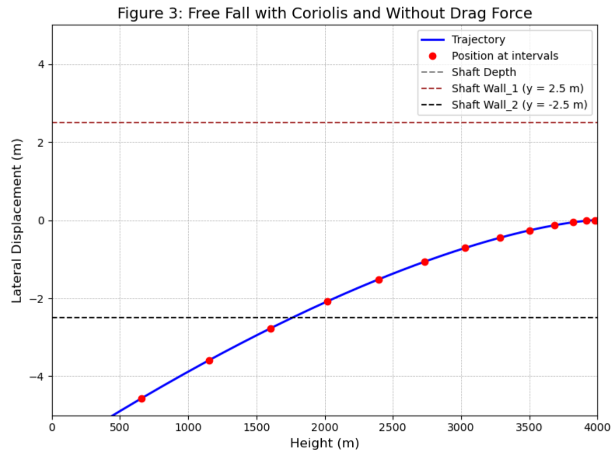
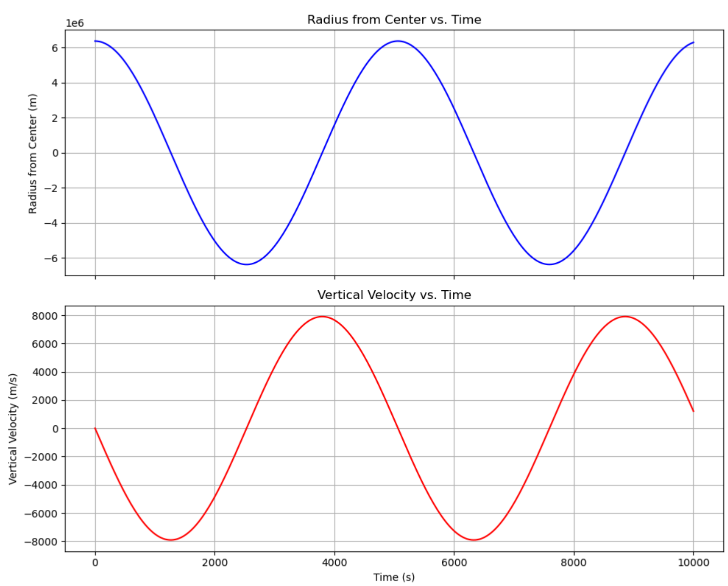


Figure 3

The red dots show how the mass moves as time goes by, we can see that the shift to the east gets faster and faster over time. Here the test mass hits the wall faster than it reaches the depth as seen in Figure 3. If we add drag force, when the test mass reaches its maximum velocity, then it would fall in the y direction way much slower as seen in the code, therefore hiting the wall faster. So yes, adding drag force makes a huge difference, making this method of measurement unreliable.

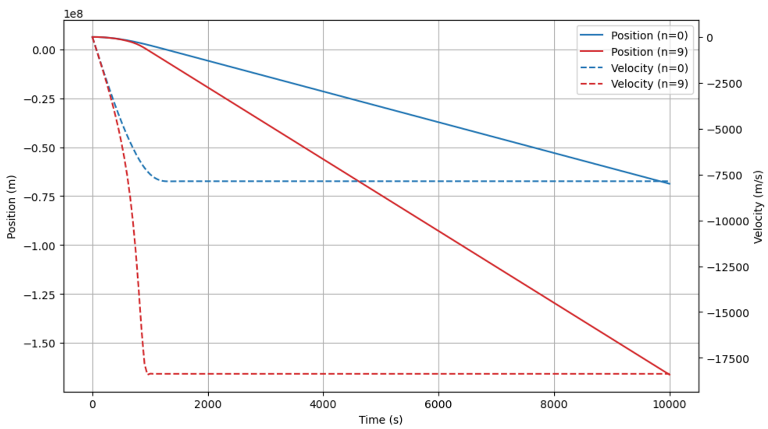
Section 4



In this section we drilled a theoretical hole from one end of the earth to the other, and we let our test mass fall, but we did it for 2 cases. Homomogeneous and non-homogeneous earth. For the first, we just drill the theoretical hole, and assume linear gravity. In this case we can see a harmonic oscillator after we throw the mass from one pole of the earth, and it reaches the other. The test mass in this case crosses the center of the Earth at 1265.77 seconds, with a velocity of 7906.36 m/s toward the south pole. Then it reaches the south pole at 2531.74 seconds. The code yields a velocity of 2.7e-13, but we can assume that this is due to computational errors, and it is actually zero due to the physics.

Now for the non-homogeneous earth we solve the density as a function of distance for the various values of n given in the lab. Using the density solutions we can calculate the mass at different depths, enabling us to know the exact gravitational force at those depths.

Figure 7: Motion Through the Earth



In the end if we consider the case for n=9vs n=0(homomogeneous earth) we can see how different the velocity and position plots are so different from each other. We can also see the values it takes for the test mass to reach the center of earth in this theoretical hole. For n = 0 the time was 1267.94 s, for n = 1 the time was 1096.47 s for n = 2 the time was 1035.13s, and for n = 9 the time was 943.75s. If we were to assume earth as a homomogeneous earth we would get the special property that the time for a mass orbiting the earth from one pole to halfway the other pole is the same as for n=0.

Conclusion

Our results show that using fall time to measure shaft depth is unreliable under realistic conditions. While variable gravity had minimal effect, drag significantly increased the fall time, and Coriolis forces caused the test mass to hit the wall before reaching the bottom. This suggests the method is not practical without major corrections. Key assumptions included a spherical Earth, constant drag coefficient, and no lateral forces.

Future work could improve accuracy by modeling Earth's true shape, atmospheric variation, and more complex dynamics, also another shape for the test mass that reduces aerodynamic resistance. Lastly, we found that fall time in trans-planetary tunnels depends more on density distribution than on planetary size, highlighting the broader importance of internal structure in gravitational motion.

Signature: *Christopher Sosa Quesada*