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1

```
static int algoritmo (String a, String b){ // 1
  int i, j;
  int m, n;
  n = a.length(); // 1
  m= b.length(); // 1
  for (i=0; i <=(n-m); i++){ // (n-m+2)
                                // (n-m+1)
    j=0;
    while ((j<m) && (a.charAt(i+j)==b.charAt(j))){ // (n-m+1)(m+1)
      j++; // (n-m+1)(m)
      if (j==m){ // (n-m+1)(m)
        return (i); // 1
      }if
    }while
  }for return (-1); // 1
} algoritmo
```

$$T(n,m) = 3+(n-m+2)+(n-m+1)+(n-m+1)(m+1)+(n-m+1)(m)+(n-m+1)(m)+2$$

$$= 8+n-\cancel{m}+n-\cancel{m}+nm-m^2+\cancel{m}+n-\cancel{m}+1+nm-m^2+\cancel{m}+nm-m^2+\cancel{m}$$

$$= 9+3n+3nm-3m^2 = -3m^2+3nm+3n+9$$

comparando cual crece mas: $(n \geq m)$

| n | m | $m^2$ | nm | n |
|---|---|---|---|---|
| 15 | 5 | 25 | 25 | 5 |
| 10 | 5 | 25 | 50 | 10 |
| 6 | 5 | 25 | 30 | 6 |
| 7 | 6 | 36 | 42 | 7 |

$\therefore T(n,m) \in O(nm)$

(2)

$$T(n)=\begin{cases}1 & \text{si } n=1 \text{ (caso base)}\\ 16T(n/2)+(1/n^4) & \text{e.o.c}\end{cases}$$

$\frac{n}{2}\cdot\frac{1}{2}$ $\quad n\cdot\frac{1}{2}$ $\quad \frac{n}{2^2}\cdot\frac{1}{2}=\frac{n}{2^3}$

$\boxed{\text{Dado que } \frac{1}{n^4} \text{ no es un polinomio no se puede aplicar el Teo. Master para polinomios.}}$

$$T(n)=16T(n/2)+1/n^4 \quad K=1$$

$$=16\left[16T(n/2^2)+1/(n/2)^4\right]+1/n^4=16^2T(n/2^2)+\frac{16}{\frac{n^4}{2^4}}+\frac{1}{n^4}=16^2T(n/2^2)+\frac{2^8}{n^4}+\frac{1}{n^4} \quad K=2$$

$$=16^2\left[16T(n/2^3)+1/(n/2^2)^4\right]+\frac{2^8}{n^4}+\frac{1}{n^4}=16^3T(n/2^3)+\frac{16^2\cdot 2^8}{\frac{n^4}{2^8}}+\frac{2^8}{n^4}+\frac{1}{n^4}$$

$$=16^3T(n/2^3)+\frac{2^{16}}{n^4}+\frac{2^8}{n^4}+\frac{1}{n^4} \quad K=3$$

forma gral:

$$T(n)=16^KT(n/2^K)+\frac{1}{n^4}\left(\frac{2^{8K}-1}{2^8-1}\right)$$

$$=16^KT(n/2^K)+n^{-4}\left(\frac{2^{8K}-1}{2^8-1}\right)$$

$$=2^{4K}T(n/2^K)+\frac{2^{8K}n^{-4}-n^{-4}}{2^8-1}$$

CA|

$2^0 \quad 2^8 \quad 2^{16}$

$1 \quad 2^8 \quad 2^{16}$

$2^8 \quad 2^8$

$a_1=1;\ r=2^8$

$$S_K=a_1\frac{r^K-1}{r-1}=\frac{(2^8)^K-1}{2^8-1}$$

igualando con el caso base:

$$\frac{n}{2^K}=1 \Rightarrow n=2^K // \log_2 \Rightarrow \log_2 n=K$$

$n=2^K //()^4 \qquad n=2^K//()^8$

$n^4=2^{4K} \qquad n^8=2^{8K}$

reemplazando:

$$T(n)=n^4\cancel{T(1)}^{1}+\frac{n^8n^{-4}-n^{-4}}{2^8-1}=n^4+\frac{n^4-n^{-4}}{2^8-1}$$

$n \quad n^4 \quad n^{-4}$

$3 \quad 81 \quad 1/81$

$$\therefore T(n)\in O(n^4)$$

③

$$T(n)=\begin{cases} 1 & si\ n=1 \\ 5 & si\ n=2 \\ 11 & si\ n=3 \\ T(n-1)+4T(n-2)+2T(n-3) & si\ n\geq 4 \end{cases}$$

$$T(n)=T(n-1)+4T(n-2)+2T(n-3)$$

C.V $T(n)=r^n$

$$r^n = r^{n-1}+4r^{n-2}+2r^{n-3} \quad // \frac{1}{r^{n-3}}$$

$$r^3 = r^2+4r+2$$

$$r^3 - r^2 - 4r - 2 = 0$$

por. Ruffini:

| | 1 | -1 | -4 | -2 | Grado ③ |
|---|---|---|---|---|---|
| -1 | | -1 | +2 | +2 | |
| | 1 | -2 | -2 | 0 | Grado ② |

$$(r+1)(r^2-2r-2)=0$$

$$(r+1)=0 \quad \wedge \quad \underset{a}{r^2}\underset{b}{-2r}\underset{c}{-2}=0$$

$$r_1=-1 \qquad r_{2,3}=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}$$

$$r_{2,3}=\frac{2\pm\sqrt{4+8}}{2}$$

$$r_2 = 1+\sqrt{3} \quad \wedge \quad r_3 = 1-\sqrt{3}$$

como $r_1 \neq r_2 \neq r_3$ entonces la sol grol es:

$$T(n) = C_1 r_1^n + C_2 r_2^n + C_3 r_3^n$$

$$= C_1(-1)^n + C_2(1+\sqrt{3})^n + C_3(1-\sqrt{3})^n \quad (*)$$

igualando con los casos base: (*)

$$\begin{cases} T(1) = C_1 r_1^1 + C_2 r_2^1 + C_3 r_3^1 \\ T(2) = C_1 r_1^2 + C_2 r_2^2 + C_3 r_3^2 \\ T(3) = C_1 r_1^3 + C_2 r_2^3 + C_3 r_3^3 \end{cases}$$

$$\Rightarrow \begin{cases} 1 = -C_1 + (1+\sqrt{3})C_2 + (1-\sqrt{3})C_3 \\ 5 = C_1 + (1+\sqrt{3})^2 C_2 + (1-\sqrt{3})^2 C_3 \\ 11 = -C_1 + (1+\sqrt{3})^3 C_2 + (1-\sqrt{3})^3 C_3 \end{cases}$$

$$\left(\begin{array}{cccc} -1 & 1+\sqrt{3} & 1-\sqrt{3} & 1 \\ 1 & 4+2\sqrt{3} & 4-2\sqrt{3} & 5 \\ -1 & 10+6\sqrt{3} & 10-6\sqrt{3} & 11 \end{array}\right) \begin{array}{l} F_2+F_3 \\ F_2+F_1 \end{array} \sim$$

$$\left(\begin{array}{cccc} 0 & 5+3\sqrt{3} & 5-3\sqrt{3} & 6 \\ 1 & 4+2\sqrt{3} & 4-2\sqrt{3} & 5 \\ 0 & 14+8\sqrt{3} & 14-8\sqrt{3} & 16 \end{array}\right) F_1 \leftrightarrow F_2 \sim$$

$$\left(\begin{array}{cccc} 1 & 4+2\sqrt{3} & 4-2\sqrt{3} & 5 \\ 0 & 5+3\sqrt{3} & 5-3\sqrt{3} & 6 \\ 0 & 14+8\sqrt{3} & 14-8\sqrt{3} & 16 \end{array}\right) \begin{array}{l} -F_2+F_1 \\ -3F_2+F_3 \end{array} \sim$$

$$\left(\begin{array}{cccc} 1 & -1-\sqrt{3} & -1+\sqrt{3} & -1 \\ 0 & 5+3\sqrt{3} & 5-3\sqrt{3} & 6 \\ 0 & -1-\sqrt{3} & -1+\sqrt{3} & -2 \end{array}\right) \begin{array}{l} 5F_3+F_2 \\ -F_3+F_1 \end{array} \sim$$

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & -2\sqrt{3} & 2\sqrt{3} & -4 \\ 0 & -1-\sqrt{3} & -1+\sqrt{3} & -2 \end{array}\right) -\frac{1}{2}F_2$$

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} & 2 \\ 0 & -1-\sqrt{3} & -1+\sqrt{3} & -2 \end{array}\right) \begin{array}{l} F_2+F_3 \\ \frac{1}{\sqrt{3}}F_2 \end{array} \sim \left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2/\sqrt{3} \\ 0 & -1 & -1 & 0 \end{array}\right) F_2+F_3 \sim$$

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2/\sqrt{3} \\ 0 & 0 & -2 & 2/\sqrt{3} \end{array}\right) -\frac{1}{2}F_3 \sim \left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2/\sqrt{3} \\ 0 & 0 & 1 & -1/\sqrt{3} \end{array}\right)$$

$$\overset{F_3+F_2}{\sim}\left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & -1/\sqrt{3} \end{array}\right) \Rightarrow \begin{array}{l} C_1 = 1 \\ C_2 = 1/\sqrt{3} \\ C_3 = -1/\sqrt{3} \end{array}$$

reemplazando $C_1, C_2, C_3$ en (☆):

$$T(n) = 1 \cdot (-1)^n + \frac{1}{\sqrt{3}}(1+\sqrt{3})^n - \frac{1}{\sqrt{3}}(1-\sqrt{3})^n$$

$$= (-1)^n + \frac{1}{\sqrt{3}}(1+\sqrt{3})^n - \frac{1}{\sqrt{3}}(1-\sqrt{3})^n$$

verificando si esta bien:

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| e.o.c | 1 | 5 | 11 | 33 | 87 |
| $T(n)$ | 1 | 5 | 11 | 33 | 87 |

$$\therefore T(n) \in O\left((1+\sqrt{3})^n\right)$$