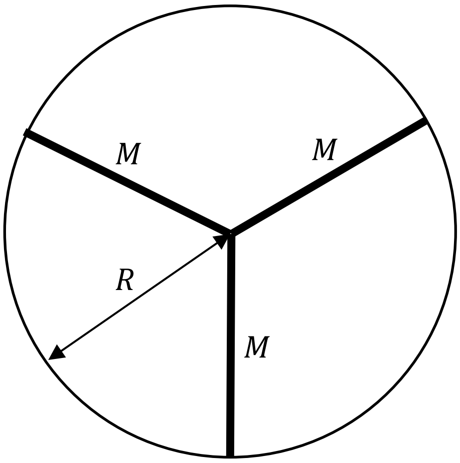
Problem 1.



Use the equivalent of Newton's second law for rotating bodies where the angular acceleration is calculated from the change in angular velocity and the time interval.

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = 168.75\ \text{N}\cdot\text{m}$$

Problem 2.

We express Newton's second law for rotations, where the only present torque is from the axe. The angular acceleration is calculated from the change in the angular speed (remember to convert to rad/s) in the unknown time interval.

$$\tau = I\alpha$$

$$\tau = FR = \mu NR$$

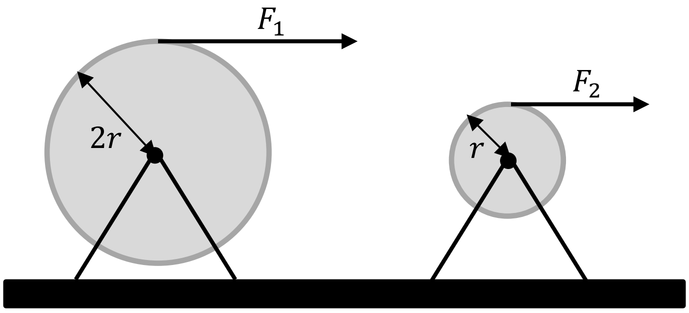
$$I\alpha = \frac{1}{2}MR^2\frac{\Delta\omega}{\Delta t}$$

$$\mu NR = \frac{1}{2}MR^2\frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{1}{2}MR^2\frac{\Delta\omega}{\mu NR} = 2.618\ \text{s}$$

The correct answer is B.

Problem 3.



We write up Newton's second law for rotation for the two cylinders. They have the same mass, but different radii, so they have different moments of inertia.

$$\frac{1}{2}m(2r)^2\alpha = F_1(2r)$$

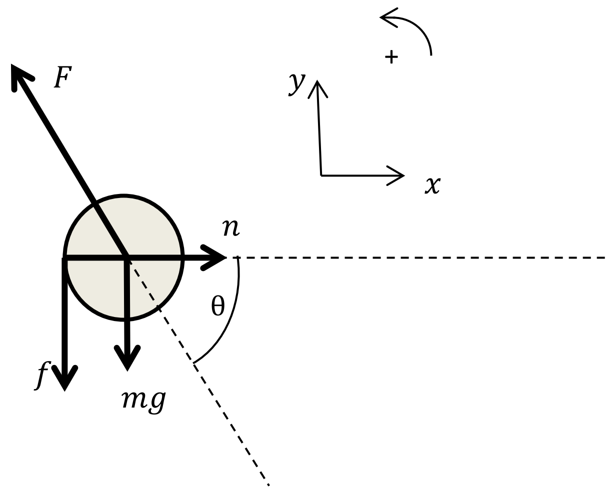
$$\frac{1}{2}m(r)^2\alpha = F_2 r$$

Comparing the two equations results in $F_1 = 2F_2$.

The correct answer is E.

Problem 4.

a) The force-body diagram is shown to the right. There are four forces: the known force F, gravitational force mg, normal force n and friction force f.



b)

Known: m, R, g, F, θ

Unknown: n, f, a, α

N2(x): $n - F\cos\theta = 0$

N2(y): $ma = F\sin\theta - f - mg$

RN2(cm): $I\alpha = \frac{1}{2}mR^2\alpha = fR$

GC: $a = R\alpha$

Insert GC into RN2 (cm) and eliminate the radius. Add this equation with N2(y).

$$\tfrac{3}{2}ma = F\sin\theta - mg$$

$$a = \tfrac{2}{3}\left(\frac{F\sin\theta}{m} - g\right)$$

The static friction obeys the inequality $f \le \mu_s n$, which requires $\mu_s \ge \dfrac{f}{n}$. We must express f and n using the known quantities. The normal force is found directly from N2(x) and the friction force from RN2(cm) by insertion of the expression for acceleration.

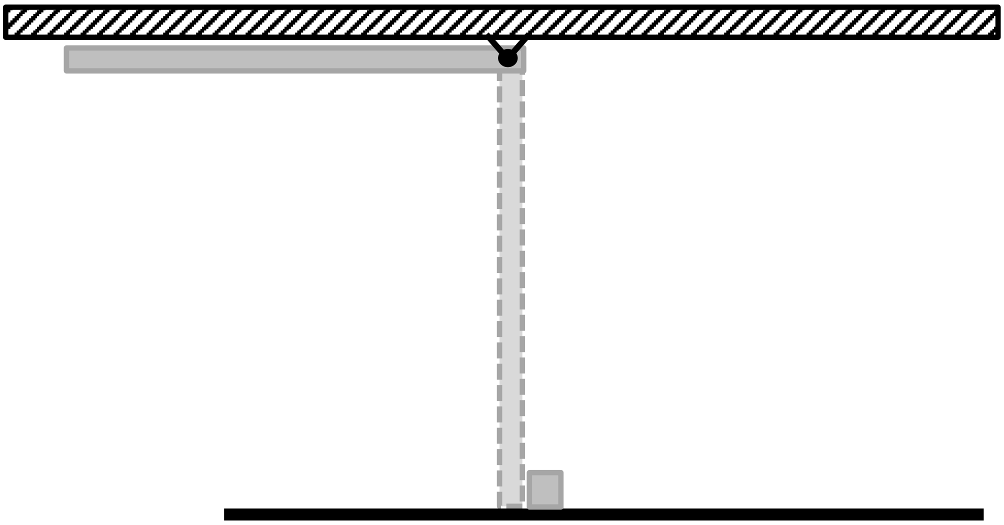
$$n = F\cos\theta$$

$$f = F\sin\theta - mg - ma = F\sin\theta - mg - \frac{2}{3}F\sin\theta - \frac{2}{3}mg = \frac{1}{3}(F\sin\theta - mg)$$

$$\mu_s \ge \frac{f}{n} = \frac{\frac{1}{3}(F\sin\theta - mg)}{F\cos\theta} = \frac{1}{3}\left(\tan\theta - \frac{mg}{F\cos\theta}\right)$$

$$\mu_s \ge \tfrac{1}{3}\left(\tan\theta - \frac{mg}{F\cos\theta}\right)$$

Problem 5.



a) We can use conservation of energy during the rod's rotation before the collision.

$$I_1 = \frac{1}{3} 6mL^2 = 2mL^2$$

$$0 + 6mgL = \frac{1}{2} 2mL^2 \omega_1^2 + 6mg\frac{L}{2}$$

Angular velocity just before the collision is $\omega_1 = \sqrt{\frac{3g}{L}}$.

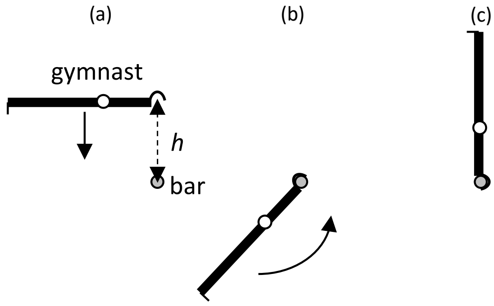
b) We can use angular momentum conservation. The contributing moment of inertia is the rod's, and after the collision it is the rod's plus the block's moment of inertia.

$$I_2 = I_1 + mL^2 = 3mL^2$$

$$I_1\omega_1 = I_2\omega_2$$

The angular velocity just after collision is $\omega_2 = \frac{2}{3}\sqrt{\frac{3g}{L}} = \frac{2}{\sqrt{3}}\sqrt{\frac{g}{L}}$

Problem 6.



a) We have conservation of mechanical energy from start until just before the gymnast reaches the bar. The zero point for the potential energy is set at the bar's height above the ground.

Energy consv.(a→b-): $U_a + K_a = U_{\text{b-}} + K_{\text{b-}} = mgh + 0 = 0 + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}.$

During the collision, there is conservation of angular momentum with respect to the bar; the collision is so short that the torque from gravity can be ignored, and since the contact force is applied from the bar, the arm is zero.

Ang. momentum consv.(b-→b+): $L_{\text{b-}} = L_{\text{b+}} \Rightarrow \frac{L}{2}mv = I\omega_{\text{b+}}$

$$\frac{L}{2}m\sqrt{2gh} = \frac{1}{3}mL^2\omega_{\text{b+}}$$

$$\omega_{\text{b+}} = \frac{3}{2L}\sqrt{2gh}$$

b) Like above except we now also have the motion after the collision, where we have conservation of mechanical energy.

Energy consv.(a→b-): $U_a + K_a = U_{\text{b-}} + K_{\text{b-}} = mgh + 0 = 0 + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}.$

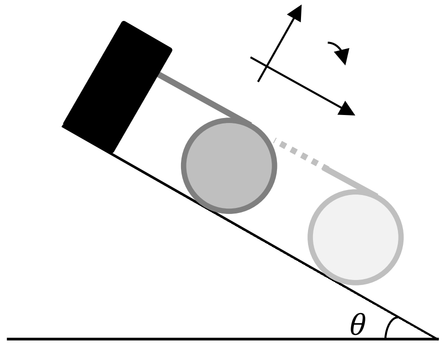
Ang. momentum consv.(b-→b+): $L_{\text{b-}} = L_{\text{b+}} \Rightarrow \frac{L}{2}mv = I\omega_{\text{b+}}$

The collision itself is inelastic, and neither energy, nor momentum is conserved.

From (b) to (c) energy conservation can again be used. With the zero point at the bar's height above the ground we get:

Energy consv.(b+→c): $U_{\text{b+}} + K_{\text{b+}} = U_c + K_c \Rightarrow 0 + \frac{1}{2}I\omega_{\text{b+}}^2 = mg\frac{L}{2} + 0.$

Combining all gives $h = \frac{2}{3}L$.



a)

The correct answer is A and D.

b)

The correct answer is A and C.

c) Look at the signs of the acceleration and the angular acceleration. They have opposite signs.

$a_x = -r\alpha$

d) Write up Newton's second law for translations and rotations for the center of mass, and the geometric relation.

N2: $ma_x = mg\sin\theta - S$

RN2: $\frac{1}{2}mr^2\alpha = -Sr$

GC: $a_x = -r\alpha$

Insert GC into RN2 and eliminate the radius.

$$\frac{1}{2}ma_x = S$$

Add N2 and the above.

$$\frac{3}{2}ma_x = mg\sin\theta$$

$$a_x = \frac{2}{3}g\sin\theta$$