SDP for SRFLP

$$\min \quad K - \sum_{i<j} \frac{c_{ij}}{2} \left[\sum_{k<i} l_k X_{ki,kj} - \sum_{i<k<j} l_k X_{ik,kj} + \sum_{i<k} l_k X_{ik,jk} \right] \tag{1}$$

$$s.t. \quad X_{ij,jk} - X_{ij,ik} - X_{ik,jk} = -1 \quad \text{for all triples} i < j < k \tag{2}$$

$$diag(X) = e \tag{3}$$

$$rank(X) = 1 \tag{4}$$

$$X \succeq 0 \tag{5}$$

The constraints (3) is the equivalent to $X_{ij,ij} \in \{-1, 1\}$. The constraint (4) ensures that the matrix X has rank 1, which is part of the optimality condition for SDP. The constraint (5) ensures that the matrix X is positive semidefinite.

Solving the indexing problem

Assume we want the first part of the index of a matrix X for a pair (a, b) and lets assume all pairs (a, b) where $a < b$ are ordered the following:

$\{(1,2);(1,3);(1,4);\ldots;(1,n);(2,3);(2,4)\ldots(2,n);\ldots;(n-1,n)\}$

Then with help of the Gauss summation we get the following equation:

$$n(a-1) - \frac{a^2 + a}{2} + (b - a) - 1 \tag{6}$$

To obtain the index starting with zero.

$$\begin{bmatrix} 1 & y_{1,2} & \cdots & y_{1,n} \\ y_{1,2} & y_{1,2}y_{1,2} & \cdots & y_{1,2}y_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{1,n} & y_{1,n}y_{1,2} & \cdots & y_{1,n}y_{1,n} \end{bmatrix}$$

Abstract

The Single Row Facility Location Problem (SRFLP) is a combinatorial optimization problem that arises in the context of locating facilities in a network. The problem is to find the optimal location of a single facility on a line network, such that the total transportation cost is minimized. For this problem semi definite programming relaxations have proven to be effective. In this paper we build on this approach and apply it to the Multi-Objective Single Row Facility Location Problem (MOSRFLP). We present a semi definite programming framework with for the MOSRFLP and compare it to state-of-the-art integer programming formulation used in literature for the single objective SRFLP.