SCHOOL OF MATHEMATICS AND STATISTICS

MAST30013 Techniques in Operations Research Semester 1, 2018

Assignment 3 **Due: 10am, Monday, 14 May**

- Submission via the OneNote page.
- All assignments must have 'typeset quality'.
- Show all necessary working.

1. Consider the quadratic program

$$\begin{array}{ll} \min & \frac{1}{2}x^T Q x + c^T x \\ \text{s.t.} & Ax \leq b, \end{array}$$

where $x, c \in \mathbb{R}^n$, $b \in \mathbb{R}^m \geq 0$ $A \in \mathbb{R}^{m \times n}$.

   (a) Find all points satisfying the KKT conditions.

   (b) Show that the KKT conditions are necessary and sufficient for a global minimizer if $Q = Q^T > 0$.

2. Consider the nonlinear program

$$\begin{array}{ll} \min & (x_1 - 2)^4/4 + x_2^4 + 4 \\ \text{s.t.} & x_1 - x_2 \leq 8 \\ & x_1 - x_2^2 \geq 4. \end{array}$$

   (a) Write down the KKT conditions and find all points that satisfy the conditions.

   (b) Check that one of the constraint qualifications holds.

   (c) Check whether or not the point(s) in (a) is a local minimizer.

(d) Make a graphical illustration of the NLP. Verify that there is one minimizer and one active constraint. (Hint: sketch level curves at 8, 100, 500 and curves $x_1 - x_2 - 8 = 0$, $x_1 - x_2^2 - 4 = 0$.)

(e) Show that the objective function is convex for all x in the constraint set.

(f) Show that the Lagrangian Saddle Point inequalities hold for all x in the constraint set and $\lambda \geq 0$, that is

$$L(x^*, \lambda) \;\leq\; L(x^*, \lambda^*) \;\leq\; L(x, \lambda^*).$$

3. Consider the nonlinear program

$$\begin{aligned} \min \quad & x_2^3 - x_1 - 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \;\leq\; 1 \\ & x_1,\; x_2 \;\geq\; 0. \end{aligned}$$

(a) Write down the log barrier penalty function $P_k(\boldsymbol{x})$ with penalty parameter k.

(b) Write down $\nabla P_k(x)$, and solve $\boldsymbol{\nabla} P_k(x) = 0$ to find any stationary points $x^k = (x_1^k, x_2^k)$ for $P_k(x)$.

(c) Find the limit $x^* = \lim\limits_{k\to\infty} x^k$.

(d) Write down an estimate λ^k of the optimal Lagrange multiplier, and find the limit $\lambda^* = \lim\limits_{k\to\infty} \lambda^k$.