Tarea 07: Unidad 03-B | splines cúbicos

Métodos Numéricos

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Tabla de Contenidos

1 CONJUNTO DE EJERCICIOS

Link Repositorio: https://github.com/Chrissisx/TAREAS-MN/tree/3be63c24143e2f0f9e561bf978bb7b3fe4d8bd74
MN

1.1 1. Dados los puntos $(0,1),(1,5),(2,3)$, determine el *spline* cúbico.

```python
import numpy as np

xs = [0, 1, 2]
ys = [1, 5, 3]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]

import matplotlib.pyplot as plt

x_vals = np.linspace(min(xs), max(xs), 200)
y_vals = np.zeros_like(x_vals)

for i, S in enumerate(splines):
    # Each spline is valid in [xs[i], xs[i+1]]
    mask = (x_vals >= xs[i]) & (x_vals <= xs[i+1])
    y_vals[mask] = [float(S.subs('x', x)) for x in x_vals[mask]]

plt.plot(x_vals, y_vals, label='Cubic Spline')
plt.scatter(xs, ys, color='red', label='Data Points')
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Spline Interpolation')
plt.show()
```

```
1 5 1.0 -4.5 1.5
0 1 5.5 0.0 -1.5
```

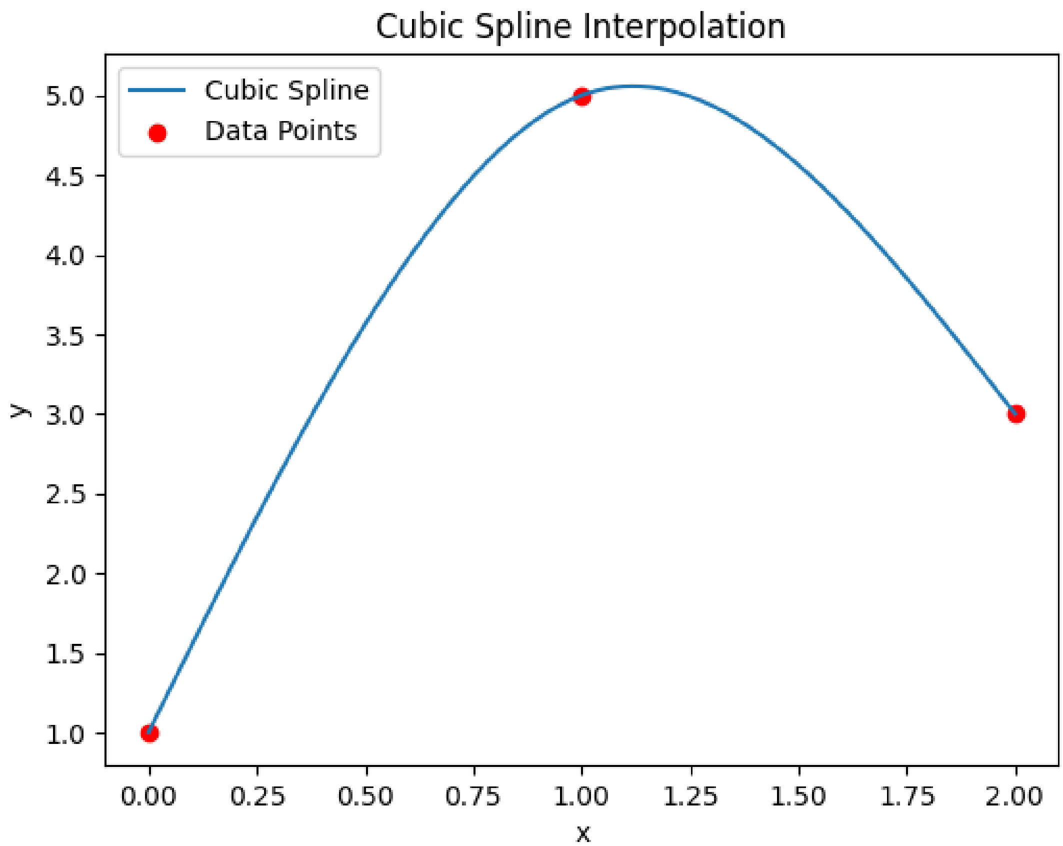
$$-1.5x^3 + 5.5x + 1$$

$$1.0x + 1.5\left(x-1\right)^3 - 4.5\left(x-1\right)^2 + 4.0$$

```
______
```

$$-1.5x^3 + 5.5x + 1$$

$$1.5x^3 - 9.0x^2 + 14.5x - 2.0$$



1.2 2. Dados los puntos $(-1, 1), (1, 3)$, determine el *spline* cúbico sabiendo que $f'(x_0) = 1, f'(x_n) = 2$.

```
xs = [-1, 1]
ys = [1, 3]

splines = cubic_spline_clamped(xs=xs, ys=ys, d0=1, dn=2)

_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]

import matplotlib.pyplot as plt

x_vals = np.linspace(min(xs), max(xs), 200)
y_vals = np.zeros_like(x_vals)

for i, S in enumerate(splines):
    # Each spline is valid in [xs[i], xs[i+1]]
```

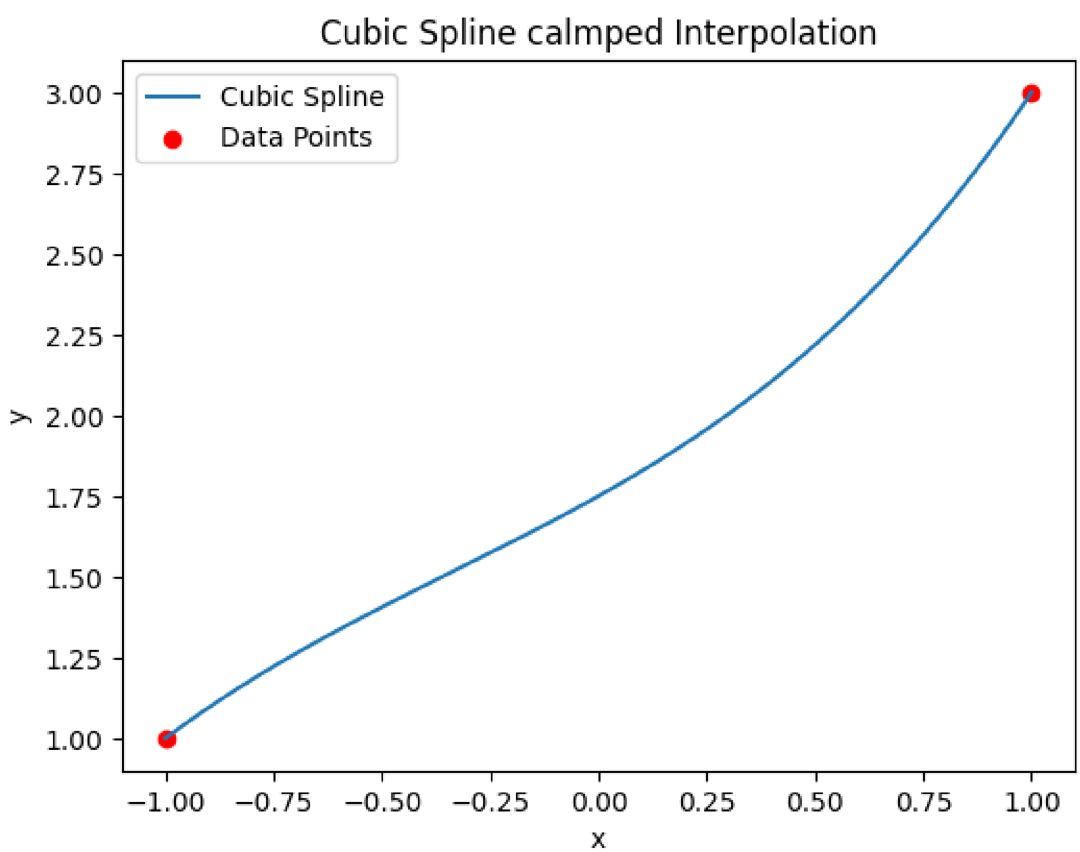
```
    mask = (x_vals >= xs[i]) & (x_vals <= xs[i+1])
    y_vals[mask] = [float(S.subs('x', x)) for x in x_vals[mask]]

plt.plot(x_vals, y_vals, label='Cubic Spline')
plt.scatter(xs, ys, color='red', label='Data Points')
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Spline calmped Interpolation')
plt.show()
```

```
0 1 1.0 -0.5 0.25
```

$1.0x + 0.25\left(x+1\right)^3 - 0.5\left(x+1\right)^2 + 2.0$

```
------
```

$0.25x^3 + 0.25x^2 + 0.75x + 1.75$



1.3 3. Diríjase al pseudocódigo del *spline* cúbico con frontera natural provisto en clase, en base a ese pseudocódigo complete la siguiente función:

```
import sympy as sym
from IPython.display import display


# ####################################################################
def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    """
    Cubic spline interpolation ``S``. Every two points are interpolated by a cubic polyn
    ``S_j`` of the form ``S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3.

    xs must be different  but not necessarily ordered nor equally spaced.

    ## Parameters
    - xs, ys: points to be interpolated

    ## Return
    - List of symbolic expressions for the cubic spline interpolation.
    """

    points = sorted(zip(xs, ys), key=lambda x: x[0])  # sort points by x

    xs = [x for x, _ in points]
    ys = [y for _, y in points]

    n = len(points) - 1  # number of splines

    h = [xs[i + 1] - xs[i] for i in range(n)]  # distances between  contiguous xs

    alpha = [0] * (n + 1)  # coefficients for the tridiagonal system
    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [1]
    u = [0]
    z = [0]

    for i in range(1, n):
        l += [2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]]
        u += [h[i] / l[i]]
```

```
        z += [(alpha[i]-h[i-1]*z[i-1])/l[i]]

    l.append(1)
    z.append(0)
    c = [0] * (n + 1)

    x = sym.Symbol("x")
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d = (c[j + 1] - c[j]) / (3 * h[j])
        a = ys[j]

        print(j, a, b, c[j], d)
        S= a + b * (x - xs[j]) + c[j] * (x - xs[j])**2 + d * (x - xs[j])**3
        splines.append(S)
    splines.reverse()
    return splines
```

1.4 4. Usando la función anterior, encuentre el spline cúbico para:

$$xs = [1, 2, 3]$$

$$ys = [2, 3, 5]$$

```
xs = [1, 2, 3]
ys = [2, 3, 5]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]

import matplotlib.pyplot as plt

x_vals = np.linspace(min(xs), max(xs), 200)
y_vals = np.zeros_like(x_vals)

for i, S in enumerate(splines):
    # Each spline is valid in [xs[i], xs[i+1]]
    mask = (x_vals >= xs[i]) & (x_vals <= xs[i+1])
```

```
    y_vals[mask] = [float(S.subs('x', x)) for x in x_vals[mask]]

plt.plot(x_vals, y_vals, label='Cubic Spline')
plt.scatter(xs, ys, color='red', label='Data Points')
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Spline Interpolation')
plt.show()
```

```
1 3 1.5 0.75 -0.25
0 2 0.75 0.0 0.25
```

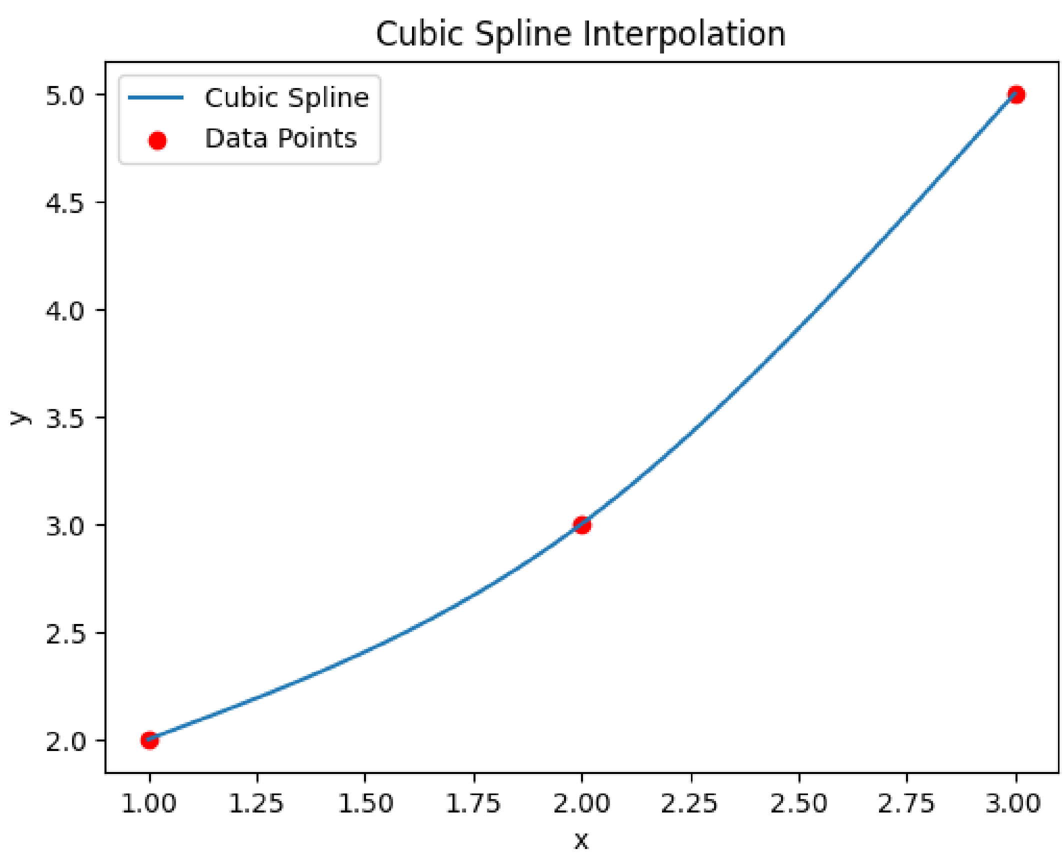
$0.75x + 0.25\,(x-1)^3 + 1.25$

$1.5x - 0.25\,(x-2)^3 + 0.75\,(x-2)^2$

```
------
```

$0.25x^3 - 0.75x^2 + 1.5x + 1.0$

$-0.25x^3 + 2.25x^2 - 4.5x + 5.0$



1.5 5. Usando la función anterior, encuentre el spline cúbico para:

$$xs = [0, 1, 2, 3]$$

$$ys = [-1, 1, 5, 2]$$

```
xs = [0, 1, 2, 3]
ys = [-1, 1, 5, 2]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]

import matplotlib.pyplot as plt

x_vals = np.linspace(min(xs), max(xs), 200)
y_vals = np.zeros_like(x_vals)

for i, S in enumerate(splines):
    # Each spline is valid in [xs[i], xs[i+1]]
    mask = (x_vals >= xs[i]) & (x_vals <= xs[i+1])
    y_vals[mask] = [float(S.subs('x', x)) for x in x_vals[mask]]

plt.plot(x_vals, y_vals, label='Cubic Spline')
plt.scatter(xs, ys, color='red', label='Data Points')
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.title('Cubic Spline Interpolation')
plt.show()
```

```
2 5 1.0 -6.0 2.0
1 1 4.0 3.0 -3.0
0 -1 1.0 0.0 1.0
```

$1.0x^3 + 1.0x - 1$

$4.0x - 3.0\,(x-1)^3 + 3.0\,(x-1)^2 - 3.0$

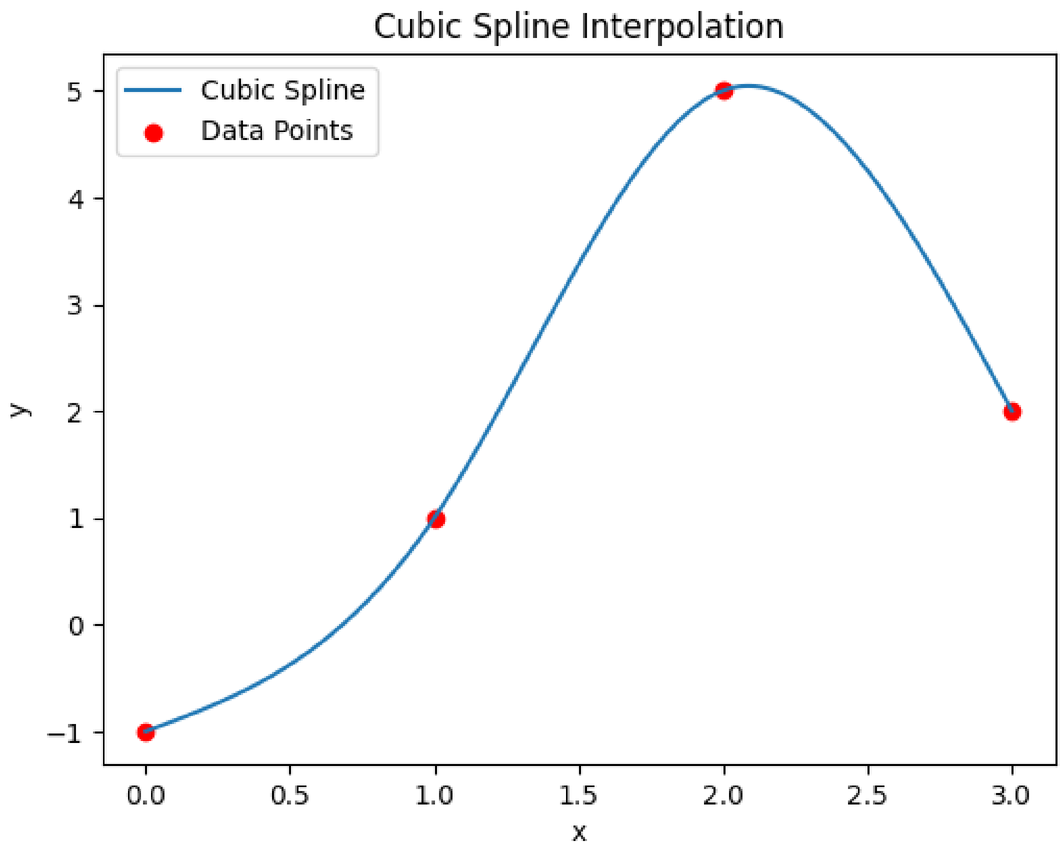
$1.0x + 2.0\,(x-2)^3 - 6.0\,(x-2)^2 + 3.0$

```
______
```

$1.0x^3 + 1.0x - 1$

$-3.0x^3 + 12.0x^2 - 11.0x + 3.0$

$2.0x^3 - 18.0x^2 + 49.0x - 37.0$



1.6 6. Use la función cubic_spline_clamped, provista en el enlace de Github, para graficar los datos de la siguiente tabla.

Curva 1				Curva 2				Curva 3			
i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	−1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	−4.0				
7	13	6.7									
8	17	4.5	−0.67								

Figura 1: TablaDeber.png

```
import sympy as sym
from IPython.display import display


# ####################################################################
def cubic_spline_clamped(
    xs: list[float], ys: list[float], d0: float, dn: float
) -> list[sym.Symbol]:
    """
    Cubic spline interpolation ``S``. Every two points are interpolated by a cubic polyn
    ``S_j`` of the form ``S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3.

    xs must be different  but not necessarily ordered nor equally spaced.

    ## Parameters
    - xs, ys: points to be interpolated
    - d0, dn: derivatives at the first and last points

    ## Return
    - List of symbolic expressions for the cubic spline interpolation.
    """

    points = sorted(zip(xs, ys), key=lambda x: x[0])  # sort points by x
    xs = [x for x, _ in points]
    ys = [y for _, y in points]
    n = len(points) - 1  # number of splines
```

```python
    h = [xs[i + 1] - xs[i] for i in range(n)]  # distances between  contiguous xs

    alpha = [0] * (n + 1)  # prealloc
    alpha[0] = 3 / h[0] * (ys[1] - ys[0]) - 3 * d0
    alpha[-1] = 3 * dn - 3 / h[n - 1] * (ys[n] - ys[n - 1])

    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [2 * h[0]]
    u = [0.5]
    z = [alpha[0] / l[0]]

    for i in range(1, n):
        l += [2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]]
        u += [h[i] / l[i]]
        z += [(alpha[i] - h[i - 1] * z[i - 1]) / l[i]]

    l.append(h[n - 1] * (2 - u[n - 1]))
    z.append((alpha[n] - h[n - 1] * z[n - 1]) / l[n])
    c = [0] * (n + 1)  # prealloc
    c[-1] = z[-1]

    x = sym.Symbol("x")
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d = (c[j + 1] - c[j]) / (3 * h[j])
        a = ys[j]
        print(j, a, b, c[j], d)
        S = a + b * (x - xs[j]) + c[j] * (x - xs[j]) ** 2 + d * (x - xs[j]) ** 3

        splines.append(S)
    splines.reverse()
    return splines
```

1.6.1 Primera Curva

```python
xs = [1, 2, 5, 6, 7, 8, 10, 13, 17]
ys = [3, 3.7, 3.9, 4.2, 5.7, 6.6, 7.1, 6.7, 4.5]
```

```python
splines = cubic_spline_clamped(xs=xs, ys=ys, d0=1, dn=-0.67)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

```
7 6.7 -0.3381314976116886 -0.07593425119415571 0.0057417813992694635
6 7.1 0.04846024164091059 -0.052929661890044014 -0.0025560654782346335
5 6.6 0.5472201929380908 -0.19645031375854607 0.023920108644750342
4 5.7 1.4091093003652708 -0.665438793668634 0.15632949330336265
3 4.2 1.0163426056008245 1.0582054884330803 -0.5745480940339047
2 3.9 -0.07447972276856785 0.03261683993631198 0.3418628828322561
1 3.7 0.4468099653460711 -0.20638006930785827 0.02655521213824114
0 3 1.0 -0.3468099653460706 0.046809965346070785
```

$1.0x + 0.0468099653460708\left(x-1\right)^3 - 0.346809965346071\left(x-1\right)^2 + 2.0$

$0.446809965346071x + 0.0265552121382411\left(x-2\right)^3 - 0.206380069307858\left(x-2\right)^2 + 2.80638006930786$

$-0.0744797227685678x + 0.341862882832256\left(x-5\right)^3 + 0.032616839936312\left(x-5\right)^2 + 4.27239861384284$

$1.01634260560082x - 0.574548094033905\left(x-6\right)^3 + 1.05820548843308\left(x-6\right)^2 - 1.89805563360495$

$1.40910930036527x + 0.156329493303363\left(x-7\right)^3 - 0.665438793668634\left(x-7\right)^2 - 4.1637651025569$

$0.547220192938091x + 0.0239201086447503\left(x-8\right)^3 - 0.196450313758546\left(x-8\right)^2 + 2.22223845649527$

$0.0484602416409106x - 0.00255606547823463\left(x-10\right)^3 - 0.052929661890044\left(x-10\right)^2 + 6.61539758359089$

$-0.338131497611689x + 0.00574178139926946\left(x-13\right)^3 - 0.0759342511941557\left(x-13\right)^2 + 11.095709468952$

```
______
```

$0.0468099653460708x^3 - 0.487239861384283x^2 + 1.83404982673035x + 1.60638006930786$

$0.0265552121382411x^3 - 0.365711342137305x^2 + 1.5909927882364x + 1.7684180949705$

$0.341862882832256x^3 - 5.09532640254753x^2 + 25.2390680902875x - 37.6450407417814$

$-0.574548094033905x^3 + 11.4000711810434x^2 - 73.7333174112578x + 160.299730261309$

$0.156329493303363x^3 - 3.94835815303925x^2 + 33.7056879273205x - 90.3912821953733$

$0.0239201086447503x^3 - 0.770532921232554x^2 + 8.28308607286689x - 22.5976772501638$

$-0.00255606547823463x^3 + 0.023752302456995x^2 + 0.340233835971401x + 3.87849687282113$

$0.00574178139926946x^3 - 0.299863725765665x^2 + 4.54724220286598x - 14.3518727170554$

1.6.2 Segunda Curva

```
xs = [17,20,23,24,25,27,27.7]
ys = [4.5,7.0,6.1,5.6,5.8,5.2,4.1]

splines = cubic_spline_clamped(xs=xs, ys=ys, d0=3, dn=-4.0)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

```
5 5.2 -0.4011781849199465 0.1258152222202451 -2.568002126658778
4 5.8 0.1539868142803838 -0.4033977218204103 0.08820215734010924
3 5.6 -0.11137135038117751 0.6687558864819717 -0.35738453610079396
2 6.1 -0.6085014127556733 -0.17162582410747595 0.2801272368631492
1 7.0 -0.19787464681108174 0.03475023545927881 -0.022930673285194974
0 4.5 3.0 -1.1007084510629728 0.12616207628025017
```

$3.0x + 0.12616207628025(x - 17)^3 - 1.10070845106297(x - 17)^2 - 46.5$

$-0.197874646811082x - 0.022930673285195(x - 20)^3 + 0.0347502354592788(x - 20)^2 + 10.9574929362216$

$-0.608501412755673x + 0.280127236863149(x - 23)^3 - 0.171625824107476(x - 23)^2 + 20.0955324933805$

$-0.111371350381178x - 0.357384536100794(x - 24)^3 + 0.668755886481972(x - 24)^2 + 8.27291240914826$

$0.153986814280384x + 0.0882021573401092(x - 25)^3 - 0.40339772182041(x - 25)^2 + 1.9503296429904$

$-0.401178184919947x - 2.56800212665878(x - 27)^3 + 0.125815222220245(x - 27)^2 + 16.0318109928386$

```
______
```

$0.12616207628025x^3 - 7.53497434135573x^2 + 149.806607471118x - 984.439023122068$

$-0.022930673285195x^3 + 1.41059063257098x^2 - 29.1046920074162x + 208.302973401493$

$0.280127236863149x^3 - 19.5004051676648x^2 + 451.848211398006x - 3479.00261937341$

$-0.357384536100794x^3 + 26.4004424857391x^2 - 649.772132283688x + 5333.96013008014$

$0.0882021573401092x^3 - 7.0185595223286x^2 + 185.702917918006x - 1628.33195493397$

$-2.56800212665878x^3 + 208.133987481581x^2 - 5623.41585118756x + 50653.7369670161$

1.6.3 Tercera Curva

```
xs = [27.7,28,29,30]
ys = [4.1,4.3,4.1,3.0]

splines = cubic_spline_clamped(xs=xs, ys=ys, d0=0.33, dn=-1.5)
_ = [display(s) for s in splines]
print("______")
_ = [display(s.expand()) for s in splines]
```

```
2 4.1 -0.7653465346534649 -0.26930693069306927 -0.06534653465346556
1 4.3 0.6613861386138599 -1.1574257425742556 0.2960396039603954
0 4.1 0.3299999999999999 2.2620462046204524 -3.799413274660778
```

$0.33x - 3.79941327466078\left(x - 27.7\right)^3 + 2.26204620462045\left(x - 27.7\right)^2 - 5.041$

$0.66138613861386x + 0.296039603960395\left(x - 28\right)^3 - 1.15742574257426\left(x - 28\right)^2 - 14.2188118811881$

$-0.765346534653465x - 0.0653465346534656\left(x - 29\right)^3 - 0.269306930693069\left(x - 29\right)^2 + 26.2950495049505$

```
______
```

$-3.79941327466078x^3 + 317.993289328931x^2 - 8870.74279427938x + 82483.079611294$

$0.296039603960395x^3 - 26.0247524752475x^2 + 761.762376237622x - 7420.30198019801$

$-0.0653465346534656x^3 + 5.41584158415843x^2 - 150.014851485149x + 1393.54455445545$