Problem 1.
The correct answer is A. Each of the three doors has $I = \frac{1}{3}MR^2$.

Problem 2.
The correct answer is C. Calculate the moment of inertia of the cylinder without a hole. Calculate the moment of inertia of the "hole" and use the parallel-axis theorem on it. Then, subtract the two values from each other.

The moment of inertia of the cylinder without the hole is:

$$I_1 = \frac{1}{2}M(6R)^2 = 18MR^2$$

The moment of inertia of the "massive" hole with respect to its CM is

$$I_2 = \frac{1}{2}\frac{M}{36}R^2 = \frac{1}{72}MR^2$$

The moment of inertia of the "massive" hole with the respect to the CM of the cylinder is

$$I_3 = I_2 + \frac{M}{36}R^2 = \frac{17}{24}MR^2$$

The moment of inertia of the cylinder with the hole is

$$I = I_1 - I_3 = \frac{415}{24}MR^2$$

Problem 3.
a) Use conservation of mechanical energy. The moment of inertia is

$$I_1 = \frac{1}{3}6M(2L)^2 = 8ML^2$$

$$U_1 + K_1 = U_2 + K_2$$

$$6MgL + 0 = 0 + \frac{1}{2}8ML^2\omega_1^2$$

$$\omega_1 = \sqrt{\frac{3}{2}\frac{g}{L}}$$

b) Use conservation of mechanical energy.

$$6Mg\frac{2L}{2} + 0 = 6Mg\frac{2L}{2}(1 - \cos\theta) + \frac{1}{2}8ML^2\omega_2^2$$

$$\omega_2 = \sqrt{\frac{3}{2}\cos\theta\,\frac{g}{L}}$$

$$v_{\mathrm{cm}} = L\omega_2 \qquad v_{\mathrm{cm}} = L\omega_2 = \sqrt{\frac{3}{2}\cos\theta\, gL}$$

Problem 4.
We have an uncertainty on the speed and use both the smallest and the largest value, $v = 4.40\ \mathrm{m/s}$ og $v = 4.70\ \mathrm{m/s}$. The work-energy theorem gives the growth in kinetic energy from the work done by gravity, $\Delta K = MgH$. The moment of inertia can be written as $I = cMR^2$ where c needs to be determined. The kinetic energy gained is $K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ and with the geometric constaint, $v = R\omega$, we get $K = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{2}cMv^2 = \frac{1}{2}(1+c)Mv^2$. We have $MgH = \frac{1}{2}(1+c)Mv^2$ and therefore, $c = \frac{2gH}{v^2} - 1$. We determine c for the two speeds by insertion, giving $c = 0.60$ and $c = 0.83$. The five formulas have $c = 1$, $c = \frac{5}{8}$, $c = \frac{1}{2}$, $c = \frac{2}{5}$ and $c = \frac{2}{3}$. The second and fifth lie within the inteval, so the body can be a hollow sphere, or a thick ring with inner radius $R/2$. The correct answer is B and E.

Problem 5.

a) The work done by F is $W = FL$ and the growth in kinetic energy is $\Delta K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{1}{2}MR^2\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2$. We also have the geometrical rolling relation, $v = R\omega$.

The work-energy theorem $\Delta K = W$ gives $\frac{3}{4}MR^2\omega^2 = FL$ where $\omega = \frac{2}{\sqrt{3}}\sqrt{\frac{FL}{MR^2}}$ is a solution.

b) Notice that the force has now been applied over a displacement of $2L$. Otherwise, the situation is the same as before,

$\frac{3}{4}MR^2\omega^2 = 2FL$ giving $\omega = \frac{2\sqrt{2}}{\sqrt{3}}\sqrt{\frac{FL}{MR^2}}$.

Problem 6.

a) Use conservation of mechanical energy.

$$U_1 + K_{1,\text{trans}} + K_{1,\text{rot}} = U_2 + K_{2,\text{trans}} + K_{2,\text{rot}}$$

$$mgh + 0 + 0 = 0 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

There is a geometrical constraint between velocity and angular velocity.

$$v = R\omega$$

Conservation of mechancal energy:

$$mgh + 0 + 0 = 0 + \frac{1}{2}mR^2\omega^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$mgh = \left(\frac{1}{2}m + \frac{1}{4}M\right)R^2\omega^2$$

$$\omega = \sqrt{\frac{mgh}{\left(\frac{1}{2}m + \frac{1}{4}M\right)R^2}}$$