Deep Sequence Modeling

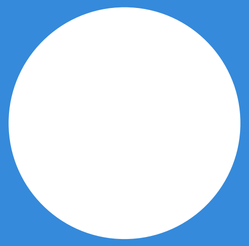
MIT 6.S191

Ava Soleimany

January 28, 2019

MIT EECS

Given an image of a ball,
can you predict where it will go next?



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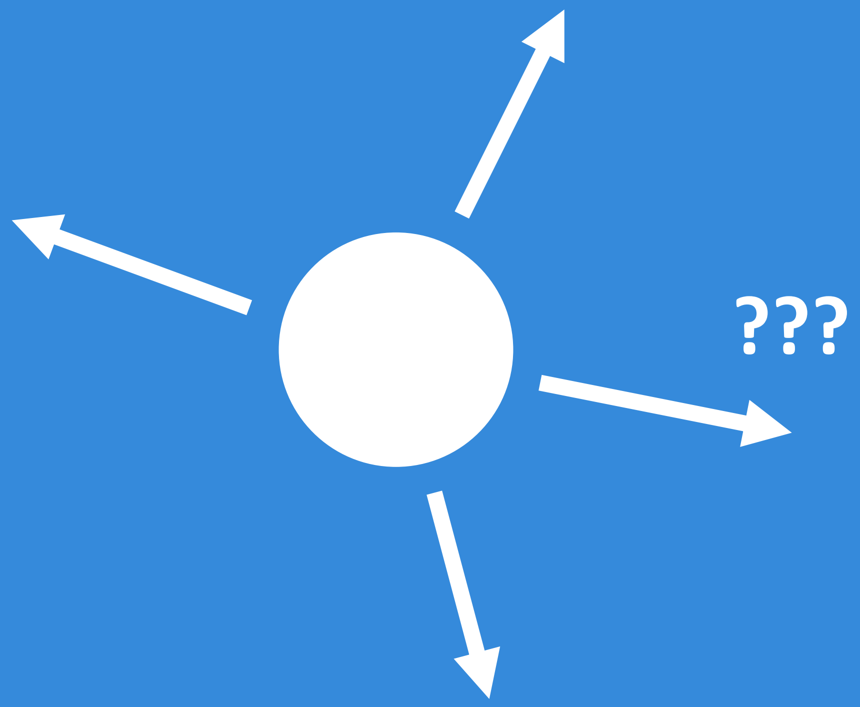

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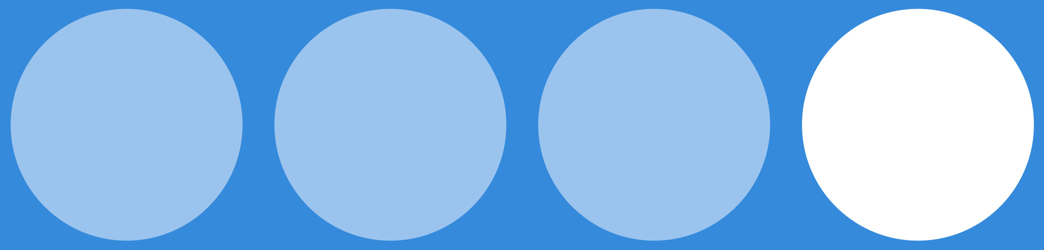

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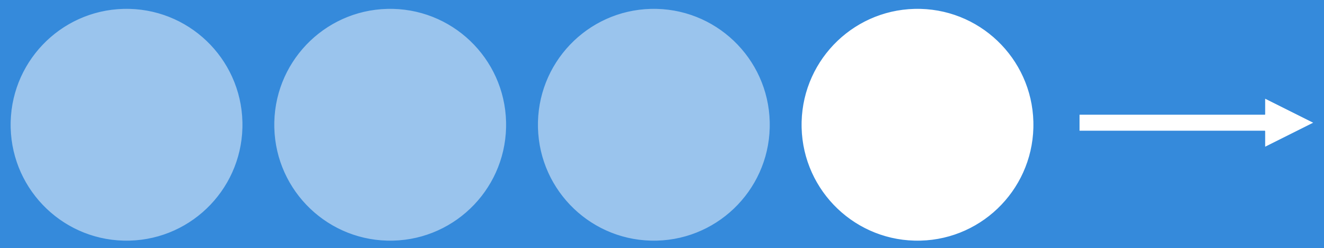

Sequences in the wild

Audio

Sequences in the wild

character:

6.S191 Introduction to Deep Learning

word:

Text

A Sequence Modeling Problem:
Predict the Next Word

A sequence modeling problem: predict the next word

"This morning I took my cat for a walk."

Adapted from H. Suresh, 6.S191 2018

A sequence modeling problem: predict the next word

"This morning I took my cat for a walk."

given these words

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A sequence modeling problem: predict the next word

"This morning I took my cat for a walk."

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predict the next word

Adapted from H. Suresh, 6.S191 2018

Idea #1: use a fixed window

"This morning I took my cat for a walk."

given these two words

predict the next word

Adapted from H. Suresh, 6.S191 2018

Idea #1: use a fixed window

"This morning I took my cat for a walk."

given these two words

predict the next word

One-hot feature encoding: tells us what each word is

[1 0 0 0 0 0 1 0 0 0]

for a

prediction

Adapted from H. Suresh, 6.S191 2018

Problem #1: can't model long-term dependencies

"**France** is where I grew up, but I now live in Boston. I speak fluent ___."

J'aime 6.S191!

We need information from **the distant past** to accurately predict the correct word.

Adapted from H. Suresh, 6.S191 2018

Idea #2: use entire sequence as set of counts

"This morning I took my cat for a"

↓

"bag of words"

[0 1 0 0 1 0 0 … 0 0 1 1 0 0 0 1]

↓

prediction

Adapted from H. Suresh, 6.S191 2018

Problem #2: counts don't preserve order

The food was good, not bad at all.

vs.

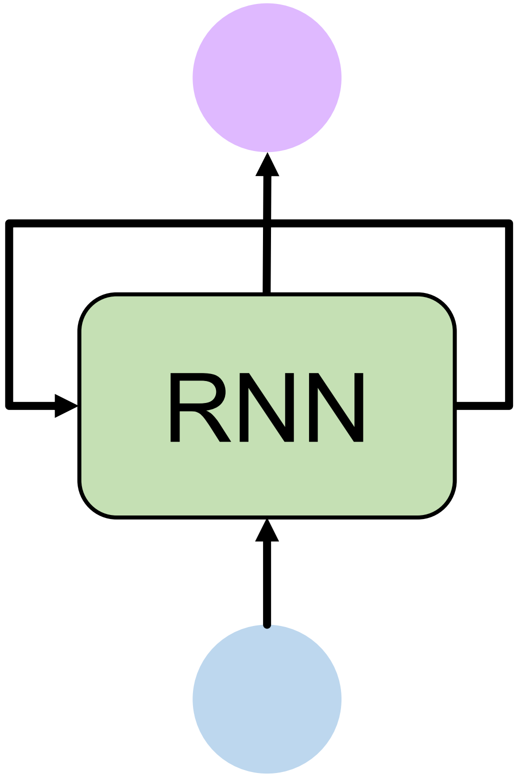
The food was bad, not good at all.

Adapted from H. Suresh, 6.S191 2018

Sequence modeling: design criteria

To model sequences, we need to:

1. Handle **variable-length** sequences
2. Track **long-term** dependencies
3. Maintain information about **order**
4. **Share parameters** across the sequence

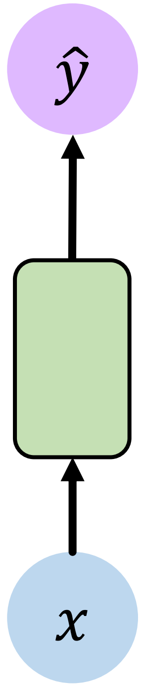


Today: **Recurrent Neural Networks (RNNs)** as an approach to sequence modeling problems

Adapted from H. Suresh, 6.S191 2018

Recurrent Neural Networks (RNNs)

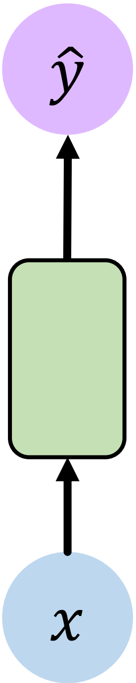
Standard feed-forward neural network



One to One
"Vanilla" neural network

[1]

Recurrent neural networks: sequence modeling



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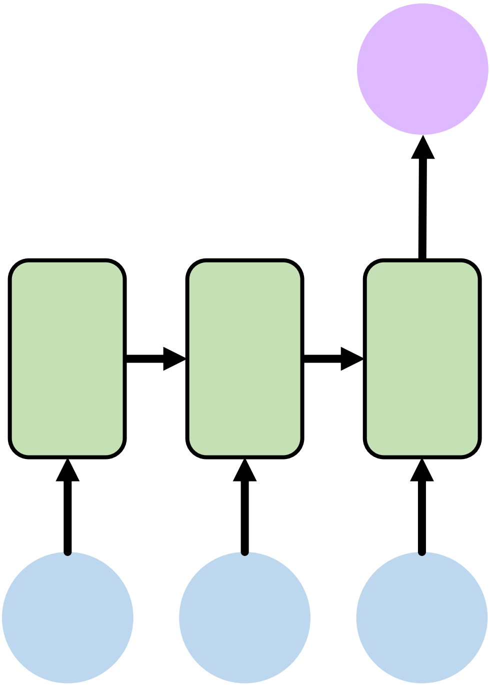
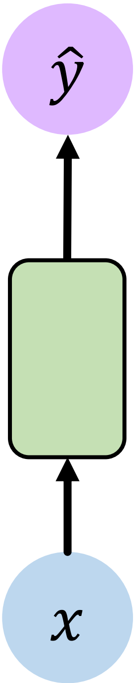
"Vanilla" neural network



Many to One
Sentiment Classification

[1]

Recurrent neural networks: sequence modeling



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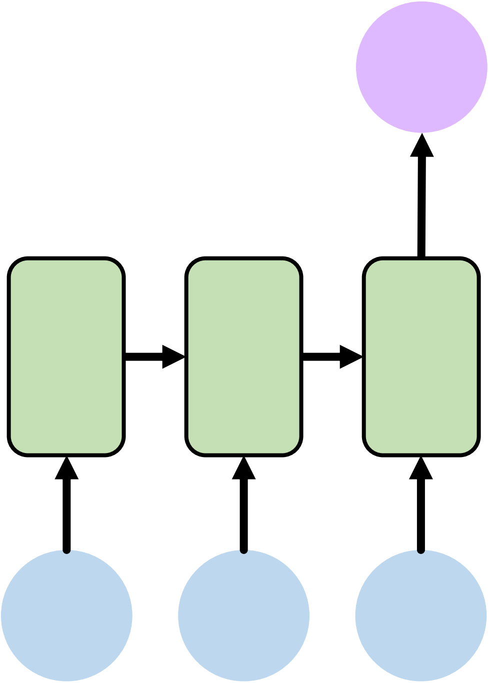

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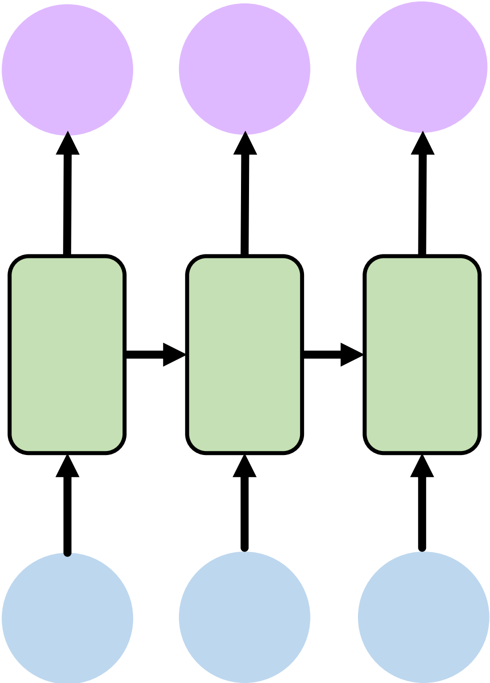
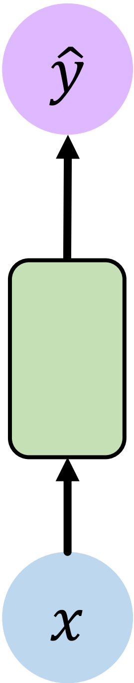


Many to Many
Music Generation

6.S191 Lab!

[1]

Recurrent neural networks: sequence modeling



One to One
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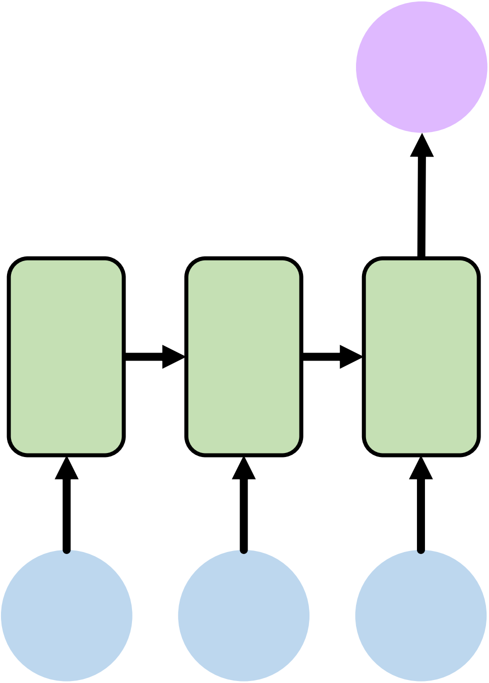

Many to One
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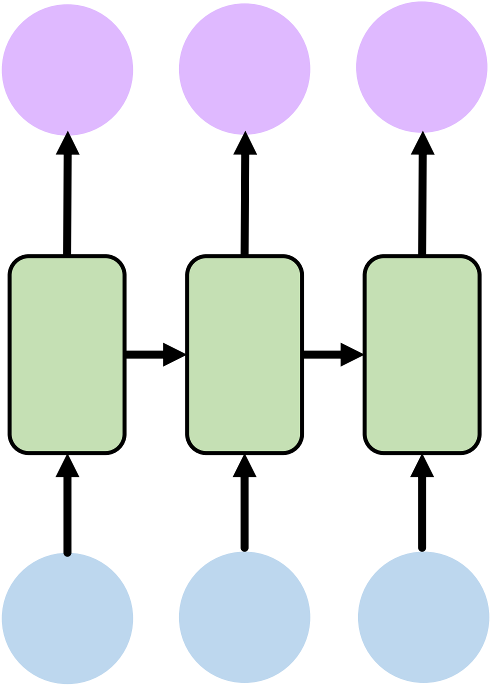


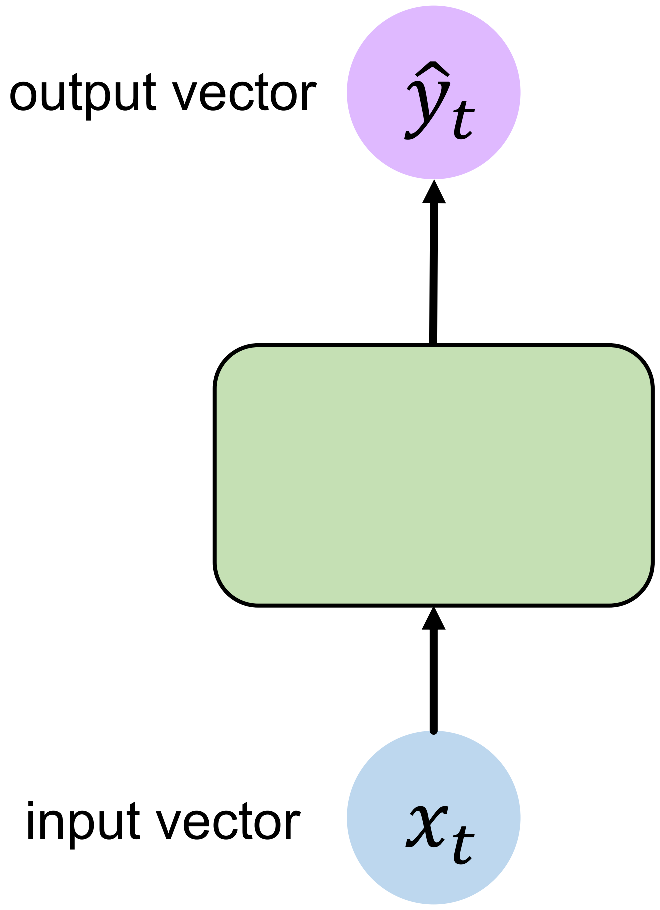
Many to Many
Music Generation

6.S191 Lab!

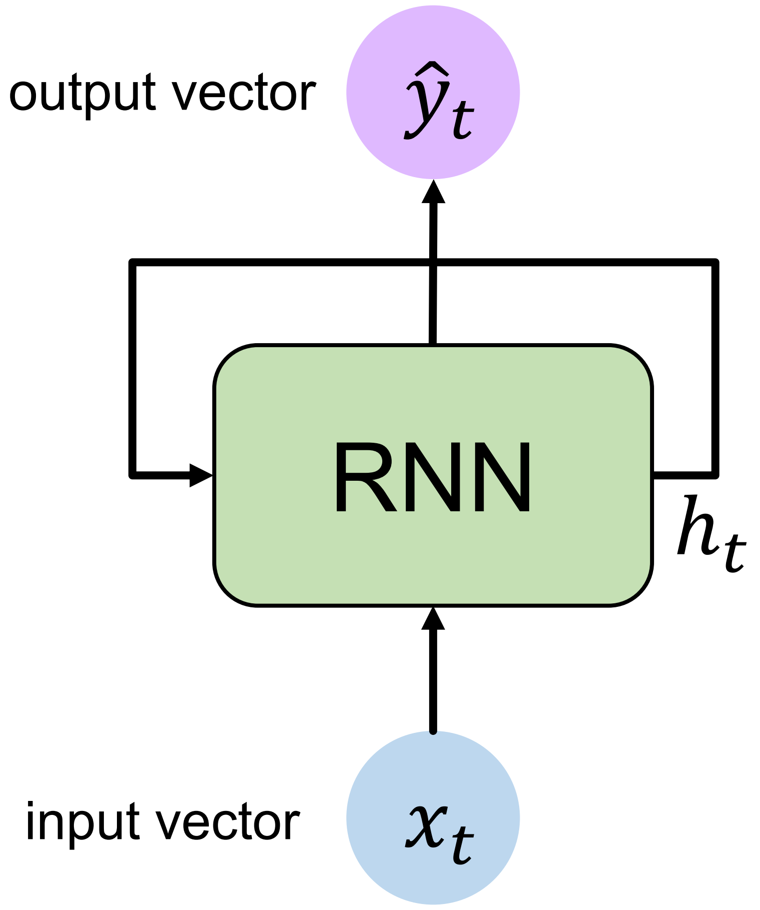
… and many other architectures and applications

[1]

A standard "vanilla" neural network

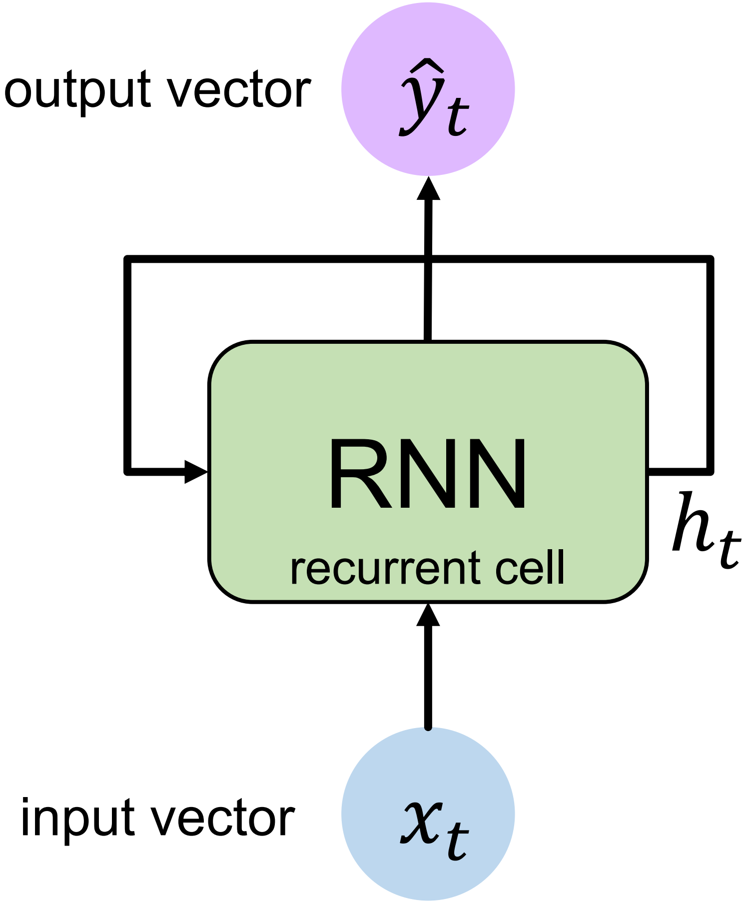


A recurrent neural network (RNN)



[2]

A recurrent neural network (RNN)



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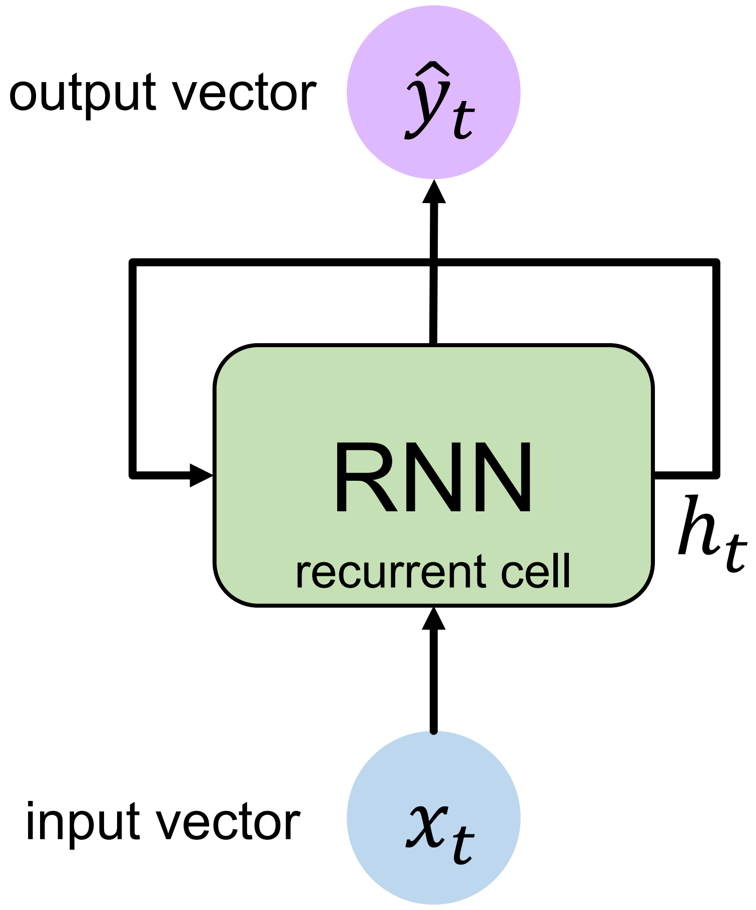
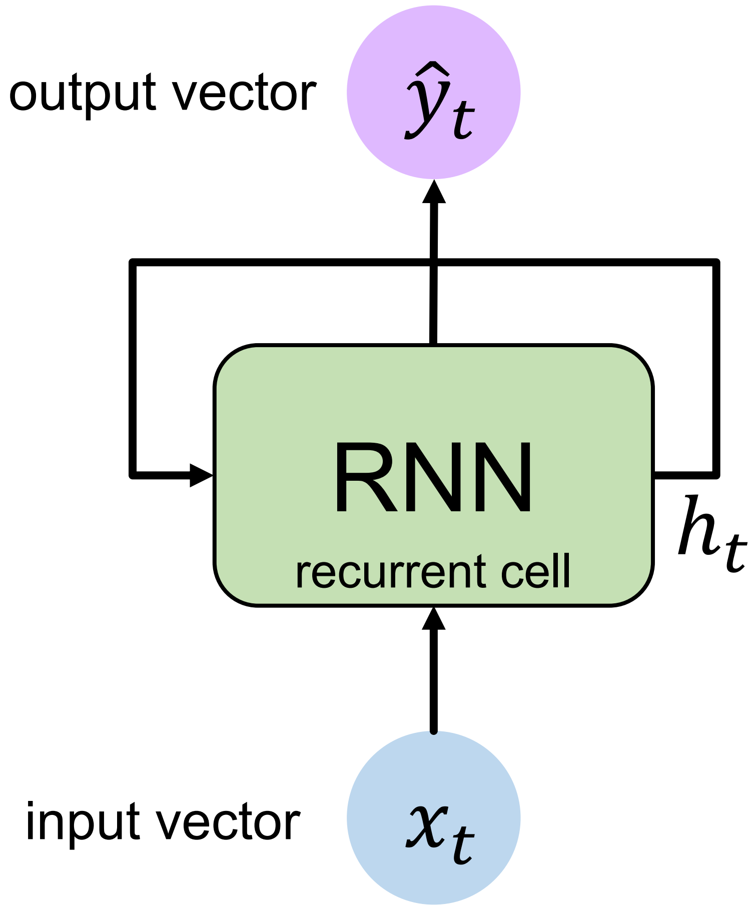


Apply a **recurrence relation** at every time step to process a sequence:

A recurrent neural network (RNN)



Apply a **recurrence relation** at every time step to process a sequence:

$$\boxed{h_t} = \boxed{f_W}(\boxed{h_{t-1}}, \boxed{x_t})$$

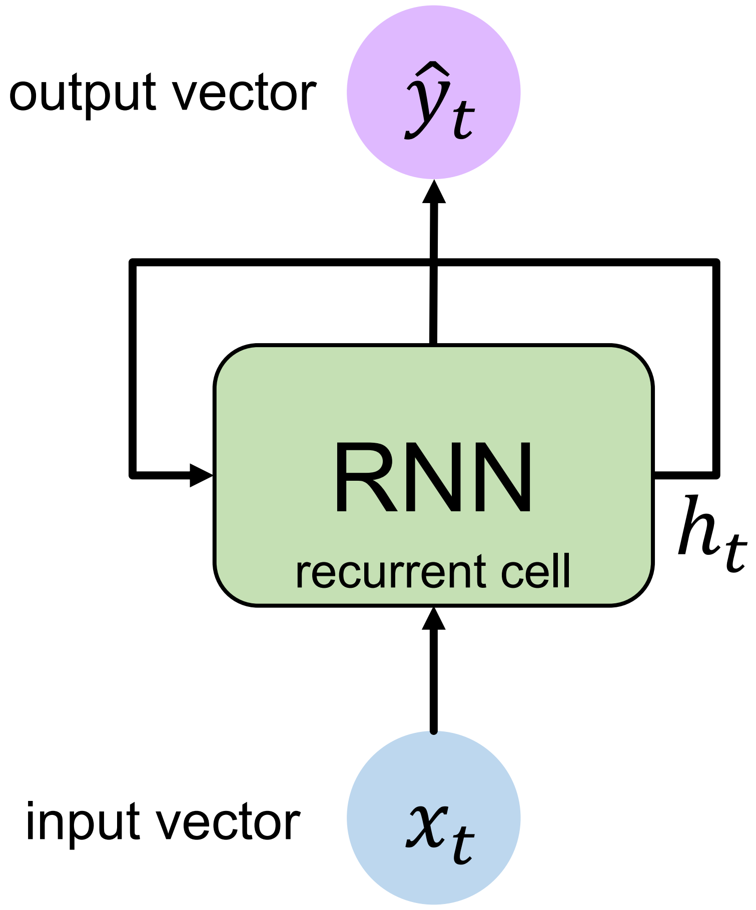
cell state

function parameterized by W

old state

input vector at time step *t*

A recurrent neural network (RNN)



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new state

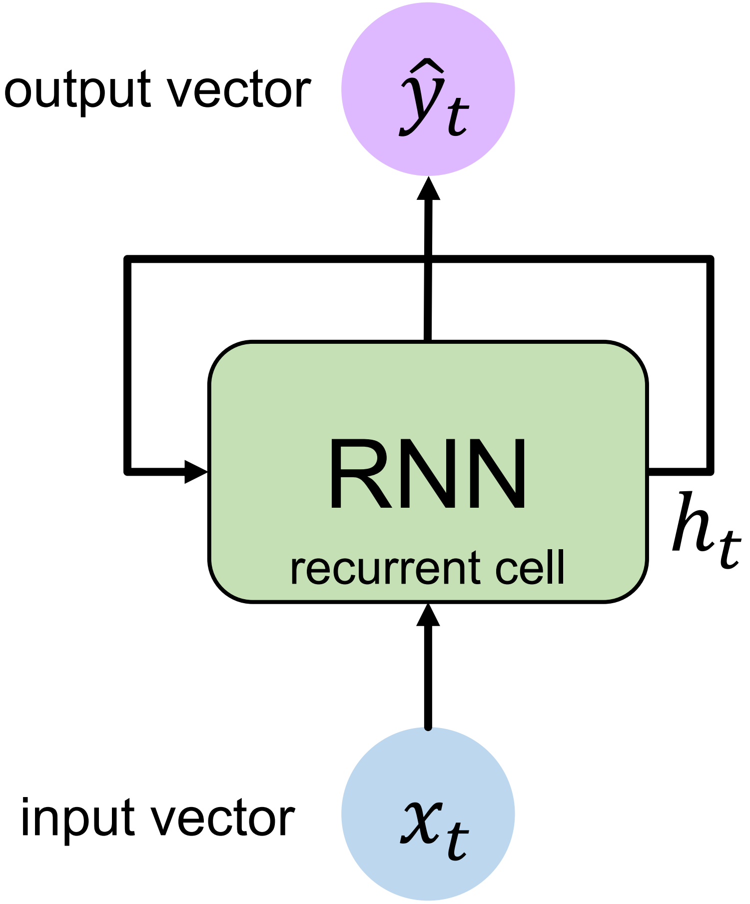
function parameterized by W

old state

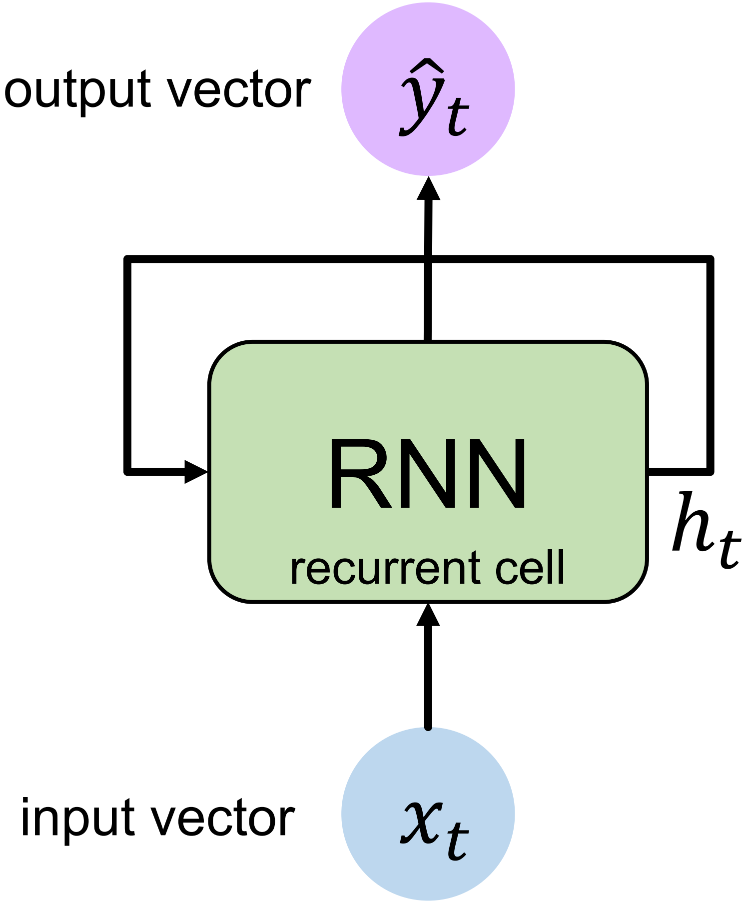
input vector at time step *t*

Note: the same function and set of parameters are used at every time step

RNN state update and output

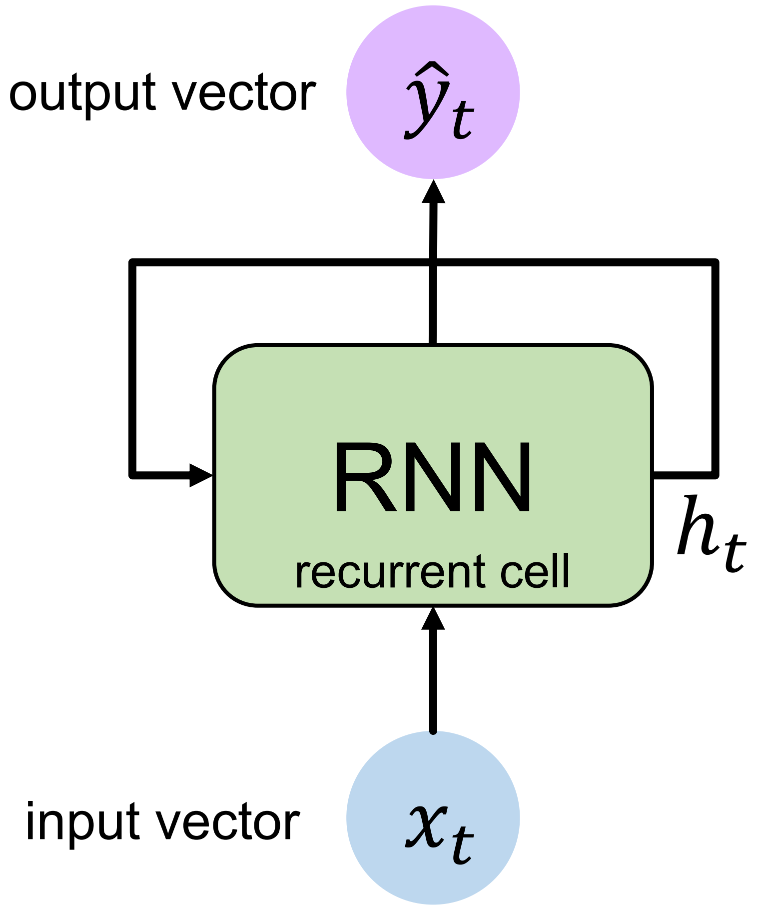


RNN state update and output



Input Vector

RNN state update and output

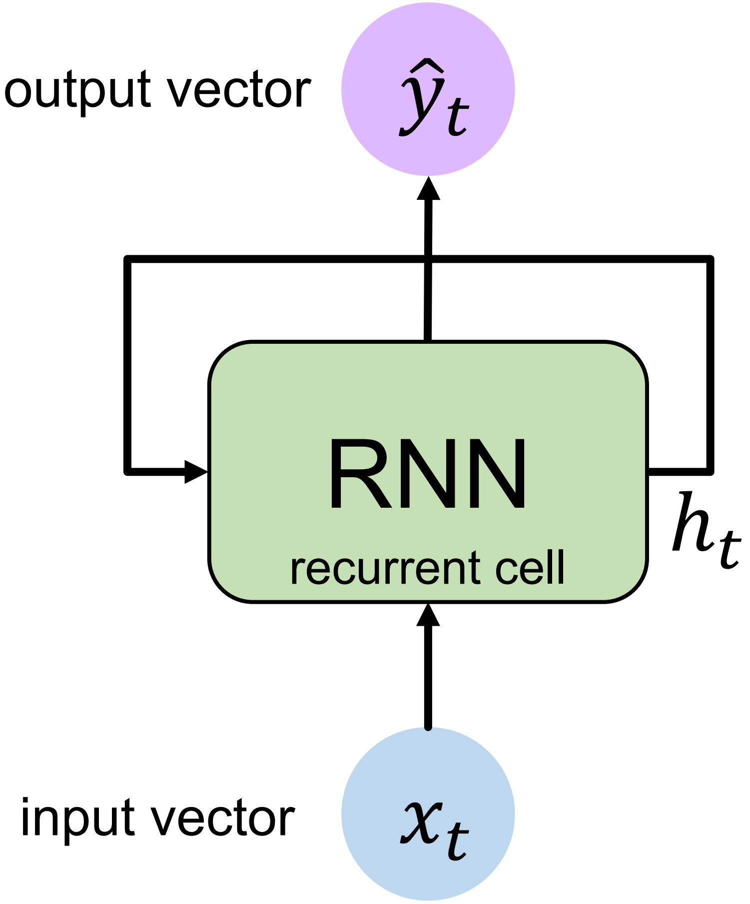


Update Hidden State

$$h_t = \tanh(\boldsymbol{W_{hh}} h_{t-1} + \boldsymbol{W_{xh}} x_t)$$

Input Vector

RNN state update and output



Output Vector

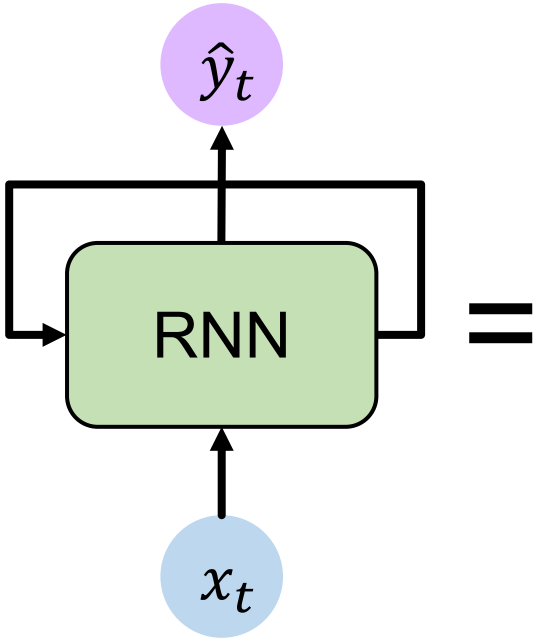
$$\hat{y}_t = \boldsymbol{W}_{\boldsymbol{hy}} h_t$$

Update Hidden State

$$h_t = \tanh(\boldsymbol{W}_{\boldsymbol{hh}} h_{t-1} + \boldsymbol{W}_{\boldsymbol{xh}} x_t)$$

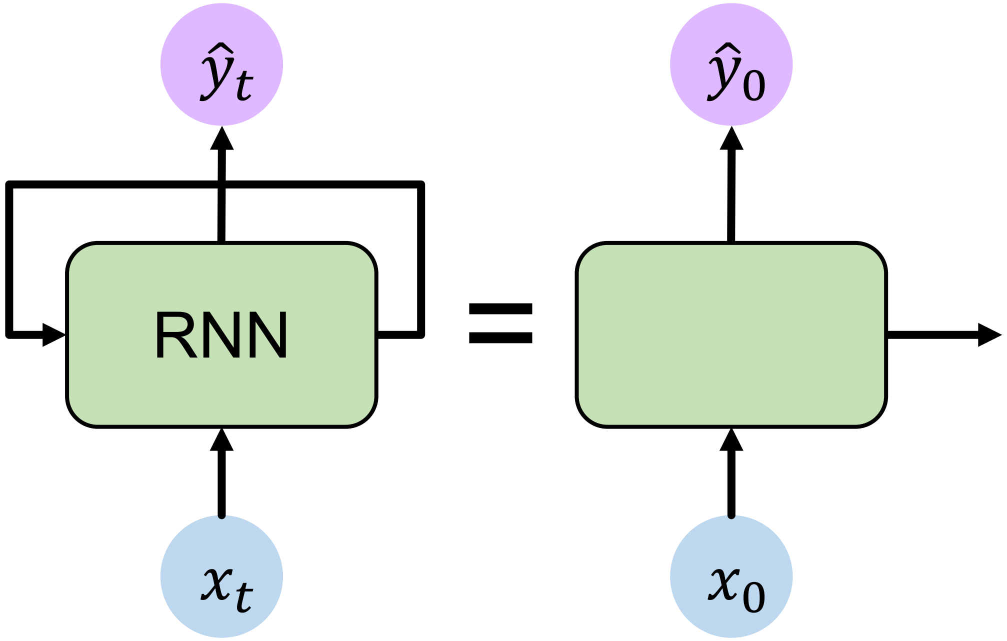
Input Vector

RNNs: computational graph across time

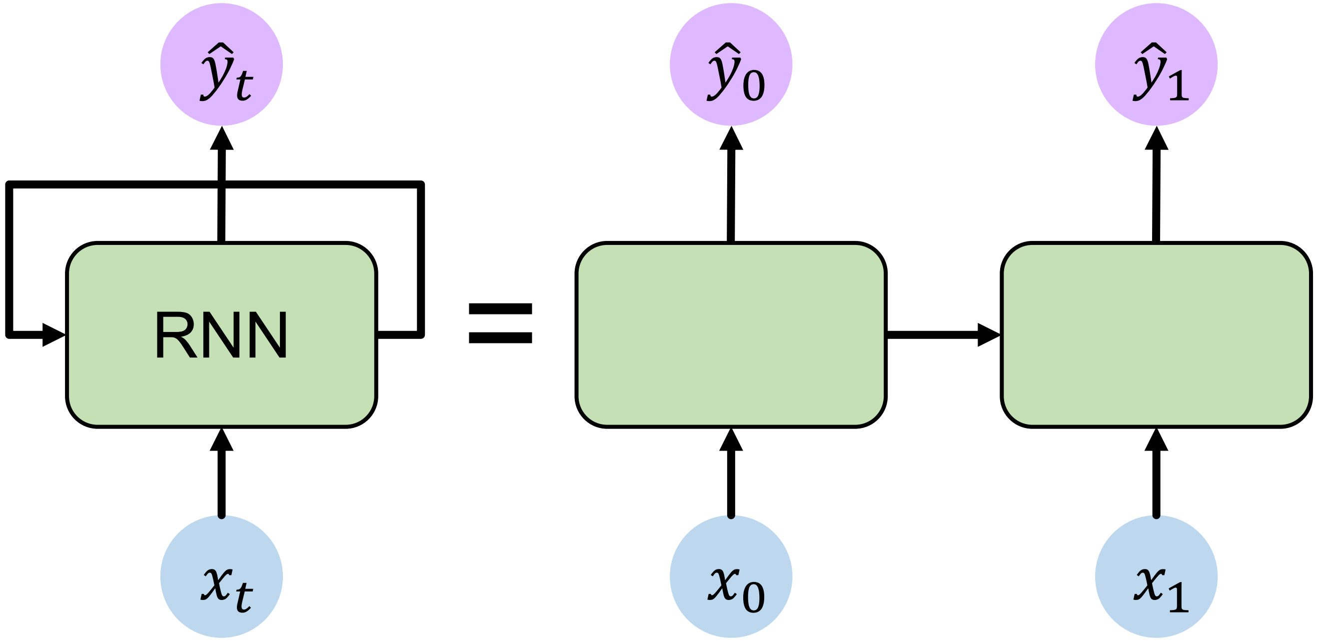


Represent as computational graph unrolled across time

RNNs: computational graph across time



RNNs: computational graph across time



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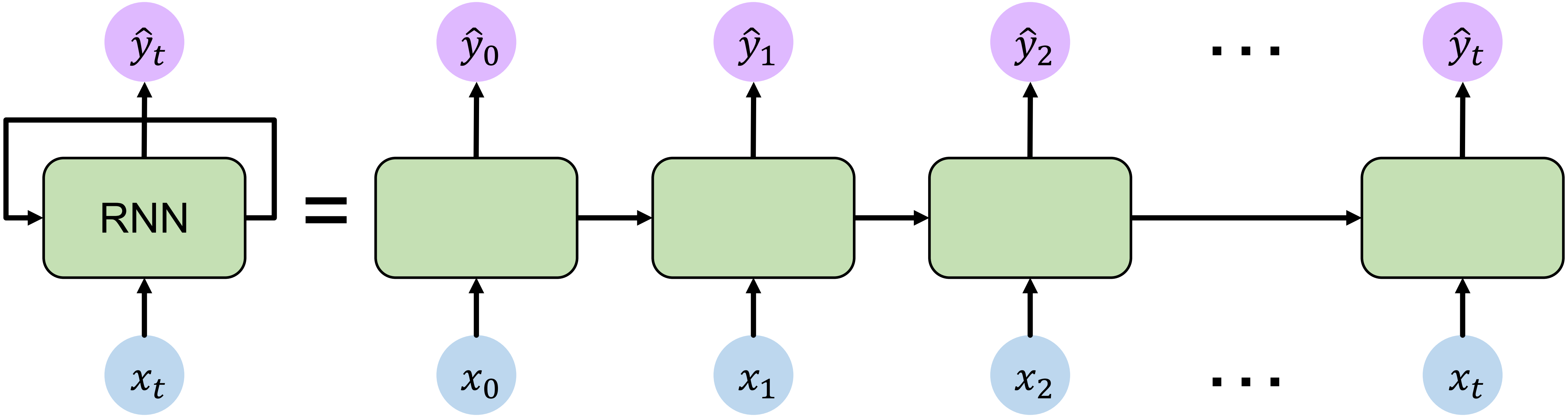

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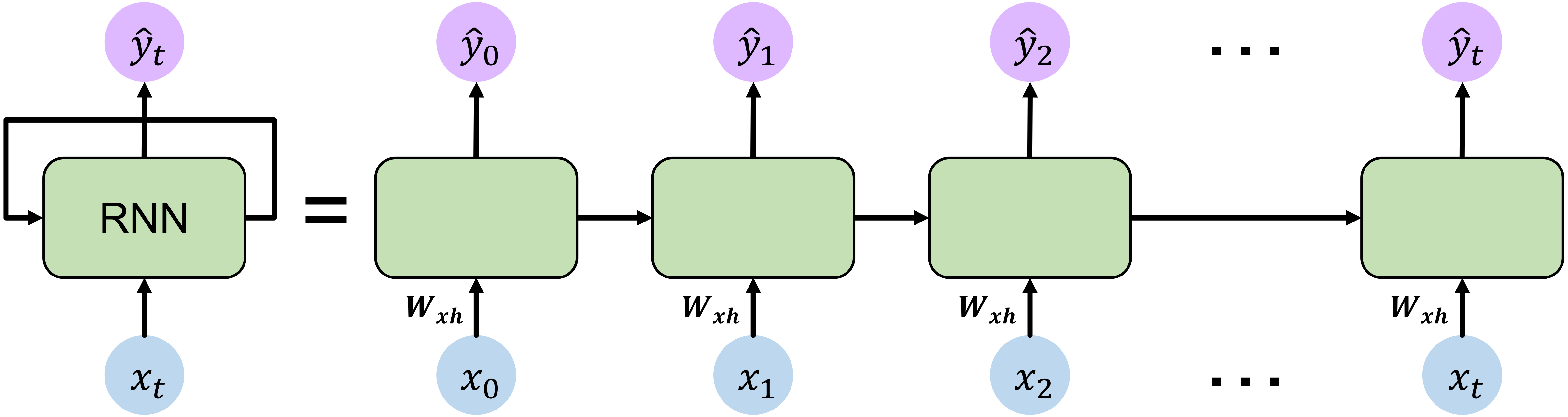

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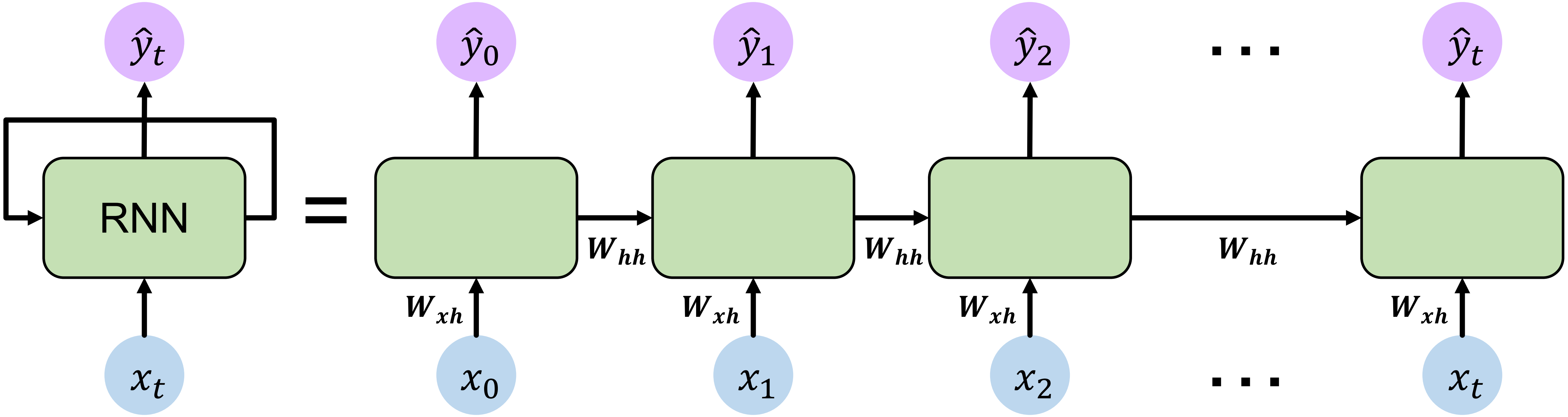

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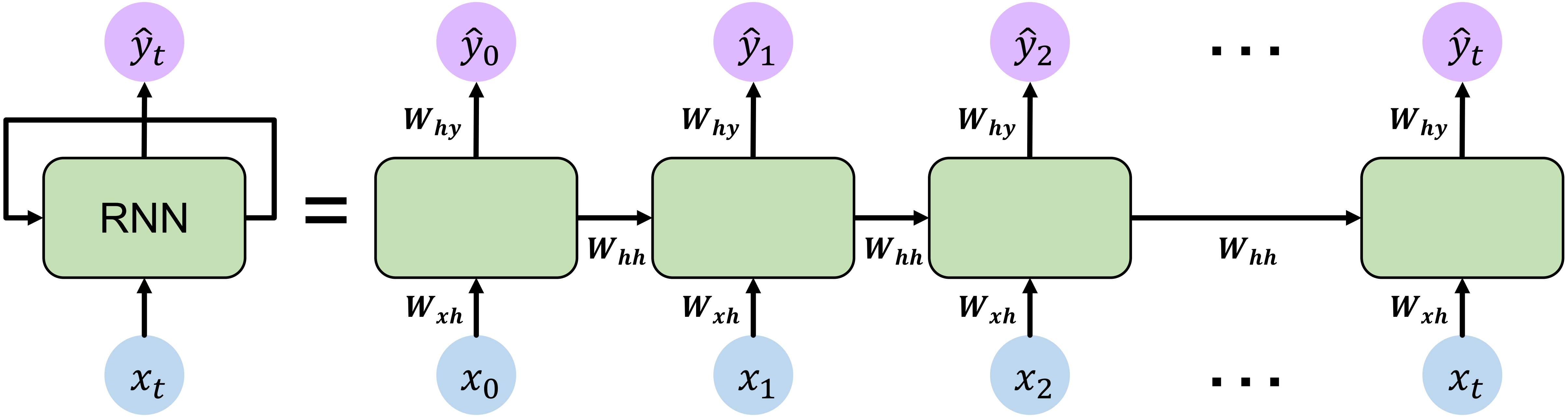
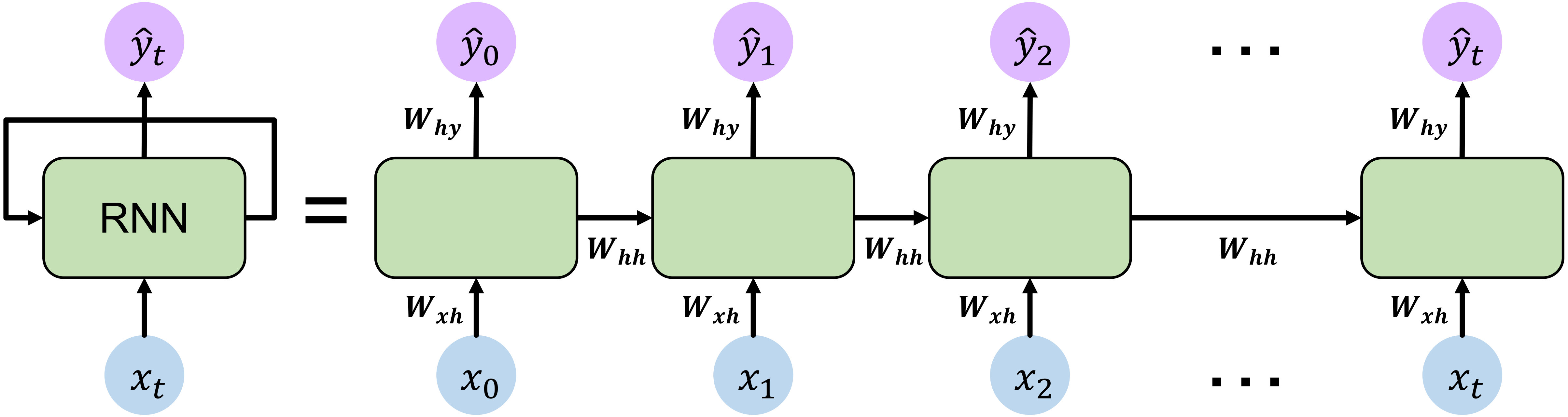


RNNs: computational graph across time

Re-use the **same weight matrices** at every time step



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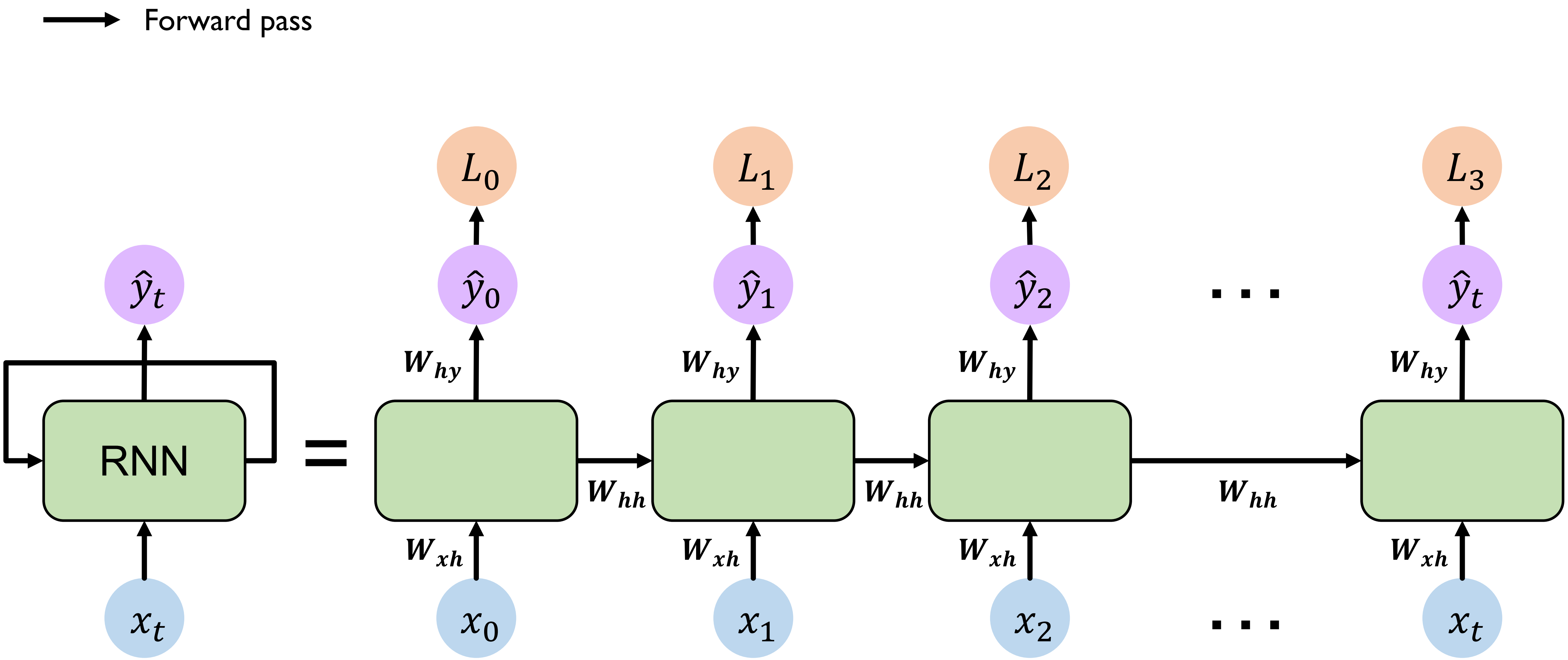
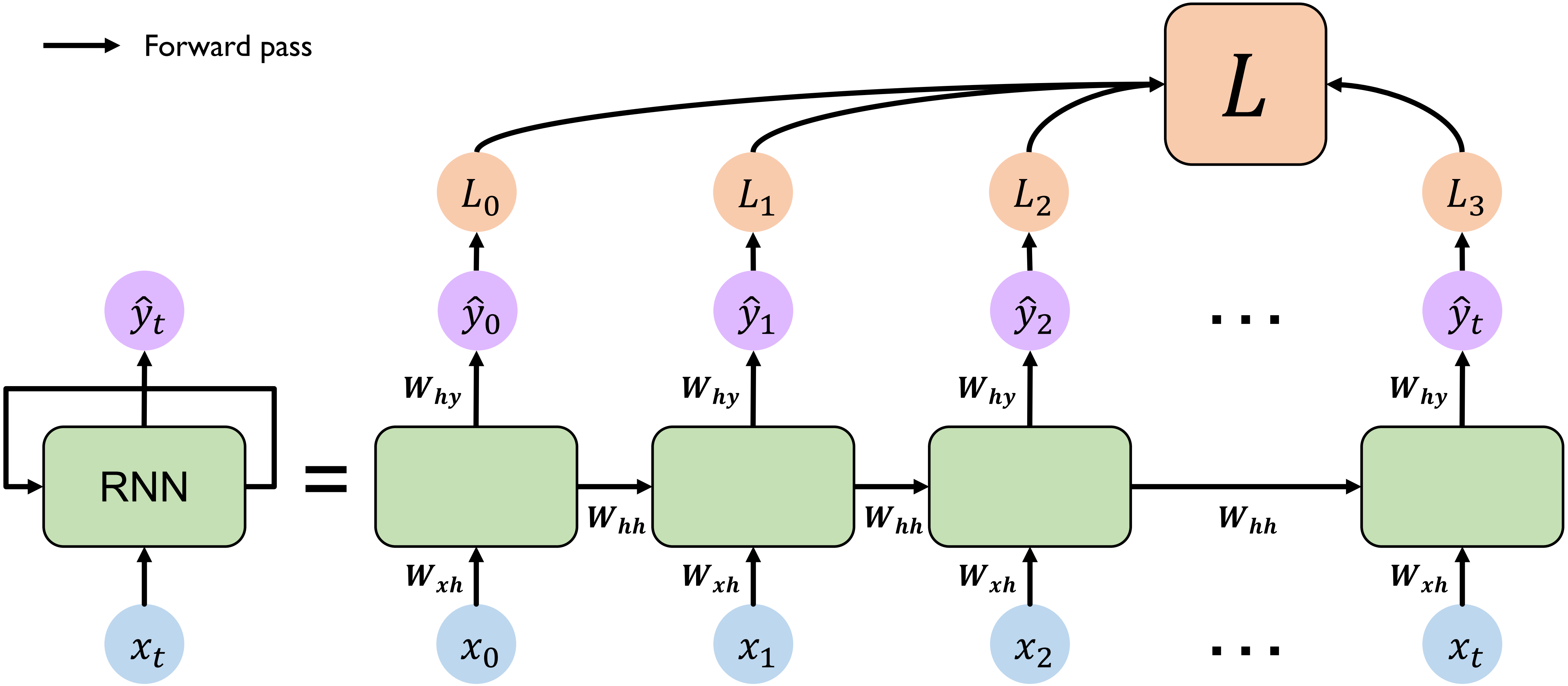
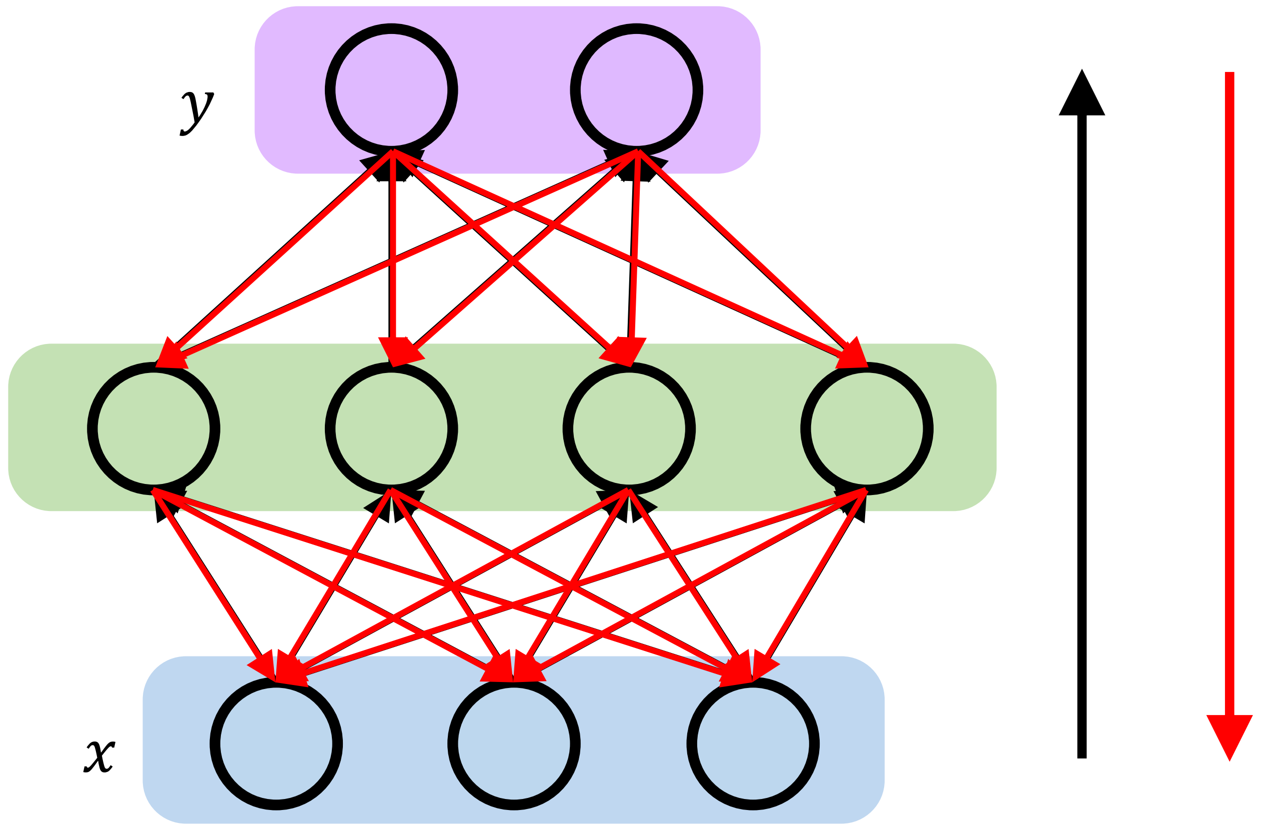


RNNs: computational graph across time



Backpropagation Through Time (BPTT)

Recall: backpropagation in feed forward models

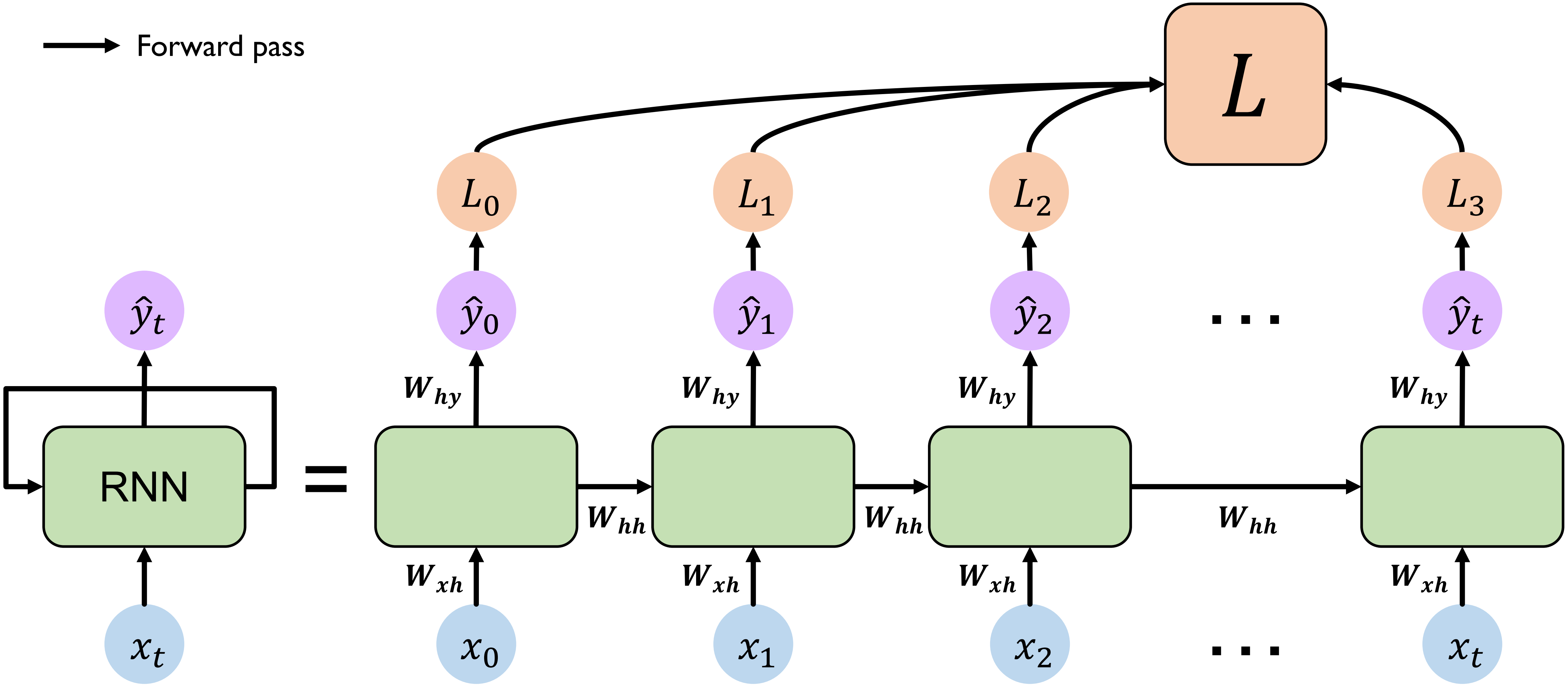


Backpropagation algorithm:

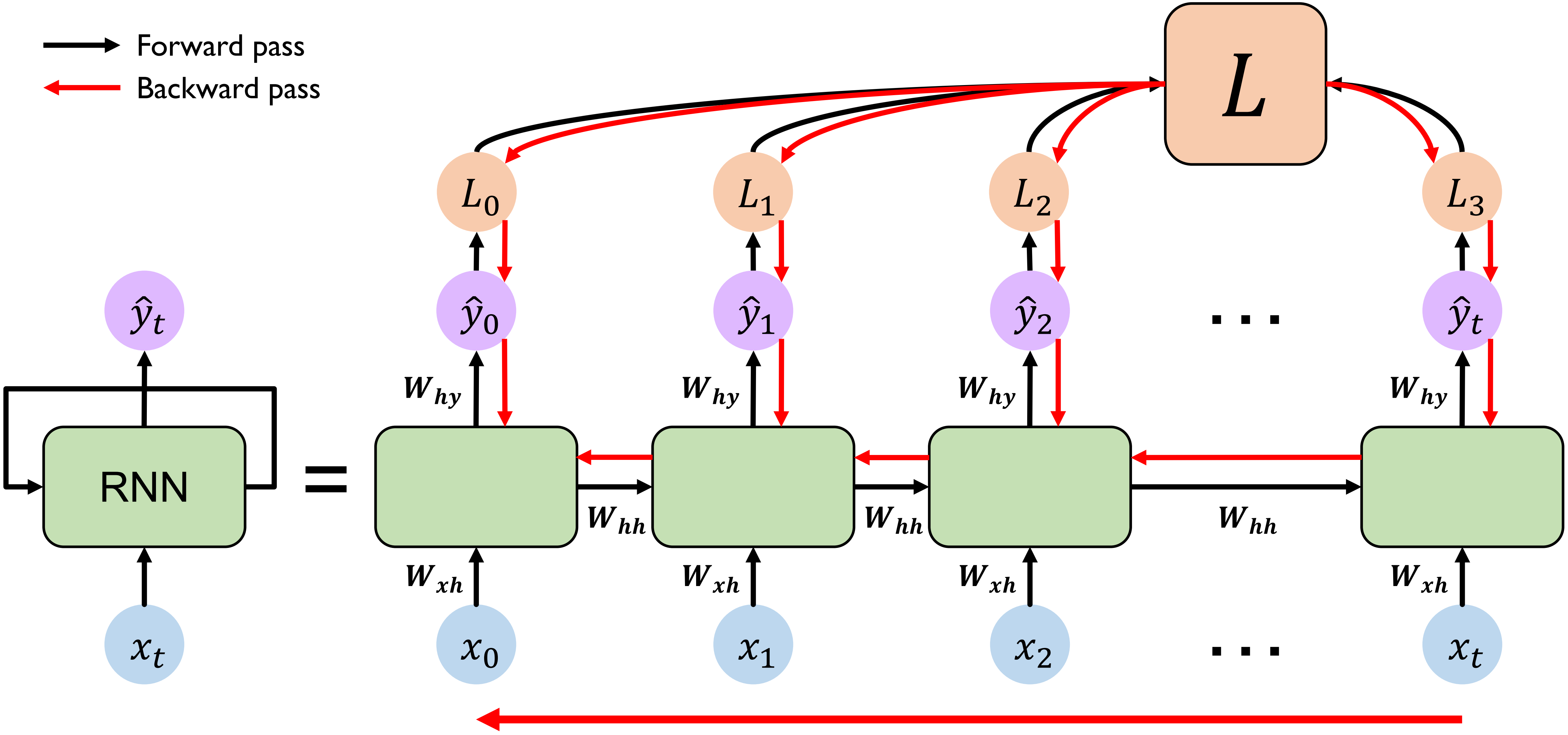
1. Take the derivative (gradient) of the loss with respect to each parameter
2. Shift parameters in order to minimize loss

[3]

RNNs: backpropagation through time

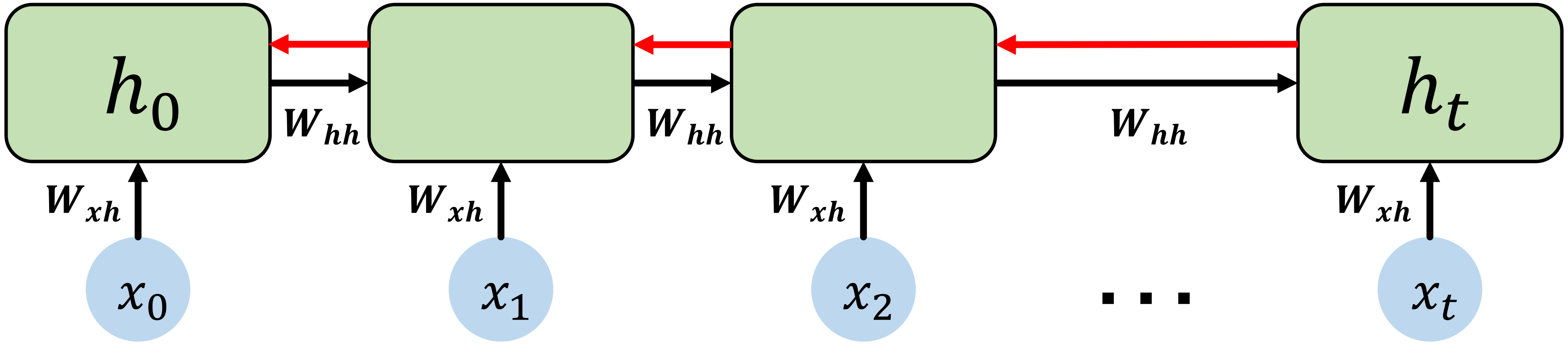


RNNs: backpropagation through time



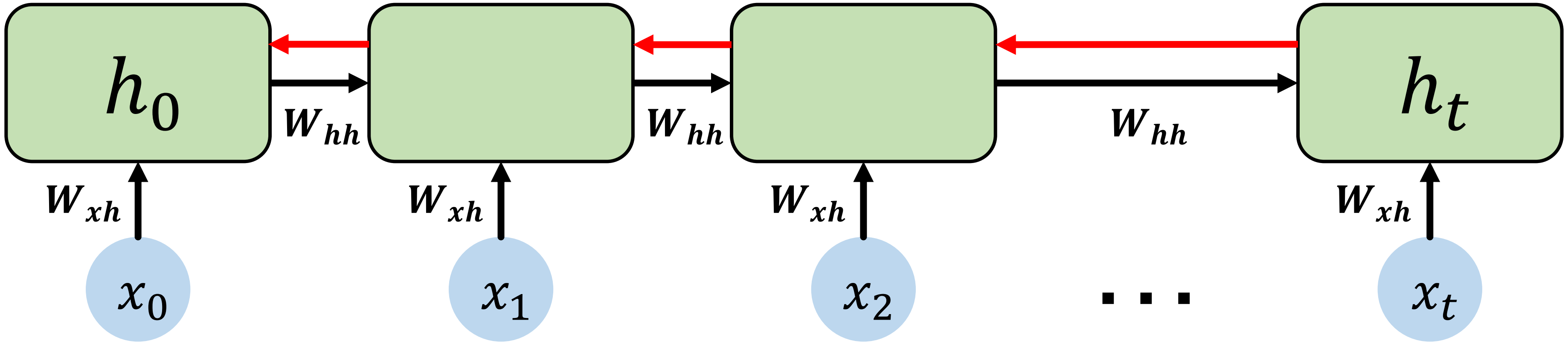
[4]

Standard RNN gradient flow



[1]

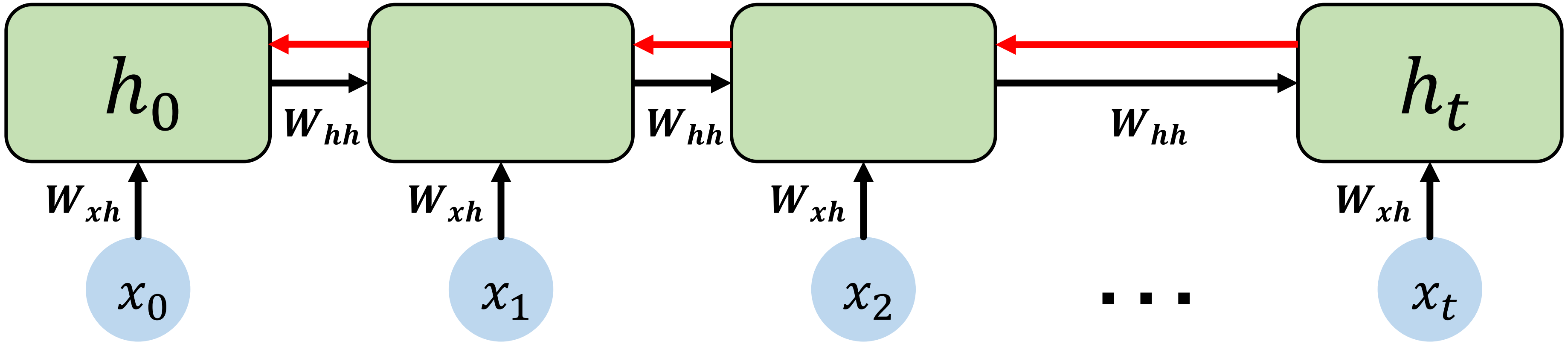
Standard RNN gradient flow



Computing the gradient wrt h_0 involves **many factors of** W_{hh} (and repeated f'!)

[1]

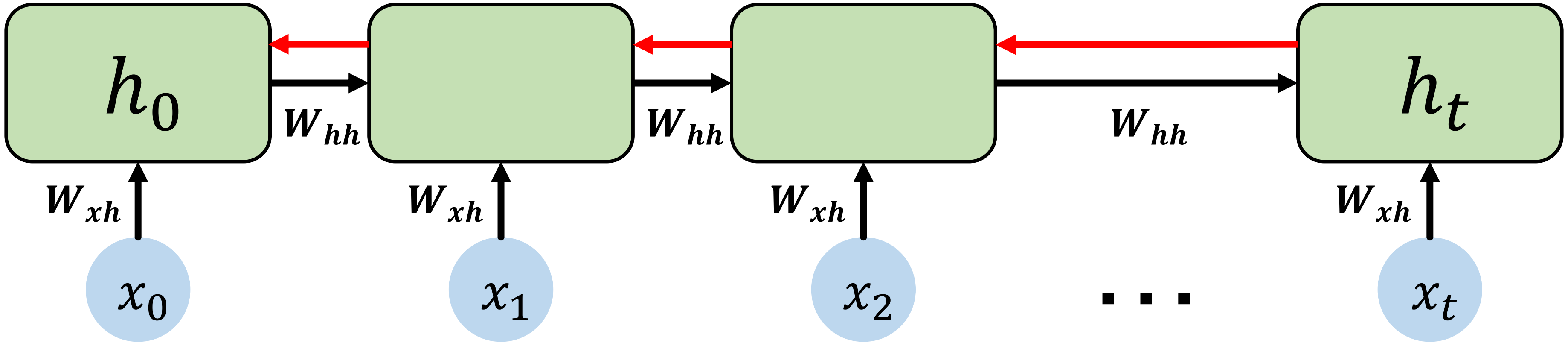
Standard RNN gradient flow: exploding gradients



Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Many values > 1:
exploding gradients

Standard RNN gradient flow: exploding gradients



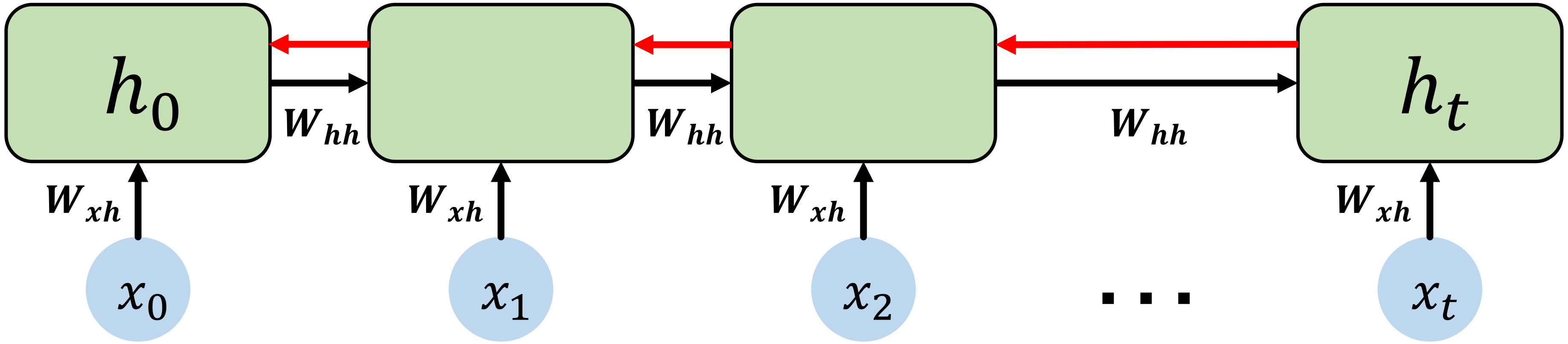
Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Many values > 1:
exploding gradients

Gradient clipping to
scale big gradients

[1]

Standard RNN gradient flow: vanishing gradients



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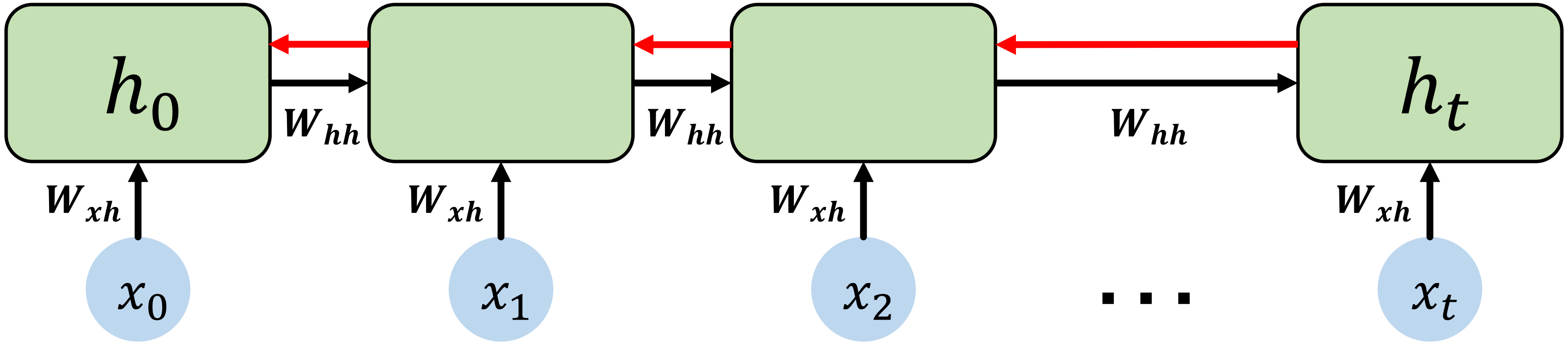
Many values > 1:
exploding gradients

Gradient clipping to scale big gradients

Many values < 1:
vanishing gradients

[1]

Standard RNN gradient flow: vanishing gradients



Computing the gradient wrt h_0 involves **many factors of W_{hh}** (and repeated f'!)

Largest singular value > 1:
exploding gradients

Gradient clipping to scale big gradients

Largest singular value < 1:
vanishing gradients

1. Activation function
2. Weight initialization
3. Network architecture

[1]

The problem of long-term dependencies

Why are vanishing gradients a problem?

The problem of long-term dependencies

Why are vanishing gradients a problem?

Multiply many **small numbers** together

The problem of long-term dependencies

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Multiply many **small numbers** together

↓

Errors due to further back time steps
have smaller and smaller gradients

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Bias network to capture short-term dependencies

The problem of long-term dependencies

"The clouds are in the ___"

Why are vanishing gradients a problem?

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Bias network to capture short-term dependencies

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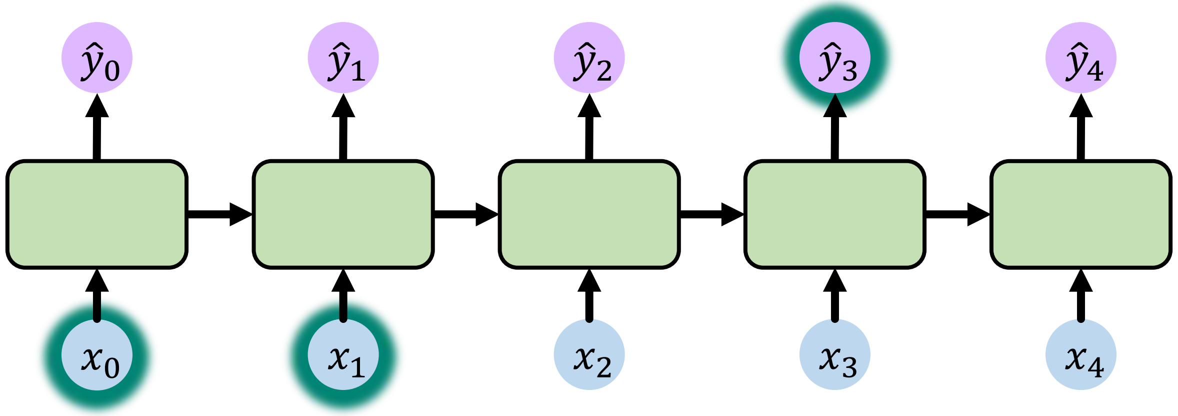
Bias parameters to capture short-term dependencies

"The clouds are in the ___"



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Bias parameters to capture short-term dependencies

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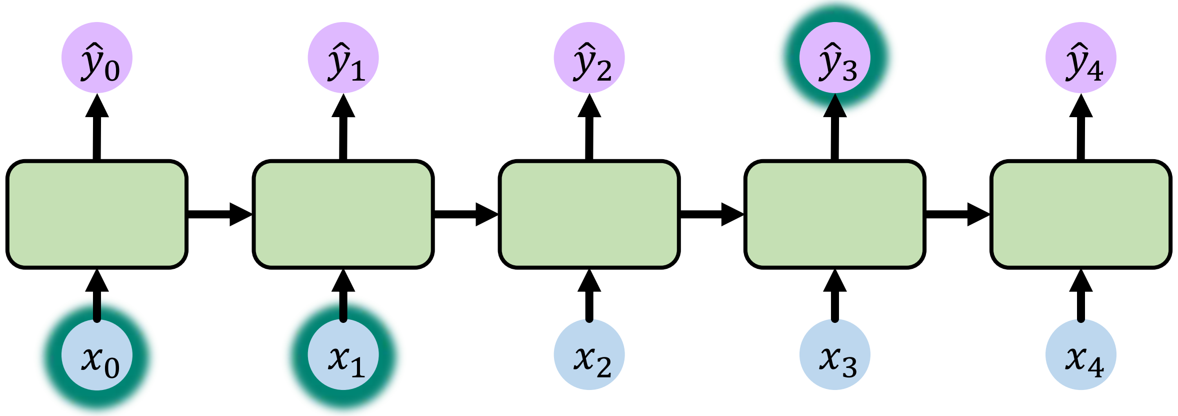


"I grew up in France, … and I I speak fluent___ "

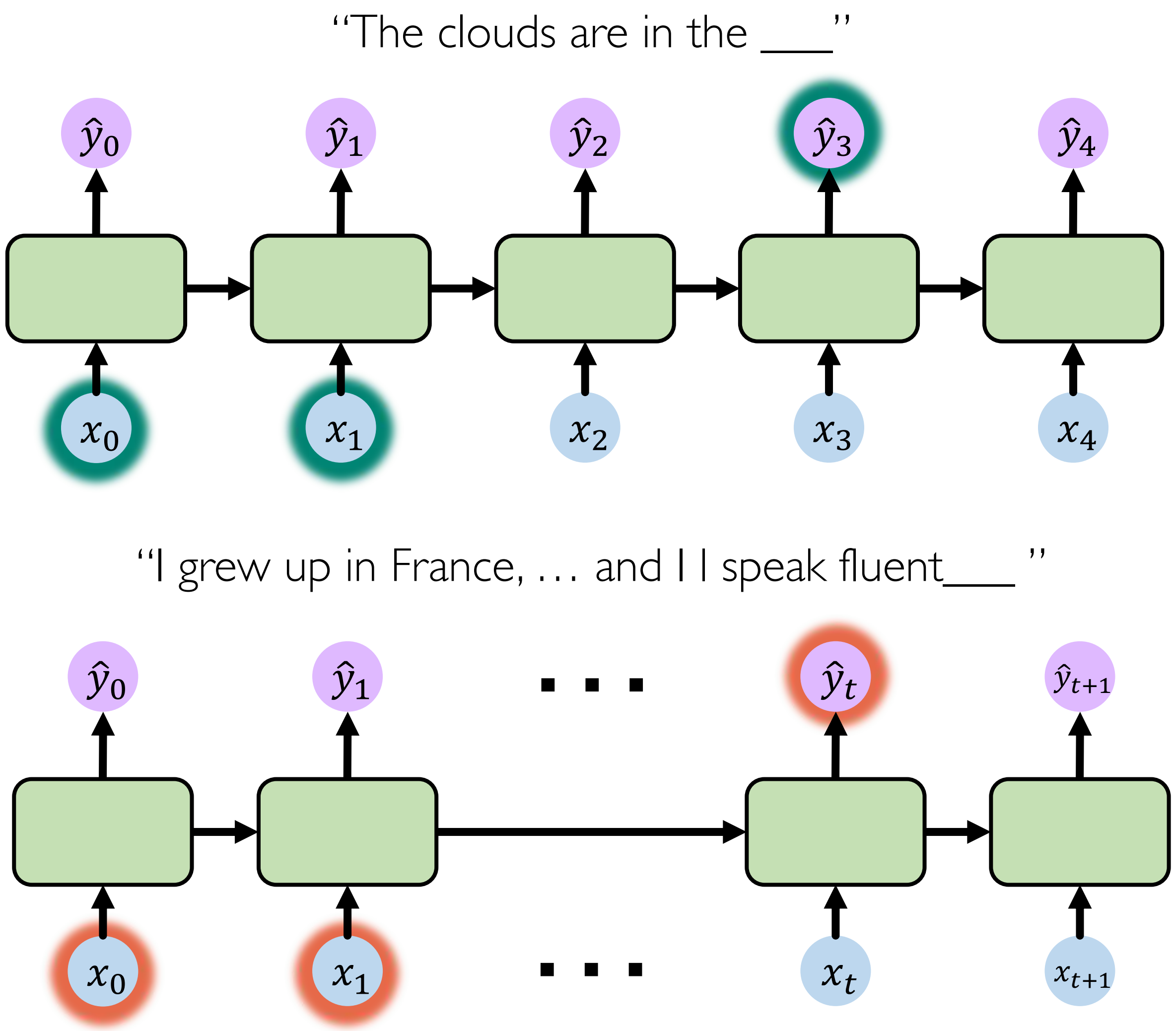
The problem of long-term dependencies

Why are vanishing gradients a problem?

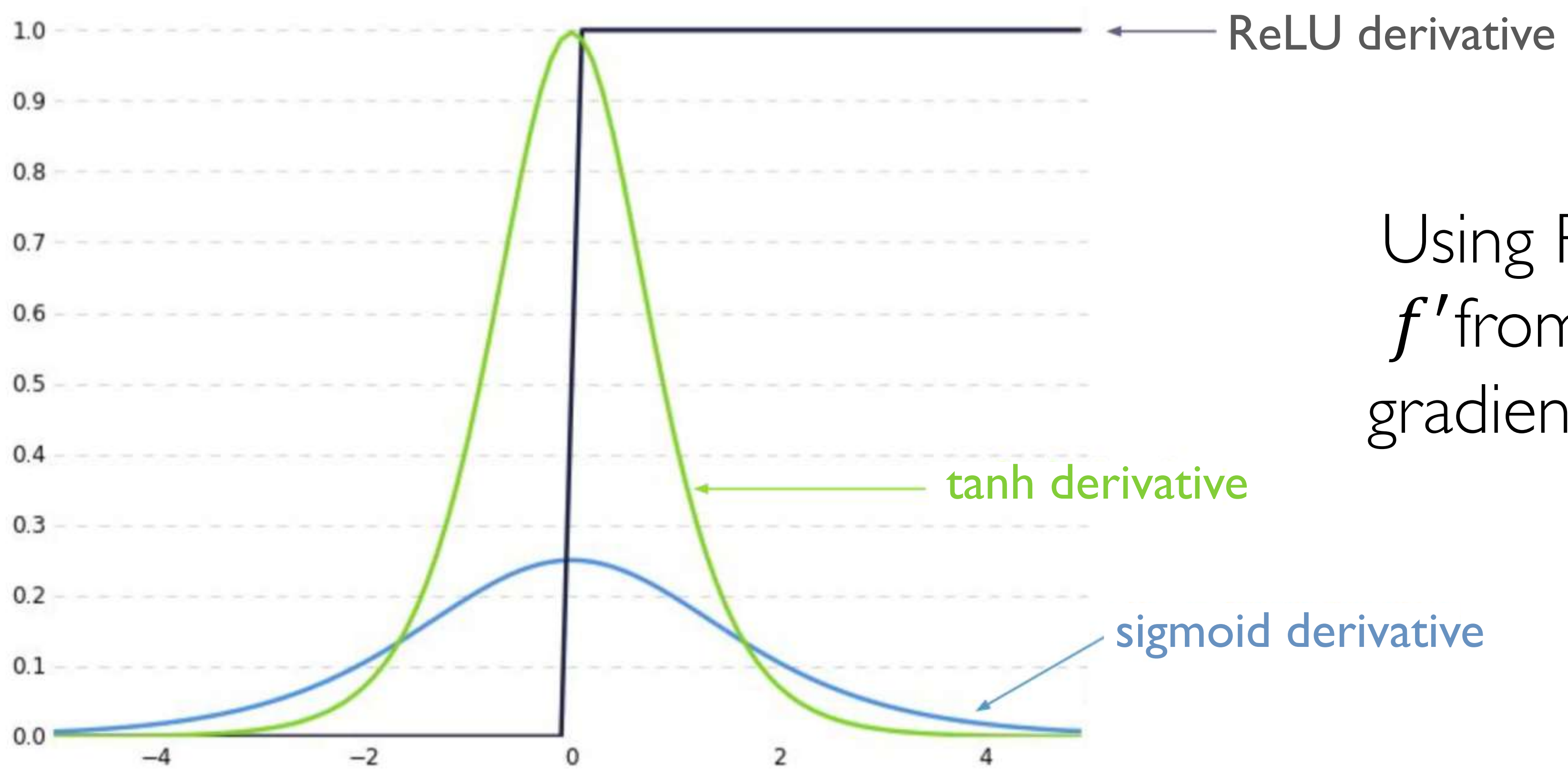
Multiply many **small numbers** together

Errors due to further back time steps have smaller and smaller gradients

Bias parameters to capture short-term dependencies



Trick #1: activation functions



Using ReLU prevents f' from shrinking the gradients when $x > 0$

Adapted from H. Suresh, 6.S191 2018

Trick #2: parameter initialization

Initialize **weights** to identity matrix

Initialize **biases** to zero

$$I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$$

This helps prevent the weights from shrinking to zero.

Adapted from H. Suresh, 6.S191 2018

Solution #3: gated cells

Idea: use a more **complex recurrent unit with gates** to control what information is passed through

gated cell

LSTM, GRU, etc.

Adapted from H. Suresh, 6.S191 2018

Solution #3: gated cells

Idea: use a more **complex recurrent unit with gates** to control what information is passed through

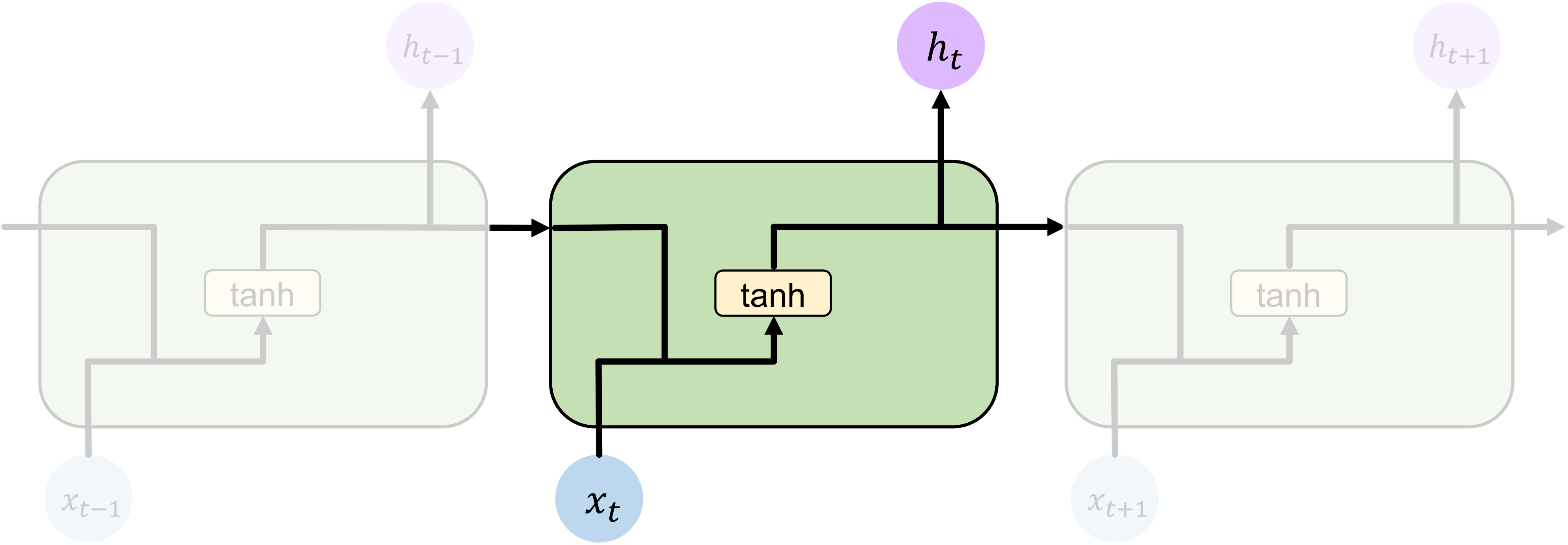
Long Short Term Memory (LSTMs) networks rely on a gated cell to track information throughout many time steps.

Adapted from H. Suresh, 6.S191 2018

Long Short Term Memory (LSTM) Networks

Standard RNN

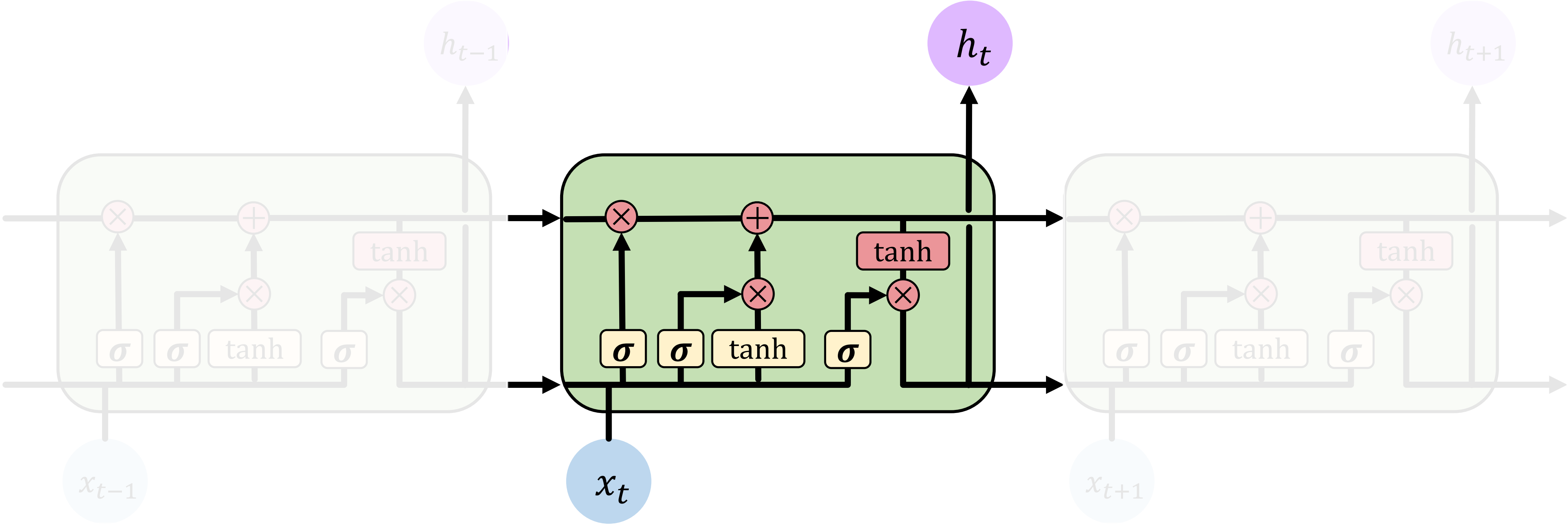
In a standard RNN, repeating modules contain a **simple computation node**



[2]

Long Short Term Memory (LSTMs)

LSTM repeating modules contain **interacting layers** that **control information flow**



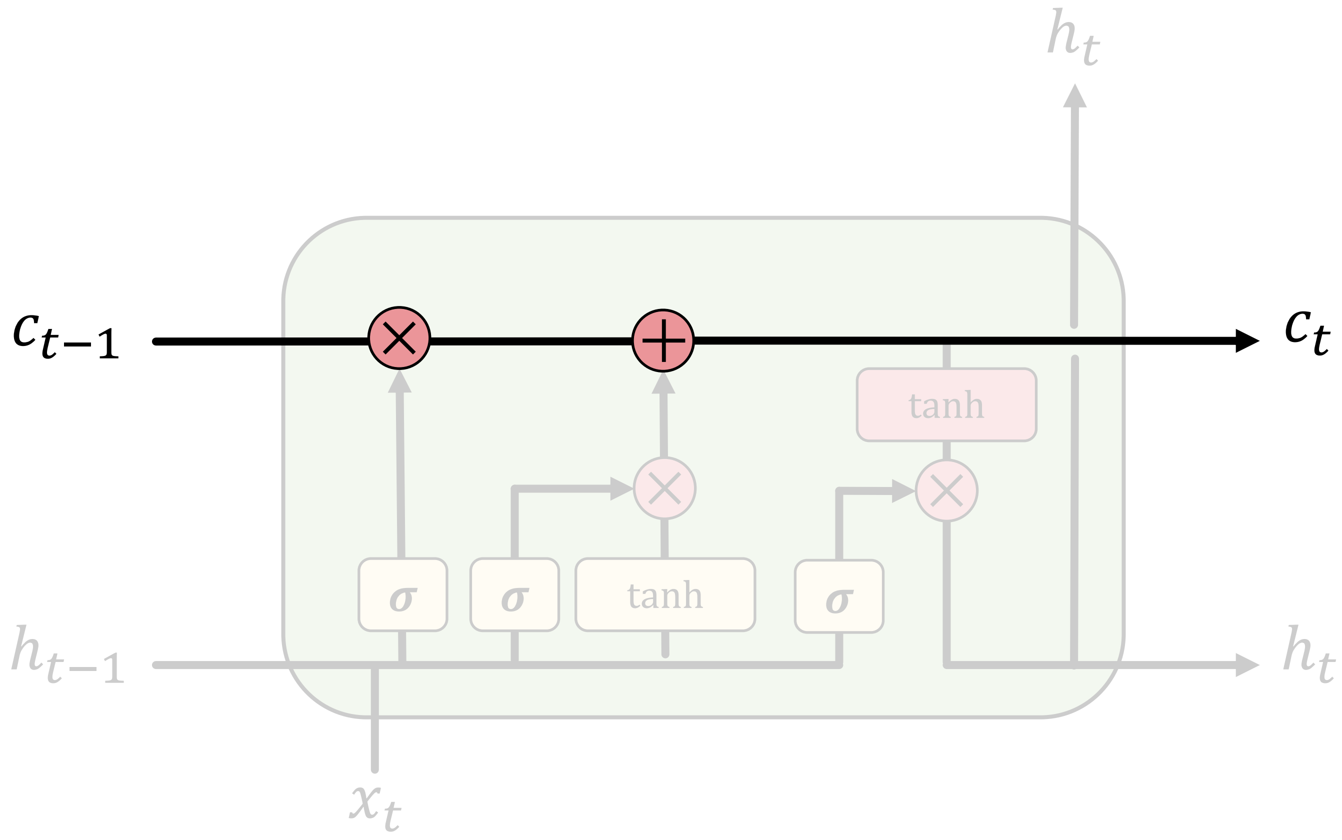
LSTM cells are able to track information throughout many timesteps

```
tf.keras.layers.LSTM(num_units)
```

reiter & Schmidhuber, 1997 [2, 5]

Long Short Term Memory (LSTMs)

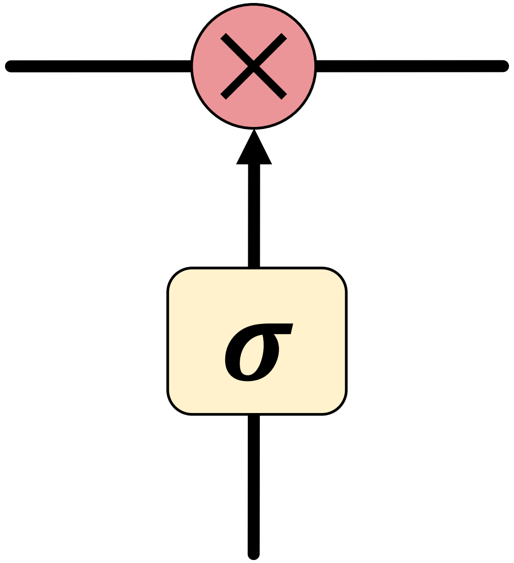
LSTMs maintain a **cell state** c_t where it's easy for information to flow



[2, 5]

Long Short Term Memory (LSTMs)

Information is **added** or **removed** to cell state through structures called **gates**

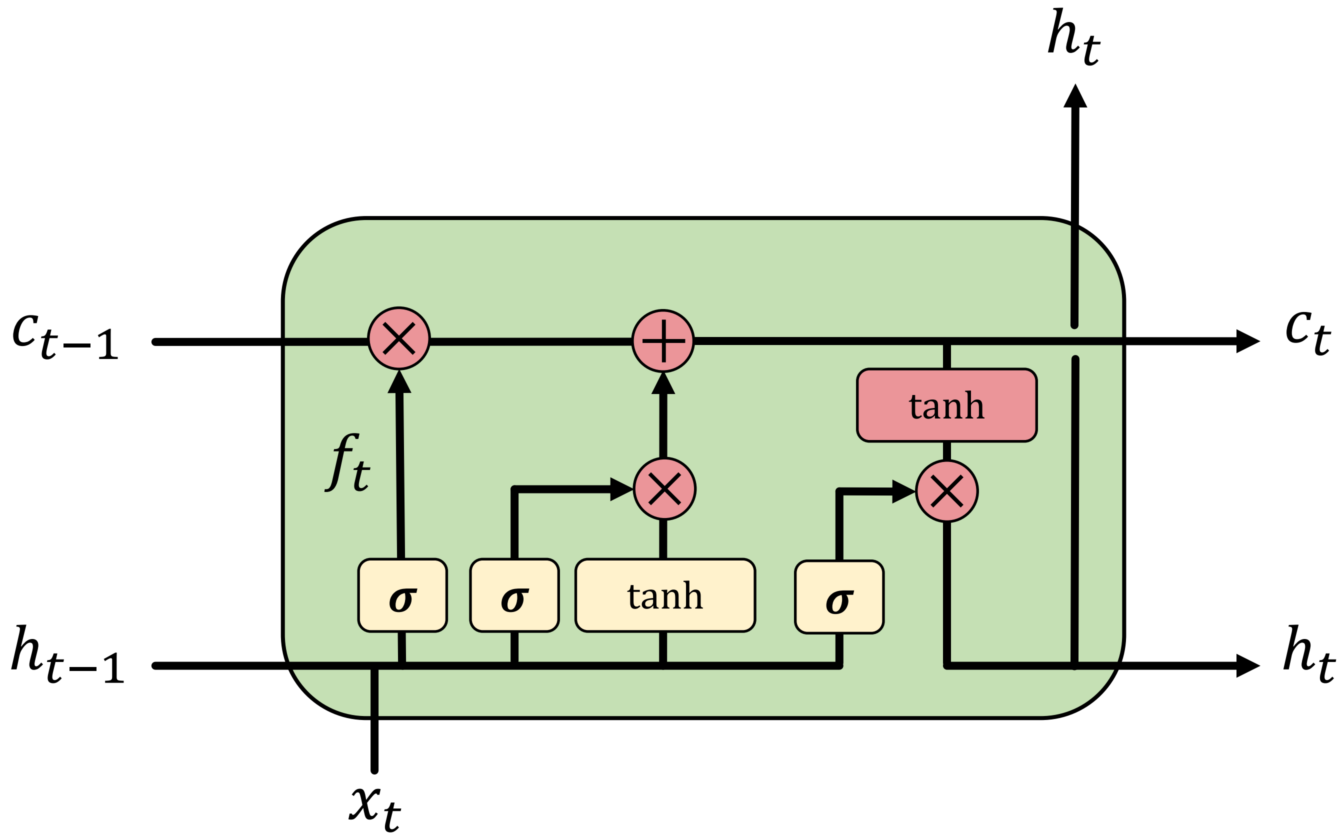


Gates optionally let information through, via a sigmoid neural net layer and pointwise multiplication

[2, 5]

Long Short Term Memory (LSTMs)

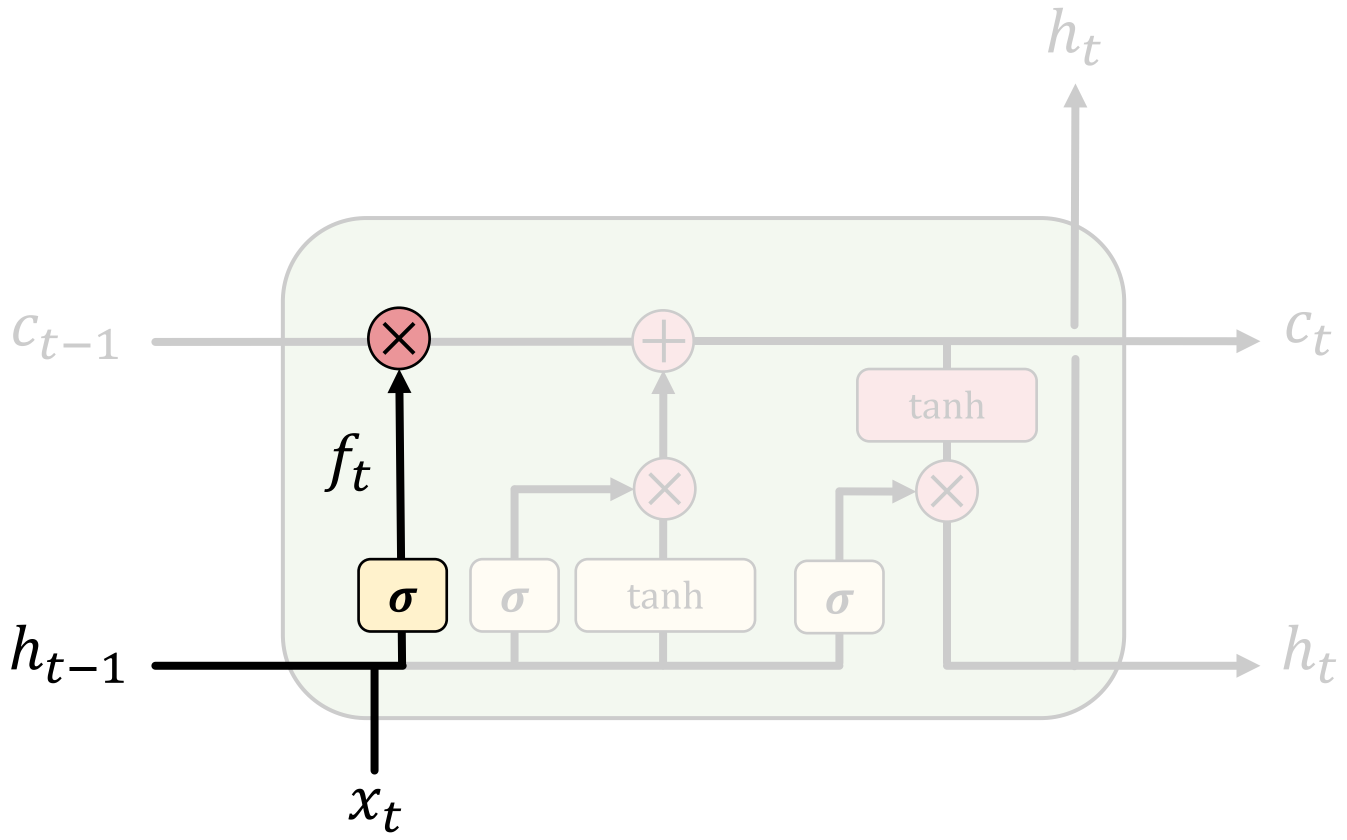
How do LSTMs work?



[2, 5]

Long Short Term Memory (LSTMs)

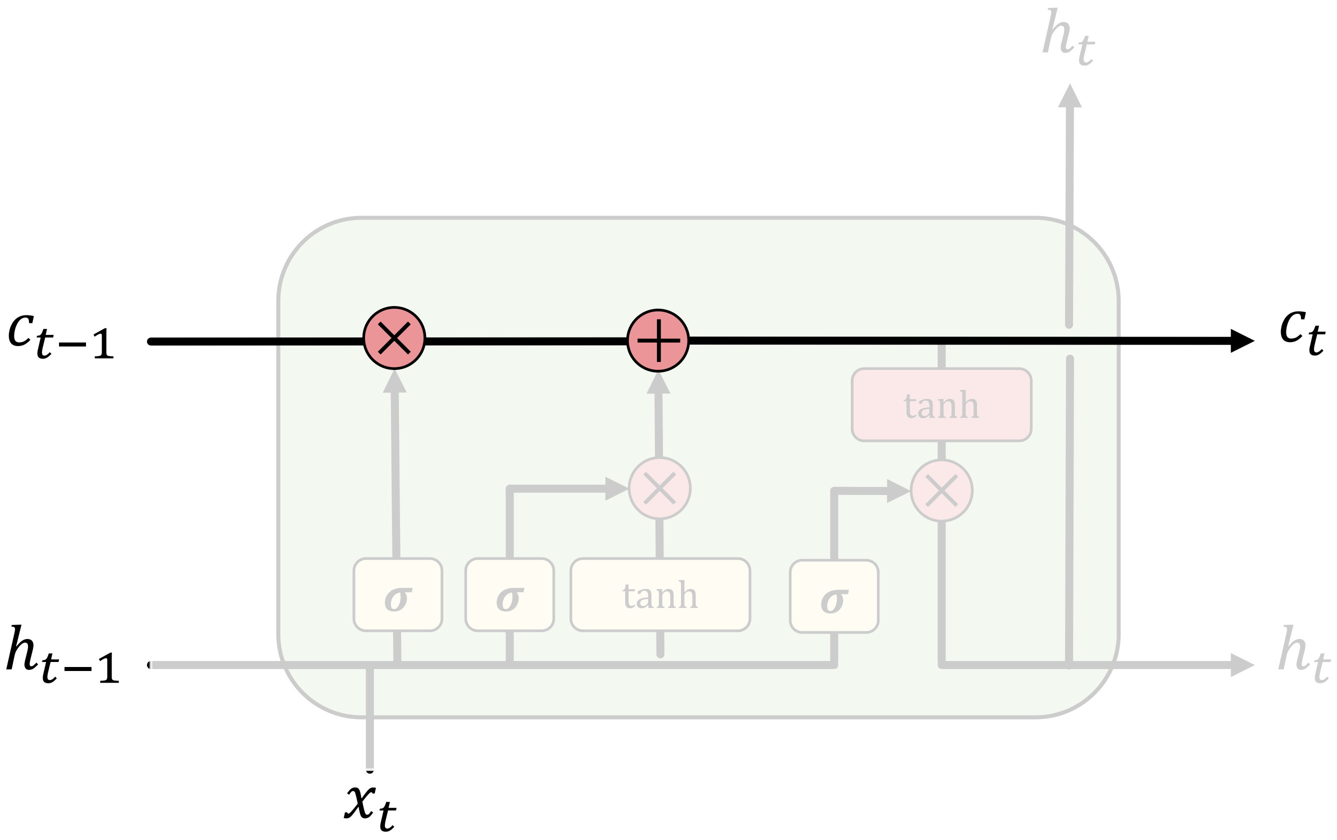
LSTMs **forget irrelevant** parts of the previous state



[2, 5]

Long Short Term Memory (LSTMs)

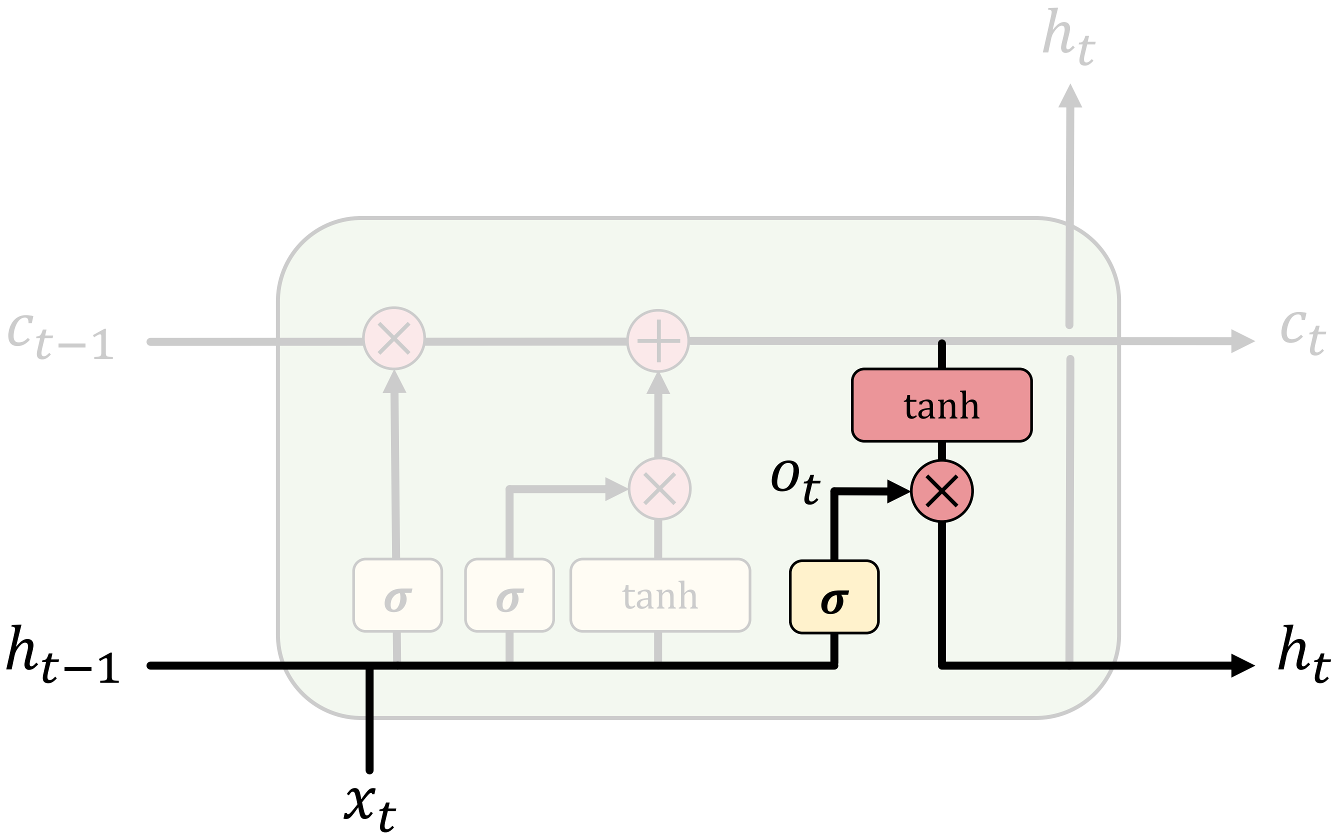
LSTMs **selectively update** cell state values



[2, 5]

Long Short Term Memory (LSTMs)

LSTMs use an **output gate** to output certain parts of the cell state

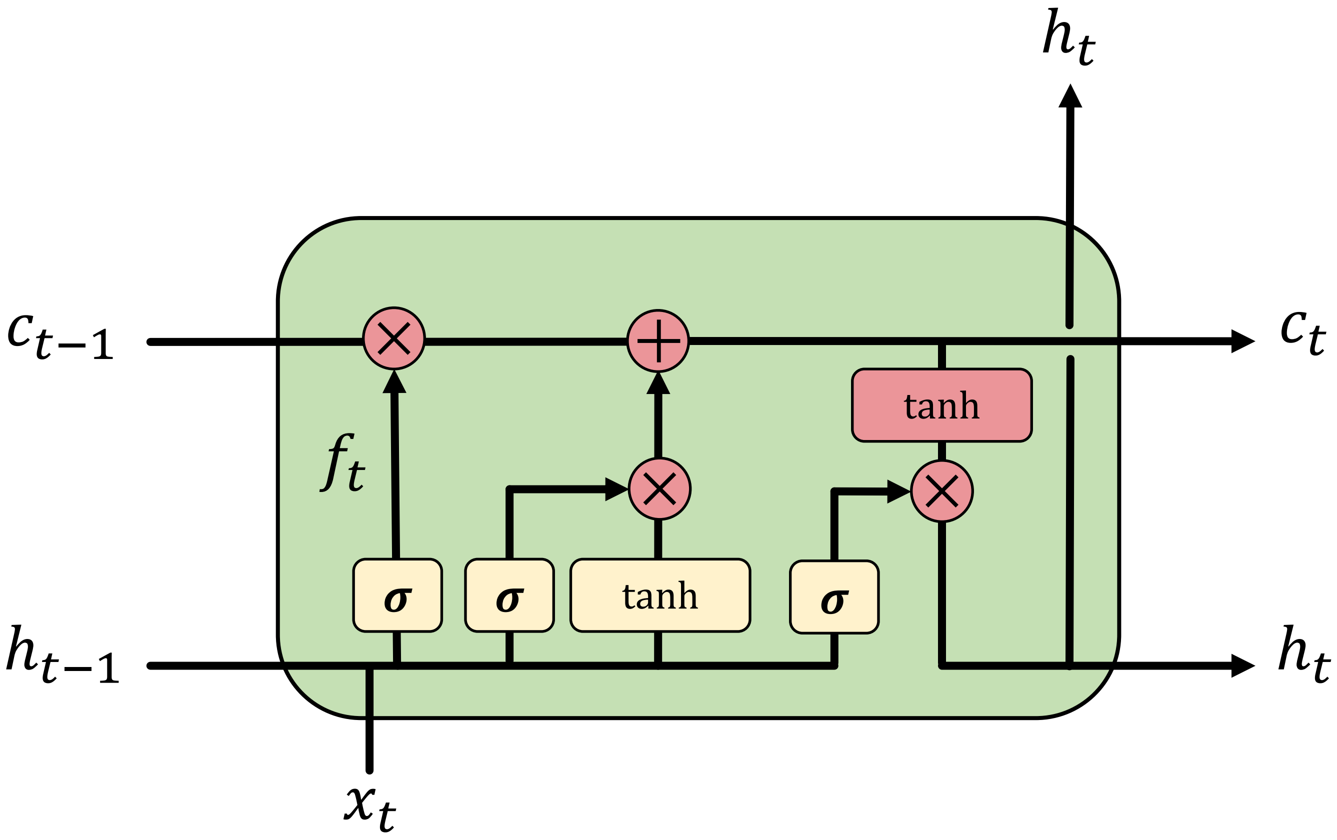


[2, 5]

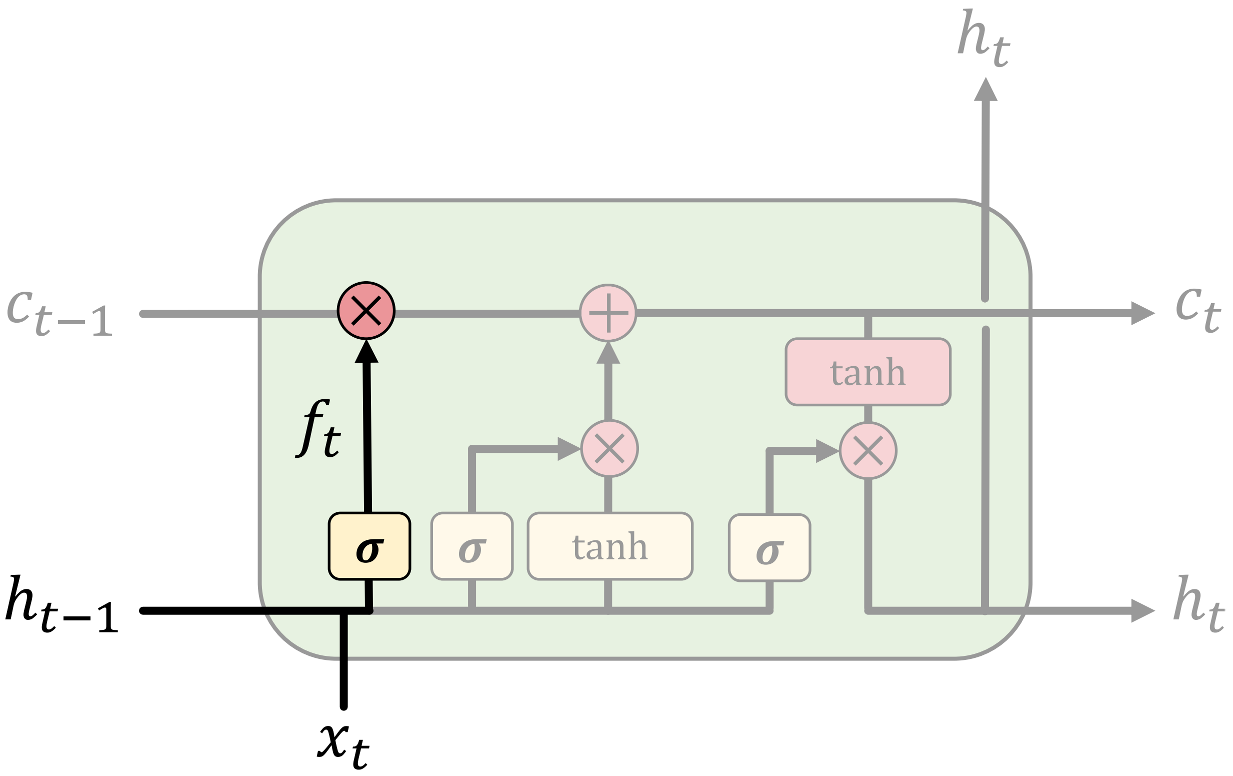
Long Short Term Memory (LSTMs)

How do LSTMs work?

1) Forget 2) Update 3) Output



LSTMs: forget irrelevant information



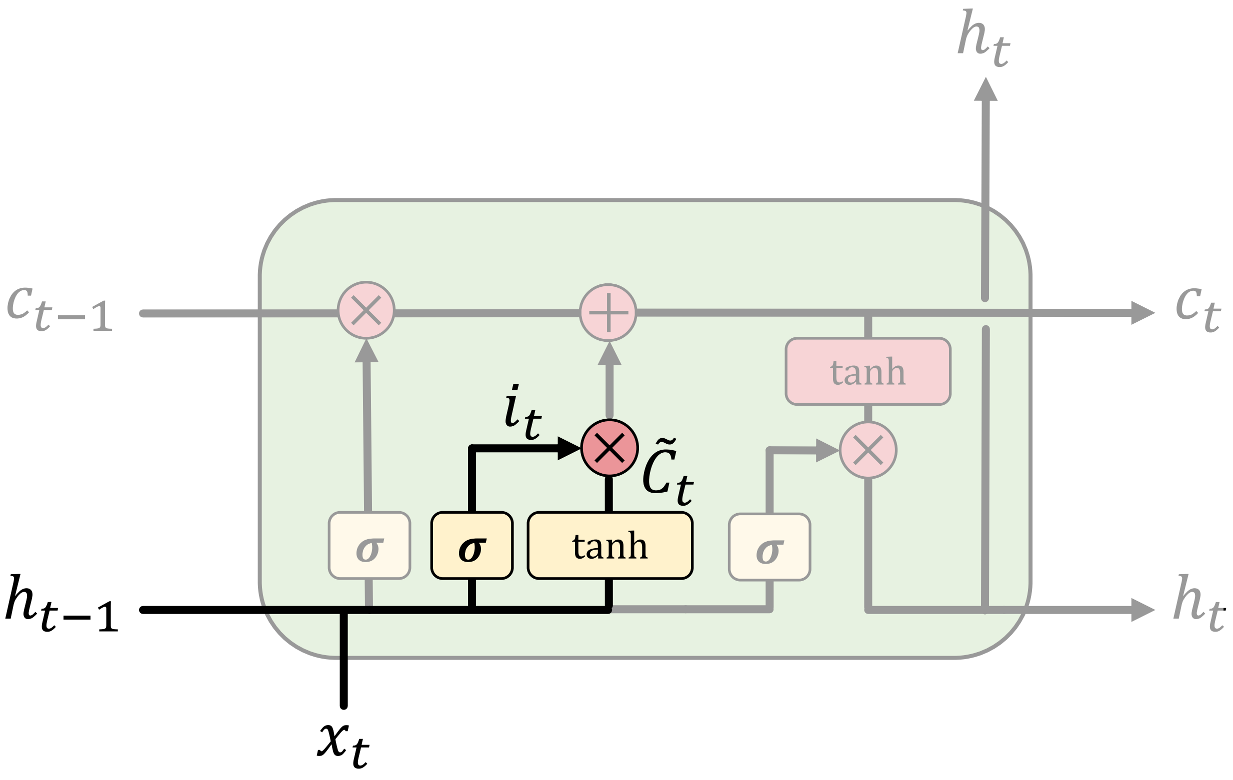
$$f_t = \sigma(\boldsymbol{W_i}[\, h_{t-1}, x_t \,] + b_f)$$

- Use previous cell output and input
- Sigmoid: value 0 and 1 – "completely forget" vs. "completely keep"

ex: Forget the gender pronoun of previous subject in sentence.

[2, 5]

LSTMs: identify new information to be stored



$$i_t = \sigma(\boldsymbol{W_i}[\, h_{t-1}, x_t \,] + b_i)$$

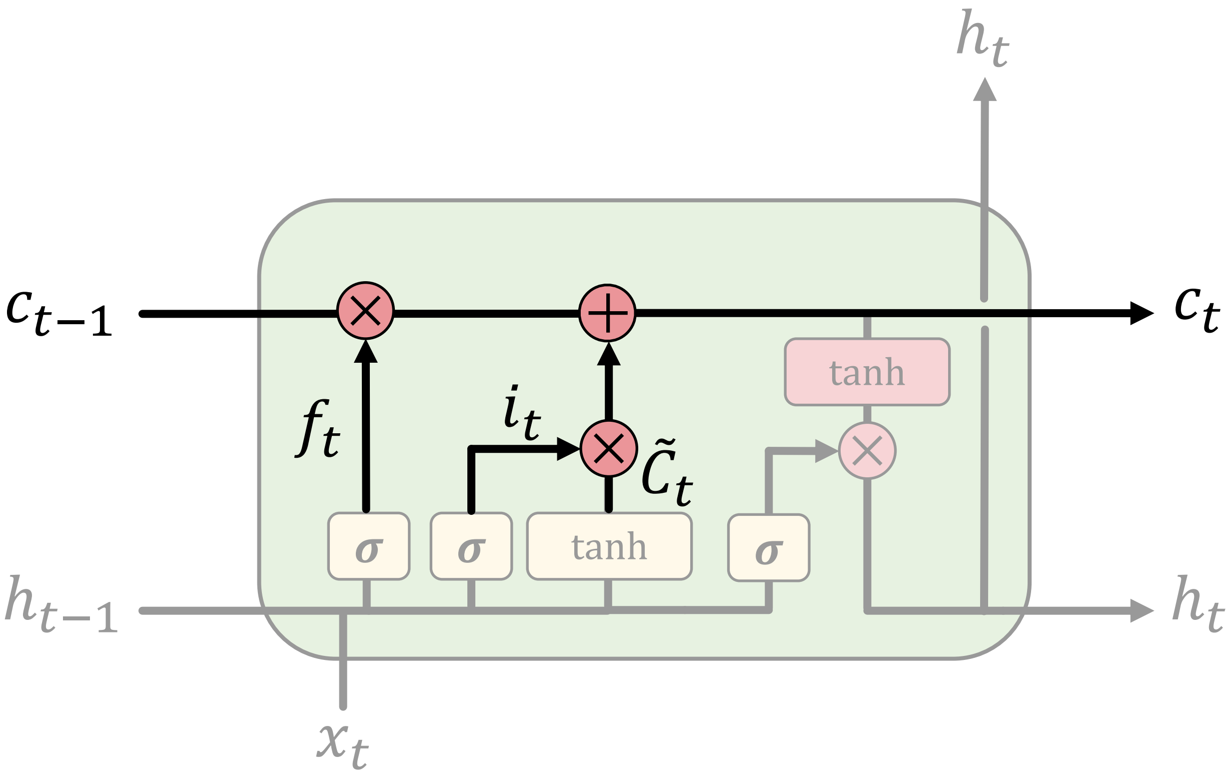
$$\tilde{C}_t = \tanh(\boldsymbol{W_C}[\, h_{t-1}, x_t \,] + b_C)$$

- Sigmoid layer: decide what values to update
- Tanh layer: generate new vector of "candidate values" that could be added to the state

ex: Add gender of new subject to replace that of old subject.

[2, 5]

LSTMs: update cell state



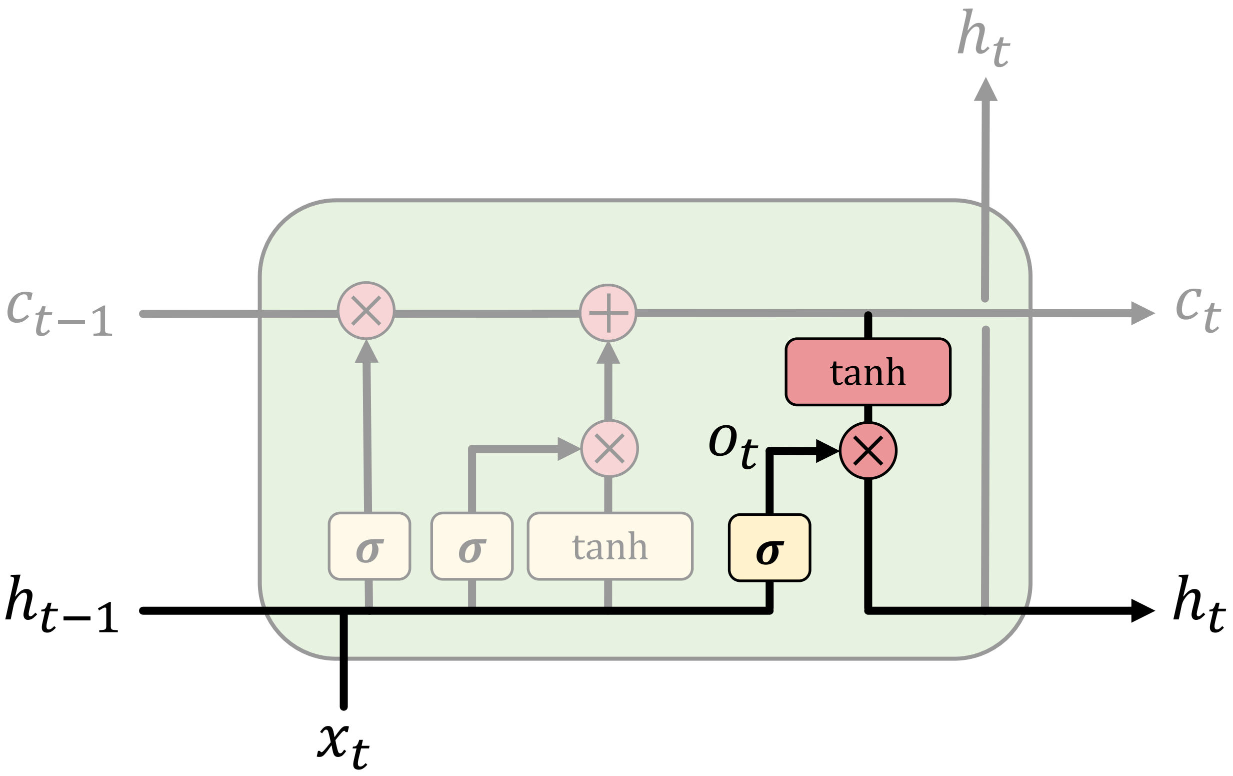
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

- Apply forget operation to previous internal cell state: $f_t * C_{t-1}$
- Add new candidate values, scaled by how much we decided to update: $i_t * \tilde{C}_t$

ex: Actually drop old information and add new information about subject's gender.

[2, 5]

LSTMs: output filtered version of cell state



$$o_t = \sigma(\boldsymbol{W_o}[\, h_{t-1}, x_t \,] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

- Sigmoid layer: decide what parts of state to output
- Tanh layer: squash values between -1 and 1
- **$o_t * \tanh(C_t)$: output filtered version of cell state**

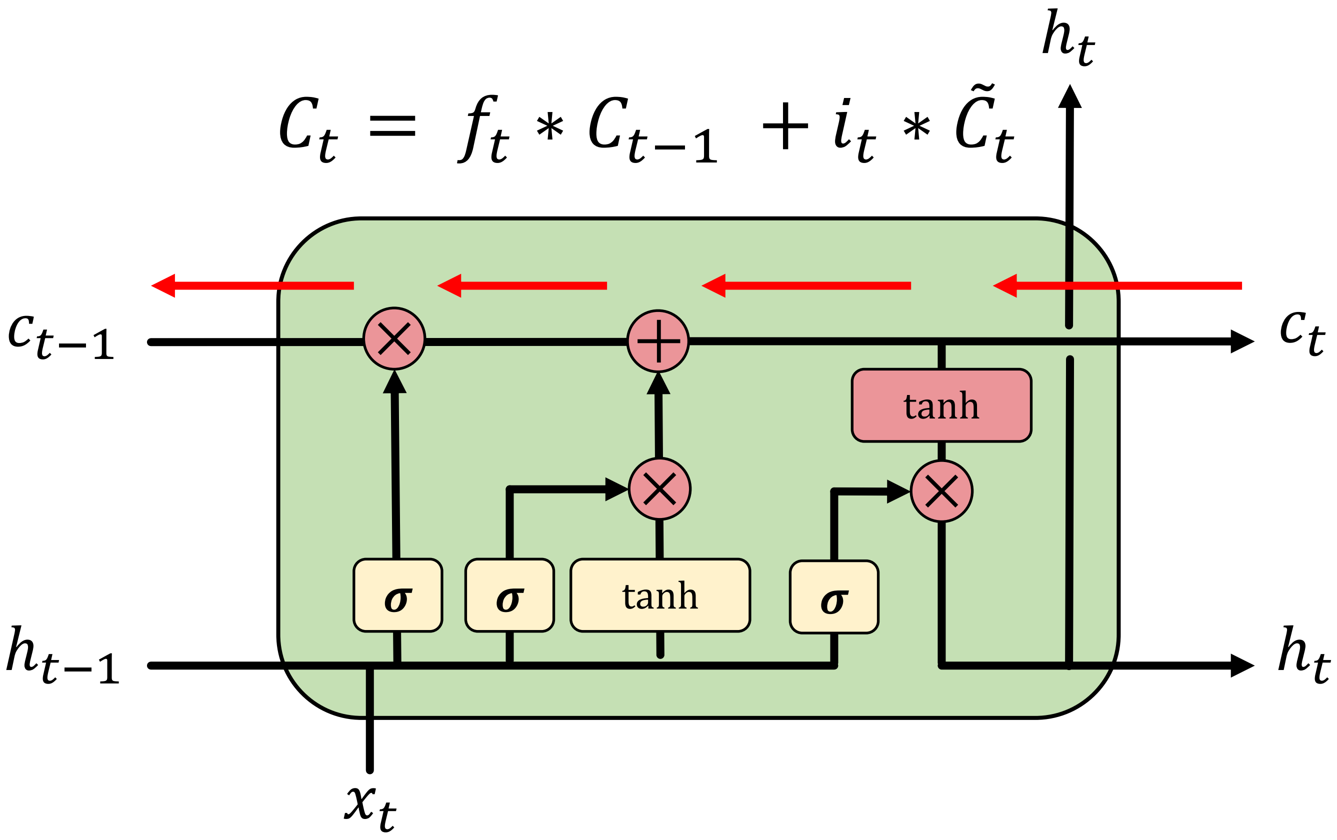
ex: Having seen a subject, may output information relating to a verb.

[2, 5]

LSTM gradient flow

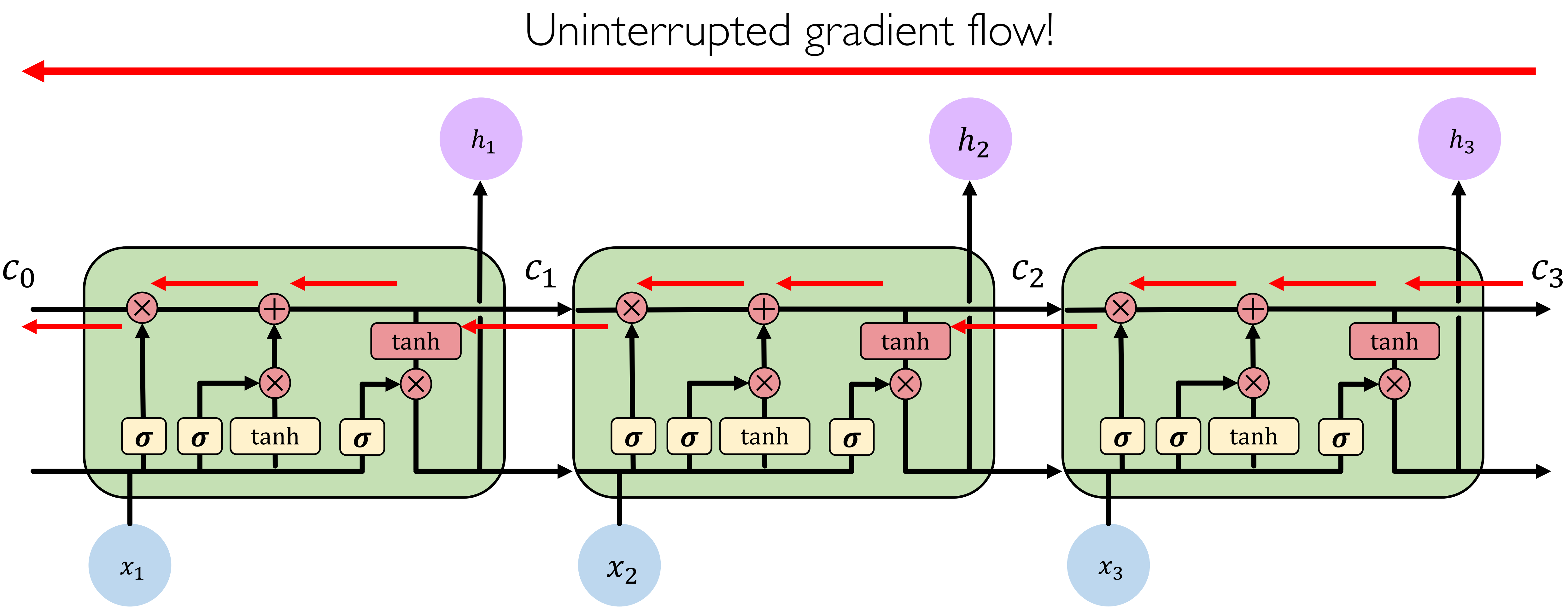
Backpropagation from C_t to C_{t-1} requires only elementwise multiplication!
No matrix multiplication → avoid vanishing gradient problem.

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



[2, 5]

LSTM gradient flow

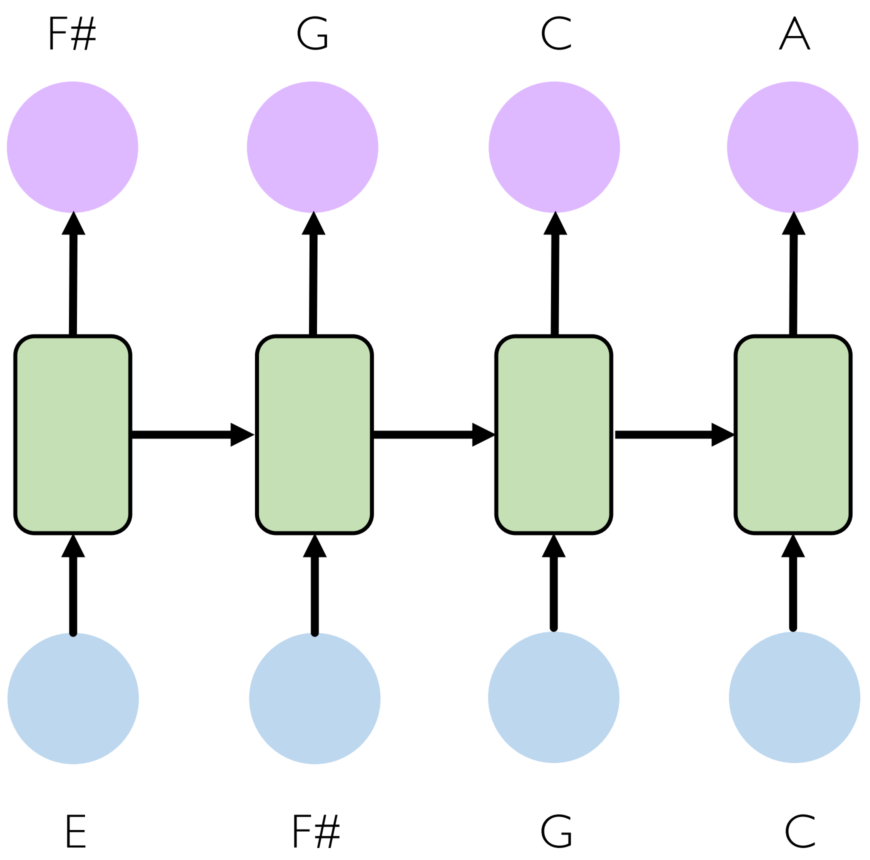


LSTMs: key concepts

1. Maintain a **separate cell state** from what is outputted
2. Use **gates** to control the **flow of information**
   - Forget gate gets rid of irrelevant information
   - Selectively update cell state
   - Output gate returns a filtered version of the cell state
3. Backpropagation from c_t to c_{t-1} doesn't require matrix multiplication: **uninterrupted gradient flow**

RNN Applications

Example task: music generation



Input: sheet music

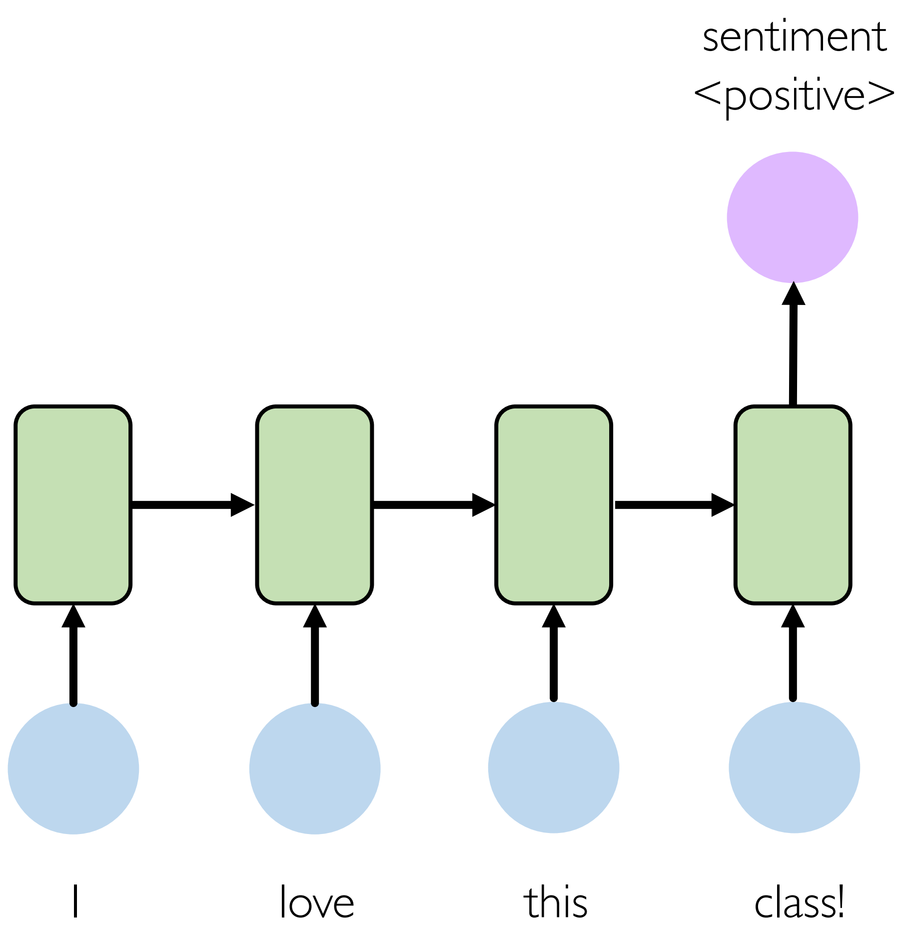
Output: next character in sheet music

6.S191 Lab!

Adapted from H. Suresh, 6.S191 2018

[6]

Example task: sentiment classification



Input: sequence of words

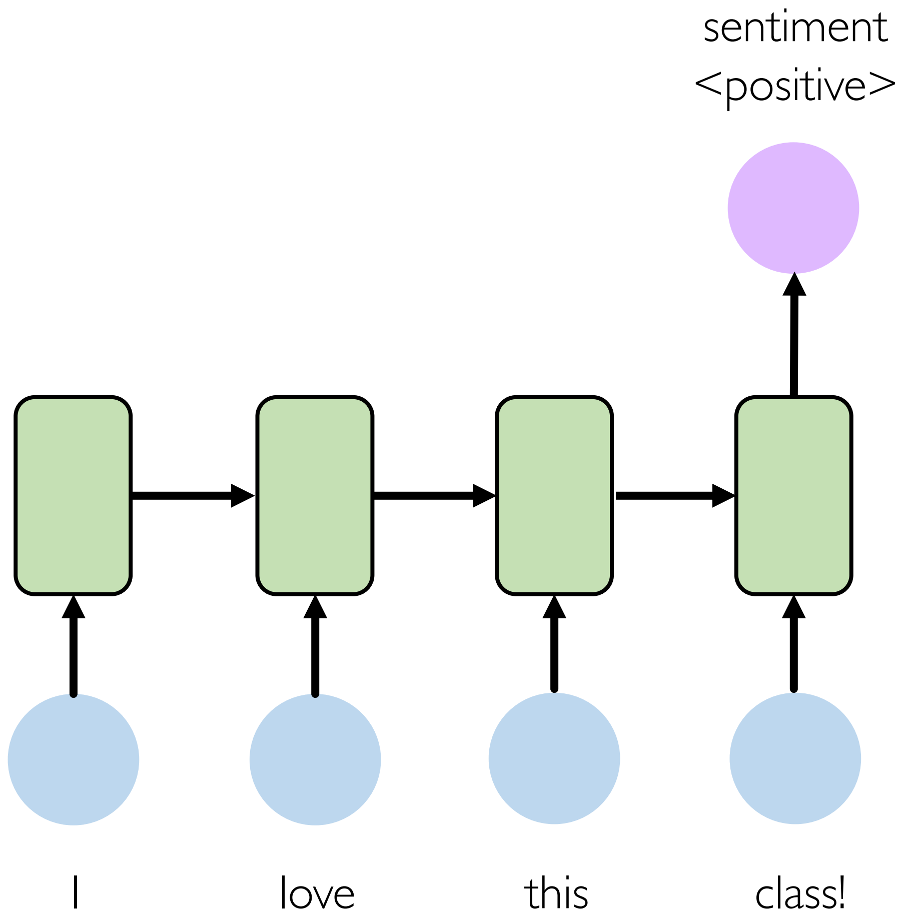
Output: probability of having positive sentiment

```
loss = tf.nn.softmax_cross_entropy_with_logits(
    labels=model.y, logits=model.pred
)
```

Adapted from H. Suresh, 6.S191 2018

[7]

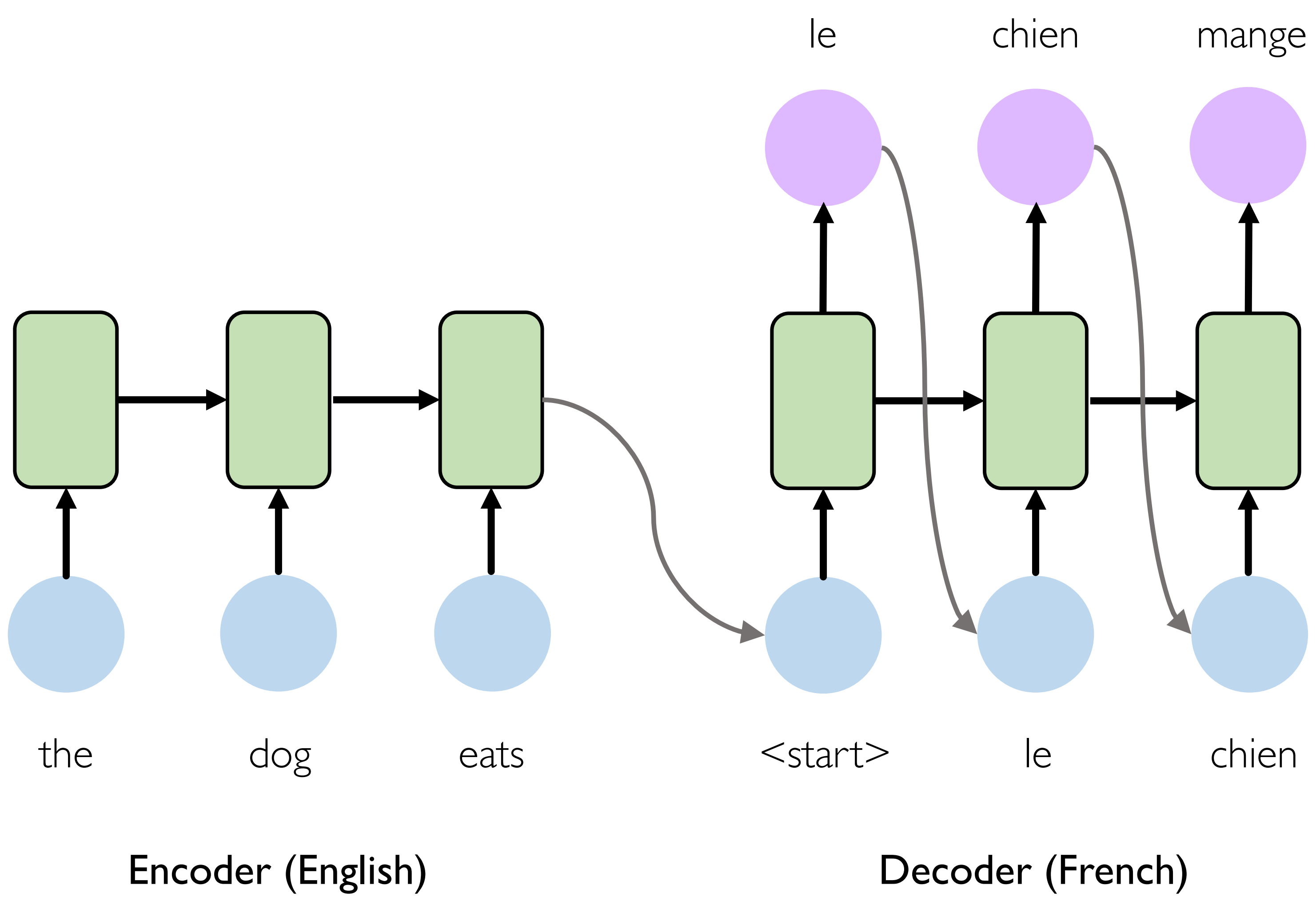
Example task: sentiment classification



Tweet sentiment classification

Adapted from H. Suresh, 6.S191 2018

Example task: machine translation



Adapted from H. Suresh, 6.S191 2018

[8,9]