SDP for SRFLP

$$
\begin{aligned}
\min \quad & K - \sum_{i<j} \frac{c_{ij}}{2} \left[\sum_{k<i} l_k X_{ki,kj} - \sum_{i<k<j} l_k X_{ik,kj} + \sum_{i<k} l_k X_{ik,jk} \right] && (1) \\
s.t. \quad & X_{ij,jk} - X_{ij,ik} - X_{ik,jk} = -1 \quad \text{for all triples} i < j < k && (2) \\
& diag(X) = e && (3) \\
& rank(X) = 1 && (4) \\
& X \succeq 0 && (5)
\end{aligned}
$$

The constraints (3) is the equivalent to $X_{ij,ij} \in \{-1, 1\}$. The constraint (4) ensures that the matrix X has rank 1, which is part of the optimality condition for SDP. The constraint (5) ensures that the matrix X is positive semidefinite.

Solving the indexing problem

Assume we want the first part of the index of a matrix X for a pair (a, b) and lets assume all pairs (a, b) where $a < b$ are ordered the following:

$\{(1, 2); (1, 3); (1, 4); \ldots ; (1, n); (2, 3); (2, 4) \ldots (2, n); \ldots ; (n - 1, n)\}$

Then with help of the Gauss summation we get the following equation:

$$n(a - 1) - \frac{a^2 + a}{2} + (b - a) - 1 \qquad (6)$$

To obtain the index starting with zero.