DTU

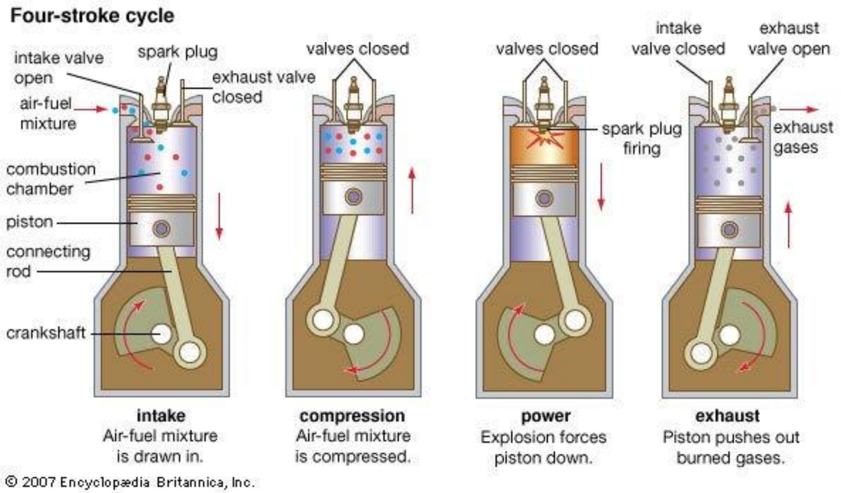
10060 Physics (Polyteknisk grundlag)

The Kinetic Theory of Gases

Gases are everywhere

To what pressure should fill your oxygen tank based on the dive depth?

What is the gas pressure on a piston after combustion?



To what temperature should you keep your air ballon to have lift?

… Olympics trivia

Why do endurance athletes go to altitude training camps to improve performance?

Discuss with your neighbour…
what is the physics involved?

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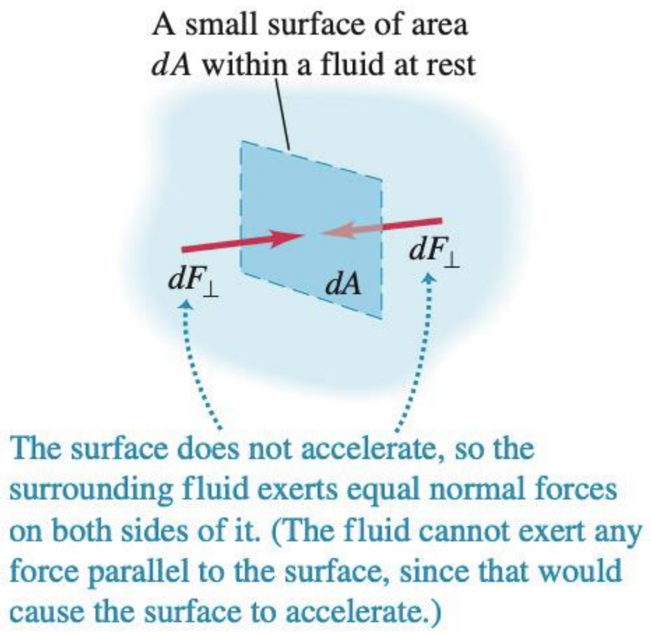
Experimental evidences on gases

Short Intro on Pressure in Fluids/Gases

- Fluids/Gases exert a force perpendicular to any surface they contact

To visualize "pressure" consider a tiny patch in a fluid:



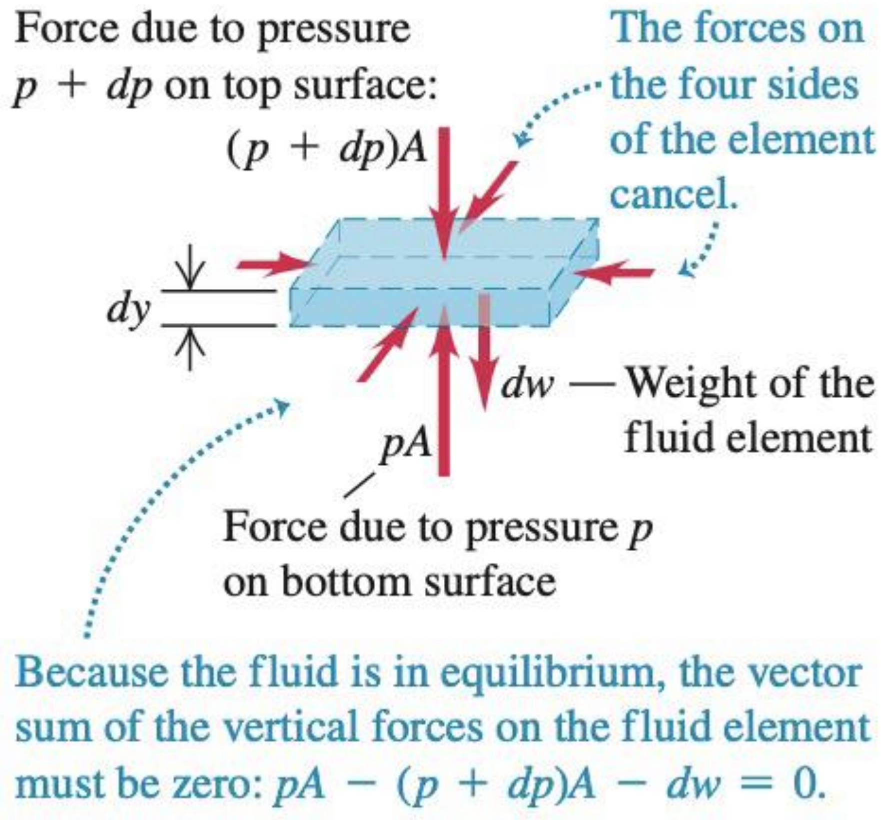
$$p = \frac{dF_\perp}{dA} = \frac{N}{m^2} = Pa$$

Pressure is often reported in atmospheres:

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$
$$= 1.013 \text{ bar} = 1013 \text{ mbar}$$

Pressure and depth

- Pressure increases with depth to satisfy force balance: $\sum F_y = 0$



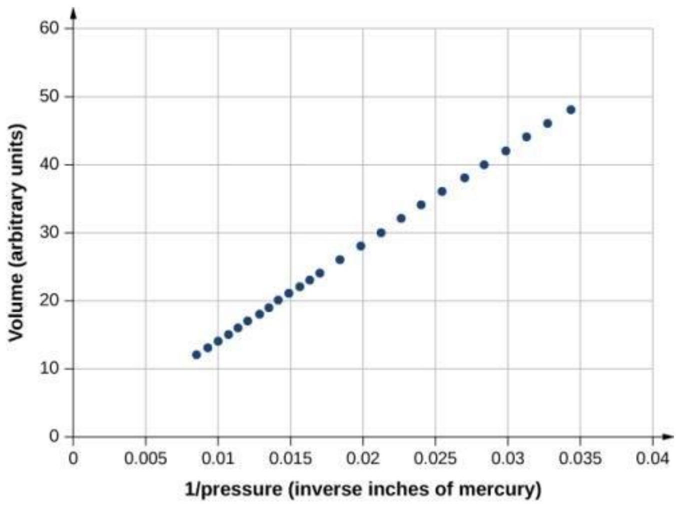
$$g\rho A dy = A\,dp \rightarrow \frac{dp}{dy} = -\rho g$$

Integrate w.r.t. dy:

$$p(y) = p_0 + \rho g y$$

Pioneering experiments on Gases

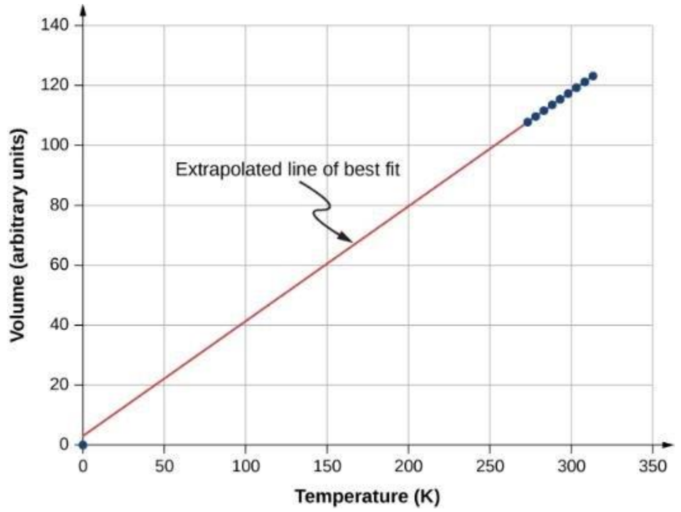
Boyle's law (1662)



* Constant T and N of molecules

$$pV = const$$

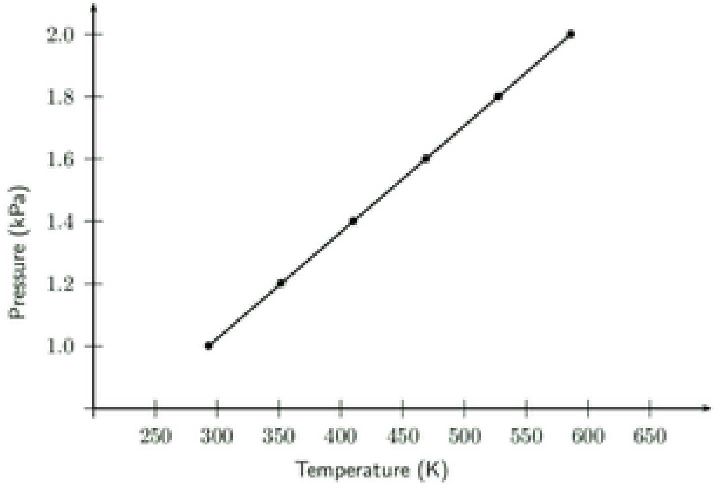
Charles's law (1787)



* Constant P and N of molecules

$$\frac{V}{T} = const$$

Lussac's law (1808)



* Constant V and N of molecules

$$\frac{P}{T} = const$$

Ideal Gas Law

Boltzmann constant

Volume

Pressure

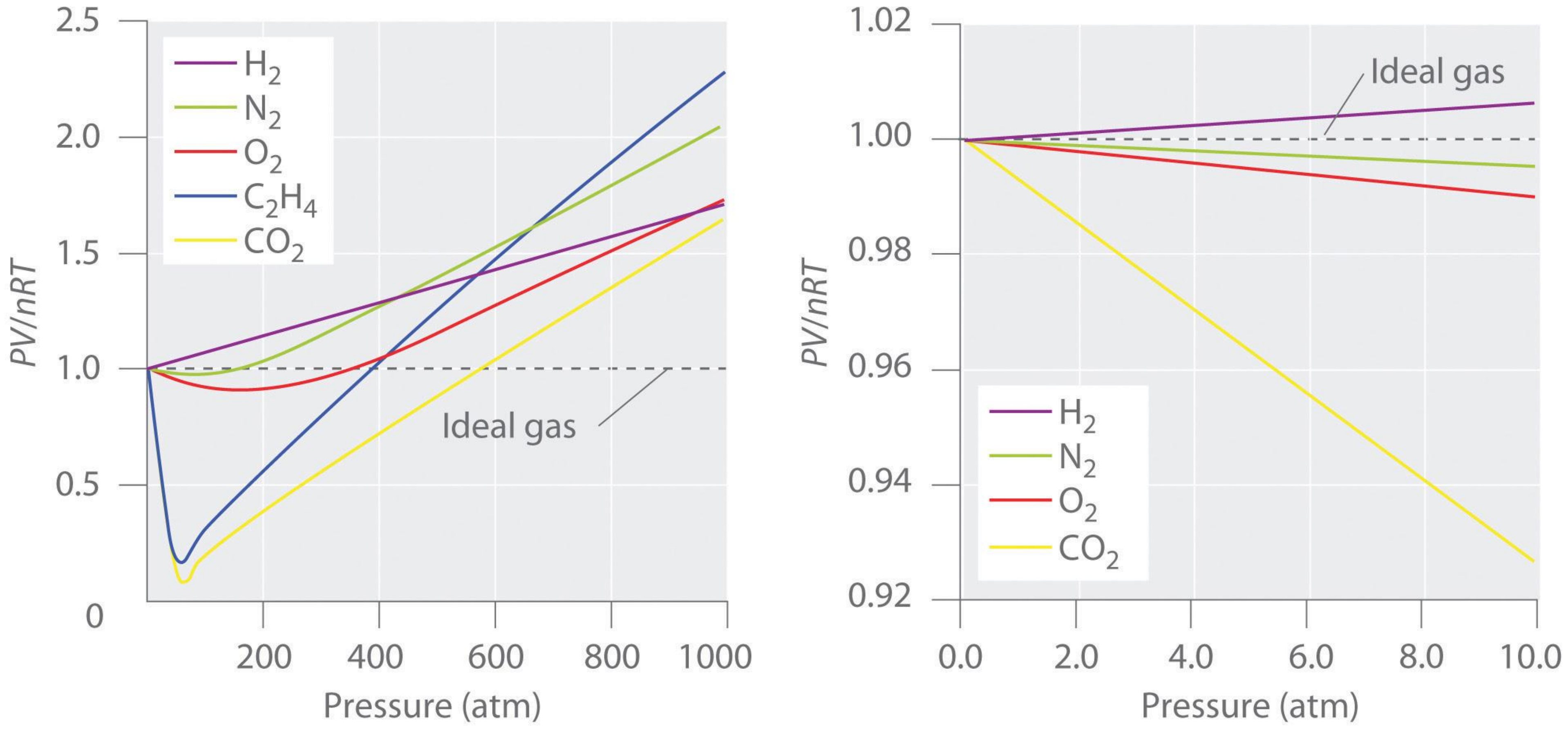
$$pV = Nk_BT$$

\# gas molecules

Temperature

$$k_B = 1.38 \times 10^{-23}\ \text{J/K}.$$

Gases tend to behave in the same way at low densities



(a) *PV/nRT* at high pressures (b) *PV/nRT* at low pressures

Ref. Link

How many molecules are in your bike tyre?

A bicycle tire is inflated to a pressure of 4 atmospheres at a temperature of 20˚ C. The volume of air inside the tire is 2 liters. Assuming the air behaves as an ideal gas and is composed mainly of nitrogen, estimate the number of nitrogen molecules in the tire.

$$N = \frac{pV}{K_b T} \rightarrow$$

$$N = \frac{4.052 \cdot 10^5\ Pa\ \times\ 2 \cdot 10^{-3} m^3}{1.38 \cdot 10^{-23} \frac{J}{K}\ \times\ 293\ K} \sim 2 \cdot 10^{23}$$

Avocadro's Number and Moles

$$N_A = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$$

- Avogadro's number relates the mass of an amount of substance in grams to the number of protons and neutrons in an atom or molecule (12 for a carbon-12 atom)
- A carbon-12 atom has a mass of exactly 12 u, with u the unified atomic mass unit
- N_A is also the conversion from u to grams: $6.02 \cdot 10^{23}\ u = 1\ g$

Ideal Gas Law (using moles)

Defining the universal gas constant R:

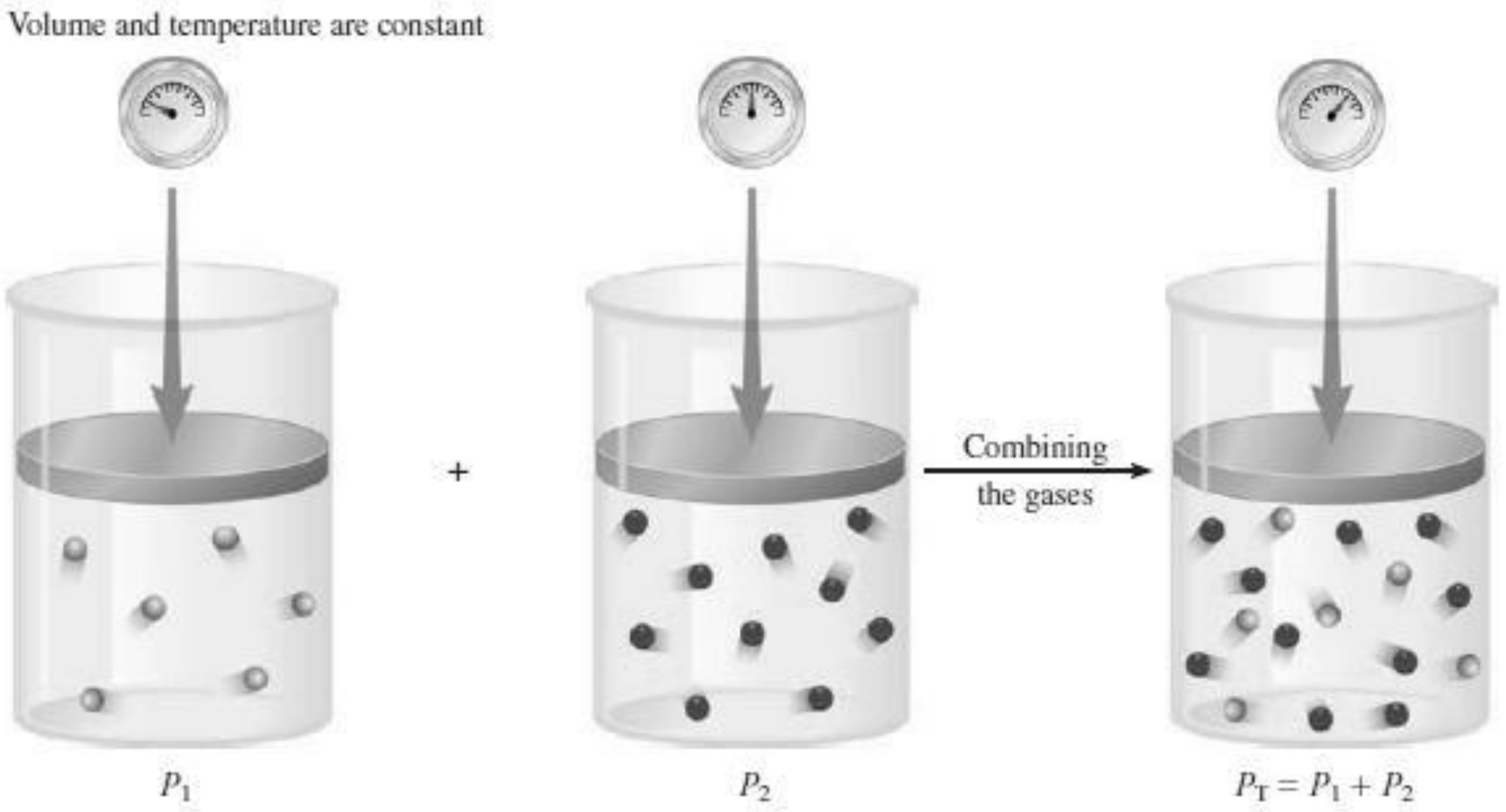
$$R = N_\mathrm{A} k_\mathrm{B} = \left(6.02 \times 10^{23}\ \mathrm{mol}^{-1}\right)\left(1.38 \times 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}\right) = 8.31\ \frac{\mathrm{J}}{\mathrm{mol} \cdot \mathrm{K}}.$$

Pressure Volume Boltzmann constant # moles of gas Universal gas constant

$$pV = Nk_B T = nRT$$

gas molecules Temperature

Dalton’s law



In general

$$P_{Total} = \sum_{i=1}^{n} P_i$$

… back to altitude training

At sea level and 2000 meters, the volume (V) and temperature (T) are assumed to be constant. This means the ratio of number of moles of oxygen per breath between sea level and altitude is equal to the ratio in pressure:

$$\frac{n_{sea}}{n_{alt}} = \frac{P_{sea}}{P_{alt}}$$

$P_{sea} = 101.3\ kPa,$

* Follows from Boyle's law, i.e. PV = const.

$$dP = -\rho_{air} g dy \quad \rightarrow \quad \frac{dP}{dy} = -\rho_{air} g, \qquad \rho_{air} = \rho_{air}^{sea} \frac{P}{P_{sea}}$$

$$\frac{dP}{dy} = -\rho_{air}^{sea} \frac{P}{P_{sea}} g \quad \rightarrow \quad P(y) = P_{sea} e^{-\frac{\rho_{air}^{sea}}{P_{sea}} g y}, \qquad \rho_{air}^{sea} = 1.225 \frac{kg}{m^3} \quad \rightarrow \quad P_{alt}(2000m) \sim 79.8\ kPa \qquad \rightarrow \quad \frac{n_{sea}}{n_{alt}} \sim 1.27$$

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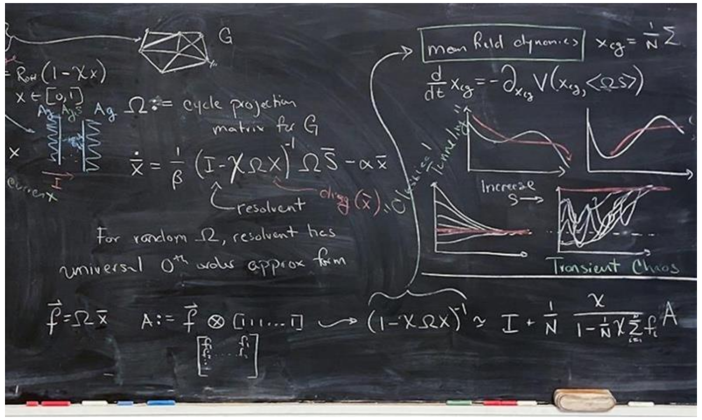
Microscopic Theory of Gases

A very introductory dive in Statistical Mechanics

Statistical Mechanics: is a branch of physics that combines the principles of statistics and classical or quantum mechanics to explain and predict the behavior of systems composed of a large number of particles, such as atoms or molecules. It provides a bridge between the microscopic properties of individual particles (such as their positions, velocities, and energies) and the macroscopic properties of the system as a whole (such as temperature, pressure, and entropy).

Kinetic theory of gases

Black Board Time

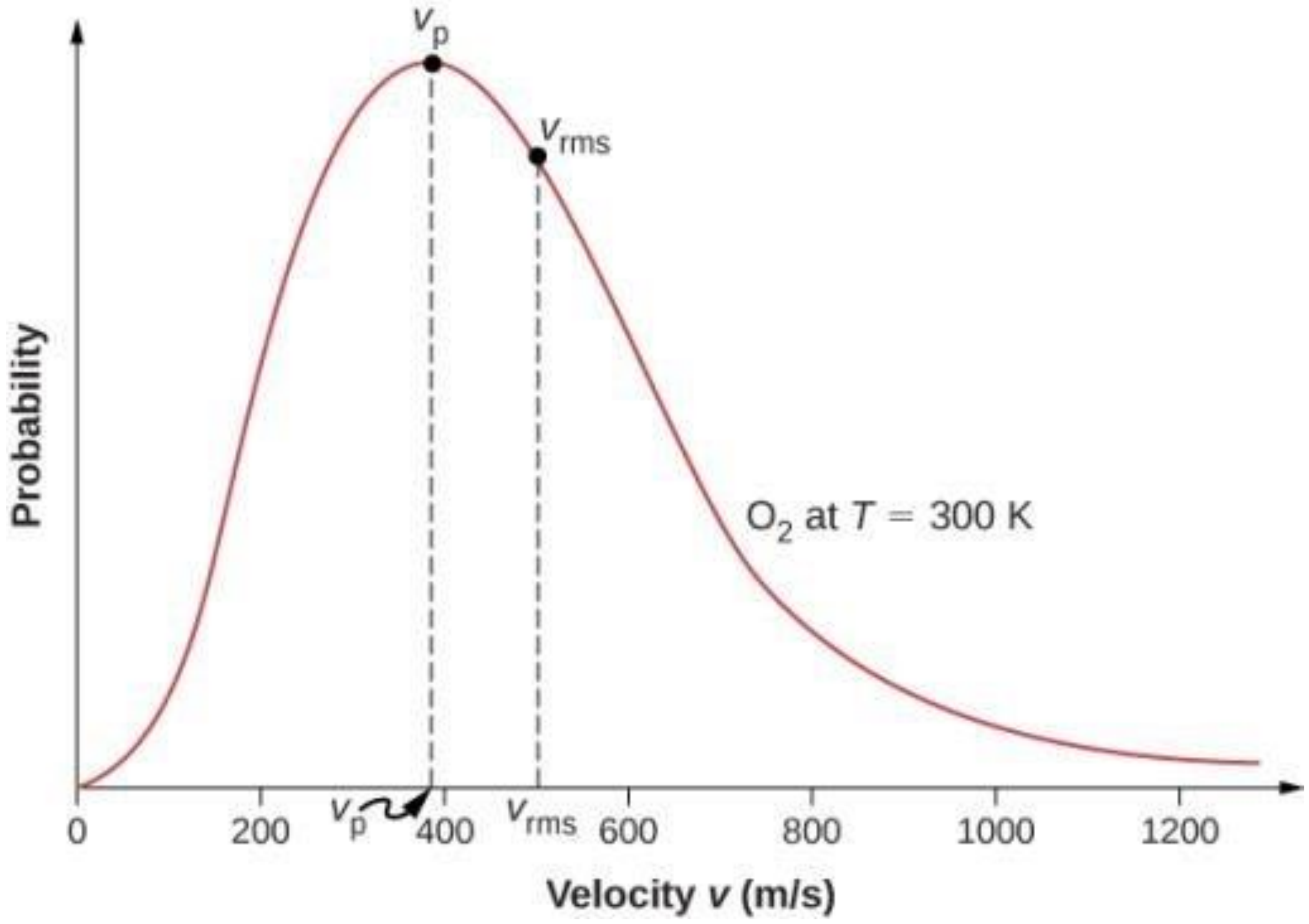


Let’s derive the equation
of state for ideal gases

Maxwell-Boltzmann Distribution of Molecular speeds

… with a bit more hardcore statistical mechanics one can derive the velocity distribution of an ideal gas

$$f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T} \right)^{3/2} v^2 e^{\left(-mv^2/\left(2k_B T\right)\right)}$$



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Heat Capacity and Equipartition of Energy

Heat Capacity for Ideal Gases

Remember the definition of heat capacity: $Q = mc\Delta T$ $c = (1/m)Q/\Delta T$

It is more convenient to work with moles:

$$C_V = \frac{1}{n}\frac{Q}{\Delta T}, \text{ with } V \text{ held constant.}$$

Can we calculate it explicitly?

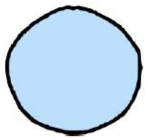
$E_{int} = \frac{3nRT}{2} \rightarrow \Delta E_{int} = \frac{3nR\Delta T}{2}$ and $\Delta E_{int} = Q \rightarrow Q = \frac{3nR\Delta T}{2} \rightarrow C_V = \frac{3R}{2}$

@ constant volume

Equipartition of Energy

Each degree of freedom gets: $\frac{1}{2} K_B T$

Helium (He):

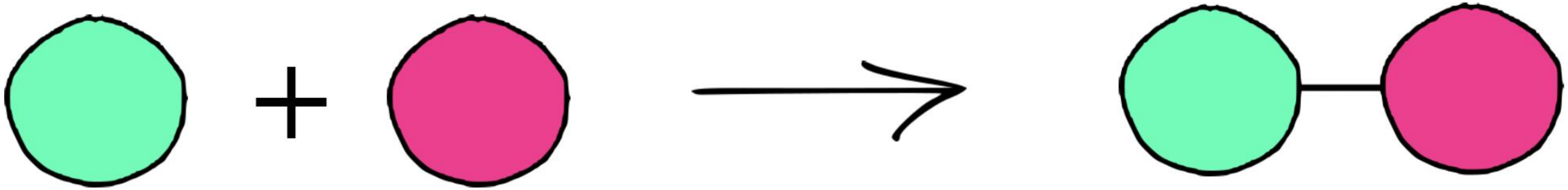


- "Monatomic"
- Symmetry about 3 axes
- 3 degrees of freedom
- $U = N\frac{3}{2}k_B T$

Question

Two monatomic gases react to form a diatomic gas. Suppose the reaction is performed with 1 mole of each gas, and in a thermally isolated chamber.

What happens to the temperature of the system?



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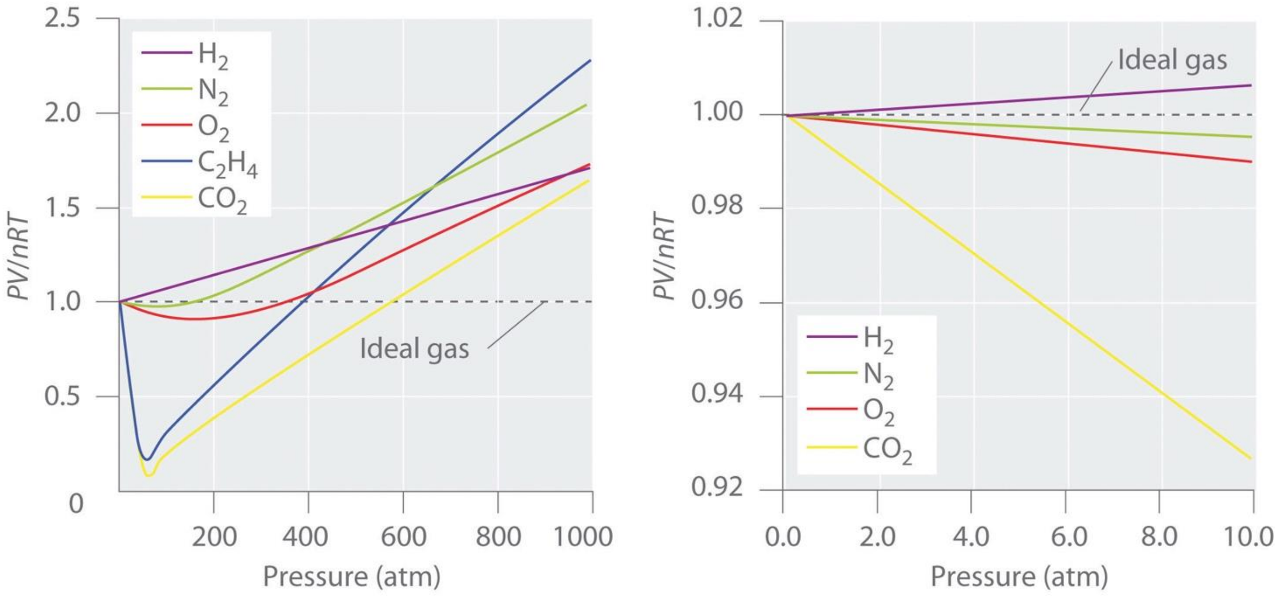
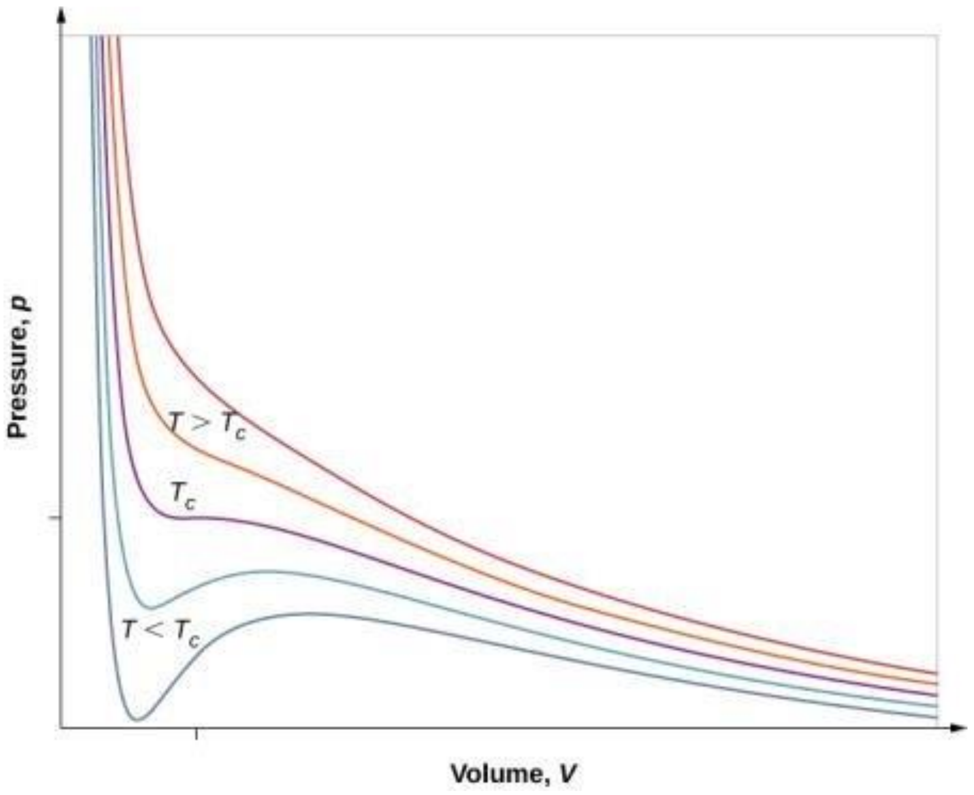
Real Gases

When are gases no longer ideal?

Discuss with your neighbour…



(a) *PV/nRT* at high pressures **(b) *PV/nRT* at low pressures**

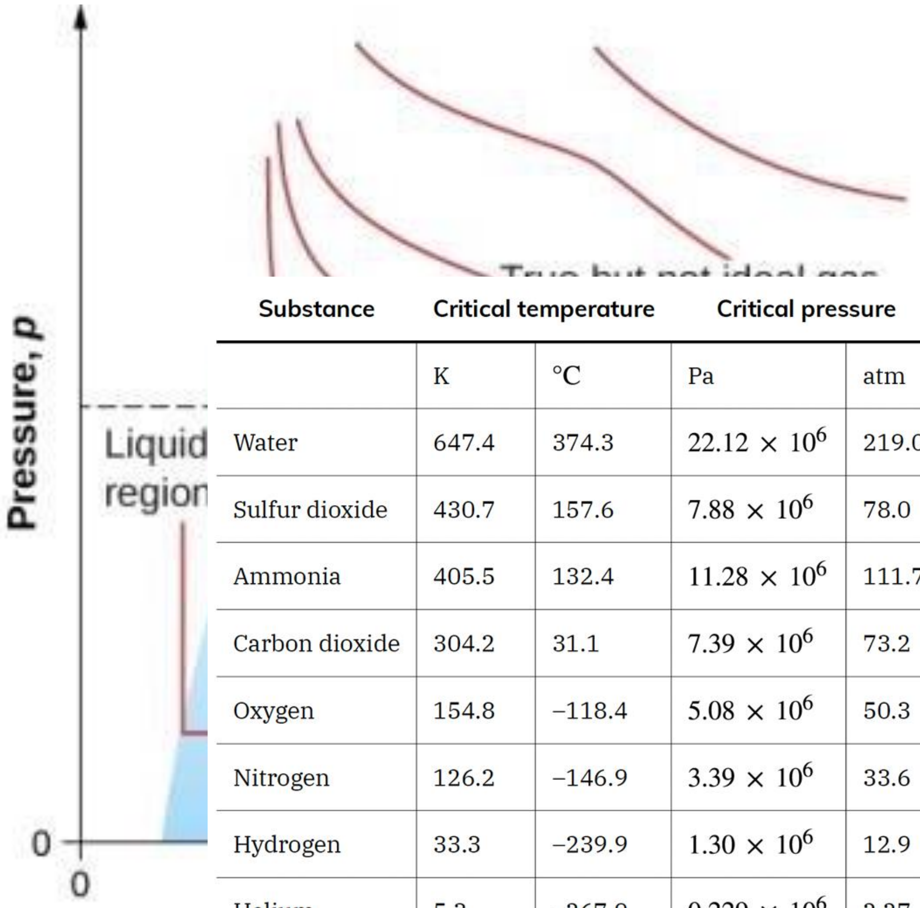


Van der Waals Equation of State

$$\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$$

Effective pressure renormalization

Effective volume renormalization



Substance	Critical temperature		Critical pressure	
	K	°C	Pa	atm
Water	647.4	374.3	22.12×10^6	219.0
Sulfur dioxide	430.7	157.6	7.88×10^6	78.0
Ammonia	405.5	132.4	11.28×10^6	111.7
Carbon dioxide	304.2	31.1	7.39×10^6	73.2
Oxygen	154.8	−118.4	5.08×10^6	50.3
Nitrogen	126.2	−146.9	3.39×10^6	33.6
Hydrogen	33.3	−239.9	1.30×10^6	12.9
Helium	5.3	−267.9	0.229×10^6	2.27

Table 2.1 Critical Temperatures and Pressures for Various Substances