Generalized Formalization of Games

Christiaan van de Sande Tanner Reese

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1 Introduction

2 Games

3 Moves

4 Rules

4.1 Motivation and Definition

Definition 4.1.1. Let A be a set and let $(B, \circ)$ be a group with action on A. Let $r : A \to \mathcal{P}(B)$. Then $r \in \mathcal{R}(A, B, \circ)$.

4.2 Elementary Rules

Definition 4.2.1. Let $(G, \cdot)$ be a Group with action on A. Define r to be the rule **induced** by G (written $r = R(G, \cdot)$). Let $x \in A$. Then $r(x) = G$.

Definition 4.2.2. Let A and B be sets where $A \subseteq B$. Let $x \in B$. Then $P_A \in \mathcal{R}(B, \{e\}, \cdot)$ where $P_A = \begin{cases} \varnothing & x \notin A \\ \{e\} & x \in A \end{cases}$ and e denotes the identity element of the trivial group. For

4.3 Operations on Rules

$$\begin{aligned} r &: A \to B \\ s &: C \to D \\ r \times s &: A \times C \to B \times D \\ (r \times s)(x, y) &= r(x) \times s(y) \end{aligned}$$

$$
\begin{aligned}
r &: A \to B \\
s &: A \to C \\
r \wedge s &: A \to B \cap C \\
(r \wedge s)(x) &= r(x) \cap s(x)
\end{aligned}
$$

$$
\begin{aligned}
r &: A \to B \\
s &: A \to C \\
r \vee s &: A \to B \cup C \\
(r \vee s)(x) &= r(x) \cup s(x)
\end{aligned}
$$

$$
\begin{aligned}
r &: A \to B \\
s &: A \to C \\
r \cdot s &: A \to B \times C \\
(r \cdot s)(x) &= \{z \circ y : y \in s(x), z \in r(y \cdot x)\}
\end{aligned}
$$

$$
\begin{aligned}
r &: A \to B \\
\widehat{r} &: \mathcal{P}(A) \to A \times B \\
\widehat{r}(X) &= \bigsqcup \{r(x) : x \in X\}
\end{aligned}
$$

$$
\begin{aligned}
n &\in \mathbb{N} \\
r &: A \to B \\
r^n &: A \to B \\
r^n &= \begin{cases} P & n = 0 \\ r \cdot r^{n-1} & n > 0 \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
r &: A \to B \\
r^{-1} &: A \to B \\
r^{-1} &= \{y^{-1} : y \in r(x)\}
\end{aligned}
$$

$$
\begin{gathered}
r : A \to B \\
s : C \to D \\
r \times s : A \times C \to B \sqcup D \\
(r + s) = (r \times P) \vee (P \times s)
\end{gathered}
$$

4.4 Construction of Fundamental Rules

5 Evaluators

6 Conclusion