9.3 HW. Partial Sums.

Find the Sum of the Partial Geometric Sequence.

67. $\sum_{n=1}^{7} 4^{n-1}$ $n=7$ $r=4$ $a_1=1$

$S_7 = 1\left(\frac{1-4^7}{1-4}\right)$

$S_7 = 1\left(\frac{-16383}{-3}\right)$

$S_7 = 1(5461)$

$\boxed{S_7 = 5461}$

69. $S_6 = ?$ $1, -7, 49, -343, \ldots$ $n=6$ $r=-7$ $a_1=1$

$S_6 = 1\left(\frac{1+7^6}{1+7}\right)$

$S_6 = 1\left(\frac{117{,}650}{8}\right)$

$S_6 = 1(14{,}706.25)$

$\boxed{S_6 = 14{,}706.25}$

87. $10 + 30 + 90 + \ldots + 7290$ $r=3$ $a_1=10$ $n=?$

$a_n = a_1 r^{n-1}$

$7{,}290 = 10(3)^{n-1}$

$729 = 3^{n-1}$ → tested on calculator

$729 = 3^{7-1}$

$\boxed{n=7}$

$S_7 = 10\left(\frac{1-3^7}{1-3}\right)$

$S_7 = 10\,\frac{-2186}{-2}$

$S_7 = 10(1{,}093)$

$\boxed{S_7 = 10{,}930}$

85. $\sum_{n=1}^{10} 5\left(-\frac{1}{3}\right)^{n-1}$ $r=5$ $a_1=5$ $n=10$

$S_{10} = 5\left(\frac{1+\frac{1}{3}^{10}}{1+\frac{1}{3}}\right)$

$S_{10} = 5\left(\frac{1.000017}{1\frac{1}{3}}\right)$

$S_{10} = 5(0.75)$

$\boxed{S_{10} = 3.75}$

Find the Sum of the infinite Geometric Sequence $S = \frac{a_1}{1-r}$

95. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

$S = \frac{1}{1+\frac{1}{2}}$

$S = \frac{1}{1\frac{1}{2}}$

$\boxed{S = 0.67}$

97. $\sum_{n=1}^{\infty} 4\left(\frac{1}{4}\right)^{n-1}$

$S = \frac{4}{1-\frac{1}{4}}$

$S = \frac{4}{0.75}$

$\boxed{S = 5.34}$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} \quad \text{-or-} \quad = \sum_{i=0}^{n-1} a_1 r^{i}$$

what term # we are starting @

PreCalc Partial Sum of Geometric Series

formulas to know..

finding the infinite Sum

$.4 + .04 + .004 + \ldots$ $r = \frac{.04}{.4}$ $r = 0.1$

$S = \frac{a_1}{1-r} \rightarrow S = \frac{.4}{1-0.1} \rightarrow S = \frac{.4}{.9}$ $\boxed{S = 0.\overline{4}}$

common ratio: $r = \frac{a_2}{a_1} = \frac{a_n}{a_{n-1}}$

$a_n = a_1 r^{n-1}$

$S = \frac{a_1}{1-r}$ (if $|r| < 1$)

Geometric

Partial Sum ⇒ like a sum of set terms (ex: 1-10)

Formula: $S_n = a_1\left(\frac{1-r^n}{1-r}\right) = \sum_{i=1}^{n} a_1 r^{i-1}$

first term/ starting point

Ex:

3, 6, 12, 24

$S_7 = 3\left(\frac{1-2^7}{1-2}\right)$

$S_7 = 3\left(\frac{1-128}{-1}\right)$

$S_7 = 3\left(\frac{-127}{-1}\right)$

~~$S_7 = 3(127)$~~

$\boxed{S_7 = 381}$

$= \sum_{i=1}^{7} 3(2^{1-1})$

$= \sum_{i=1}^{7} 3(2^0)$

$= \sum_{i=1}^{7} 3(1)$

$= \sum_{i=1}^{7} 3$

Geometric Mean

multiply terms

$\underbrace{2, 4, 8}_{2}, 16, 32$

$\sqrt{2 \cdot 8} = 4$

$\sqrt[3]{2 \cdot 4 \cdot 8}$ = the geometric mean

ALL FORMULAS

Arithmetic

$a_n = a_1 + (n-1)d$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$ → partial sum

Geometric:

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}, r \neq 0$

$a_n = a_1 r^{n-1}$

$S = \frac{a_1}{1-r}$ → infinite Sum

$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = \sum_{i=1}^{n} a_1 r^{i-1}$

Find the Sum of the Partial Geometric Sequence.

67. $\sum_{n=1}^{7} 4^{n-1}$ $a_1 = 1$, $n = 7$, $r = 4$

$S_7 = 1\left(\frac{1-4^7}{1-4}\right)$

$= 1\left(\frac{-16{,}383}{-3}\right)$

$\boxed{= 5{,}461.\overline{3}}$

69. $S_6 = ?$ $1, -7, 49, -343, \ldots$ $r = -7$

$S_6 = 1\left(\frac{1+7^6}{1+7}\right)$

$= 1\left(\frac{117{,}650}{8}\right)$

$\boxed{= 14{,}706.25}$

87. $10 + 30 + 90 + \ldots + 7290$ $a_1 = 10$, $n = 7$, $r = 3$

$S_7 = 10\left(\frac{1-3^7}{1-3}\right)$

$= 10\left(\frac{-2{,}186}{-2}\right)$

$= 10(1{,}093)$

$\boxed{= 10{,}930}$

85. $\sum_{n=1}^{10} 5\left(-\frac{1}{3}\right)^{n-1}$ $a_1 = 5$, $r = -\frac{1}{3}$, $n = 10$

$S_{10} = 5\left(\frac{1+\frac{1}{3}^{10}}{1+\frac{1}{3}}\right)$

Find the Sum of the infinite Geometric Sequence

$S = \frac{a_1}{1-r}$

95. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

$S = \frac{1}{1+\frac{1}{2}}$

$= \frac{1}{1.5}$

$\boxed{= 0.\overline{6}}$

97. $\sum_{n=1}^{\infty} 4\left(\frac{1}{4}\right)^{n-1}$

$S = \frac{4}{1-\frac{1}{4}}$

$= \frac{4}{0.75}$

$\boxed{= 5.\overline{3}}$

Notes

Summation Notation

$\sum_{n=0}^{6}(n) = 0 + 1 + 2 + 3 + 4 + 5 + 6$

$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = \sum_{i=0}^{n-1} a_1 r^i$

$n = 21$ $r = -\frac{1}{4}$ $a_1 = 1$

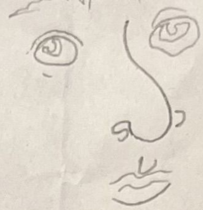
$\sum_{n=0}^{20}\left(-\frac{1}{4}\right)^n = S_{21} = \left(\frac{1-\left(-\frac{1}{4}\right)^{21}}{1-\left(-\frac{1}{4}\right)}\right)$

$\doteq$ small

$n = 7$ $r = \frac{3}{2}$ $a_1 = 3$

$\sum_{n=1}^{7} 3\left(\frac{3}{2}\right)^{n-1} = S_7 = 3\left(\frac{1-\left(\frac{3}{2}\right)^7}{1-\frac{3}{2}}\right) = 3\left(\frac{-16.086}{-0.5}\right) = \boxed{96.516}$

PreCalc Partial Sum of Geometric Series



Formulas to know..

$$S = \frac{10}{1-\frac{1}{2}} = \frac{10}{\frac{1}{2}} = 20$$

Common ratio: $r = \frac{a_2}{a_1} = \frac{a_n}{a_{n-1}}$

$a_n = a_1 r^{n-1}$

$S = \frac{a_1}{1-r}$ (if $|r|<1$)

Σ

infinite sums

$.4 + .04 + .004$ $\quad r = \frac{a_2}{a_1} = \frac{.04}{.4} = \frac{1}{10} = .1$

$$S = \frac{.4}{1-.1} = \frac{.4}{.9} = \frac{4}{9} = \boxed{.\overline{4}}$$

partial sums - geometric

$S_n = (a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^{n-1})$ $\times r$

$rS_n = a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^n$

$S_n - rS_n = a_1 - a_1 r^n$

$$\boxed{S_n = a_1\left(\frac{1-r^n}{1-r}\right)}$$ partial sum for geometric

$\times 2 \quad \times 2 \quad \times 2$

$3, 6, 12, 24$ $\quad r = \frac{a_2}{a_1} = \frac{6}{3} = 2$

$$S_7 = 3\left(\frac{1-2^7}{1-2}\right) = 3\left(\frac{1-128}{-1}\right) = 3\left(\frac{-127}{-1}\right) = 3(127) = \boxed{381}$$

counting to $\rightarrow$ $\sum_{i=1}^{n} a_1 r^{i-1}$ $\leftarrow$ starting point

$\sum_{i=1}^{7} 3(2^{i-1})$

Σ 3

Arithmetic (no infinite)

$a_n = a_1 + (n-1)d$

$S_n = n\left(\frac{a_1 + a_n}{2}\right) \rightarrow$ partial sum

Geometric

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_n}{a_{n-1}}, \; r \neq 0$

$a_n = a_1 r^{n-1}$

$S = \frac{a_1}{1-r}$ $\quad |r|<1 \longrightarrow$ infinite sum

$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \rightarrow$ partial sum

$= \sum_{i=1}^{n} a_1 r^{i-1}$

9.3 HW. Partial Sums.

Find the Sum of the Partial Geometric Sequence.

67. $\sum_{n=1}^{7} 4^{n-1}$

$\boxed{5461}$? $S_n = 1\left(\frac{1-4^7}{1-4}\right)$

n	1	2	3	4	5	6	7
$S_n = 4^{n-1}$	4^0	4^{2-1}	4^2	4^3	4^4	4^5	4^6
Σ	1 +	4 +	16 +	64 +	256 +	1024	4096

69. $S_6 = ?$ 1, -7, 49, -343, ...

$\sum_{n=1}^{6} 1(-7)^{n-1}$

$\boxed{S_6 = -14706}$

85. $\sum_{n=1}^{10} 5\left(-\frac{1}{3}\right)^{n-1}$

$\boxed{3.75}$

87. $10 + 30 + 90 + \ldots + 7290$

$7290 = 10(3)^{n-1}$

$7290 = a_7$

$6 = n-1$

$\sum_{n=1}^{7} 10(3)^{n-1}$ $\boxed{10930}$

Find the Sum of the infinite Geometric Sequence

95. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

$S = \frac{1}{1.5} = \frac{2}{3}$

97. $\sum_{n=1}^{\infty} 4\left(\frac{1}{4}\right)^{n-1}$

$S = \frac{4}{0.75} = \frac{16}{3}$

PreCalc. Partial Sum of Geometric ~~Series~~ Sequences

formulas to know..

common ratio: $r = \frac{a_2}{a_1} = \frac{a_n}{a_{n-1}}$

$a_n = a_1 r^{n-1}$

$S_\infty = \frac{a_1}{1-r}$ (if $|r|<1$)

$a_n = a_1 + (n-1)d$

$S_n = n\left(\frac{a_n + a_1}{2}\right)$

$.4, .04, .004$

$r = \frac{.04}{.4} = .1$

infinite → $S = \frac{.4}{1-.1} = \frac{4}{9} = .\overline{4}$

partial → $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$\sum_{i=1}^{n}$ means from i to n added sort of

unnecessary $\left\{ \frac{.\overline{a}}{x} = \frac{1}{x} \quad \forall x : (x \neq 0) \right.$

also works if >1 → $S_n = a_1\left(\frac{1-r^n}{1-r}\right) = \sum_{i=1}^{n} a_1 r^{i-1}$

$= \sum_{i=0}^{n-1} a_1 r^i$

3, 6, 12

$S_7 = 3\left(\frac{1-2^7}{1-2}\right)$

$S_7 = 381$

$\sum_{i=1}^{7} 3(2)^{i-1} \to 3$

$+$

$\sum_{i=2}^{7} 3(2)^{i-1} \to 6$

$+$

$\sum_{i=3}^{7} 3(2)^{i-1} \to 12$

...

→ 381

9.3 HW. Partial Sums.

Find the Sum of the Partial Geometric Sequence.

67. $\sum_{n=1}^{7} 4^{n-1}$

5461

69. $S_6 = ?$ $1, -7, 49, -343, \ldots$ $r = -7$

-14706

85. $\sum_{n=1}^{10} 5\left(-\frac{1}{3}\right)^{n-1}$

3.749936649

87. $10 + 30 + 90 + \ldots + 7290$

$a_1 r^{n-1} = a_n$

$10(3)^{(n-1)} = 7290$

$(3)^{(n-1)} = 729$

$\log_3(729) = 6$

$n = 7$

$\sum_{n=1}^{7} (10)(3)^{(n-1)}$

10930

Find the Sum of the infinite Geometric Sequence

95. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

$\frac{a_1}{1-r}$

$2/3 = .6667$

$S = \frac{1}{1.5} = \frac{2}{3} = .66\overline{6}$

97. $\sum_{n=1}^{\infty} 4\left(\frac{1}{4}\right)^{n-1}$

$S = \frac{4}{.75} = \frac{16}{3} = 5.33333\overline{3}$

PreCalc Partial Sum of Geometric Series

Common ratio: $r = \frac{a_2}{a_1} = \frac{a_n}{a_{n-1}}$

$a_n = a_1 r^{n-1}$

$S = \frac{a_1}{1-r}$ (if $|r|<1$)

$.4 + .04 + .004 + ...$

$r = \frac{a_2}{a_1} = \frac{.04}{.4} = \frac{1}{10} = .1$

$S = \frac{.4}{1-(.1)} = \frac{.4}{.9} = \frac{4}{9} = .\overline{4}$

$S_{n-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 ... + a_1 r^{n-1}$

$\times r$

$S_n = a_1 r + a_1 r^2 +$

$S_n - r S_n = a_1 - a_1 r^n$

$$\boxed{S_n = a_1\left(\frac{1-r^n}{1-r}\right)}$$

3, 6, 12, 24, .. 48, 96, 192

192

381

$\sum_{i=1}^{n} (a_1)(r)^{i-1}$

$\sum_{i=1}^{7} 3(2^{i-1})$