Problem 1

a) The velocity of the wolf is zero at the points A, E, and H. The answer is option B.

b) The acceleration of the wolf is zero in the points where the slope of the $v - t$ curve is 0. This occurs at points C and F. The answer is option C.

c) The acceleration is constant when the velocity changes *linearly* during some time interval. This condition is fulfilled in the intervals D-E and G-H. The answer is option C.

d) The distance from the starting point is determined by integrating the $v - t$ curve, i.e., summing the area between the curve and the x-axis. An area below the x-axis is subtracted from the total area, so the largest *positive* area (and hence largest *positive* distance traveled) can be found at point E. The answer is option B.

Problem 2

The velocity is increasing in the beginning, constant in the midsection, until the velocity eventually decreases.

The answer is option A.

Problem 3

In a $x - t$ curve the acceleration is determined by the *curvature* of the graph (or sign of the second derivative). The curvature of the curve is negative at points D and E.

The answers are options D and E.

Problem 4

a) Each car has $l = \frac{85}{2}\text{m} = 42.5\text{m}$ to decrease their velocity to 0. The breaking distance Δx is calculated using

$$v^2 = v_0^2 + 2a\Delta x \Leftrightarrow$$
$$\Delta x = \frac{v^2 - v_0^2}{2a}$$

By inserting $v = 0\frac{\text{m}}{\text{s}}$, $v_0 = 88\frac{\text{km}}{\text{h}} = 24.4\frac{\text{m}}{\text{s}}$, and $a = -8\frac{\text{m}}{\text{s}^2}$, the breaking distance is

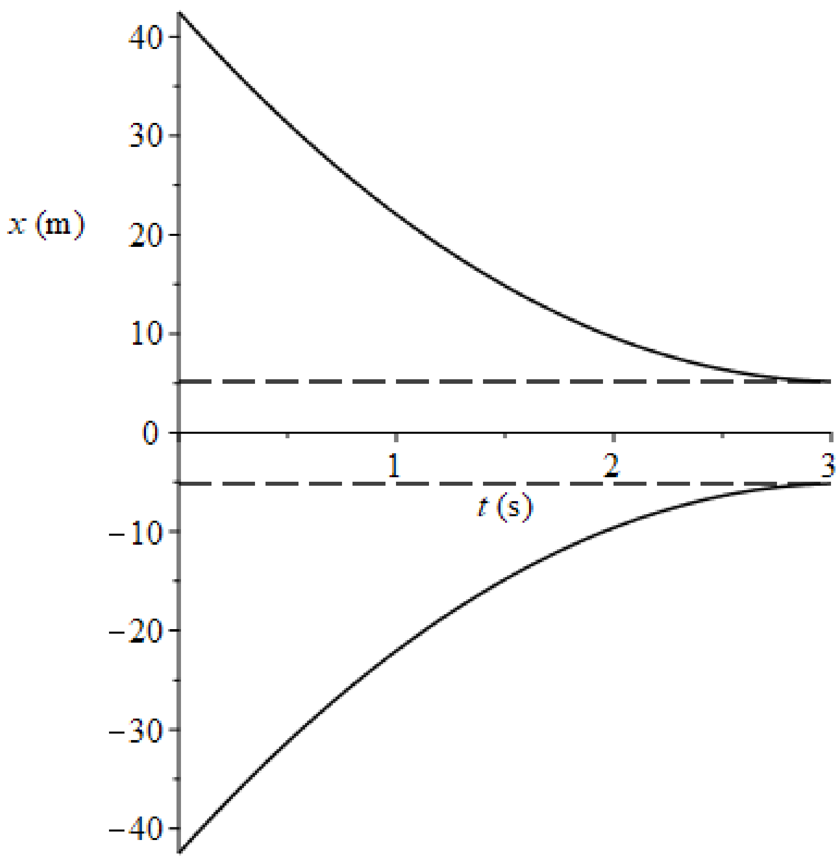
$$\Delta x = 37.3\text{ m}$$

Since $\Delta x < l$ the cars will not crash.

c) When the cars stop, the distance L from each other is:

$$L = 2(l - \Delta x) = 10.4\text{ m}$$

d) A graph of the positions of the cars can be seen below. The graphs are parabolas which end with a horizontal tangent, corresponding to zero velocity – at which point (time) the cars have stopped.



Problem 5

Each car has $l = 50$ m to decrease their velocity to 0. The breaking distance Δx is calculated by:

$$v^2 = v_0^2 + 2a\Delta x \Leftrightarrow \Delta x = \frac{v^2 - v_0^2}{2a}$$

By inserting $v = 0\,\frac{\mathrm{m}}{\mathrm{s}}$, $v_0 = 50\,\frac{\mathrm{km}}{\mathrm{h}} = 13.9\,\frac{\mathrm{m}}{\mathrm{s}}$ and $a = -5.00\,\frac{\mathrm{m}}{\mathrm{s}^2}$:

$$\Delta x = 19.3\ \mathrm{m}$$

Since $\Delta x < l$ the cars will not crash.

When the cars stop, their distance L from each other is:

$$L = 2(l - \Delta x) = 61.4\ \mathrm{m}$$

The cars come to a complete stop at time t:

$$v = v_0 + a \cdot t \Leftrightarrow t = -\frac{v_0}{a} = 2.78\ \mathrm{s}$$

The answers are options B and H.

Problem 6

We list the known variables and convert them to SI-units:

Speed:	$v = 95\frac{\text{km}}{\text{h}} = 26.4\frac{\text{m}}{\text{s}}$
Distance to pedestrian:	$L = 60\text{ m}$
Reaction time:	$t_\text{R} = 1.5\text{ s}$
Acceleration:	$a_\text{B} = -8.0\frac{\text{m}}{\text{s}^2}$

Because of the reaction time, the car travels the distance x_R before the driver starts breaking:

$$x_\text{R} = v \cdot t_\text{R} = 39.6\text{ m}$$

The velocity of the car after breaking can be determined

$$v_\text{B}^2 = v^2 + 2 \cdot a_\text{B} \cdot (L - x_\text{R}) \Leftrightarrow$$
$$v_\text{B} = \sqrt{v^2 + 2 \cdot a_\text{B} \cdot (L - x_\text{R})} = 19.2\frac{\text{m}}{\text{s}} = 69.2\frac{\text{km}}{\text{h}}$$

Alternatively after calculating x_R one could solve for the time it takes to travel the remaining distance $L - x_R = 20.4$

$$L - x_R = v_0 \cdot t - \frac{1}{2}at^2 = 20.4\text{ m}$$

Solved for t gives $t = 0.895\ s$. The velocity at this point is $v = v_0 - a \cdot t = 19.23$ m/s=69.2 km/h.

The answer is option B.

Problem 7

A) The area below the curve (distance traveled) when the velocity is positive, is larger compared to when the velocity is negative, so A must be true.

B) When $t = 2$ s the velocity is $10\,\frac{\mathrm{m}}{\mathrm{s}}$, so B is **not** true.

C) The area below the curve is the distance traveled. For the time interval $0 - 4$s the area is (dividing the area into two triangles and one rectangle)

$$A_{t=0-4\mathrm{s}} = \frac{1}{2} \cdot (2\ \mathrm{s}) \cdot \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right) + \frac{1}{2} \cdot (2\ \mathrm{s}) \cdot \left(5\,\frac{\mathrm{m}}{\mathrm{s}}\right) + (2\ \mathrm{s}) \cdot \left(5\,\frac{\mathrm{m}}{\mathrm{s}}\right) = 25\ \mathrm{m}$$

Hence C is **not** true.

D) Movement in the negative x-direction occurs in the interval $6\ \mathrm{s} - 8\ \mathrm{s}$. The area below the curve is:

$$A_{t=6\mathrm{s}\,-8\mathrm{s}} = \frac{1}{2} \cdot (2\ \mathrm{s}) \cdot \left(-5\,\frac{\mathrm{m}}{\mathrm{s}}\right) = \ -5\ \mathrm{m}$$

Option D must be true.

E) Acceleration is given by the slope of the curve. At time $t = 1$ s the acceleration is $a = 5\,\frac{\mathrm{m}}{\mathrm{s}^2}$. At time $t = 6$ s the acceleration is $a = -5\,\frac{m}{s^2}$. Option E is true.

F) The particle is not at rest, since the velocity is positive, $v_{4s-5s} = 5\ m/s$ (even though the slope is 0). Answer F is **not** true.

The correct answers are A, D and E.

Problem 8

The first car has travelled the distance

$$x_1 = \frac{1}{2}aT^2$$

The second car has only travelled for $\frac{T}{2}$, so it has only travelled the distance

$$x_2 = \frac{1}{2}a\left(\frac{T}{2}\right)^2$$

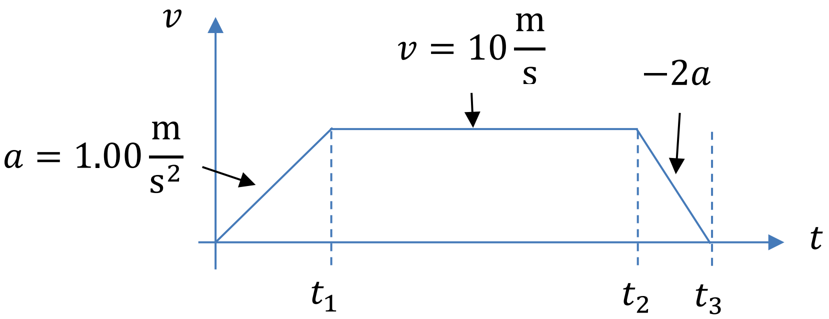
The difference in distance travelled L is

$$L = x_1 - x_2 = \frac{1}{2}aT^2 - \frac{1}{2}a\left(\frac{T}{2}\right)^2 = \frac{1}{2}aT^2\left(1 - \frac{1}{4}\right) = \frac{3}{8}aT^2$$

The answer is option G.

Problem 9

a) We sketch the velocity of the car below. In the beginning the acceleration is $a = 1.00\,\frac{\mathrm{m}}{\mathrm{s}^2}$, and $-2a$ when it breaks in the end. The maximum velocity is $v = 10.0\,\frac{\mathrm{m}}{\mathrm{s}}$.



b) To determine the end time t_3, the equation for the distance traveled L is:

$$L = \underbrace{\frac{1}{2}\cdot v\cdot t_1}_{\text{acceleration}} + \underbrace{v\cdot(t_2 - t_1)}_{Const.\ \text{vel.}} + \underbrace{\frac{1}{2}\cdot v\cdot(t_3 - t_2)}_{\text{deceleration}}$$

In the above expression we have used the following equation to calculate the distance traveled during acceleration and deceleration:

$$x - x_0 = \frac{v_0 + v}{2}t$$

To determine t_3 we need expressions for both t_1 and t_2 to eliminate them from the equation. We consider the velocity during $0 \to t_1$, where

$$v = v_0 + a\cdot t_1 = 0 + a\cdot t_1 = a\cdot t_1 \Rightarrow t_1 = \frac{v}{a}$$

Similarly for the velocity in the interval $t_2 \to t_3$:

$$0 = v - 2\cdot a\cdot(t_3 - t_2) \Rightarrow t_2 = t_3 - \frac{v}{2a}$$

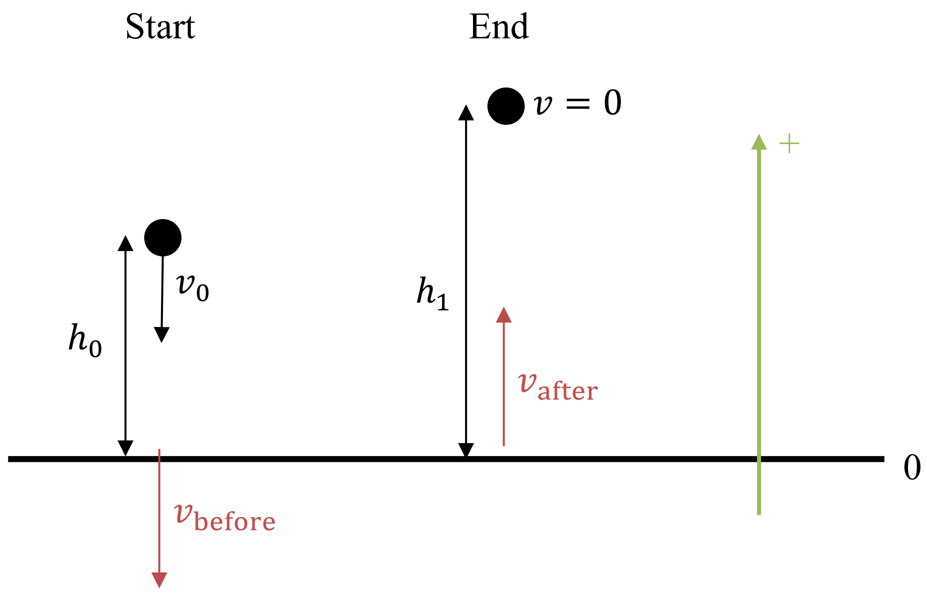
Inserting the expressions for both t_1 og t_2 in the equation for L:

$$L = \frac{1}{2}v\left(\frac{v}{a}\right) + v\left(t_3 - \frac{v}{2a} - \frac{v}{a}\right) + \frac{1}{2}v\left(t_3 - t_3 + \frac{v}{2a}\right) \Leftrightarrow$$
$$L = \frac{1}{2}\frac{v^2}{a} + vt_3 - \frac{3}{2}\frac{v^2}{a} + \frac{1}{4}\frac{v^2}{a} = vt_3 - \frac{3}{4}\frac{v^2}{a} \Leftrightarrow$$
$$t_3 = \frac{L}{v} + \frac{3v}{4a} = \frac{115}{2}\ \mathrm{s} = 57.5\ \mathrm{s}$$

It takes 57.5 s for the car to travel the distance $L = 500m$.

Problem 10

We draw a sketch of the problem:



The ball is falling with constant acceleration, until it hits the ground. Here there is conservation of energy, so the speed of the ball is the same before and after the impact:

$$v_{before} = v_{after}$$

Calculating the velocity at impact with the ground (positive axis points up):

$$v_{before}^2 = v_0^2 - 2g(0 - h_0) = v_0^2 + 2gh_0 \qquad (1)$$

The movement after the impact is similarly described by:

$$0 = v_{after}^2 - 2g(h_1 - 0) \Leftrightarrow$$
$$v_{after}^2 = 2gh_1 \qquad (2)$$

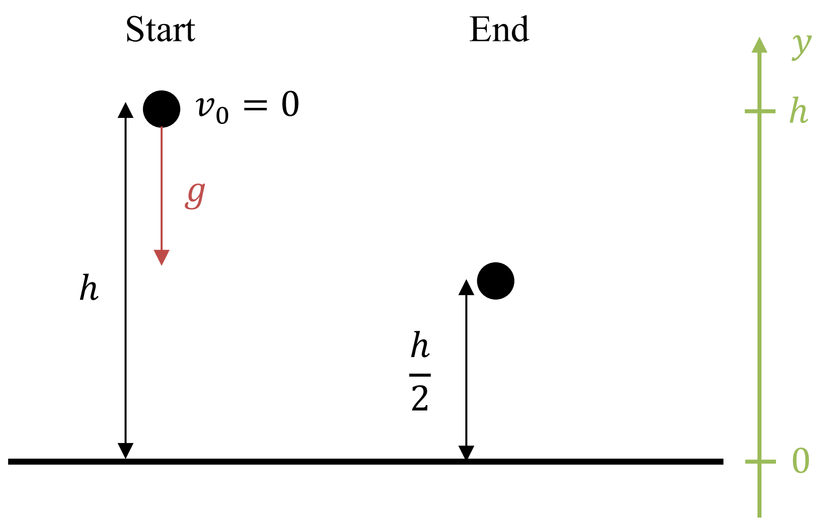
By inserting equations (1) and (2) into $v_{before} = v_{after}$:

$$v_0^2 + 2gh_0 = 2gh_1 \Leftrightarrow$$
$$v_0^2 = 2gh_1 - 2gh_0 = 2g(h_1 - h_0) \Leftrightarrow$$
$$v_0 = \sqrt{2g(h_1 - h_0)}$$

The answer is option D.

Problem 11

We consider the athlete from the top of the jump $y = h$ and determine the time t_1 it takes for the athlete to fall down to the height $y = h/2$, see sketch below.



At the highest point the velocity of the athlete is $v_0 = 0$, so motion in the y-direction is given by

$$y = h - \frac{1}{2}gt^2$$

The time t_1 it takes to fall to $y = \frac{h}{2}$ is determined by:

$$\frac{h}{2} = h - \frac{1}{2}gt_1^2 \Leftrightarrow t_1 = \sqrt{\frac{h}{g}}$$

The time t_2 it takes to reach the ground $y = 0$ is

$$0 = h - \frac{1}{2}gt_2^2 \Leftrightarrow t_2 = \sqrt{\frac{2h}{g}}$$

Notice that t_2 is also the time it takes to jump from the ground $y = 0$ to maximum height $y = h$. Similarly, it takes t_1 while mid-jump from $y = \frac{h}{2}$ to $y = h$.

The time it takes to complete the jump is

$$t_{\text{jump}} = 2 \cdot t_2$$

The time t_{up} the athlete spends above the height $y > \frac{h}{2}$ is

$$t_{\mathrm{up}} = 2 \cdot t_1$$

The ratio of the jump above $y > \frac{h}{2}$ to the time it takes to complete the jump is

$$\frac{t_{\mathrm{up}}}{t_{\mathrm{jump}}} = \frac{2t_1}{2t_2} = \frac{\sqrt{\frac{h}{g}}}{\sqrt{\frac{2h}{g}}} = \sqrt{\frac{1}{2}} = 0.707 = 70.7\ \%$$

The athlete spends 70.7% of the time near their maximum jump height, which makes it seem like they are "hanging" in the air.

Problem 12

a) Motion with constant acceleration means we can calculate the final velocity v of the rock after falling the distance R, by using the formula

$$v^2 = v_0^2 + 2a\Delta x$$

Inserting $v_0 = 0$ (rock initially at rest), $a = g$ and $\Delta x = R$. We find that

$$v^2 = 2gR \Rightarrow v = \sqrt{2gR}$$

b) The acceleration is no longer constant $a(x)$ and is a function of x. We should then integrate $a(x) = -\frac{x}{R}g$ from $x = R$ to $x = 0$, while the velocity is integrated from $v = 0$ (rest) to the final velocity v:

$$\int_{x=R}^{x=0} a(x)\,\mathrm{d}x = \int_0^v v\,\mathrm{d}v \Leftrightarrow$$
$$\int_R^0 \left(-\frac{x}{R}g\right)\mathrm{d}x = \frac{1}{2}v^2 \Leftrightarrow$$
$$\frac{g}{R}\left(\frac{1}{2}R^2\right) = \frac{1}{2}v^2 \Leftrightarrow$$
$$v = \sqrt{gR}$$