ME 383

Lab group B3

Lab # 5

Group Members:

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Introduction:

The product we chose to analyze for the class is an Agri-fab Reel Lawn mower. This lawnmower is intended to be used by average household consumers on a weekly basis. Unlike other lawn mowers, this reel mower has no engine to turn the cutting blade. The blade is turned by two gears, one of the gears is integrated into the wheel and the other is attached to the blade shaft. Without the assistance of an engine to rotate the blade, the force needed to propel the lawn mower increases due to the added resistance from the gears rotating the blade.

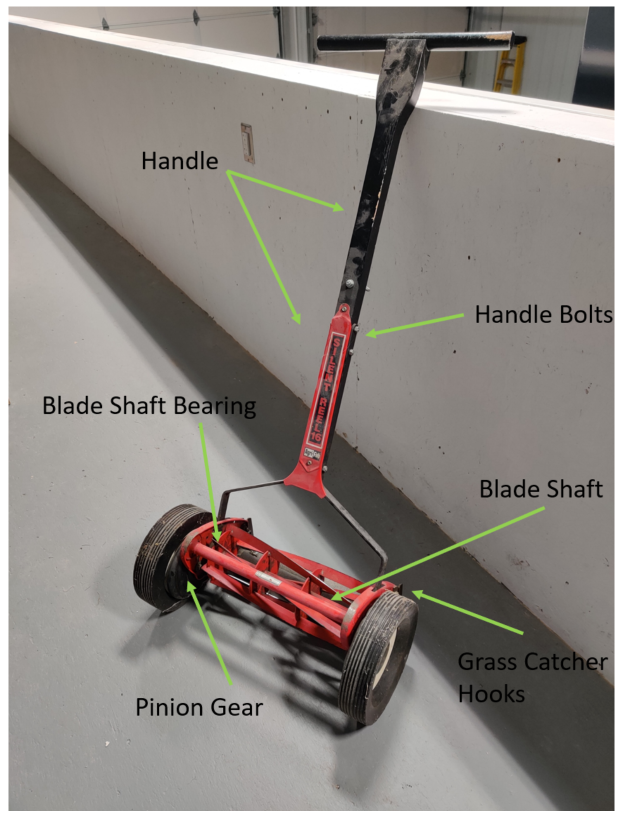
	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 6	Spec 7	Spec 8	Spec 9	Spec 10	
	Turn Radius	Average length of grass after cutting	Blade height range	Force required to push	Assembled weight	Handle length	Rate of corrosion in salt spray test	Collection Volume	Cycles to failure	Blade clearance	
	in	in	in	lbs	lbs	in	Mil per year	in3	Cycles	in	
Be able to turn corners	5										5
Cut at a constant height		5									5
Adjust the cutting height			5								5
Easy to push				5	2						7
Light weight				2	5	2					9
Comfortable Handle Length						5					5
Weather resilient							5				5
Able to collect clippings				2				5			7
Low maintenance							2	2	5		9
Able to cut lawn	2		2	2					2	5	11
	5	3	4	50	70	66	20	200	~	3/16	Max
	0	1	0	~	~	48	~	100	500, 000	0	Min
	0	2	3	10-15	50	54	10	144	1,000,000	1/16	Target

When choosing customer requirements, we examined the lawnmower and considered what worked well and what could be improved. With this, we came up with CR's such as low maintenance, adjustable cutting height and able to collect lawn clippings. These are all things that we wouldnt change. For the CR's that we would change from the original design, we came up with adjustable handle height, easy to push and light weight. We take these customer requirements into consideration when giving our design recommendations. Coming up with engineering specifications gives us a way to quantifiably evaluate the customer requirements.

The components we chose to evaluate were

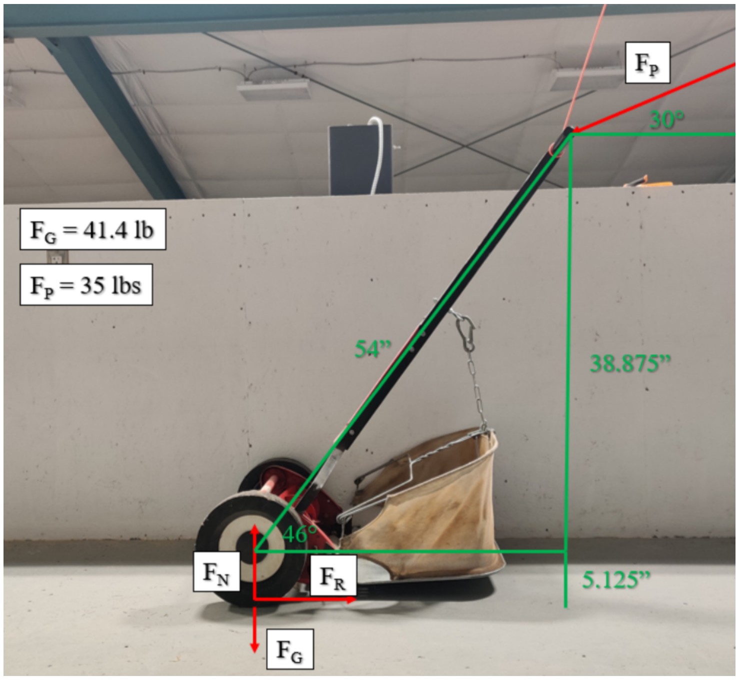
- Handle
- Handle bolts
- Pinion gear
- Blade shaft bearing
- Bladeshaft
- Grass catcher hooks

These are all crucial components of the lawn mower and fit well within the scope of the class. After we chose our components, we put them into a FMEA to analyze the failure modes and the severity of these failures.



Results:

Overall FBD Case 1:



Case 1: Normal Loading

Sum of forces in x direction:

$$\sum F_x = F_R - F_P * \cos(30) = 0$$

Solve for F_R:

$$F_R = F_P * \cos(30)$$

$$F_R = 35 * \cos(30) = 30.3\ lb$$

Sum of forces in y direction:

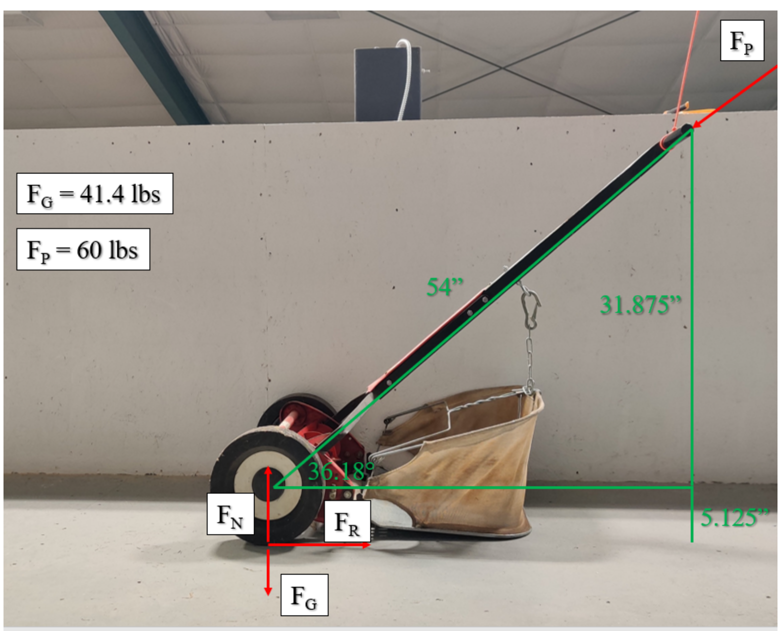
$$\sum F_y = F_N - F_P * \sin(30) - F_G = 0$$

Solve for F_N:

$$F_N = F_P * \sin(30) + F_G$$

$$F_N = 35 * \sin(30) + 41.4 = 58.9\ lb$$

Overall FBD Case 2:



Case 2: Maximum Loading

Sum of forces in x direction:

$$\sum F_x = F_R - F_P * \cos(36.18) = 0$$

Solve for F_R:

$$F_R = F_P * \cos(36.18)$$

$$F_R = 60 * \cos(36.18) = 48.4\ lb$$

Sum of forces in y direction:

$$\sum F_y = F_N - F_P * \sin(36.18) - F_G = 0$$

Solve for F_N:

$$F_N = F_P * \sin(36.18) + F_G$$

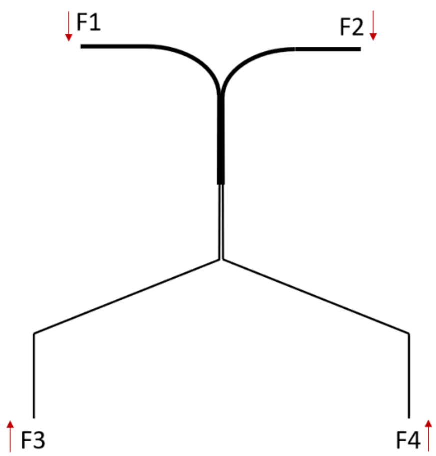
$$F_N = 60 * \sin(36.18) + 41.4 = 76.8\ lb$$

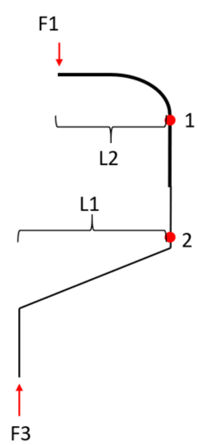
Handle:

The handle is composed of four parts. There are two bent tubes on top and two bent metal strips on the bottom. They are bolted together in the center. The top part of the original handle broke, and has been replaced with wood. We are analyzing the original handle. The handle was analyzed with the force from Case 1, 35 lbs, acting directly down the handle for simplicity. There is normal stress due to compression, which adds with the normal stress due to the moment on the inside edge of the curve of each piece of the handle.

FMEA:

Component	Function	Failure Mode	Failure Effect	Failure Cause	Severity [1 - 10]	Occurrence [1 - 10]	Detection [1 - 10]	Risk Priority Number
Overall handle	Transfers push into motion of the wheels	Wearing Bolt Holes	Handle will become more difficult to use	Long term usage of handles (constant force applied)	9	2	3	54
		Fatigue Failure - Bending	Replacement of wrangled part required	Excessive amount of force applied to handle	3	5	2	30





The maximum stress was analyzed and compared at points 1 and 2, which are symmetrical with the other side. The overall FBD and one side of the handle (composed of two pieces) are shown. The maximum shear was analyzed at other points in the handle, but it did not compare to the stress at points 1 and 2. The maximum stress was found to be at point 2.

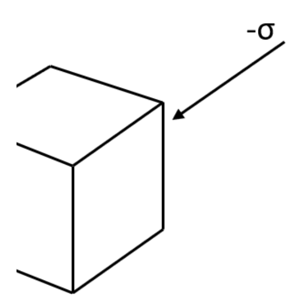
Moment on point 2:

$$Moment = F * L = F_3 * L_1 = 30 * 7.09 = 212.7\ lb * in\ CW$$

The greatest stress will occur on the inside of the curve. First we solve for the stress due to the moment on the outside of the shaft:

$$I = \left(\frac{1}{12}\right) bh^3 = \left(\frac{1}{12}\right) * 0.98 * 0.26^3 = 0.001435 in^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{212.7 * \left(\frac{0.26}{2}\right)}{0.001435} = 19.26\ kpsi$$



Next we solve for the normal stress due to the reaction force:

$$\sigma = \frac{F}{A} = \frac{F_3}{bh} = \frac{30}{0.98 * 0.26} = 0.118\ kpsi$$

And add the two to get the total normal stress:

$$\sigma = 0.118\ kpsi + 19.26\ kpsi = 19.4\ kpsi$$

Factors of Safety:

The material was assumed to be SAE 1020 Hot Rolled steel, with a yield strength of 30 ksi and an ultimate tensile strength of 55 ksi. There were no manufacturer specifications or way to perform a density test from which to more accurately determine the material, so a common type of steel was chosen. The handle has a large static factor of safety. The cycle was chosen to be alternating between zero and full loading with a period of 1 second. Under this loading scenario it achieves infinite life with the Goodman factor of safety.

Static Loading:

Because there is no shear stress,

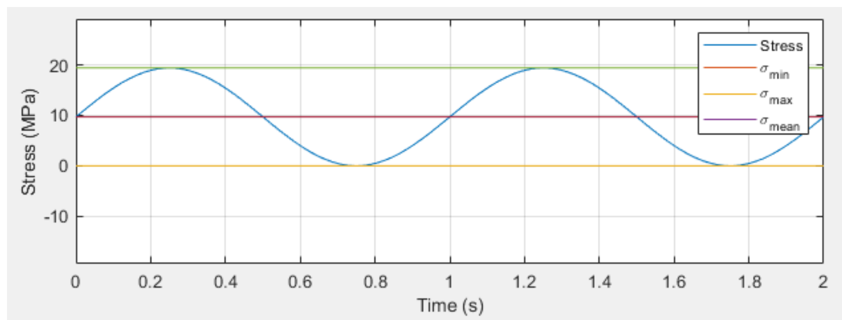
$$\sigma_1 = \sigma_2 = 19.4\ ksi$$

$$\sigma_3 = 0\ ksi$$

$$\sigma' = 19.4\ ksi$$

$$n = \frac{S_y}{\sigma'} = \frac{30}{19.4} = 1.55$$

Stress over time:



Dynamic Loading:

$$S_e = k_a k_b k_c k_d k_e S'_e$$

Surface factor – hot rolled

$$k_a = aS_{ut}^b = 11.0 * 55^{-0.650} = 0.813$$

Size factor – nonrotating rectangular cross section

$$d_e = 0.808\sqrt{bh} = 0.808 * \sqrt{0.98 * 0.26} = 0.4079$$

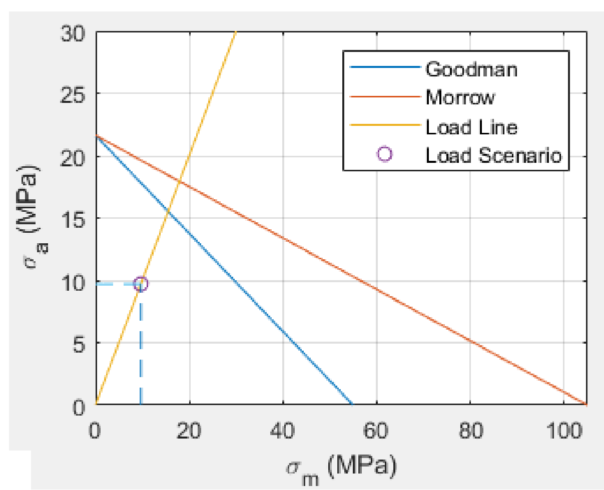
$$k_b = 0.879 * d_e^{-0.107} = 0.879 * 0.4079^{-0.107} = 0.9675$$

Loading factor, temperature factor, reliability factor

$$k_c = k_d = k_e = 1$$

$$\sigma_a = \sigma_m = \frac{19.4}{2} = 9.7\ ksi$$

Stress vs. Strain:



Infinite life fatigue Failure Criterion using goodman factor of safety:

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\frac{9.7}{21.63} + \frac{9.7}{55}\right)^{-1} = 1.6$$

Design Recommendations:

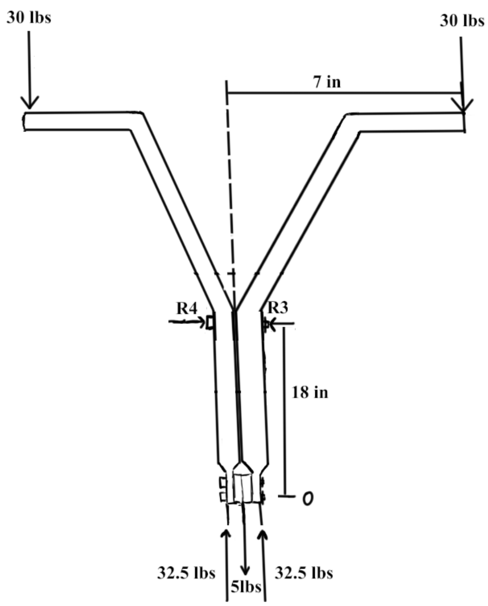
Based on the analysis of the handle, a simple improvement could be made. Making the upper and lower part of each side of the handle out of one piece of the same rectangular cross section as the bottom would reduce stress and complexity. Currently, four pieces of metal of two different shapes are bolted together with three bolts in the center. Making each side of the handle out of one piece would reduce the number of parts to two and reduce the necessary bolts to two. It would reduce total stress, because the circular cross section has a higher stress due to bending at the edges than the rectangular cross section.

Handle Bolts:

Three bolts hold the handle assembly together. One bolt is located at the top, near the grip, while the other two are located farther down, towards the center of the lawnmower's frame. The upper bolt keeps the two upper handle pieces together, while the other two connect the upper and lower components. The handle is constructed in such a way that the top bolt experiences the most tensile stress from the applied load, while the bottom two bolts withstand the majority of the shear and bending stress. The material of the bolts is assumed to be SAE Grade No. 1 low or medium carbon steel.

FMEA:

Function	Failure Mode	Failure Effect	Failure Cause	Severity [1 - 10]	Occurrence [1 - 10]	Detection [1 - 10]	Risk Priority Number
Hold handle pieces together	Thread Stripping	Maintainence more difficult, replacement of bolts required	Improper maintainace methods	2	3	6	36
	Fatigue Failure - Bending	Handle no longer possible to use correctly	Regular use over time, excessive forces applied instantaniously	7	2	1	14



The FBD shows the force from the load on bolt 1. The reactions at the bolt are calculated using a sum of moments from point O:

$$\sum M_o = 0$$

$$R_3 = R_4$$

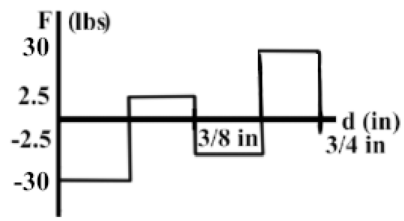
$$0 = R_3\,(18\ in) - 30\ lbs\,(7\ in)$$

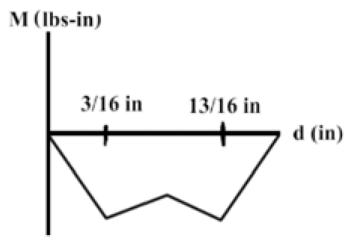
$R_3 = R_4 =$ 11.7 lbs

The maximum bending moment on bolts 2 and 3 can be found using their shear and moment diagrams.

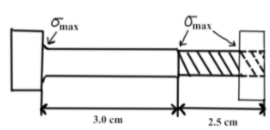
$$M_{max} = 30\ lbs\,(3/16\ in)$$

$$M_{max} = 5.63\ lbs$$

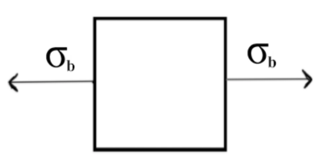




Bolt 1



*Shigley's, Page 434



The stresses experienced by bolt 1 are due to a combination of the preload due to the tightening of the bolts as well as the load itself. Bolts 2 and 3 experience the preload stress as well as bending and shear stresses. For the preload, it is assumed that 30 lbs of force exerted by the user with a standard 8 inch wrench should be sufficient.

$P = 11.7\ lbs$ $\quad A = 0.239\ in^2$ $\quad T = 30\ lbs\ (8/12\ ft)$

$A_t = 0.0404\ in^2$ $\quad T = KF_i d$

$k_m = \frac{AE}{l}$ $\quad k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$ $\quad d_m = \frac{d + d_i}{2} = 0.237\ in$

$K = \frac{d_m}{2d}\left(\frac{tan(\lambda) + sec(\alpha)}{1 - ftan(\lambda)sec(\alpha)}\right) + 0.625 f_c$ $\quad \lambda = tan^{-1}\left(\frac{1}{\pi d_m N}\right)$

$C = \frac{k_b}{k_b + k_m}$ $\quad N = 3$

$P_b = CP$ $\quad P_b = 2.5\ lbs$ $\quad f = f_c = 0.237\ in$

$\alpha = 30^o$

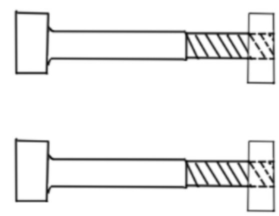
$\sigma_b = \frac{P_b + F_i}{A_t}$ $\quad \sigma_b = 4.74\ kpsi$ $\quad F_i = 189\ lbs$

Bending Stress	Shear Stress	Axial Stress
$I = \frac{\pi(d/2)^4}{4}$	$\tau = \frac{F}{A}$	$\sigma_b = \frac{F_i}{A_t}$
$\sigma_x = \frac{Md}{2I}$	$\tau_{max} = 0.263\ kpsi$	$\sigma_b = 4.68\ kpsi$
$\sigma_x = 2.73\ kpsi$	At max bending:	
	$\tau = 5.2\ psi$	

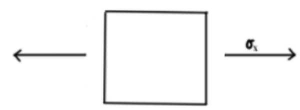
Safety Factors:

Bolt 1	Bolt 2
$\sigma_a = \frac{C(P_{max} - P_{min})}{2A_t} = 15.6\ psi$	$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 0.5\ kpsi$
$\sigma_m = \frac{C(P_{max} + P_{min})}{2A_t} + \frac{F_i}{A_t} = 4.73\ kpsi$	$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 5.99\ kpsi$
$S_y = 36\ kpsi$	$n_p = \frac{S_p}{\sigma_a + \sigma_m} = 5.1$
$S_p = 33\ kpsi$ $\quad S_e = 17.1\ kpsi$	$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m + \sigma_i)} = 4.5$
$n_p = \frac{S_p}{\sigma_a + \sigma_m} = 6.95$	

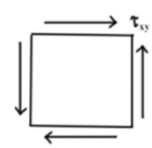
Bolts 2 and 3



Maximum at 3/16 in and 13/16

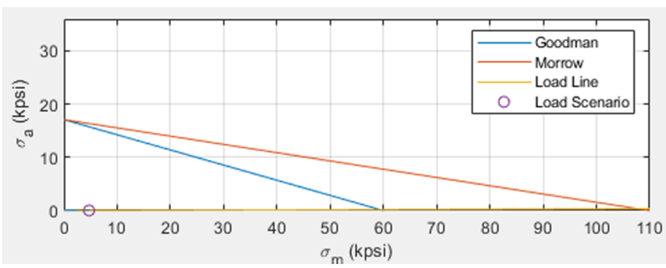


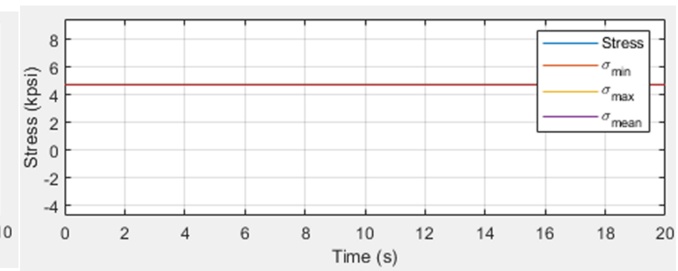
Maximum at 0 and 3/4 in



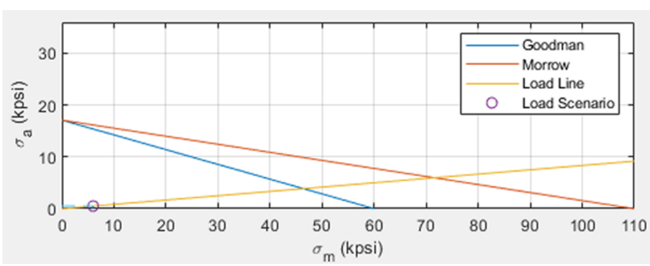
$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m + \sigma_i)} = 5.84$$

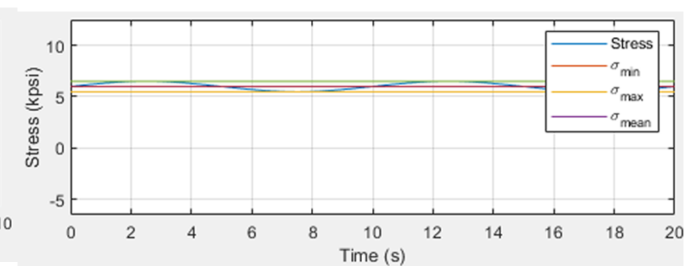
Bolt 1 Goodman and Stress Over Time Graphs



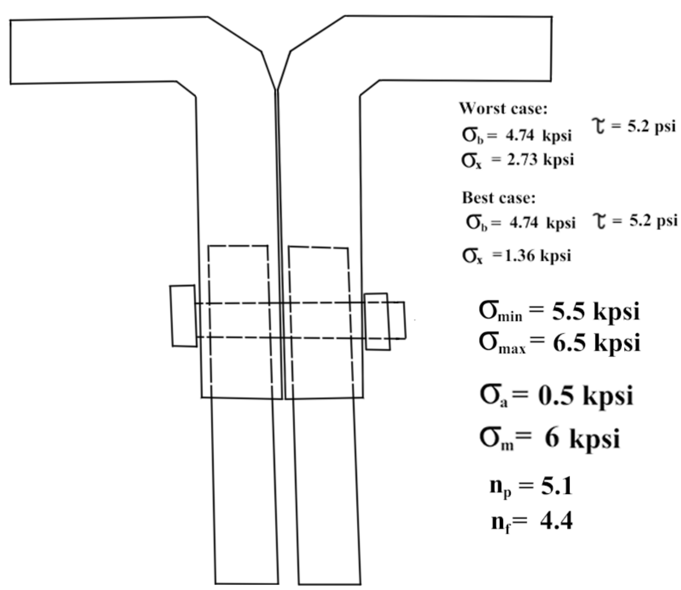


Bolts 2 and 3 Goodman and Stress Over Time Graphs





It is recommended that the top bolt is removed from the design. Neglecting the preload, the bolt sees a very small tensile stress and it is likely that any advantage of having the top bolt is offset by the cost of adding in the additional bolt for each unit shipped. The handle, being made of steel, is unlikely to separate on its own due to the force applied to the handle by the user. Alternatively, if the bolt is kept, it

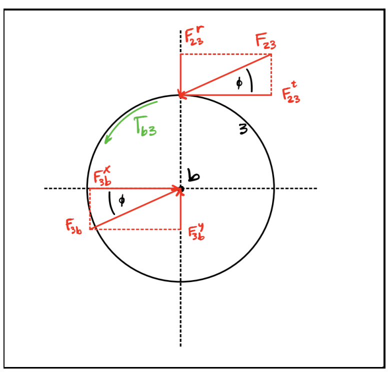


could be used to create height settings for the handle. If the upper pieces of the handle were to be redesigned in order to have one piece able to slide over the other, then bolt 1 could be used to hold the preferred height setting. Bolt 1 would then share the same bending and shear stresses along with bolts 2 and 3.

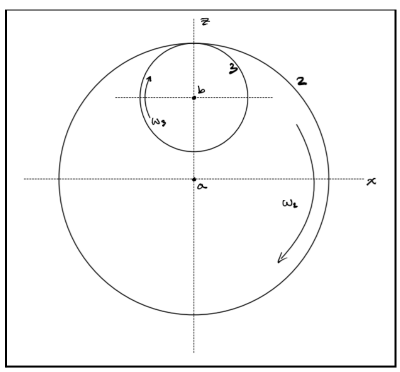
Pinion Gear:

The pinions are located on the interior of each wheel. The pinion interfaces with the outer gear as the operator pushes the mower forwards. This rotational force on the gear causes the blade to rotate and cut grass. This component is important to analyse because of its utility in the machine. It is the main driving element behind the spin of the blade shaft. If this component fails the machine will not run. Examining a scenario when the blade is caught on an obstruction we can see the maximum theoretical stress that the pinion will experience. For the dynamic scenario a constant force is applied to the handle causing a constant velocity forwards, the pushing force cancelling with the resistive force.

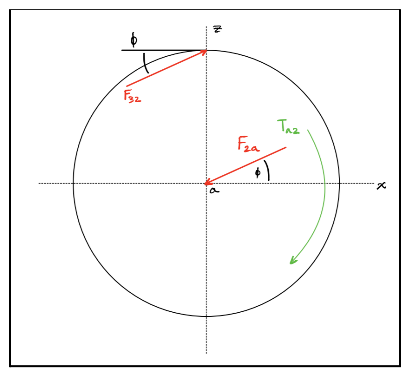
Pinion Gear

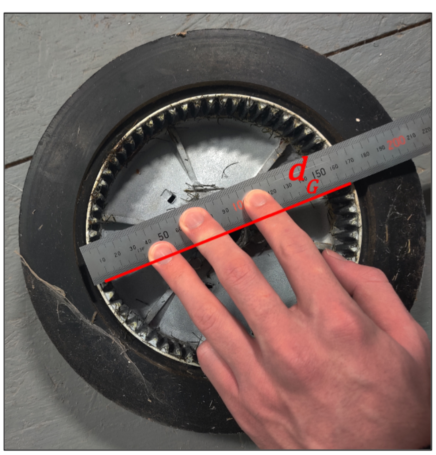


Gear Interface

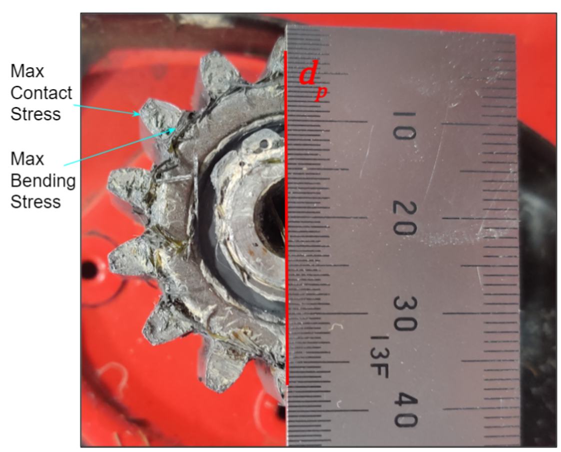


Outer Gear





$N_G = 62$



$N_P = 14$

FMEA

Part	Function	Failure Mode	Failure Effect	Failure Cause	Severity	Occurrence	Detection	Risk Priority Number
Pinion Gear	Transfers motion of the wheels to motion of the blade	Excessive tooth loads	Fracture of gear teeth leading to inoperable drive train	Excessive force applied by user	10	1	1	10
		Surface Fatigue (Pitting)	Increase risk of tooth fracture	Prolonged use with constant stress	8	1	3	24

Assumptions:

- The gear tooth can be treated as a rectangular Cantilever beam (Lewis equation)
- The gear and pinion are both made from 6061 Aluminum
- The system is in static equilibrium
- Maximum bending stress occurs in the base of the tooth
- Maximum surface stress occurs at the contact point between the gears
- Static Scenario: The mower is stopped on an obstruction and full force is applied (70 lbs)
- Dynamic Scenario: The mower is free to move and full force is applied (70 lbs)

Measurements:

$W_t = F_{23} cos\phi = 66.9 lbf$

$d_p = 0.984\ in$

$F = 0.649\ in$

$d_g = 6.30\ in$

$Y = 0.277$

$E_p = E_G = 10\ Mpsi$

$\nu_p = \nu_G = 0.33$

$K_v = 1$ For a stopped gear

$$\boxed{F_{3b} = \sqrt{\frac{4T^2}{d_2^2}(1 - tan^2\phi)}}$$

$$\boxed{T_{b3} = F_{23}^t d_3}$$

Case 2: Maximum Loading

$F_{3b} = 72.1$ lbs

$T_{b3} = 62.9$ lbs-in

Stress Calculations:

Equations taken from Shigley's Chapter 14-3: AGMA Stress Equations

$$\sigma = W^t K_O K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

$$\sigma_c = C_p \sqrt{W^t K_O K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}}$$

$K_m = C_{mf} = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e)$

$C_{mc} = 1$ For uncrowned teeth

$C_{pf} = \frac{F}{10 d_p} - 0.025 \qquad F \le 1 in$

$C_{pm} = 1$

$C_{ma} = A + BF + CF^2$

$A = 0.127 \qquad B = 0.0158 \qquad C = -0.930 \times 10^{-4}$

$C_e = 1$

$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = 1$

$K_O = 1$

$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$

$I = \frac{cos\phi sin\phi}{2 m_N} \frac{m_G}{m_G + 1}$

$J = \frac{Y}{K_f m_N}$

$m_N = 1$ For spur gears

$K_B = 1 \qquad t_R \gg h_t$

$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M$

$H = 0.34 \quad 0.4583662\,\phi$

$K_O = 1$

$K_v = 1$

$K_s = 1$

$K_m = 1.1443$

$K_B = 1$

$K_f = 1.079$

$\sigma = 6.7\ kpsi$

$\sigma_c = 41\ kpsi$

$J = 0.2482$

$I = 0.1319$

$C_f = 1$

Material:

The material for the gear and pinion are thought to be 6061 Aluminum. This is due to two rudimentary tests. First was a magnetism test. We held a magnet against the pinion and felt only attraction coming from the shaft it is attached to. Second, to rule out nitrided steel, was a scratch test. The surface of the material easily gave way to a steel file. This left the most possible material to be Aluminum. The common 6061 Aluminum for gears was chosen.

Fatigue calculations:

$$S_F = \frac{\frac{S_t Y_N}{K_T K_R}}{\sigma} = \frac{corrected\ bending\ strength}{bending\ stress}$$

$$S_H = \frac{\frac{S_c Z_N C_H}{K_T K_R}}{\sigma_c} = \frac{corrected\ surface\ strength}{surface\ stress}$$

$S_c = 322HB + 29100$

$S_t = 77.3HB + 12800$

$C_H = 1$

$Z_N = 1.4488N^{-0.023}$

$Y_N = 1.6831N^{-0.0323}$

$K_T = 1$

$K_R = 0.5 - 0.109ln(1 - R)$

$\mathrm{R} = 0.99$

$HB = 90$

Dynamic Scenario

$K_v = 1.11$

$\sigma = 7.3\ ksi$

$\sigma_c = 42.6\ ksi$

Safety factor for N = 10^6

$S_F = 2.96$

$S_H = 1.47$

Cycles until failure

$N = 2.2 \times 10^{13}$

$S_H = 0.999$

Design recommendations:

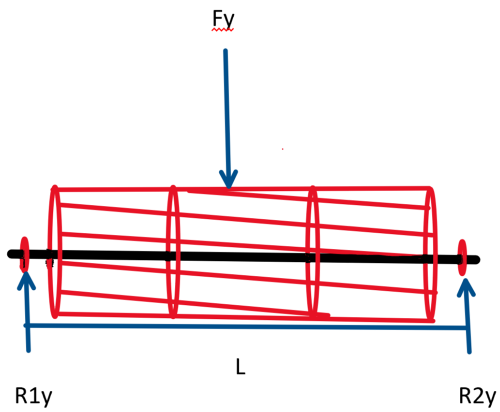
Material: The safety factors based on Aluminum 6061 show a fatigue lifetime of over 10^{13} cycles. The gear ratio being 4.6, this means that the lawn mower could travel far over 1 million miles before the pinion fails. This suggests that a lighter and material could be used such as Aluminum 5052.
Gearing: The gear ratio is a compromise between being able to push the lawn mower easily, and the cutting speed. We recommend a larger pinion. A pinion that is 25% larger would decrease the required pushing force proportionally by 25%.

Blade Shaft Bearing:

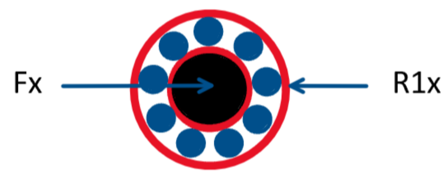
Having bearings on the blade shaft helps satisfy our customer requirement, easy to push. Without these bearings in place, the friction between the blade shaft and the frame of the lawnmower would increase, resulting in an increase of the force needed to operate the lawnmower.

Part	Function	Failure Mode	Failure Effect	Failure Cause	Severity (1-10)	Occurrence (1-10)	Detection (1-10)	Risk Priority Number (1-10)
Blade shaft bearing	Allows blade to spin freely	Lubrication failure	Increases friction, Blade turns slower	Loss of seal	4	3	2	24
		Spalling (fatigue)	Inner race fractures and causes bearing to seize	Excessive force	6	2	4	48

In our FMEA we came up with a risk priority number of 48 for the bearings. If these bearings were to fail the lawn mower could potentially still operate, but it would not be as efficient. Some of the ways failure could occur would be a loss of lubrication or spalling of the races.



Front view of Reel Mower



Side view

Forces:

- Fy: 4 lbs
- Fx: 48.43 lbs
- R1y: 2 lbs
- R2y: 2lbs
- R1x: 24.22 lbs
- R2x: 24.22 lbs

$$\Sigma MR1y = 0 = -Fy\left(\frac{L}{2}\right) + R2y * L$$

$$R2y = \frac{Fy}{2}$$

$$\Sigma Fy = 0 = R1y - Fy + R2y$$

$$R1y = Fy - R2y$$

$$\Sigma MR1x = 0 = Fx\left(\frac{L}{2}\right) - R2x * L$$

$$R2x = \frac{Fx}{2}$$

$$\Sigma Fx = 0 = -R1x + Fx - R2x$$

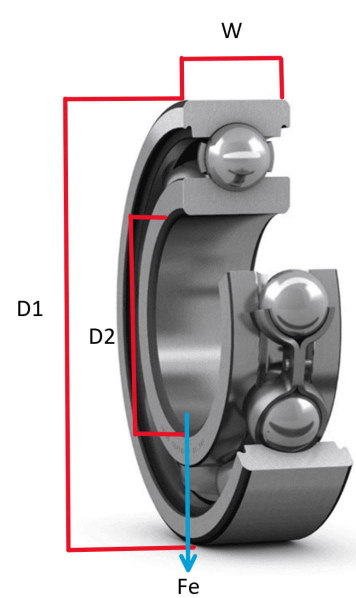
$$R1x = Fx - R2x$$

L: 18.5 in

In the free body diagram, I was able to calculate the forces the bearings would see. In normal use the bearing has the weight of the blade shaft as the main force which is 2 lbs. per bearing. In extreme conditions, such as the mower hitting a rock or being stuck on an object, the bearings could see a force of up to 24.22 lbs. each. Since the extreme case would be a rare occurrence it is unlikely for this to be the main cause of failure of the bearings.

Bearing Dimensions:

- D1:1.37 in
- D2:0.59 in
- W:0.43 in



Assumptions:

- Mow lawn once a week
- Mow lawn maximum of 35 times per year
- Average of 30 minutes to mow lawn
- Average walking speed of 3 mph

- Lawnmower lifetime of 10 years

Wheel Circumference: $10.25\ in\ *\ \pi\ =\ 32.20\ in$

Gear ratio:$62t/14t\ =\ 4.43$

Blade rotation: $4.43\ rev/32.20\ in\ =\ 0.1375\ rev/in$

Rpm: $0.1375\ rev/in\ \ *\ 63360\ in/mi\ \ *\ 3\ mi/hr\ \ *\ 1\ hr/60\ min\ =\ 435.65\ rev/min$

Revolutions per mow: $435.65\ rev/min\ \ *\ 30\ min/mow\ =\ 13069.5\ rev/mow$

Lawn mows per life
$1\ mow/week\ \ *\ 4.375\ weeks/month\ *\ 8\ months/year\ *\ 10\ years/life\ =\ 350\ mows/life$

Revolutions per life: $350\ mows/life\ \ *\ 13069.5\ rev/\ mow\ =\ 4574325\ rev/life$

$L_{10} = (C_{10}/F_e)^a * 10^6$

$L_{10}\ =\ (7.80\ *\ 10^3 N/8.9N)^3 * 10^6\ =\ 6.73\ *\ 10^{14}$

Dynamic load capacity C_{10}= 1753.5 lbs.

Static load capacity C_0=798 lbs

Fe=R1y

Ball Bearing a=3

Static factor of safety $F_s = C_o/F_e$

Using the dimensions of the bearing I was able to look up the C10 and C0 values in the Shigley's textbook. When calculating the lifetime cycles and static factor of safety for the bearing I computed a L10 value of 6.73*10^14 and a static safety factor of 398. Based on these values, it is highly unlikely the bearing will fail due to wear. It is more likely to fail due to loss of lubrication.

Design Recommendations:

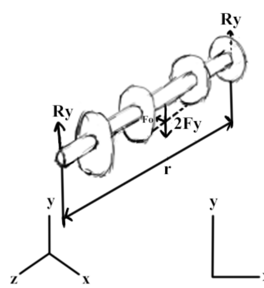
Since heat and rpm are low in this application, this bearing could be replaced with a lubrication free bearing. The bearing is in a dirty environment, so a shielded bearing is best for this application. The bearing in the lawn mower is made of 52100 chrome steel.To save money this could be replaced with a high carbon steel bearing. A replacement bearing, I can suggest would be a 6202 z. This bearing is made of high carbon steel, it is lubrication free and shielded to keep out dust. This bearing is also cheaper than the 6202 rs that is currently being used in the lawn mower now.

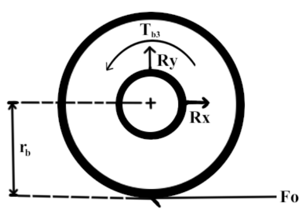
Blade Shaft:

The blade shaft is driven by two pinion gears and allows the blades to revolve around it and cut the grass while the lawn mower is pushed by the operator. It rests on the blade shaft bearing which allows for decreased friction while the shaft is rotating. In the worst case scenario, the blades are abruptly stopped by an object while the operator attempts to clear the debris by continuing to push the machine. The blade shaft is assumed to be made from low-carbon, cold drawn, 1020 steel.

FMEA:

Function	Failure Mode	Failure Effect	Failure Cause	Severity [1 - 10]	Occurrence [1 - 10]	Detection [1 - 10]	Risk Priority Number
Transfer motion from pinion gear to blades	Fatigue Failure - Bending	Machine will no longer able to efficently cut, blades will also have to be replaced	Hitting hard obstacles such as rocks	7	3	3	63
	Fatigue Failure - Torsion	Destruction of the part, user will need to replace immediately	Uneven force applied to blade shaft	6	2	4	48





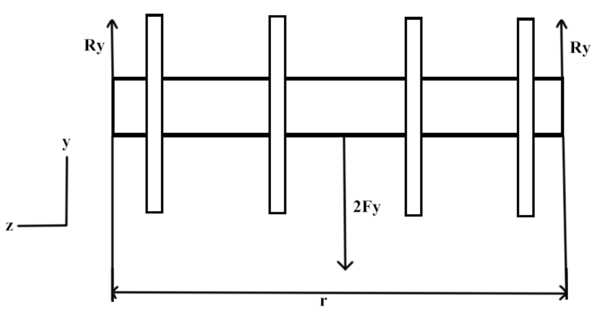
$F_y = 2\ lbs \qquad T_R = T_{b3} = 62.9\ lbs - in$

$r = 21.5\ in \qquad r_b = 2.5\ in$

At one of the attachment points:

$2F_y = 2R_y \qquad F_o = \frac{T_{b3}}{r_b} = 25.2\ lbs$

$R_y = 2\ lbs$



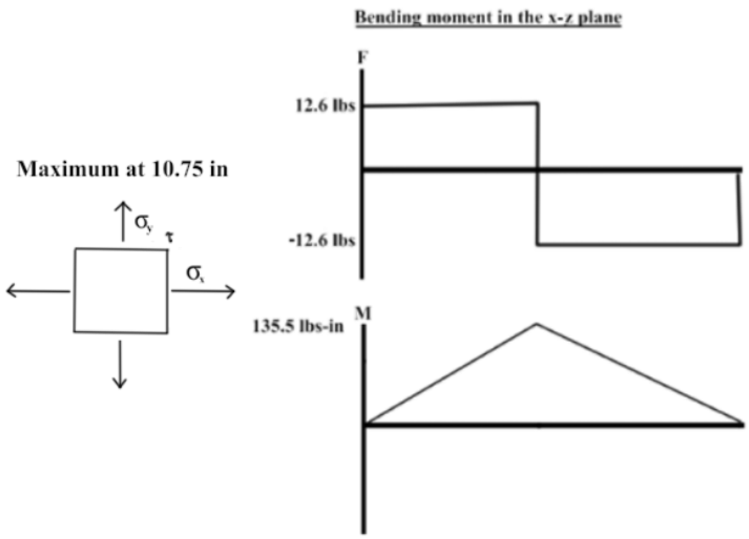
Bending moments

Vertical:

$M_{bv} = 2F_y(\frac{r}{2}) = 43\ lbs - in$

Horizontal:

$M_{bh} = F_o(\frac{r}{2}) = 271\ lbs - in$



Bending Stress - Horizontal:

$r_s = 0.75\ in \quad I = \frac{\pi(r_s/2)^4}{4}$

$\sigma_x = \frac{Mr_s}{2I} = 0.815\ kpsi$

Bending Stress - Vertical:

$\sigma_y = \frac{Mr_s}{2I} = 0.065\ kpsi$

Torsional Stress:

$\tau = \frac{16T_R}{\pi r_s} = 1.52\ kpsi$

Safety Factors:

$S_{ut} = 68\ kpsi \quad S_y = 57\ kpsi \quad S_e = 13.9\ kpsi \quad K_f = K_{fs} = 1$

$\sigma_a = K_f \frac{32M_a}{\pi d^3} = 3.26\ kpsi \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3} = 9.78\ kpsi$

$\tau_a = \frac{16T_a}{\pi d^3} = 0.380\ kpsi \qquad \tau_m = \frac{16T_m}{\pi d^3} = 1.14\ kpsi$

$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = 3.33\ kpsi \sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = 9.98\ kpsi$

A = $(4(K_f M_a)^2 + 3(K_{fs}T_a)^2)^{1/2} = 275.4$

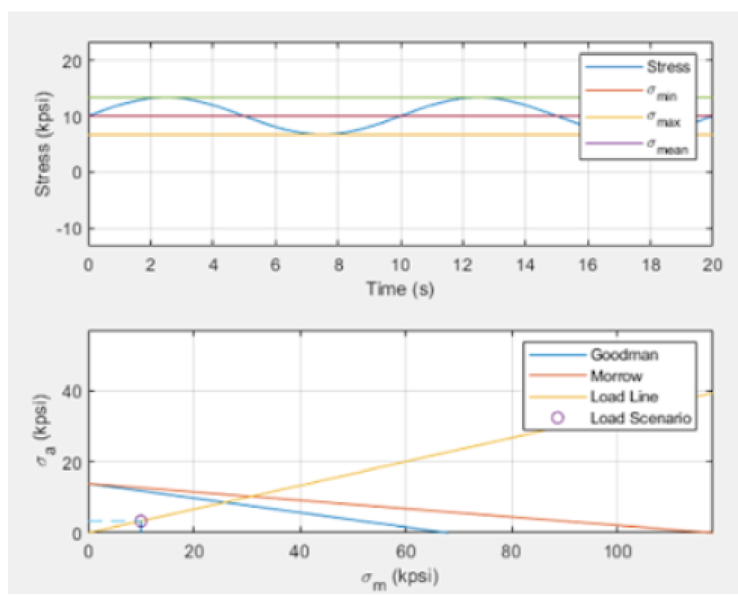
<u>Stress over Time and Goodman Graphs</u>

B = (4(

$K_f M_m)^2 + 3(K_{fs}T_m)^2)^{1/2} = 826.4$

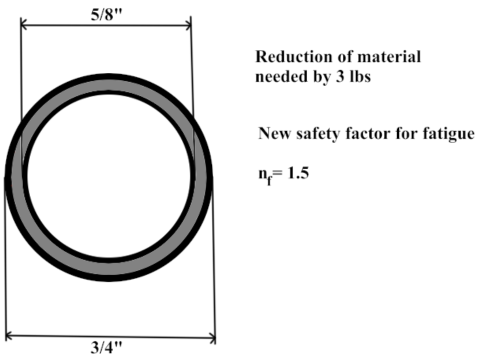
$n_f = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} = 2.6$

$n_y = \frac{S_y}{\sigma_{max}'} = 4.3$



Recommendations:

Reducing the diameter of the blade shaft from ¾” to ⅝” will reduce the overall weight and material needed to produce one unit by about 3 lbs while keeping a safety factor of 1.5.



Grass Catcher Hooks:

The grass catcher hooks connect the grass catcher to the main body of the push mower. The FMEA shows that the risk priority for the grass catcher hooks are rather low, although this component is very important regarding user experience. Our calculated values of stress logically should be low because our maximum force is relatively low. The factors of safety are rather high, which tells us that our grass catcher hooks are not very prone to failure in this scenario. In the design analysis we can determine a way to lower these factors that would help save money in producing this component.

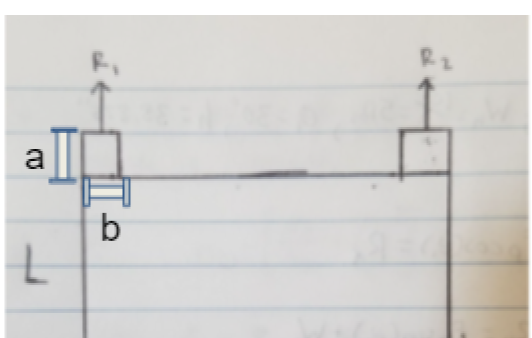
Part	Function	Failure Mode	Failure Effect	Failure Cause	Severity [1 - 10]	Occurrence [1 - 10]	Detection [1 - 10]	Risk Priority Number
Grass catcher hooks	Hold grass carrying bin in place	Fatigue Failure - Bending	Replace hooks, grass catcher easily slides off hooks	Excessive weight inside grass catcher	5	2	2	20
		Fatigue Failure - Torsion	Replace hooks, hooks become twisted and unusable	Excessive force while turning mower	4	3	2	24

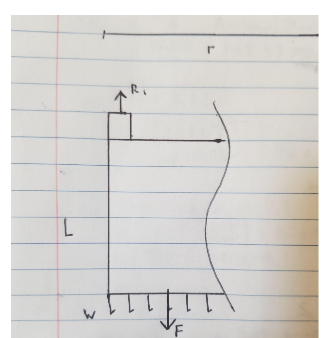
Assumptions:

- Equally distributed load
- Symmetric system ($R_1 = R_2$)
- Grass catcher filled with maximum volume of wet grass (heavier than dry grass)
- Steel Alloy 1020 - Hot rolled (Good weldability, non-expensive)
- Transverse loading (no axial loading)

Measurements:

- L = 17.5 inches
- r = 21.5 inches





- w = 1.64 lb/in
- a = 0.4724in
- b = 0.1378in
- Sy = 30ksi
- Sut = 55ksi
- a = 11ksi
- b = -0.650
- kc = 0.59
- kd = 1
- ke = 1
- d = 0.4724in

Calculations:

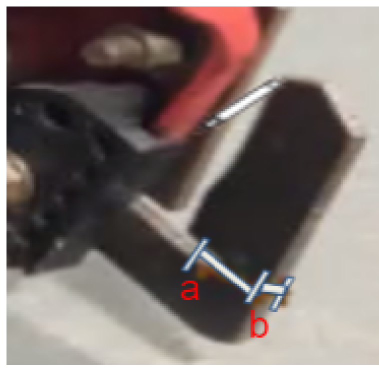
Reaction Forces:

$$\sum F_z = R_1 = F$$

F = w(r/2)

$$R_1 = w(r/2)$$

$R_1 = 17.63$ lbs, $R_2 = 17.63$ lbs



Stress Analysis:

Shear Stress:

$$\tau_a = k_f[\frac{F_a}{A}], \tau_m = k_f[\frac{F_m}{A}]$$

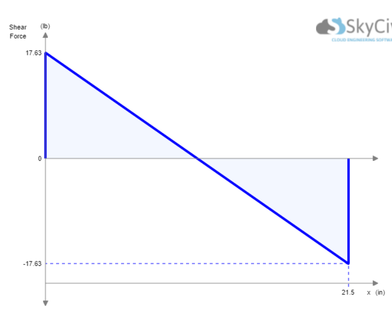
$$F_a = 17.63lbs,\ F_m = 8.815lbs$$

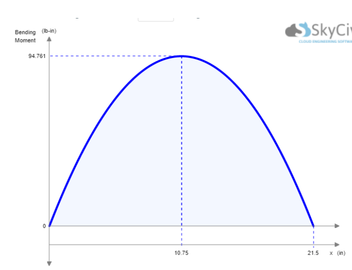
$$k_f = 1 + q(k_t - 1), q = 0.68, k_t = 2.1$$

$$k_f = 1 + 0.68(2.1 - 1) = 1.748$$

$$\tau_a = 1.748[\frac{17.63}{0.065}] = 473.41psi, \tau_m = 1.748[\frac{8.815}{0.065}] = 237.06psi$$

$\tau_{max} = \tau_{xy} = \tau_a + \tau_m$ = 710.47psi





Von-Mises Stress:

$$\sigma_x = 0, \sigma_y = 0$$

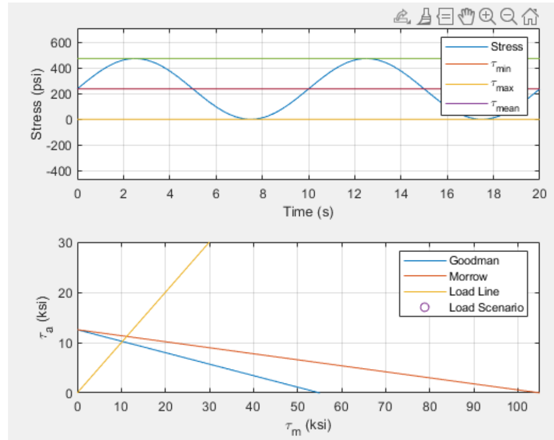
$$\sigma' = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$$

$$\sigma' = \sqrt{(3 * (710.47psi)^2}$$

$$\sigma' = 1230.57psi$$

Maximum shear force of 17.63lb occurs at x=0 and x=r

Maximum bending moment occurs at x=10.75 which is the same as r/2



Static Factor of Safety Calculation:

$$n_s = \frac{S_y}{\sigma'} = \frac{30{,}000psi}{1230.57psi} = 24.38$$

Fatigue Factor of Safety Calculation:

$$n_f = \left(\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{ut}}\right)^{-1}$$

$$S_e = S_e'k_ak_bk_ck_dk_e,\ S_e' = 0.55S_{ut} = 27.5ksi$$

$$k_a = aS_{ut}^{\ b} = 0.813,$$

$$k_b = (d/0.3)^{-0.107} = 0.953$$

$$S_e = 27.5ksi * 0.813 * 0.953 * 0.59 = 12.57ksi$$

$$n_f = \left(\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{ut}}\right)^{-1} = \left(\frac{0.473ksi}{12.57ksi} + \frac{0.237ksi}{55ksi}\right)^{-1} = 23.84$$

Design Recommendations

Reduce the amount of material (Steel Alloy 1020) in hooks by 60%.

New values using same equations with reduced area of 60%

- τxy=4.438ksi
- σ’=7.687ksi
- ns=3.90
- nf=4.16

Benefits:

- Reduced cost
- Low risk of failure
- Weldability of material

Calculations:

Shear Stress:

$$\tau_a = k_f\left[\frac{F_a}{A}\right],\ \tau_m = k_f\left[\frac{F_m}{A}\right]$$

$F_a = 17.63 lbs,\ F_m = 8.815 lbs$

$k_f = 1 + q(k_t - 1), q = 0.68, k_t = 2.1$

$k_f = 1 + 0.68(2.1 - 1) = 1.748$

$\boldsymbol{\tau}_a = 1.748[\frac{17.63}{0.0104}] = 2958.79 psi,\ \boldsymbol{\tau}_m = 1.748[\frac{8.815}{0.0104}] = 1479.40 psi$

$\boldsymbol{\tau}_{max} = \boldsymbol{\tau}_{xy} = \boldsymbol{\tau}_a + \boldsymbol{\tau}_m = 4438.19\text{psi}$

Von-Mises Stress:

$\boldsymbol{\sigma}_x = 0, \boldsymbol{\sigma}_y = 0$

$\boldsymbol{\sigma}' = \sqrt{\boldsymbol{\sigma}_x^2 - \boldsymbol{\sigma}_x\boldsymbol{\sigma}_y + \boldsymbol{\sigma}_y^2 + 3\boldsymbol{\tau}_{xy}^2}, \boldsymbol{\sigma}' = \sqrt{(3 * (4438.19 psi)^2}$

$\boldsymbol{\sigma}' = 7687.17 psi$

Static Factor of Safety Calculation:

$n_s = \frac{S_y}{\sigma'} = \frac{30{,}000 psi}{7687.17 psi} = 3.90$

Fatigue Factor of Safety Calculation:

$n_f = (\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{ut}})^{-1}$

$S_e = S_e' k_a k_b k_c k_d k_e, S_e' = 0.55 S_{ut} = 27.5 ksi$

$k_a = a S_{ut}^{\ b} = 0.813, k_b = (d/0.3)^{-0.107} = 1.05$

$S_e = 27.5 ksi * 0.813 * 1.05 * 0.59 = 13.86 ksi$

$n_f = (\frac{\tau_a}{S_e} + \frac{\tau_m}{S_{ut}})^{-1} = (\frac{2.959 ksi}{13.86 ksi} + \frac{1.479 ksi}{55\ ksi})^{-1} = 4.16$

Another design recommendation would be to reallocate the material distribution on the grass catcher hooks. Increasing the area where the grass catcher attaches to the hooks could allow for lower maximum shear stress and higher factors of safety. Using this recommendation, you would be able to safely reduce the amount of material in the hooks by even more.

Evaluation:

This product is subject to relatively low stresses, so it is feasible to achieve high factors of safety in many of the components while remaining simple and affordable. Our design recommendations show that there are in fact several unique designs that could improve our product. The benefits of these designs include factors such as increasing our factors of safety and reducing component weights and costs.