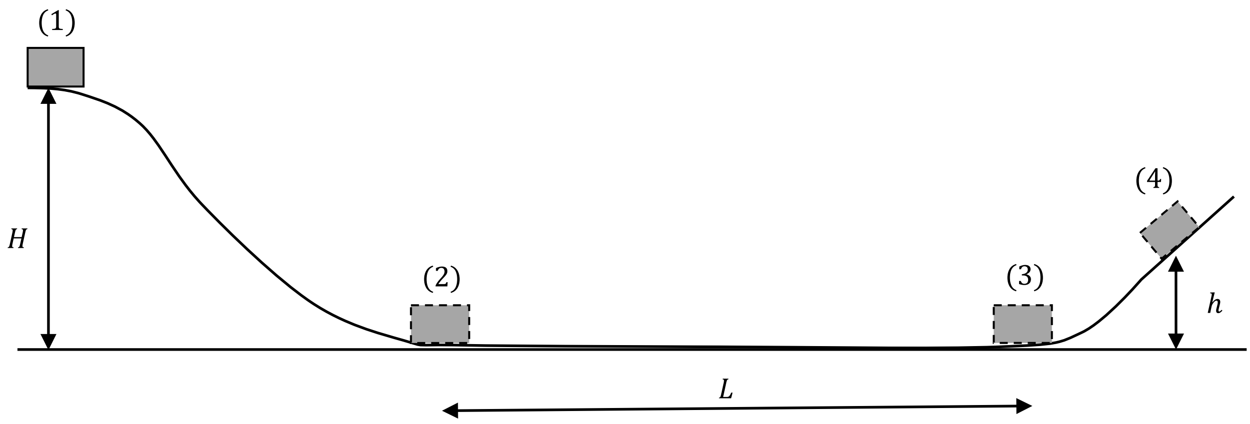
Problem 1.

The snowboarder starts and ends at rest, so $v = K = 0$, which means options A and D are incorrect.

The initial velocity is zero, so the potential energy during the motion can never exceed the initial potential energy. This excludes option B.

The answer is option C.

Problem 2.



a) We set up the equation for conservation of energy from (3)→(4). The reference point for the potential energy is in position (3).

Conservation of energy (3→4):

$$U_3 + K_3 = U_4 + K_4 \Leftrightarrow$$
$$0 + \frac{1}{2}mv_3^2 = mgh + 0 \Leftrightarrow$$
$$v_3 = \sqrt{2gh}$$

The answer is option D.

b) We set up the conservation of energy from (1)→(3), where the work done by friction W is included.

Conservation of energy (1→3):

$$U_1 + K_1 + W = U_3 + K_3 \Leftrightarrow$$
$$mgH + 0 + W = 0 + \frac{1}{2}mv_3^2 \Leftrightarrow$$
$$W = \frac{1}{2}mv_3^2 - mgH$$

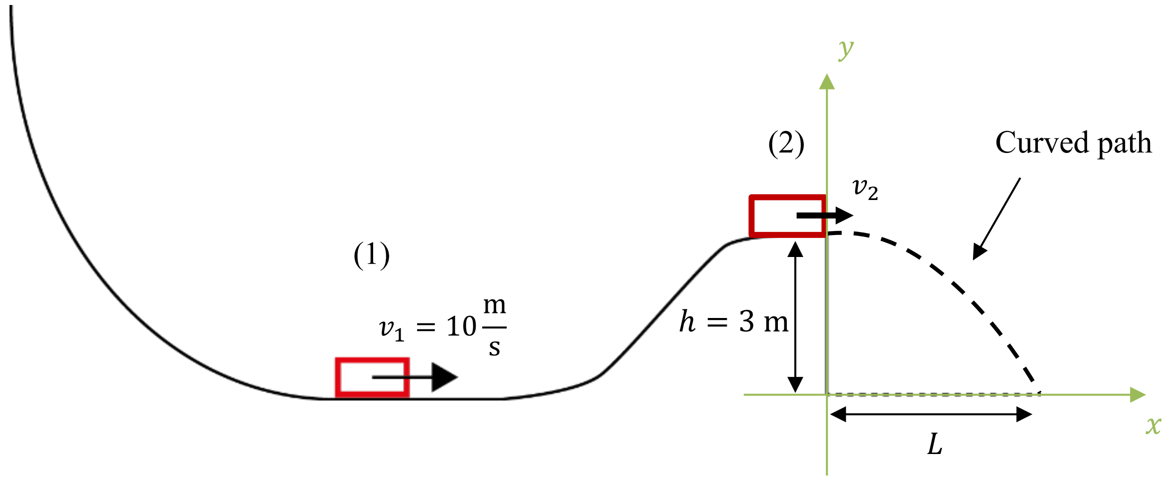
We insert the expression for v_3 from question a) and find

$$W = \frac{1}{2}m(2gh) - mgH = mg(h - H)$$

The answer is option C.

Problem 3.

The sled performs a projectile motion from the top of the hill until it hits the ground, see sketch below.



We set up the projectile motion with the initial position as shown in the coordinate system in the sketch. The initial velocity v_2 only has a x-component so the angle is 0°:

Projectile motion: $y(x) = h - \frac{g}{2v_2^2}x^2$

The point of impact for the sled is $(x, y) = (L, 0)$, where L is unknown.

$$0 = h - \frac{g}{2v_2^2}L^2 \Leftrightarrow$$
$$L^2 = \frac{2h}{g}v_2^2$$

To determine L we must know the speed v_2. Conservation of energy for the motion (1)→(2) gives

Conservation of energy (1→2): $U_1 + K_1 = U_2 + K_2$

$$0 + \frac{1}{2}mv_1^2 = mgh + \frac{1}{2}mv_2^2 \Leftrightarrow$$
$$v_2^2 = v_1^2 - 2gh$$

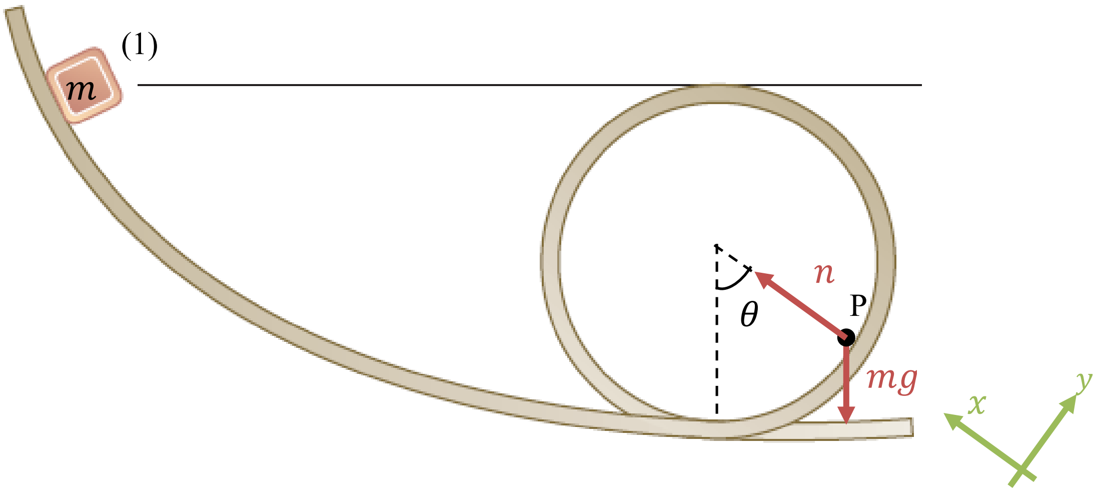
We can now determine L

$$L^2 = \frac{2h}{g}(v_1^2 - 2gh) \Leftrightarrow$$
$$L = \sqrt{\frac{2h}{g}(v_1^2 - 2gh)} = 5.0\ \mathrm{m}$$

The answer is option A.

Problem 4.

a) We draw a force diagram with a coordinate system, where the x-axis points along the radial direction. Both the force of gravity and a normal force is acting on the block.



b) We set up the equations for conservation of energy for the motion from the initial position (1) to the position P. We set the reference for potential energy to be at the bottom of the loop.

Conservation of energy. (1→P): $U_1 + K_1 = U_P + K_P \Leftrightarrow$

$$mg2R + 0 = mg(R - R\cos\theta) + \frac{1}{2}mv_P^2 \Leftrightarrow$$

$$v_P = \sqrt{2gR(1+\cos\theta)}$$

c) We set up Newton's second law in the radial direction and use $a_{rad} = \frac{v_P^2}{R}$.

N2(x): $m\frac{v_P^2}{R} = n - mg\cos\theta$

The block loses contact when the normal force is zero $n = 0$. We find

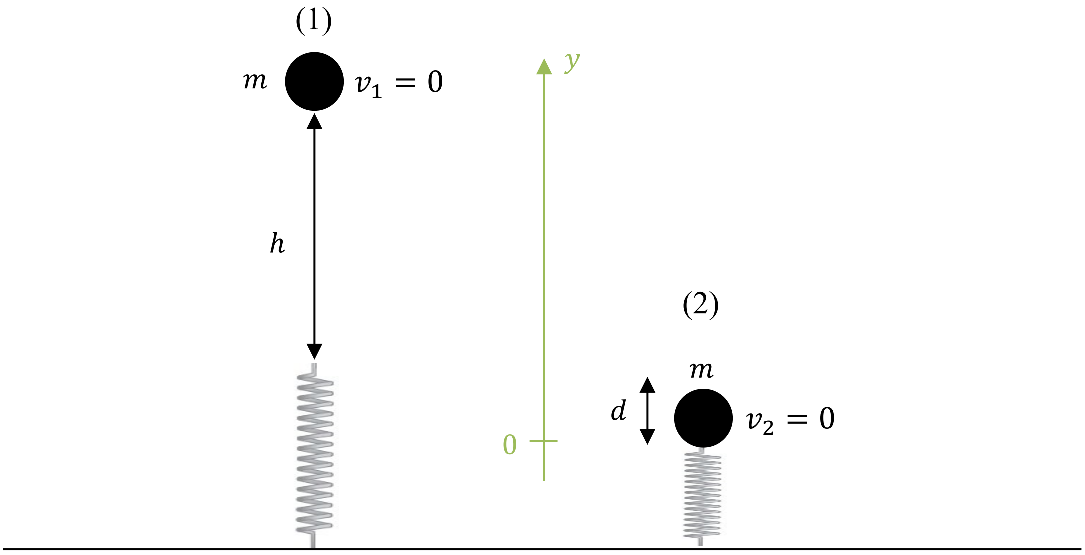
$$m\frac{v_P^2}{R} = -mg\cos\theta$$

We insert the expression for v_P from question b) to determine the angle θ

$$m\frac{2gR(1+\cos\theta)}{R} = -mg\cos\theta \Leftrightarrow$$
$$2 + 2\cos\theta = -\cos\theta \Leftrightarrow$$
$$\cos\theta = -\frac{2}{3} \Leftrightarrow$$
$$\theta = \cos^{-1}\left(-\frac{2}{3}\right) = 131.8°$$

Problem 5.

We draw a sketch of the situation



We set up the conservation of energy for the motion (1)→(2):

Conservation of energy (1→2): $U_{\text{gravity},1} + U_{\text{spring},1} + K_1 = U_{\text{gravity},2} + U_{\text{spring},2} + K_2 \Leftrightarrow$

$$mg(h+d) + 0 + 0 = 0 + \frac{1}{2}kd^2 + 0 \Leftrightarrow$$

$$kd^2 - 2mgd - 2mgh = 0$$

This is a quadratic equation in d. We solve for the positive solution

$$d = \frac{2mg + \sqrt{(2mg)^2 - 4k(-2mgh)}}{2k} \Leftrightarrow$$

$$d = \frac{1}{k}\left(mg + \sqrt{m^2g^2 + 2mgkh}\right) \Leftrightarrow$$

$$d = 0.12\ \text{m}$$

Problem 6.

We use conservation of energy, where the work done by the friction force W is included. The initial position of the block (1) is as shown in the exercise, and the final position (2) is located at the height h. The velocity is 0 in both positions. The reference point for potential energy is at the lowest point in the landscape.

Conservation of energy (1→2):

$$U_1 + K_1 + W = U_2 + K_2 \Leftrightarrow$$
$$mg\left(d + \frac{d}{2}\right) + 0 + W = mgh + 0 \Leftrightarrow$$
$$\frac{3}{2}mgd + W = mgh$$

The work done by the friction force is

$$W = f_{\mathrm{k}}\Delta s\cos(180°) = -\mu_{\mathrm{k}}N\Delta s$$

Here $\mu_{\mathrm{k}} = \frac{1}{2}$ is given in the exercise, and Δs is the sum of the horizontal paths

$$\Delta s = d + \frac{d}{2} = \frac{3}{2}d$$

The normal force N on the horizontal paths are determined from Newton's first law

$$N = mg$$

We find the expression for the work W to be

$$W = -\frac{1}{2}mg\frac{3}{2}d = -\frac{3}{4}mgd$$

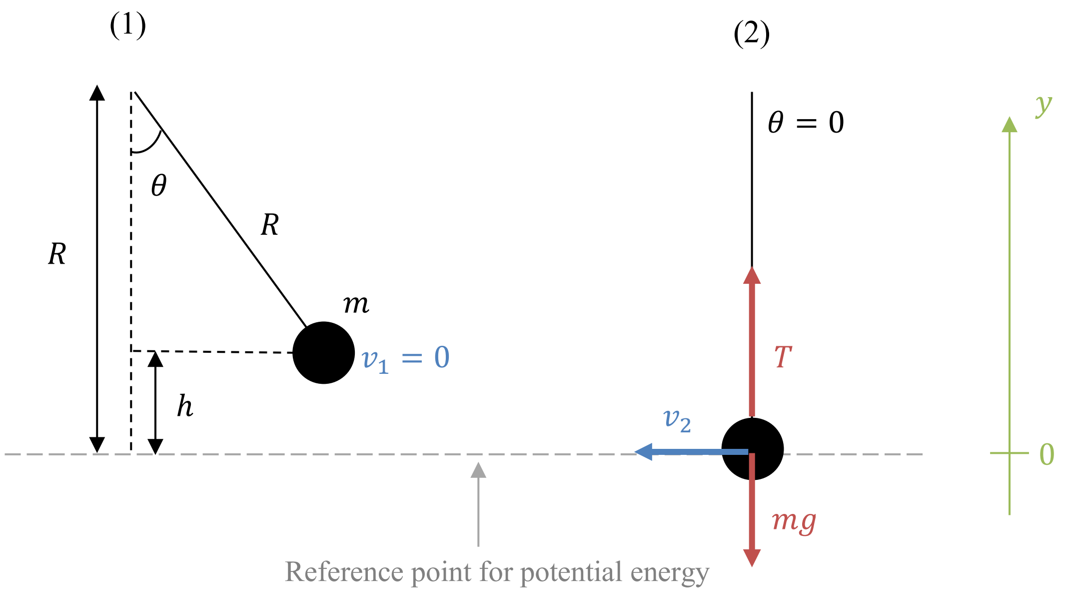
We insert this into the equation for conservation of energy and solve for h:

$$\frac{3}{2}mgd - \frac{3}{4}mgd = mgh \Leftrightarrow$$
$$h = \frac{3}{4}d$$

The answer is option D.

Problem 7.

a) We draw a sketch of the initial position (1), where the string is at an angle θ with respect to the vertical position, along with the final position (2), where the angle is zero for the first time.



We set up conservation of energy for (1)→(2)

Conservation of energy (1→2):

$$U_1 + K_1 = U_2 + K_2 \Leftrightarrow$$
$$mgh + 0 = 0 + \frac{1}{2}mv_2^2 \Leftrightarrow$$
$$v_2 = \sqrt{2gh}$$

Considering the right angle in the sketch

$$h = R - R\cos\theta = R(1 - \cos\theta)$$

The speed of the weight is

$$v_2 = \sqrt{2gR(1 - \cos\theta)}$$

b) We set up Newton's second law in the radial direction for position (2). Since the weight performs a circular motion, we have that $a_{\text{rad}} = \frac{v_2^2}{R}$

N2(y):

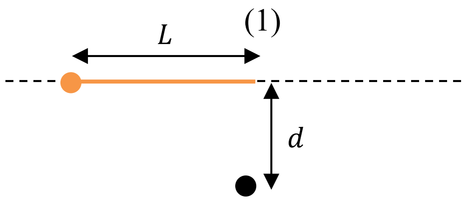
$$m\frac{v_2^2}{R} = T - mg \Leftrightarrow$$
$$T = m\frac{v_2^2}{R} - mg$$

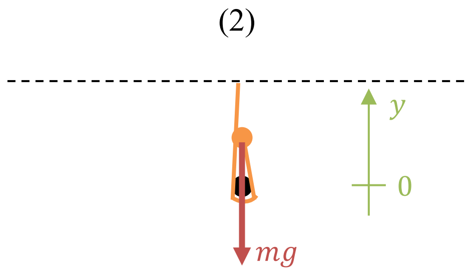
We insert the expression for v_2 from question a) and find

$$T = mg(3 - 2\cos\theta)$$

Problem 8.

a) We draw a sketch of the initial position (1), where the angle is $\theta = 90°$, and the final position (2), where the pendulum is in the vertical position.





If the pendulum has just enough energy to complete the circular motion, then the string tension will be 0 in (2) and the particle will only be affected by the force of gravity. Newton's second law for the y-direction gives

N2(y):
$$ma_{\text{rad}} = mg \Leftrightarrow$$
$$a_{\text{rad}} = g$$

The pendulum performs a circular motion around the nail with a radius of $R = L - d$. The centripetal acceleration is therefore

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{v^2}{L-d}$$

Inserting the acceleration into N2(y) gives

N2(y):
$$\frac{v^2}{L-d} = g$$

The speed v at the top position can be determined from conservation of energy. The reference point for potential energy is set to the same height as the nail. The pendulum is at rest in position (1).

Conservation of energy. (1→2):
$$U_1 + K_1 = U_2 + K_2 \Leftrightarrow$$
$$mgd + 0 = mg(L-d) + \frac{1}{2}mv^2 \Leftrightarrow$$
$$v^2 = g(4d - 2L)$$

We insert the expression for v^2 into N2(y) to determine d

$$\frac{g(4d-2L)}{L-d} = g \Leftrightarrow$$
$$4d - 2L = L - d \Leftrightarrow$$
$$d = \frac{3}{5}L$$