Generalized Formalization of Game Rules

Christiaan van de Sande   Tanner Reese

September 14, 2020

1 Introduction

2 Rule Defenition

$R(A, B) = (\lambda, \mu, \phi, \rho)$

2.1 The Four Functions

$$\lambda : A \to \mathcal{P}(B)$$
$$\mu : A \times B \to B$$
$$\phi : A \times B \to A$$

$$\phi(\mu(a, b), b) = b$$

2.2 Equivalence in A and B

2.3 Repeatablity of a Rule

2.4 Independence between Rulse

Existence: $\forall\ b_R \in B_R,\ \forall\ b_S \in \lambda_S \circ \phi(b_R),\ \exists\ b'_S \in \lambda_S \circ \mu_R(b_R), b'_S \sim b''_S$
Uniquenes: $\forall\ b'_S \in \lambda \circ \mu_R(b_R),\ b'_S \sim b''_S \implies \mu_S(b'_S) = \mu_S(b''_S)$

2.5 Operations on Rules

2.5.1 Negation of a Repeatable Rule

$$S(A, A) = -R(A, A)$$
$$\lambda_S(a) = \begin{cases} \varnothing & \lambda_R(a) \neq \varnothing \\ \{a\} & \lambda_R(a) = \varnothing \end{cases}$$
$$\mu_S(a, b) = \mu_R(a, b)$$
$$\phi_S(a, b) = \phi_R(a, b)$$

2.5.2 Union between Rules

$$T(A, B) = R(A, B) \cup S(A, B)$$
$$\lambda_T(a) = \lambda_R(a) \cup \lambda_S(a)$$
$$\mu_T(a, b) = \mu_R(a, b) = \mu_S(a, b)$$
$$\phi_T(a, b) = \phi_R(a, b) = \phi_S(a, b)$$

2.5.3 Intersection between Rules

$$\begin{aligned}
T(A,B) &= R(A,B) \cap S(A,B) \\
\lambda_T(a) &= \lambda_R(a) \cap \lambda_S(a) \\
\mu_T(a,b) &= \mu_R(a,b) = \mu_S(a,b) \\
\phi_T(a,b) &= \phi_R(a,b) = \phi_S(a,b)
\end{aligned}$$

2.5.4 Indetpendent Product between Rules

The independent product ($\times$) is only defined for rules that are Independent.

$$\begin{aligned}
T(A,(B \times C)) &= R(A,B) \times S(A,C) \\
\lambda_T(a) &= \lambda_R(a) \times \lambda_S(a) \\
\mu_T(a,(b,c)) &= \mu_R(\mu_S(a,c),b) \\
\phi_T(a,(b,c)) &= \phi_S(\phi_R(a,b),c)
\end{aligned}$$

2.5.5 Dependent Product between Rules

The dependent product between two rules $S \cdot R$ is only defined if $A_S = B_R$.

$$\begin{aligned}
T(A,(B \times C)) &= R(A,B) \cdot S(A,B) \\
\lambda_T(a) &= \{(b,c) | b \in \lambda_R(a), c \in \lambda_S(b)\} \\
\mu_T(a,(b,c)) &= \mu_R(a, \mu_S(b,c)) \\
\phi_T(a,(b,c)) &= \phi_R(a, \phi_S(b,c))
\end{aligned}$$

2.5.6 Patterning of a Repeatable Rule over a Range

$$\begin{aligned}
V(A,B) &= R(B,B)\Big|_{S(A,B)}^{T(A,C)} \\
\nu_V(b) &= \begin{cases} \{b\} & \lambda_T(b) \neq \varnothing \\ \{b\} \cup (\widehat{\nu}_V \circ \lambda_R(x)) & \lambda_T(b) = \varnothing \end{cases} \\
\lambda_V(a) &= \widehat{\nu}_V \circ \lambda_S(a) \\
\mu_V(a,b) &= \mu_S(a, \mu_R(b,b)) \\
\phi_V(a,b) &= \phi_S(a, \phi_R(b,b))
\end{aligned}$$

3 Implementation

4 Implications

4.1 Game Creation

4.2 Game Analysis

5 Conclusion