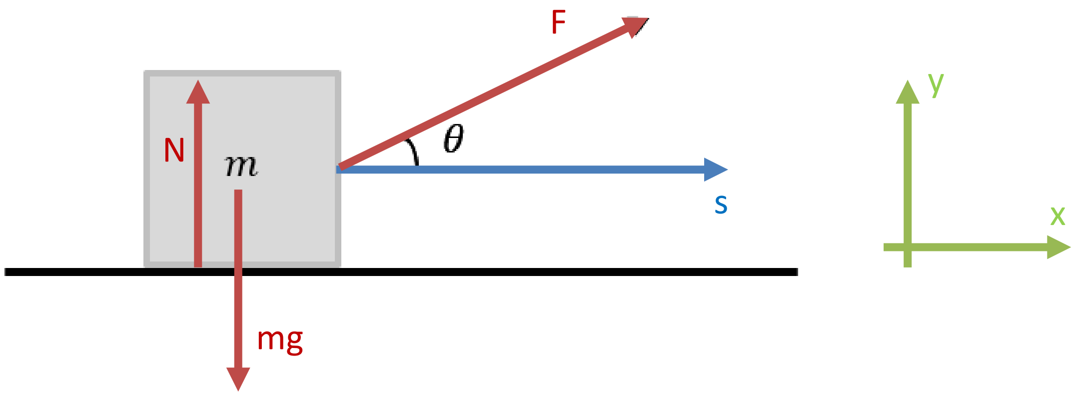
Problem 1.

The force is constant, which means that the acceleration must be constant. A constant acceleration means that the velocity changes linearly in time. Therefor the effect, $P = F \cdot v$, must also be increasing linearly as a function of time.

The answer is option A.

Problem 2.

a) We draw a force diagram, a coordinate system, and the displacement of the block.



The normal force and the force of gravity are perpendicular to the displacement, so they do not do any work. Only F does work, and the displacement $\vec{s}$ points along the x-axis and has the length L. The work done by F can be calculated as.

$$W_{\mathrm{F}} = \vec{F} \cdot \vec{s} = F\cos\theta\, L$$

We use the work-energy theorem to determine the final velocity v. The block starts from rest:

$$W_{\mathrm{tot}} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}m0^2 = \frac{1}{2}mv^2$$

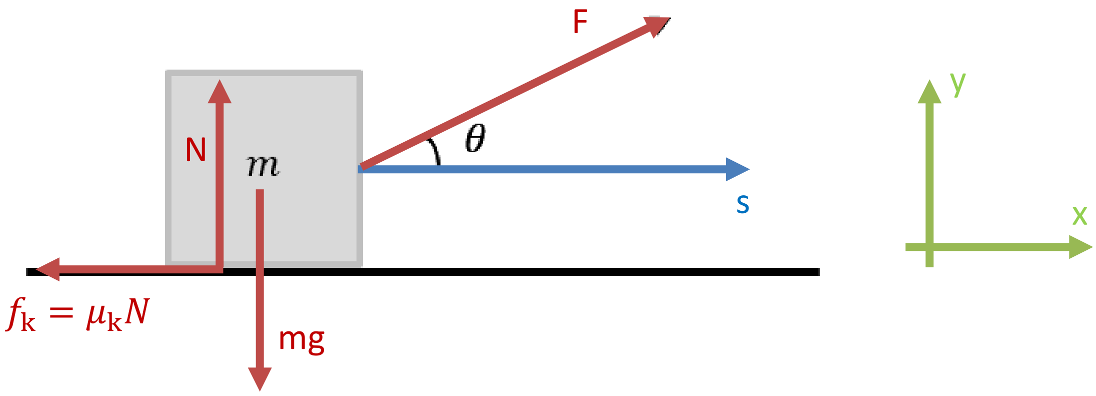
Only the force F does any work on the block, so we know the relationship between the work done by all forces and F, $W_{\mathrm{tot}} = W_{\mathrm{F}}$.

$$F\cos\theta\, L = \frac{1}{2}mv^2$$

Isolating v:

$$v = \sqrt{\frac{2F\cos\theta\, L}{m}}$$

b) We draw a force diagram and include the friction force.



The friction force is in the opposite direction with respect to the displacement (the angle is 180°), so the work done by the friction force is negative. To determine the final velocity, we must calculate the work done by the friction force. The size of the friction force is dependent on the normal force, so we set up Newton's second law in the y-direction.

N1(y):

$$0 = N - mg + F \sin\theta$$
$$N = mg - F\sin\theta$$

Now we can calculate the work done by the friction force:

$$W_\mathrm{f} = f_k L \cos(180°) = -f_k L = -\mu_k N L = -\mu_k L(mg - F\sin\theta)$$

The total work done by all forces is:

$$W_\mathrm{tot} = W_\mathrm{F} + W_\mathrm{f} = F\cos\theta\, L - \mu_k L(mg - F\sin\theta)$$
$$W_\mathrm{tot} = (F(\cos\theta + \mu_k \sin\theta) - \mu_k mg)L$$

We use the work energy theorem to determine the final velocity v. The block is initially at rest:

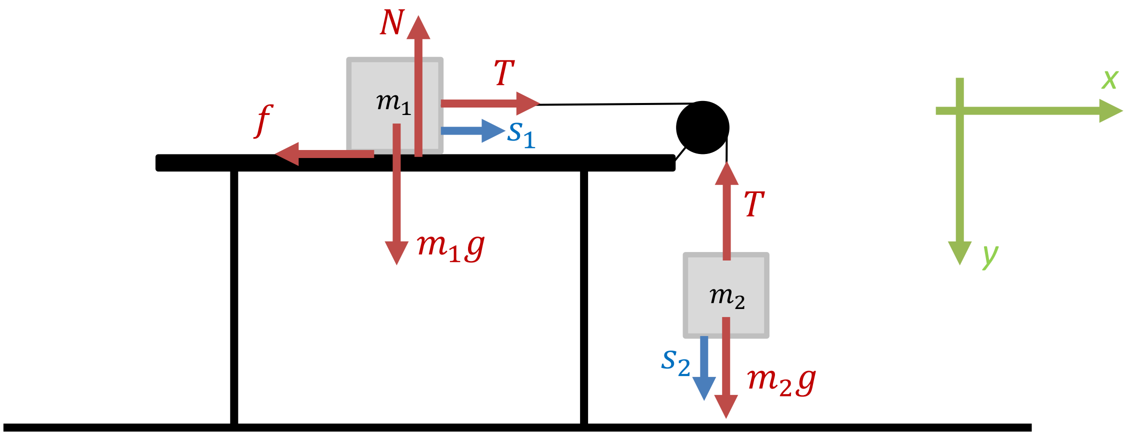
$$W_\mathrm{tot} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}m0^2 = \frac{1}{2}mv^2$$

The expression for the total work is inserted on the left side, and v is expressed in terms of the given quantities.

$$v = \sqrt{\frac{2(F(\cos\theta + \mu_k\sin\theta) - \mu_k mg)L}{m}}$$

Problem 3.

We draw the forces for each block along with a coordinate system that is used for both blocks. We also draw the displacement for both blocks. Notice that the direction of the displacement is different, but the length is the same for the blocks.



a) To determine the work from the friction force and the string tension, we need to determine the size of the forces.

The tension can be determined from block 2. As both blocks move with constant velocity, we can set up Newton's first law in the y-direction for block 2:

N1(m_2, y): $0 = m_2 g - T$

$T = m_2 g$

The friction force can be determined by setting up Newton's first law in the x-direction for block 1:

N1(m_1, x): $0 = T - f = m_2 g - f$

$f = m_2 g$

Even though both forces are similar in magnitude, they have different directions. Therefore their work is not equal. We calculate the work done by both forces on block 1:

$$W_\mathrm{T} = \vec{T} \cdot \vec{s_1} = T\cos(0°)\, d = m_2 g d$$
$$W_\mathrm{f} = \vec{f} \cdot \vec{s_1} = f\cos(180°) d = -m_2 g d$$

Notice that the sum of the total work $W_{\mathrm{tot}} = W_\mathrm{T} + W_\mathrm{f}$ is equal to 0, which according to the Work-Energy theorem means that there is no change in the kinetic energy. This makes sense as the block is moving with constant velocity during the displacement.

b) We know both the tension and the force of gravity acting on block 2. Therefore we can calculate the work done by both forces. We should notice that the second block is moving along the y-direction, so the coordinates of the direction/path has changed.

$$W_{\mathrm{T}} = \vec{T} \cdot \overrightarrow{s_2} = T\cos(180°)\, d = -Td = -m_2 g d$$
$$W_{\mathrm{gravity}} = m_2 \vec{g} \cdot \overrightarrow{s_2} = m_2 g \cos(0°)\, d = m_2 g d$$

Problem 4.

a) We use the Work-Energy Theorem on the motion from point 1 to 2.

$$\Delta K_{1,2} = W_{\text{tot}}$$

There is both kinetic energi in the first and second position:

$$\Delta K_{1,2} = K_2 - K_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The force of gravity and the friction force contribute to the work done:

$$W_{\text{tot}} = W_{\text{gravity}} + W_{\text{friction}} = mgh + W_{\text{friction}}$$

From this equation the work done by the friction force can be calculated as

$$W_{\text{friction}} = W_{\text{tot}} - mgh = \Delta K_{1,2} - mgh = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 - mgh$$

With the given values:

$$W_{\text{friction}} = -401.7 \text{ J}$$

b) We consider the motion of the block from 2 to 3, and set up the Work-Energy Theorem. From 2 to 3 only the friction force contributes to the work done on the block as the force of gravity and the normal force act perpendicular to the direction of motion. The friction force is anti-parallel (parallel but opposite in direction) to the direction of the motion. From this we get

$$\Delta K_{2,3} = K_3 - K_2 = \tfrac{1}{2}mv_3^2 - \tfrac{1}{2}mv_2^2 = W_{\text{friction}} = \vec{f} \cdot \vec{s} = f\cos(180°)l = -fl$$

The kinematic friction force is given by

$$f = \mu_k N$$

The normal force is expressed by setting up Newton's first law in the vertical direction:

N1(vertical): $0 = mg - N$

$$N = mg$$

The friction force is then $f_k = \mu_k mg$ and this expression is inserted into the Work-Energy Theorem:

$$\tfrac{1}{2}mv_3^2 = \tfrac{1}{2}mv_2^2 - \mu_{\text{k}} mgl$$

We isolate the speed v_3 and insert the given values:

$$v_3 = \sqrt{v_2^2 - 2\mu_{\text{k}} gl} = 5.89 \text{ m/s}$$

c) We consider the motion of the block from 3 to 4 and set up the Work-Energy Theorem. The work is performed by the friction force and the spring. The spring is fully compressed when the velocity of the block is 0, so the change in kinetic energy is given by:

$$W_{\text{tot}} = \Delta K_{3,4} = K_4 - K_3 = 0 - \frac{1}{2}mv_3^2 = -\frac{1}{2}mv_3^2$$

The total work done by the friction force and the spring:

$$W_{\text{tot}} = W_{\text{friction}} + W_{\text{spring}}$$

The friction force is calculated as in question b), where the distance d is from 3 to 4:

$$W_{\text{friction}} = -\mu_k mgd$$

The work performed by the spring force can be calculated from

$$W_{\text{spring}} = \frac{1}{2}kx_{\text{start}}^2 - \frac{1}{2}kx_{\text{end}}^2$$

Here x is the displacement from the spring equilibrium. As the spring is in equilibrium in position 3 and compressed the distance d in position 4, we insert $x_{\text{start}} = 0$ og $x_{\text{end}} = d$. The work done by the spring force is:

$$W_{\text{spring}} = -\frac{1}{2}kd^2$$

We put this into the Work-Energy Theorem:

$$-\mu_k mgd - \frac{1}{2}kd^2 = -\frac{1}{2}mv_3^2$$

$$\frac{1}{2}kd^2 + \mu_k mgd - \frac{1}{2}mv_3^2 = 0$$

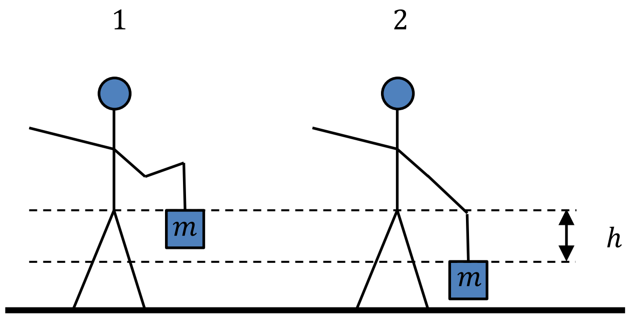
This is a quadratic equation in d, where the solution is:

$$d = \frac{-\mu_k mg + \sqrt{\mu_k^2 m^2 g^2 + m\,k\,v_3^2}}{k}$$

By inserting the given values, the distance can be calculated:

$$d = 0.400\text{ m}$$

Problem 5.



a) The block is affected by the force of gravity and the tension in the string. We set up Newton's second law in the vertical direction, with the positive direction pointing downwards.

N2(↓): $ma = m\frac{g}{4} = mg - S$

$$S = \frac{3}{4}mg$$

The answer is option F.

b) The person performs work on the block by the string. We calculate the work done on the block by the string. The tension is known from question a) and the direction is opposite the direction of motion of the block, so the work is negative.

$$W_{\text{string}} = -Sh = -\frac{3}{4}mgh$$

The answer is option D.

c) We set up the Work-Energy Theorem where only the string tension and the force of gravity performs work on the block. The block is initially at rest. The speed in situation 2 is denoted v.

$$W_{\text{tot}} = \Delta K = K_2 - K_1 = \frac{1}{2}mv^2 - \frac{1}{2}m0^2 = \frac{1}{2}mv^2$$

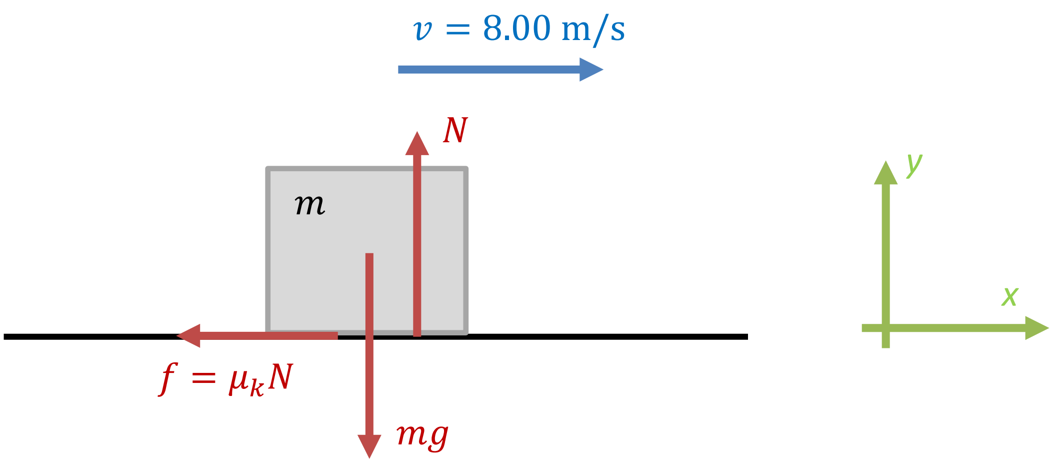
$$W_{\text{tot}} = W_{\text{gravity}} + W_{\text{string}} = mgh - \frac{3}{4}mgh = \frac{1}{4}mgh$$

We compare the equations and solve for the speed

$$\frac{1}{4}mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{1}{2}gh}$$

Problem 6.
We draw a sketch of the system with the applied forces and a coordinate system.



We consider the motion of the block from situation 1, which is sketched above to the point where the block is no longer moving.

The average power that the friction force produces is given by

$$P = \frac{W_{\text{friction}}}{\Delta t}$$

Here W_{friction} is the work that the friction force does on the block during deceleration and Δt is the time it takes for the block to stop moving.

As the force of gravity and the normal force are perpendicular to the direction of motion, it is only the friction force that does any work on the block. The block has the initial velocity v and ends at rest:

$$W_{\text{tot}} = W_{\text{friction}} = \Delta K = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$$

$$W_{\text{friction}} = -\frac{1}{2}mv^2$$

We only need to determine the time Δt, it takes for the block to stop moving. We set up Newton's law in the x- and y-directions to determine the acceleration of the block:

N1(y):
$$0 = N - mg$$
$$N = mg$$

N2(x):
$$ma = -f_k = -\mu_k N = -\mu_k mg$$
$$a = -\mu_k g$$

From these equations the acceleration of the block is constant. The speed of the block as a function of time is given by:

$$v_{\text{block}}(t) = v + at = v - \mu_k g t$$

The time it takes for the block to stop moving is $v_{\text{block}}(\Delta t) = 0$, can now be determined

$$0 = v - \mu_k g \Delta t$$
$$\Delta t = \frac{v}{\mu_k g}$$

We can now calculate the power

$$P = \frac{-\frac{1}{2}mv^2}{\frac{v}{\mu_k g}} = -\frac{1}{2}\mu_k g m v = -157\ \text{W}$$

Problem 7.

a) The ball is only affected by gravity as soon as it is released from the shot putter's hand. The motion of the ball is described by the formula for a 2-dimensional projectile motion. The motion starts at the origin of the chosen coordinate system, and is described by the parabola:

$$y(x) = \tan(\theta)\, x - \frac{g}{2v^2\cos^2(\theta)} x^2$$

We use a coordinate system where the origin is the point where the shot putter releases the ball, so h above the ground. The coordinates for the point where the ball lands on the ground will be $(x, y) = (L, -h)$, which is inserted into the formula above:

$$-h = \tan(\theta)\, L - \frac{g}{2v^2\cos^2(\theta)} L^2$$

We find the expression for v:

$$v = \sqrt{\frac{gL^2}{2\cos^2(\theta)\,[\tan(\theta)L + h]}}$$

By inserting the given values:

$$v = 13.3\ \mathrm{m/s}$$

b) The ball starts at rest on the ground, where the ball is lifted by the shot putter. The change in kinetic energy is given by:

$$\Delta K = K_2 - K_1 = \frac{1}{2}mv^2 - \frac{1}{2}m0^2 = \frac{1}{2}mv^2$$

The Work-Energy theorem relates the change in kinetic energy of the ball to the total work performed on the ball by all forces:

$$W_{\text{tot}} = \Delta K = \frac{1}{2}mv^2$$

It is only the shot putter and the force of gravity that performs work on the ball. We can write the total work as:

$$W_{\text{tot}} = W_{\text{gravity}} + W_{\text{shotputter}} = \frac{1}{2}mv^2$$

$$W_{\text{shotputter}} = \frac{1}{2}mv^2 - W_{\text{gravity}}$$

The work done by gravity depends only on the difference between the initial and final height of the ball. Since the ball is lifted from the ground ($y_1 = -h$) to a final height of $y_2 = 0$ (cf. coordinate system defined in question a) we find:

$$W_{\text{gravity}} = mg(y_1 - y_2) = mg(-h - 0) = -mgh$$

The work done by the shot putter is given as

$$W_{\text{shotputter}} = \frac{1}{2}mv^2 + mgh$$

By inserting the given values, numerical value of the work is:

$$W_{\text{shotputter}} = 770\ \text{J}$$

Problem 8.

The force diagram and setup for Newton's second law are the same as in the solution for Problem 3. We will reference the solution for Problem 3 here.

This exercise is more easily solved by using the Work-Energy Theorem for the system containing both blocks. Both blocks are initially at rest and they will both have the same speed v after moving the distance l. The change in kinetic energy of the system will be:

$$\Delta K = K_{\text{after}} - K_{\text{before}} = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 - 0 = \frac{1}{2}(m_1 + m_2)v^2$$

The change in kinetic energy is related to the work done by all forces on both blocks. The normal force and the force of gravity perform no work on block 1 as they are perpendicular to the displacement of block 1. The contributions to the total work comes from the tension in the string and the force of gravity on block 2, and the friction and string tension on block 1.

$$W_{\text{tot}} = W_{\text{gravity,2}} + W_{\text{string,2}} + W_{\text{string,1}} + W_{\text{friction,1}}$$

The tension in the string is equal in magnitude on both blocks, but for block 1 the force from the string is parallel to the direction of motion, while it is in the opposite direction to the motion on block 2. Therefore the work done on both blocks will be equal in magnitude, but have different signs: $W_{\text{string,1}} = -W_{\text{string,2}}$. This is the solution to Problem 3. The sum of all work is reduced to:

$$W_{\text{tot}} = W_{\text{gravity,2}} + W_{\text{friction,1}}$$

The work done by the force of gravity on block 2 is only related to the vertical displacement of the block:

$$W_{\text{gravity,2}} = m_2 g l$$

The friction force is antiparallel to the direction of motion, so the work on block 1 is given by

$$W_{\text{friction,1}} = f\cos(180°)\, l = -fl = -\mu_k N l = -\mu_k m_1 g l$$

Now we see the sum of all work done on the system of both blocks:

$$W_{\text{tot}} = \Delta K$$
$$m_2 g l - \mu_k m_1 g l = \frac{1}{2}(m_1 + m_2)v^2$$

We isolate the speed v:

$$v = \sqrt{\frac{2(m_2 - \mu_\text{k} m_1)gl}{m_1 + m_2}}$$