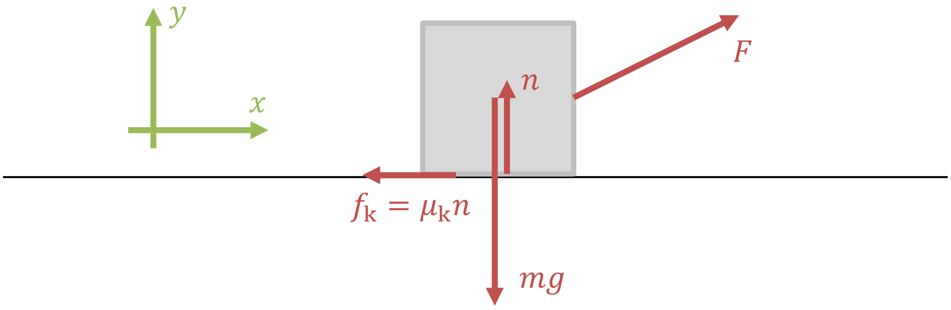
Problem 1.

a) The forces acting on the box are the force of gravity, the normal force, the kinematic friction force and the pulling force. A force diagram can be seen below.



b) We set up Newton's second law in the x- and y-directions. There is no acceleration in the y-direction.

N2(y): $$m \cdot 0 = F\sin\theta + n - mg \Rightarrow n = mg - F\sin\theta$$

N2(x): $$ma = F\cos\theta - f_k = F\cos\theta - \mu_k n$$

We insert the expression for the normal force in N2(x) and solve for the acceleration a

$$\begin{aligned} ma &= F\cos\theta - \mu_k(mg - F\sin\theta) \Leftrightarrow \\ ma &= F(\cos\theta + \mu_k\sin\theta) - \mu_k mg \Leftrightarrow \\ a &= \frac{F}{m}(\cos\theta + \mu_k\sin\theta) - \mu_k g \end{aligned}$$

c) The box is moving with constant velocity, so the acceleration in the x-direction must be 0. This observation can be utilized to determine the force F in the expression for the acceleration found in question b).

$$\begin{aligned} 0 &= \frac{F}{m}(\cos\theta + \mu_k\sin\theta) - \mu_k g \Leftrightarrow \\ \mu_k mg &= F(\cos\theta + \mu_k\sin\theta) \Leftrightarrow \\ F &= \frac{\mu_k mg}{(\cos\theta + \mu_k\sin\theta)} \end{aligned}$$

d) The minimum of F occurs when the denominator attains its maximum value. The maximum value for the denominator is found by differentiating with respect to θ and setting:

$$\frac{d}{d\theta}(\cos\theta + \mu_k\sin\theta) = -\sin\theta + \mu_k\cos\theta = 0 \Leftrightarrow \tan\theta = \mu_k \Rightarrow \theta = \tan^{-1}(\mu_k)$$

Problem 2.

Both blocks are affected by the force of gravity and the tension of the string (this excludes D). The tension in both strings must have the same size (this excludes both A and B).

The answer is option C.

Problem 3.

The curling stone is affected by gravity (↓), the normal force (↑), the friction force (←) and the external force pushing down and to the right.

- The external pushing force, E, should be included.
- The vertical component of the force E and the force of gravity should equal the normal force. This fits the size and direction of C.
- The horizontal component of the external force E should be larger than the friction force, otherwise the curling stone will not accelerate to the right. This condition is met for vector F.

The correct answers are B, C, E and F.

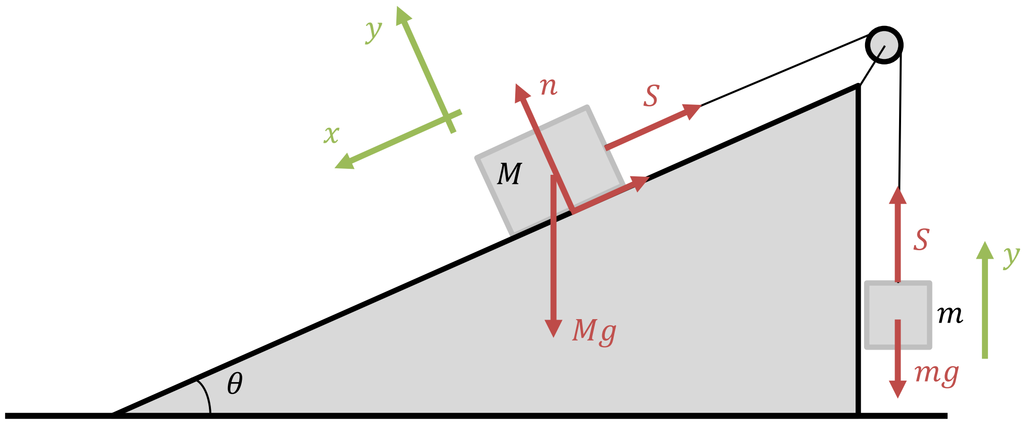
Problem 4.

As the box is accelerating up the inclined surface with an angle of 45° then *the sum of all forces* should have the same angle as the inclined surface due to Newtons second law. This is only the case for force diagram D.

The answer is option D.

Problem 5.

We draw the force diagram and a separate coordinate system for each box. We include the friction force in the diagram.



a) Here $f_{\mathrm{k}} = 0$! We set up Newtons 2. law in the direction of motion for each box:

N2(M, x): $Ma = Mg\sin\theta - S$ (1)

N2(m, y): $ma = S - mg$ (2)

Notice that the boxes has the same acceleration, as they are connected by a string. We add equation (1) to equation (2) and find an expression for the acceleration a

$$Ma + ma = Mg\sin\theta - S + (S - mg) \Leftrightarrow$$
$$(m + M)a = (M\sin\theta - m)g \Leftrightarrow$$

$$a = g\frac{M\sin\theta - m}{m + M}$$

b) Now the friction force is included $f_{\mathrm{k}} = \mu_{\mathrm{k}} n$. We set up Newtons second law for the boxes in both directions

N1(M, y): $M \cdot 0 = n - Mg\cos\theta \Rightarrow n = Mg\cos\theta$

N2(M, x):
$$Ma = Mg\sin\theta - S - f_{\mathrm{k}} \Leftrightarrow$$
$$Ma = Mg\sin\theta - S - \mu_{\mathrm{k}} n \Leftrightarrow$$
$$Ma = Mg\sin\theta - S - \mu_{\mathrm{k}} Mg\cos\theta \Leftrightarrow$$
$$Ma = Mg(\sin\theta - \mu_{\mathrm{k}}\cos\theta) - S \quad (3)$$

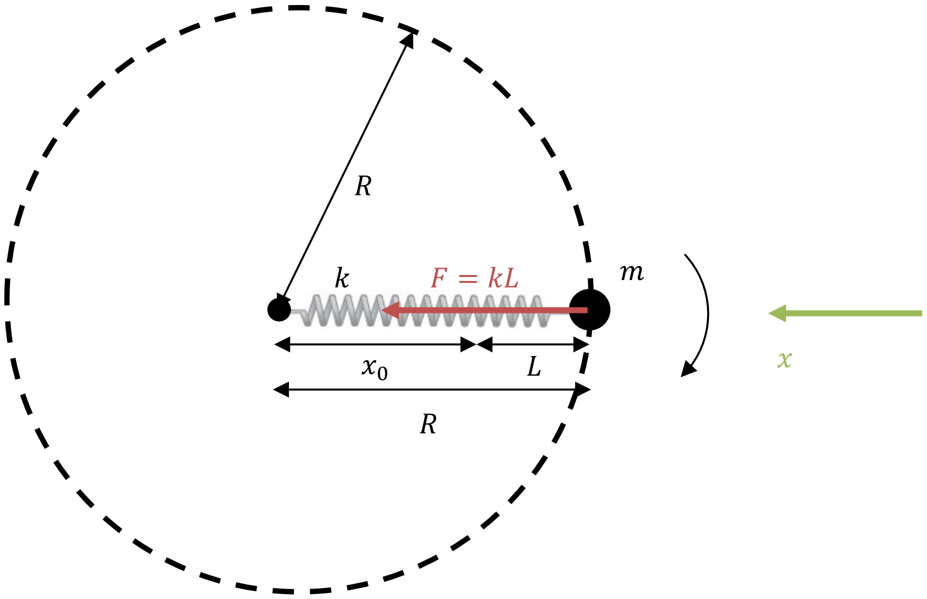
N2(m, y): $ma = mg - S$ (4)

We add (3) to equation (4) and find the expression for the acceleration a

$$a = g\frac{M(\sin\theta - \mu_{\mathrm{k}}\cos\theta) - m}{m + M}$$

Problem 6.

We draw a force diagram for the particle as well as a coordinate system. We choose the x-direction to be the radial direction. We neglect the force of gravity in this exercise.



The spring is extended from its equilibrium x_0 by the length L. From the drawing it can be seen that $x_0 + L = R \Rightarrow L = R - x_0$. The spring force is given by Hooke's law

$$F = kL = k(R - x_0)$$

The particle performs a circular motion with constant velocity v, which means that the particle's acceleration is given by $a_c = \frac{v^2}{R}$. Now we set up Newtons 2. law in the x-direction

N2(x): $$ma_c = F$$

$$m\frac{v^2}{R} = k(R - x_0)$$

The velocity of the particle v is expressed by

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R}{T}$$

This is inserted into the expression for N2(x) and then the equation is solved for x_0

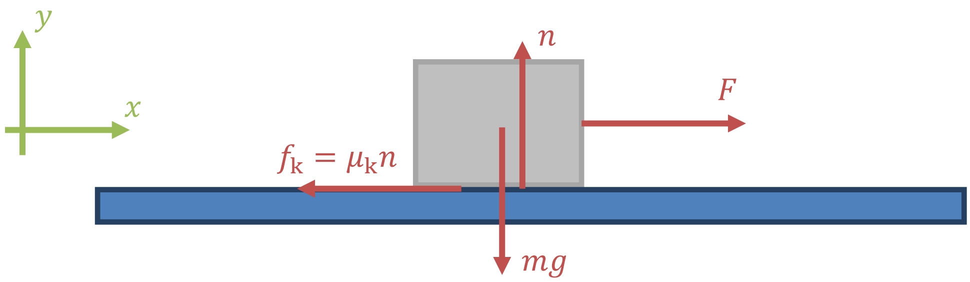
$$m\frac{\left(\frac{2\pi R}{T}\right)^2}{R} = k(R - x_0) \Leftrightarrow$$

$$x_0 = R - \frac{4\pi^2 mR}{kT^2} = 0.84 \text{ m}$$

The correct answer is option C.

Problem 7.

We draw a force diagram and a coordinate system.



a) We set up Newton's second law in the x- and y-directions. There is no acceleration in the y-direction.

N1(y): $0 = n - mg \Rightarrow n = mg$

N2(x): $ma = F - f_k$

If the block is moving with constant velocity, then the acceleration must be zero, $a = 0$:

$$0 = F - f_k \Rightarrow f_k = F$$

The answer is option C.

b) A force $3F$ is now pulling the box in the positive x-direction, so Newton's second law in the x-direction is

N2(x): $ma = 3F - f_k$

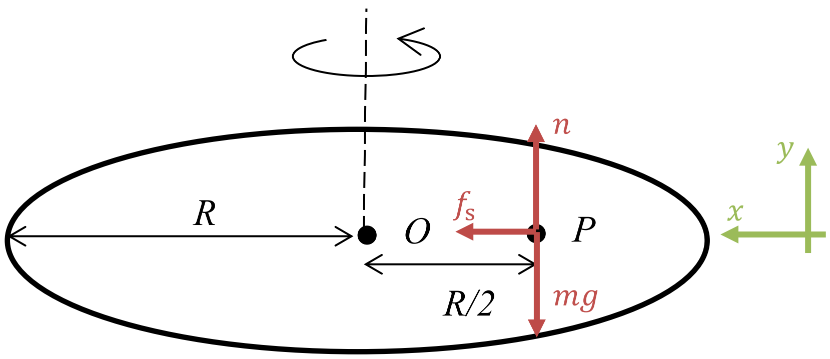
From question a) we know that $f_k = F$ and therefore

N2(x): $ma = 3F - F = 2F \Rightarrow a = \frac{2F}{m}$

The answer is option C.

Problem 8.

We draw a force diagram for the coin and a coordinate system. The x-axis is in the radial direction.



We set up Newton's second law in the x- and y-directions. There is no acceleration in the y-direction.

N1(y): $0 = n - mg \Rightarrow n = mg$

N2(x): $ma = f_s$

The coin performs a circular motion with constant velocity, so the acceleration is given by the centripetal acceleration.

$$a = \frac{v^2}{\frac{R}{2}} = \frac{2v^2}{R}$$

The velocity of the coin is given by

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi\frac{R}{2}}{T} = \frac{\pi R}{T}$$

The acceleration can now be given as a function of the revolution time T

$$a = \frac{2}{R}\left(\frac{\pi R}{T}\right)^2 = \frac{2\pi^2 R}{T^2}$$

The expression above can be inserted into Newton's second law in the x-direction

N2(x): $m\frac{2\pi^2 R}{T^2} = f_s$

The static friction force is given by the inequality

$$f_s \leq \mu_s n = \mu_s mg$$

The coin starts to slide when the friction force f_s attain its maximum value. At this point the friction force is no longer strong enough to keep the coin in a circular motion.

$$f_\mathrm{s}^\mathrm{max} = \mu_\mathrm{s} m g$$

We insert the maximum value of the friction force into N2(x) and solve for T:

N2(x): $$m\frac{2\pi^2 R}{T^2} = \mu_\mathrm{s} m g \Leftrightarrow$$

$$T = 2\pi\sqrt{\frac{R}{2\mu_\mathrm{s} g}}$$

The answer is option A.