Formelsamling - Formler for fysik

Fysik (Danmarks Tekniske Universitet)

Alle Key Equations Volume 1 & 2

Volume 1

Chapter 1 Units and Measurement

Percent uncertainty

$$\text{Percent uncertainty} = \frac{\delta A}{A}$$

Chapter 3 Motion Along a Straight Line

Displacement

$$\Delta x = x_\mathrm{f} - x_\mathrm{i}$$

Total displacement

$$\Delta x_\text{Total} = \sum \Delta x_\mathrm{i}$$

Avegrage velocity (for constant acceleration)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Instantaneous velocity

$$v(t) = \frac{dx(t)}{dt}$$

Average speed

$$\text{Average speed} = \bar{s} \; = \frac{\text{Total distance}}{\text{Elapsed time}}$$

Instantenous speed

$$\text{Instantenous speed} = |v(t)|$$

Average acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$$

Instantaneous acceleration

$$a(t) = \frac{dv(t)}{dt}$$

Position from average velocity

$$x = x_0 + \bar{v}t$$

Average velocity

$$\bar{v} = \frac{v_0 + v}{2}$$

Velocity from acceleration

$$v = v_0 + at \text{ (constant a)}$$

Position from velocity and acceleration

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant a)}$$

Velocity from distance

$$v^2 = v_0^2 + 2a(x - x_0)\text{(constant a)}$$

Velocity of free fall

$$v = v_0 - gt\text{(positive upward)}$$

Height of free fall

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

Velocity of free fall from height

$$v^2 = v_0^2 - 2g(y - y_0)$$

Velocity from acceleration

$$v(t) = \int a(t)dt + C_1$$

Position from velocity

$$x(t) = \int v(t)dt + C_2$$

Chapter 4 Motion in Two and Three Dimensions

Position vector

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

Displacement vector

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

Velocity vector

$$\vec{v}(t) = \lim_{x\to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt}$$

Velocity in terms of components

$$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}$$

Velocity components

$$v_x(t) = \frac{dx(t)}{dt}, v_y(t) = \frac{dy(t)}{dt}, v_z(t) = \frac{dz(t)}{dt}$$

Average velocity

$$\vec{v}_{\text{avg}} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

Instantaneous acceleration

$$\vec{a}(t) = \lim_{t\to 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt}$$

Instantaneous acceleration, component form

$$\vec{a}(t) = \frac{dv_x(t)}{dt}\hat{i} + \frac{dv_y(t)}{dt}\hat{j} + \frac{dv_z(t)}{dt}\hat{k}$$

Instantaneous acceleration as second derivatives of position

$$\vec{a}(t) = \frac{d^2v_x(t)}{dt^2}\hat{i} + \frac{d^2v_y(t)}{dt^2}\hat{j} + \frac{d^2v_z(t)}{dt^2}\hat{k}$$

Time of flight

$$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}$$

Trajectory

$$ y= (\text{tan}\theta _0)x - \left[\frac{g}{2(v_0\text{cos}\theta _0)^2}\right] x^2

Range

$$R = \frac{v_0^2 \text{sin} 2\theta_0}{g}$$

Centripetal acceleration

$$a_C = \frac{v^2}{r}$$

Position vector, unifrom circular motion

$$ \vec r(t)= A\text{cos} , \omega t, \hat i + A\text{sin} , \omega t , \hat j

Velocity vector, uniform circular motion

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = -A\omega \sin \omega t \hat{i} + A\omega \cos \omega t \hat{j}$$

Acceleration vector, uniform circular motion

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = -A\omega^2 \cos \omega t \hat{i} + A\omega^2 \sin \omega t \hat{j}$$

Tangential acceleraion

$$a_{\mathrm{T}} = \frac{d|\vec{v}|}{dt}$$

Total acceleration

$$\vec{a} = \vec{a}_{\mathrm{C}} + \vec{a}_{\mathrm{T}}$$

Position vector in frame S is the position vector in frame S' plus the vector from the origin S to the origin of S'

$$\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$$

Relative velocity equation connecting two refrence frames

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$$

Relative velocity equation connecting more than two reference frames

$$\vec{v}_{PC} = \vec{v}_{PA} + \vec{v}_{AB} + \vec{v}_{BC}$$

Relative acceleration equation

$$\vec{a}_{PS} = \vec{a}_{PS'} + \vec{a}_{S'S}$$

Chapter 5 Newton's Laws of Motion

Net external force

$$\vec{F}_{\text{net}} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$

Newton's first law

$$\vec{v} = \text{constant when } \vec{F}_{\text{net}} = \vec{0}N$$

Newton's second law, vector form

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

Newton's second law, scalar form

$$\vec{F}_{\text{net}} = ma$$

Newton's second law, component form

$$\sum \vec{F}_x = m\vec{a}_x, \sum \vec{F}_y = m\vec{a}_y, \text{and} \sum \vec{F}_z = m\vec{a}_z$$

Newton's second law, momentum form

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

Definiton of weight, vector form

$$\vec{w} = m\vec{g}$$

Definiton of weight, scalar form

$$w = mg$$

Newton's third law

$$\vec{F}_{\text{AB}} = -\vec{F}_{\text{BA}}$$

Normal force on an object resting on a horizontial surface, vector form

$$\vec{N} = -m\vec{g}$$

Normal force on an object resting on a horizontial surface, scalar form

$$N = mg$$

Normal force on an object resting on an inclined plane, scalar form

$$N = mg\cos\theta$$

Tension in a cable supporting an object of mass m at rest, scalar form

$$T = w = mg$$

Chapter 6 Apllication of Newton's Laws

Magnitude of static friction

$$f_s \leq \mu_s N$$

Magnitude of kinetic friction

$$f_k = \mu_k N$$

Centripetal force

$$F_c = m^{\frac{v^2}{r}},\ or\ F_c = mr\omega^2$$

Ideal angle of a banked curve

$$\tan\ \theta = \frac{v^2}{rg}$$

Drag force

$$F_D = \frac{1}{2}C\rho Av^2$$

Stokes' law

$$F_s = 6\pi r\eta v$$

Chapter 7 Work and Kinetic Energy

Work done by a force over an infinitesimal displacement

$$dW = \vec{F} \cdot d\vec{r} = |\vec{F}||d\vec{r}|cos\theta$$

Work done by a force acting along a path a path from A to B

$$W_{AB} = \int_{\text{pathAB}} \vec{F} \cdot d\vec{r}$$

Work done by a constant force of kinetic friction

$$W_{fr} = -f_k|l_{AB}|$$

Work done going from A to B by Earth's gravity, near it's surface

$$W_{grav,AB} = -mg(y_B - y_A)$$

Work done going from A to B by one-dimensional spring force

$$W_{spring,AB} = -(\frac{1}{2}k)\ (x_B^2 - x_A^2)$$

Kinetic energy of a non-relativistic particle

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Work-energy theorem

$$W_{net} = K_B - K_A$$

Power as a rate of doing work

$$P = \frac{dW}{dt}$$

Power as the dot product of force and velocity

$$P = \vec{F} \cdot \vec{v}$$

Chapter 8 Potential Energy and Conservation of Energy

Difference in potential engergy

$$\Delta U_{AB} = U_B - U_A = -W_{AB}$$

Potential energy with respect to zero of potential at $\vec{r}_0$

$$\Delta U = U(\vec{r}) - U(\vec{r}_0)$$

Gravitational potential energy near Earth's surface

$$U(y) = mgy + const.$$

Potential energy for an ideal spring

$$U(x) = \frac{1}{2}kx^2 + const.$$

Work done by conservative force over a closed path

$$W_{\text{closed path}} = \int \vec{F}_{\text{cons}} \cdot d\vec{r} = 0$$

Condition for conservative force in two dimensions

$$\left(\frac{dF_x}{dy}\right) = \left(\frac{dF_y}{dx}\right)$$

Conservative force is the negative energy of potential energy

$$F_l = -\frac{dU}{dl}$$

Conservation of energy with no non-conservative forces

$$0 = W_{nc,AB} = \Delta(K+U)_{AB} = \Delta E_{AB}$$

Chapter 9 Linear momentum and Collisions

Definition of momentum

$$\vec{p} = m\vec{v}$$

Impulse

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt,\ or\ \vec{J} = \vec{F}_{ave}\Delta t$$

Impusle-momentum theorem

$$\vec{J} = \Delta\vec{p}$$

Average force from momentum

$$\vec F = \frac{\Delta \vec p}{dt}

Instantaneous force from momentum (Newton's second law)

$$\vec{F}(t) = \frac{d\vec{p}}{dt}$$

Conservation of momentum

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \text{ or } \vec{p}_1 + \vec{p}_2 = \text{constant}$$

Generalized conservation of momentum

$$\sum_{j=1}^{N} \vec{p}_j = \text{constant}$$

Conservation of momentum in two dimensions

$$p_{f,x} = p_{1,i,x} + p_{2,i,x}$$

$$p_{f,y} = p_{1,i,y} + p_{2,i,y}$$

External forces

$$\vec{F}_{ext} = \sum_{j=1}^{N} \frac{d\vec{p}_j}{dt}$$

Newton's second law for an extended object

$$\vec{F} = \frac{d\vec{p}_{CM}}{dt}$$

Acceleration of the center of mass

$$\vec{a}_{CM} = \frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{r}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{a}_j$$

Position of the center of mass for a system of particles

$$\vec{r}_{CM} \equiv \frac{1}{M}\sum_{j=1}^{N} m_j \vec{r}_j$$

Velocity of the center of mass

$$\vec{v}_{CM} = \frac{d}{dt}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{r}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{v}_j$$

Position of the center of mass of a continuous object

$$\vec{r}_{CM} \equiv \frac{1}{M}\int \vec{r}\, dm$$

Rocket equation

$$\Delta v = u \ \ln\left(\frac{m_1}{m}\right)$$

Chapter 10 Fixed-axis rotation

Angular Position

$$\theta = \frac{s}{r}$$

Angular Velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Tangential Speed

$$v_t = r\omega$$

Angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Tangential Acceleration

$$a_t = r\alpha$$

Average Angular Velocity

$$\overline{\omega} = \frac{\omega_0 + \omega_f}{2}$$

Angular Displacement

$$\theta_f = \theta_0 + \overline{\omega}_t$$

Angular Velocity with Constant Angular Acceleration

$$\omega_f = \omega_0 + \alpha t$$

Relationship Between Angular Velocity and Displacement

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Change in angular velocity

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$

Total Acceleration

$$\vec{a} = \vec{a}_c + \vec{a}_t$$

Rotational Kinetic Energy

$$K = \frac{1}{2}\left(\sum_j m_j r_j^2\right)\omega^2$$

Moment of Inertia

$$I = \sum_j m_j r_j^2$$

Rotational kinetic energy in terms of of the moment of inertia of a rigid body

$$K = \frac{1}{2} I \omega^2$$

Moment of inertia of a continous object

$$I = \int r^2 \ dm$$

Parallel-Axis Theorem

$$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$$

Moment of inertia of a compound object

$$I_{total} = \sum_i I_i$$

Torque vector

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Magnitude of torque

$$|\vec{\tau}| = r \perp F$$

Total torque

$$\vec \tau _{net} = \sum_i |\vec \tau _i|

Newton's second law for rotation

$$\sum_i \tau_i = I\alpha$$

Incremental work done by a torque

$$dW = \left(\sum_i \tau_i \right) d\theta$$

Work-energy theorem

$$W_{AB} = K_B - K_A$$

Rotational work done by net force

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta$$

Rotational power

$$P = \tau\omega$$

Chapter 11 Angular Momentum

Velocity of center of mass of rolling object

$$v_{CM} = R\omega$$

Acceleration of center of mass of rolling object

$$a_{CM} = R\alpha$$

Displacement of center of mass of rolling object

$$d_{CM} = R\theta$$

Acceleration of an object rolling without slipping

$$a_{CM} = \frac{mg\,sin\,\theta}{m + (I_{CM}/r^2)}$$

Angular momentum

$$\vec{l} = \vec{r} \times \vec{p}$$

Derivative of angular momentum equals torque

$$\frac{d\vec{l}}{dt} = \sum \vec{\tau}$$

Angular momentum of a system of particles

$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N$$

For a system of particles, derivative of angular momentum equals torque

$$\frac{d\vec{L}}{dt} = \sum \vec{\tau}$$

Angular momentum of a rotating rigid body

$$L = I\omega$$

Conservation of angular momentum

$$\frac{d\vec{L}}{dt} = 0$$

Conservation of angular momentum

$$\vec{L} = \vec{I}_1 + \vec{I}_2 + \cdots + \vec{I}_N = \text{constant}$$

Precessional angular velocity

$$\omega_P = \frac{rMg}{I\omega}$$

Volume 2

Chapter 1 Temperature and Heat

Linear thermal expansion

$$\Delta L = \alpha L \Delta T$$

Thremal expansion in two dimensions

$$\Delta A = 2\alpha A \Delta T$$

Thermal expansion in three dimesnions

$$\Delta V = \beta V \Delta T$$

Heat transfer

$$Q = mc\Delta T$$

Transfer of heat in calorimeter

$$Q_{cold} + Q_{hot} = 0$$

Heat due to phase change (melting and freezing)

$$ Q = mL_{\text{f}}

Heat due to phase change (evaporation and condensation)

$$Q = mL_{\text{v}}$$

Rate of conductive heat transfer

$$ P = \frac{kA \left(T_h-Tc\right)}{d}

Net rate of heat transfer by radiation

$$P_{\text{net}} = \sigma e A \left(T_2^4 - T_1^4\right)$$

Chapter 2 The Kinetic Theory of Gasses

Ideal gas law in terms of molecules

$$pV = Nk_BT$$

Ideal gas law ratios if the amount of gas is constant

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

Ideal gas law in terms of moles

$$pV = nRT$$

Van der Waals equation

$$\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$$

Pressure, volume eand molecular speed

$$pV = \frac{1}{3}Nm\vec{v}^2$$

Root-mean-square speed

$$v_{\text{rms}}\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{m}}$$

Mean free path

$$\lambda = \frac{V}{4\sqrt{2}\pi r^2 N} = \frac{k_BT}{4\sqrt{2}\pi r^2 p}$$

Mean free time

$$\tau = \frac{k_BT}{4\sqrt{2}\pi r^2 p v_{\text{rms}}}$$

The following two equations apply only to a monatomic ideal gas:

Average kinetic energy of a molecule

$$\overline{K} = \frac{3}{2}k_BT$$

Internal energy

$$E_{\text{int}} = \frac{3}{2}Nk_BT$$

Heat in terms of molar heat capacity at constant volume

$$Q = nC_V\Delta T$$

Molar heat capacity at constant volume for an ideal gas with d degrees of freedom

$$C_V = \frac{d}{2}R$$

Maxwell-Boltzmann speed distribution

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2}v^2e^{-mv^2/2k_BT}$$

Average velocity of a molecule

$$ \overline v = \sqrt {\frac{8}{\pi} , \frac{k_BT}{m}} = \sqrt {\frac{8}{\pi} , \frac{RT}{M}}

Peak velocity of a molecule

$$ v_p = \sqrt {\frac{k_BT}{m}} = \sqrt {\frac{2RT}{M}}

Chapter 3 The First Law of Thermodynamics

Equation of state for a closed system

$$f(p, V, T) = 0$$

Net work for a finite change in volume

$$W = \int_{V_1}^{V_2} pdV$$

Internal energy of a system (average total energy)

$$E_{\text{int}} = \sum_i(\overline{K}_i + \overline{U}_i)$$

Internal energy of a monatomic ideal gas

$$E_{\text{int}} = nN_A\left(\frac{3}{2}k_BT\right) = \frac{3}{2}nRT$$

First law of thermodynamics

$$\Delta E_{\mathrm{int}} = Q - W$$

Molar heat capacity at constant pressure

$$C_p = C_V + R$$

Ratio of molar heat capacities

$$\gamma = C_p / C_V$$

Condition for an ideal gas in a quasi-static adiabatic process

$$pV^{\gamma} = \text{constant}$$

Chapter 4 The Second Law of Thermodynamics

Result of energy conservation

$$W = Q_{\mathrm{h}} - Q_{\mathrm{c}}$$

Efficiency of a heat engine

$$e = \frac{W}{Q_{\mathrm{h}}} = 1 - \frac{Q_{\mathrm{c}}}{Q_{\mathrm{h}}}$$

Coefficient of performance of a refrigerator

$$K_{\mathrm{R}} = \frac{Q_{\mathrm{c}}}{W} = \frac{Q_{\mathrm{c}}}{Q_{\mathrm{h}} - Q_{\mathrm{c}}}$$

Coefficient of performance of heat pump

$$K_{\mathrm{P}} = \frac{Q_{\mathrm{h}}}{W} = \frac{Q_{\mathrm{h}}}{Q_{\mathrm{h}} - Q_{\mathrm{c}}}$$

Resulting effeciency of Carnot cycle

$$e = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}$$

Performance coefficient of a reversible refrigerator

$$K_{\mathrm{R}} = \frac{T_{\mathrm{c}}}{T_{\mathrm{h}} - T_{\mathrm{c}}}$$

Performance coefficient of a reversible heat pump

$$K_{\mathrm{P}} = \frac{T_{\mathrm{h}}}{T_{\mathrm{h}} - T_{\mathrm{c}}}$$

Entropy of a system undergoing a reversible process at a constant temperature

$$\Delta S = \frac{Q}{T}$$

Change of entropy of a system under reversible process

$$\Delta S = S_B - S_A = \int_A^B dQ/T$$

Entropy of a system undergoing any complete reversible process

$$\oint dS = \oint \frac{dQ}{T} = 0$$

Change of entropy of a closed system under an irreversible process

$$\Delta S \geq 0$$

Change in entropy of the system along an isotherm

$$\lim_{T \to 0} (\Delta S)_T = 0$$

Chapter 5 Electric Charges and Fields

Coulumb's law

$$\vec{F}_{12}(r) = \frac{1}{4\pi\epsilon_0} \frac{q1q2}{r_{12}^2} \hat{r}_{12}$$

Superposition of electric forces

$$\vec{F}(r) = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i$$

Electric force due to an electric field

$$\vec{F} = Q\vec{E}$$

Electric field at point P

$$\vec{E}(P) \equiv \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i$$

Field of an infinite wire

$$\vec{E}(z) = \equiv \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} \hat{k}$$

Field of an infinte plane

$$ \vec E = \frac {\sigma}{2\epsilon _0} \hat k

Dipole moment

$$\vec p \equiv q\vec d$$

Torque on dipole in external E- field

$$\vec \tau = \vec p \times \vec E$$

Chapter 6 Gauss's Law

Definiton of electric flux, for uniform electric field

$$\Phi = \vec E \cdot \vec A \implies EA\cos\theta$$

Electric flux through an open surface

$$\Phi = \int_S \vec E \cdot \hat n d A = \int_S \vec E \cdot d \vec A

Electric flux through a closed surface

$$\Phi = \oint_S \vec E \cdot \hat n dA = \oint_S \vec E \cdot d\vec A$$

Gauss's law

$$\Phi = \oint_S \vec E \cdot \hat n dA = \frac{q_{\text{enc}}}{\epsilon_0}$$

Gauss's Law for systems with symmetry

$$\Phi = \oint_S \vec E \cdot \hat n dA = E\oint_S dA = \frac{q_{\text{enc}}}{\epsilon_0}$$

The magnitude of the electric field just outside the surface of a conductor

$$E = \frac{\sigma}{\epsilon_0}$$

Chapter 7 Electric potentiel

Potentential energy of a two-charge system

$$U(r) = k\frac{qQ}{r}$$

Work done to assemble a system of charges

$$W_{12\cdots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}} \text{for } i \neq j$$

Potential difference

$$\Delta V = \frac{\Delta U}{q} \text{or } \Delta U = q\Delta V$$

Electric potential

$$V = \frac{U}{q} = -\int_{R}^{P}\vec{E}\cdot d\vec{l}$$

Potential difference between two points

$$\Delta V_{BA} = V_B - V_A = -\int_{A}^{B}\vec{E}\cdot d\vec{l}$$

Electric potential of a point charge

$$V = \frac{kq}{r}$$

Electric potential of a system of point charges

$$V_P = k\sum_{l}^{N}\frac{q_i}{r_i}$$

Electric dipole moment

$$\vec{p} = q\vec{d}$$

Electric potential due to a dipole

$$V_P = k\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Electric potential of a continous charge distrobution

$$V_P = k\int\frac{dq}{r}$$

Electric field components

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

Del operator in Cartesian coordinates

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial_x}, \hat{j}\frac{\partial}{\partial_y}, \hat{k}\frac{\partial}{\partial_z}$$

Electric field as gradient of potentiel

$$\vec E =- \vec \nabla V

Del operator in cylindrical coordinates

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial_r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial_\varphi} + \hat{z}\frac{\partial}{\partial_z}$$

Del operator in spherical coordiantes

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial_r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial_\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial_\varphi}$$

Chapter 8 Capacitance

Capacitance

$$C = \frac{Q}{V}$$

Capacitance of a parallel-plate capacitor

$$C = \epsilon_0 \frac{A}{d}$$

Capacitance of a vacuum spherical capacitor

$$C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

Capacitance of a vacuum cylindrical capacitor

$$C = \frac{2\pi\epsilon_0 l}{\ln(R_2/R_1)}$$

Capacitance of a series combination

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

Capacitance of a parallel combination

$$C_P = C_1 + C_2 + C_3 + \cdots$$

Energy density

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Energy stored in a capacitor

$$U_C = \frac{1}{2}V^2C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Capacitance of a capacitor with dielectric

$$C = kC_0$$

Energy stored in an isolated capacitor with dielectric

$$U = \frac{1}{k}U_0$$

Dielectric constant

$$k = \frac{E_0}{E}$$

Induced electrical field in a dielectric

$$\vec{E}_{\mathrm{i}} = \left(\frac{1}{k} - 1\right)\vec{E}_0$$

Chapter 9 Current and Resistance

Average electrical current

$$I_{\mathrm{ave}} = \frac{\Delta Q}{\Delta t}$$

Definition of an ampere

$$1\ \mathrm{A} = 1\ \mathrm{C/s}$$

Electrical current

$$I = \frac{dQ}{dt}$$

Drift velocity

$$v_d = \frac{I}{nqA}$$

Current density

$$I = \iint_{\text{area}} \vec{J} \cdot d\vec{A}$$

Resistivity

$$\rho = \frac{E}{J}$$

Common expression of Ohm's law

$$V = IR$$

Resistivity as a function of temperature

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

Definition of resistance

$$R \equiv \frac{V}{I}$$

Resistance of a cylinder material

$$R = \rho\frac{L}{A}$$

Temperature dependence of resistance

$$R = R_0(1 + \alpha\Delta T)$$

Electric power

$$P = IV$$

Power dissipated by a resistor

$$P = I^2R = \frac{V^2}{R}$$

Chapter 10 Direct-Current Circuits

Terminal voltage of a single voltage source

$$V_{\text{terminal}} = \epsilon - Ir_{\text{eq}}$$

Equivalent resistance of a series circuit

$$R_{\text{eq}} = R_1 + R_2 + R_3 \cdots R_{N-1} + R_N = \sum_{i=1}^{N} R_i$$

Equivalent resistance of a parallel circuit

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}$$

Junction rule

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Loop rule

$$\sum V = 0$$

Terminal voltage of N voltage sources in series

$$V_{\text{terminal}} = \sum_{i=1}^{N} \epsilon_i - I \sum_{i=1}^{N} r_i = \sum_{i=1}^{N} \epsilon_i - I r_{\text{eq}}$$

Terminal voltage of N voltage sources in parallel

$$V_{\text{terminal}} = \epsilon - I \sum_{i=1}^{N} \left(\frac{1}{r_i}\right)^{-1} = \epsilon - I r_{\text{eq}}$$

Charge on a charging capacitor

$$q(t) = C\epsilon \left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$$

Time constant

$$\tau = RC$$

Current during charging of a capacitor

$$I = \frac{\epsilon}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

Charge on a discharging capacitor

$$q(t) = Qe^{-\frac{t}{\tau}}$$

Current during discharging of a capacitor

$$I(t) = -\frac{Q}{RC} e^{-\frac{t}{\tau}}$$

Chapter 11 Magnetic Forces and Fields

Force on a charge in a magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$

Magnitude of a magnetic force

$$F = qvB\sin\theta$$

Radius of a particle's path in a magnetic field

$$r = \frac{mv}{qB}$$

Period of a particle's motion in a magnetic field

$$T = \frac{2\pi m}{qB}$$

Force on a current-carrying wire in a uniform magnetic field

$$\vec{F} = I\vec{l} \times \vec{B}$$

Magnetic dipole moment

$$\vec{\mu} = NIA\hat{n}$$

Torque on a current loop

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Energy of a magnetic dipole

$$U = -\vec{\mu} \cdot \vec{B}$$

Drift velocity in crossed electric and magnetic fields

$$v_d = \frac{E}{B}$$

Hall potential

$$V = \frac{IBl}{neA}$$

Hall potential in terms of drift velocity

$$V = Blv_d$$

Charge-to-mass ratio in a mass spectrometer

$$\frac{q}{m} = \frac{E}{BB_0R}$$

Maximum speed of a particle in a cyclotron

$$v_{\max} = \frac{qBR}{m}$$

Chapter 12 Sources of Magnetic Fields

Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \mathrm{T} \cdot \mathrm{m/A}$$

Contribution to magnetic field from a current element

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl\, \sin\theta}{r^2}$$

Biot-Savart law

$$\vec{B} = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{Id\vec{l} \times \hat{r}}{r^2}$$

Magnetic field due to a long straight wire

$$B = \frac{\mu_0 I}{2\pi R}$$

Force between two parallel currents

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi R}$$

Magnetic field of a current loop

$$B = \frac{\mu_0 I}{2R} (\text{at center of loop})$$

Ampère's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Magnetic field strength inside a solenoid

$$B = \mu_0 n I$$

Magnetic field strength inside a toroid

$$B = \frac{\mu_0 NI}{2\pi R}$$

Magnetic permeability

$$\mu = (1 + x)\mu_0$$

Magnetic field of solenoid filled with paramagnetic material

$$B = \mu nI$$

Chapter 13 Electromagnetic Induction

Magnetic flux

$$\Phi_{\mathrm{m}} = \int_S \vec{B} \cdot \hat{n} dA$$

Faraday's law

$$\epsilon = -N \frac{d\Phi_{\mathrm{m}}}{dt}$$

Motionally induced emf

$$\epsilon = Blv$$

Motional emf around a circuit

$$\epsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_{\mathrm{m}}}{dt}$$

Emf produced by an electric generator

$$\epsilon = N\,B\,A\,\omega \sin(\omega t)$$

Chapter 14 Inductance

Mutual inductance by flux

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_1 \Phi_{12}}{I_2}$$

Mutual inductance in circuits

$$\epsilon_1 = -M \frac{dI_2}{dt}$$

Self-inductance in terms of magnetic flux

$$N\Phi_{\mathrm{m}} = LI$$

Self-inductance in terms of emf

$$\epsilon = -L\frac{dI}{dt}$$

Self-inductancce of a solenoid

$$L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l}$$

Self-inductance of a toroid

$$L_{\text{toroid}} = \frac{\mu_0 N^2 h}{2\pi}\ln\frac{R_2}{R_1}$$

Energy stored in an inductor

$$U = \frac{1}{2}LI^2$$

Current as a function of time for a RL circuit

$$I(t) = \frac{\epsilon}{R}\left(1 - e^{-\frac{t}{\tau}L}\right)$$

Time constant for a RL circuit

$$\tau_L = L/R$$

Charge oscillation in LC circuits

$$q(t) = q_0\cos(\omega t + \phi)$$

Angular frequency in LC circuits

$$\omega = \sqrt{\frac{1}{LC}}$$

Current oscillation in LC circuits

$$i(t) = -\omega q_0 \sin(\omega t + \phi)$$

Charge as a function of time in RLC circuit

$$q(t) = q_0 e^{-Rt/2L}\cos(\omega' t + \phi)$$

Angular frequency in RLC circuit

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Chapter 15 Alternating-Current Circuits

AC voltage

$$v = V_0 \sin \omega t$$

AC current

$$i = I_0 \sin \omega t$$

capacitive reactance

$$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$$

rms voltage

$$V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}}$$

rms current

$$I_{\mathrm{rms}} = \frac{I_0}{\sqrt{2}}$$

inductance reactance

$$\frac{V_0}{I_0} = \omega_L = X_L$$

Phase angle of an RLC series circuit

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

AC version of Ohm's law

$$I_0 = \frac{V_0}{Z}$$

Impedance of an RLC series circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Average power associated with a circuit element

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi$$

Average power dissipated by a reistor

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R$$

Resonant angular frequency of a circuit

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Quality factor of a circuit

$$Q = \frac{\omega_0}{\Delta\omega}$$

Quality factor of a circuit in terms of the circuit parameters

$$Q = \frac{\omega_0 L}{R}$$

Transformer equation with voltage

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Transformer equation with current

$$I_S = \frac{N_P}{N_S} I_P$$

Chapter 16 Electromagnetic Waves

Displacement current

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Gauss's law for magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's law

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_m}{dt}$$

Ampère-Maxwell law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\phi_E}{dt}$$

Wave equation for plane EM wave

$$\frac{\partial^2 E_y}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2}$$

Speed of EM waves

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Ratio of E field to B field in electromagnetic wave

$$c = \frac{E}{B}$$

Energy flux (Poynting) vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Average intensity of an electromagnetic wave

$$I = S_{\text{avg}} = \frac{c\epsilon_0 E_0^2}{2} = \frac{cB_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0}$$

Radiation pressure

$$p = \begin{cases} I/c & \text{Perfect absorber} \\ 2I/c & \text{Perfect reflector} \end{cases}$$