Problem 1.

a) We use conservation of momentum for the system consisting of both block and projectile. Situation 1 is before the collision and situation 2 is after the collision. The block is at rest before the collision.

Conservation of momentum: $mv_1 + M \cdot 0 = (m+M)v_2 \Leftrightarrow$

$$v_2 = \frac{m}{m+M} v_1$$

b) After the collision the block and projectile move as one body only affected by the force of gravity. Therefore we consider conservation of energy. Situation 3 is the top point of the motion, where the speed will be 0. The reference point of potential energy is in situation 2. For a coordinate system where the y-axis is pointing upwards, we get

Conservation of energy: $U_2 + K_2 = U_3 + K_3 \Leftrightarrow$

$$0 + \frac{1}{2}(m+M)v_2^2 = (m+M)gh + 0 \Leftrightarrow$$

$$\frac{1}{2}(m+M)\left(\frac{m}{m+M}v_1\right)^2 = (m+M)gh \Leftrightarrow$$

$$h = \left(\frac{m}{m+M}\right)^2 \frac{v_1^2}{2g}$$

Problem 2.

We use the formula for the impulse.

$$\Delta p = p_{\text{final}} - p_{\text{start}} = \int_0^{10} F\,dt \Leftrightarrow$$
$$p_{\text{final}} = p_{\text{start}} + \int_0^{10} F\,dt \Leftrightarrow$$
$$mv_{\text{final}} = mv_0 + \int_0^{10} F\,dt \Leftrightarrow$$
$$v_{\text{final}} = v_0 + \frac{1}{m}\int_0^{10} F\,dt$$

We are given the initial velocity v_0 and the mass m. This is now a question of calculating the integral of the force F with respect to time from 0 to 10 s. This is done by calculating the area below the $F - t$ curve. We split the integral into a rectangle and a triangle

$$\int_0^{10} F\,dt = 10\text{N} \cdot 5\text{s} + \frac{1}{2} 10\text{N} \cdot 5\text{s} = 75\ \text{N} \cdot \text{s}$$

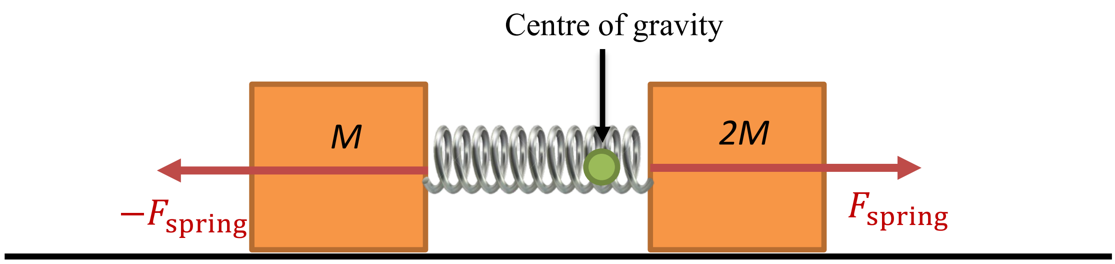
The final velocity is then

$$v_{\text{final}} = 10\frac{\text{m}}{\text{s}} + \frac{1}{10.0\ \text{kg}} \cdot 75\ \text{N} \cdot \text{s} = 17.5\frac{\text{m}}{\text{s}}$$

The answer is option D.

Problem 3.

Let us start by drawing a force diagram in the horizontal direction. The spring is compressed and exerts a force F_{spring} on both blocks:



The most important thing is that the spring force always acts on both blocks with the same magnitude. This is because the coupled system (composed of the two blocks and the spring) does not accelerate in the horizontal direction. Each block accelerates in different directions, but the coupled system does not, as the centre of gravity (marked by the green dot) does not move. There is no external force to move the centre of gravity.

We can now determine the momemtum of each block by considering the impulse exerted by the spring force. We consider the movement of each block from the initial time $t_0 = 0$ to some arbitrary final time t_{f}. The analysis is the same for both blocks, but let us consider the impulse on the large block (*2M*) first

$$\Delta p_{2M} = p_{2M,\text{final}} - p_{2M,\text{start}} = \int_0^{t_{\text{f}}} F_{\text{spring}}\, dt$$

The large block starts at rest so $p_{2M,\text{start}} = 0$, so the momentum of the block at time t_{f} will be

$$p_{2M,\text{final}} = \int_0^{t_{\text{f}}} F_{\text{spring}}\, dt$$

By performing the same analysis for the small block (*M*) we find

$$p_{M,\text{final}} = -\int_0^{t_{\text{f}}} F_{\text{spring}}\, dt$$

Since the spring force has the same magnitude on both blocks during the entire movement, we find

$$p_{2M,\text{final}} = -p_{M,\text{final}} \quad (1)$$

We can therefore conclude that two blocks receive the same amount of momentum from the spring (option A), but the momenta are in opposite directions (remember that momentum is a vector, so we must keep track of the signs even in one-dimensional motion!).

Let us name the final velocities v_{2M} og v_M and rewrite equation (1) as:

$$2Mv_{2M} = -Mv_M \Leftrightarrow$$
$$v_M = -2v_{2M}$$

From this we see that the small block (*M)* attains a larger velocity than the larger block (*2M*).

We can now calculate the kinetic energy of each block:

$$K_{2M} = \frac{1}{2}(2M)v_{2M}^2 = Mv_{2M}^2$$
$$K_M = \frac{1}{2}Mv_M^2 = \frac{1}{2}M(-2v_{2M})^2 = 2Mv_{2M}$$

We can now see that the kinetic energy of the small block (*M*) is twice as big as the kinetic energy of the large block (*2M*). From this we see that answer E is correct.

The answers are options A og E.

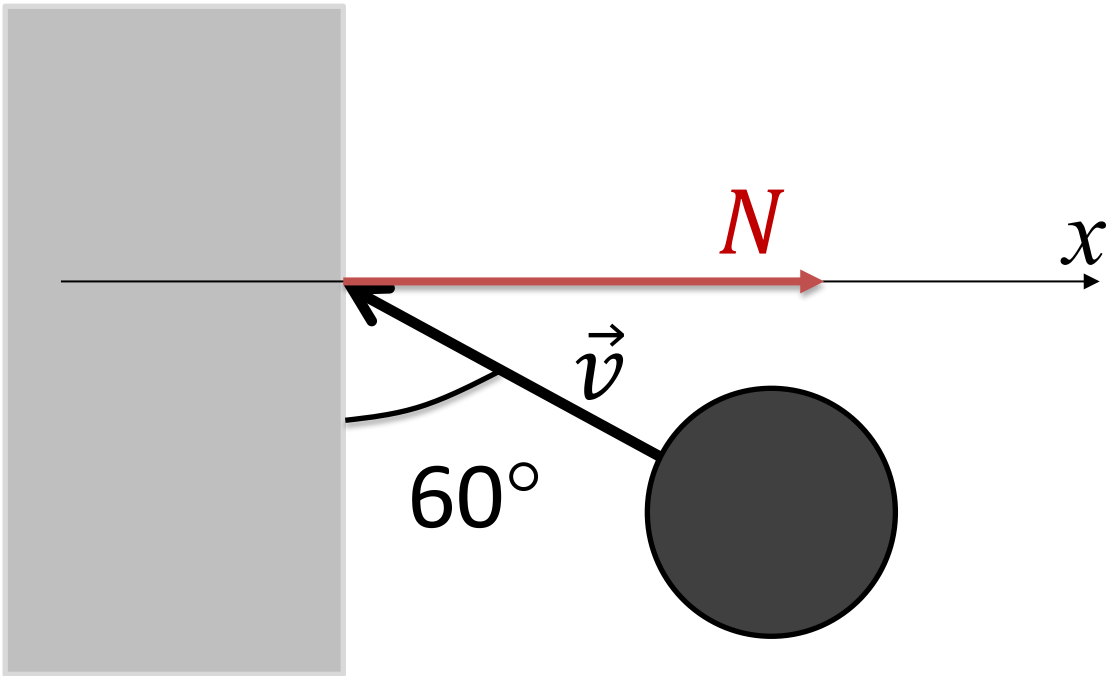
Problem 4.

When the ball collides with the wall, it will apply a force to the ball that changes the momentum of the ball during the collision. The average force $\vec{F}_{\text{av}}$ during the collision is related to the change in momentum $\Delta\vec{p}$:

$$\vec{F}_{\text{av}}\Delta t = \Delta\vec{p}$$

Here Δt is the duration of the collision, which is given in the exercise.

It is worth to note that the above equation is a *vector* equation, so there could be contributions to the average force in both the x and y-directions. Since there is no friction during the collision, it will only be the normal force from the wall that changes the momentum of the ball. The normal force is not constant during the collision, but changes as a function of time (the sketch below can be seen as a ”snapsnot”).



The normal force is perpendicular to the wall, so in the x direction and we can reduce the vector equation to

$$F_{\text{av}}\Delta t = \Delta p_x \Leftrightarrow$$
$$F_{\text{av}} = \frac{\Delta p_x}{\Delta t} = \frac{p_{2x} - p_{1x}}{\Delta t}$$

The momentum of the ball in the x-direction before the collision is

$$p_{1x} = mv_{1x} = -mv\sin(\theta)$$

The minus sign is included as the velocity is in the negative x-direction. Notice that we only need the x-component of the momentum for the calculations and that is why we multiply by $\sin(\theta)$.

We are told that the collision is elastic, so there is conservation of kinetic energy. From this information we use the following relation

$$(v_{\text{wall2}} - v_{2x}) = -(v_{\text{wall1}} - v_{1x})$$

Here $v_{\text{wall 1}}$ and $v_{\text{wall 2}}$ are the velocities of the wall before and after the collision. The wall does not move during the collision so both these quantities are 0. Now there is only the relationship between the velocity of the ball before and after the collision

$$v_{2x} = -v_{1x}$$

The momentum of the ball after the collision is

$$p_{2x} = mv_{2x} = -mv_{1x} = mv\sin(\theta)$$

Now we can calculate the average force that is applied to the ball during the collision:

$$F_{\text{av}} = \frac{p_{2x} - p_{1x}}{\Delta t} = \frac{mv\sin(\theta) - (-mv\sin(\theta))}{\Delta t} = \frac{2mv\sin(\theta)}{\Delta t} = 1.7\ \text{kN}$$

The answer is option D.

Problem 5.

a) We choose the direction of motion of block A to be the positive direction. Block B is therefore moving with a negative velocity. Before the collision, the velocity of block is $v_A = u$ and the velocity of block B is $v_B = -u$.

After the collision, the blocks move a one body with an unknown velocity v. There is conservation of momentum in the inelastic collision.

Conservation of momentum:

$$
\begin{aligned}
& m_\mathrm{A} v_\mathrm{A} + m_\mathrm{B} v_\mathrm{B} = (m_\mathrm{A} + m_\mathrm{B})v \Leftrightarrow \\
& mu + 4m(-u) = (m + 4m)v \Leftrightarrow \\
& -3mu = 5mv \Leftrightarrow \\
& v = -\frac{3}{5}u
\end{aligned}
$$

b) Before the collision, the sum of kinetic energy is the sum of the kinetic energy of block A and block B.

$$K_\mathrm{before} = K_\mathrm{A} + K_\mathrm{B} = \frac{1}{2} m_\mathrm{A} v_\mathrm{A}^2 + \frac{1}{2} m_\mathrm{B} v_\mathrm{B}^2 = \frac{1}{2} mu^2 + \frac{1}{2}(4m)(-u)^2 = \frac{5}{2} mu^2$$

After the collision there is only one body moving with the velocity v found in question (a). The kinetic energy is:

$$K_\mathrm{after} = \frac{1}{2}(m_\mathrm{A} + m_\mathrm{B})v^2 = \frac{1}{2}(m + 4m)\left(-\frac{3}{5}u\right)^2 = \frac{9}{10} mu^2$$

The relationship between the kinetic energy before and after the collision is:

$$\frac{K_\mathrm{after}}{K_\mathrm{before}} = \frac{\frac{9}{10} mu^2}{\frac{5}{2} mu^2} = \frac{18}{50} = \frac{9}{25}$$

The answer is option B.

Problem 6.

We split the problem into two collisions. The first collision with block 1 is inelastic, while the other collision with block 2 is completely inelastic as the projectile and block 2 move as one body after the collision. Momentum is conserved in both collisions.

First collision: We consider the projectile and block 1 as a system. Before the collision, only the projectile is moving and it is the only thing that contributes to the system momentum, as the block has no velocity. After the collision both the projectile and block 1 have non-zero velocities. We know from the exercise that block 1 has the velocity v_2 after the collision.

Conservation of momentum ($1 \rightarrow 2$):

$$p_1 = p_2 \Leftrightarrow$$
$$p_{1,\text{projectile}} = p_{2,\text{projectile}} + p_{2,\text{block1}} \Leftrightarrow$$
$$mv = mv_1 + M_1 v_2 \qquad (1)$$

Second collision: In the second collision we consider the projectile (now with decreased momentum) and block 2 as a system. Before the collision is denoted 3, where only the projectile has any momentum considering block 2 has no velocity. The situation after the collision is denoted 4, where projectile and block 2 are moving as one body with common velocity v_2.

Conservation of momentum ($3 \rightarrow 4$):

$$p_3 = p_4$$
$$p_{2,\text{projectile}} = p_{4,\text{projectile+block2}}$$
$$mv_1 = (m + M_2)v_2 \qquad (2)$$

There are two equations (1) and (2) with two unknowns, v_1 and v_2. We solve the two equations

$$v_1 = \frac{M_2 + m}{M_1 + M_2 + m} v$$

$$v_2 = \frac{m}{M_1 + M_2 + m} v$$

Problem 7.

a) The collision conserves momentum in the x (eastern) and y (northern) direction. The first car is denoted car A, with the initial velocity $v_{A1x} = 25.0 \text{ m/s}$ in the x-direction. The other car (B) has the initial velocity $v_{B1y} = 20.0 \text{ m/s}$ in the y-direction.

After the collision, the cars move as one body. The collision is completely inelastic, and the cars have both x- and y-components of the final velocity, v_{2x} and v_{2y}. We set up the conservation of momentum in both directions:

Conservation of momentum in x: $m_A v_{A1x} = (m_A + m_B)v_{2x}$

Conservation of momentum in y: $m_B v_{B1y} = (m_A + m_B)v_{2y}$

We solve the two equations with two unknowns:

$$v_{2x} = \frac{m_A v_{A1x}}{m_A + m_B} = 9.38 \text{ m/s}$$

$$v_{2y} = \frac{m_B v_{B1y}}{m_A + m_B} = 12.5 \text{ m/s}$$

The magnitude of the velocity vector is

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = 15.6 \text{ m/s}$$

The direction of the velocity is given by (the angle with respect to the x-axis)

$$\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right) = 53.1°$$

Problem 8.

We know that the collision is elastic, and therefore we know that there is conservation of both momentum and kinetic energy. We define the positive direction to be to the right. Before the collision, the speed of block A is $u_A = u$, while the speed of block B is $u_B = -u$. After the collision the blocks have the unknown velocities v_A and v_B. We set up conservation of momentum and conservation of kinetic energy.

Conservation of momentum:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B \Leftrightarrow$$
$$mu - 4mu = mv_A + 4mv_B \Leftrightarrow$$
$$-3u = v_A + 4v_B \qquad (1)$$

Conservation of kinetic energy:

$$(u_A - u_B) = -(v_A - v_B) \Leftrightarrow$$
$$\big(u - (-u)\big) = -(v_A - v_B) \Leftrightarrow$$
$$2u = -(v_A - v_B) \quad (2)$$

From equations (1) and (2) there are two unknowns, v_A and v_B. We solve the equations and find

$$v_A = -\frac{11}{5}u$$
$$v_B = -\frac{1}{5}u$$

The answer is option A.

Problem 9.

Block A and block B are in an elastic collision, so there is both conservation of momentum and kinetic energy. Situation 1 is just before the collision between block A and B, while situation 2 is just after the collision. The velocity of block A before the collision is v_{A1}. We set up the conservation laws.

Conservation of momentum (1 → 2):

$$p_{A1} + p_{B1} = p_{A2} + p_{B2} \Leftrightarrow$$
$$mv_{A1} + 0 = mv_{A2} + 2mv_{B2} \Leftrightarrow$$
$$mv_{A1} = mv_{A2} + 2mv_{B2} \qquad (1)$$

Conservation of kinetic energy (1 → 2):

$$v_{B2} - v_{A2} = -(v_{B1} - v_{A1}) \Leftrightarrow$$
$$v_{B2} - v_{A2} = -(0 - v_{A1}) \Leftrightarrow$$
$$v_{B2} - v_{A2} = v_{A1} \qquad (2)$$

Block B makes a completely inelastic collision with block C. Here only conservation of momentum applies, and the blocks move as one body with one final velocity. Situation 3 is after the collision between block B and block C.

Conservation of momentum (2 → 3):

$$p_{B2} + p_{C2} = p_{B3} + p_{C3} \Leftrightarrow$$
$$2mv_{B2} + 0 = 2mv_3 + mv_3 \Leftrightarrow$$
$$2mv_{B2} = 3mv_3 \qquad (3)$$

We now have 3 equations (1), (2) og (3) with 3 unknowns, v_{A2},v_{B2} and v_3. We solve these and find

$$v_{A2} = -3\frac{\text{m}}{\text{s}}$$
$$v_{B2} = 6\frac{\text{m}}{\text{s}}$$
$$v_3 = 4\frac{\text{m}}{\text{s}}$$

We were asked to find the final velocity of block C which is given by v_3.

The answer is option D.